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QUARTERLY PROGRESS REPORT
FOR APRIL, MAY, AND JUNE 1970
Under Contract N00024-70-C-1279

NAVAL SHIP SYSTEMS COMMAND
Contract N00024-70-C-1279
Proj. Ser. No. SF 11552-001, Task 8118

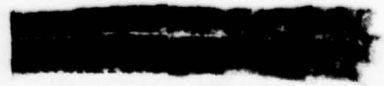


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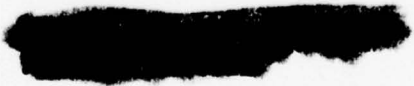
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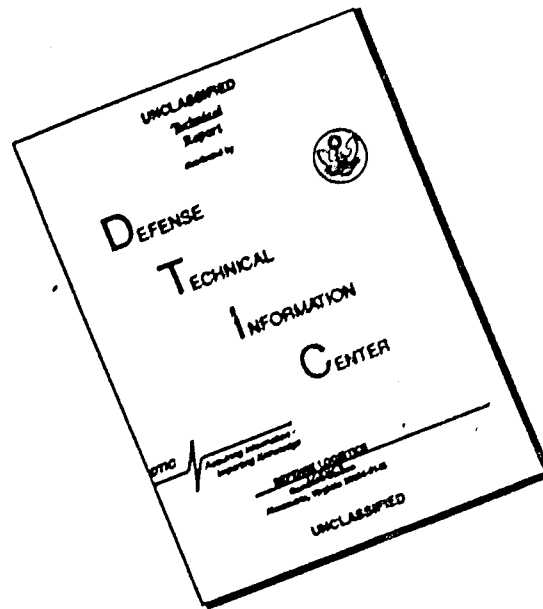
ABSTRACT

In Section I it is shown by using the simplest reflecting surface--an infinite, pressure release plane surface--that at least the Fresnel phase approximation and a reasonable directivity pattern approximation are necessary before accurate predictions in surface reflection and scattering problems can be made. The average Rayleigh reflection coefficient for random penetrable surfaces of various roughnesses is calculated in Section II. In Section III an integral expression is derived for the acoustic field of a point source in a surface duct with a rough boundary. The integral was obtained by Green's functions and includes the rough surface condition by means of an effective impedance condition which also depends on the properties of the medium.

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I. THE REFLECTION OF AN ACOUSTIC BEAM
BY AN INFINITE, PRESSURE RELEASE PLANE SURFACE

The prediction of the field scattered by a known surface configuration or, conversely, the prediction of the surface configuration from its scattered field is a difficult and complex problem. Due to the extreme complexity of the problem, all of the solutions of the integral equations obtained to date have contained numerous approximations. One of the fundamental difficulties associated with surface scattering has been the determination of the ranges of validity and the interaction of the various approximations.

The usual approximations can be put into five categories: (1) formulation, (2) boundary values, (3) source directivity, (4) phase approximation, and (5) surface representation. It was shown in the Final Report under Contract N00024-69-C-1275 (April 1970) that the potential formulation of reflection and scattering problems has distinct advantages over the Helmholtz' and Green's functions formulations and, in the limit of a plane surface, reduces to the Rayleigh-Sommerfeld formula. The boundary value for a plane, pressure release surface is known exactly. The boundary values for arbitrary penetrable surfaces will be the subject of later progress reports. Surface representations for randomly rough surfaces will also be treated further in future reports.

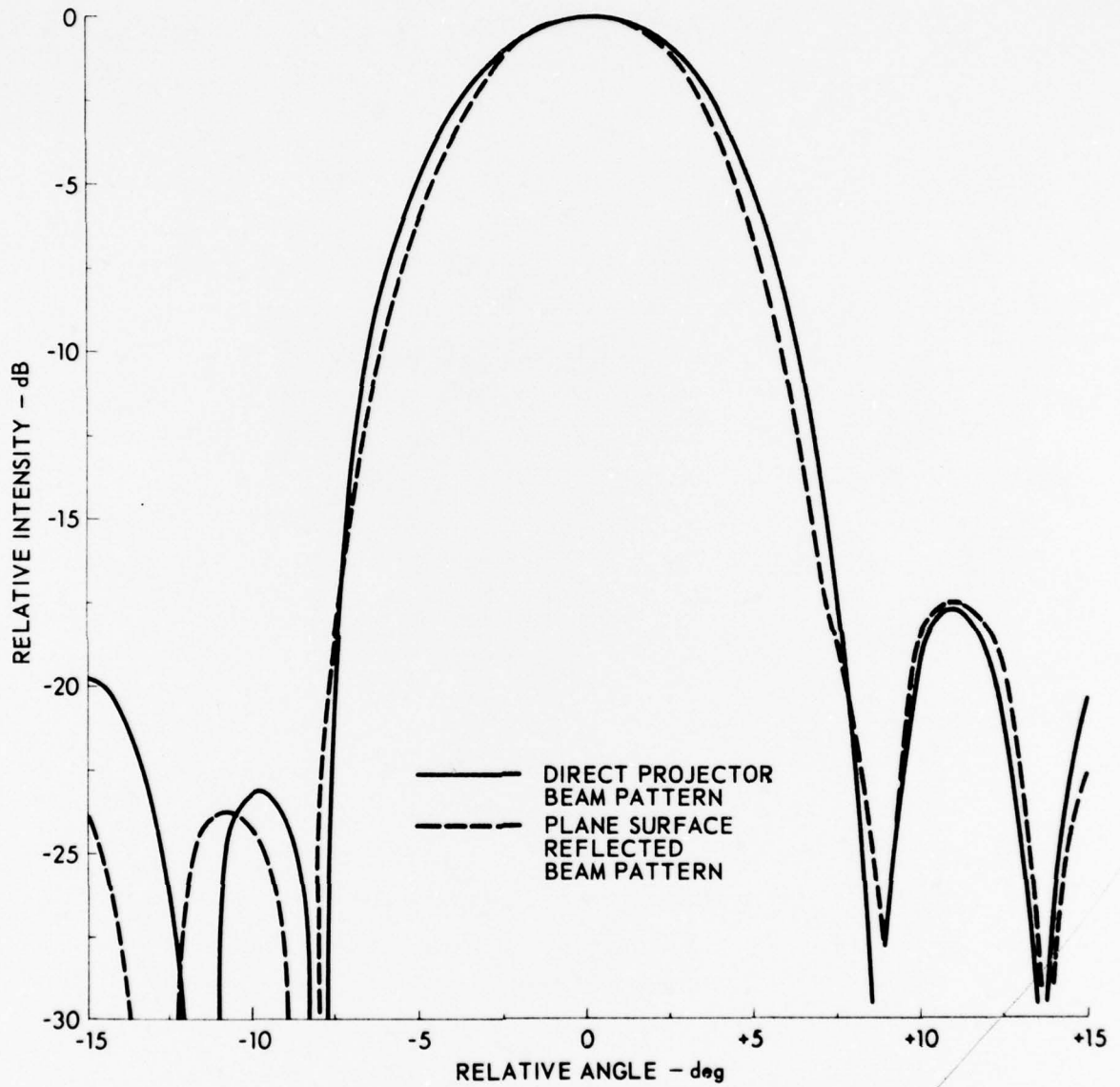
At this time only an infinite, pressure release plane surface will be considered, since this is the simplest of surfaces and the results are easily verified by the method of images. By treating a plane surface, attention can be focused exclusively on the phase and

source directivity approximations that will be necessary in the treatment of arbitrary penetrable surfaces and the inverse problem, and certain misconceptions about the phase and directivity approximations can be exposed.

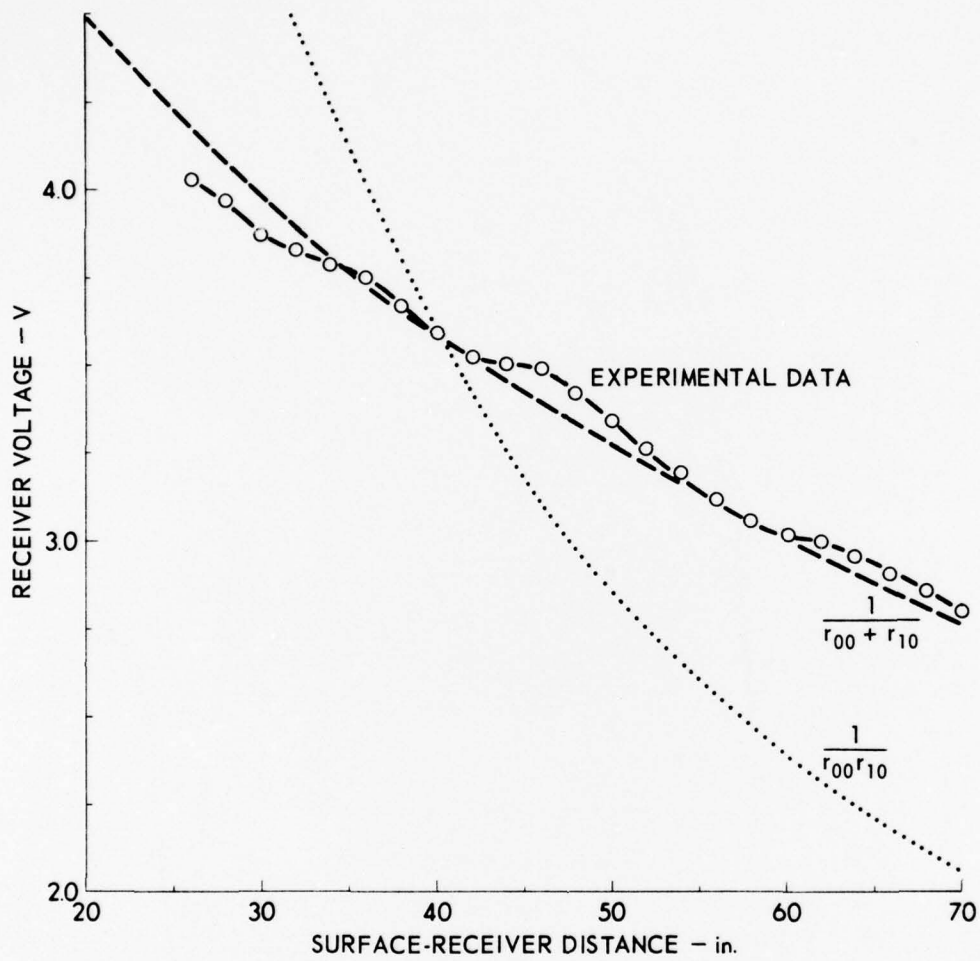
To gain insight into what approximations might be necessary, it is useful to recall two experimentally established properties of the intensity reflected by a pressure release plane surface: (1) the range dependence is that predicted by geometrical acoustics, namely $1/(r_{oo} + r_{lo})$, and (2) the results predicted by the image technique are indeed obtained. Drawings AS-69-478 and AS-69-479 demonstrate these properties. In light of the experimental facts, two conclusions are inescapable: (1) the Fraunhofer phase approximation is never correct when one is concerned with surface reflection or scattering since it predicts a $1/r_{oo} r_{lo}$ spreading loss; and (2) the analytic solution should reduce to the image solution. The first conclusion applies to arbitrarily rough surfaces since in the limit as the roughness approaches zero, the results must reduce to a plane surface. Hence, second and possibly higher order terms must be used in the expansion of r_o and r_l in the phase (Fresnel approximation); otherwise, the correct range dependence will never be obtained.

In previous reports (see, for example, Final Report under Contract N00024-70-C-1275), it has been shown that the pressure reflected from a plane pressure release surface to some receiver at A is given by:

$$p(A) = \frac{ik}{2\pi} \iint_{\Sigma'} P_o \frac{e^{ik(r_o + r_l)}}{r_o r_l} \hat{n} \cdot \hat{e}_1 ds \quad , \quad (1)$$



COMPARISON OF DIRECT AND PLANE SURFACE REFLECTED PROJECTOR BEAM PATTERNS



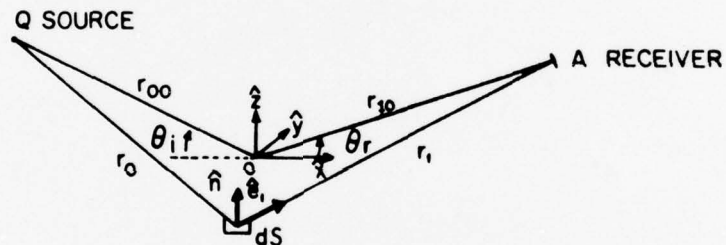
COMPARISON OF THE EXPERIMENTALLY MEASURED RANGE DEPENDENCE OF THE PLANE SURFACE REFLECTED PRESSURE FIELD AND RANGE DEPENDENCES OF $\frac{1}{r_{00}r_{10}}$ AND $\frac{1}{r_{00} + r_{10}}$

ARL - UT
AS-69-479
HGF - RFO
5 - 21 - 69

where

- 1) if the amplitude factor is suppressed, P_0 is simply the directivity pattern of the source,
- 2) r_0 and r_1 are the distances from the source and the receiver respectively to the surface element dS ,
- 3) \hat{n} is the unit normal out of the surface at the point enclosed by dS ,
- 4) \hat{e}_1 is the unit vector pointing from the surface element dS along r_1 to the receiver, and
- 5) Σ' represents the insonified surface area.

The only assumption contained in this result is $kr \gg 1$. The geometry is depicted in the accompanying diagram.



The origin is fixed at the point where the axis of the acoustic beam intersects the plane. From simple trigonometry, it can be shown for a plane:

$$\hat{n} \cdot \hat{e}_1 = \frac{r_{10} \sin \theta_r}{r_1} \quad , \quad (2)$$

and

$$dS = dx dy \quad . \quad (3)$$

Note that r_{10} , r_{00} , θ_r , and θ_i are constants, while r_1 and r_0 are variables.

In general, Eq. (1) cannot be evaluated without making certain approximations. Previously, it was pointed out that at least the Fresnel approximation was required in the phase. The Fresnel phase approximation is given by the equation¹

$$ik(r_1+r_0) = ik(r_{10} + r_{00} + ax + \frac{1}{R_1} x^2 + \frac{1}{R} y^2) + \text{higher order terms} \quad , \quad (4)$$

where

$$a = \cos\theta_i - \cos\theta_r \quad ,$$

$$R = 2r_{00}r_{10}/(r_{00}+r_{10}) \quad ,$$

$$R_1 = 2r_{00}r_{10}/(r_{00}\sin^2\theta_r + r_{10}\sin^2\theta_i) \quad .$$

Elsewhere r_0 and r_1 will be replaced by r_{00} and r_{10} .

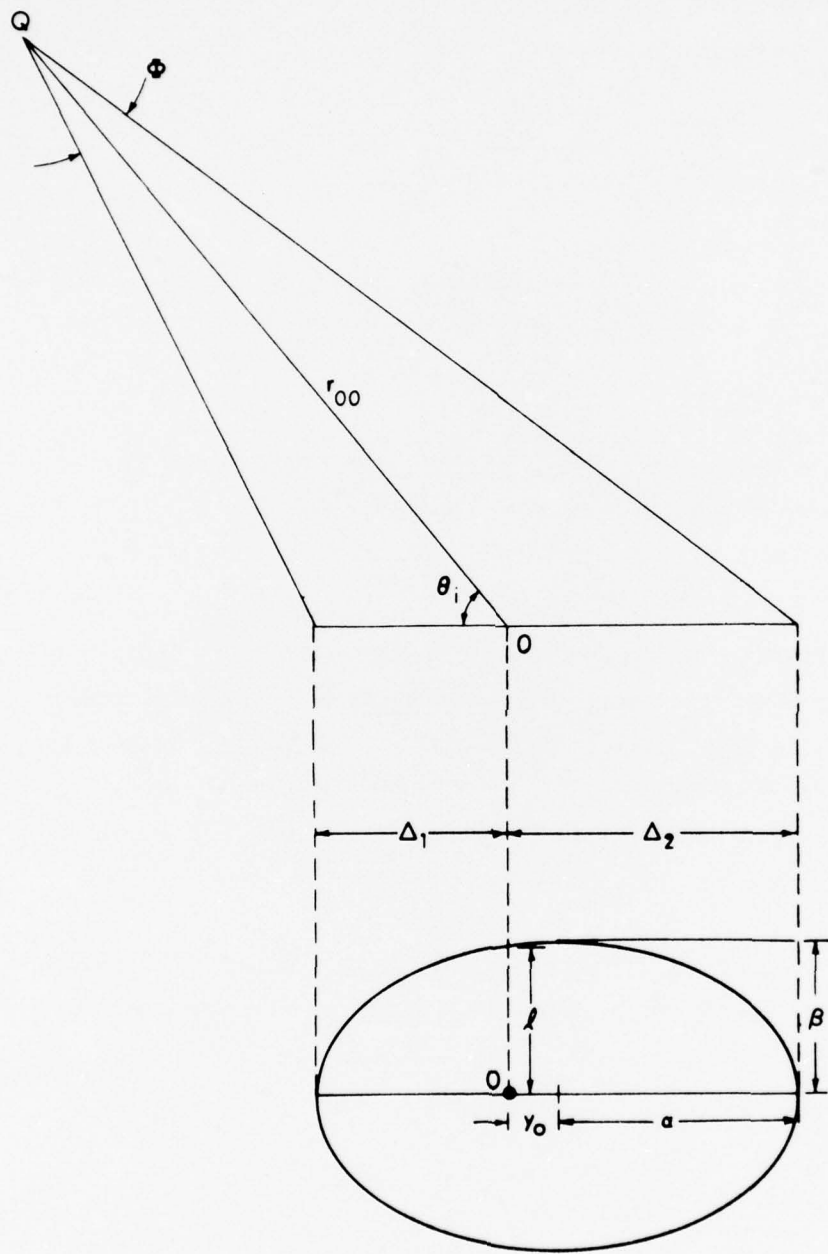
All that remains now is to find a suitable approximation to the directivity pattern. For a nonisotropic source there are essentially two tractable approximations for the directivity pattern. The first is the simple aperture type function where it is assumed the insonification is a constant over the region of the surface defined by the -3 dB points of the source and zero elsewhere on the surface. The geometry is shown in Dwg. AS-69-635. If the beamwidth of the source is Φ deg, then the following expressions hold:

$$\Delta_1 = r_{00} \sin(\theta/2) / \sin(\theta_i + \Phi/2) \quad , \quad (5)$$

$$\Delta_2 = r_{00} \sin(\Phi/2) / \sin(\theta_i - \Phi/2) \quad , \quad (6)$$

$$l = r_{00} \tan(\Phi/2) \quad , \quad \text{and} \quad (7)$$

$$y_0 = \frac{\Delta_2 - \Delta_1}{2} \quad . \quad (8)$$



INSONIFIED AREA GEOMETRY

Then:

$$K = 3/20 \log_{10} e \quad , \quad (9)$$

$$\alpha = \frac{\Delta_2 + \Delta_1}{2} \quad , \quad \text{and} \quad (10)$$

$$\beta = \frac{l\alpha}{\sqrt{y_0^2 + \alpha^2}} \quad (11)$$

where

- 1) K is a constant which provides the -3 dB value at the appropriate point off the beam axis,
- 2) α is the semimajor axis of the insonified ellipse, and
- 3) β is the semiminor axis.

Note that the effect of the range differences from one point on the ellipse to another has been ignored in setting the -3 dB points. These expressions are calculated in detail in the Quarterly Progress Report under Contract N00024-69-C-1275 for April - June 1969. In the calculations, a beamwidth, Φ , of 8 deg will be assumed since this is the beamwidth of the source used in the experimental program.

If now the ellipse is approximated by a rectangle of dimensions $2l$, $\Delta_2 + \Delta_1$, then the pressure reflected by a plane surface where the directivity pattern is assumed to be a simple aperture may be calculated.¹ Using Eqs. (2), (3), and (4) in Eq. (1) gives

$$p(A) = \frac{ik \sin\theta_r}{2\pi r_{00} r_{10}} e^{ik(r_{00} + r_{10})} \int_{-\Delta_1}^{\Delta_2} \int_{-l}^l e^{ik\left(ax + \frac{x^2}{R_1} + \frac{y^2}{R}\right)} dx dy \quad . \quad (12)$$

The integrals in Eq. (12) are by definition Fresnel integrals and upon forming the product of $p(A)$ and its complex conjugate, the intensity is obtained:

$$I_p = p^*(A)p(A) = \frac{\left[C_\ell(C_{\Delta_1} + C_{\Delta_2}) - S_\ell(S_{\Delta_1} + S_{\Delta_2}) \right]^2 + \left[S_\ell(C_{\Delta_1} + C_{\Delta_2}) + C_\ell(S_{\Delta_1} + S_{\Delta_2}) \right]^2}{(r_{00} + r_{10})(r_{00} + r_{10} \sin^2 \theta_i / \sin^2 \theta_i)} \quad (13)$$

where

$$C(x) = \int_0^x \cos\left(\frac{\pi}{2} u^2\right) du \quad , \quad (14)$$

and

$$S(x) = \int_0^x \sin\left(\frac{\pi}{2} u^2\right) du \quad . \quad (15)$$

Then

$$C_\ell = C\left[\ell \sqrt{\frac{2k}{\pi R}}\right] \quad , \quad (16)$$

$$C_{\Delta_1} = C\left[\left(\Delta_1 - \frac{1}{2} aR_1\right) \sqrt{\frac{2k}{\pi R_1}}\right] \quad , \quad (17)$$

$$C_{\Delta_2} = C\left[\left(\Delta_2 + \frac{1}{2} aR_1\right) \sqrt{\frac{2k}{\pi R_1}}\right] \quad , \quad (18)$$

$$S_\ell = S\left[\ell \sqrt{\frac{2k}{\pi R}}\right] \quad , \quad (19)$$

$$S_{\Delta_1} = S\left[\left(\Delta_1 - \frac{1}{2} aR_1\right) \sqrt{\frac{2k}{\pi R_1}}\right] \quad , \quad \text{and} \quad (20)$$

$$S_{\Delta_2} = S\left[\left(\Delta_2 + \frac{1}{2} aR_1\right) \sqrt{\frac{2k}{\pi R_1}}\right] \quad . \quad (21)$$

It is, however, possible to make a considerably better approximation to the directivity pattern. If the side lobes may be ignored, a suitable functional form is

$$P_o = \exp\left[-K\psi^2/(\phi/2)^2\right] , \quad (22)$$

where

- 1) K and ϕ have been previously defined, and
- 2) ψ is the angular displacement measured from the beam axis.

Since the integration is to be performed over the surface, it is necessary to express Eq. (22) as a function of the surface variables x and y . A useful, analytically tractable approximation to Eq. (22) is

$$P_o = \exp\left[-K\left(\frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2}\right)\right] . \quad (23)$$

This form was suggested by Clay,² and Horton and Muir.³ The directivity pattern given by Eq. (23) represents a single lobe* source whose surface insonification is an ellipse of semimajor axis α , and semiminor axis β . With the introduction of Eq. (23) the integration limits may now be taken as infinite for most practical cases. This consideration can be very important in connection with arbitrary surfaces, since often the integrability of the equations will depend on the extent of the limits.

* It is possible to generalize Eq. (23) to a multi-lobe directivity pattern; for an N -lobe source the equation becomes

$$P_o = \sum_{i=1}^N e^{-K_i \left(\frac{x^2 - \delta_i^2}{\alpha_i^2} + \frac{y^2 - \epsilon_i^2}{\beta_i^2} \right)}$$

In Dwg. AS-70-800, the experimentally measured directivity pattern of the 100 kHz source used in scattering experiments is compared with Eq. (22). Drawing AS-70-809 compares Eqs. (22) and (23) for various source grazing angles

Now the intensity reflected from a plane, pressure release surface can be calculated. Substituting Eqs. (4) and (23) into Eq. (1) and taking the product of p(A) with its complex conjugate gives

$$I_p = p^*(A)p(A) = \left(\frac{k \sin \theta_r}{2\pi r_{oo} r_{lo}} \right)^2 \iiint_{-\infty}^{+\infty} \iiint_{-\infty}^{+\infty} e^{-K \left(\frac{x^2 + x'^2}{\alpha^2} + \frac{y^2 + y'^2}{\beta^2} \right)} e^{i \left[ka(x-x') + \frac{k}{R_1} (x^2 - x'^2) + \frac{k}{R} (y^2 - y'^2) \right]} dx dy dx' dy' \quad (24)$$

Make the transformations $\delta = x-x'$, $\eta = y-y'$, $\bar{x} = (x+x')/2$, and $\bar{y} = (y+y')/2$ then the integrations follow easily, yielding

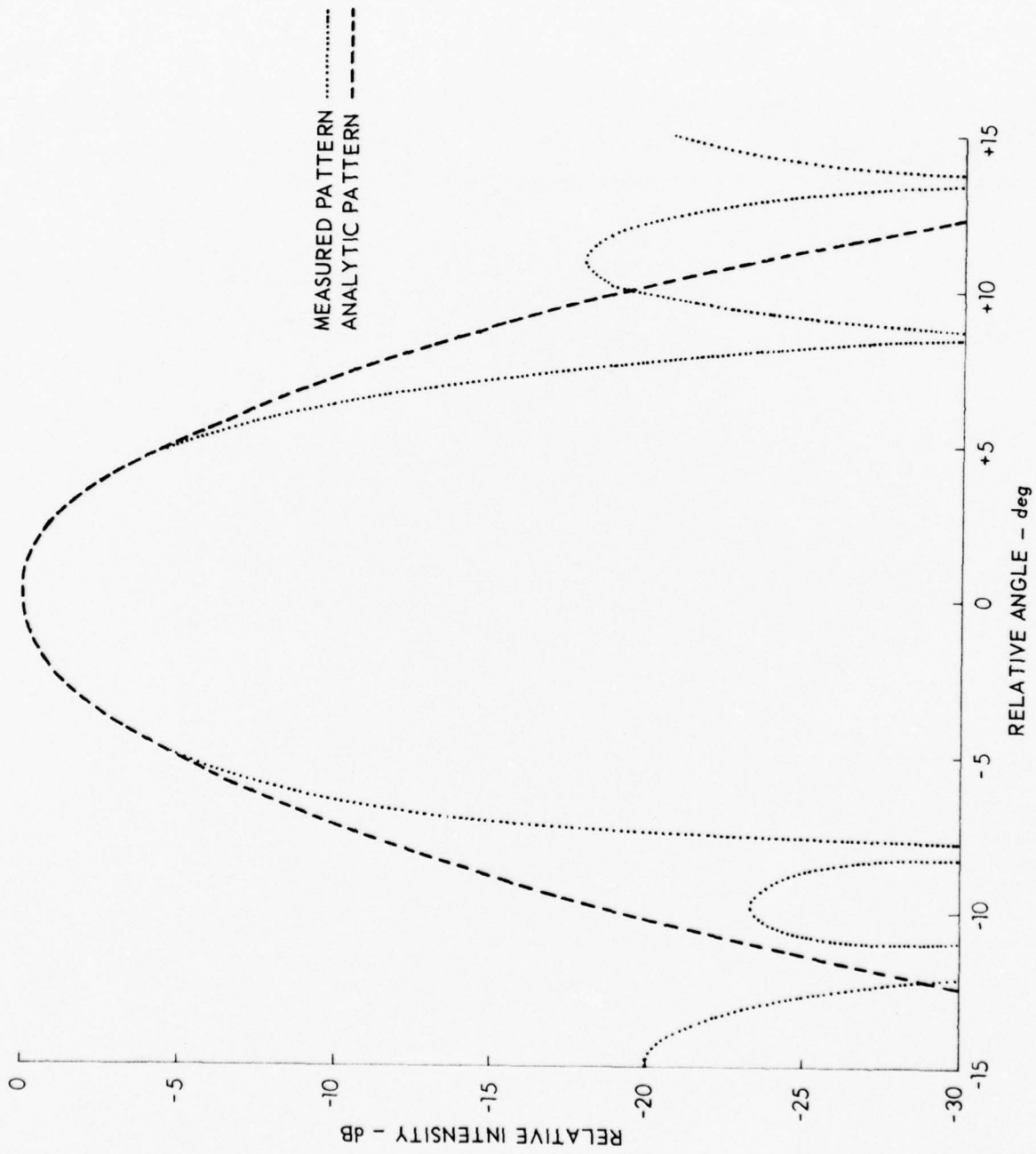
$$I_p = \frac{\exp \left[-KR_1^2 a^2 / 2\alpha^2 (1+P) \right]}{(r_{oo} + r_{lo})(r_{oo} + r_{lo} \sin^2 \theta_i / \sin^2 \theta_r) \sqrt{(1+P)(1+Q)}} \quad (25)$$

where

$$P = \left(KR_1 / k\alpha^2 \right)^2, \text{ and}$$

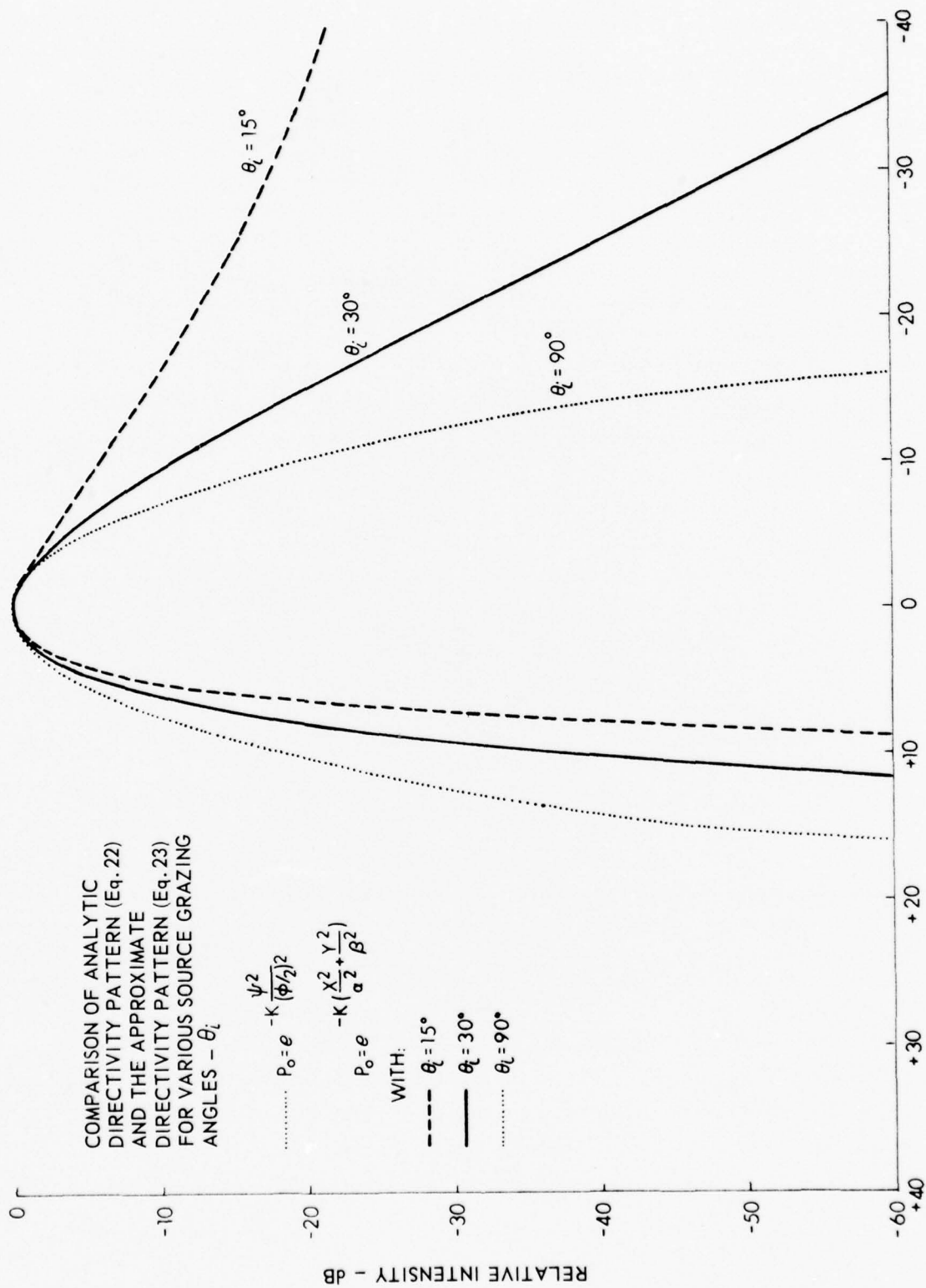
$$Q = \left(KR / k\beta^2 \right)^2.$$

P and Q are approximately zero for the case of interest here. (This is merely a restatement of $kr \gg 1$.)



COMPARISON OF MEASURED DIRECTIVITY PATTERN
AND ANALYTIC DIRECTIVITY PATTERN

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AS - 70 - 800
PJW - ORS
7 - 3 - 70

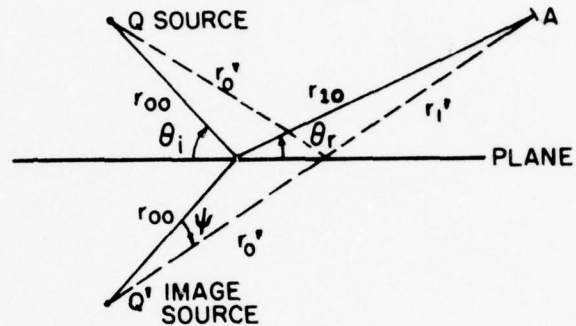


RELATIVE ANGLE FROM BEAM AXIS - deg

The solution obtained using image techniques is

$$I_p^{\text{image}} = \frac{P_o^2}{(r_o' + r_l')^2} = \frac{\exp[-2K\psi^2 / (\phi/2)^2]}{(r_o' + r_l')^2}, \quad (26)$$

where ψ is now the angle to the receiver measured from the beam axis of the image source. The geometry is given in the accompanying diagram. Through simple geometry, ψ is found in terms of r_{oo} , r_{lo} , θ_i , and θ_r :



$$\psi = \text{Arctan} \left[\frac{r_{lo} \sin(\theta_i - \theta_r)}{r_{oo} + r_{lo} \cos(\theta_i - \theta_r)} \right], \quad (27)$$

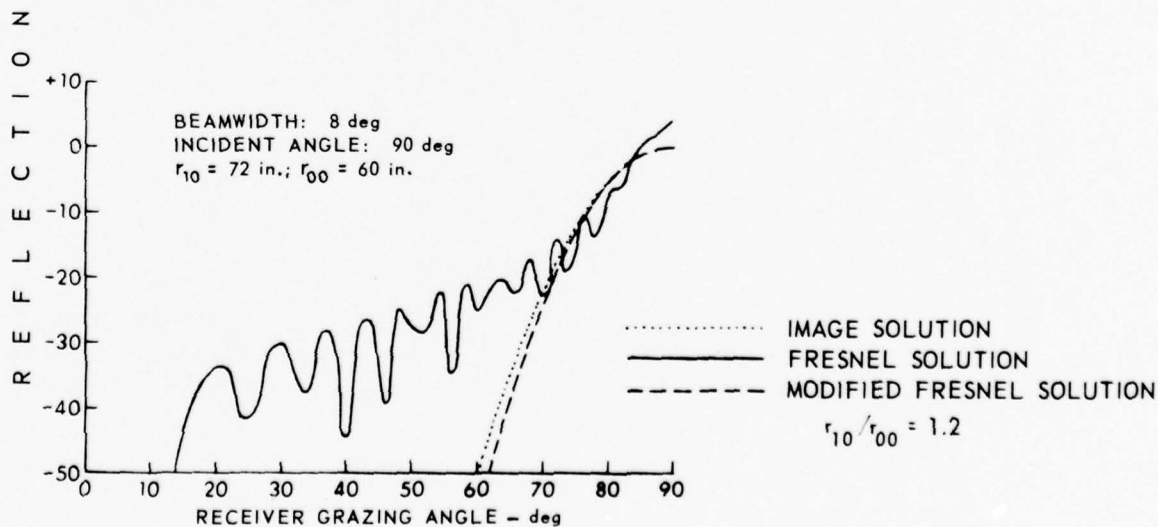
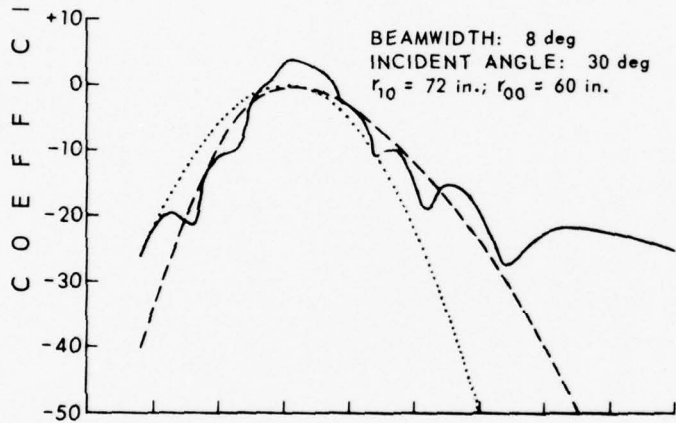
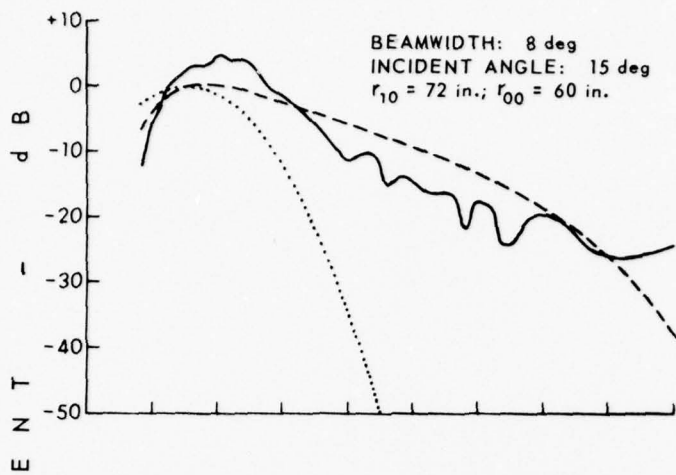
and by the Law of Cosines

$$(r_o' + r_l')^2 = r_{oo}^2 + r_{lo}^2 + 2r_{oo} r_{lo} \cos(\theta_i - \theta_r). \quad (28)$$

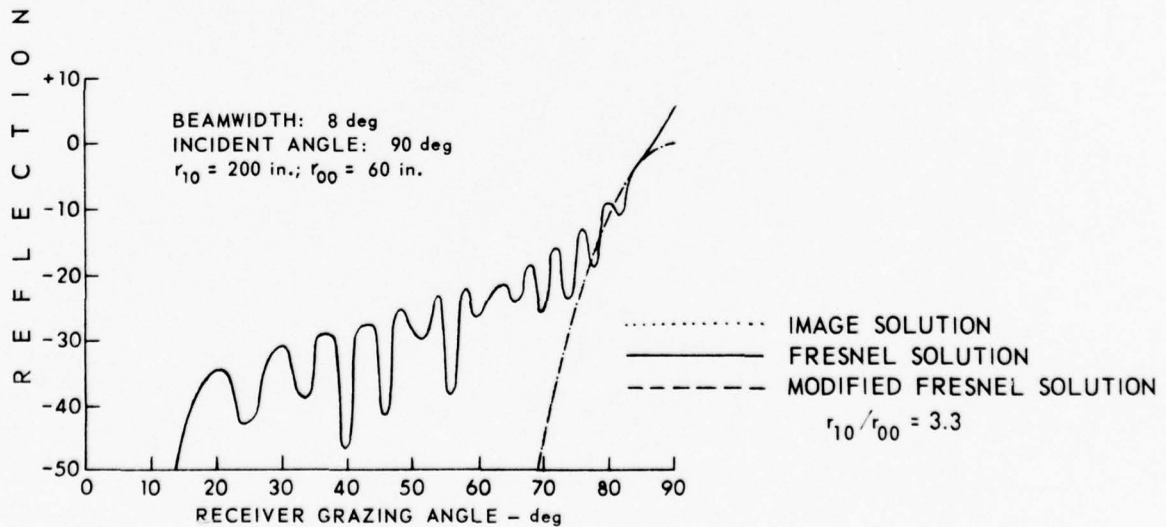
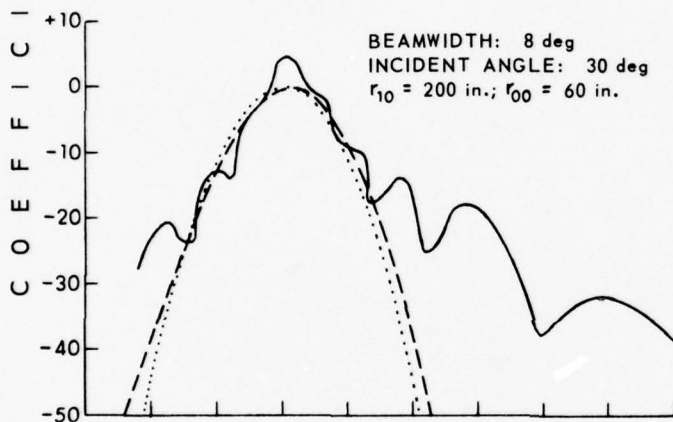
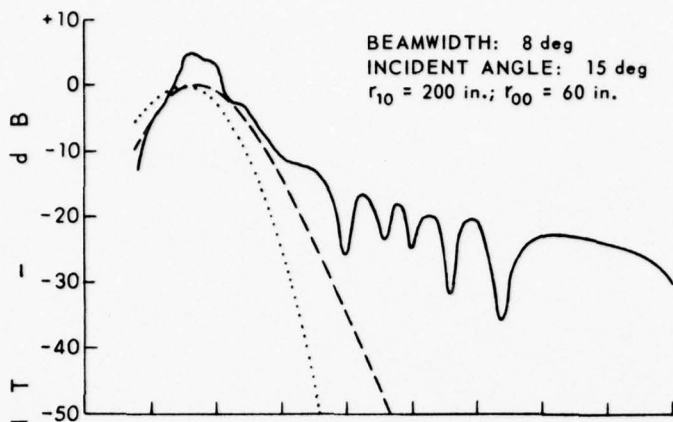
In the specular direction, examination of Eqs. (25) and (26) reveals

$$I_p = I_p^{\text{image}} = \frac{1}{(r_{oo} + r_{lo})^2}. \quad (29)$$

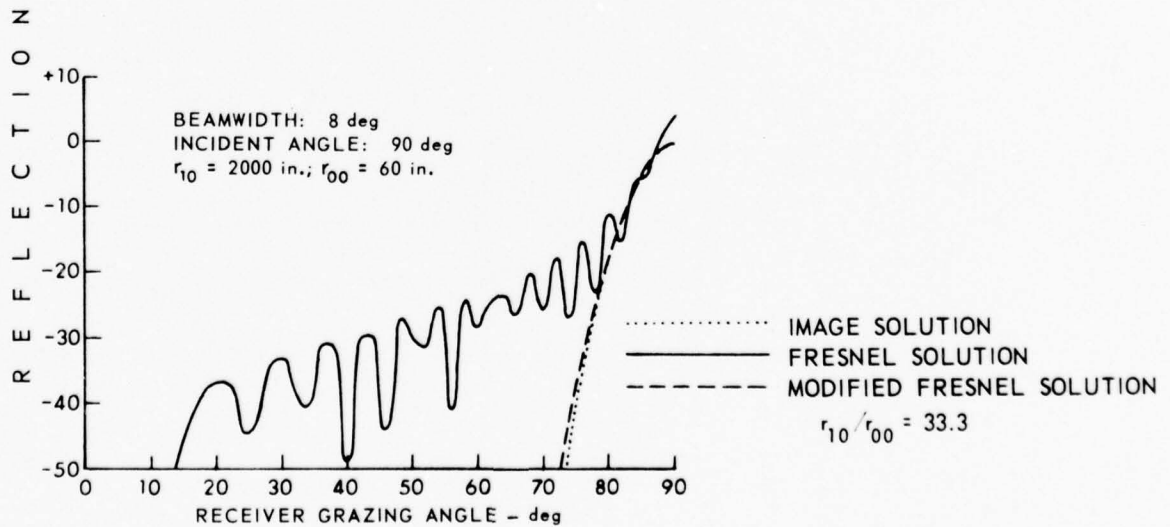
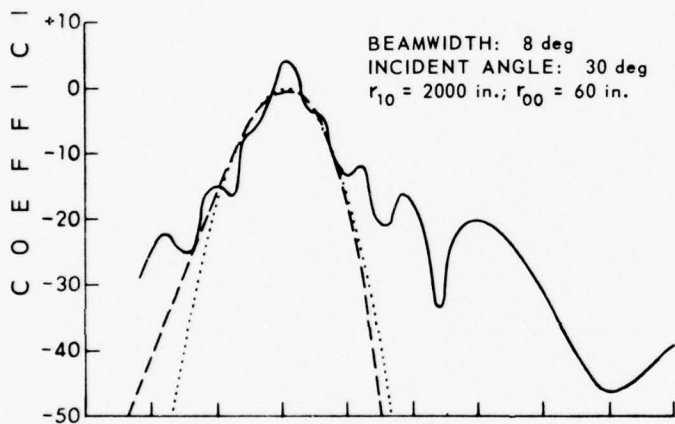
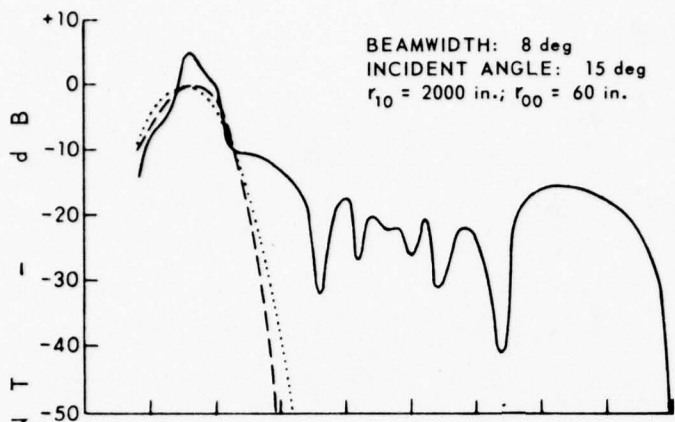
The image solution Eq. (26) is compared with Eqs. (13) and (25) for various source grazing angles and receiver ranges in Dwgs. AS-70-801, AS-70-802, and AS-70-803. The Fresnel solution with the realistic directivity pattern (Modified Fresnel solution) is clearly superior to



COMPARISON OF THE IMAGE SOLUTION WITH THE FRESNEL
AND MODIFIED FRESNEL SOLUTION FOR A PLANE



COMPARISON OF THE IMAGE SOLUTION WITH THE FRESNEL
 AND MODIFIED FRESNEL SOLUTION FOR A PLANE



COMPARISON OF THE IMAGE SOLUTION WITH THE FRESNEL AND MODIFIED FRESNEL SOLUTION FOR A PLANE

the Fresnel solution with an aperture type directivity pattern (Fresnel solution). It is interesting to note that at low grazing angles and for receiver ranges of the same magnitude as the source range, the distortion in the Modified Fresnel solution corresponds closely to the distortion in the directivity pattern (Eq. 23), but is reflected. As r_{10}/r_{00} increases, the distortion in the Modified Fresnel solution decreases until for large r_{10}/r_{00} the Modified Fresnel is nearly equal to the image solution.

II. CALCULATION OF REFLECTION COEFFICIENTS
FOR ROUGH ATTENUATING BOTTOMS

In this section the Rayleigh reflection coefficient is modified to account for bottom roughness. The method of Mackenzie⁴ is followed so that the effects of roughness will be easily observable. From Dwg. AS-70-795 it is obvious that to include the effects of roughness it is only necessary to express the local incident and refracted angles, ϕ_1 and ϕ_2 , in terms of the angles from the mean plane, θ_1 and θ_2 , at each point on the surface. These are related through the angle β which, for a randomly rough surface, will be a stochastic variable. From the drawing it is clear that

$$\phi_1 = \theta_1 + \beta \quad , \quad \phi_2 = \theta_2 + \beta \quad . \quad (30)$$

It is assumed that at any point on the surface, the reflection coefficient is given by the usual Rayleigh coefficient referenced to the tangent plane. Following Mackenzie,⁴ this expression is

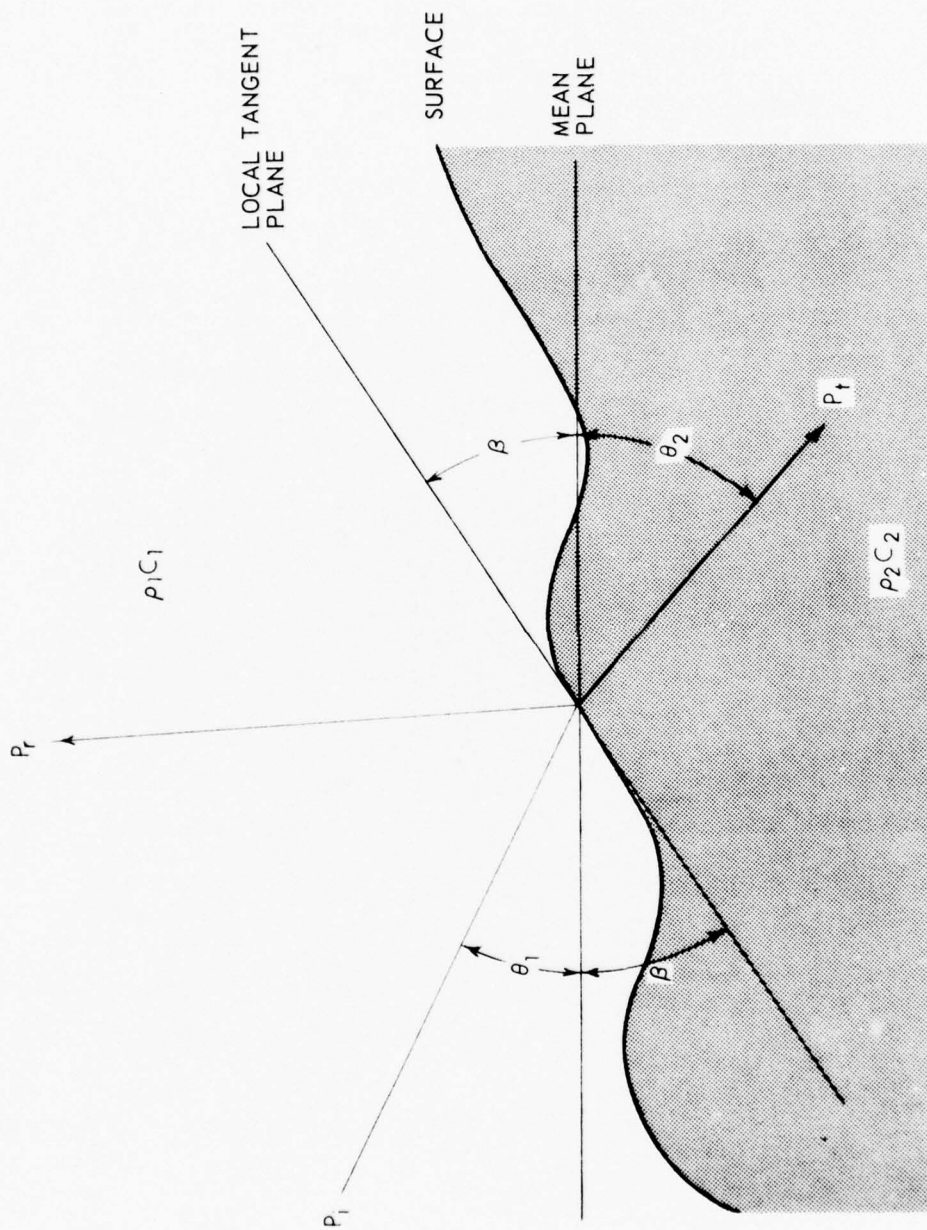
$$R = \frac{p_r}{p_i} = \frac{1-S}{1+S} \quad , \quad (31)$$

where

$$S = \frac{\rho_1 c_1 \sin \phi_2}{\rho_2 c_2 \sin \phi_1} \quad , \quad (32)$$

so that

$$R = \frac{\rho_2 c_2 \sin \phi_1 - \rho_1 c_1 \sin \phi_2}{\rho_2 c_2 \sin \phi_1 + \rho_1 c_1 \sin \phi_2} \quad . \quad (33)$$



PENETRABLE ROUGH SURFACE GEOMETRY

Using Eq. (30) in Eq. (33) and the appropriate trigonometric identities gives

$$R = \frac{\rho_2 c_2 (\sin\theta_1 \cos\beta + \cos\theta_1 \sin\beta) - \rho_1 c_1 (\sin\theta_2 \cos\beta + \cos\theta_2 \sin\beta)}{\rho_2 c_2 (\sin\theta_1 \cos\beta + \cos\theta_1 \sin\beta) + \rho_1 c_1 (\sin\theta_2 \cos\beta + \cos\theta_2 \sin\beta)} \quad (34)$$

Now let

$$\eta = \tan\beta \quad , \quad (35)$$

so that η is the slope at the given point on the surface, and let

$$\begin{aligned} A_- &= \rho_2 c_2 \sin\theta_1 - \rho_1 c_1 \sin\theta_2 \quad , \\ A_+ &= \rho_2 c_2 \sin\theta_1 + \rho_1 c_1 \sin\theta_2 \quad , \\ B_- &= \rho_2 c_2 \cos\theta_1 - \rho_1 c_1 \cos\theta_2 \quad , \text{ and} \\ B_+ &= \rho_2 c_2 \cos\theta_1 + \rho_1 c_1 \cos\theta_2 \quad . \end{aligned} \quad (36)$$

This gives

$$R = \frac{A_- + \eta B_-}{A_+ + \eta B_+} = \frac{(A_- + \eta B_-)}{A_+} \left(1 + \frac{B_+}{A_+} \eta \right)^{-1} \quad (37)$$

Expanding this in terms of η gives

$$R = \frac{A_-}{A_+} + \left(\frac{A_- B_+ - A_+ B_-}{A_+^2} \right) \sum_{K=1}^{\infty} (-1)^K \left(\frac{B_+}{A_+} \right)^{K-1} \eta^K \quad , \quad (38)$$

where A_-/A_+ is the Rayleigh coefficient for a plane surface.

The parameter S may now be redefined in terms of the angles θ_1 and θ_2 as

$$S' = \frac{\rho_1 c_1 \sin \theta_2}{\rho_2 c_2 \sin \theta_1} \quad (39)$$

After a little algebraic manipulation the Rayleigh coefficient becomes

$$R = \left| \frac{1-S'}{1+S'} \right| + \left\{ \frac{2S' \left(\frac{\tan \theta_1}{\tan \theta_2} - 1 \right)}{\tan \theta_1 [1+S']^2} \right\} \sum_{K=1}^{\infty} (-1)^K \eta^K \left\{ \tan \theta_1 \frac{\left[\frac{\tan \theta_1}{1 + \frac{\tan \theta_1}{\tan \theta_2} S'} \right]^{K-1}}{[1+S']^K} \right\} \quad (40)$$

Since η is a random variable, to find $\langle R \rangle$ it is only necessary to replace η by $\langle \eta \rangle$ in the series above. It is assumed that η has a zero mean Gaussian distribution. This implies that all odd moments $\langle \eta^{2K+1} \rangle$ are zero.

To include the effects of attenuation in the bottom it is now necessary to allow S' to be complex:

$$S' = x + iy \quad (41)$$

The quantities x and y are related to Mackenzie's parameters g and h by

$$x + iy = \frac{h + ig}{\sigma \sin \theta_1} \quad (42)$$

where σ is the impedance ratio at the boundary. If only the first nonzero term of the series is retained, $\langle R \rangle$ may be put in the form

$$\langle R \rangle = \frac{A' + iB'}{C' + iD'} \quad (43)$$

where

$$\begin{aligned}
 A' &= \left\{ \left[(1+x)^2 - y^2 \right] (1-x) + 2y^2(1+x) + 2\langle \eta^2 \rangle \left(\frac{\tan \theta_1}{\tan \theta_2} - 1 \right) \right. \\
 &\quad \left. \cdot \left[x + \frac{\tan \theta_1}{\tan \theta_2} (x^2 - y^2) \right] \right\} , \\
 B' &= \left\{ -y \left[(1+x)^2 - y^2 \right] + 2y^2(1-x^2) + 2\langle \eta^2 \rangle \left(\frac{\tan \theta_1}{\tan \theta_2} - 1 \right) \left[y + \frac{\tan \theta_1}{\tan \theta_2} 2xy \right] \right\} , \\
 C' &= \left\{ \left[(1+x)^2 - y^2 \right] (1+x) - 2y^2(1+x) \right\} , \text{ and} \tag{44} \\
 D' &= \left\{ y \left[(1+x)^2 - y^2 \right] + 2y(1+x)^2 \right\} .
 \end{aligned}$$

Finally, the magnitude of the reflection coefficient including attenuation and roughness is given by

$$|\langle R \rangle|^2 = \left(\frac{1}{C'^2 + D'^2} \right)^2 \left[(A'C' + B'D')^2 + (B'C' - A'D')^2 \right] . \tag{45}$$

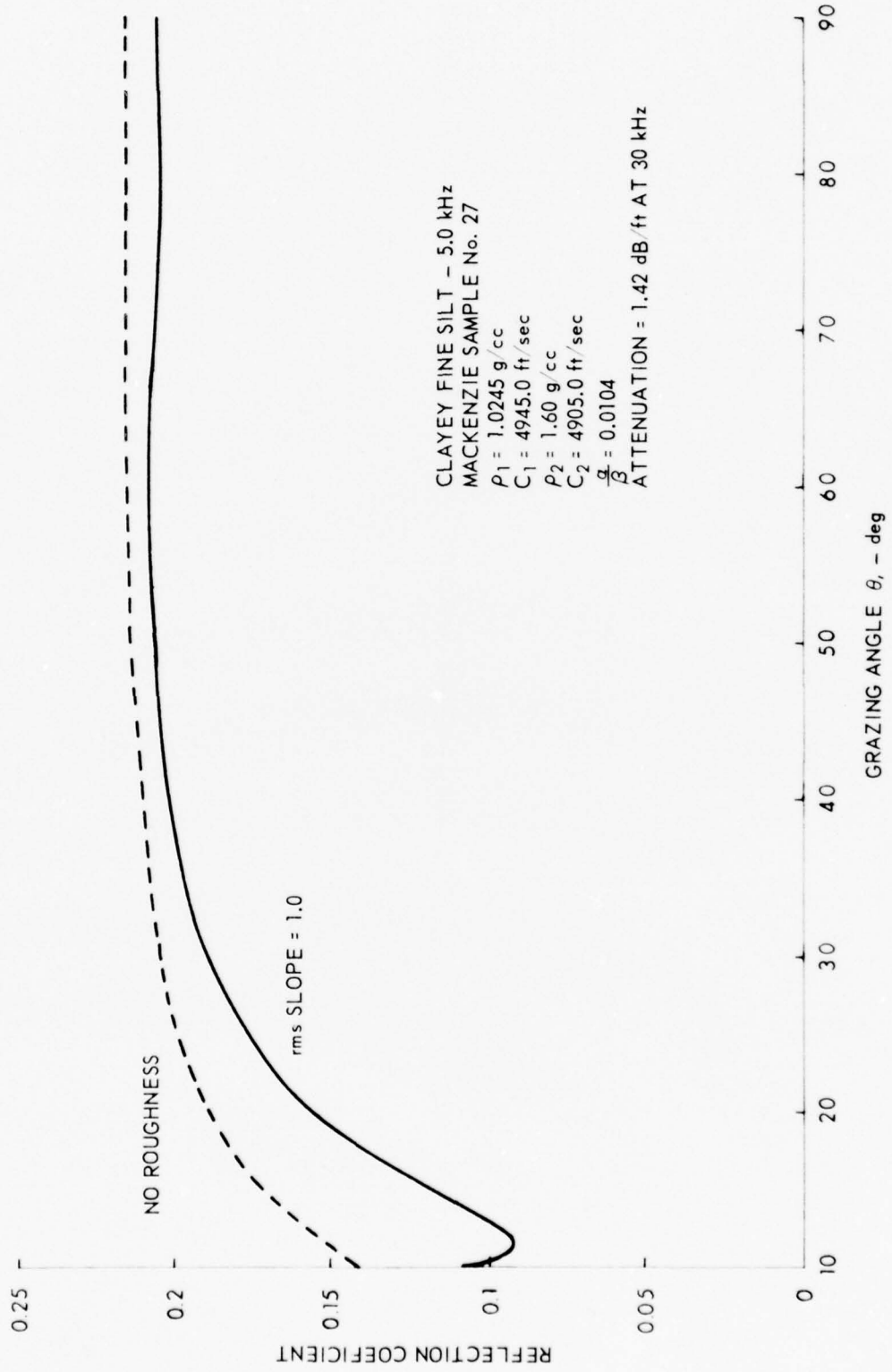
In calculating the RMS slope $\langle \eta^2 \rangle$, it was assumed that the surface was shadowed at all points where the slope was less than the tangent of the incident angle. This assumption results in a simple change in the limits of the integral that defines $\langle \eta^2 \rangle$ and gives a value of $\langle \eta^2 \rangle$ that significantly differs from the true RMS slope only at small grazing angles.

Three separate fluid bottoms with different densities and velocities were considered. These correspond to three of the samples used by Mackenzie. The calculated quantity is the magnitude of the reflection coefficient as given by Eq. (45). This equation does not

include the angular distribution of the scattered field, so it is not an equation for the full scattering coefficient. It simply represents the change in the reflection coefficient resulting from the introduction of random roughness into the geometry of the problem. In subsequent reports the angular distribution will be combined with these results to give an equation for the scattering coefficient for penetrable rough surfaces.

All of the calculated curves were evaluated at 5.0 kHz for ease of comparison with Mackenzie's data. Several different values of RMS slope were calculated, but only the one which produced the largest variations in the reflection coefficient was plotted. Drawings AS-70-796 and AS-70-797 show cases where $c_2/c_1 < 1$. For these cases it is possible to have an angle of intromission; however, neither of the curves shown exhibit this behavior because of the assumed attenuation. Drawing AS-70-798 shows a case where $c_2/c_1 > 1$. Again, the critical angle behavior of this bottom is absent due to the attenuation. It should be noted that where $c_2/c_1 < 1$, the reflection coefficient that includes roughness lies, for the most part, below the plane reflection coefficient, while for $c_2/c_1 > 1$, the opposite is true.

In conclusion, the overall effect of including roughness in the reflection coefficient is very slight. In none of the cases considered was the variation more than about 2 dB for the roughest bottoms and, indeed in most cases, the variation was only a few tenths of a decibel.



CLAYEY FINE SILT - 5.0 kHz
 MACKENZIE SAMPLE No. 27
 $\rho_1 = 1.0245 \text{ g/cc}$
 $C_1 = 4945.0 \text{ ft/sec}$
 $\rho_2 = 1.60 \text{ g/cc}$
 $C_2 = 4905.0 \text{ ft/sec}$
 $\frac{\alpha}{\beta} = 0.0104$
 ATTENUATION = 1.42 dB/ft AT 30 kHz

REFLECTION COEFFICIENT AS A FUNCTION OF GRAZING ANGLE FOR A CLAYEY FINE SILT BOTTOM

SAND - SILT - CLAY - 5.0 kHz
MACKENZIE SAMPLE No. 13

$\rho_1 = 1.0245 \text{ g/cc}$

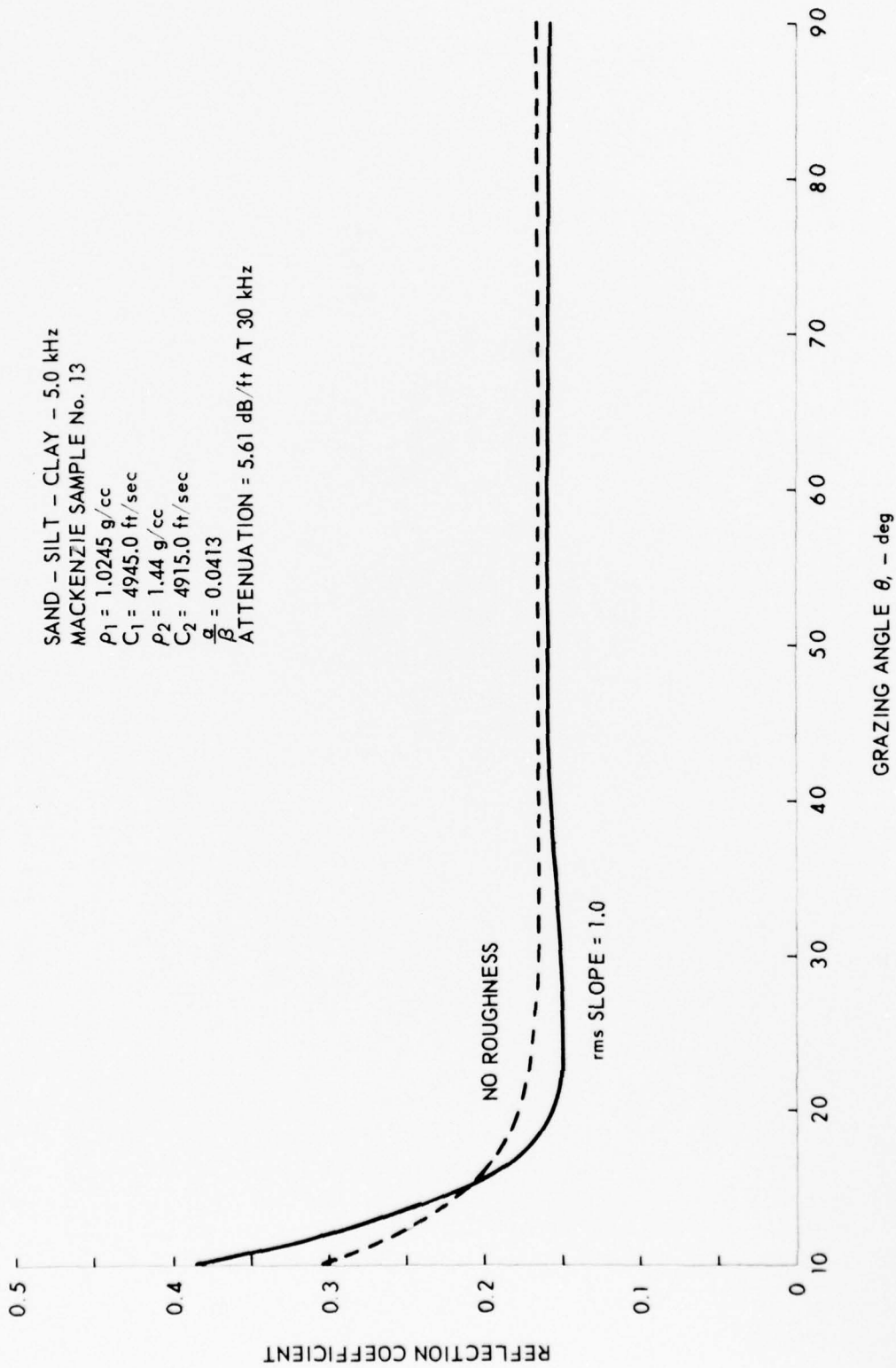
$C_1 = 4945.0 \text{ ft/sec}$

$\rho_2 = 1.44 \text{ g/cc}$

$C_2 = 4915.0 \text{ ft/sec}$

$\frac{\alpha}{\beta} = 0.0413$

ATTENUATION = 5.61 dB/ft AT 30 kHz



ARL - UT
AS-70-797
PJW - WDC
7-7-70

REFLECTION COEFFICIENT AS A FUNCTION OF GRAZING
ANGLE FOR A SAND - SILT - CLAY BOTTOM

SILTY VERY FINE SAND - 5.0 kHz
MACKENZIE SAMPLE No. 28

$$\rho_1 = 1.0245 \text{ g/cc}$$

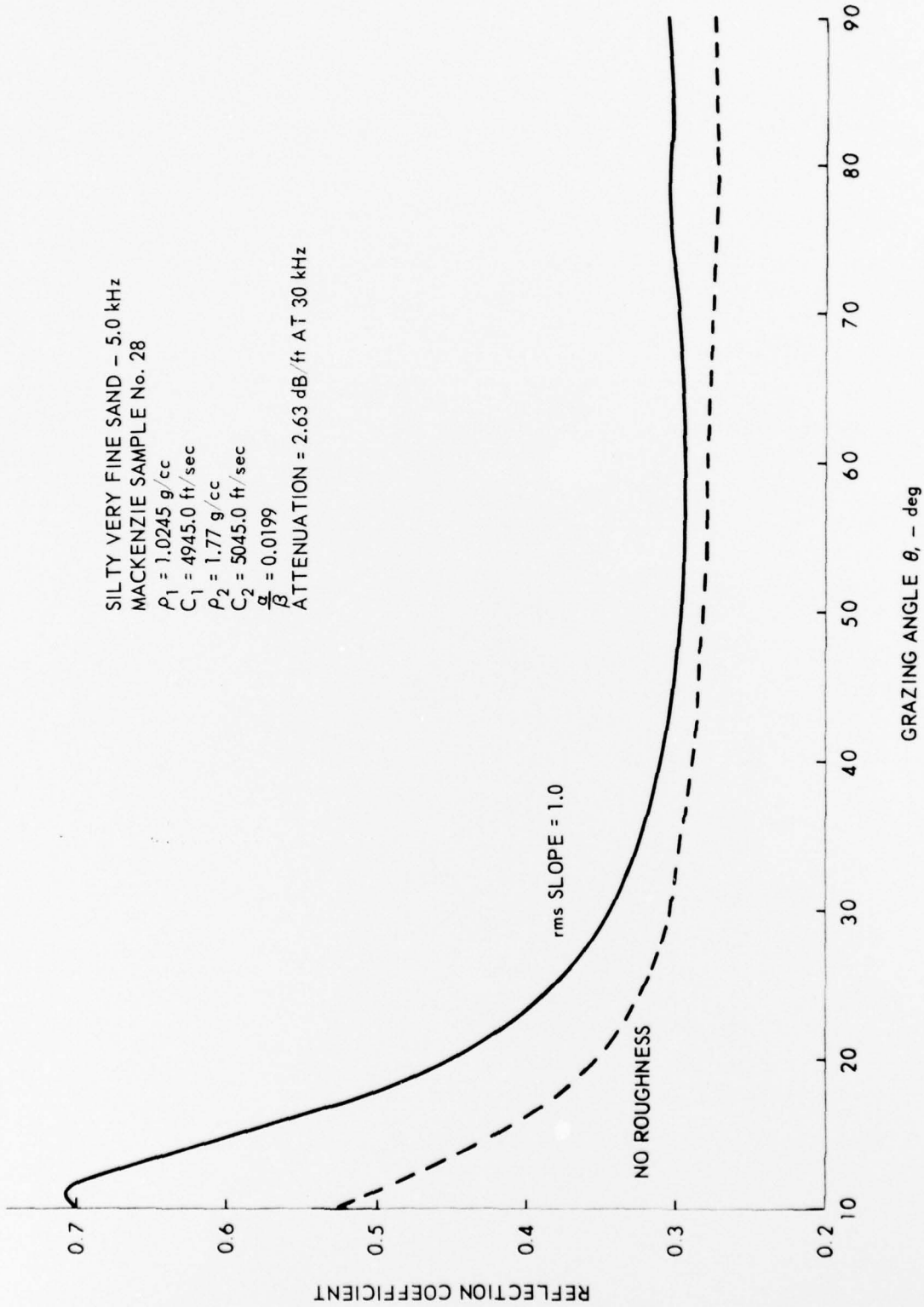
$$C_1 = 4945.0 \text{ ft/sec}$$

$$\rho_2 = 1.77 \text{ g/cc}$$

$$C_2 = 5045.0 \text{ ft/sec}$$

$$\frac{\alpha}{\beta} = 0.0199$$

ATTENUATION = 2.63 dB/ft AT 30 kHz



REFLECTION COEFFICIENT AS A FUNCTION OF GRAZING ANGLE FOR A SILTY VERY FINE SAND BOTTOM

III. ACOUSTIC FIELD IN A SURFACE DUCT WITH A ROUGH BOUNDARY

A. Introduction

The propagation of acoustic waves in a surface duct with an irregular boundary is treated by means of Green's functions. Ray theory has been applied to this problem by various authors,⁵⁻¹⁰ but for shallow water, low-frequency propagation ray theory is inadequate. Normal mode wave theory has also been used¹¹⁻²² in examining this problem. A useful survey of both ray theory and wave approaches has recently been given by Schulkin.²³

The following treatment will be very similar to that of Bucker's²² but will use Green's functions and a different impedance condition in the basic integral. The impedance condition now depends on the nature of the medium as well as the random boundary. A rough boundary can be approximated by a flat boundary with some effective impedance (or reflection coefficient) in the case of wave guide propagation, but the expression for this impedance (or reflection coefficient) will be completely different from the forms for reflection from a single uneven surface.^{14,17,19,20}

When the rough surface is replaced by an impedance condition (or effective reflection coefficient), the propagation problem can be solved by classical methods. As Bucker²² indicates, this approach is useful for the case where both source and receiver are in the duct. This case is referred to as first-order scattering (scattering out of a mode). When the source is in the duct and the receiver is below the

duct, then one must consider second-order scattering (scattering into a mode). The treatments of second-order scattering given by Schweitzer⁷ and Van Ness⁸ are believed to be in error. The present treatment will only consider the first-order effects of the rough boundary.

B. Green's Function Solution

The wave equation for a point monopole source of unit strength, with angular frequency ω , is given by

$$\Delta^2 p - \frac{1}{c^2(z)} \frac{\partial^2 p}{\partial t^2} = -4\pi \delta(\vec{r}-\vec{r}_0) e^{-i\omega t} \quad , \quad (46)$$

where

- 1) p represents the pressure,
- 2) $c(z)$ is the sound velocity (variable in the coordinate z), and
- 3) $\delta(\vec{r}-\vec{r}_0)$ is the three dimensional Dirac delta function.

If cylindrical coordinates (r, θ, z) with azimuthal symmetry are assumed and the time factor of $\exp(-i\omega t)$ is suppressed, then

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial G}{\partial r} \right) + \frac{\partial^2 G}{\partial z^2} + k^2(z)G = -\frac{2}{r} \delta(r) \delta(z-z_0) \quad , \quad (47)$$

where 1) $k(z) = \omega/c(z)$ represents the wave number, and 2) the source is located at $z=z_0$ and $r=0$, as indicated in Dwg. AS-70-767. The vertical depth coordinate z varies from $0 \leq z \leq \infty$, and the range coordinate r varies from $0 < r < \infty$.

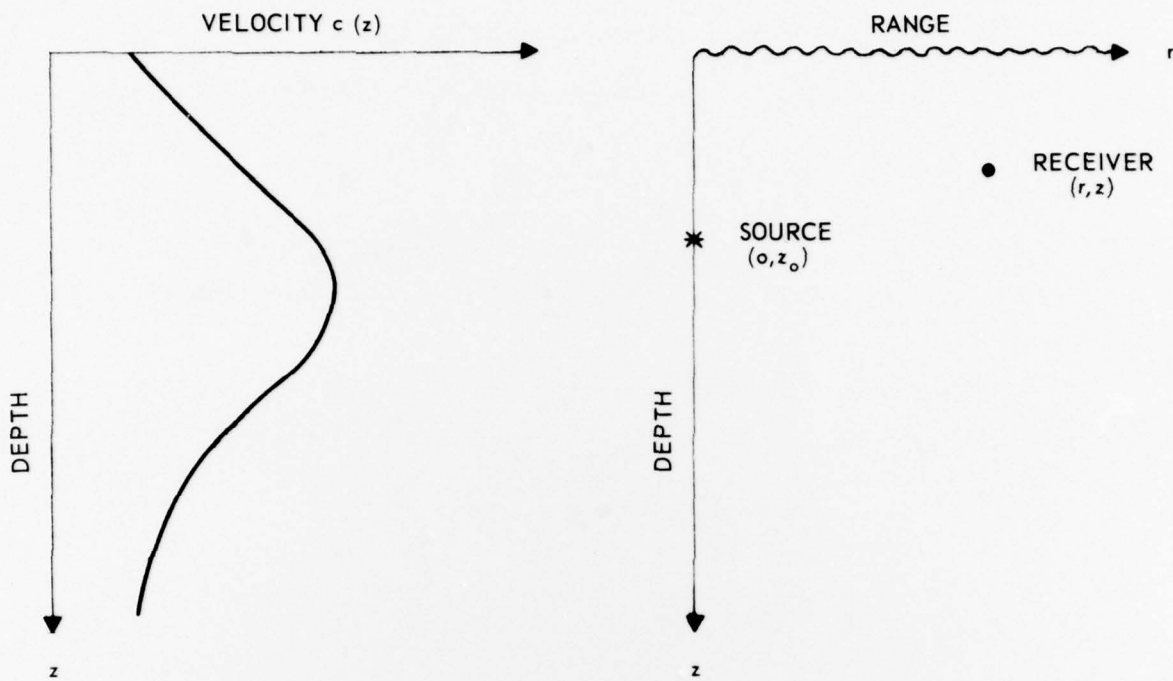


DIAGRAM OF WAVEGUIDE SHOWING LOCATION OF SOURCE AND RECEIVER

The pressure is then $\exp(-i\omega t)$ times the Green's function G of Eq. (47). The boundary conditions imposed on Eq. (47) are

a) G must satisfy a radiation condition for $r \rightarrow \infty$ and $z \rightarrow \infty$.

$$b) \quad G / \left. \frac{\partial G}{\partial z} \right|_{z=0} = \gamma \quad , \quad (48)$$

where γ represents the impedance (or corresponding effective reflection coefficient) condition found by the prescription given by Lysanov.²⁰

In the Green's function approach, Eq. (47) is separated into the following forms:

$$[L_r + \lambda_1 r] G_1(r, \lambda_1) = -2\delta(r) \quad , \quad (49)$$

$$[L_z - \lambda_1] G_2(z, z_0, -\lambda_1) = -\delta(z - z_0) \quad , \quad (50)$$

where the differential operators are defined by

$$L_r = \frac{d}{dr} \left(r \frac{d}{dr} \right) \quad , \quad \text{and} \quad (51)$$

$$L_z = \frac{d^2}{dz^2} + k^2(z) \quad , \quad \text{and} \quad (52)$$

where λ_1 is the separation constant. G_1 and G_2 are defined as the resolvent Green's functions when they satisfy Eq. (49) and Eq. (50), respectively, and are subject to the previously stated boundary conditions. The separation constant is a complex parameter so restricted as to assure a unique solution for Eqs. (49-50).

The Green's function G of Eq. (47) is now given by a complex convolution of G_1 and G_2 :

$$G(r, z, z_0) = \frac{1}{2\pi i} \int_C G_1(r, \lambda_1) G_2(z, z_0, -\lambda_1) d\lambda_1, \quad (53)$$

where the contour C separates the singularities of G_1 and G_2 and can be closed around those singularities of either one which allow a uniqueness condition to be satisfied for G_1 and G_2 .

The solutions to Eq. (49) and Eq. (50) that satisfy the required boundary conditions will be given now. For the $\exp(-i\omega t)$ time factor, these solutions are given by

$$G_1(r, \lambda_1) = i\pi H_0^1(\xi r) (\xi = \lambda_1^{1/2}, 0 < \arg \lambda_1 < 2\pi), \quad (54)$$

and

$$G_2(z, z_0, -\lambda_1) = \frac{n_1(z, \xi) [n_2(z_0, \xi) - X^- n_1(z_0, \xi)]}{W(n_2, n_1)} \quad z_0 < z < \infty, \quad (55)$$

$$G_2(z, z_0, -\lambda_1) = \frac{n_1(z_0, \xi) [n_2(z, \xi) - X^- n_1(z, \xi)]}{W(n_2, n_1)} \quad 0 < z < z_0, \quad (56)$$

where $H_0^1(\xi r)$ is the Hankel function of the first kind, and

$$X^- = \frac{n_2(0, \xi) - \gamma n_2'(0, \xi)}{n_1(0, \xi) - \gamma n_1'(0, \xi)}. \quad (57)$$

The Wronskian of n_2 and n_1 is represented by $W(n_2, n_1)$, where n_1 is a solution to the homogeneous part of Eq. (50) which has outgoing waves

at $z \rightarrow +\infty$, and n_2 which has outgoing waves at $z \rightarrow -\infty$. The primes indicate differentiation with respect to z .

When Eq. (54) and Eq. (55) are used in Eq. (53), the Green's function for Eq. (47) is obtained:

$$G(r, z, z_0) = \int_C \frac{n_1(z_< \xi) [n_2(z_>, \xi) - X^- n_1(z_>, \xi)] H_0^1(\xi r) \xi d\xi}{W(n_2, n_1)}, \quad (58)$$

where $z_<$ and $z_>$ denote the smaller or larger, respectively, of the variables z and z_0 .

When Eq. (58) is integrated by Cauchy's residue theorem, the normal modes (plus any branch line integrals) are obtained. The poles yielding the normal modes are given by the zeroes of the expression

$$[n_1(0, \xi) - \gamma n_1'(0, \xi)].$$

Equation (58) represents the desired formula for the propagation of sound in a surface channel with a rough boundary. It should be noted, however, that because of the formulation, only first-order effects of the rough surface are considered.

Future work will deal with the application of Eq. (58) to specific velocity-depth profiles, the first being the bilinear profile. The results will then be compared with Bucker's,²² who also used the bilinear profile with a different interpretation for the impedance condition.

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21 July 1970

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13. ABSTRACT In Section I it is shown by using the simplest reflecting surface--an infinite, pressure release plane surface-- that at least the Fresnel phase approximation and a reasonable directivity pattern approximation are necessary before accurate predictions in surface reflection and scattering problems can be made. The average Rayleigh reflection coefficient for random penetrable surfaces of various roughnesses is calculated in Section II. In Section III an integral expression is derived for the acoustic field of a point source in a surface duct with a rough boundary. The integral was obtained by Green's functions and includes the rough surface condition by means of an effective impedance condition which also depends on the properties of the medium. (U-[REDACTED])			

14 KEY WORDS	LINK A		LINK B		LINK C	
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Reflection from Plane Surface						
Fresnel Phase Approximation						
Gaussian Directivity Pattern						
Aperture Directivity Pattern						
Fraunhofer Phase Approximation						
Rayleigh-Sommerfeld Formulation						
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