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STRESSES PRODUCED IN A LONG CYLINDER BY A NUCLEAR THERMAL ENVIR--ETC(U)
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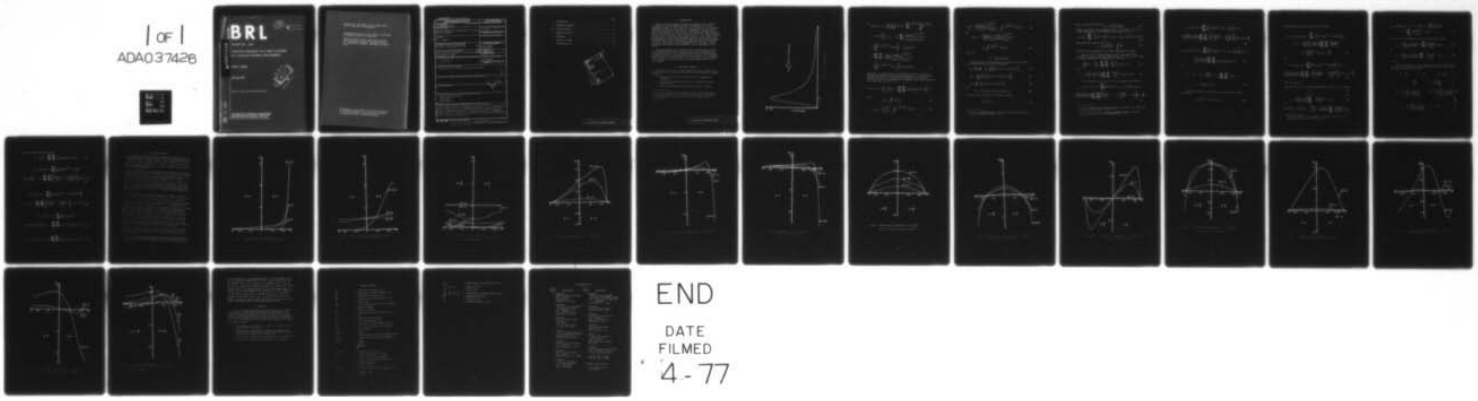

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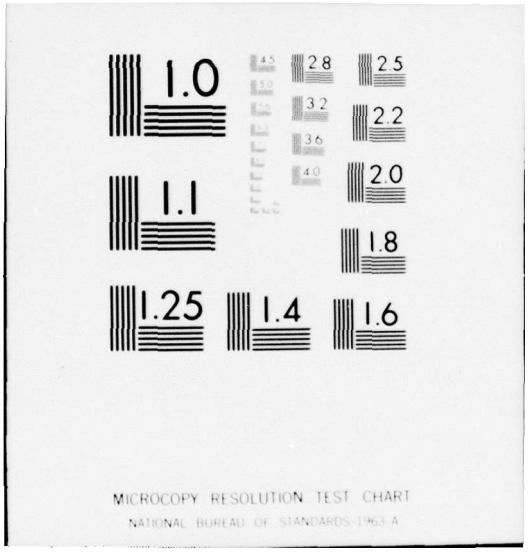
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STRESSES PRODUCED IN A LONG CYLINDER BY A NUCLEAR THERMAL ENVIRONMENT

Ennis F. Quigley

February 1977

USA BALLISTIC RESEARCH LABORATORY
ABERDEEN PROVING GROUND, MARYLAND

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This report contains the plane strain derivation of a set of non-dimensional equations for the quasi-static, thermal stress field in a long, unrestrained, isotropic, homogeneous, solid cylinder whose lateral surface is subjected to heating by a nuclear thermal pulse. Included are the results of a parametric analysis of the stress field as a function of the rise time of the thermal pulse and the total energy of the pulse.

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AB

I. INTRODUCTION

When the lateral surface of a long, unrestrained, solid cylinder is exposed to a nuclear thermal radiation environment such as that indicated by Figure 1, both a transient, two dimensional, non axisymmetric temperature field[1] and a transient, three dimensional, thermal stress field are produced within the cylinder. Knowledge of the magnitude and type of these stresses is essential not only for thermal response analysis but also for the subsequent blast response analysis. This report describes a derivation of a set of non-dimensional equations for the stress field. Such equations will provide a convenient means for calculating the values of these stresses and for performing a parametric study of the equation variables. The results of such a parametric study involving t_o and Q, two of the three nuclear pulse parameters, are also presented in this report.

The stress equations are derived within the framework of classical linear thermoelasticity theory using the plane strain assumption, and the approach is quasi-static in which inertia effects and thermoelastic coupling are neglected. Although the use of the plane strain assumption does not result in a solution valid for all the regions of the cylinder, it does provide a solution valid in the regions away from the ends of the cylinder.

II. TEMPERATURE EQUATION

The equation[2] for the transient temperature field in a long, solid cylinder whose lateral surface is heated by a nuclear thermal environment for which

1. the thermal properties of the cylinder are independent of temperature,
2. convection and radiation heat losses by the cylinder can be neglected, and
3. the initial temperature field in the cylinder is uniform.

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[1] *E.F. Quigley, "Transient Temperatures Produced in Solid Cylinders by a Nuclear Thermal Pulse", BRL Report No. 1937, August, 1976, US Army Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland. (AD #A030703)*

[2] *Ibid.*

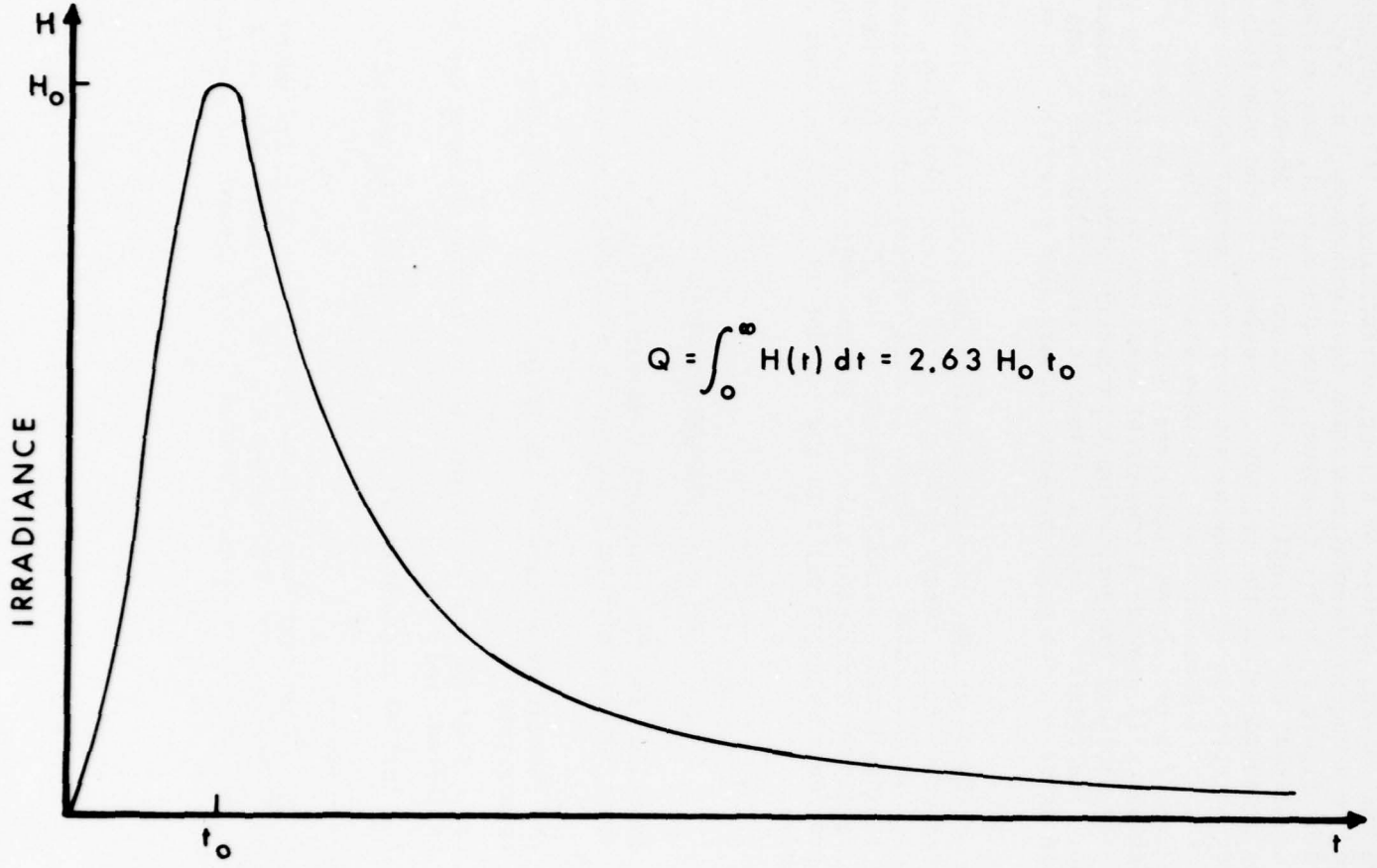

Figure 1. Nuclear Thermal Pulse

is:

$$T(r,\theta,t) = T_o + \frac{2H_o}{\Pi\rho c r_o}\left\{\int_o^t f(t)dt + \sum_{n=1}^{\infty} e^{-\lambda_{on}^2 \frac{\kappa}{\rho c} t}\left[\int_o^t e^{\lambda_{on}^2 \frac{\kappa}{\rho c} t'} \right.\right.$$

$$\left.\left. X \; f(t')dt'\right]\right] J_o(\lambda_{on} r) + \frac{\Pi}{2}\sum_{n=1}^{\infty} \frac{[\lambda_{1n} r_o]^2 \, e^{-\lambda_{1n}^2 \frac{\kappa}{\rho c} t}}{([\lambda_{1n} r_o)^2 - 1] J_1(\lambda_{1n} r_o)}$$

$$X \left[\int_o^t e^{\lambda_{1n}^2 \frac{\kappa}{\rho c} t'} f(t')dt'\right] \quad J_1(\lambda_{1n} r) \cos\theta$$

$$-2\sum_{m=2}^{\infty}\sum_{n=1}^{\infty} \frac{[\lambda_{mn} r_o]^2 \cos(\frac{m\Pi}{2}) \, e^{-\lambda_{mn}^2 \frac{\kappa}{\rho c} t}}{[m^2-1]\,[\lambda_{mn} r_o)^2 - m^2] J_m(\lambda_{mn} r_o)}$$

$$X \left[\int_o^t e^{\lambda_{mn}^2 \frac{\kappa}{\rho c} t'} f(t')dt'\right] J_m(\lambda_{mn} r) \cos m\theta \qquad (1)$$

where $H_o f(t)$ is the functional representation of the curve in Figure 1. Although the temperature field is non-axisymmetric, it is readily seen from (1) that the field is symmetric with respect to θ. Since the temperature equation is needed for the development of the stress equations, it is convenient to rewrite (1) as

$$T(r,\theta,t) = T_o + \frac{2H_o}{\Pi\rho c r_o}\left\{A_{oo}(t) + \sum_{m=0}^{\infty}\sum_{n=1}^{\infty} A_{mn}(t) J_m(\lambda_{mn} r)\cos m\theta\right\} \qquad (2)$$

where

$$A_{oo}(t) = \int_o^t f(t')dt' \qquad (3)$$

$$A_{on}(t) = \frac{e^{-\lambda_{on} \frac{\kappa}{\rho c} t}}{J_o(\lambda_{on} r_o)} \int_o^t e^{\lambda_{on}^2 \frac{\kappa}{\rho c} t'} f(t')dt' \qquad (4)$$

$$A_{1n}(t) = \frac{\pi[\lambda_{1n}r_o]^2 \, e^{-\lambda_{1n}^2 \frac{\kappa}{\rho c} t}}{2[(\lambda_{1n}r_o)^2-1]\, J_1(\lambda_{1n}r_o)} \int_o^t e^{\lambda_{1n}^2 \frac{\kappa}{\rho c} t'} f(t')dt' \qquad (5)$$

and

$$A_{mn}(t) = -\frac{2[\lambda_{mn}r_o]^2 \cos\left(\frac{m\pi}{2}\right) e^{-\lambda_{mn} \frac{\kappa}{\rho c} t}}{[m^2-1]\,[(\lambda_{mn}r_o)^2-m^2]\, J_m(\lambda_{mn}r_o)}$$

$$\times \int_o^t e^{\lambda_{mn}^2 \frac{\kappa}{\rho c} t'} f(t')dt' \qquad (6)$$

for $m \geq 2$.

III. STRESS EQUATIONS

Under the plane strain assumption, the stresses in the cylinder can be determined from the following formulae[3]:

$$T_{rr} = \left\{\frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\right\}\left\{\Omega_1(r,\theta,t) - 2G\,\Omega_2(r,\theta,t)\right\} \qquad (7)$$

$$T_{r\theta} = -\frac{\partial^2}{\partial\theta\partial r}\left[\frac{1}{r}\left\{\Omega_1(r,\theta,t) - 2G\Omega_2(r,\theta,t)\right\}\right] \qquad (8)$$

$$T_{\theta\theta} = \frac{\partial^2}{\partial r^2}\left\{\Omega_1(r,\theta,t) - 2G\,\Omega_2(r,\theta,t)\right\} \qquad (9)$$

and

$$T_{zz} = \nu(T_{rr} + T_{\theta\theta}) - 2G(1+\nu)\alpha\,\Phi(r,\theta,t) \qquad (10)$$

where Ω_1 is the solution of the biharmonic equation,

$$\Delta^2\,\Delta^2\,\Omega_1 = 0 \qquad (11)$$

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[3] *W. Nowacki, Thermoelasticity, Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1962, p. 389.*

Ω_2 is a particular solution of

$$\Delta^2 \Omega_2 = \left(\frac{1 + \nu}{1 - \nu}\right) \alpha \Phi \tag{12}$$

and Φ is the change in the temperature field. A solution of (11)[4] which is well behaved at r=0 and is symmetric with respect to θ is

$$\Omega_1 = b_o r^2 + \sum_{\ell=1}^{\infty} \left[(a_\ell r^\ell + b_\ell r^{\ell+2}) \cos \ell \theta + (c_\ell r\ell + d_\ell r^{\ell+2}) \sin \ell \theta \right] \tag{13}$$

and a particular solution of (12)[5] is

$$\Omega_2 = \left(\frac{1 + \nu}{1 - \nu}\right) \frac{\alpha \kappa}{\rho c} \int_{t_i}^{t} \Phi \, dt \tag{14}$$

when the initial temperature or the final temperature is uniform.

Since the initial temperature and the final temperature of the cylinder is uniform, we can define Φ as either T(t) - T(o) or T(t) - T(∞). By choosing the latter and allowing f(t')=0 for t' > t,

$$\int_{t_\infty}^{t} \Phi \, dt = - \frac{2H_o}{\Pi \kappa r_o} \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} \frac{A_{mn}(t)}{\lambda^2_{mn}} J_m(\lambda_{mn} r) \cos m \theta \tag{15}$$

and

$$\Omega_2 = - \frac{2\alpha H_o}{\Pi \rho c r_o} \left(\frac{1 + \nu}{1 - \nu}\right) \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} \frac{A_{mn}(t)}{\lambda^2_{mn}} J_m(\lambda_{mn} r) \cos m \theta \tag{16}$$

The substitution of (13) and (16) into (7) through (10) results in

$$T_{rr} = 2b_o + 2b_1 r \cos \theta + \sum_{m=2}^{\infty} \left[(m-m^2) a_m r^{m-2} + (2 + m-m^2) b_m r^m \right] \cos m \theta$$

$$+ 2G \left\{ \frac{2\alpha H_o}{\Pi \rho c r_o} \left(\frac{1 + \nu}{1 - \nu}\right) \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} \left[\frac{J'_m(\lambda_{mn} r)}{\lambda_{mn} r} - m^2 \frac{J_m(\lambda_{mn} r)}{(\lambda_{mn} r)^2} \right] A_{mn}(t) \right\} \cos m \theta \tag{17}$$

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[4] *Y.C. Fung, Foundations of Solid Mechanics, Prentic-Hall, Inc., Englewood, Cliffs, New Jersey, 1965, p. 246.*

[5] *B.A. Boley and J.H. Weiner, Theory of Thermal Stresses, John Wiley & Sons, Inc., New York, New York, 1960, p. 83.*

$$T_{r\theta} = 2b_1 r \sin\theta + \sum_{m=2}^{\infty} \left[(m^2-m) a_m r^{m-2} + (m^2+m) b_m r^m \right] \sin m\theta$$

$$+ 2G \left\{ \frac{2\alpha H_o}{\Pi \rho c r_o} \left(\frac{1+\nu}{1-\nu} \right) \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} m \left[\frac{J_m'(\lambda_{mn} r)}{\lambda_{mn} r} - \frac{J_m(\lambda_{mn} r)}{(\lambda_{mn} r)^2} \right] A_{mn}(t) \sin m\theta \right\} \tag{18}$$

$$T_{\theta\theta} = 2b_o + 6b_1 r \cos\theta + \sum_{m=2}^{\infty} \left[(m^2-m) a_m r^{m-2} + (m^2+3m+2) b_m r^m \right] \cos m\theta$$

$$+ \frac{4\alpha G H_o}{\Pi \rho c r_o} \left(\frac{1+\nu}{1-\nu} \right) \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} A_{mn}(t) J_m(\lambda_{mn} r) \cos m\theta \tag{19}$$

and

$$T_{zz} = \nu (T_{rr} + T_{\theta\theta}) - \frac{4\alpha G H_o}{\Pi \rho c r_o} (1+\nu) \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} A_{mn}(t)$$

$$\times J_m(\lambda_{mn} r) \cos m\theta \tag{20}$$

Since the cylinder is unrestrained, the stresses at the surface must satisfy

$$T_{rr}(r_o, \theta, t) = T_{r\theta}(r_o, \theta, t) = 0 \tag{21}$$

By substituting (21) into (17) and (18) one obtains

$$0 = 2b_o + 2b_1 r_o \cos\theta + \sum_{\ell=2}^{\infty} \left[(\ell-\ell^2) a_\ell r_o^{\ell-2} + (2+\ell+\ell^2) b_\ell r_o^{\ell}\right] \cos \ell\theta + 2G \left\{ \frac{2\alpha H_o}{\Pi\rho c r_o} \left(\frac{1+\nu}{1-\nu}\right) \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} \frac{J_m'(\lambda_{mn} r_o)}{\lambda_{mn} r_o} - m^2 \frac{J_m(\lambda_{mn} r_o)}{(\lambda_{mn} r_o)} A_{mn}(t) \cos m\theta \right\} \tag{22}$$

and

$$0 = 2b_1 r_o \sin\theta + \sum_{\ell=2}^{\infty} \left[(\ell^2-\ell) a_\ell r_o^{\ell-2} + (\ell^2+\ell) b_\ell r_o^{\ell}\right] \sin \ell\theta + 2G \left\{ \frac{2\alpha H_o}{\Pi\rho c r_o} \left(\frac{1+\nu}{1-\nu}\right) \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} m \left[\frac{J_m'(\lambda_{mn} r_o)}{\lambda_{mn} r_o} - \frac{J_m(\lambda_{mn} r_o)}{(\lambda_{mn} r_o)^2} \right] A_{mn}(t) \sin m\theta \right\} \tag{23}$$

where $J_m'(\lambda_{mn} r_o) = 0$*

Equations (22) and (23) will be satisfied for all values of ℓ, m, and n if

$$\ell = m \tag{24}$$

$$b_o = 0 \tag{25}$$

$$b_1 = \frac{2\alpha H_o}{\Pi\rho c r_o} \left(\frac{1+\nu}{1-\nu}\right) \sum_{n=1}^{\infty} \frac{J_1(\lambda_{1n} r_o)}{\lambda_{1n} r_o} A_{1n}(t) \tag{26}$$

and

$$(m^2-m) r_o^{m-2} a_m + (m^2-m) r_o^{m} b_m = \frac{4G\alpha H_o}{\Pi\rho c r_o} \left(\frac{1+\nu}{1-\nu}\right) m \sum_{n=1}^{\infty} \frac{J_m(\lambda_{mn} r_o)}{(\lambda_{mn} r_o)^2} A_{mn}(t) \tag{27}$$

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* *This is a consequence of satisfying the boundary conditions in the deviation of the temperature field equation.*

and $$(m-m^2)r_o^{m-2}a_m + (2 + m - m^2)r_o^m b_m = \frac{4G\alpha H_o}{\Pi\rho c r_o}\left(\frac{1+\nu}{1-\nu}\right)$$

$$\times\ m^2 \sum_{n=1}^{\infty} \frac{J_m(\lambda_{mn}r_o)}{(\lambda_{mn}r_o)^2} A_{mn}(t) \qquad (28)$$

for $m \geq 2$. The simultaneous solution of (27) and (28) yields

$$a_m = \frac{2G\alpha H_o}{\Pi\rho c r_o^{m-2}}\left(\frac{1+\nu}{1-\nu}\right)(2+m) \sum_{n=1}^{\infty} \frac{J_m(\lambda_{mn}r_o)}{(\lambda_{mn}r_o)^2} A_{mn}(t) \qquad (29)$$

and

$$b_m = \frac{2G\alpha H_o}{\Pi\rho c r_o^{m+1}}\left(\frac{1+\nu}{1-\nu}\right) m \sum_{n=1}^{\infty} \frac{J_m(\lambda_{mn}r_o)}{(\lambda_{mn}r_o)^2} A_{mn}(t) \qquad (30)$$

for $m \geq 2$.

Both the temperature and stress equations, (2), (17), (18), (19), and (20), can be reduced to a dimensionless form through the use of the following dimensionless quantities:

$$r^* = \frac{r}{r_o} \qquad t^* = \frac{t}{t_o} \qquad \rho_{mn} = \lambda_{mn} r_o$$

$$\beta^* = \frac{\kappa t_o}{\rho c r_o^2} \qquad T^* = \frac{T - T_o}{0.24\,\frac{Q}{\rho c r_o}}$$

$$a_m^* = \frac{a_m}{0.24\,\frac{\alpha G Q}{\rho c r_o}\left(\frac{1+\nu}{1+\nu}\right)} \qquad b_m^* = \frac{b_m}{0.24\,\frac{\alpha G Q}{\rho c r_o}\left(\frac{1+\nu}{1-\nu}\right)} \qquad (31)$$

$$T_{ij}^* = \frac{T_{ij}}{0.24\,\frac{\alpha G Q}{\rho c r_o}\left(\frac{1+\nu}{1-\nu}\right)} \qquad i,j = r,\theta,z$$

The use of these quantities yields

$$T^* = A_{oo}(t^*) + \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} A_{mn}(t^*) J_m(\rho_{mn} r^*) \cos m\theta \qquad (32)$$

$$T^*_{rr} = 2b_1^* r^* \cos\theta + \sum_{m=2}^{\infty} \left[(m-m^2) a_m^* (r^*)^{m-2} + (2+m-m^2) \right.$$

$$\left. \times b_m^* (r^*)^m \right] \cos m\theta + 2 \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} \left[\frac{J_m'(\rho_{mn} r^*)}{\rho_{mn} r^*} - m^2 \frac{J_m(\rho_{mn} r^*)}{(\rho_{mn} r^*)^2} \right] A_{mn}(t^*) \cos m\theta \qquad (33)$$

$$T^*_{r\theta} = 2b_1^* r^* \sin\theta + \sum_{m=2}^{\infty} \left[(m-m^2) a_m^* (r^*)^{m-2} - (m^2+m) b_m^* (r^*)^m \right]$$

$$\times \sin m\theta + 2 \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} m \left[\frac{J_m'(\rho_{mn} r^*)}{\rho_{mn} r^*} - \frac{J_m(\rho_{mn} r^*)}{(\rho_{mn} r^*)} \right] A_{mn}(t^*) \sin m\theta \qquad (34)$$

$$T^*_{\theta\theta} = 6b_1^* r^* \cos\theta + \sum_{m=2}^{\infty} \left[(m^2-m) a_m^* (r^*)^{m-2} \right.$$

$$\left. + (m^2+3m+2) b_m^* (r^*)^m \right] \cos m\theta + 2 \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} A_{mn}(t^*) J_m''(\rho_{mn} r^*) \cos m\theta \qquad (35)$$

$$T^*_{zz} = \nu(T^*_{rr} + T^*_{\theta\theta}) - 2(1-\nu) \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} A_{mn}(t^*) J_m(\rho_{mn} r^*) \cos m\theta \qquad (36)$$

IV. PARAMETRIC ANALYSIS

The values of the non-dimensional temperatures and stresses were calculated for $\beta^* = 0.01$, 0.1, and 1.0; for $t^* = 1.0$ and 10.0; and for $\theta = 0, \frac{\pi}{2}, \pi$, and $\frac{3\pi}{2}$. Because ν appears in the right hand side of (36), $\nu = 0.33$ was used in calculating T^*_{zz}. Figures 2,3 and 4 are the plots of the temperature distributions and Figures 5 through 14 are the plots of the stress distributions.

For a given material of a given radius, the influence of β^* (and consequently, t_o) and Q on the temperature field can be seen from Figures 2,3 and 4. At $t^* = 1.0$ and $t^* = 10.0$, the surface temperature for $\theta = 0$ decreases as β^* increases if Q remains constant. For $\theta = \frac{\pi}{2}$ and consequently, $\frac{3\pi}{2}$, the surface temperature increases as β^* increases at $t^* = 1.0$ and $t^* = 10.0$. It can be concluded from Figures 3 and 4 that, for $\beta^* = 1.0$, the temperature in the cylinder is uniform for all values of Q. For a given β^*, it follows from (31) that the temperature at any point in the cylinder will increase as Q increases and will decrease as Q decreases.

The effect of β^* and Q on the thermal stresses at $t^* = 1.0$ and $t^* = 10.0$ can be seen from Figures 5 through 14. Except for T^*_{rr} on $\theta = \frac{\pi}{2}$ and $\theta = \frac{3\pi}{2}$ (See Figure 8), an increase in β^* results in a decrease in stresses if Q remains constant. Since the lateral surface of the cylinder is traction free, T^*_{rr} and $T^*_{r\theta}$ are zero for $r_o = 1$. Consequently the maximum value of these stresses occurs at an interior point of the cylinder as is shown in Figures 5, 10 and 12. In addition to influencing the maximum value of these stresses, β^* also effects the location of the maximum value. At $t^* = 1.0$ and $t^* = 10.0$, an increase in β^* shifts the maximum value of T^*_{rr} towards the rear lateral surface of the cylinder (See Figures 5 and 12); and an increase in β^* shifts the maximum values of $T^*_{r\theta}$ towards the center of the cylinder (See Figure 10). As in the case of the temperature distribution, it follows from (33), (34), (35) and (36) that, for a given β^*, the stresses at any point in the cylinder will increase as Q increases and will decrease as Q decreases.

Using simple beam theory, this author[5] had shown that the maximum axial stress is simply supported rectangular beams exposed to a nuclear thermal environment did not occur at $t^* = 1.0$ but at some later t^*.

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[5] *E.F. Quigley, "Thermoelastic Response of Aluminum Alloy Beams and Plates Exposed to Nuclear Thermal Environments", BRL-MR 2427, December 1974, US Army Ballistic Research Laboratory, Aberdeen Proving Ground, MD 21005. (AD #B001134L)*

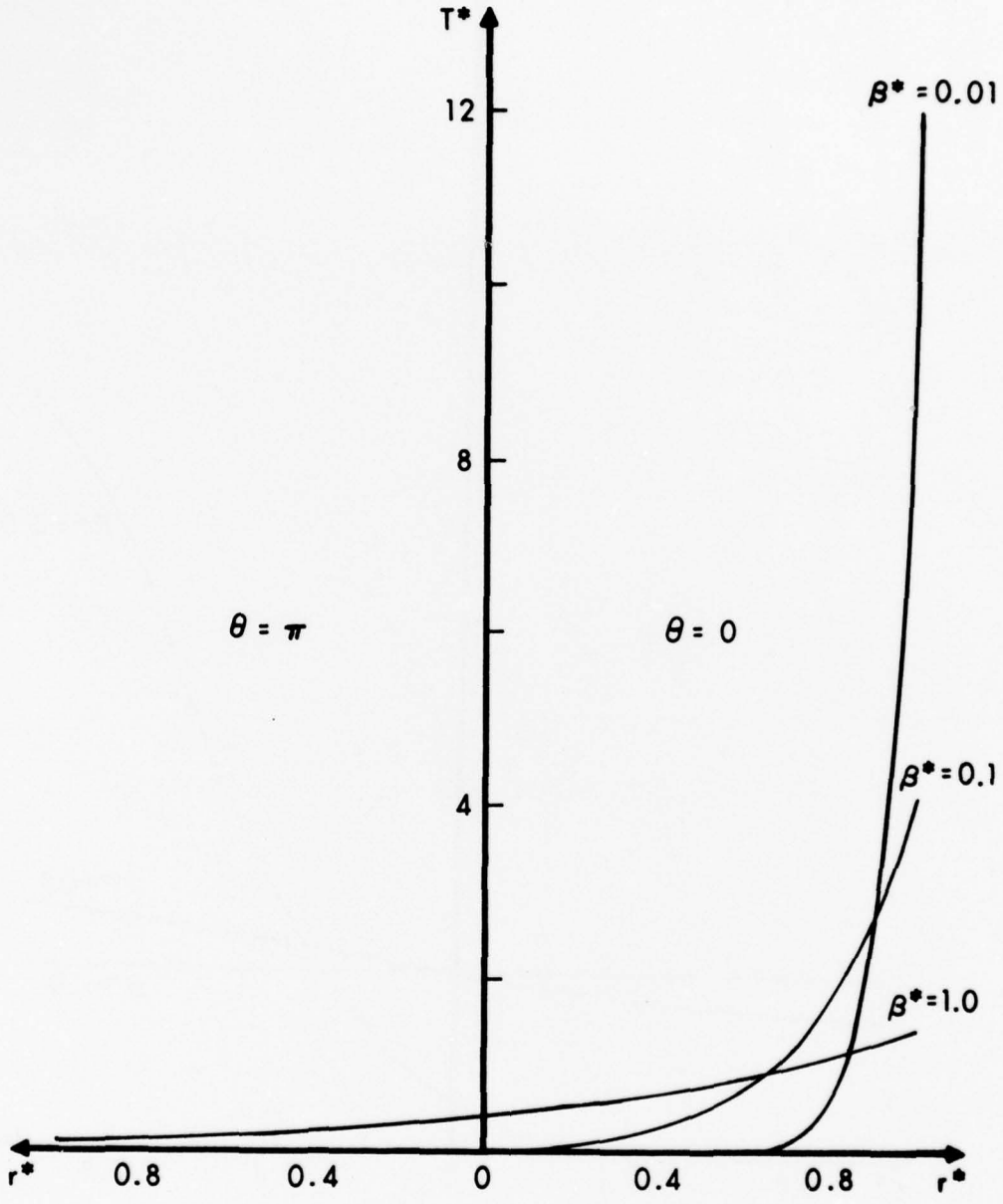

Figure 2. Radial Temperature Distribution on $\theta = 0$ and $\theta = \pi$ for $t^* = 1.0$ and $\beta^* = 0.01$, 0.1, and 1.0

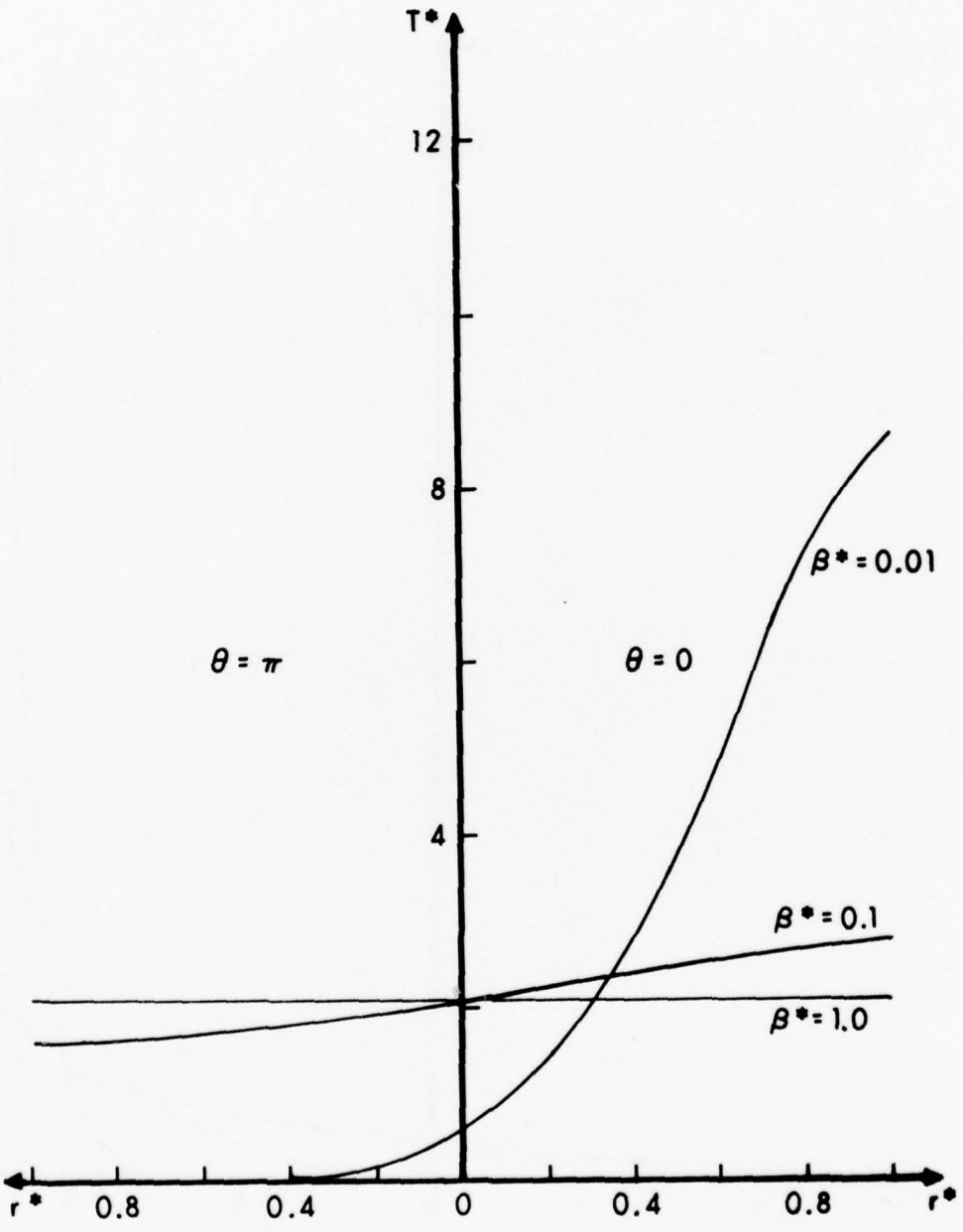

Figure 5. Radial Temperature Distribution on $\theta = 0$ and π for $t^* = 10.0$ and $\beta^* = 0.01$, 0.1, and 1.0

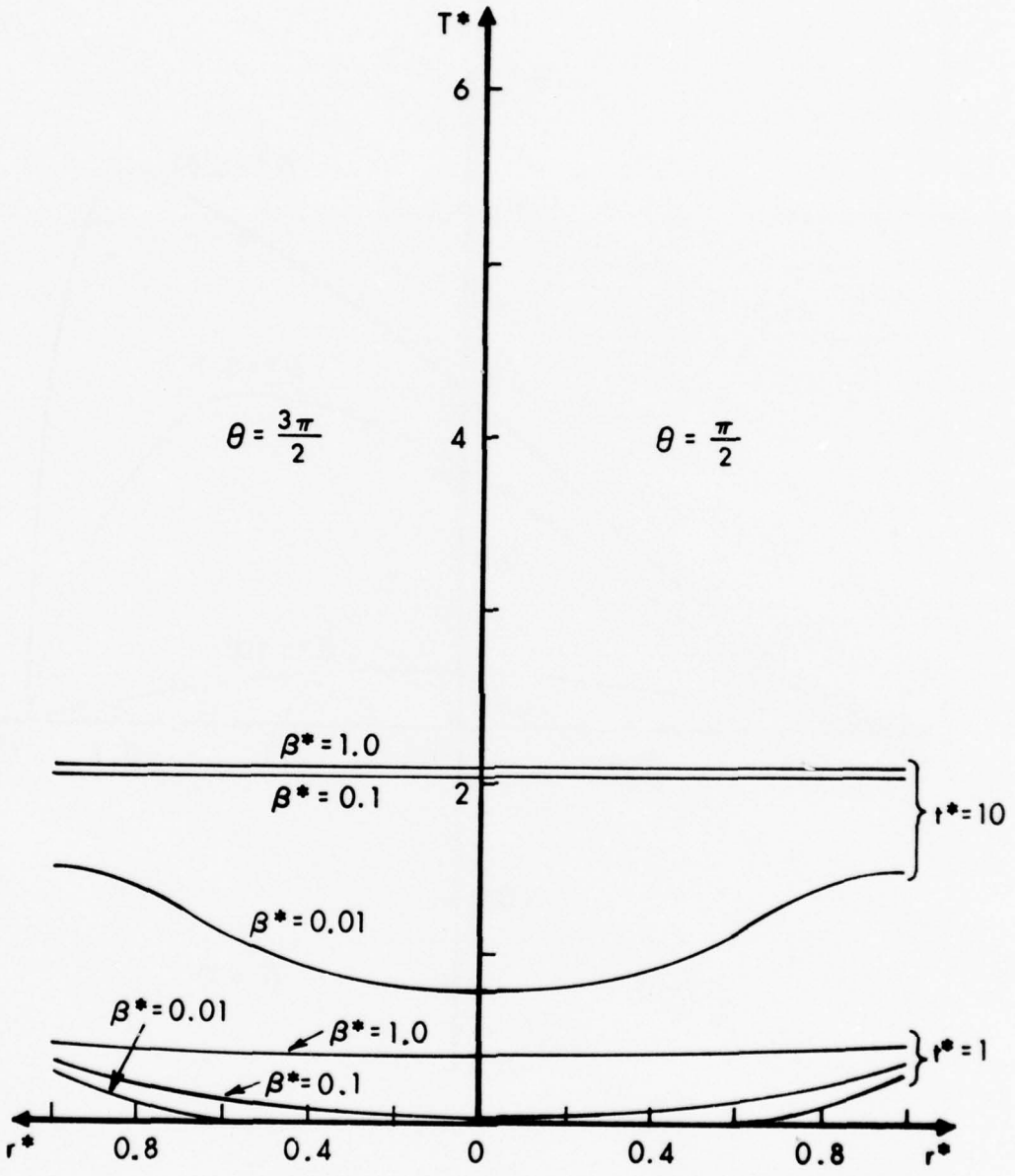

Figure 1. Radial Temperature Distribution on $\theta = \frac{\pi}{2}$ and $\frac{3\pi}{2}$ for $t^* = 1.0$ and 10.0 and $\beta^* = 0.01$, 0.1, and 1.0

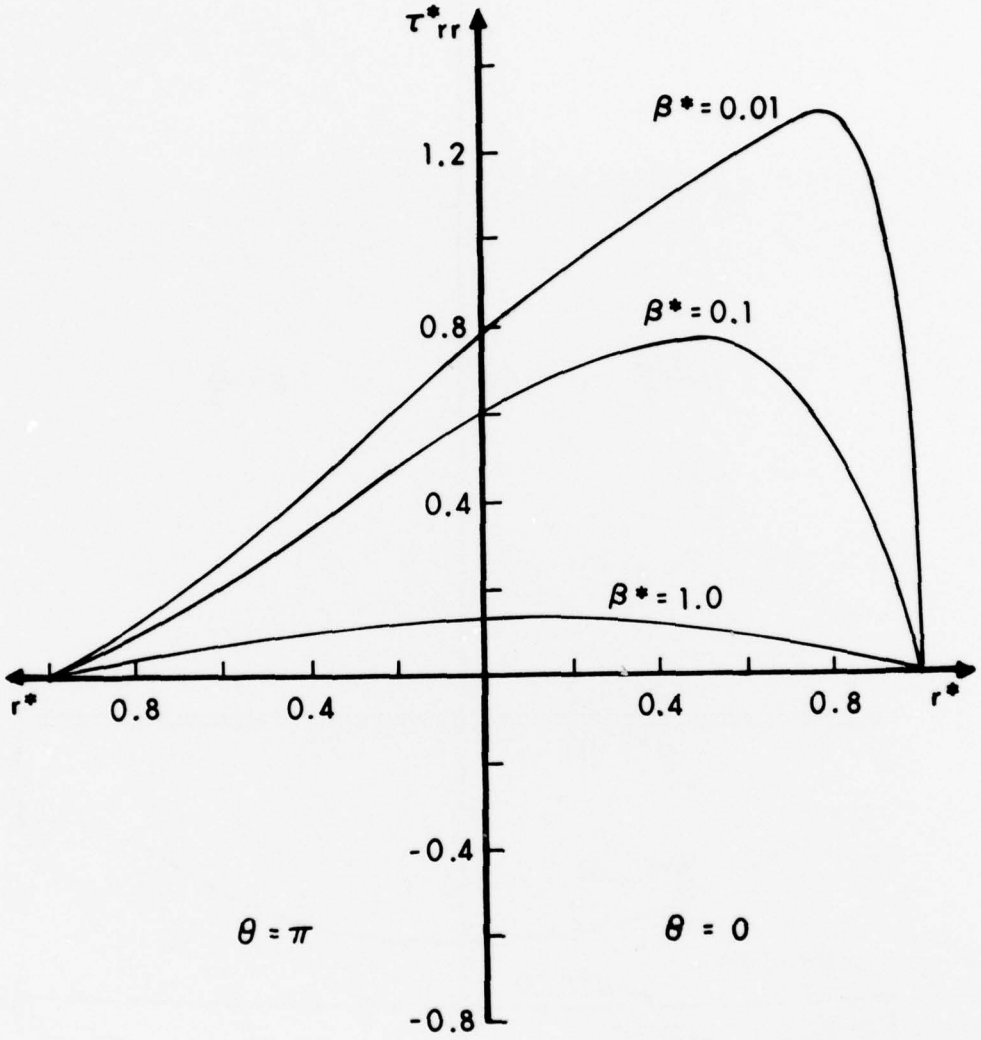

Figure 5. Radial Stress Distribution on $\theta = 0$ and π for $t^* = 1.0$ and $\beta^* = 0.01$, 0.1, and 1.0

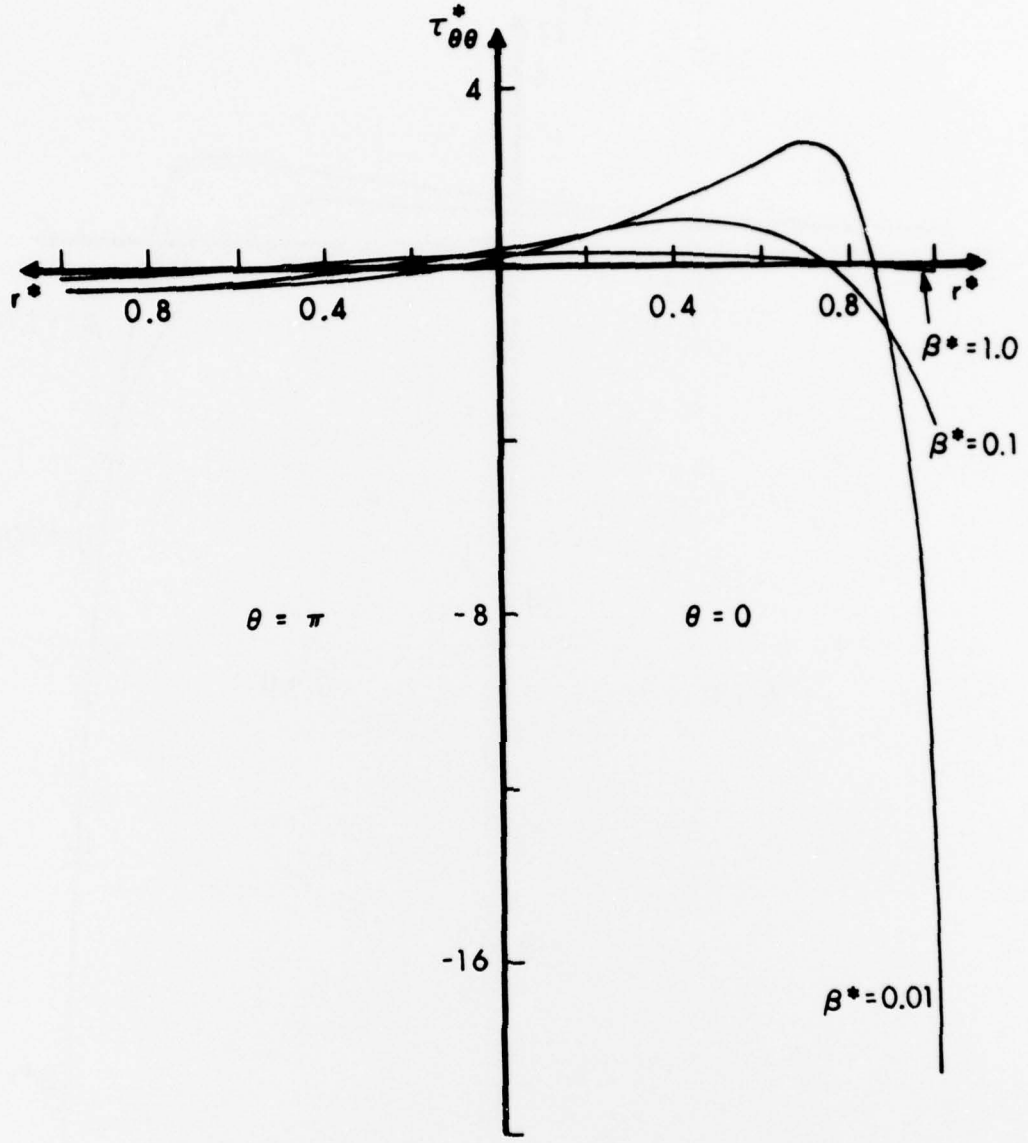

Figure 6. Tangential Stress Distribution on $\theta = 0$ and π for $t^* = 1.0$ and $\beta^* = 0.01$, 0.1, and 1.0

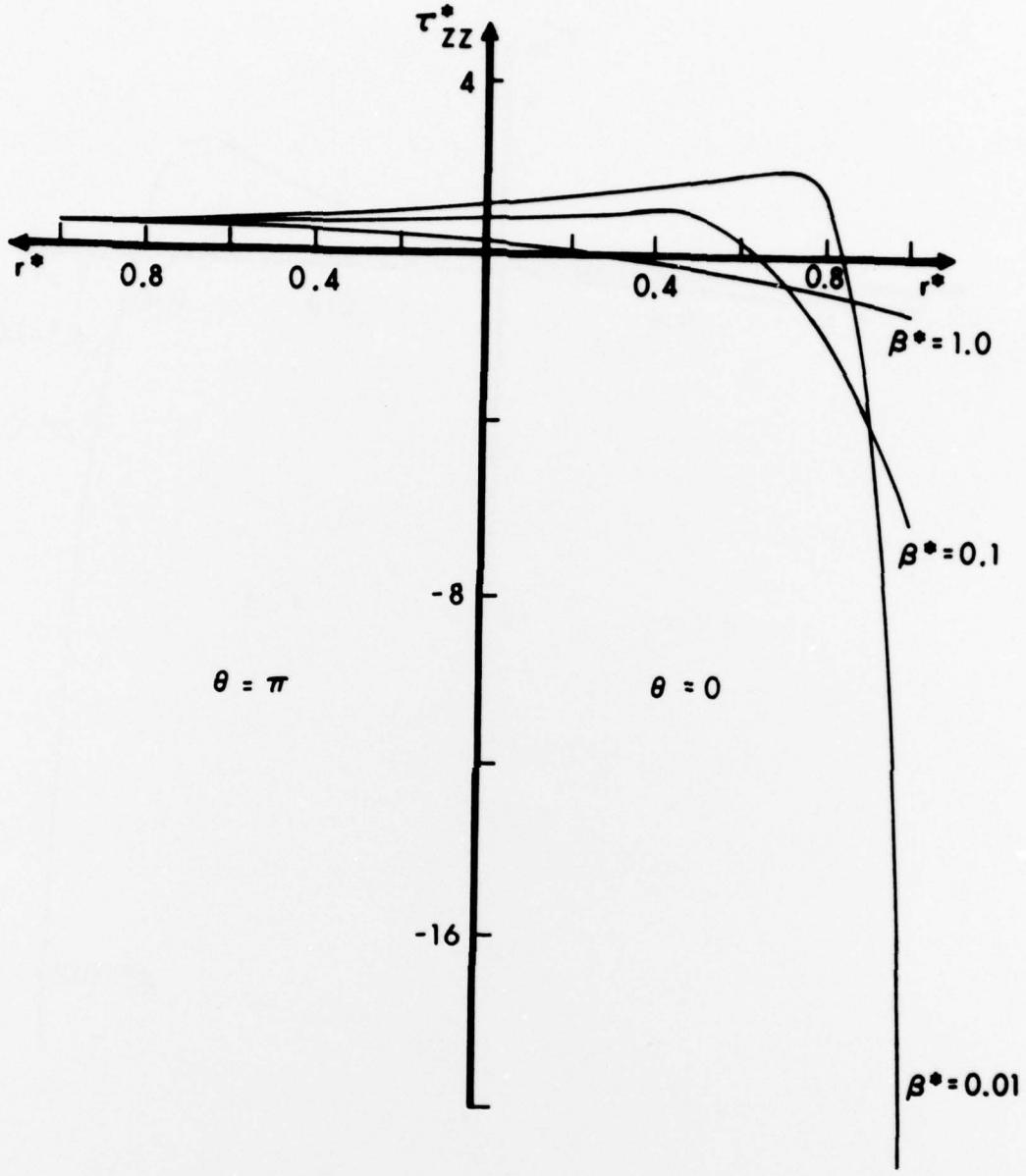

Figure 7. Axial Stress Distribution on $\theta = 0$ and π for $t^* = 1.0$ and $\beta^* = 0.01$, 0.1, and 1.0

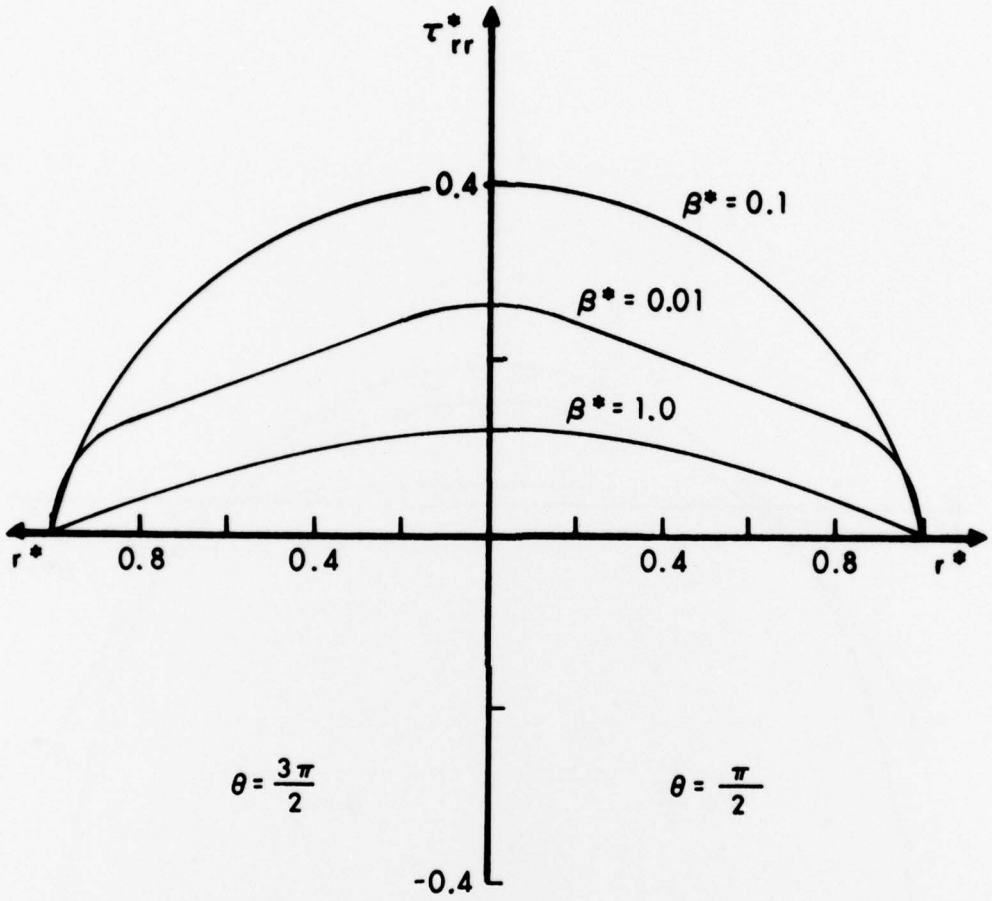

Figure 8. Radial Stress Distribution on $\theta = \frac{\pi}{2}$ and $\frac{3\pi}{2}$ for $t^* = 1.0$ and $\beta^* = 0.01$, 0.1, and 1.0

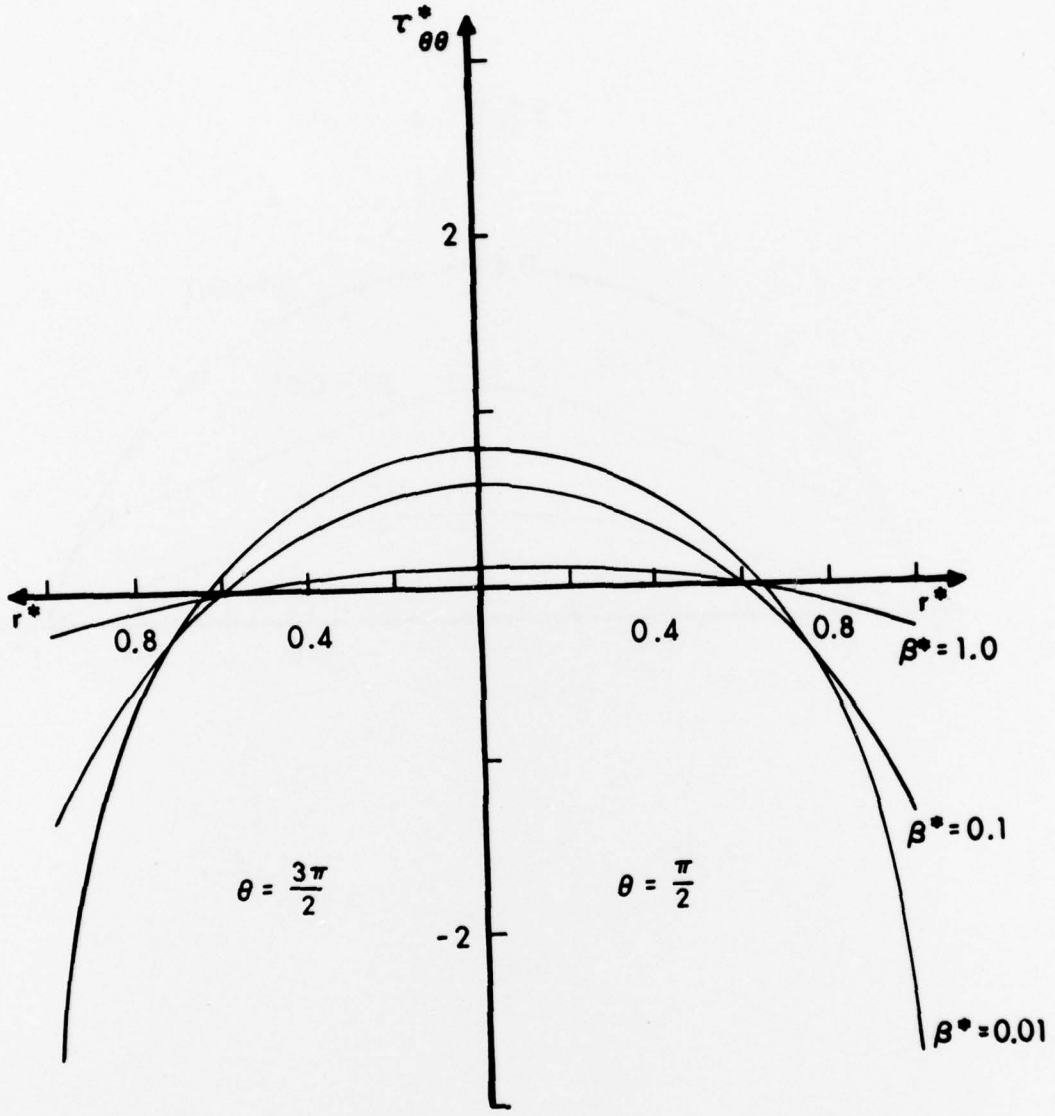

Figure 9. Tangential Stress Distribution on $\theta = \frac{\pi}{2}$ and $\frac{3\pi}{2}$ for $t^* = 1.0$ and $\beta^* = 0.01$, 0.1, and 1.0

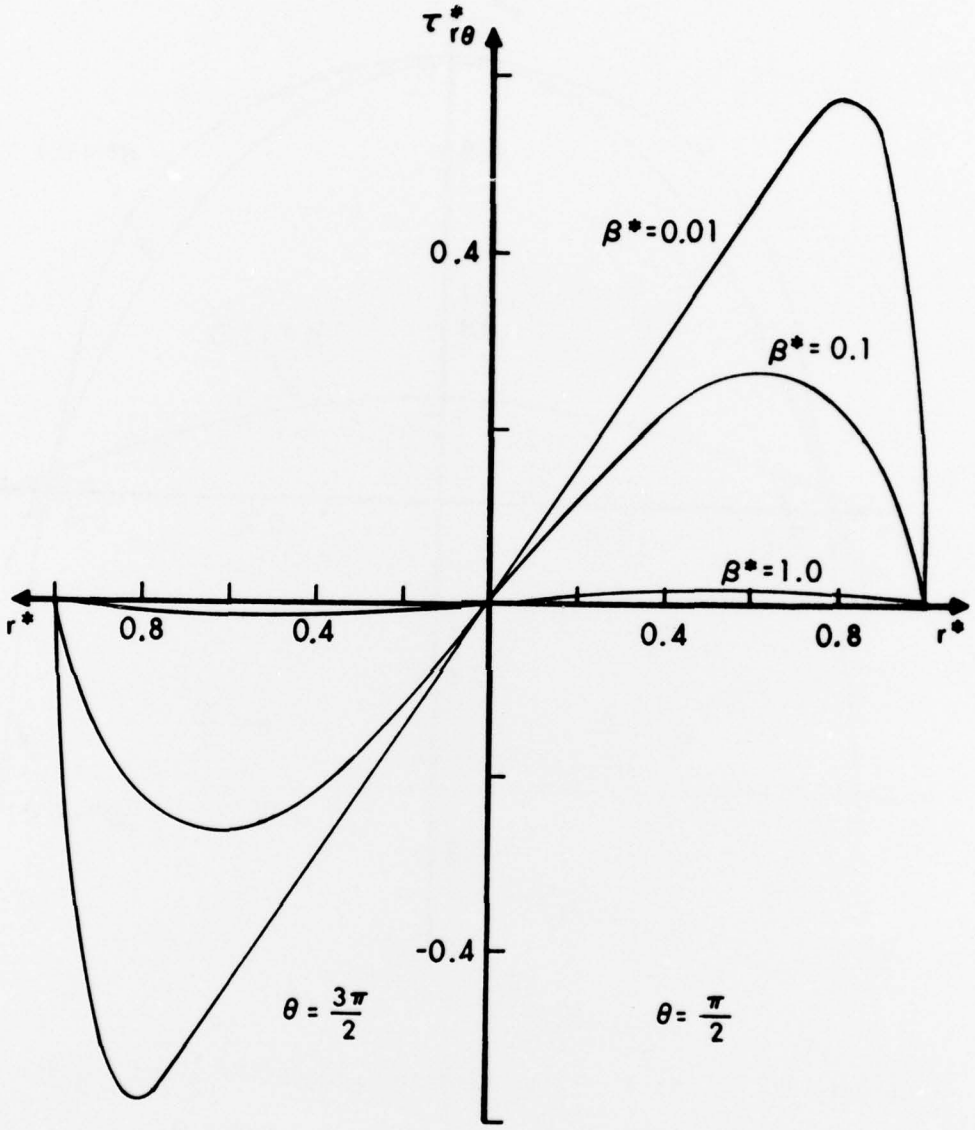

Figure 10. Shear Stress Distribution on $\theta = \frac{\pi}{3}$ and $\frac{3\pi}{2}$ for $t^* = 1.0$ and $\beta^* = 0.01$, 0.1, and 1.0

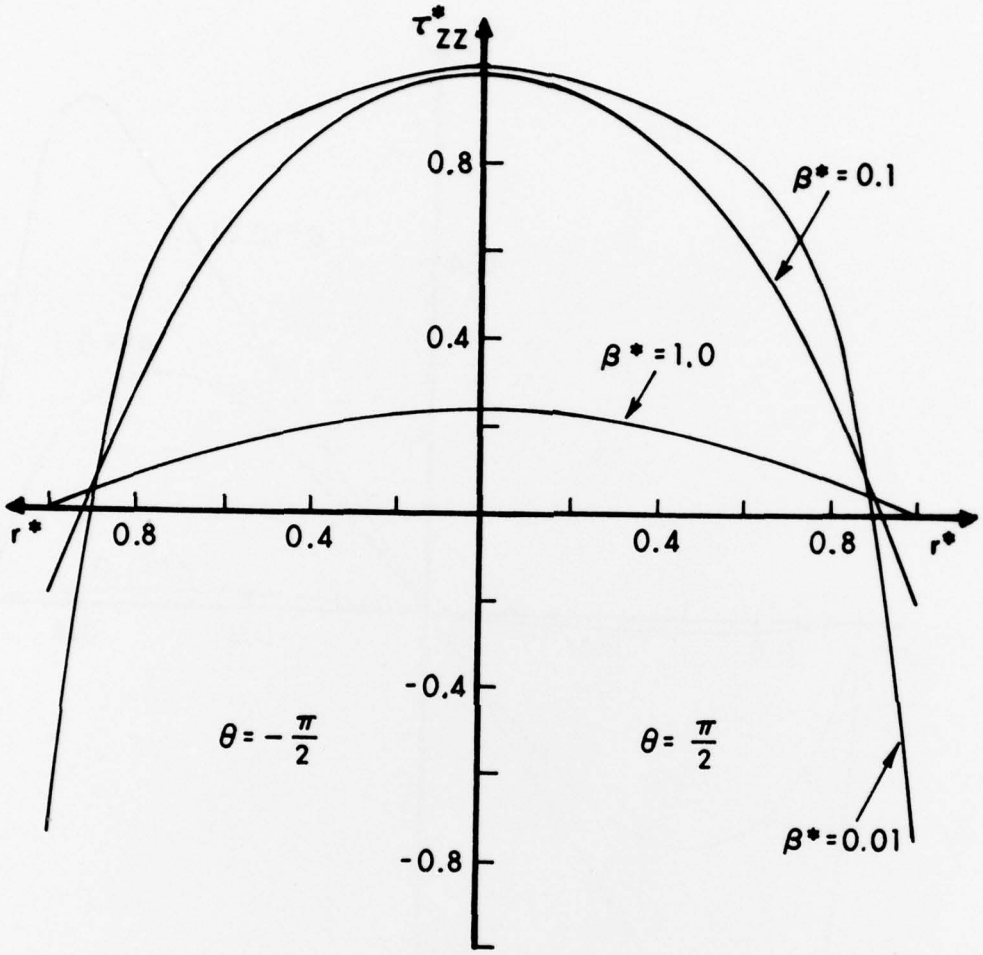

Figure 11. Axial Stress Distribution on $\theta = \frac{\pi}{2}$ and $-\frac{\pi}{2}$ for $T^* = 1.0$ and $\beta^* = 0.01$, 0.1 and 1.0

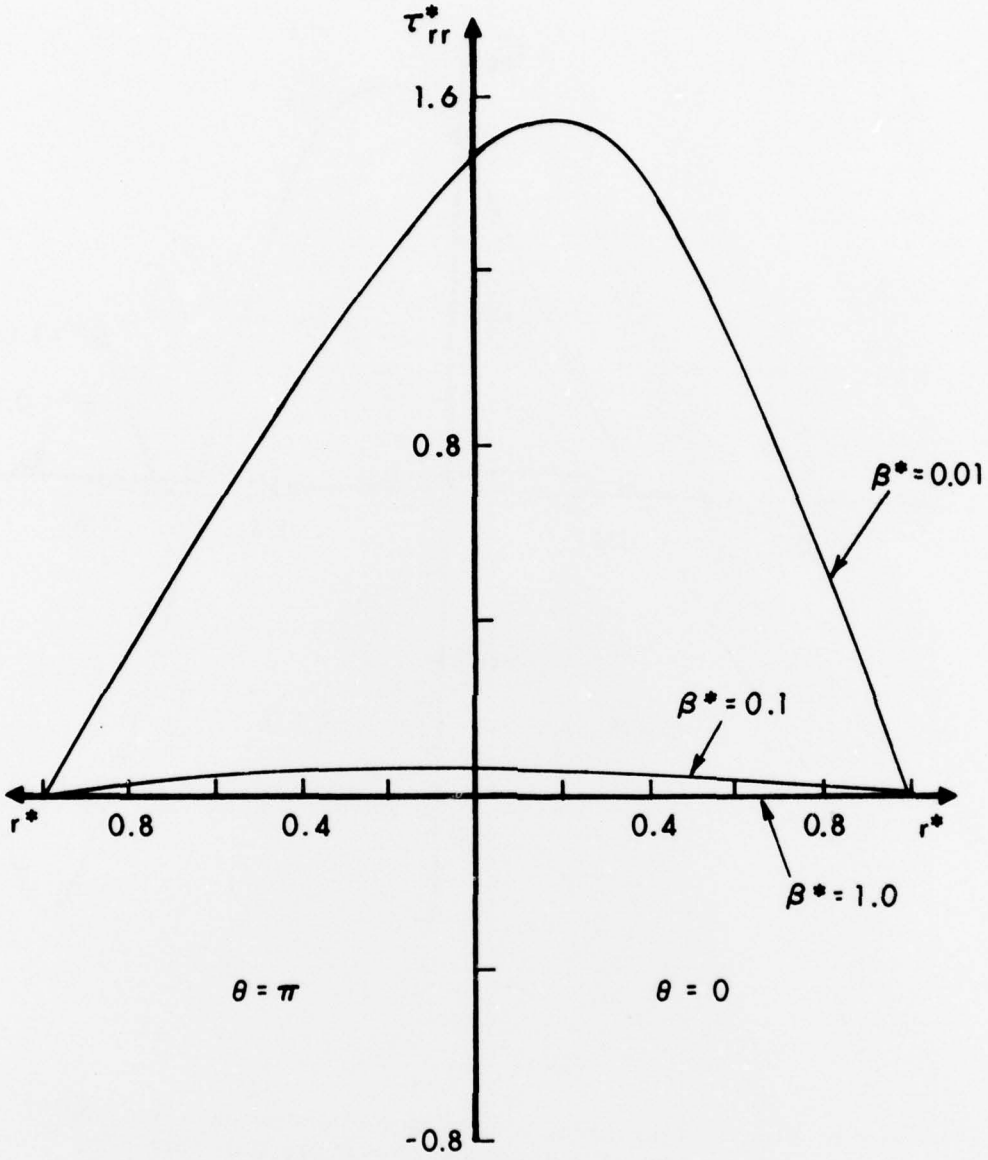

Figure 12. Radial Stress Distribution on $\theta = 0$ and π for $t^* = 10.0$ and $\beta^* = 0.01$, 0.1, and 1.0

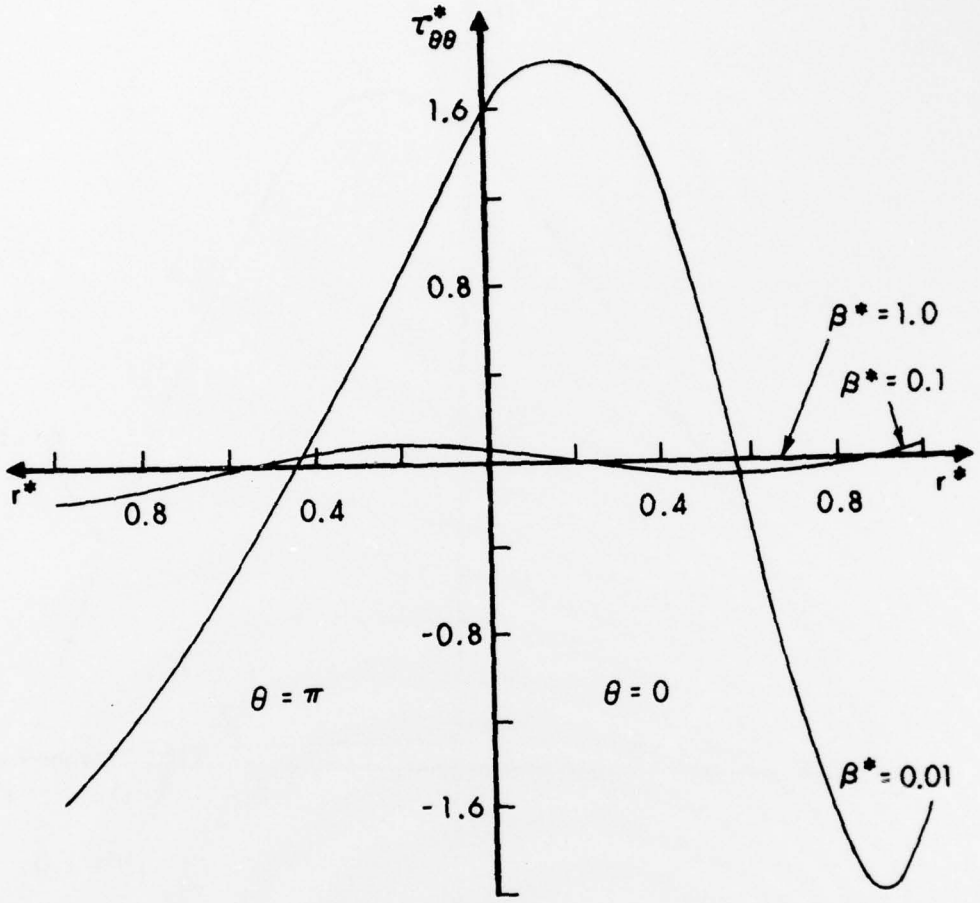

Figure 13. Tangential Stress Distribution on $\theta = 0$ and π for $t^* = 10.0$ and $\beta^* = 0.01$, 0.1, and 1.0

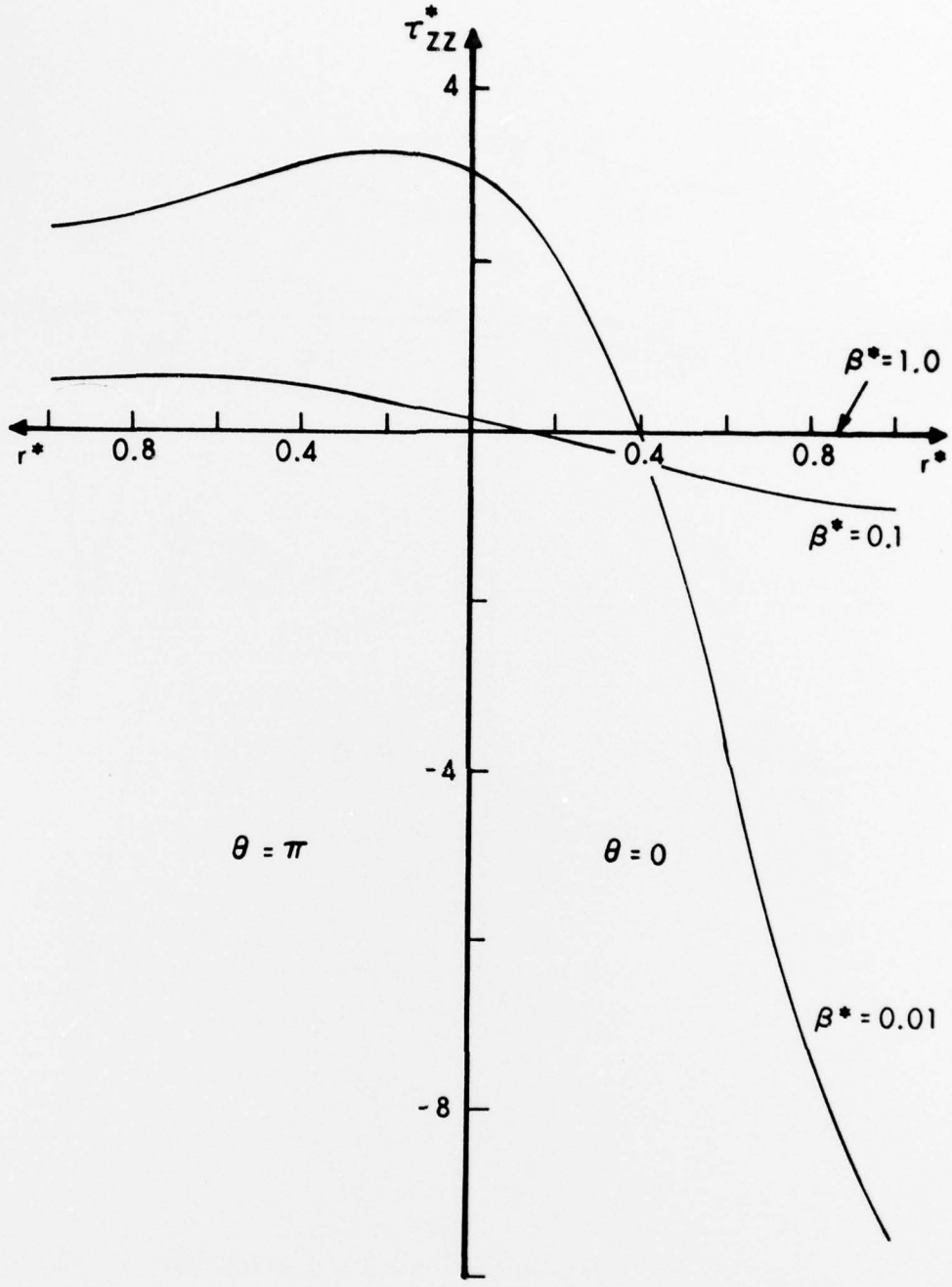

Figure 11. Axial Stress Distribution on $\theta = 0$ and π for $t^* = 10.0$ and $\beta^* = 0.01$, 0.1, and 1.0

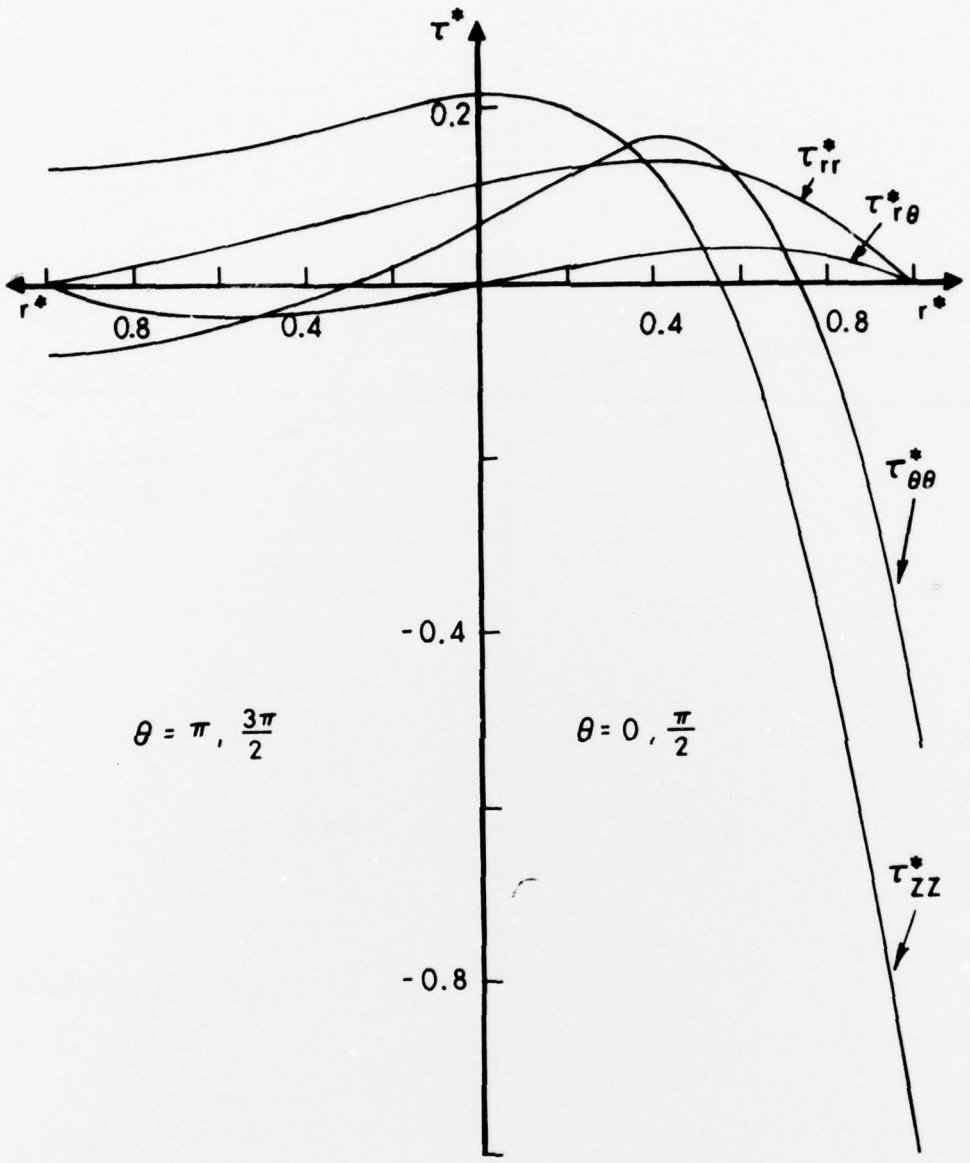

Figure 15. Normalized T^*_{rr}, $T^*_{\theta\theta}$, and T^*_{zz} on $\theta = 0$ and π and Normalized $T^*_{r\theta}$ on $\theta = \frac{\pi}{2}$ and $\frac{3\pi}{2}$ at the Time of their Maximum Value for $\beta^* = 0.1$

By trial and error, it was determined for $\beta^* = 0.1$ that maximum value of T^*_{rr}, $T^*_{r\theta}$, and T^*_{zz} occurred at $t^* = 1.35$ and that the maximum value of $T^*_{\theta\theta}$ occurred at $t^* = 1.10$. Figure 15 is a composite plot of T^*_{rr}, $T^*_{\theta\theta}$, T^*_{zz} and $T^*_{r\theta}$, normalized to the maximum value of T^*_{zz}, at the time of their maximum value. It is seen from Figure 15 that the maximum values of T^*_{rr} and $T^*_{r\theta}$ are small compared to the maximum value of T^*_{zz}. However, it is seen from this figure that the maximum value of $T^*_{\theta\theta}$ is not small compared to the maximum value of T^*_{zz}; the latter being only twice as large as the former. Also, it can be seen that the axial stress is the largest compression stress and the largest tension stress.

V. CONCLUSION

A set of non-dimensional equations for the quasi-static, thermal stress field in a long, unrestrained, isotropic, homogeneous, solid cylinder whose lateral surface is subjected to heating by a nuclear thermal environment have been derived under the plane strain assumption. The results of a parametric analysis of the stress field involving the rise time of the thermal pulse, t_o, and the total energy of the thermal environment, Q, shows:

1. For a given Q, an increase in t_o results in a decrease in the maximum value of the stress
2. For a given Q, an increase in t_o shifts the location of the maximum value of the stress for those stresses which are zero on the lateral surface of the cylinder, and
3. For a given t_o, the stresses at any point in the cylinder will increase as Q increases and will decrease as Q decreases.

GLOSSARY OF TERMS

a_m = Coefficients defined by (29)

a_m^* = Dimensionless variable defined by (31)

b_m = Coefficients defined by (30)

b_m^* = Dimensionless variable defined by (31)

c = Specific heat

$f(t)$ = Time dependent portion of curve in Figure 1

r = Radial coordinate

r_o = Radius of cylinder

r^* = Dimensionless variable defined by (31)

t = Time

t' = Dummy integration variable

t_o = Rise time of thermal pulse

t^* = Dimensionless variable defined by (31)

A_{mn} = Coefficients defined by (3) through (6)

G = Shear modulus

$H, H(t)$ = $H_o f(t)$

H_o = Maximum irradiance of nuclear thermal pulse

$J_m(u)$ = Ordinary Bessel function of argument u

$J_m'(u)$ = $\dfrac{dJ_m(u)}{du}$

Q = $\displaystyle\int_o^\infty H(t)dt$

$T, T(r,\theta,t)$ = Temperature in cylinder

T_o = Initial temperature in cylinder

α = Coefficient of linear expansion

β^* = Dimensionless variable defined by (31)

θ = Angular coordinate

κ = Thermal conductivity

$\lambda_{mn} r_o$ = The n-th positive roots of $J_m'(\lambda_{mn} r_o) = 0$

ν = Possion's ratio

ρ = Density

ρ_{mn}	=	Dimensionless variable defined by (31)
T_{rr}, $T_{\theta\theta}$, T_{zz}	=	Normal stresses
$T_{r\theta}$	=	Shear stress
T^*_{rr}, $T^*_{\theta\theta}$, T^*_{zz}, $T^*_{r\theta}$	=	Dimensionless variables defined by (31)
Φ	=	Temperature difference
Ω_1	=	Solution of (11)
Ω_2	=	Particular solution of (12)

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