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ACOUSTIC PRESSURE AT AN INFINITE PLATE WITH PARALLEL STIFFENERS--ETC(U)
MAR 67 D YARMUSH, V MANGULIS
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10 D. Yarmush
V. Mangulis

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ABSTRACT

↓ The acoustic pressure in water due to a plane wave incident on an elastic plate is considered. The plate is backed by air and by an infinite array of parallel stiffeners (or mechanical resonators). The acoustic pressure on the plate is calculated as a function of the spacing between the stiffeners or resonators, the plate thickness, the angle of incidence, and frequency. ↗

ACKNOWLEDGMENT

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The authors are indebted to Denyse Desmond for programming the numerical calculations for a computer.

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INTRODUCTION

In 1961 TRG showed¹ that the observed inability to steer a planar sonar array to endfire is caused by the loss in pressure at the hull of the ship due to the finite rigidity of steel plates of practical thickness. This theoretical deduction was later supported by experimental measurements.² TRG suggested¹ that the use of mechanical resonators (also called stiffeners) fastened to the inner surface of the hull could increase the impedance of the hull to a satisfactory level over a bandwidth sufficient for practical purposes. The effectiveness of resonators was later verified experimentally.^{2,3,4,5}

Theoretical studies of the effectiveness of resonators have considered either single resonators⁶ or resonators distributed uniformly and continuously⁷ behind an infinite elastic plate. The study reported here will be concerned with an infinite set of resonators or parallel stiffeners distributed at regular intervals behind an infinite plate in contact with water. The results for this discrete distribution approach the results for the uniform and continuous distribution when the spacing between resonators approaches zero. The purpose of this theoretical study is to find the maximum spacing between resonators at which the plate is still sufficiently rigidized. We will calculate the pressure at the plate due to an incident plane wave; the plate will be considered to be sufficiently rigidized when this pressure at the plate does not differ appreciably from the pressure at an ideally rigid plate. It has been shown experimentally that a plate can be rigidized with practical spacings between resonators;^{2,5} however, we may have used smaller spacings and more resonators than necessary, which motivates this study. One purpose of this work is to provide a more thorough analytic foundation for the design of resonant stiffeners.

The numerical results are presented in the next section. The details of the mathematical derivations are given in two appendices.

NUMERICAL RESULTS

Consider an elastic plate of thickness h in contact with water occupying the half-space $x \leq 0$, see Figure 1. Resonators or parallel stiffeners are attached to the plate at $ns-b < y < ns$, $n = 0, \pm 1, \pm 2 \dots$, where b is the width of the resonator attachment (assumed to be very small as compared to a wavelength in water or flexural wavelength in the plate), and s is the spacing between resonators. The resonators are assumed to be infinite in the z -direction (normal to the plane of the paper in Fig. 1).

The equations describing the pressure at the surface of the plate and the plate velocity in the x -direction are derived in Appendices A and B (both appendices deal with the same problem, but from slightly different points of view). The effects of resonators are given by specifying: 1) the force which the resonator exerts on the plate at the attachment point; this force is proportional to the plate displacement; 2) the moment which the resonator exerts on the plate; this moment is proportional to the slope of the plate at the resonator attachment point. Various designs of resonators are possible; for example, a simple and effective device used frequently consists of a short stud, which acts as a spring, with a larger piece, which acts as a mass, at the end. An infinite number of different resonator designs will give the same force and moment at the attachment point; therefore for simplicity in the calculations presented below we assumed that the resonator forces and moments are $1/10$ of those for a simple steel rod of circular cross section with the first axial and second transverse vibration resonance at 2500 cps. While the dimensions, which are determined by the resonant frequency, of such a simple resonator then must be very large (length 42 cms, radius 6 cms), one should remember that an infinite number of other designs will give the same forces and moments, and a more efficient resonator (such as the stud plus mass mentioned above) will be much smaller in size⁷. Note also that we are actually considering a resonator only $1/10$ of the size of the rod, since the forces and moments are assumed to be $1/10$ of

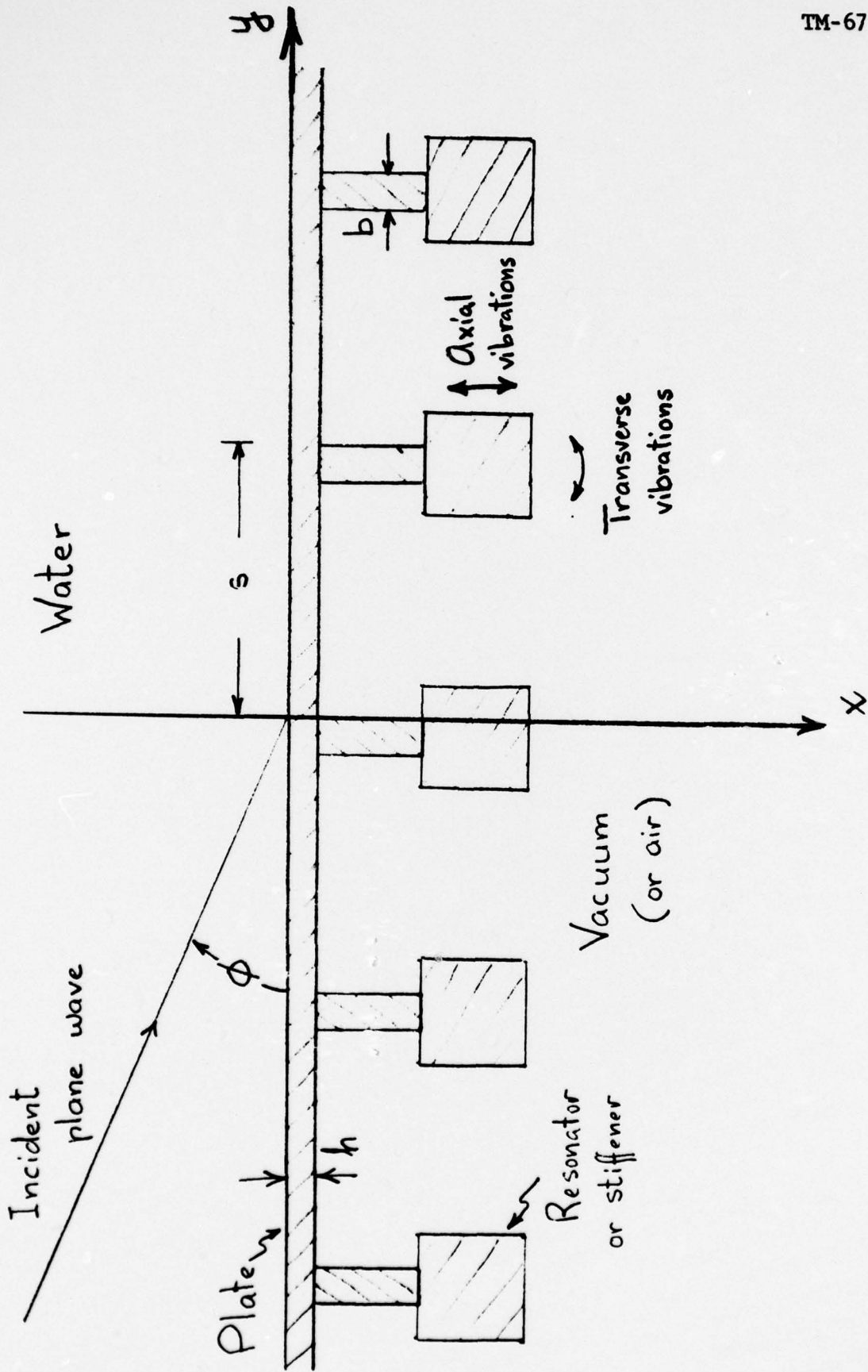


Fig. 1. Plate with resonators.

those for the rod. Most calculations were performed with the 1/10 simple rod resonator only because the mathematical expressions for forces and moments were easy to obtain and easy to compute.

Figure 2 shows the average pressure magnitude on the plate vs. the spacing between resonators. The plate is 1/4 inch thick, and the frequency is 2500 cps. An approximation in which the resonators are assumed to be distributed uniformly and continuously⁷ behind the infinite plate gives 0 db for the pressure for all spacings and angles in Fig. 2. The pressure reduction for $s \geq 7$ cms is easily understood: the wavelength for free flexural vibrations is 14 cms (for $f = 2500$ cps, $h = 1/4$ inch), therefore stiffeners placed at half-wavelength intervals ($s = 7$ cms) cannot be very effective. While we have plotted the average pressure in Fig. 2 (and also in the subsequent figures) instead of the actual pressure distribution on the plate, the actual distribution rarely differs by more than 0.01 db from the average pressure, unless the pressure levels are very low.

Figure 3 shows the average pressure vs. the angle of incidence θ for resonator spacings $s=4,5$, and 6 cms. Figures 2 and 3 indicate that a resonator spacing of 4 cms is acceptable; the pressure is reduced by less than 2 db for $\theta \geq 2^\circ$.

Figure 4 shows the average pressure vs. the frequency. The corresponding resonator or stiffener strength vs. the frequency is shown in Table I, where F and G are quantities proportional to the force and moment exerted by the resonator as defined in Appendix B, Eqs. (B-12), (B-13), (B-35) and (B-36). Although the stiffener itself exerts the maximum force and moment on the plate at 2500 cps, as shown in Table I, at $\theta = 5^\circ$ Figure 4 shows that the plate is most rigid between 2200 cps and 2400 cps, and the pressure is lower at 3000 cps than at 2000 cps. Consequently, an improvement in the pressure level in a total bandwidth of 1000 cps could be obtained by either shifting this band to lower frequencies

(from 1800 cps to 2800 cps) with a stiffener resonance at 2500 cps as in Figure 4, or by changing the stiffener resonance frequency to 2700 cps and keeping the band from 2000 cps to 3000 cps. In either case one would not lose more than 2 db in pressure at the extreme frequencies of the band with the stiffener spacing $s = 4$ cms and $\theta \geq 5^\circ$.

Figure 5 shows the average pressure vs. the thickness of the plate for a resonator spacing of 4 cms.

Figure 6 shows the average pressure vs. the resonator strength, where we have fixed the relative values of F and G in such a way that $G = -2 \times 10^{-3}F$. For small values of F and G the pressure magnitudes approach the values for a 1/4 inch thick plate without resonators, which are:

- 27.0 db at $\theta = 5^\circ$,
- 12.1 db at $\theta = 30^\circ$,
- 6.8 db at $\theta = 90^\circ$,

For a 1/2 inch thick plate without resonators the pressure levels are:

- 21.1 db at $\theta = 5^\circ$,
- 6.9 db at $\theta = 30^\circ$,
- 2.9 db at $\theta = 90^\circ$,

Figures 7-10 show the same data for a half inch thick plate as in the previous figures for a quarter inch thick plate.

Since the wavelength for free flexural vibrations for the 1/2 inch thick plate is 20 cms, the pressure vs. resonator spacing in Fig. 7 shows a loss at the half-wavelength spacing, $s = 10$ cms. Very small losses are shown in Fig. 8 at grazing angles, which should be compared with Fig. 3 for the 1/4 inch thick plate. Figures 7 and 8 indicate that at $f = 2500$ cps a resonator spacing of 6 cms for the 1/2 inch thick plate is acceptable.

However, if we now compare Figures 9 and 4, which show pressure vs. frequency at a resonator spacing of 4 cms, we do not see any improvement with the thicker plate. If we compare Figures 10 and 6, which show pressure vs. the resonator strength,

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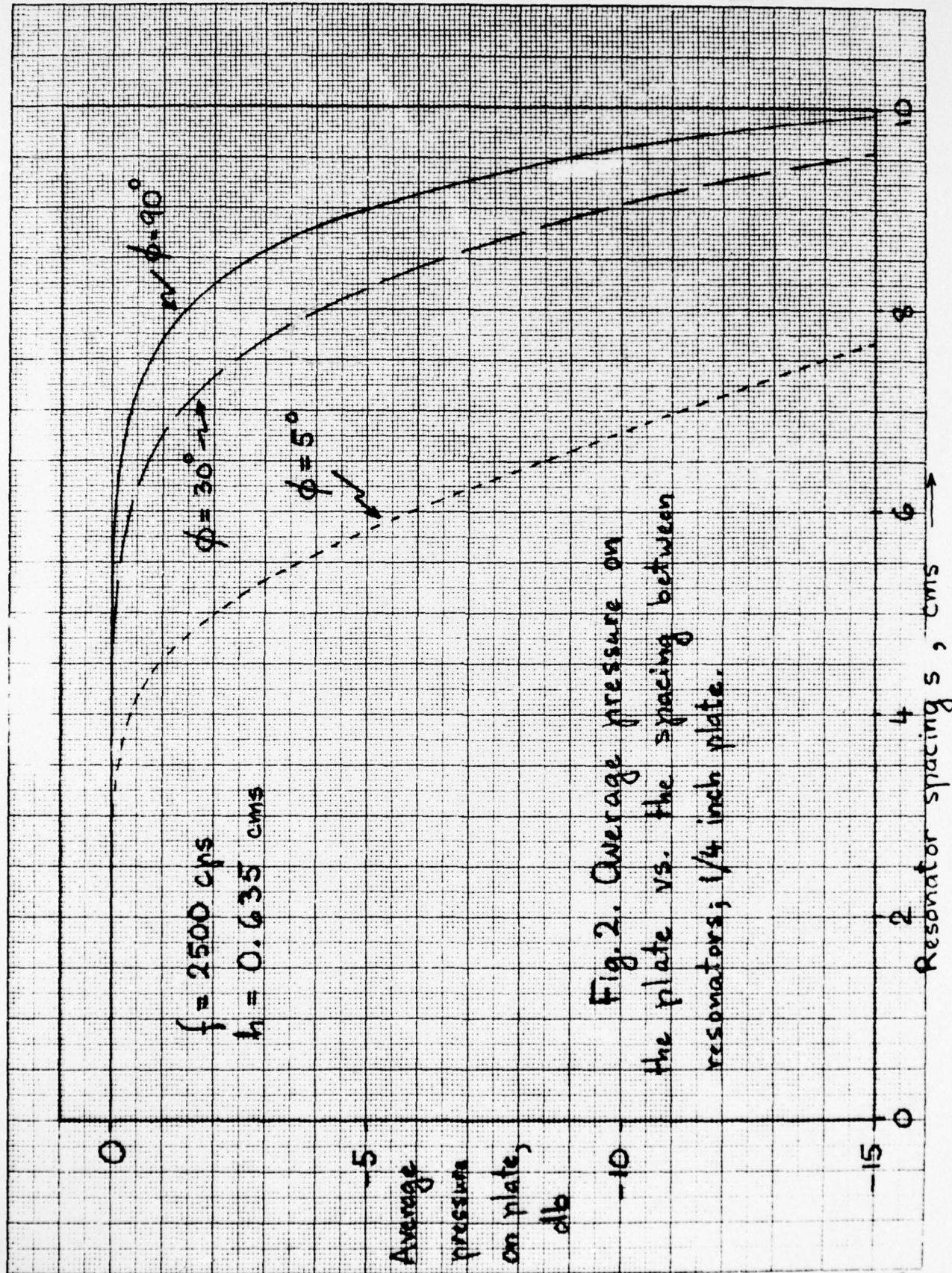


Fig. 2. Average pressure on the plate vs. the spacing between resonators; 1/4 inch plate.

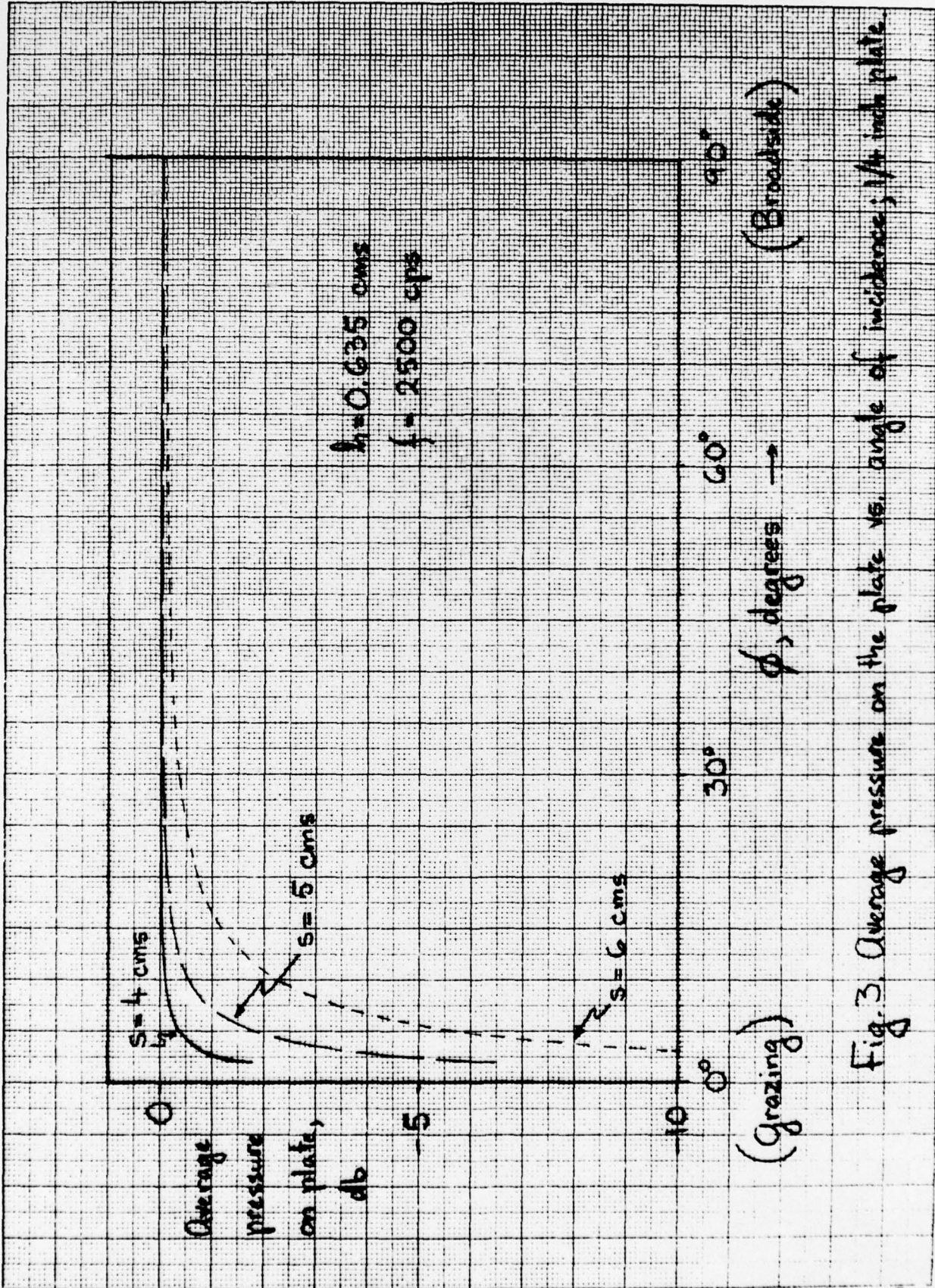


Fig. 3. Average pressure on the plate vs. angle of incidence; 1/4 inch plate.

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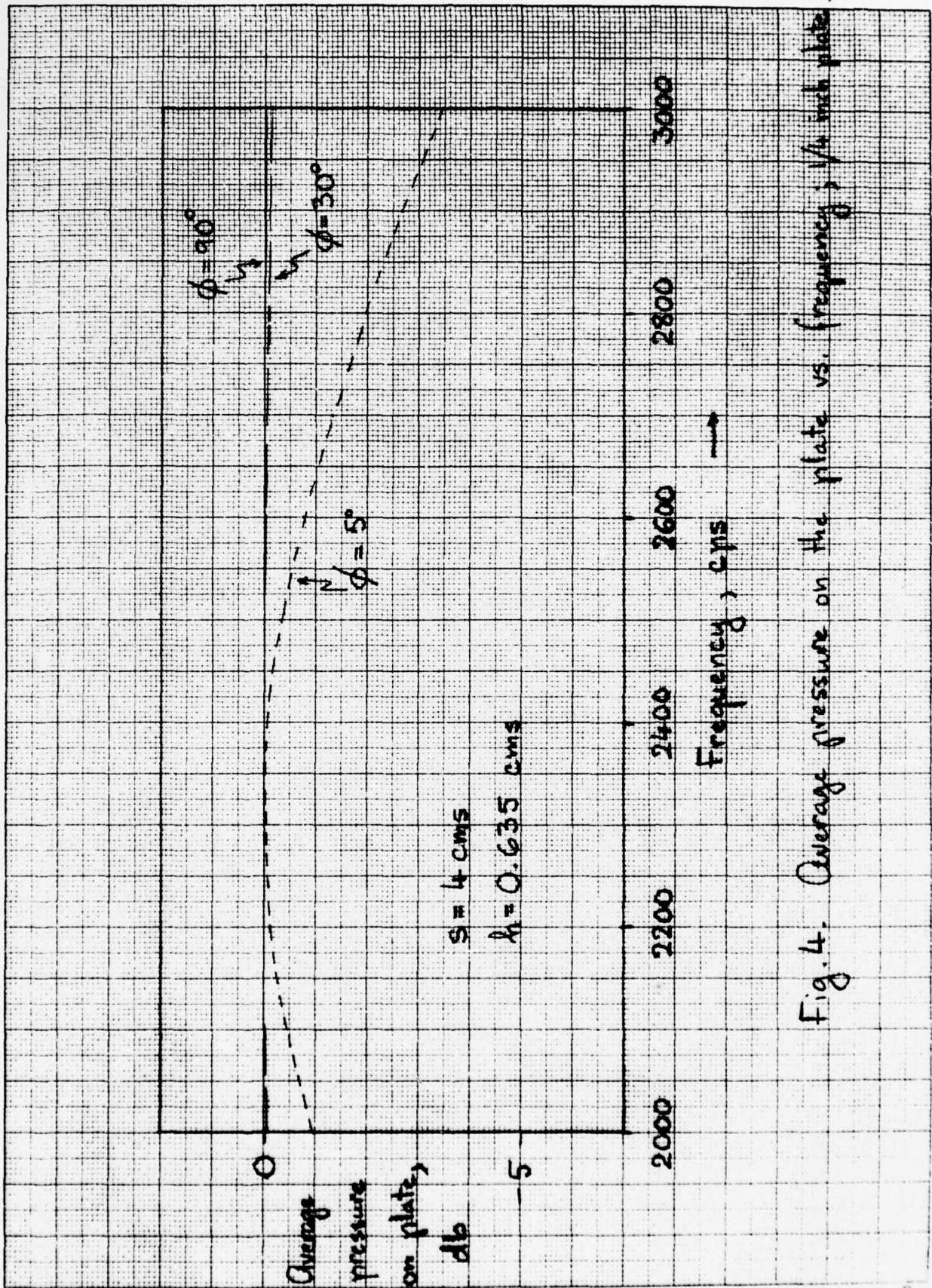


Fig. 4. Average pressure on the plate vs. frequency, $1/4$ inch plate

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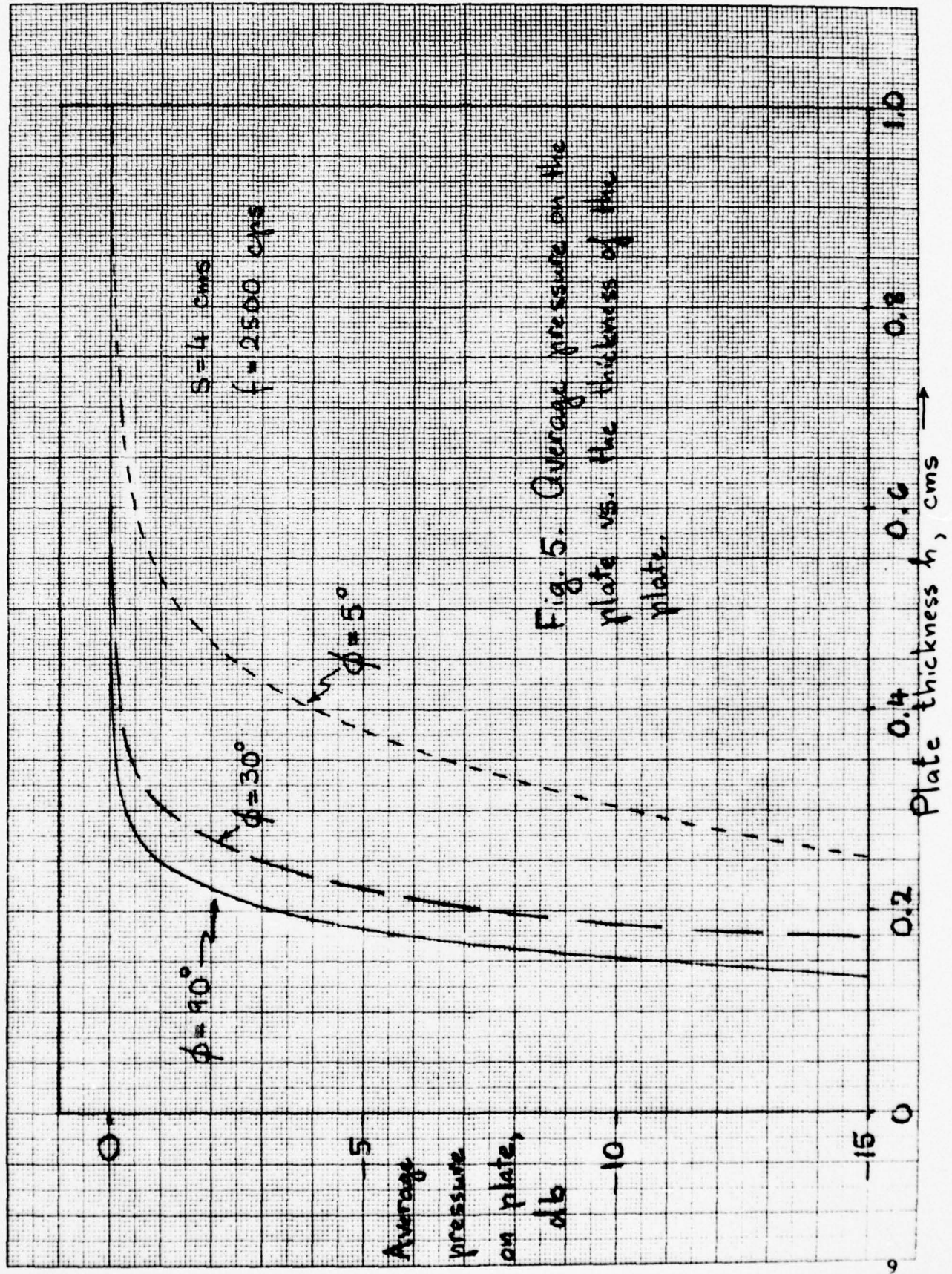
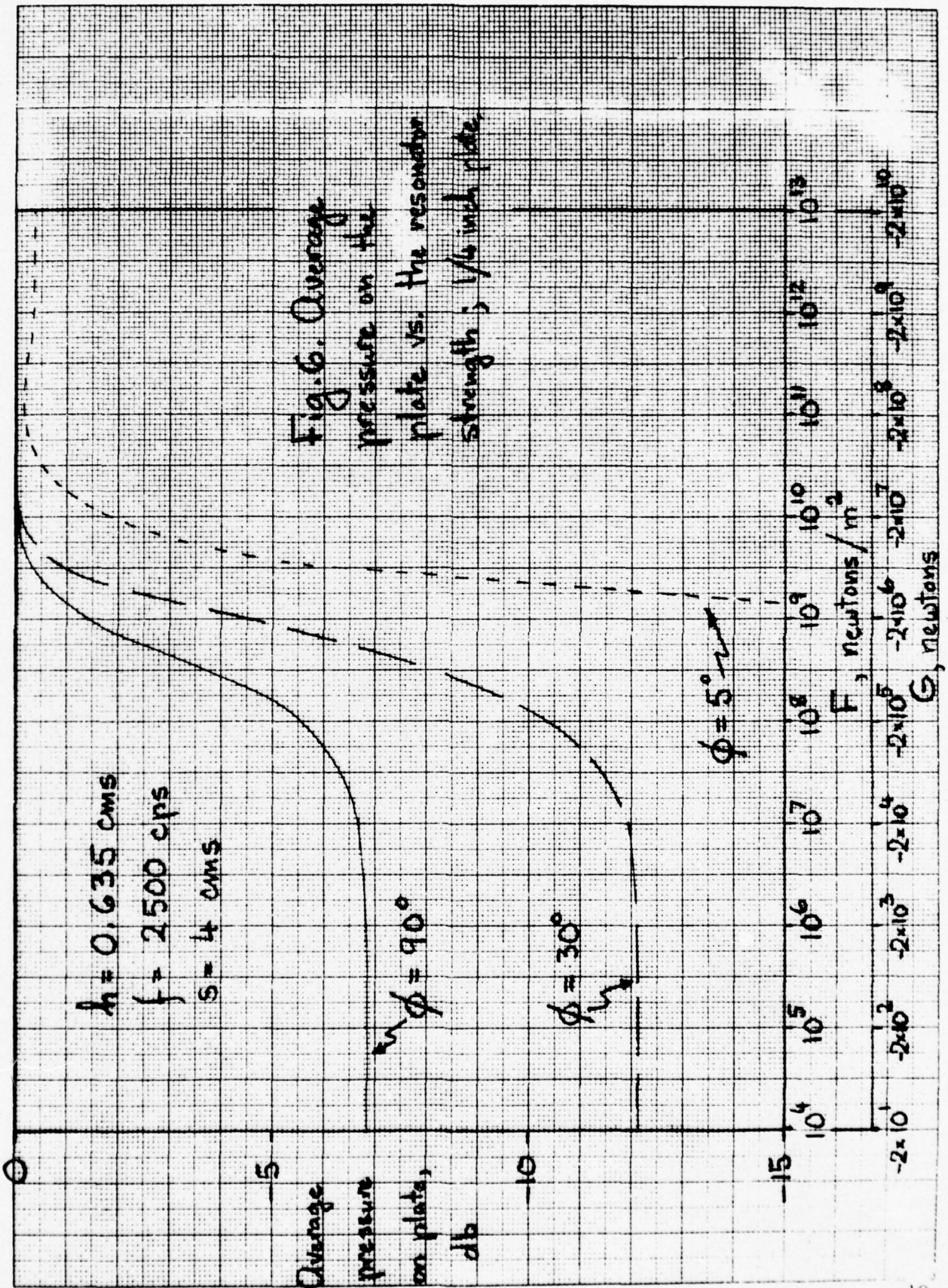


Fig. 5. Average pressure on the plate vs. the thickness of the plate.



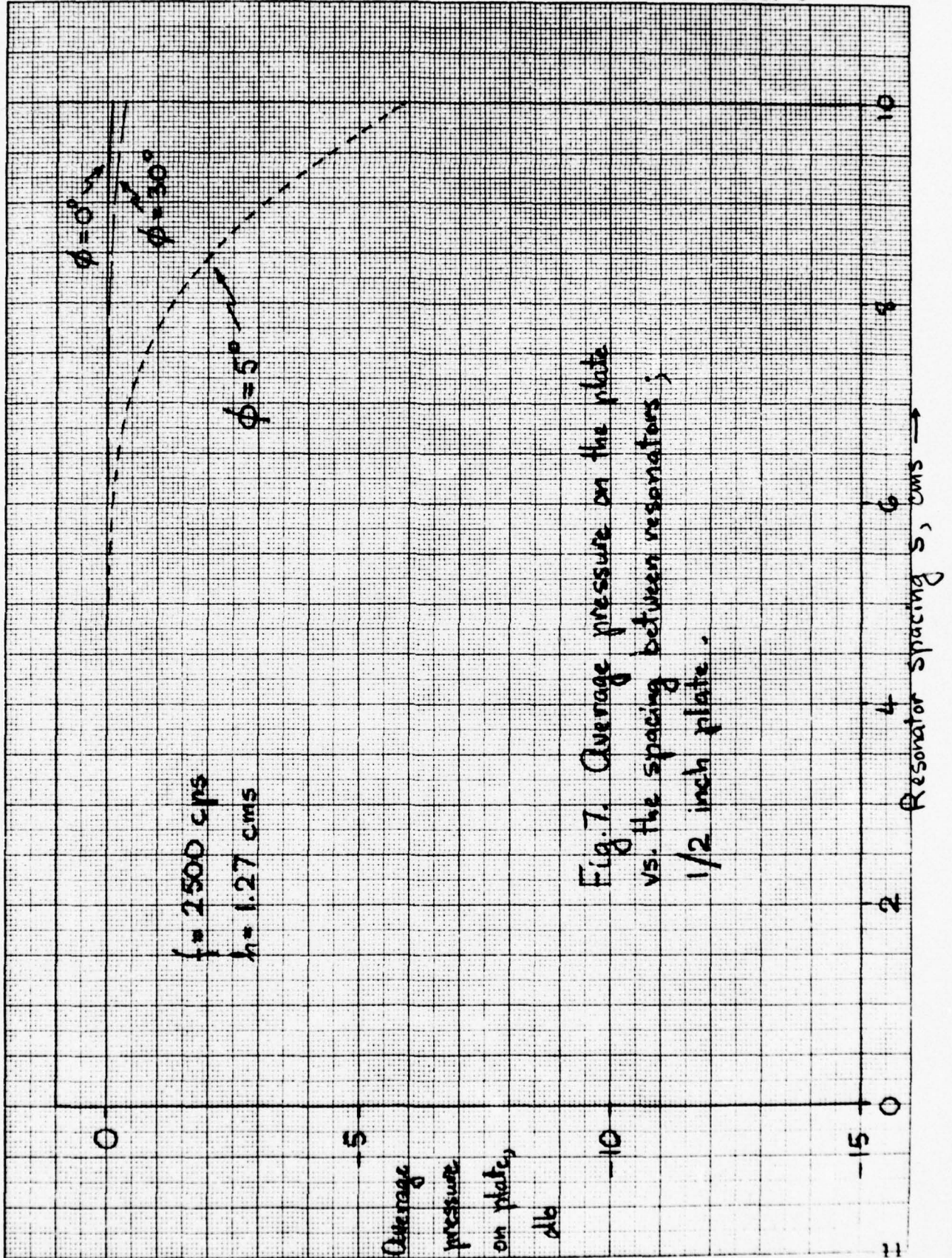


Fig. 7. Average pressure on the plate vs. the spacing between resonators; 1/2 inch plate.

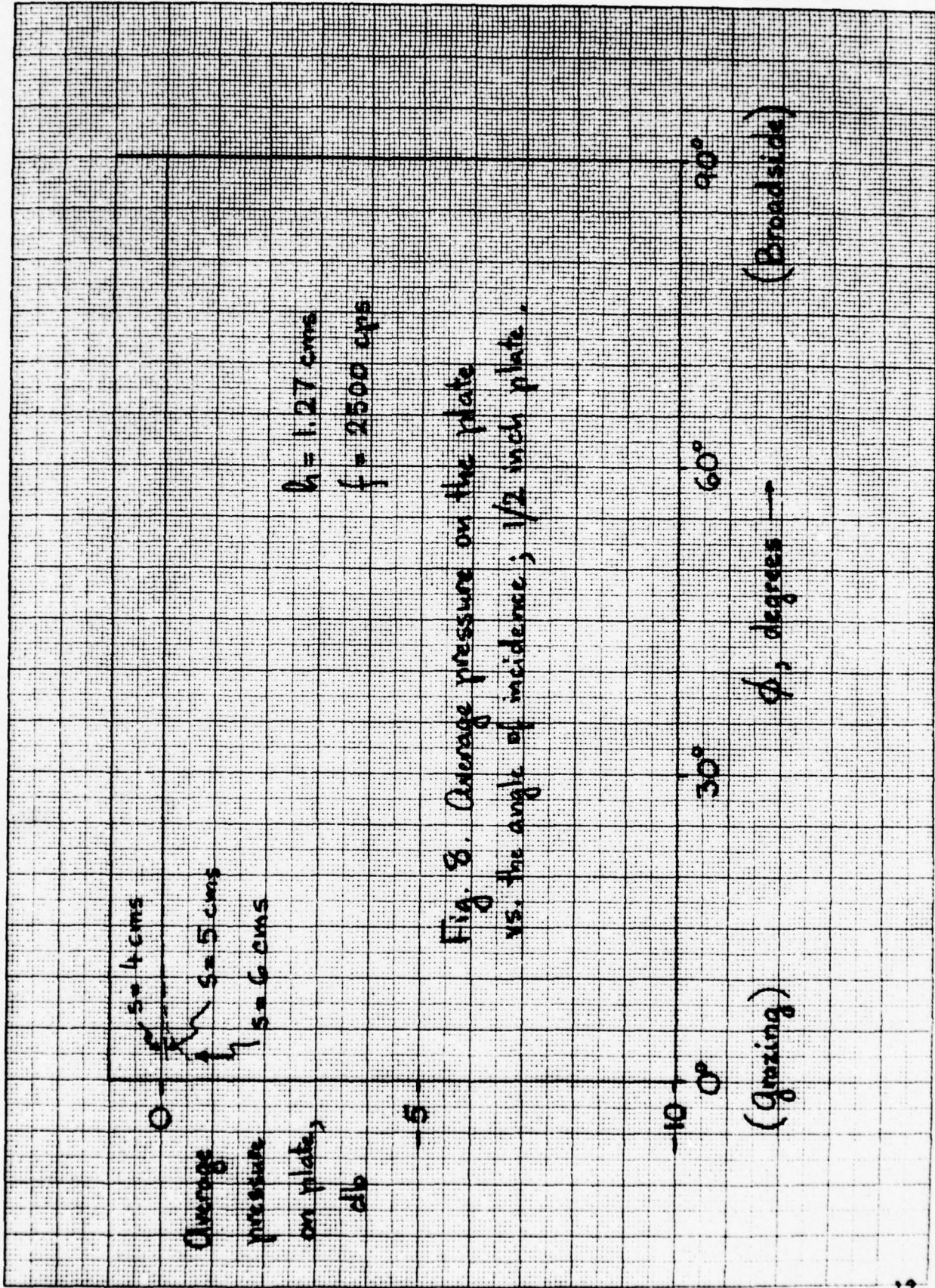


Fig. 8. Average pressure on the plate vs. the angle of incidence; 1/2 inch plate.

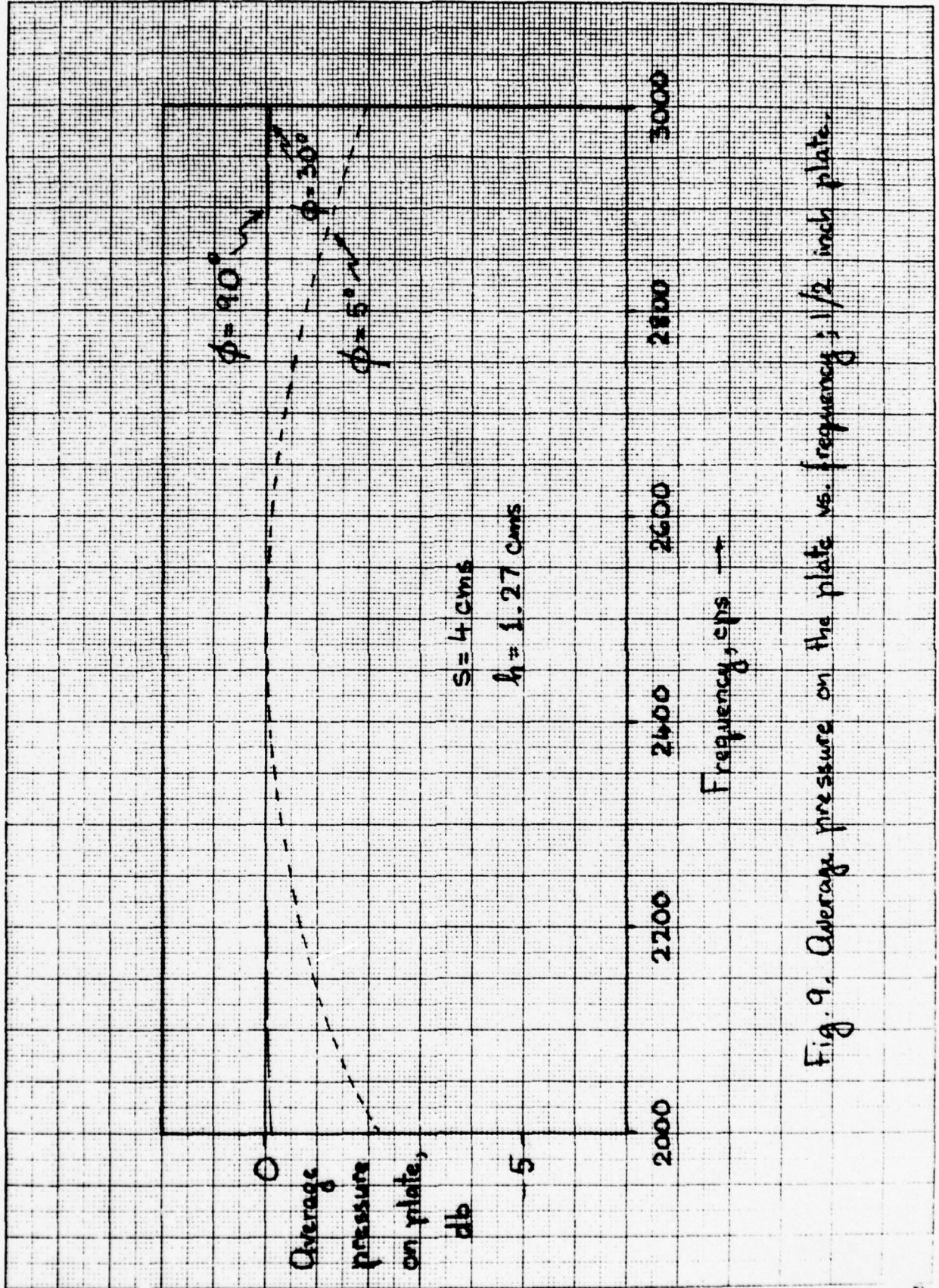


Fig. 9. Average pressure on the plate vs. frequency; $1/2$ inch plate.

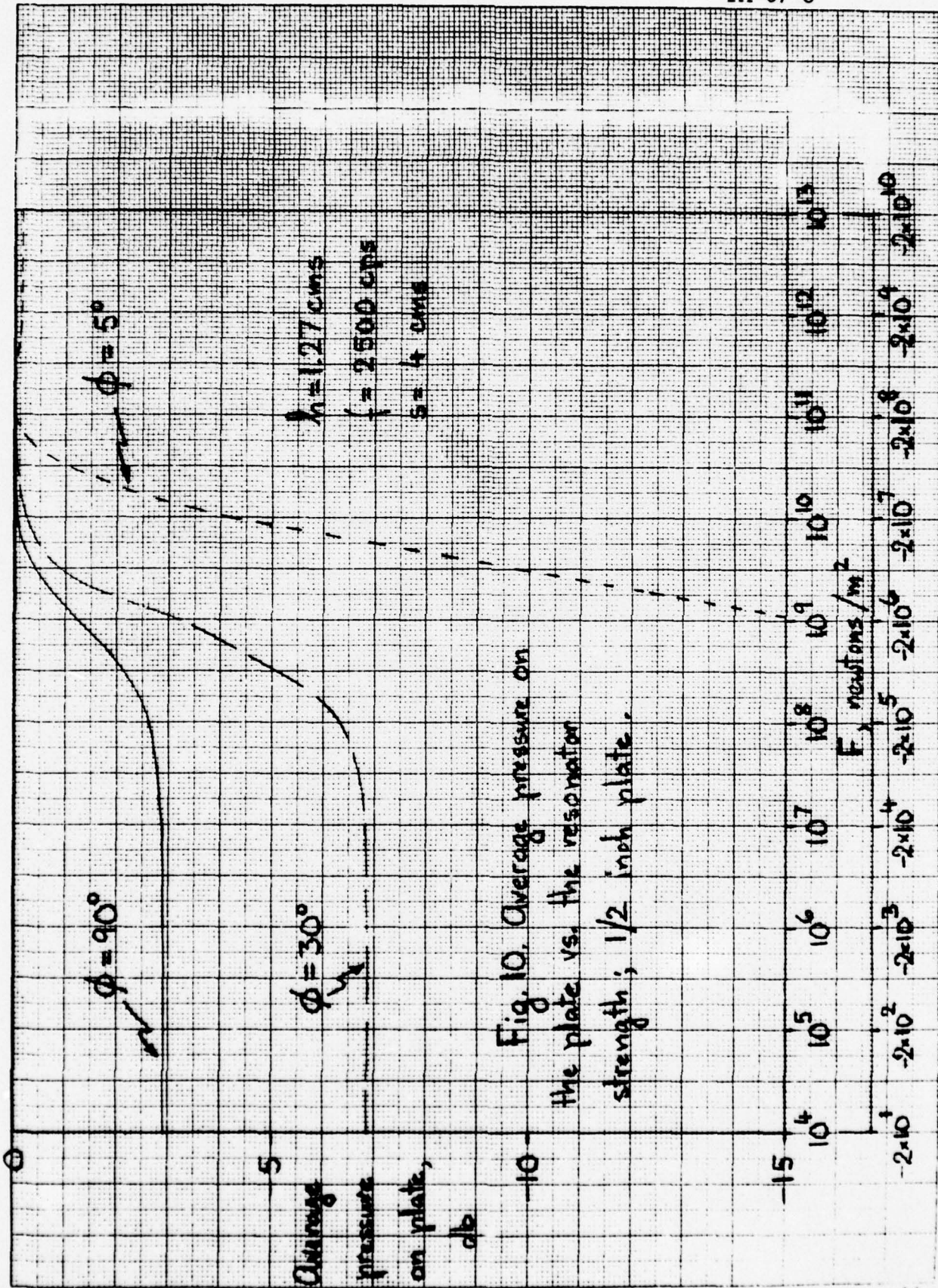


Fig. 10. Average pressure on the plate vs. the resonator strength; 1/2 inch plate.

we see that the thicker plate has a higher pressure level at low resonator strengths and at high resonator strengths, but that at intermediate resonator strengths, where the transition from a flexible to a rigid plate takes place, the slope of the pressure curve is steeper for the thinner plate, and as one increases the resonator strength, the thinner plate becomes rigidized sooner than the thicker plate. Apparently the rigidization of the plate depends on the ratio of the plate mass to the resonator strength. This explains the lack of improvement in Figure 9 as compared to Figure 4, because at the extreme frequencies of the band the resonator strengths are lower than in the middle of the band, see Table I, and the thinner plate is actually better rigidized than the thicker plate.

TABLE I

$\frac{1}{10}$ of Resonator or Stiffener Strength vs. Frequency

| Frequency cps | F, 10^{10} newtons/m ² | G, 10^7 newtons |
|------------------|--|----------------------|
| 2000 | 1.14 | -0.85 |
| 2100 | 1.51 | -1.49 |
| 2200 | 2.10 | -2.52 |
| 2300 | 3.23 | -4.48 |
| 2400 | 6.30 | -9.65 |
| 2500 | 46.83 | -64.71 |
| 2600 | -9.49 | 19.01 |
| 2700 | -4.48 | 9.34 |
| 3800 | -2.99 | 6.58 |
| 2900 | -2.27 | 5.26 |
| 3000 | -1.84 | 4.48 |

CONCLUSIONS

The calculations show that resonators or stiffeners spaced at distances of 4 cms (or less) rigidize a 1/4 or 1/2 inch thick plate sufficiently so that the pressure levels are acceptable over a bandwidth of 1000 cps.

One should note that in an actual baffle design the spacings between resonators will probably be irregular, in which case the pressure levels may be improved, because a regular spacing (which was assumed in the above calculations) permits the development of flexural waves in the plate and supports various resonance effects which may be suppressed by an irregular spacing.

One might think that the performance of the plate could be just as well improved by adding the mass of the resonators to the plate, thus making the plate thicker. However, an increase in the thickness of the plate also increases the flexural wavelength, and if the wavelength in water divided by $\cos \theta$ matches the flexural wavelength, the pressure at the plate is zero. For example, if we had added the resonator mass at 4 cm spacing to the 1/2 inch plate, then at 2500 cps the wavelength in water would match the flexural wavelength in the plate at endfire, and the thick plate would act as a pressure release surface near endfire.

APPENDIX A

Vibrations of a Plate with Parallel Stiffeners

D. Yarmush

The following discussion is largely abstracted from Ref. 6.

We first derive the equation of motion of a uniform infinite plate that bears a continuous but non-uniform distribution of axial and transverse resonators. These are idealized as being attached at points, rather than over areas. The plate is in the (y,z) -plane.

Let $A(y,z)$ be the density of axial point resonators at the point (y,z) . Each resonator is taken as of unit strength. This means that the force exerted on an element of area $dy dz$ in response to the displacement η is $\eta A(y,z) dy dz$. (It is understood that the time dependence has been factored out of η , and that A depends on the frequency.) Let $P(y,z)$ be the net pressure of the water on the plate. Then the equation of motion is

$$D(\nabla^4 \eta - k^4 \eta) = P(y,z) + A(y,z) \eta$$

where D is the plate stiffness, and k is the wavenumber for free flexural vibrations.

For transverse resonators, the motions in perpendicular planes will be handled independently. That is, each physical resonator, which can vibrate in many planes, is considered as resulting from the superposition of two kinds, say the xy and xz kinds. An xy resonator responds only to the slope of the cross-section of the plate that lies in the xy plane, and the moment it exerts on the plate lies in the same plane. Similarly for the xz resonators. These two types are assumed to have the same density $T(y,z)$ over the plate.

We will study the effect of the xy resonators only. Then the dimensionality of the system is lowered, and thin-rod theory can be used.

Consider a beam with a continuous distribution of transverse resonators. The distribution is described by a density function $T(y)$ whose units are moment/unit rotation/unit length. On an increment dy of the beam the increment of moment dm is therefore:

$$dm = T(y) \frac{d\eta}{dy} dy$$

But from simple beam theory,

$$EI \frac{d^2\eta}{dy^2} = m,$$

and therefore the contribution of the transverse resonators to the shear force per unit length is

$$EI \frac{d^4\eta}{dy^4} = \frac{d}{dy} \left(\frac{dm}{dy} \right) = \frac{d}{dy} \left(T(y) \frac{d\eta}{dy} \right).$$

For the xz resonators, the contribution is, by a similar argument,

$$\frac{d}{dz} \left(T(z) \frac{d\eta}{dz} \right)$$

The sum of the two terms can be written in the form

$$\text{div} (T(y,z) \text{grad } \eta(y,z))$$

Therefore the equation of motion for a plate bearing axial point resonators with strength-density $A(y,z)$ and transverse point resonators with strength-density $T(y,z)$ is

$$D(\nabla^4 - k^4)\eta = P + A\eta + \text{div}(T \text{grad } \eta),$$

where P is the driving pressure distribution.

We now consider the special case in which the distributions A and T are concentrated along the lines $y = ns$, for $n = 0, \pm 1, \pm 2, \dots$, and have uniform linear density along these lines. The distance s can then be called the resonator spacing.

The axial strength per unit length at frequency ω will be $F(\omega)$ and the transverse strength $G(\omega)$. We also assume that the pressure of the water on the plate is constant along each line $y = \text{const}$. Then the equation of motion becomes

$$\begin{aligned} D \frac{d^4 \eta}{dy^4} - k^4 \eta &= P(y) + F(\omega) \eta \sum \delta(y - ns) \\ &+ G(\omega) \sum \frac{d}{dy} \left[\delta(y - ns) \frac{d\eta}{dy} \right] \end{aligned}$$

We now will introduce the Fourier expansion for a regular array of δ -functions:

$$\sum \delta(y - ns) = (u/2\pi) \sum e^{inuy}$$

where $u = 2\pi/s$, and we assume that the net pressure and the displacement have expansions of the forms

$$\begin{aligned} P &= \sum b_p e^{ipuy + ivy} \\ \eta &= \sum a_p e^{ipuy + ivy} \end{aligned}$$

(All sums on p will run from $p = -\infty$ to $p = +\infty$). That is, we assume that each of $P(y)$ and $\eta(y)$ is the product of the factor e^{ivy} and a function that is periodic in y , with period s , and we expand this function in a complex Fourier series. The coefficients a_p and b_p depend on u and v . It will be seen later that

$$b_p = \omega^2 \rho^* ((pu + v)^2 - k^{*2})^{-1/2}$$

where ρ^* is the water density, and k^* is the wavenumber for acoustic waves in the water.

We define the function

$$f(w) = D(w^4 - k^4) - \omega^2 \rho^* (w^2 - k^2)^{-1/2}$$

and set:

$$F_0 = uF/2\pi = F/s$$

$$G_0 = uG/2\pi = G/s$$

Then the equation of motion becomes:

$$\begin{aligned} & \sum_p a_p f(pu+v) e^{(pu+v)iy} \\ = & e^{ivy} + F_0 \sum_p a_p \sum_n e^{(nu+pu+v)iy} \\ & - G_0 \sum_p a_p (pu+v) \sum_n (pu+nu+v) e^{(nu+pu+v)iy} \end{aligned}$$

The factor e^{-ivy} can be divided out. In the sums on the right, we make $m = n+p$ a new summation variable, in place of n , and change the order of summation. Then we rename the summation variable on the left as m . Now the coefficients of corresponding terms e^{imuy} can be equated:

$$a_m f(mu+v) = \delta_{m0} + F_0 \sum_p a_p - G_0 \sum_p a_p (mu+v) (pu+v)$$

$$m = -\infty, \dots, +\infty.$$

Now setting

$$A = \sum_p a_p, \quad P = \sum_p a_p (pu+v),$$

we have

$$a_m = f^{-1}(v) \delta_{m0} + f^{-1}(mu+v) (F_0 A - G_0 (mu+v) P)$$

$$m = -\infty, \dots, +\infty.$$

To determine A, we sum this expression for a_m over m; to determine P, we sum after multiplying by $mu+v$. Then we obtain

$$A = f^{-1}(v) + U_0 F_0 A - G_0 U_1 P$$

$$P = v f^{-1}(v) + U_1 F_0 A - G_0 U_2 P$$

where we have set

$$U_i = \sum_m f^{-1}(mu+v) (mu+v)^i \quad i = 0, 1, 2.$$

Thus we have obtained a system of two simultaneous equations for A and P:

$$(1 - F_0 U_0) A + G_0 U_1 P = f^{-1}(v)$$

$$- F_0 U_1 A + (1 + G_0 U_2) P = v f^{-1}(v)$$

If we define

$$T_i = \sum_m f^{-1}(mu+v) m^i,$$

then the determinant Δ of this system can be written in the alternative forms

$$\begin{aligned} \Delta &= 1 - F_0 U_0 + G_0 U_2 + F_0 G_0 (U_1^2 - U_0 U_2) \\ &= 1 - F_0 T_0 + G_0 (u^2 T_2 + 2uv T_1 + v^2 T_0) + F_0 G_0 u^2 (T_1^2 - T_0 T_2) \end{aligned}$$

We then have

$$A = f^{-1}(v) \Delta^{-1} (1 + G_0 (u^2 T_2 + uv T_1))$$

$$P = f^{-1}(v) \Delta^{-1} (v \Delta F_0 u T_1)$$

The final result is:

$$\eta = e^{ivy} f^{-1}(v) \left\{ 1 + \Delta^{-1} (F_0 - v^2 G_0 + u^2 F_0 G_0 T_2) \sum_p f^{-1}(pu+v) e^{ipuy} - \Delta^{-1} (uv G_0 + u^2 F_0 G_0 T_1) \sum_p f^{-1}(pu+v) p e^{ipuy} \right\}$$

It remains to find P, and to justify the form used for b_p . Consider a plate with water on one side, but no resonators. Suppose that the plate executes vibrations with wave number w:

$$\eta = e^{iwy}$$

at $x = 0$. Then the velocity potential for the acoustic displacement at depth y is:

$$-i\omega (w^2 - k^2)^{-1/2} e^{iwy} e^{-(w^2 - k^2)^{1/2} x}$$

and therefore the pressure at the interface is

$$\omega^2 \rho (w^2 - k^2)^{-1/2} e^{iwy}$$

Thus if an arbitrary displacement is Fourier-analyzed into terms with wavenumber $v+pu$, the corresponding pressure term will have b_p as coefficient. Thus the form taken for $f(p)$ is correct.

To determine the displacement, it was not necessary to specify whether the plate is set into motion by an incident acoustic wave in the water or by a traveling wave of force, distributed sinusoidally and having wavenumber v , applied to the back of the plate (the air side). This distinction is essential in determining the net pressure on the plate. If a component of displacement at wavenumber w is due to a force on the back, the corresponding component of pressure is found by multiplying by $\omega^2 \rho (w^2 - k^2)^{-1/2}$, as we have just seen. The responses of the regular array of resonators can be analyzed into such traveling force waves, and thus the effect on the total pressure is known.

The remaining contribution to the pressure can be found by setting $F_0 = G_0 = 0$, since this contribution is independent of F_0 and G_0 . Thus if we define

$$g(w) = f(w) \sqrt{k^{*2} - w^2},$$

we have

$$p(y) = e^{ivy} \left[1 + (i\omega^2 \rho^* / g(v)) + \frac{i\omega^2 \rho^*}{f(v)\Delta} \left\{ (F_0 - v^2 G_0 + u^2 F_0 T_0 T_2) \Sigma \frac{e^{ipuy}}{g(pu+v)} \right. \right. \\ \left. \left. + (uv G_0 + u^2 F_0 G_0 T_1) \Sigma \frac{pe^{ipuy}}{g(pu+v)} \right\} \right]$$

APPENDIX B
 DERIVATION OF THE THEORETICAL EXPRESSIONS
 V. Mangulis

Let the incident plane wave pressure $p_{inc} e^{i\omega t}$ in water be given by

$$p_{inc}(x,y) = \frac{1}{2} \exp(-ikx \sin\theta -iky \cos\theta), \quad (B-1)$$

which is normalized in such a way that the pressure magnitude would be equal to one on the plate itself if the plate were ideally rigid. Since the structure repeats itself at intervals s , the total pressure $p(x,y)$ in water and the velocity component in the x -direction $v_x(x,y)$ should also repeat, except for a phase factor, i.e., we should have for integers m (suppressing a time factor $e^{i\omega t}$)

$$p(x,y+ms) = p(x,y) \exp(-ikms \cos\theta), \quad (B-2)$$

$$v_x(x,y+ms) = v_x(x,y) \exp(-ikms \cos\theta). \quad (B-3)$$

Consequently we can expand the total pressure in a normal mode (or waveguide mode) series:

$$p(x,y) = p_{inc}(x,y) + \exp(-iky \cos\theta) \sum_{n=-\infty}^{\infty} A_n \exp(in_{\mu}y + \eta_n x) \quad (B-4)$$

where A_n are unknown coefficients (to be determined by the boundary conditions on the plate),

$$\mu = 2\pi/s, \quad (B-5)$$

$$\eta_n = \begin{cases} i[k^2 - (n_{\mu} - k \cos\theta)^2]^{\frac{1}{2}}, & \text{if } 0 \leq |n_{\mu} - k \cos\theta| \leq k; \\ [(n_{\mu} - k \cos\theta)^2 - k^2]^{\frac{1}{2}}, & \text{if } |n_{\mu} - k \cos\theta| > k. \end{cases} \quad (B-6)$$

In particular, on the plate itself, when $x=0$,

$$p(0,y) = \exp(-iky \cos\theta) \sum_{n=-\infty}^{\infty} (A_n + \frac{1}{2} \delta_{no}) \exp(in\mu y) \quad (B-7)$$

where δ_{no} is the Kronecker delta, $\delta_{no} = 1$ if $n=0$, and $\delta_{no} = 0$ if $n \neq 0$.

The velocity component in the x-direction $v_x(x,y)$ and the total pressure $p(x,y)$ are related by

$$v_x(x,y) = (i/\rho\omega) [\partial p(x,y)/\partial x], \quad (B-8)$$

where ρ is the density of water, and ω is the radian frequency. At the plate

$$v_x(0,y) = (i/\rho\omega) \exp(-iky \cos\theta) \sum_{n=-\infty}^{\infty} (A_n - \frac{1}{2} \delta_{no}) \eta_n \exp(in\mu y) \quad (B-9)$$

which is also the velocity of the plate itself in the x-direction.

On the plate the velocity v_x and the total pressure p must satisfy the differential equation ⁷ (obtained from the equilibrium of forces and moments on the plate)

$$D \frac{\partial^4 v_x}{\partial y^4} = i\omega \left(p + \frac{F_L}{b} + \frac{1}{b} \cdot \frac{\partial M_T}{\partial y} \right) + \rho_p h \omega^2 v_x \quad (B-10)$$

where F_L is the force per unit length, due to a resonator (in the x-direction), and M_T is the moment per unit length due to a resonator;

$$D = Eh^3/12(1-\sigma^2) \quad (B-11)$$

where E is Young's modulus and σ is Poisson's ratio; h is the plate thickness, ρ_p the plate density. For the m^{th} resonator we let (note that $v_x/i\omega$ is the plate displacement in the x-direction)

$$F_L = (F v_x/i\omega) \text{rct}(ms-b, y, ms) \tag{B-12}$$

$$M_T = [G(\partial v_x/\partial y) / i\omega] \text{rct}(ms-b, y, ms) \tag{B-13}$$

$$\text{rct}(ms-b, y, ms) = \begin{cases} 0, & y < ms-b, \\ 1, & ms-b < y < ms, \\ 0, & y < ms; \end{cases} \tag{B-14}$$

where F and G are constants which depend on the properties of the resonator and the frequency.

Let us now confine our attention to the interval $0 \leq y \leq s$, and let us substitute Eqs. (B-7), (B-9), (B-12), and (B-13) into Eq. (B-10). We obtain

$$\begin{aligned} & \sum_{n=-\infty}^{\infty} (A_n - \frac{1}{2} \delta_{no}) \eta_n e^{in\mu y} \left\{ [D(n\mu - k\cos\theta)^4 - \rho_p h\omega^2] \right. \\ & \left. - \text{rct}(s-b, y, s) [(F/b) - (G/b)(n\mu - k\cos\theta)^2] \right. \\ & \left. - [\delta(y-s+b) - \delta(y-s)] (G/b) i(n\mu - k\cos\theta) \right\} \\ & = \rho\omega^2 \sum_{n=-\infty}^{\infty} (A_n + \frac{1}{2} \delta_{no}) e^{in\mu y}, \end{aligned} \tag{B-15}$$

where we have used

$$\frac{\partial}{\partial y} \text{rct}(s-b, y, s) = \delta(y-s+b) - \delta(y-s), \quad (\text{B-16})$$

where $\delta(y)$ is the Dirac delta function.

If we multiply Eq.(B-15) by

$\exp(-im\mu y + ik y \cos\theta)$, integrate over y from 0 to s , and divide by s , then we obtain for $m = 0, \pm 1, \pm 2, \dots$, the infinite set of equations

$$\begin{aligned} & (A_m - \frac{1}{2}\delta_{m0})\eta_m [D(m\mu - k\cos\theta)^4 - \rho_p h\omega^2] - \rho\omega^2 (A_m + \frac{1}{2}\delta_{m0}) \\ & = \sum_{n=-\infty}^{\infty} (A_n - \frac{1}{2}\delta_{n0})\eta_n \{ B_{nm} [(F/b) - (G/b)(n\mu - k\cos\theta)^2] \\ & \quad + iE_{nm}(G/b)(n\mu - k\cos\theta) \} \end{aligned} \quad (\text{B-17})$$

where

$$B_{nm} = \begin{cases} b/s, & \text{if } n=m, \\ \frac{1 - \exp[-i(n-m)\mu b]}{2\pi i(n-m)}, & \text{if } n \neq m; \end{cases} \quad (\text{B-18})$$

$$E_{nm} = \{ \exp[-i(n-m)\mu b] - 1 \} / s. \quad (\text{B-19})$$

We assume that b is very small compared to a wavelength in water or plate, and that b/s is very small, so that we can simplify Eqs.(B-17), (B-18), and (B-19) by letting $b \rightarrow 0$, which yields

$$B_{nm} \rightarrow b/s \text{ for all } n, m; \quad (\text{B-20})$$

$$E_{nm} \rightarrow -i(n-m)\mu(b/s); \tag{B-21}$$

and Eq. (B-17) becomes

$$\begin{aligned} & (A_m^{-\frac{1}{2}} \delta_{mo}) \{ \eta_m [D(m\mu - k\cos\theta)^4 - \rho_p h\omega^2] - \rho\omega^2 \} \\ &= \rho\omega^2 \delta_{om} + \sum_{n=-\infty}^{\infty} (A_n^{-\frac{1}{2}} \delta_{no}) \eta_n [(F/s) \\ & - (G/s)(m\mu - k\cos\theta)(n\mu - k\cos\theta)] \end{aligned} \tag{B-22}$$

While Eq. (B-17) consisted of an infinite set of equations, and the solution of that infinite set required the inversion of an infinite matrix, by letting $b \rightarrow 0$ we have simplified the solution to the point where we have to solve only a set of two equations, as shown below.

Note that the following summations are just some constants, independent of m in Eq. (B-22):

$$(F/s) \sum_{n=-\infty}^{\infty} (A_n^{-\frac{1}{2}} \delta_{no}) \eta_n = b_1 \tag{B-23}$$

$$(G/s) \sum_{n=-\infty}^{\infty} (A_n^{-\frac{1}{2}} \delta_{no}) \eta_n (n\mu - k\cos\theta) = b_2 \tag{B-24}$$

In terms of those constants we can immediately write down the solutions for the unknown coefficients A_m ,

$$A_m^{-\frac{1}{2}} \delta_{mo} = \frac{\rho\omega^2 \delta_{om} + b_1 - (m\mu - k\cos\theta)b_2}{\eta_m [D(m\mu - k\cos\theta)^4 - \rho_p h\omega^2] - \rho\omega^2} \tag{B-25}$$

It remains to evaluate the two constants b_1 and b_2 . We can write down two equations involving those two

constants by: 1) multiplying both sides of Eq. (B-25) by $(F/s)\eta_m$ and summing over m from $-\infty$ to ∞ , which will give b_1 on the left-hand side; 2) multiplying both sides by $(G/s)\eta_m$ $(m\mu - k\cos\theta)$ and summing over m from $-\infty$ to ∞ , which will give b_2 on the left-hand side. The two equations can then be solved for b_1 and b_2 , which in turn yields A_m in Eq. (B-25), which then determines the pressure and velocity at the plate.

The final result is

$$p(0,y) = e^{-i\nu y} \left\{ 1 + \frac{i\rho\omega^2}{g(\nu)} \right. \\ \left. + \frac{i\rho\omega^2}{f(\nu) \cdot \Delta} \left[C_1 \sum_{n=-\infty}^{\infty} \frac{e^{in\mu y}}{g(n\mu - \nu)} + C_2 \sum_{n=-\infty}^{\infty} \frac{ne^{in\mu y}}{g(n\mu - \nu)} \right] \right\} \quad (B-26)$$

$$v_x(0,y) = \frac{i\omega e^{-i\nu y}}{f(\nu)} \left\{ 1 + \frac{1}{\Delta} \left[C_1 \sum_{n=-\infty}^{\infty} \frac{e^{in\mu y}}{f(n\mu - \nu)} \right. \right. \\ \left. \left. + C_2 \sum_{n=-\infty}^{\infty} \frac{ne^{in\mu y}}{f(n\mu - \nu)} \right] \right\} \quad (B-27)$$

where

$$\nu = k\cos\theta \quad (B-28)$$

$$f(n\mu - \nu) = D(n\mu - \nu)^4 - \rho_p h\omega^2 - \rho\omega^2/\eta_n \quad (B-29)$$

$$g(n\mu - \nu) = i\eta_n f(n\mu - \nu) \quad (B-30)$$

$$C_1 = F/s - (G/s)\nu^2 + (G/s)\mu^2 T_2 \quad (B-31)$$

$$C_2 = (G/s)\mu [\nu + (F/s)\mu T_1] \quad (B-32)$$

$$\Delta = 1 - (F/s)T_0 + (G/s)(\nu^2 T_0 + 2\mu\nu T_1 + \mu^2 T_2) - (F/s)(G/s)\mu^2(T_0 T_2 - T_1^2) \quad (B-33)$$

where for $j = 1, 2, 3$

$$(-1)^j T_j = \sum_{m=-\infty}^{\infty} \frac{m^j}{f(m\mu - \nu)} \quad (B-34)$$

The result derived here is the complex conjugate of the one derived in Appendix A because the time dependence used here is $e^{i\omega t}$, and the one used in Appendix A is $e^{-i\omega t}$.

Equations (B-26) and B(27) have been programmed for a computer. The infinite sums were replaced by finite sums in Eqs. (B-26), B(27), and (B-34); the summation was terminated when either the next term in the sum was less than 10^{-4} times the sum of previous terms, or when a prescribed number (usually about 40) of terms had been summed. The ratio of the last term in the sum to the total sum is always printed out so that one can always ascertain whether the sum has converged. The convergence becomes poorer as the spacing s between resonators is increased.

The functions F and G for a simple circular rod resonator are ⁷

$$F = (1/2a) ES\alpha \tan\alpha L \quad (B-35)$$

$$G = \frac{EI\beta}{2a} \cdot \frac{\cos\beta L \sinh\beta L - \sin\beta L \cosh\beta L}{1 + \cos\beta L \cosh\beta L} \quad (B-36)$$

where a is the radius and L the length of the resonator,

$$\beta^4 = \omega^2 \rho_p S / EI \quad (B-37)$$

$$\alpha = \omega / (E / \rho_p)^{1/2} \quad (\text{B-38})$$

$$S = \pi a^2 \quad (\text{B-39})$$

$$I = \pi a^4 / 4 \quad (\text{B-40})$$

where we have assumed that the resonator is made from the same material as the plate.

The values which we used in our computer program are (in MKS units)

$$\rho = 10^3 \text{ kg/m}^3, \quad \rho_p = 7.7 \times 10^3 \text{ kg/m}^3,$$

$$E = 1.36 \times 10^{11} \text{ newtons/m}^2, \quad \sigma = 0.29,$$

$$C = 1500 \text{ m/sec}, \quad a = 0.06 \text{ m},$$

$$L = 0.42 \text{ m}.$$

Other parameters, such as s , h , θ , and frequency f , were allowed to vary, and are stated separately for each figure in the main section. The values of F and G actually used were 1/10 of those given by Eqs. (B-35) and (B-36) with the above values of the parameters a , L , etc.

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