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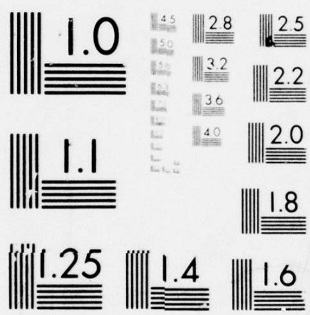

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NETWORKS FOR CROSSED-FIELD AND IN-LINE EXCITATION OF BULK AND SURFACE ACOUSTIC WAVES

(11) 1974

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were considered.

We consider, from a unified point of view, equivalent network descriptions of piezoelectrically driven devices of both surface and bulk wave kinds. Particular attention is given to the mechanism of excitation, and to the demonstration that the networks are capable of a comprehensive and detailed interpretation in which their various components are so arranged that they accurately depict the underlying physics of the devices they characterize.

Four situations are considered. The crossed-field (laterally excited) and in-line (thickness-excited) quasistatic circuits of crystal plates excited in bulk thickness modes are first interpreted in terms of their exact geometrical correspondences with the physical features of the plate resonators, particularly that of the piezotransduction mechanism. Next, the case of surface waves produced by interdigital arrays is discussed. The quasistatic equivalent network is given on an analog basis, and it is shown how the network corresponds exactly to the delta-function model. The network may be used for any surface wave mode by suitably defining appropriate mode functions associated with the transmission lines supporting the surface mode in question.

The third case considered is that of the circuit representation of bulk acoustic wave production in a piezoelectric rod by an applied microwave cavity field. It is shown how this category may be treated on the basis of a limiting form of the first case.

Case four consists of relaxing the quasistatic constraint for crystal plates subjected to crossed-field excitation. Mindlin's exact solution for the coupling of the Maxwell and acoustic fields in rotated *Y*-cut quartz plates is cast into network form, and the physical interpretation of the acoustic-electromagnetic interaction is given. The reduction of this network to that of the first case above in the quasistatic limit is also discussed.

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I. INTRODUCTION

A. Equivalent Networks for Piezoelectric Structures

Sixty years ago, Butterworth obtained the equivalent electrical circuit of a mechanically vibrating system having a single resonance. Since that time, the scope of applications of equivalent networks has increased greatly, while the concept itself has been enriched by incorporating many ideas from modern network theory. The most recent development along these lines has been that of point-by-point, exactly analogous networks, where the physical and circuit pictures match completely along a coordinate, rather than merely possessing the same governing relations at their corresponding ports. With analog networks it is possible to deal with waves in the time domain as well as with vibrations in the frequency domain; physical understanding is also enhanced.

This paper will introduce a number of new analog networks in the following sections; we will lead up to these by a short sketch of the evolution of equivalent circuits as

Presented at the Symposium on Optical and Acoustical Micro-Electronics
Polytechnic Institute of New York, April 16–18, 1974.

applied to piezoelectric devices, and by a consideration of a number of physical phenomena whose representations have to be incorporated into the networks.

Butterworth's 1914 circuit [1] was independently derived and applied to the piezoelectric crystal vibrator by Van Dyke [2]. It consisted of a series resonant circuit shunted by a capacitor, and describes the immittance of the crystal in the vicinity of a resonance. Butterworth also gave a Foster-form representation for all of the resonances associated with the harmonics of a single mode of vibration of a resonator [3]. The single resonance, lumped element equivalent network was extended in 1935 by Mason when he made provision for the inclusion of mechanical ports [4]; he then gave an exact, lumped equivalent for the resonances of a single modal type, including mechanical ports [5]. An alternate representation of the same kind is due to Roth [6]. One or another of these circuits has been used for the description of a large variety of bulk-mode, piezoelectric vibrators; a tabulation of various classes of these is listed by Onoe and Jumonji [7]. The more recent upsurge of interest in surface wave devices has led to the application of bulk-mode, lumped equivalent networks in this area also [8].

Broadband network representations of resonant systems appear in a natural fashion with the use of distributed network components. Indeed, many years ago, Mason stated that acoustic transmission lines are necessary for the adequate description of vibrators and resonant units [9]. While this was understood, the network schematic was drawn in lumped form [5], with the distributed nature implicit in the transcendental functions appearing in the lumped circuit elements. A partial-fractions expansion of the electrical port input immittance leads one back directly, in fact, to the Butterworth circuit [3].

In 1956 Redwood and Lamb redrew Mason's lumped circuit by using a piece of transmission line to replace the transcendental components, which they recognized as the tee form of the distributed network [10]. Investing the electrical schematic with a physical length was an important step forward in the development of true analog networks. Another bulk-wave representation, that of Krimholtz, Leedom, and Matthaei, uses a transmission line in a circuit quite different from that of Mason, as redrawn by Redwood and Lamb [11, 12]. The electrical port is connected in the center of an acoustic transmission line of length equal to the frequency determining length of the vibrator, and the mechanical boundary forces appear at both transmission-line ends rather than across a combination of transmission-line end and piezoelectric drive transformer. As a consequence, some of the circuit elements are frequency sensitive and appear to lack an obvious physical interpretation.

Redwood and Lamb's pictorially-distributed bulk wave equivalent circuit of a piezoelectric resonator was applied to the description of surface wave behavior in 1969 by Krairojananan and Redwood [13]. They further remarked that the piezoelectric sources are placed at the electrode edges of the interdigital array; we will make further use of their observation subsequently. More recently, Bahr and Lee applied to the surface wave problem the network of Krimholtz, Leedom, and Matthaei [14].

B. Analog Networks

The equivalent networks mentioned above all describe the operation of piezoelectrically excited acoustic structures. Each represents a single modal type, e.g., one member of the bulk mode family, or type of surface wave; some are broadband, others valid only near a single resonance. All have this in common: each describes only the port relations that are obtained in a particular device. No attempt is made to make the network schematic match up with the interior workings of the structure. Ballato, Bertoni, and Tamir have shown in 1972, that the thickness modes of piezoelectrically driven crystal plates could be represented in network form in such a fashion that the individual circuit components bore simple interpretations in terms of the underlying physics of the devices [15, 16]. Moreover, the multimode circuit schematics are exact analogs. As the physical processes of the actual problem are accurately mirrored in the quantities associated with their network equivalent, which is spatially distributed to correspond to the geometry of the structure, it is possible to visualize and trace the interplay of the waves involved simply by inspecting the circuit; in addition, the full effects arising from the coupling at the boundaries between juxtaposed plates are made evident in a succinct and readily interpretable manner.

In following sections we will present these crystal plate analogs and use them to introduce a number of new analog circuits covering surface acoustic wave propagation, bulk mode excitation by microwave cavity fields, and also piezoelectromagnetic vibrations of crystal plates, where the quasistatic constraint is removed and the full coupling of the acoustic and Maxwell fields takes place.

C. Acoustic Waves in Crystalline Dielectrics

George Green, in 1839, gave the first general theory for the propagation of plane acoustic waves in anisotropic media [17]. In 1877 Christoffel published a long article on the same subject in which he cast the equations in the form which is still used today [18]. Lawson then generalized the treatment to take into account piezoelectricity [19]. Bechmann gave a complete discussion of the unbounded propagation problem, including the effect of the piezoelectric constants on the particle motions [20]; Epstein has reformulated the problem and discussed the effect of piezoelectric stiffening on the elastic wave surfaces [21].

From these researches it is known that three plane-wave bulk modes propagate in any given direction in an arbitrary crystal, and that the inclusion of piezoelectricity modifies the three modal velocities, generally in the order of a few percent; an electric field is also produced by the wave motion, and lies in the direction of advance of the phase fronts. This component of electric field, due to the piezoelastic waves, necessitates a distinction in the circuit pictures describing excitation by driving fields along or normal to it. This will be discussed in greater detail below.

Surface and interfacial wave propagation on, and between crystals leads to more involved situations than the bulk wave case. Analytical results have been obtained by Stoneley [22], Synge [23], Deresiewicz and Mindlin [24], Burridge [25], and Gou [26]. General theoretical results are extremely difficult to obtain, however, and the numerical methods of Lim and Farnell [27] have proven particularly valuable for investigating these mode types computationally. However, numerical analysis does not provide an adequate conceptual framework for an in-depth appreciation of the various mechanisms involved, nor does it lead to a systematic procedure for the analysis and synthesis of practical devices.

The much-needed systematic procedure was provided by Oliner, Bertoni, and Li [28-30]. They have carried over to acoustical micro-electronics the sophisticated methodology of microwave network methods. Included are the concepts of the building block approach, transmission-line modal representations for transmission regions, and lumped circuits for discontinuities and junctions. The networks that appear from their formulation tie in strongly with the Redwood-Lamb variety that evolved directly from the traditional piezoelectric vibrator equivalent.

Concerning Oliner's transmission-line modal representations, whatever the mode type, and regardless of the complexity of the resulting motion, so long as the supporting structure is translationally invariant in one direction, such a representation can, in principle, be found [28]. It is upon this fact that our construction of equivalent networks is based. Associated with the transmission line carrying the mode in question are vector mode functions characteristic of the medium and geometry of the supporting structure.

We will add to the junction and discontinuity circuits those associated with piezoelectric coupling and arrange the network topology so that they provide a physically satisfying picture of the piezoelectric excitation mechanism.

D. Conditions at Boundaries

At the boundary between two crystalline materials, an acoustic plane wave impinging at normal incidence generally couples to the three modes in each medium, even in the absence of piezoelectricity. The mechanical boundary network describing the interaction has recently been given [31]. It consists of an assembly of interconnected ideal transformers, located in the analog networks, *at* the junction of the crystals, in the same manner as Oliner's [28].

For the corresponding case of piezoelectric coupling, quite a bit of material is scattered about the literature. Tiersten's exact analysis of Lawson's problem [32] disclosed that the thickness modes of plates were coupled by the piezo effect, and that the coupling took place at the plate boundaries. Transient studies by Cook [33] showed that a voltage step applied to a piezotransducer produced acoustic wave fronts that propagated inward from the two end surfaces. Slater [34] describes the use of microwave cavity fields for exciting bulk piezoelectric motion in a crystal. This effect was used by Baranskii [35] and Bömmel and Dransfeld [36] to produce hypersonic bulk acoustic waves in quartz rods and plates. Experiments and analysis, carried out in 1960 by Jacobsen led to a more complete understanding of the equivalence of piezoelectric excitation considered as arising either from a volumetric effect or from surface tractions [37]. He showed that acoustic waves are produced at all spatial discontinuities of the piezoelectric stress, the abrupt discontinuity which occurs at the end face of a rod or plate being only a special case; for us it is a very important one. He also showed that the microwave fringing field in the cavity produces a continuous distribution of acoustic sources within the piezo material, but that their effect is inconsequential compared to the delta-function-like bulk-wave source created at the end face.

Replacement of the volumetric piezoelectric driving stress by the surface forcing equivalent was fruitfully applied by Holland [38] to a variety of ceramic vibrator problems involving complex configurations. Transient problems, wherein the surface forcing effect shows up most prominently, were further considered by Redwood [39], and Peterson and Rosen [40].

Direct piezoelectric excitation of surface waves by an array of interdigital electrodes was first reported by White and Voltmer [41]. As we mentioned earlier, Krairojananan and Redwood [13] applied a bulk mode equivalent circuit to the description of this case. Milsom and Redwood [42] then modified the circuit to take into account the facts that the electric field pattern produced by the interdigital array is generally at an angle to the interface, and that the surface wave displacement pattern being driven by the field is likewise neither wholly along, nor normal to, the direction of propagation.

In 1972, Mitchell and Reilly [43] demonstrated that some of the bulk mode equivalent circuit models that had been applied to the characterization of surface wave production in piezoelectric devices were exactly equivalent to a "δ-function" model of transduction, where every electrode in the array is represented by one or more simple, independent, sound sources. The surface wave generation by the structure is determined by adding the waves produced by the separate sources. The equivalent sound sources have been placed by different authors at various positions with respect to the array fingers; for an array with few fingers it appears that the device performance simulated by the model is relatively insensitive to the placement of the sources. They have at times been located at the center of the electrode gap and at the center of the electrode fingers. As stated in [13], the sources are located at the electrode edges in their model. We shall find that this location corresponds most closely with the physics and shall arrange our analog network schematic so that the network places the excitation at the electrode edges.

Having considered briefly acoustic waves in crystalline media, and the mechanical and piezoelectric tractions that exist at boundaries, we now pass on to the network representations of various structural configurations and modal types that describe devices in an analog fashion.

II. QUASISTATIC NETWORKS OF THICKNESS-MODE PLATES

A. In-Line (Thickness) Excitation

The arbitrarily anisotropic piezoelectric plate, covered with perfectly conducting electrodes, and executing motion varying only in the thickness direction, was considered by Lawson [44], and analyzed exactly by Tiersten [32]. His results were recast in 1970 by Yamada and Niizeki [45], using a normal-coordinate transformation, as was first introduced for such problems by Basri [46]. Network realization of the plate immittance was then obtained by Onoe [47] using three circuits of the Mason type [5], in parallel.

In Fig. 1 we show the exact analog network realization of Ballato, Bertoni, and Tamir [15, 16, 31], simplified for the case where only a single thickness mode is piezoelectrically driven. The network is superimposed upon an exploded view of an electroded crystal plate to emphasize the placement of the negative capacitance and the piezoelectric drive transformers *at* the plate boundaries. The negative capacitance is introduced by the piezoelectric reaction of the electric field produced by the acoustic waves upon the electric field driving the plate vibration; it is characteristic of in-line excitation. In the figure the electrodes are assumed to be perfect conductors with no elastic properties, but only lumped surface mass. The lumped inductors and piezotransformers represent, respectively, delta-functions of inertial and piezoelectric surface traction. The absence of electrode mass leads to imposition of short circuits at the six mechanical ports. When more general mechanical boundary conditions are applied, a mechanical boundary network, consisting of interconnected ideal transformers [31], is required. Piezoelectric excitation of all three thickness modes of the plate is represented by attaching piezotransformers to the other transmission lines in a manner identical to that shown, and connecting the three primaries at each surface in parallel. The turns ratios are proportional to the piezoelectric constants referred to the normal coordinates. Observe that the piezotransformer dots are oppositely placed at the two surfaces; this is the circuit manifestation of the polar nature of the piezoelectric effect.

Each acoustic transmission line supports one of the three characteristic modes of the system. In general, the line parameters are unequal. Although drawn with the conventional circuit symbol for convenience, the shunt capacitor plates belong, of course, at the electrodes; through the capacitor flows the dielectric displacement current, while the piezoelectric polarization current flows through the two wires connecting the piezotransformer primaries in parallel. When the boundary-loadings on the plate surfaces are

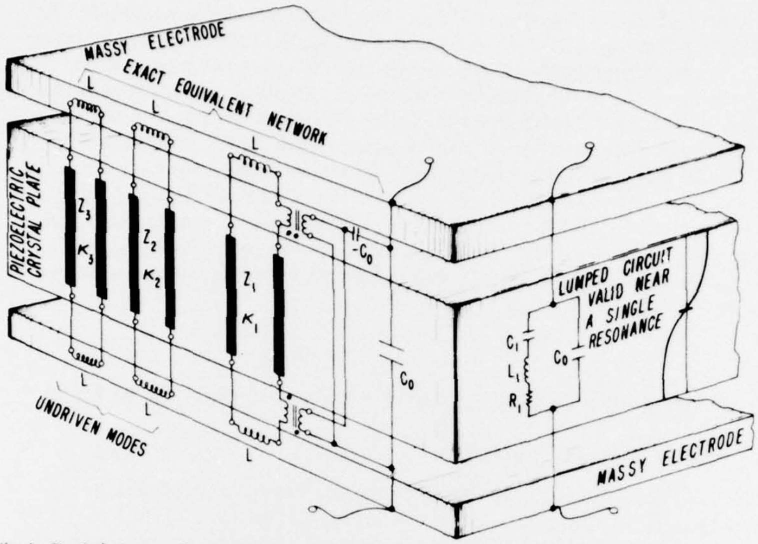

Fig. 1. Exploded view of a crystal plate with massy electrodes. The plate supports three thickness modes of which one is shown piezoelectrically driven in the equivalent electrical network superimposed. Of particular importance is the location of the piezodrive transformers at the surfaces of the plate; they correspond to surface tractions driving the motion.

not equal, the unbalance shows up in the difference in polarization current carried by the two wires.

The network of Fig. 1 is an exact analog of the physical situation. Solving the network to determine the transmission-line voltages, and other quantities at any value of the thickness coordinate, and then translating these into the corresponding physical quantities, yields exactly the same result as solving the electroelastic plate problem; the *network matches the physics at every point within the bulk and at the boundaries.* We will show in the sequel that it also provides the basis for the network interpretation of a number of other piezoacoustic problems. This is because the circuit components individually admit of physical interpretations because of the manner we have disposed them in the schematic.

At the right-hand side of the figure is the four-element Butterworth-Van Dyke circuit [1, 2], the earliest network representation.

B. Crossed-Field (Lateral) Excitation

Yamada and Niizeki also treated the crossed-field excitation of arbitrarily anisotropic piezoelectric plates in thickness modes [48]; to Onoe is due the circuit realization of their results using Mason-type networks [47].

Corresponding to their in-line analog representation, Ballato, Bertoni and Tamir [15, 16] gave the crossed-field analog network shown in Fig. 2, as specialized to the case of a single piezoelectrically driven mode; inclusion of the other two modes in the general situation follows the discussion for the in-line case above. The circuit result is obtained on the assumption that the plate is driven by a spatially constant electric field lateral to

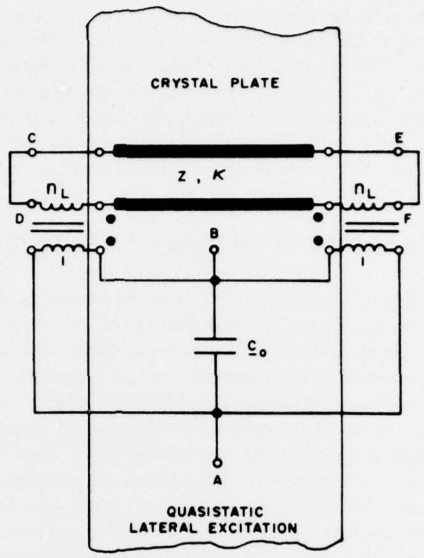

Fig. 2. Analog, quasistatic transmission-line network for a single, piezoelectrically driven, thickness mode. The driving field is lateral to the plate surface, corresponding to crossed-field excitation. Traction-free mechanical boundary conditions are represented by the short circuits at CD and EF.

the plate surfaces. This leads to the lumped capacitor with orientation shown; it represents the dielectric effect of the applied field along a unit distance of the crystal. The capacitor plates should extend across the plate thickness and be separated by one unit of length. However, that the plate motion in the quasistatic approximation is not an exact solution is evident by the fact that the plate waves produce an electric field component in the thickness direction, which contradicts the boundary conditions on the capacitor plates. We will return to this point in connection with Mindlin's problem in Section V.A.

The transmission line in Fig. 2, and the remaining two not shown, are identical to those in Fig. 1 in their parameters. As far as the acoustic modes are concerned, both problems are the same. The differences occur in the orientation and permittivity of the shunt capacitance, the presence or absence of the negative capacitance, and the piezoelectric constants entering the transformer turns ratios; the cross-field piezoconstants are likewise referred to the normal coordinates. Shown in the figure are short circuits at the two mechanical ports, representing traction-free boundaries. In general situations the mechanical boundary network [31] is required.

C. Composite Excitation

Excitation of a piezoelectric structure by an electrode arrangement producing a composite field neither along nor normal to the direction of wave propagation is an interesting problem posed by Redwood [49]. In our networks the situation would be modeled approximately by attaching, to the three modal transmission lines, electrical input networks of the types in Figures 1 and 2. Each transmission line has attached, to each end, three sources of traction: the mechanical boundary network (or short circuit in the traction-free case), and, in series with it, the series connection of two piezotransformers, one for each type of excitation. These lead off to the shunt capacitor oriented along the plate for the crossed-field component, and to negative and shunt capacitors for the in-line field component. Additionally, the two electrical ports would be connected together by a capacitive lattice, in parallel with the crystal equivalent circuit, to account for the electrostatic coupling between the electrode systems.

III. QUASISTATIC NETWORKS FOR SURFACE WAVES

Piezoelectric tractions are proportional to electric field strength. For surface wave production by interdigital arrays, the field strength is largest at the electrode edges [50, 51]. The electric field gradient, which yields the force-density, is likewise largest at the edges, and has been approximated by δ-function sources by Tancrell and Holland [52]. The piezotransformers presented in connection with Figs. 1 and 2 exert finite tractions but have no spatial extent; they correspond exactly to δ-functions of force-density. By arranging the single mode circuit discussed in Section II so that the transmission lines coincide with the direction of surface wave propagation, the piezodrive to coincide with the electrode edges, as shown in Fig. 3, we account for the principal features arising from the physics of the structure. The electrode and gap regions have been provided with separate transmission-line sections to account for the velocity and acoustic impedance differences in the two regions. Arrows along the center of the figure are symbolic of the forces produced by the piezotransformers; they alternate on either side of a gap as indicated by the transformer dots, and from gap to gap by the changing electrode polarities.

Interelectrode dielectric displacement current flows through the shunt capacitors arranged in the figure to span the gaps, while the piezoelectric polarization current carried

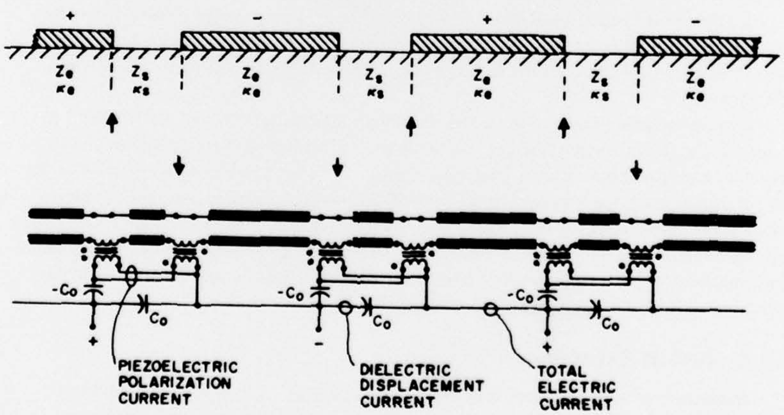

Fig. 3. Analog transmission-line network for surface wave generation by interdigital electrodes. Electric field strength peaks at the electrode edges, so the drive transformers are located there. Not shown are mechanical discontinuity networks, also located at the electrode edges, that model the production of bulk modes.

by the wave motion flows in the wires interconnecting the transformer primaries. Shown in the figure is the electrical input circuit for the in-line case, which includes a negative capacitor. Provision is made for the inadequacy of this representation [53–55], by incorporating the factor α, as introduced by Milsom and Redwood [42]. Alternatively, one may use a composite form for the electrical input, using both cross-field and in-line types, with piezotransformers in series and a capacitive lattice, as described in Section II.C. The computational complexity associated with such a network will almost never justify its use, so that Milsom and Redwood's modification appears, for many situations, the best compromise.

The transmission line associated with each region has associated with it vector mode functions that describe the particular modal type supported by the physical structure [28], so that the various types of surface and interface waves can be modeled by the same circuit configuration.

In Fig. 3 we have placed at the electrode edges only the piezotransformers. At any discontinuity, such as that at the edges, bulk-mode conversion must take place; a stricter analog network takes this fact into account by placing in series with the piezotransformers an interface network consisting of mechanical transformers attached to transmission lines normal to the interface, representing the bulk wave motions.

IV. CAVITY EXCITATION OF BULK MODES

Single-ended excitation of bulk waves by a microwave cavity field may be described in circuit form by a simple modification of our networks; historical background of this problem has been given previously in Section I.D. Consider the experimental arrangement of Figure 4. A piezoelectric crystal is used as a transducer; it is bonded to a nonpiezoelectric crystal in which it is desired to propagate acoustic waves. We show, in Fig. 5, the analog network representation. In the nonpiezoelectric crystal the three types of acoustic plane waves capable of propagating along the axis (lateral boundaries neglected),

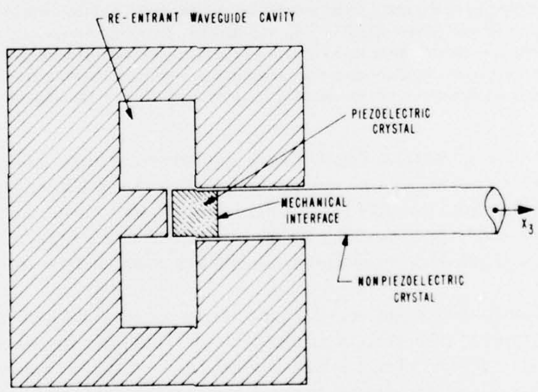

Fig. 4. Schematic of apparatus for bulk acoustic wave production by cavity field. The piezoelectric crystal end face is placed in a region of high field strength; this drops off along the length of the transducer. Bonded to the transducer is a second crystal in which it is desired to propagate acoustic waves.

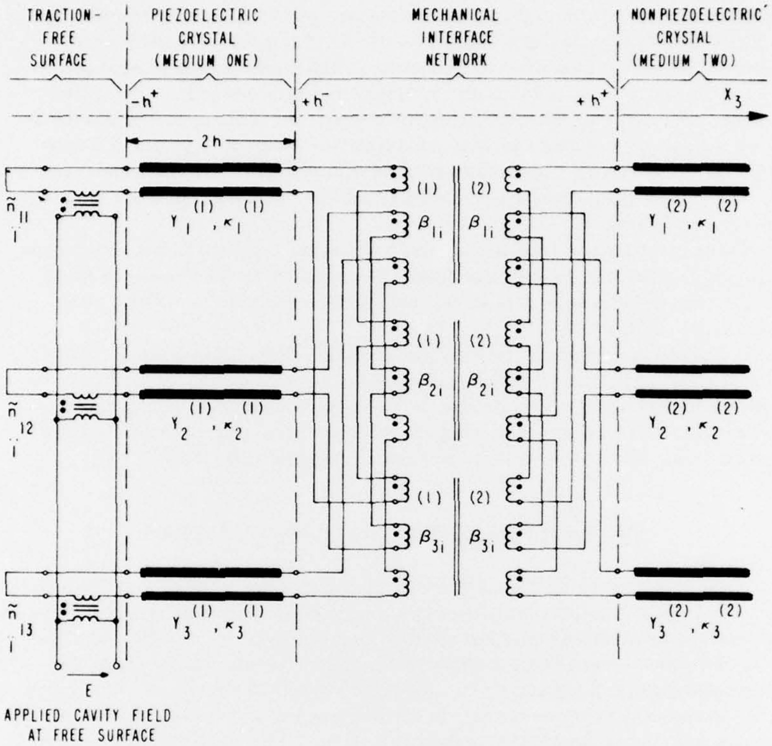

Fig. 5. Network description of the situation depicted in Figure 4. The abrupt discontinuity in piezoelectric constant occurring at the end face is modeled by the placement of piezotransformers there. These are referred to electric field strength instead of potential. Three plane-wave modes are propagated from the end face toward the second crystal. At the interface, welded-contact boundary conditions are represented by the three-dimensional mechanical interface network shown. This converts between the variables pertinent to the three characteristic modes in each medium.

are modeled by three transmission lines having appropriate parameters. The junction of the two crystals is accounted for by a mechanical boundary network interconnecting the three modal lines from each side [31]; it consists of an interconnected assembly of ideal transformers with turns ratios given by the components of the eigenvectors pertinent to the three modes in each crystal, referred to a common, laboratory coordinate system.

At the end face of the transducer crystal, the mechanical traction-free condition appears in the schematic as short circuits; placed in series with these are the piezotransformers, just as they appear in Fig. 1 at both surfaces for the plate. Now, however, three changes are required: the interconnections of the piezotransformers between boundaries, and the lumped capacitors are suppressed; also, the piezotransformer ratios are referred to surface electric field strength at the boundary rather than to applied potential between surfaces. These alterations follow from the network of Fig. 1 upon letting the plate thickness grow without bound, and recognizing that the piezotractions at the second surface are replaced now by a continuous distribution of sources in the transducer crystal. This comes about in the cavity case because of fringing effects, since the field is not uniform within the crystal, but trails off to a low value as the outside of the cavity is approached. Jacobsen's analysis and experiments have shown [37] that the spatial distribution of sources produces an inconsequential effect compared to the single lumped source at the discontinuity produced by the end face. The sum of the continuous sources is equal in strength to the lumped source which would be produced at the second boundary if the field was uniform, but distribution in space leads to an incoherence that destroys the effectiveness of these sources, and to their subsequent neglect.

In Fig. 5 the equivalent circuit for excitation by a single field component is given. When the field makes an arbitrary angle with the crystal end face, a combination of crossed-field and in-line excitation results; in the network the resultant excitation appears as three piezotransformers in series with each modal transmission line, one transformer for each field component. The effective piezoelectric constants contained in the turns ratios are the ones that arise from the corresponding thickness mode plate problem. When the end face of the transducer is not free of mechanical traction, a boundary coupling network has additionally to be placed there, instead of the short circuits, and has the effect of further coupling the transmission lines together. These network results are described further in [56], and accord completely with the treatment of cavity excitation given by Lamb and Richter [57].

V. PIEZOELECTROMAGNETIC VIBRATIONS OF THICKNESS-MODE PLATES

In 1949, Kyame [58] treated the problem of wave propagation in an infinite medium where both the acoustic and electromagnetic effects are coupled by piezoelectricity. The theory he developed incorporates the full Maxwell's equations, and predicts five plane waves, three of which become the acoustic modes, in the absence of piezoelectricity, while the remaining two become the usual electromagnetic waves. Similar analyses were carried out by Pailloux [59] in 1958, and Alda, Hruška, and Tichý [60] in 1963.

Tiersten [61] investigated the radiation and confinement of electromagnetic energy accompanying the oscillation of piezoelectric crystal plates in 1970. He found that in the quasistatic approximation (speed of light infinite) the crystal plate does not radiate electromagnetic energy. An exact solution for the coupled acoustic and electromagnetic fields in rotated Y-cut quartz plates was reported by Mindlin in 1973 [62]. In the following we will indicate how Mindlin's solution resolves the contradiction that arose in connection with the crossed-field excitation in Section II.B, and give the complete analog network for a piezoelectromagnetic plate vibrator.

A. Mindlin's Problem

Mindlin considered the pure-shear thickness mode of a rotated Y-cut quartz plate together with its coupling to the electromagnetic field via the piezoelectric effect. With the plate motion driven by specified mechanical tractions at the surfaces, his rigorous solution predicts that the plate radiates electromagnetic energy, and that the electric field lateral to the boundary surfaces of the plate is a function of the plate thickness coordinate. This situation corresponds to the case of lateral, or crossed-field excitation

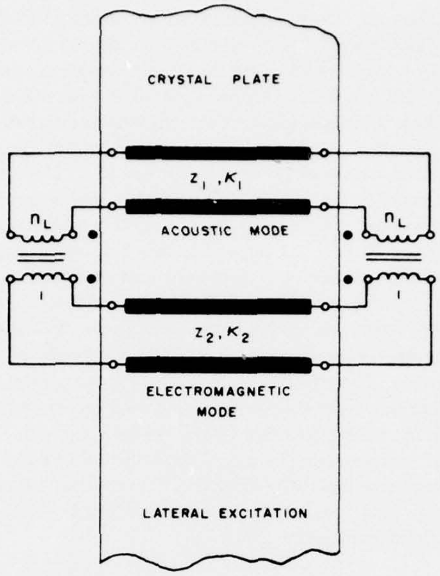

Fig. 6. Analog transmission-line network corresponding to Fig. 2 where now the quasistatic constraint is relaxed and Maxwell's equations are employed. The static capacitor of Fig. 2 represents the lowest-order approximation to the quasielectromagnetic mode transmission line. Piezocoupling is represented by the boundary transformers; the plate is mechanically traction-free, and coated with a perfect magnetic conductor to prevent radiation.

of thickness modes where only one mode is coupled by a piezoelectric constant, but where the lateral field is not constrained to be constant across the thickness. Figure 2 shows the crossed-field quasistatic network in the absence of mechanical tractions, as indicated by the short circuits across the mechanical ports at CD and EF. Retaining the full Maxwell's equations leads to replacement of the capacitor by a transmission line, as shown in Fig. 6, and to an electric field in the lateral direction that varies with the thickness coordinate.

The two transmission lines of Fig. 6 represent the normal modes of the coupled system: one quasiacoustic mode, with velocity of propagation very close to that of the line in Fig. 2, and one quasielectromagnetic mode, with velocity very close to that of light in the crystal in the absence of the piezoelectric effect. In Fig. 6 the mechanical ports are short circuited, and the plate surfaces considered to be coated with a perfect magnetic conductor [63], having no inertial or elastic properties. This provides internal reflection of the coupled modes of the plate, and no radiation of electromagnetic energy into the surrounding space. The piezotransformers are placed at the discontinuity produced by the bounding surfaces of the plate in this characteristic-mode representation; alternatively, the transmission lines could be considered to be coupled continuously along their length in a coupled-mode representation [64].

More general boundary conditions are accommodated by attaching to the two transmission lines of Fig. 6, which represent the system characteristic modes, a boundary network. Because only a single acoustic mode and a single optical mode are being

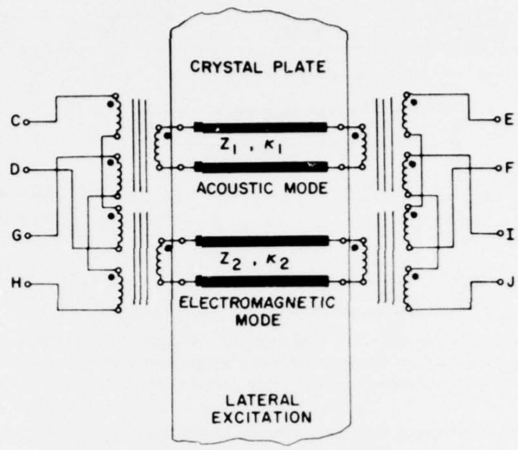

Fig. 7. Piezoelectromagnetic analog network with provision for general boundary conditions. At each surface one port represents mechanical and one electromagnetic boundary quantities; the transformer turns ratios are the components of the eigenvectors of the plate modes.

considered now, the network required is one that describes a two-dimensional rotation of coordinates [65].

Figure 7 models the more general situation. At one port on each side, say CD and EF, appears the set of purely mechanical variables, force and particle velocity, while at the other port on each side, GH and IJ, the purely electromagnetic variables appear. The turns ratios of the transformers are again the components of the eigenvectors that are appropriate to the characteristic mode representation. Because of the disparity of the velocities of the two modes (ratio of order 10^5) and the smallness of the piezoelectric coupling in most materials, including quartz, the turns ratios will be nearly unity (quasi-acoustic mode to mechanical port; quasielectromagnetic mode to optical port), or very small (quasiacoustic mode to optical port; quasielectromagnetic mode to mechanical port).

Mindlin solved the problem where the perfect magnetic conductor is absent from the plate boundaries and mechanical tractions are applied to the surfaces. This is described by Fig. 7 by attaching voltage sources to the mechanical ports CD and EF and transmission lines to the optical ports GH and IJ; these lines represent the electromagnetic radiation emitted by the oscillating plate. When the mechanical tractions vanish, the boundary network may be simplified, as shown in Figure 8. Here the transmission lines extending from the plate model the radiated optical mode. Use of standard network procedures (bisection and transverse resonance [28]), leads to the equations for the resonances of the structure in a straightforward manner.

We have concentrated on the case where a single acoustic mode is piezoelectrically coupled to a single optical mode. As Kyame [58] showed, all five modes have generally to be considered. The undriven modes in our case are very simply represented. The second optical mode in the crystal merely joins the free-space optical transmission line on each side by a direct feedthrough, while the two acoustic modes are interconnected together on each side by a two-dimensional network of the form given in Fig. 7; if the

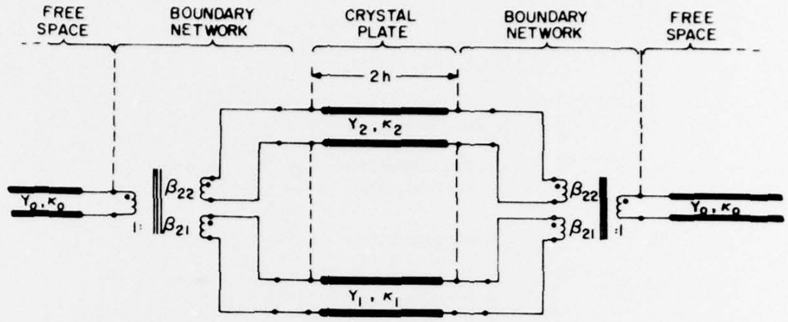

Fig. 8. Electromagnetic-mechanical coupling in a rotated Y-cut quartz plate with traction-free boundaries. Within the crystal one quasiacoustic and one quasioptical mode propagate; they are piezoelectrically coupled at the boundaries to a single optical mode that propagates in free space.

boundaries have no mechanical tractions, the acoustic modes are completely uncoupled, as shown in Fig. 1, with the inductors replaced by short circuits.

In the most general case, the five characteristic modes of the plate are coupled together at the boundary by the piezoelectric effect, and by mechanical tractions; this is represented in network fashion by a five-dimensional rotation of coordinates [65]. When the mechanical boundary tractions are absent, the coupling network for a single surface of the plate reduces to that given in Figure 9. To the left, two lines, representing the optical modes in free space, couple at the surface to all five quasiacoustic,

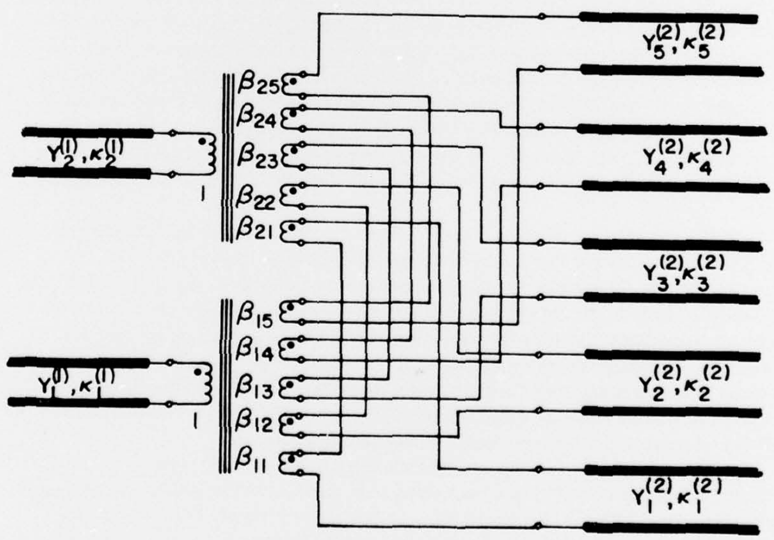

Fig. 9. General analog network representation of piezoelectric crystal-vacuum interface. Three quasiacoustic and two quasioptical modes propagate within the crystal and are coupled by piezoelectricity at the surface to two optical modes.

quasielectromagnetic modes, and to each other. The coupling is due largely to piezoelectricity; if this vanishes, three of the five turns ratios on each core also vanish. The three acoustic modes are then uncoupled from the boundary, while the two optical modes in the crystal remain coupled by a two-dimensional transformer circuit with ratios involving permittivities only.

When two crystals are joined, two five-dimensional transformation networks are simply attached together at corresponding ports, just as the mechanical interface network of Fig. 5 has been connected for the three-dimensional case. In most instances the circuits will be simplified by the absence of piezoelectric, elastic, and dielectric coupling terms characterizing the materials used. In any and all cases, the necessary modifications and simplifications may be performed with relative ease, once one is familiar with the network representations and gains some experience with circuit manipulations.

VI. CONCLUSION

We have discussed equivalent electrical networks describing, on an analog basis, the operation of piezoelectric structures of four types, with particular emphasis on the mechanism of piezoelectric transduction. The analog network representations correlate closely with the corresponding physics involved. The most distinctive feature of the new representations is the splitting up of the piezodrive transformer into parts and their removal to the discontinuities of the structures.

For the cases of the thickness modes of crystal plates, with either coupling to the Maxwell fields or in the quasistatic regime, the piezotransformers are located at the plate surfaces, while in the case of surface wave excitation by interdigital fingers, the piezodrive is located at the electrode edges. Excitation of bulk waves by a microwave cavity field requires placement of the piezotransformers only at the end face of the transducer; the small field gradient within the transducer crystal produces a continuous distribution of sources which is negligible compared to that at the end face.

Application of analog networks to these diverse situations points up the similarities in the piezoelectric excitation, and shows how matching the network to the physics on a point-for-point basis permits exploiting a building-block approach to circuit representations of novel piezoacoustic structures and devices. It also allows accurate investigations of response in the time domain.

ACKNOWLEDGEMENTS

The author wishes to acknowledge the patient guidance and stimulation of H. L. Bertoni and T. Tamir of the Polytechnic Institute of New York, and the encouragement he has received from the US Army Electronics Technology and Devices Laboratory, Fort Monmouth, NJ.

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