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THERMOELASTIC PROPERTIES OF UNIDIRECTIONAL FILAMENTARY COMPOSIT--ETC(U)
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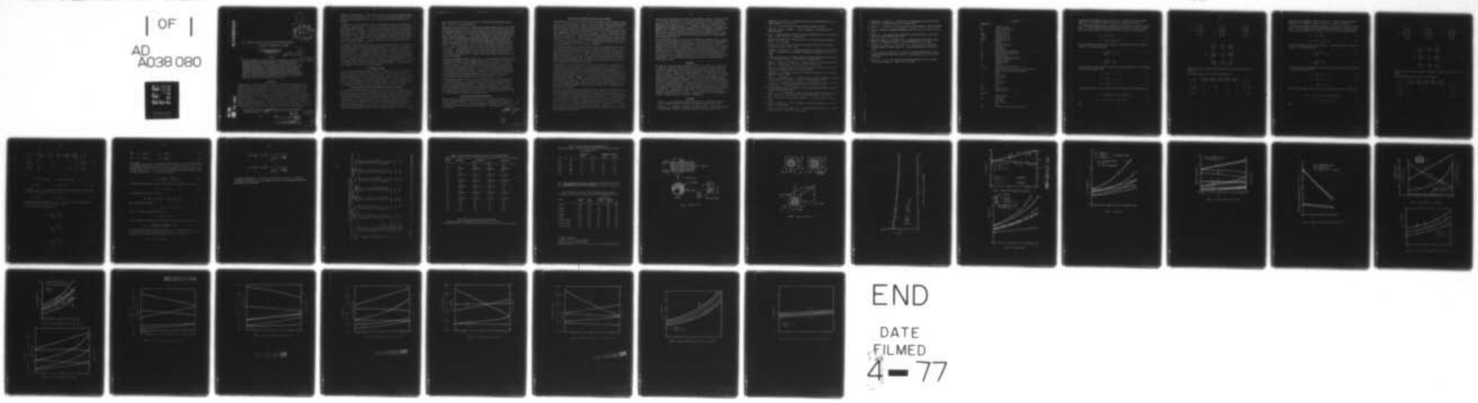

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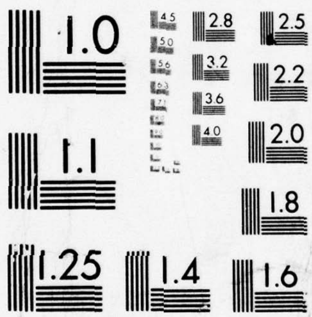

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THERMOELASTIC PROPERTIES OF UNIDIRECTIONAL FILAMENTARY COMPOSITES BY A SEMIEMPIRICAL MICROMECHANICS THEORY*

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ABSTRACT

A unified, semiempirical micromechanics theory is described which relates the thermoelastic properties of the unidirectional, filamentary composite to the quantities and to the corresponding properties of its constituent materials. The theory treats the composite, the filaments and the matrix as being generally orthotropic, linearly elastic, and accounts for the effect of voids. It is based on the equivalent section concept, on parallel and series connected elements and on the judicious incorporation of certain empirical factors, which reflect the particular fabrication process. Results are presented which demonstrate and verify application of this theory to boron, carbon and glass-filament epoxy-resin composites. Additional results are presented which exhibit the voids and in-situ matrix orthotropicity effects on the thermoelastic properties of the unidirectional composite. Finally, results are included for all the thermoelastic properties of boron, carbon and glass-filament epoxy-resin composites.

INTRODUCTION

The continuous strive for efficient materials utilization and the environment of space travel has challenged the ingenuity of both materials scientist and designer. Their combined efforts created numerous high-strength light-weight filament composite materials which are characterized by high stiffness and high strength to weight ratio. The properties of these composites depend on the properties and quantity of their constituents and on the particular fabrication process. The mathematical formalism which relates the properties of the composite to those of its constituents has been coined as "micromechanics". A reliable micromechanics theory is an invaluable tool in trade off studies and in functional designs. The alternative to micromechanics in establishing the composite's properties is the experimental approach. However, the many possible filament-matrix combinations, arising in parametric and trade off studies, would require a large number of experiments. This could be economically prohibitive and could impose severe limitations on the utilization of filament

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composites as structural members. Many of these difficulties can be conveniently bypassed by predictively correct semiempirical theories. In this paper, a semiempirical micromechanics theory which deals with the thermoelastic properties of a unidirectional filamentary composite (ply) is described.

In recent years, several micromechanics theories have been proposed to predict the thermoelastic properties of fibrous composites. These theories range from the strength of materials approach to sophisticated statistical methods. Theories which are representative of the published work are found in Refs. 1-15. Reference 14 is a critical review of the state of the art. Most of these methods are for isotropic filaments (except Ref. 8), isotropic matrices, and neglect the effect of voids (except Ref. 15). They are primarily based on theoretical considerations and are deterministic in nature. Available experimental results suggest that the unidirectional filamentary composite's thermoelastic properties are of highly statistical nature, due to the numerous factors which influence these properties (Ref. 16).

The semiempirical theory described here differs from existing theories in that: (i) it reflects the particular fabrication process through the empirical factors; (ii) it allows the in-situ matrix to be generally orthotropic; (iii) it predicts the complete thermoelastic properties of the composite, i.e. nine elastic coefficients, three thermal coefficients of expansion, three heat conductivities and the heat capacity; (iv) it is convenient for design application. Briefly then, the article covers description of the theory, verification of its application, generation and discussion of additional results. The working equations are listed in the Appendix A. The detailed derivations are not included here; however, they are given in Chapter 1 and Appendices A and B of Ref. 17. For subsequent discussion, the terms fiber and filament will be used interchangeably. The terms unidirectional composite and ply will also be used interchangeably.

PLY FUNDAMENTAL ELEMENTS

The ply has certain fundamental elements. These are: (1) Filaments which come in a roving end of yarn and are specified as number of filaments per roving end of yarn (Fig. 1); (2) Matrix; (3) Apparent volume ratio of filament; (4) Volume ratio of voids. Other ply elements which are needed for design and analysis are: (1) Actual volume fraction of filament; (2) Actual volume fraction of matrix; (3) Ply thickness; (4) Filament spacing within the ply. The last four elements can be related to the ply's fundamental elements as is shown in Appendix A. The various thermal and mechanical properties of the filaments and matrix in their bulk state are assumed to be known.

PLY-ELASTIC PROPERTIES AND PLY LINEAR THERMAL COEFFICIENTS OF EXPANSION

The assumptions on which the semiempirical theory is based are: (1) The ply is linearly elastic and generally orthotropic. (2) Both filaments and matrix are linearly elastic and generally orthotropic. (3) The axes of orthotropy of the ply, filament and matrix are coincident. (4) Both filaments and matrix could contain voids; (5) Empirical factors are introduced to account for the following simplifications: (5a) Filaments are regularly spaced in the form of a square array and are completely aligned; (5b) Filament's cross-sectional area can be represented by an equivalent square cross-sectional area (Fig. 2); (5c) Filaments and matrix are connected in parallel in the longitudinal direction and they are connected in series in the two transverse directions; (5d) Residual stresses have negligible effects on elastic and thermal properties; (5e) There is complete bond at the interface of the constituents and there is no transitional region between the constituents. (6) The ply is at uniform temperature.

The motivation for the simplification in assumption (5) stems mainly from four sources: (1) The many processing variables that affect the ply's elastic behavior, such as filament tension, matrix viscosity, temperature, humidity and air impurities at the matrix bath stage; temperature and pressure at the cure stage. Other factors such as fiber surface treatment, and various matrix additives effect the ply's thermoelastic properties. (2) The reasonable agreement of the results obtained from the square-base-filament approximation when compared with other theories and experiment (see Tables 1 and 2, Ref. 17). (3) The reasonable correlation obtained between theoretically predicted ply-thickness and measured results; Fig. 3. (4) The results of approximating spheres with cubes, Refs. 18 and 19. It is noted that simplifications 5a, d, and e are also made in the theories cited in Refs. 1-15; however, in these theories there are no provisions to compensate the effects of these simplifications. In the present theory, the effects of the simplifications are compensated by the empirical factors. The important point to be kept in mind is that simplifications 5a, d and e reflect a particular fabrication process. Therefore, the empirical factors are representative only of that fabrication process.

The geometry of the mathematical model is shown in Fig. 2. The basic hypothesis in the formulation of the semiempirical theory is that the averaged energy stored in a ply's typical-cell equals the averaged energy stored in the filaments and matrix contained in the typical cell. It is consistence with this hypothesis, then, to consider elements with finite dimensions in the typical-cell and talk about their averaged state-of-affairs. This is a modification from the concept of point description in the classical elasticity theory. The difficulties of theories seeking to describe point behavior are extensively discussed in Ref. 17, Section 1.2. The formulation is that of the strength-of-materials using three-dimensional stress-strain relationships. This avoids the assumption of completely restrained matrix in the fiber direction, Refs. 1, 5, and 15.

The averaged energy stored in a unit-cell, Fig. 2, is expressed in terms of stresses using equilibrium and continuity conditions. The desired equations are obtained by comparing corresponding coefficients in the averaged energy expression. The empirical factors are introduced in writing the equilibrium equations since the local stress is replaced by an averaged one. The resulting expressions for the normal elastic constants, thermal coefficient of expansion and shear moduli are given in Appendix A eqs. A.8, A.9 and A.17-A.19 respectively. The detailed derivations are lengthy and are omitted; however, they are given in Appendix B of Ref. 17.

PLY HEAT CAPACITY AND THERMAL CONDUCTIVITIES

The ply heat capacity is obtained by equating the heat stored in the unit-cell to the sum of its components. The resulting expression is given by Eq. A.25. Suitable equations for conductivities are presented in Refs. 3, 20 and 21. Here, these equations are either modified or rederived so that orthotropicity and empirical factors can be incorporated. The resulting expressions are given by Eqs. A.27-A.30.

VERIFICATION OF SEMIEMPIRICAL THEORY RESULTS AND DISCUSSION

The semiempirical theory results discussed subsequently were generated from the input data in Tables 1 and 2. The value of the empirical factors in Table 2 were chosen such that predicted and experimental results of plies with known filament and void ratios correlate. Comparison of the semiempirical theory with other theories and with experimental data is presented in Figs. 4-6. As can be seen in these figures, the results of the semiempirical theory are in good agreement with the experimental averaged data in the practical range of fiber content. In Fig. 7, the semiempirical theory results for all the ply elastic constants are compared with corresponding results of Ref. 10 for a glass-fiber resin-matrix ply. The comparison of these results indicates that: (i) good agreement for ($E_{\ell 11}$), ($\nu_{\ell 12}$), and ($G_{\ell 23}$); (ii) the semiempirical theory predicts higher values for ($E_{\ell 22}$) and ($G_{\ell 12}$) (this is in line with experiment); (iii) the semiempirical theory predicts higher values for ($\nu_{\ell 23}$) (no experimental results for this property are available).

The semiempirical theory results for the ply thermal properties are compared with other theories in Fig. 8 and limited experimental data in Fig. 9 for a carbon-fiber resin-matrix system. As can be seen from these figures, the correlation is good. The effects of the voids on the transverse modulus ($E_{\ell 22}$) are shown in Fig. 10. The theoretical curves in this figure were constructed by choosing the empirical factor β_m for glass-resin, so that the predicted values provide an upper bound ($k_v = 0$) on the experimental data. The important points to be noted in Fig. 10 are that the experimental results can be bounded with void content and that any theory for predicting the ply elastic properties must include the effect of voids.

The effects of in-situ matrix orthotropicity on the transverse modulus ($E_{\ell 22}$) are shown in Fig. 11. The bounds of Ref. 5 are also shown in this figure for comparison purposes. As can be seen in Fig. 11, the effects of matrix orthotropicity on the transverse modulus are quite pronounced. The important points to be made here are: (i) available experimental results can be bounded by allowing the matrix to behave orthotropically in-situ; (ii) theories on the ply's elastic properties should treat the matrix as generally orthotropic. In the work of other investigators, the in-situ matrix has been treated as isotropic. However, there are several good reasons to suspect that the in-situ matrix does not behave isotropically. Some of these reasons are: nonuniform matrix film thickness between fibers, presence of residual stress, possible preferential molecular alignment in very thin films; the weight-density dependence on the coupson size reported in Ref. 22; induced orthotropy defined in Ref. 23, page 89.

The effects of voids on the ply heat conductivities are shown in Table 3 for a glass-resin ply. The semiempirical theory results are given for two values of the empirical factor β_{kv} (0.9 and 1.0). As can be seen from Table 3, the semiempirical factors can be chosen so that the semiempirical theory is in good agreement with either other theories or with experimental data. It is important to note from Table 2 that most of the empirical factors are near unity or have values which approximately correspond to square base filaments. When the fiber volume content is near the two extremes ($k_f = 0$ and $k_f = 1.0$), the semiempirical theory will not predict reasonable results for ply thermoelastic properties which have other than unity empirical factors. However, this is no limitation on the semiempirical theory since one could have different values of the empirical factors for fiber content ranges near the two extremes.

Further verification and application versatility of the semiempirical theory is illustrated in Table 4. In this table, the thicknesses of all the composite plates were given but only the elastic properties and fiber content for the (+30) composite plate were known. The fiber content for the other composites were determined from Eq. A.4. The empirical factors were selected so that the semiempirical theory reproduces the elastic properties for the (+30) composite. Subsequently, these empirical factors and the corresponding fiber ratios were used in the semiempirical theory to generate the elastic constants and flexural rigidities for the other composite plates. As can be seen in Table 4, computed buckling loads based on these flexural rigidities are in good agreement with experimental data of Ref. 24.

Additional theoretical results for all the ply elastic properties are presented in Figs. 12-15 and for all the ply thermal properties in Figs. 16 and 17. Theoretical results for the combined matrix orthotropicity and void effects on the transverse modulus are presented in Figs. 18 and 19 for boron-epoxy and Thornel-40-epoxy plies respectively. As can be seen from Figs. 18 and 19 the matrix orthotropicity and voids effects are rather pronounced in the boron-epoxy composite. In all these theoretical results, the ply was assumed to be transversely isotropic and the input data is that given in Tables 1 and 2. It should be noted that some of the constituent properties in Table 1 are only estimates and the corresponding empirical factors should be modified as experimental values become available. This is particularly true for the case of $G_{\ell 23}$ and $\nu_{\ell 23}$ where experimental results are not available.

CONCLUSIONS

The semiempirical theory described herein provides an effective and versatile tool to predict the ply thermoelastic properties in practical ranges of fiber volume content. The ply thickness and the inter-fiber spacing depend on the fiber content and expressions relating them to the fiber content can be derived. Though results are not presented here, it was found that the ply thermal coefficients of expansion and the ply heat capacity are not sensitive to void content. The ply heat conductivity is sensitive to void content. Both voids and in-situ matrix orthotropicity have pronounced effects on the transverse ply modulus. The fabrication process should be considered in the theories for ply thermoelastic properties since there are many fabrication processing variables which could influence these properties. The empirical factors and the in-situ matrix orthotropicity provide realistic degrees of freedom for better experiment-theory correlation for the ply's thermoelastic properties. The fabricators of fiber composites should assume the responsibility to provide the designers with information shown in Tables 1 and 2. The test strain rate at which these properties are measured should also be reported. The matrix modulus is strain-rate dependent and this effect is analogous to that of in-situ matrix orthotropicity.

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APPENDIX A

Nomenclature

A	constant, Eq. A.14
A_m	area of typical unit cell
B	constant, Eq. A.14
c_f, c_m	constants, Eq. A.15
c_f, c_m	constants, Eqs.A.16
c_f', c_m'	constants, Eqs. A.20
c_f'', c_m''	constants, Eqs. A.21
d	diameter
E	Modulus of Elasticity
G	shear modulus
H_c	heat capacity
K	bulk modulus, heat conductivity
$k, \bar{k}$	apparent and actual volume ratio
k_v	void volume ratio
N_f	number of filaments per roving end
s	filament spacing Fig. 2
ΔT	temperature change
t	thickness
1,2,3	ply or composite orthotropic axes
$\beta, \tilde{\beta}, \beta', \beta''$	empirical factors for ply elastic, and thermal behavior
δ	shortest matrix distance between filaments
ε	strain
ν	Poisson's ratio
ρ	weight density
σ	stress
{ }	1-column array
⌊⌋	1-row array
[ ]	array
$[\ ]^T$	transpose of array
$[\ ]^{-1}$	inverse of array

Subscripts

f	filament property
k, k_v	conductivity
ℓ	ply property
m	matrix property
v	void
1,2,3	directions corresponding to orthotropic axes

The equations of the semiempirical theory are given herein. The detail derivations are rather lengthy and are not included here; however, they are given in Appendices A and B Ref. 17.

It is assumed that the various thermoelastic properties of the constituents in their bulk state are known. It is further assumed that the apparent filament volume ratio (k_f) is given. Using the axion that the volume (weight) of the whole equals the sum of its parts, it can be shown that

$$\bar{k}_f = (1.0 - k_v)\, k_f \qquad \text{A.1}$$

$$\bar{k}_m = (1.0 - k_f)\,(1.0 - k_v) \qquad \text{A.2}$$

where the barred quantities denote actual volume ratio. The interfiber spacing (δ) and the ply thickness (t_ℓ) are respectively given by

$$\delta = \left[\left(\frac{\pi}{4\, \bar{k}_f} \right)^{1/2} - 1 \right] d_f \qquad \text{A.3}$$

$$t_\ell = \left(\frac{\pi N_f}{\bar{k}_f} \right)^{1/2} d_f/2 \qquad \text{A.4}$$

Let the normal strain-stress-temperature relationships along the orthotropic axes for ply, filament and matrix be given respectively by

$$\{\varepsilon_\ell\} = [E_\ell]\,\{\sigma_\ell\} + \Delta T\,\{\alpha_\ell\} \qquad \text{A.5}$$

$$\{\varepsilon_f\} = [E_f]\,\{\sigma_f\} + \Delta T\,\{\alpha_f\} \qquad \text{A.6}$$

$$\{\varepsilon_m\} = [E_m]\,\{\sigma_m\} + \Delta T\,\{\alpha_m\} \qquad \text{A.7}$$

The normal elastic properties and the thermal coefficients of expansion are given respectively by

$$[E_\ell] = [C_{f\ell}]^T [E_f][C_{f\ell}]\, \bar{k}_f + [C_{m\ell}]^T [E_m][C_{m\ell}]\, \bar{k}_m \qquad \text{A.8}$$

$$\{\alpha_\ell\} = [C_{f\ell}]^T \{\alpha_f\}\, \bar{k}_f + [C_m]^T \{\alpha_m\}\, \bar{k}_m \qquad \text{A.9}$$

where

$$\{\sigma_\ell\} = \begin{Bmatrix} \sigma_{\ell 11} \\ \sigma_{\ell 22} \\ \sigma_{\ell 33} \end{Bmatrix}; \qquad \{\epsilon_\ell\} = \begin{Bmatrix} \epsilon_{\ell 11} \\ \epsilon_{\ell 22} \\ \epsilon_{\ell 33} \end{Bmatrix}; \qquad \{\alpha_\ell\} = \begin{Bmatrix} \alpha_{\ell 11} \\ \alpha_{\ell 22} \\ \alpha_{\ell 33} \end{Bmatrix}; \qquad \text{A.10}$$

$$[E_\ell] = \begin{bmatrix} \dfrac{1}{E_{\ell 11}} & -\dfrac{\nu_{\ell 22}}{E_{\ell 22}} & -\dfrac{\nu_{\ell 31}}{E_{\ell 33}} \\ \dfrac{\nu_{\ell 12}}{E_{\ell 11}} & +\dfrac{1}{E_{\ell 22}} & -\dfrac{\nu_{\ell 32}}{E_{\ell 33}} \\ \dfrac{\nu_{\ell 13}}{E_{\ell 11}} & -\dfrac{\nu_{\ell 23}}{E_{\ell 22}} & -\dfrac{1}{E_{\ell 33}} \end{bmatrix} \qquad \text{A.11}$$

Analogous equations for filament and matrix are obtained by replacing the subscripts in Eqs. A.10 and A.11.

The filament and matrix stresses ply stress relationships are

$$\begin{Bmatrix} \sigma_{f11} \\ \sigma_{f22} \\ \sigma_{f33} \end{Bmatrix} = \begin{bmatrix} \dfrac{1}{AE_{m11}k_m} & \dfrac{1}{A}\left(\dfrac{\nu_{f21}}{C_f E_{f22}} - \dfrac{\nu_{m21}}{C_m E_{m22}}\right) & \dfrac{1}{A}\left(\dfrac{\nu_{f31}}{C_f E_{f33}} - \dfrac{\nu_{m31}}{C_m E_{m33}}\right) \\ 0 & \dfrac{1}{C_f} & 0 \\ 0 & 0 & \dfrac{1}{C_f} \end{bmatrix} \begin{Bmatrix} \sigma_{\ell 11} \\ \sigma_{\ell 22} \\ \sigma_{\ell 33} \end{Bmatrix} \qquad \text{A.12}$$

The equations of the semiempirical theory are given herein. The detail derivations are rather lengthy and are not included here; however, they are given in Appendices A and B Ref. 17.

It is assumed that the various thermoelastic properties of the constituents in their bulk state are known. It is further assumed that the apparent filament volume ratio (k_f) is given. Using the axion that the volume (weight) of the whole equals the sum of its parts, it can be shown that

$$\bar{k}_f = (1.0 - k_v)\, k_f \qquad \text{A.1}$$

$$\bar{k}_m = (1.0 - k_f)\,(1.0 - k_v) \qquad \text{A.2}$$

where the barred quantities denote actual volume ratio. The interfiber spacing (δ) and the ply thickness (t_ℓ) are respectively given by

$$\delta = \left[\left(\frac{\pi}{4\, \bar{k}_f} \right)^{1/2} - 1 \right] d_f \qquad \text{A.3}$$

$$t_\ell = \left(\frac{\pi N_f}{\bar{k}_f} \right)^{1/2} d_f / 2 \qquad \text{A.4}$$

Let the normal strain-stress-temperature relationships along the orthotropic axes for ply, filament and matrix be given respectively by

$$\{\epsilon_\ell\} = [E_\ell]\{\sigma_\ell\} + \Delta T\{\alpha_\ell\} \qquad \text{A.5}$$

$$\{\epsilon_f\} = [E_f]\{\sigma_f\} + \Delta T\{\alpha_f\} \qquad \text{A.6}$$

$$\{\epsilon_m\} = [E_m]\{\sigma_m\} + \Delta T\{\alpha_m\} \qquad \text{A.7}$$

The normal elastic properties and the thermal coefficients of expansion are given respectively by

$$[E_\ell] = [C_{f\ell}]^T[E_f][C_{f\ell}]\, \bar{k}_f + [C_{m\ell}]^T[E_m][C_{m\ell}]\, \bar{k}_m \qquad \text{A.8}$$

$$\{\alpha_\ell\} = [C_{f\ell}]^T\{\alpha_f\}\, \bar{k}_f + [C_m]^T\{\alpha_m\}\, \bar{k}_m \qquad \text{A.9}$$

where

$$\{\sigma_\ell\} = \begin{Bmatrix} \sigma_{\ell 11} \\ \sigma_{\ell 22} \\ \sigma_{\ell 33} \end{Bmatrix} ; \qquad \{\epsilon_\ell\} = \begin{Bmatrix} \epsilon_{\ell 11} \\ \epsilon_{\ell 22} \\ \epsilon_{\ell 33} \end{Bmatrix} ; \qquad \{\alpha_\ell\} = \begin{Bmatrix} \alpha_{\ell 11} \\ \alpha_{\ell 22} \\ \alpha_{\ell 33} \end{Bmatrix} ; \qquad \text{A.10}$$

$$[E_\ell] = \begin{bmatrix} \frac{1}{E_{\ell 11}} & - & \frac{\nu_{\ell 22}}{E_{\ell 22}} & - & \frac{\nu_{\ell 31}}{E_{\ell 33}} \\ \frac{\nu_{\ell 12}}{E_{\ell 11}} & + & \frac{1}{E_{\ell 22}} & - & \frac{\nu_{\ell 32}}{E_{\ell 33}} \\ \frac{\nu_{\ell 13}}{E_{\ell 11}} & - & \frac{\nu_{\ell 23}}{E_{\ell 22}} & - & \frac{1}{E_{\ell 33}} \end{bmatrix} \qquad \text{A.11}$$

Analogous equations for filament and matrix are obtained by replacing the subscripts in Eqs. A.10 and A.11.

The filament and matrix stresses ply stress relationships are

$$\begin{Bmatrix} \sigma_{f11} \\ \sigma_{f22} \\ \sigma_{f33} \end{Bmatrix} = \begin{bmatrix} \frac{1}{AE_{m11}k_m} & \frac{1}{A}\left(\frac{\nu_{f21}}{C_f E_{f22}} - \frac{\nu_{m21}}{C_m E_{m22}}\right) & \frac{1}{A}\left(\frac{\nu_{f31}}{C_f E_{f33}} - \frac{\nu_{m31}}{C_m E_{m33}}\right) \\ 0 & \frac{1}{C_f} & 0 \\ 0 & 0 & \frac{1}{C_f} \end{bmatrix} \begin{Bmatrix} \sigma_{\ell 11} \\ \sigma_{\ell 22} \\ \sigma_{\ell 33} \end{Bmatrix} \qquad \text{A.12}$$

$$\begin{Bmatrix} \varepsilon_{m11} \\ \varepsilon_{m22} \\ \varepsilon_{m33} \end{Bmatrix} = \begin{bmatrix} \frac{1}{BE_{f11}\bar{k}_f} & \frac{1}{B}\left(\frac{\nu_{m21}}{C_m E_{m22}} - \frac{\nu_{f21}}{C_f E_{f22}}\right) & \frac{1}{B}\left(\frac{\nu_{m31}}{C_m E_{m33}} - \frac{\nu_{f31}}{C_f E_{f33}}\right) \\ 0 & \frac{1}{C_m} & 0 \\ 0 & 0 & \frac{1}{C_m} \end{bmatrix} \begin{Bmatrix} \sigma_{\ell 11} \\ \sigma_{\ell 22} \\ \sigma_{\ell 33} \end{Bmatrix} \qquad A.13$$

where

$$A = (1/E_{f11} + \bar{k}_m/E_{m11}\bar{k}_f) \; ; \qquad B = (1/E_{m11} + \bar{k}_f/E_{f11}\bar{k}_m) \qquad A.14$$

$$C_f = (\bar{k}_f/k_f)\,\beta_f \qquad ; \qquad C_m = (\bar{k}_m/k_m)\,\beta_m \qquad A.15$$

The parameters β_f and β_m are chosen such that theory and experiment correlate. When computing the ply thermal coefficients of expansion from Eq. A.9 use

$$\tilde{C}_f = (\bar{k}_f/k_f)\,\tilde{\beta}_f \qquad ; \qquad \tilde{C}_m = (\bar{k}_m/k_m)\,\tilde{\beta}_m \qquad A.16$$

instead of Eq. A.15 in Eqs. A.12 and A.13. Here $\tilde{\beta}_f$ and $\tilde{\beta}_m$ are chosen such that theory and experiment correlate for the thermal coefficients of expansion.

The ply shear moduli are given respectively by

$$G_{\ell 12} = \frac{G_{m12}}{\frac{G_{m12}}{C'^2_f G_{f12}}\,\bar{k}_f + \frac{\bar{k}_m}{C'^2_m}} \qquad A.17$$

$$G_{\ell 13} = \frac{G_{m13}}{\frac{G_{m13}}{C'^2_f G_{f13}}\,\bar{k}_f + \frac{\bar{k}_m}{C'^2_m}} \qquad A.18$$

$$G_{\ell 23} = \frac{G_{m23}}{\frac{G_{m23}}{C''^2_m G_{f23}}\,\bar{k}_f + \frac{\bar{k}_m}{C'^2_m}} \qquad A.19$$

where $C'_f = (\bar{k}_f/k_f)\,\beta'_f$; $C'_m = (\bar{k}_m/k_m)\,\beta'_m$ A.20

and $C''_f = (\bar{k}_f/k_f)\,\beta''_f$; $C''_m = (\bar{k}_m/k_m)\,\beta''_m$ A.21

The parameters β'_f, β'_m, β''_f and β''_m are empirical factors and are to be chosen such that predicted and experimental results for the shear moduli correlate. The empirical factors β'_f and β'_m are to be chosen from horizontal-shear-beam tests where the filaments are parallel to the beam axis. The empirical factors β_f'' and β_m'' are to be chosen from horizontal-shear-beam tests where the filaments are normal to the beam axis. It is noted that the condition

$$G_{\ell 23} = \frac{E_{\ell 33}}{2(1+\nu_{\ell 32})} = \frac{E_{\ell 22}}{2(1+\nu_{\ell 23})} \qquad \text{A.22}$$

should be satisfied when the ply possesses transverse isotropy in the 2-3 plane. For this case

$$E_{\ell 33} = E_{\ell 22} \quad \text{and} \quad \nu_{\ell 32} = \nu_{\ell 23} \qquad \text{A.23}$$

The hydrostatic ply bulk modulus is given by

$$\frac{1}{K_\ell} = \left(\frac{1}{E_{\ell 11}} + \frac{1}{E_{\ell 22}} + \frac{1}{E_{\ell 33}}\right) - 2\left(\frac{\nu_{\ell 21}}{E_{\ell 11}} + \frac{\nu_{\ell 23}}{E_{\ell 22}} + \frac{\nu_{\ell 13}}{E_{\ell 11}}\right) \qquad \text{A.24}$$

The ply heat capacity is given by

$$H_{C\ell} = \frac{1}{\rho_\ell}\left[H_{Cf}\,\rho_f\,\bar{k}_f + H_{Cm}\,\rho_m\,\bar{k}_m\right] \qquad \text{A.25}$$

where ρ_ℓ is the ply weight density and is given by

$$\rho_\ell = \left[\rho_f\,\bar{k}_f + \rho_m\,\bar{k}_m\right] \qquad \text{A.26}$$

The heat conductivity of the matrix containing randomly distributed spherical voids is given by

$$\bar{K}_{m\alpha\alpha} = K_{m\alpha\alpha}\left[\frac{2\beta_{kv}\,K_{m\alpha\alpha} + K_v - 2k_v(K_{m\alpha\alpha} - K_v)}{2K_{m\alpha\alpha} + K_v - k_v(K_{m\alpha\alpha} - K_v)}\right] \qquad \text{A.27}$$

where K_v denotes the air conductivity and the parameter β_{kv} is the empirical factor which is to be chosen such that predicted and experimental results correlate. The subscripts ($\alpha\alpha$) refer to the orthotropic axes of the matrix. The heat conductivities of the ply are given by:

$$K_{\ell 11} = \beta_{k1}\,\bar{k}_f\,K_{f11} + k_m\,\bar{K}_{m11} \qquad \text{A.28}$$

$$K_{\ell 22} = \overline{K}_{m22} \left[1 - \beta_{k2}\sqrt{\overline{k}_f} + \frac{1}{\dfrac{1}{\beta_{k2}\sqrt{\overline{k}_f}} - \left(1 - \dfrac{\overline{K}_{m22}}{K_{f22}}\right)}\right]$$

$$K_{\ell 33} = \overline{K}_{m33} \left[1 - \beta_{k3}\sqrt{\overline{k}_f} + \frac{1}{\dfrac{1}{\beta_{k3}\sqrt{\overline{k}_f}} - \left(1 - \dfrac{\overline{K}_{m33}}{K_{f33}}\right)}\right]$$

It is important to note that k_m is used in Eq. A.27 instead of $\overline{k}_m$ since $\overline{K}_{m\alpha\alpha}$ includes the void effects. The parameters β_{k1} etc. are semiempirical factors and are chosen such that predicted and experimental results correlate.

Table 1: Mechanical and Thermal Properties of Constituents

Property	Units	Boron-Resin Filament	Boron-Resin Matrix	Carbon-Resin Filament (TH-25)	Carbon-Resin Filament (TH-40)	Carbon-Resin Matrix	Glass-Resin Filament	Glass-Resin Matrix
ρ	lb/cu.in.	0.085	0.040	0.052	0.056	0.040	0.090	0.040
E_{11}	10^6 psi	60.0	0.60	25.0	40.0	0.50	10.6	0.50
E_{22}	10^6 psi	60.0	0.60	2.0	1.1*	0.50	10.6	0.50
E_{33}	10^6 psi	60.0	0.60	2.0	1.1*	0.50	10.6	0.50
ν_{12}	Ratio	0.20	0.35	0.20	0.20	0.35	0.22	0.35
ν_{23}	Ratio	0.20	0.35	0.25	0.25	0.35	0.22	0.35
ν_{13}	Ratio	0.20	0.35	0.20	0.20	0.35	0.22	0.35
G_{12}	10^6 psi	25.0	0.22	2.0	1.50*	0.185	4.34	0.185
G_{23}	10^6 psi	25.0	0.22	0.80	0.80*	0.185	4.34	0.185
G_{13}	10^6 psi	25.0	0.22	2.0	1.50*	0.185	4.34	0.185
α_{11}	10^{-6}in/in-°F	2.8	32.0	-0.55	-0.55*	43.8	2.8	32.0
α_{22}	10^{-6}in/in-°F	2.8	32.0	-5.56	-5.60*	43.8	2.8	32.0
α_{33}	10^{-6}in/in-°F	2.8	32.0	5.56*	5.60*	43.8	2.8	32.0
H_c	Btu/lb-°F	0.31	0.25	0.17	0.225	0.25	0.17	0.25
K_{11}	$\frac{Btu}{Hr\text{-}ft^2\text{-}°F/in}$	22.3	1.70	580.0	580.0	1.25	20.8	1.70
K_{22}	$\frac{Btu}{Hr\text{-}ft^2\text{-}°F/in}$	22.3	1.70	58.0*	58.0*	1.25	20.8	1.70
K_{33}	$\frac{Btu}{Hr\text{-}ft^2\text{-}°F/in}$	22.3	1.70	58.0*	58.0*	1.25	20.8	1.70

* Estimates

Table 2: Ply-Basic-Elements and Empirical Factors for Elasto-Thermal Behavior

Property	Boron/Resin	Carbon/Resin		Glass/Resin
		TH-25	TH-40	
N_f	1	1440	1420	204
d_f	0.004 IN	0.000291 IN	0.00027 IN	0.00036 IN
k_v	0	0	0	0
k_f	$0.3<k_f<0.8$	$0.3<k_f<0.8$	$0.3<k_f<0.8$	$0.3<k_f<0.8$
β_f	1.0	1.0	1.0	1.0
β_m	$(1.0/k_m)1/4$	$(1.0/k_m)1/4$	$(1.0/k_m)1/4$	$(1.0/k_m)1/4$
β'_f	1.0	1.0	1.0	1.0
β'_m	$(1.0/k_m)1/2$	$(1.0/k_m)1/2$	$(1.0/k_m)1/2$	1.60
β''_f	1.0	1.0	1.0	1.0
β''_m	$(1.0/k_m)1/4$	$(1.0/k_m)1/4$	$(1.0/k_m)1/4$	$(1.0/k_m)1/4$
$\tilde{\beta}_f$	1.0	1.0	1.0	1.0
$\tilde{\beta}_m$	$(1.0/k_m)1/2$	$(1.0/k_m)1/2$	$(1.0/k_m)1/2$	$(1.0/k_m)1/2$
β_{k1}	1.0*	1.0	1.0	1.0
β_{k2}	1.0*	1.05	1.05	0.90
β_{k3}	1.0*	1.05	1.05	0.90
β_{kv}	1.0*	1.0*	1.0*	0.90

The Fractions Following the Parantheses Are Exponents

* Estimates - (Modify as needed when experimental results become available.)

Table 3: Effect of Voids on Ply Heat Conductivities

		$K_{\ell 22}$ Btu/hr-ft^2-°F/in			
		Reference 21		Semiempirical Theory	
k_f	k_v	Theor.	Exp.	β_{kv}=0.9	β_{kv}=1.0
0.58	0.17	2.1	2.5	2.1	2.3
0.70	0.28	1.9	2.2	2.2	2.2
0.58	0.17	2.1	2.3	2.1	2.3
0.70	0.25	2.2	1.9	2.1	2.3
0.56	0.16	2.2	1.9	2.0	2.3

Note: The constituent thermal properties from which the semiempirical theory results were computed, are from Table 1, Reference 21.

Table 4: Semiempirical Theory Verification Through Plate Buckling Loads

Plate	Thickness Inches	Fiber Content	BUCKLING LOAD Measured Ref. 24	Predicted Ref. 29 **	***
20(0°)	0.096	0.572	278	349	300
20(+30°)*	0.106	0.517	394	449	461
20(+45°)	0.095	0.586	372	493	446
20(+60")	0.106	0.517	433	417	430
20(90°)	0.096	0.572	251	264	239
10(±30°),10(∓30°)	0.110	0.482	662	606	632
10(±45°),10(∓45°)	0.101	0.570	602	717	698
10(±60°),10(∓60°)	0.110	0.482	661	609	630

* Known Ply Properties

** With k_f = 0.517 Constant for all Plates

*** With k_f as given in the third column

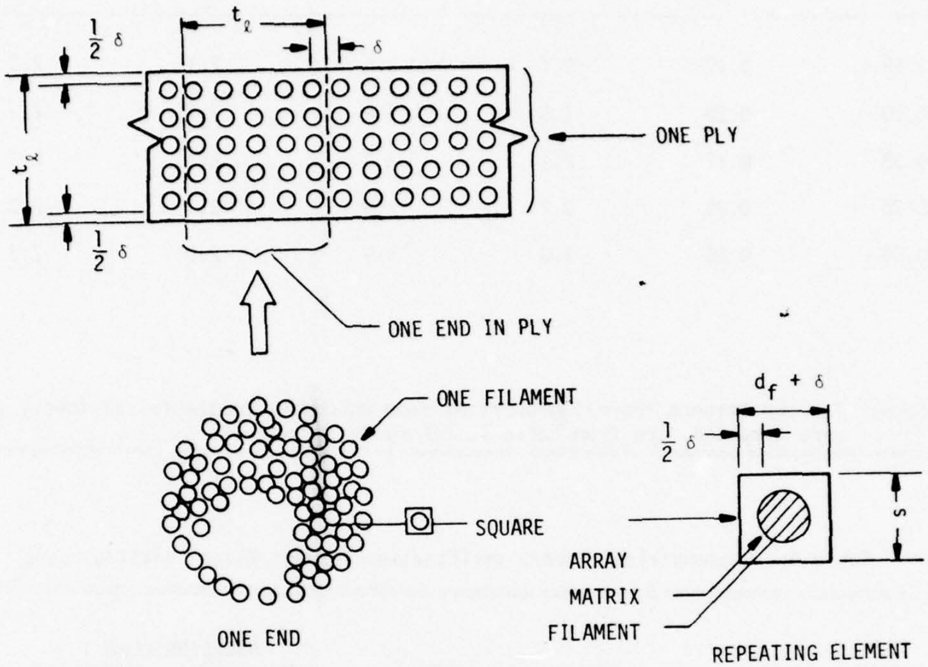
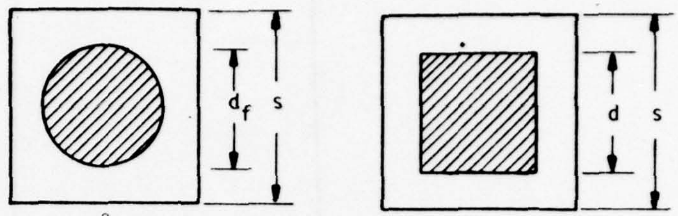


FIGURE 1. SCHEMATIC OF PLY



where $d^2 = \frac{\pi d_f^2}{4} = \bar{k}_f s^2 \rightarrow (d/s) = \sqrt{\bar{k}_f} \rightarrow \bar{k}_f^q$ { FOR DEGREE OF FREEDOM

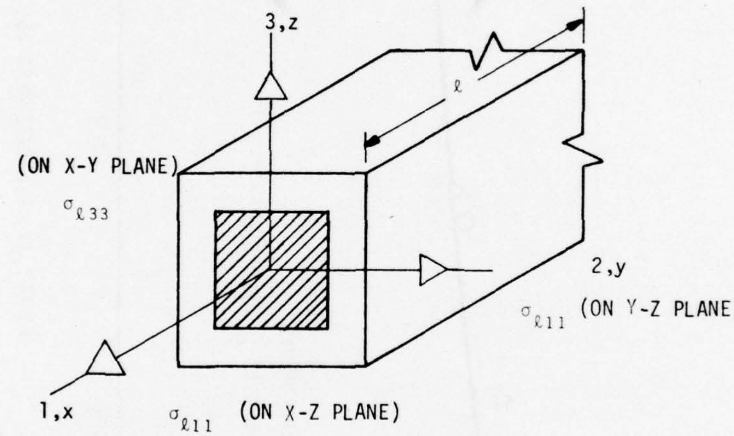


FIGURE 2. TYPICAL PLY UNIT CELL

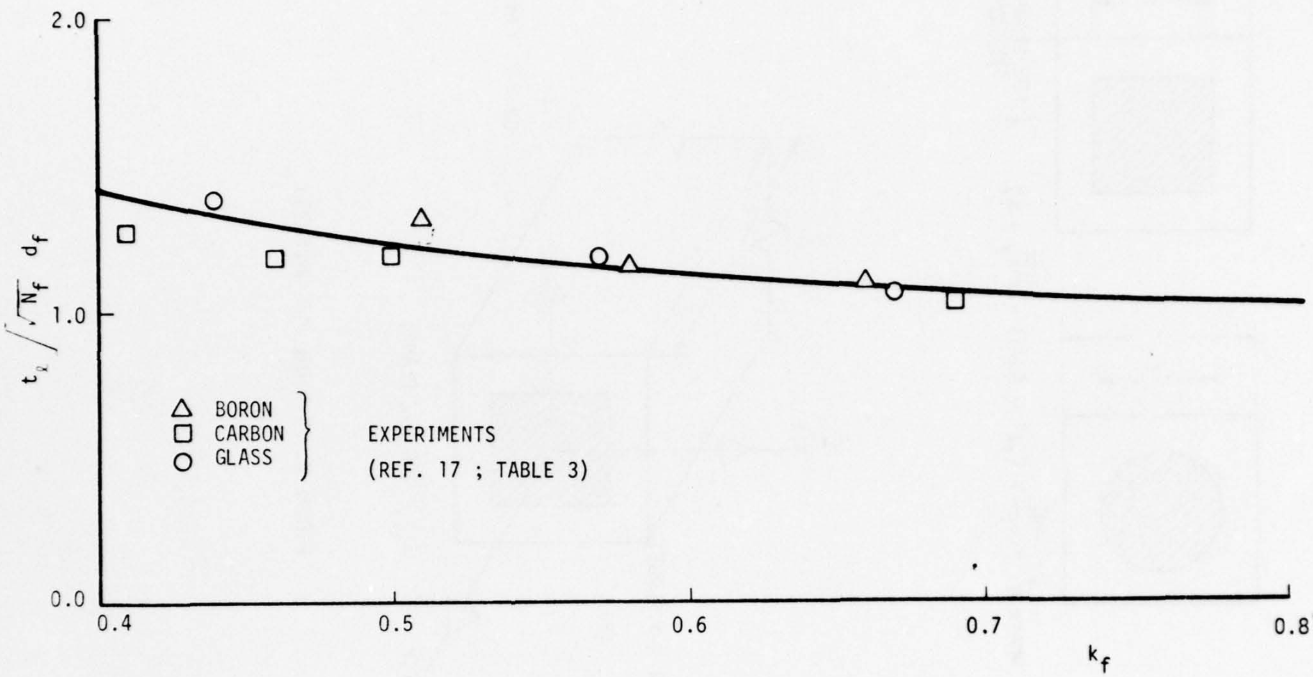


FIGURE 3. PLY THICKNESS VS. FIBER CONTENT

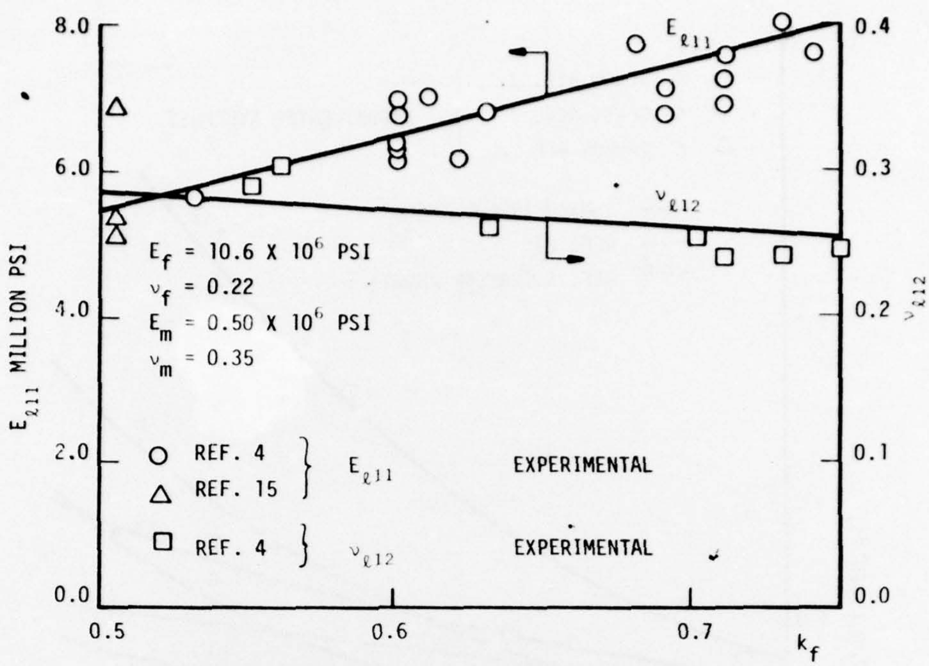


FIGURE 4. MODULUS AND POISSON RATIO (GLASS-EPOXY)

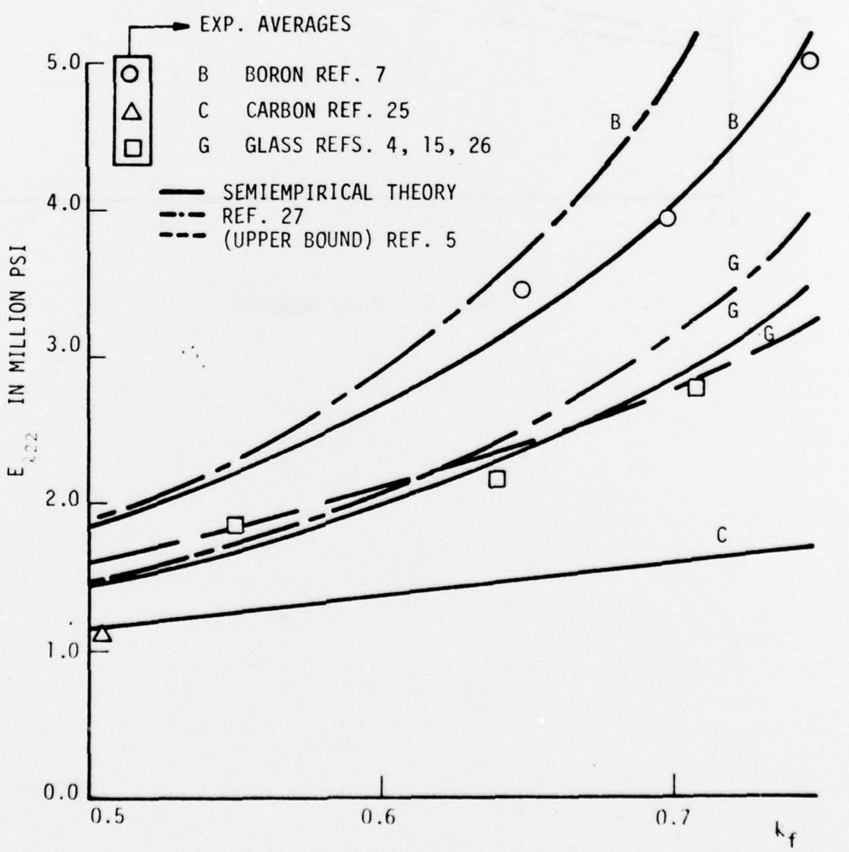


FIGURE 5. TRANSVERSE MODULUS

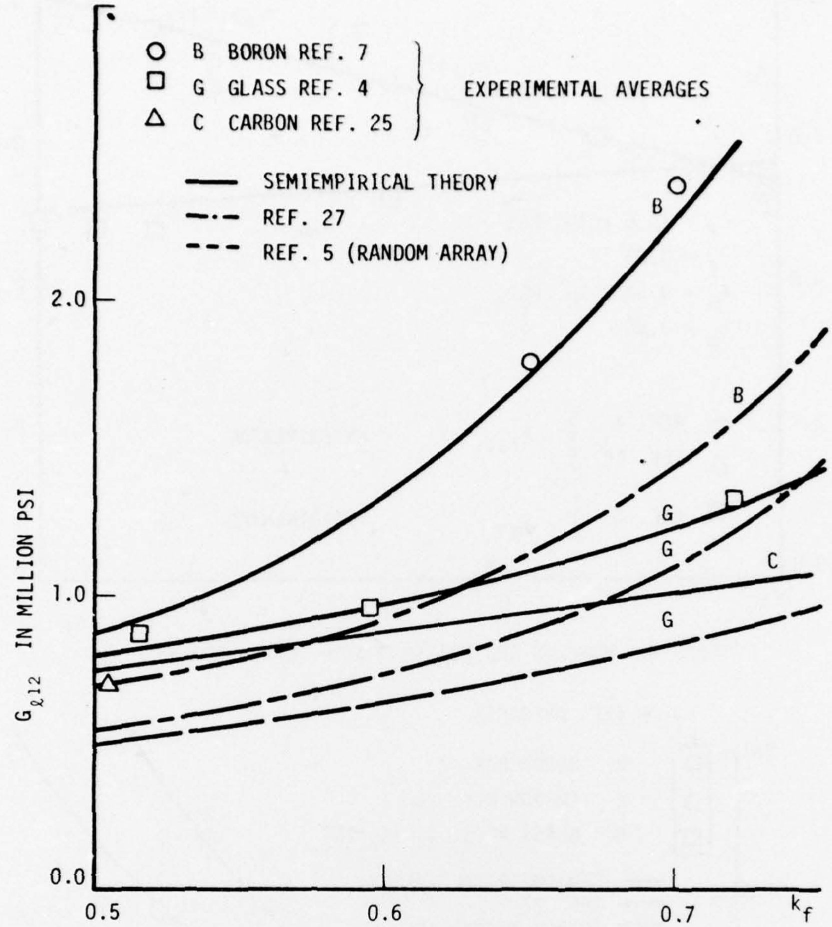


FIGURE 6. SHEAR MODULUS

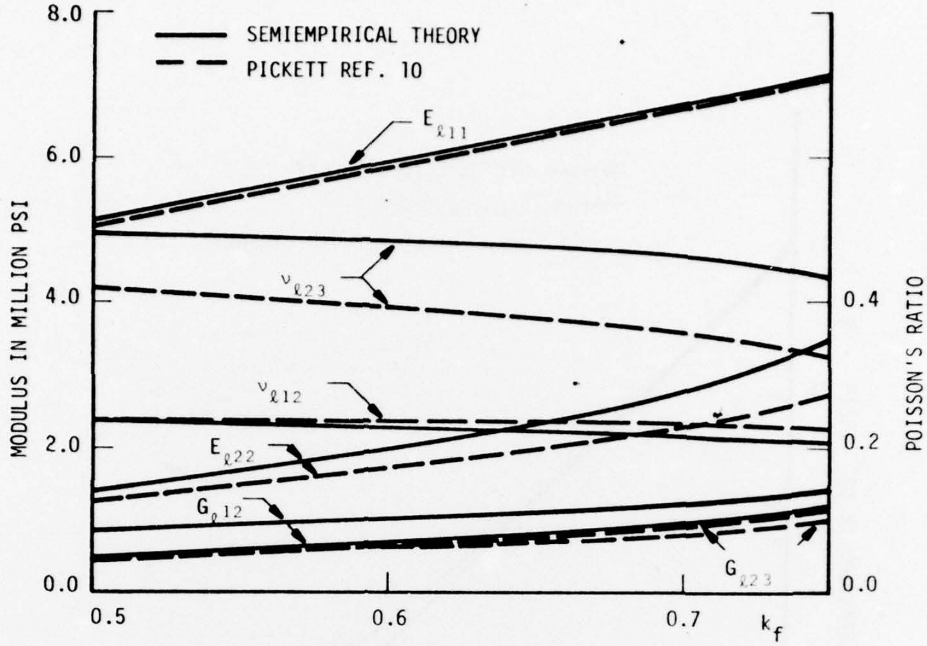


FIGURE 7. PLY ELASTIC PROPERTIES (GLASS-EPOXY)

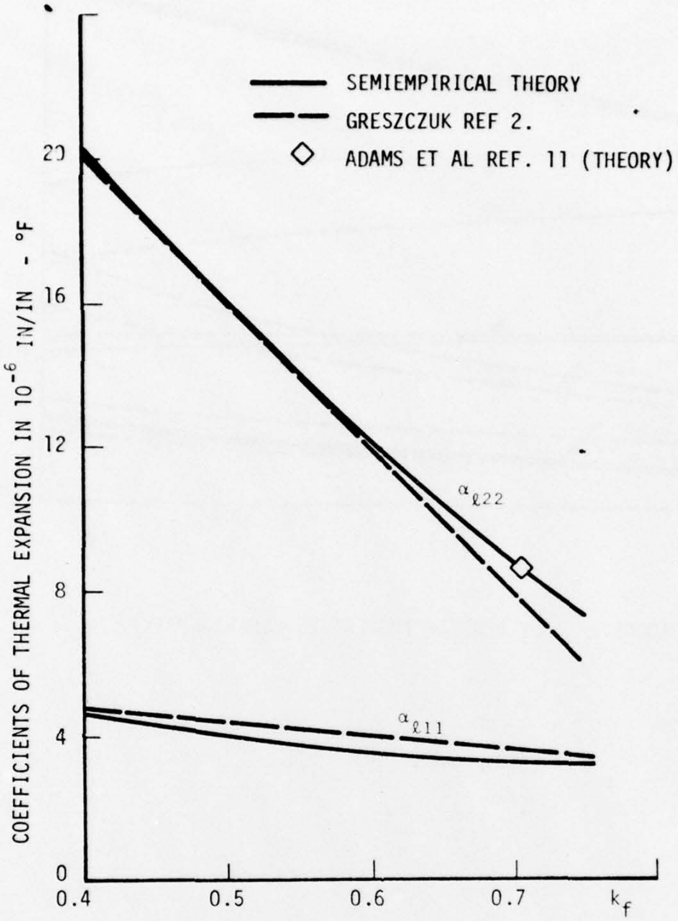


FIGURE 8. THERMAL COEFFICIENT OF EXPANSION (GLASS-EPOXY)

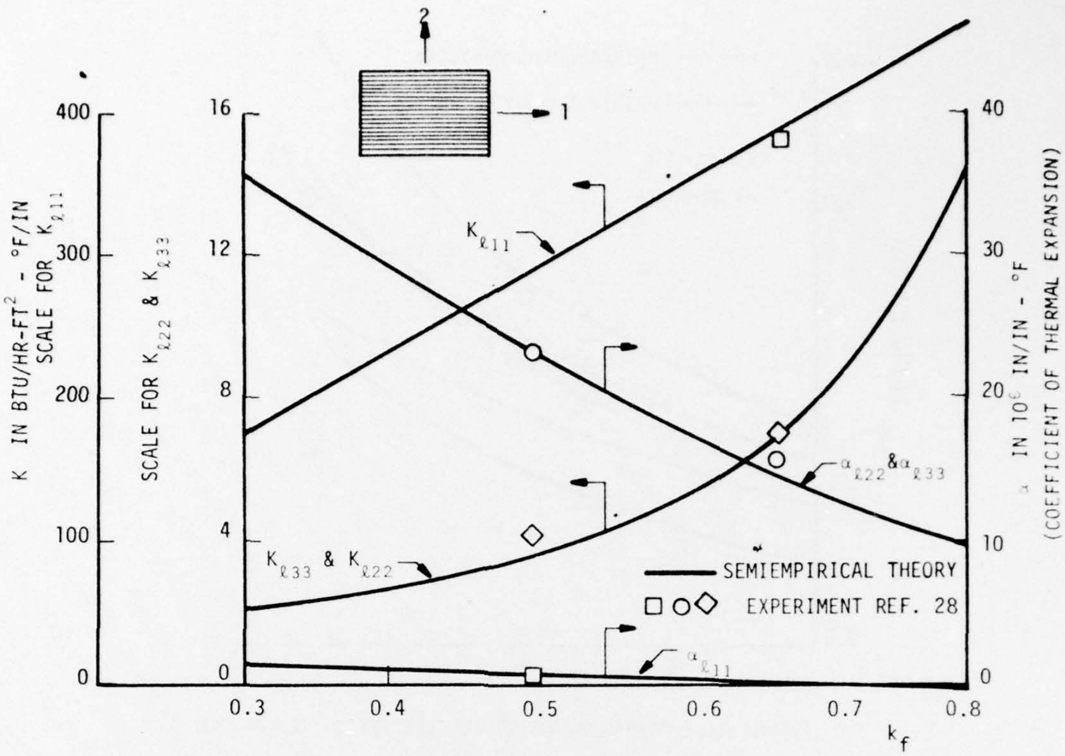


FIGURE 9. THERMAL PROPERTIES (CARBON-EPOXY)

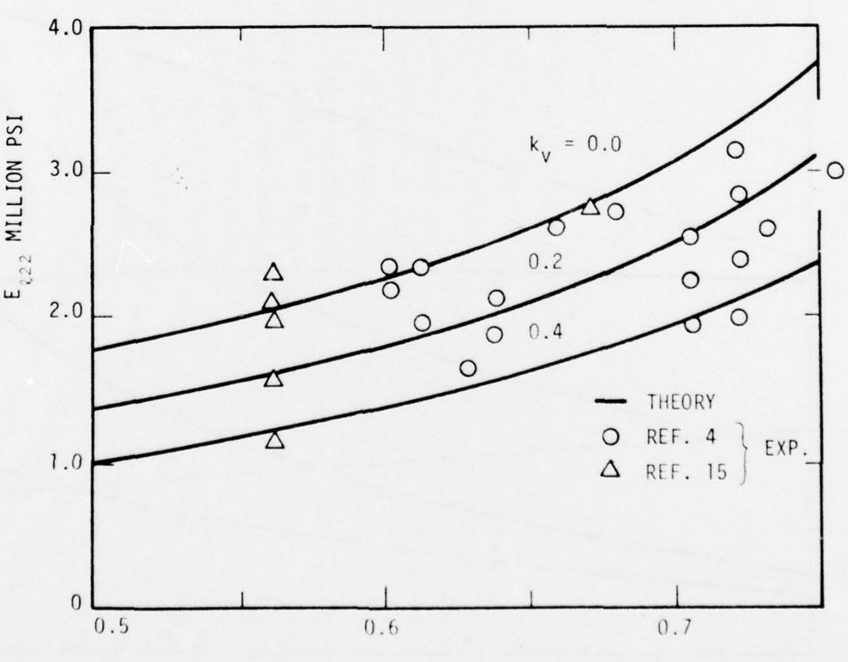


FIGURE 10. VOID EFFECTS ON TRANSVERSE MODULUS (GLASS-EPOXY)

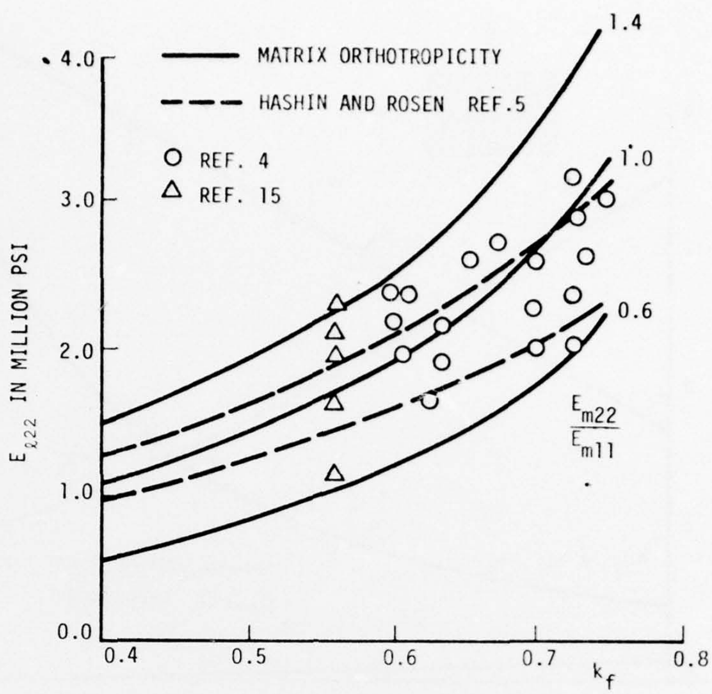


FIGURE 11. MATRIX ORTHOTROPICITY EFFECTS (GLASS-EPOXY)

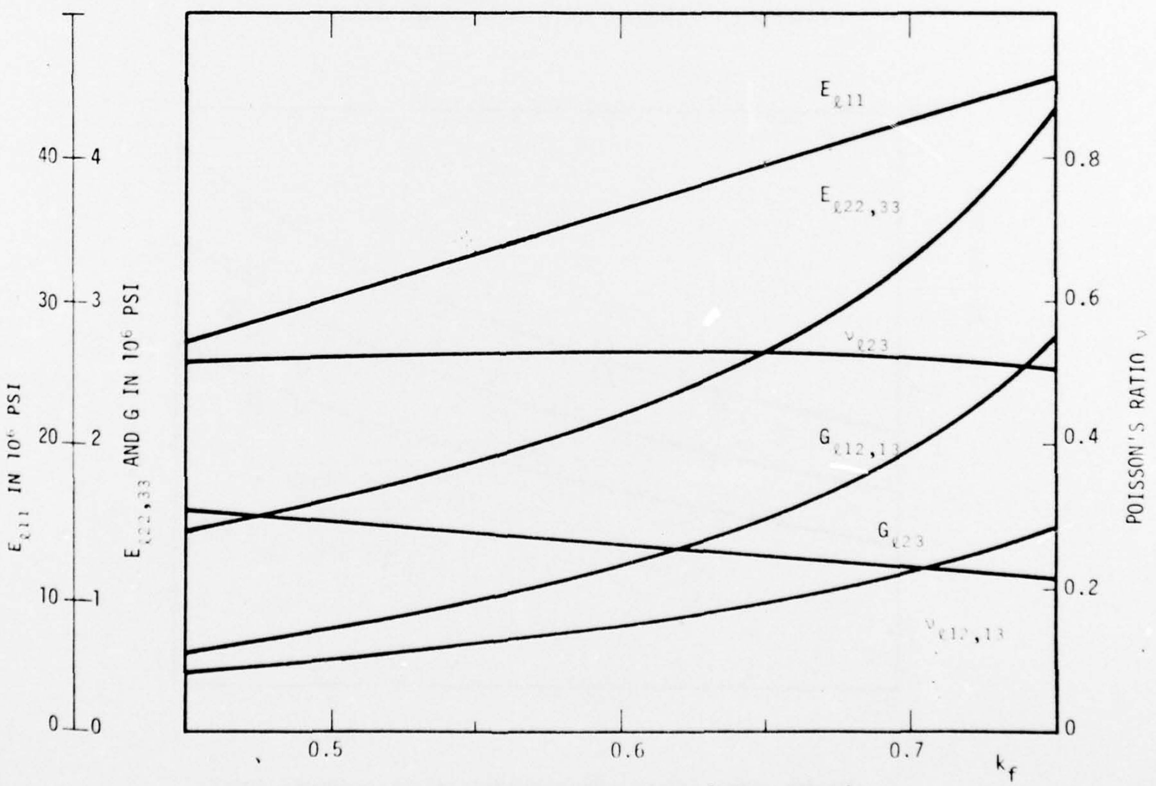


FIGURE 12. PLY ELASTIC PROPERTIES (BORON-EPOXY)

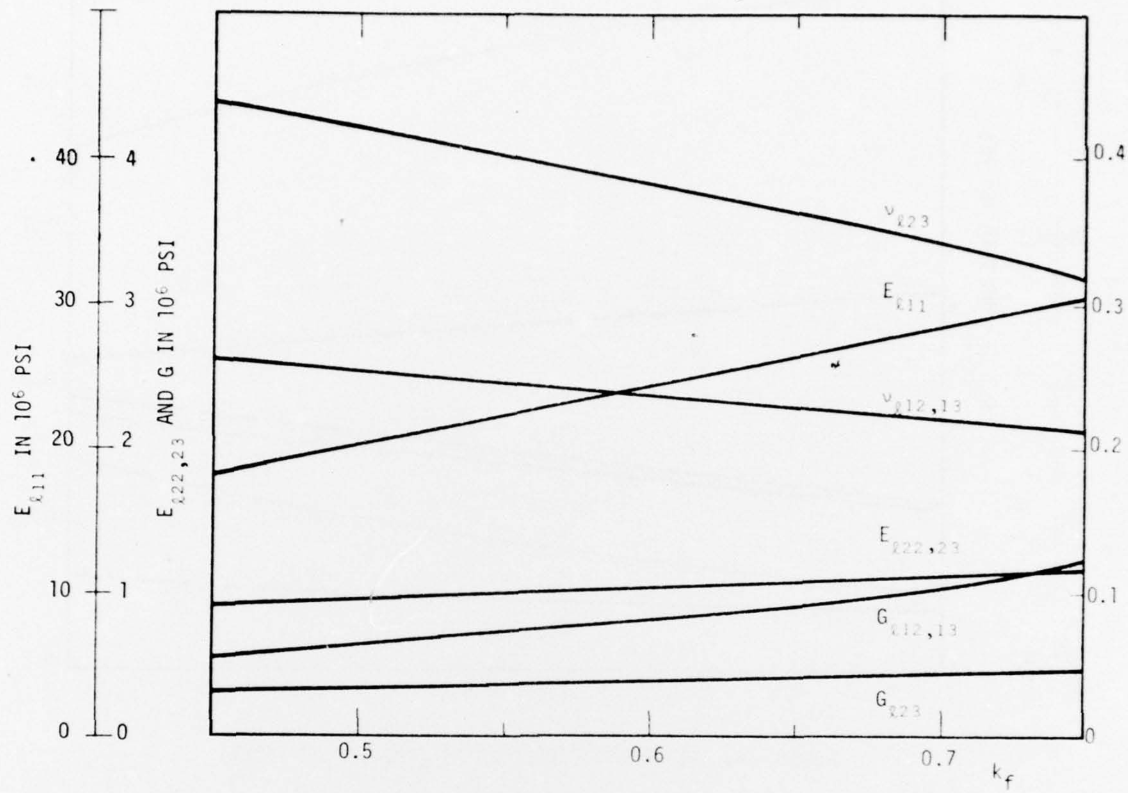


FIGURE 13. PLY ELASTIC PROPERTIES (THORNEL-40-EPOXY)

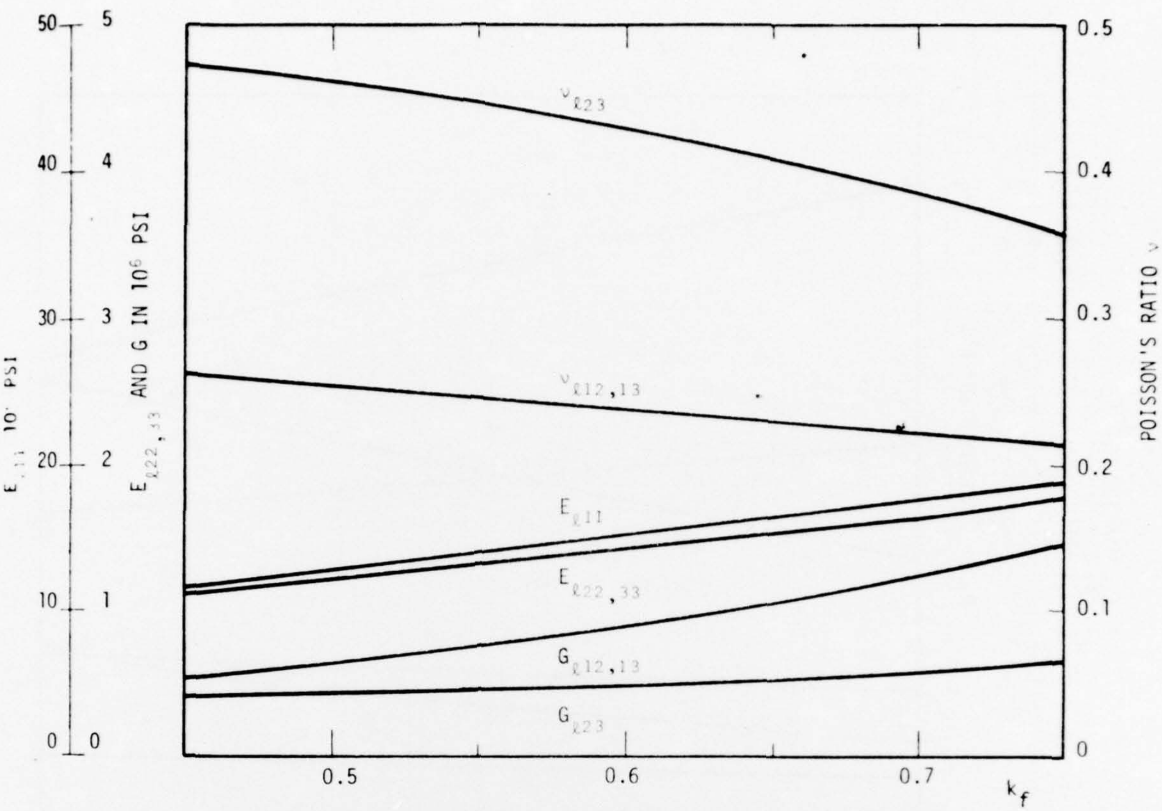


FIGURE 14. PLY ELASTIC PROPERTIES (THORNEL 25-EPOXY)

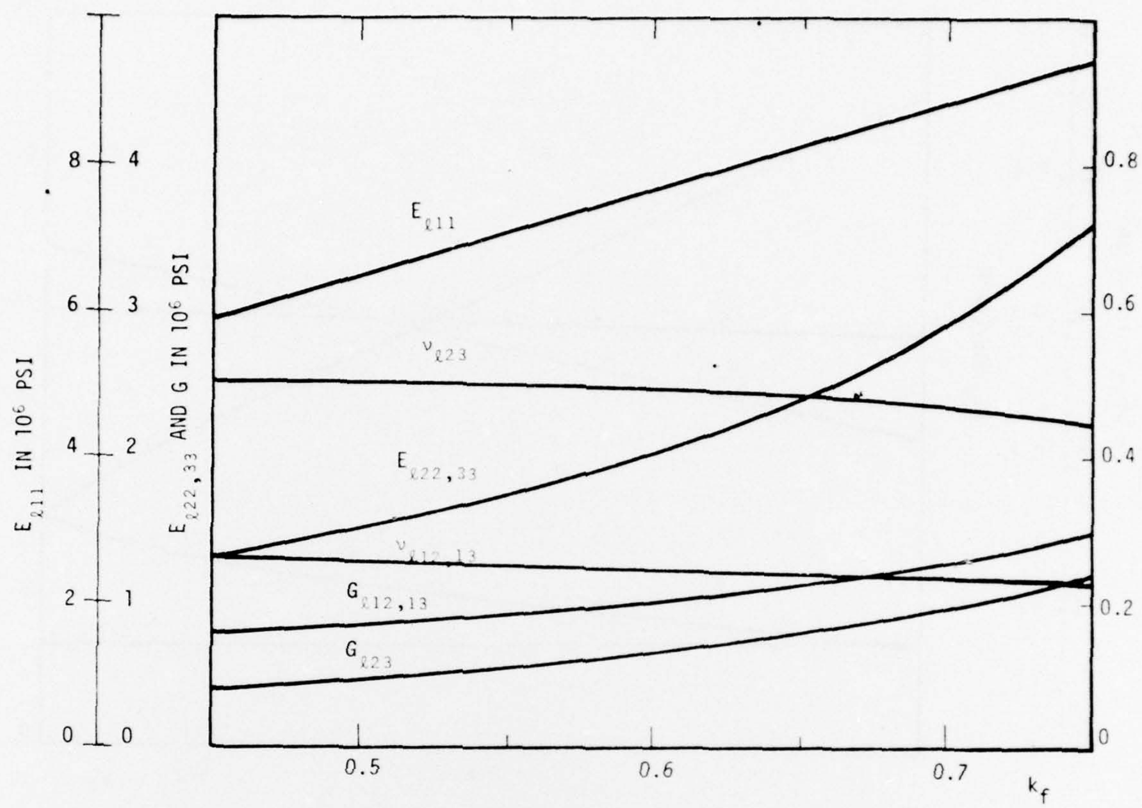


FIGURE 15. PLY ELASTIC PROPERTIES (S-GLASS-EPOXY)

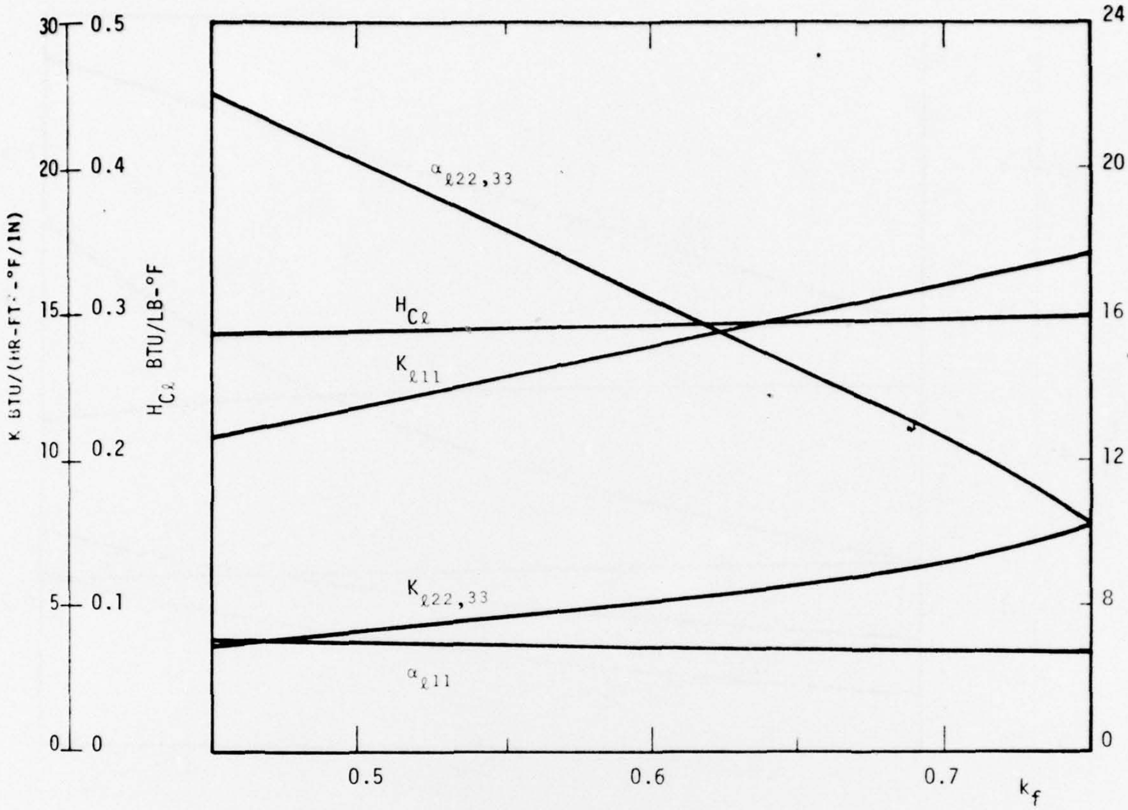


FIGURE 16. PLY THERMAL PROPERTIES (BORON-EPOXY)

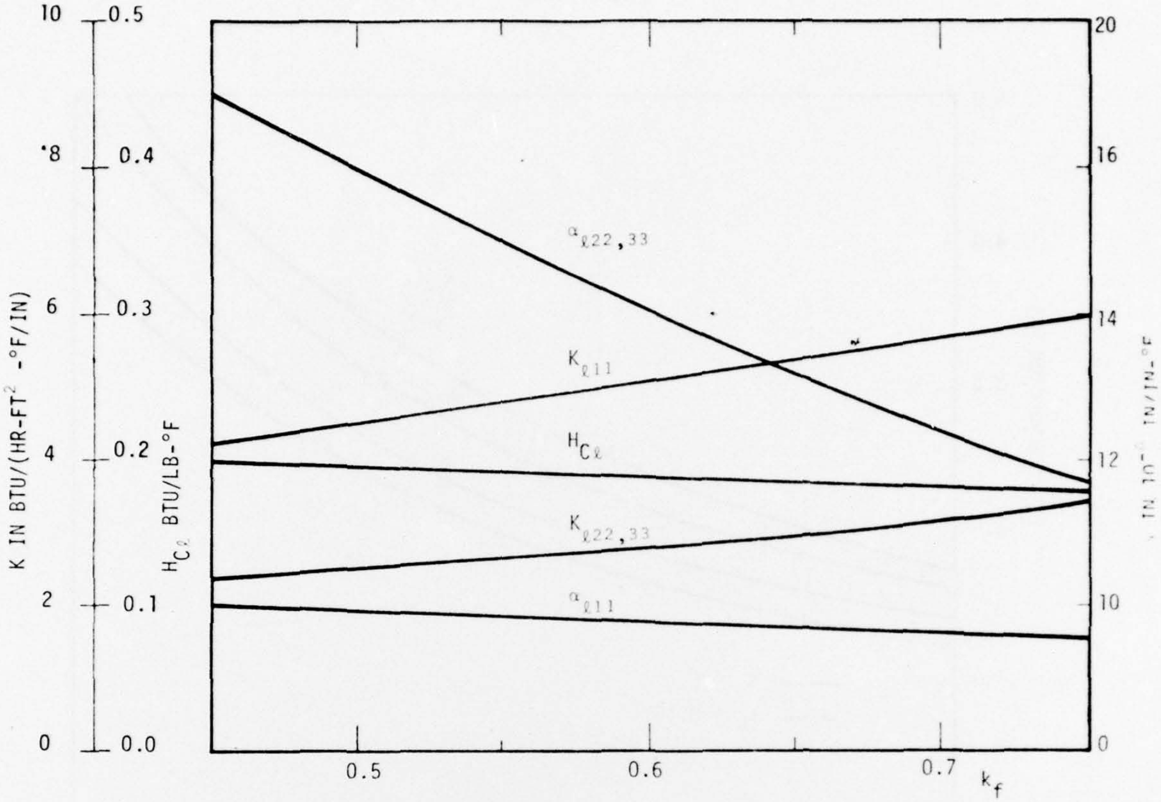


FIGURE 17. PLY THERMAL PROPERTIES (S ~ E-GLASS EPOXY)

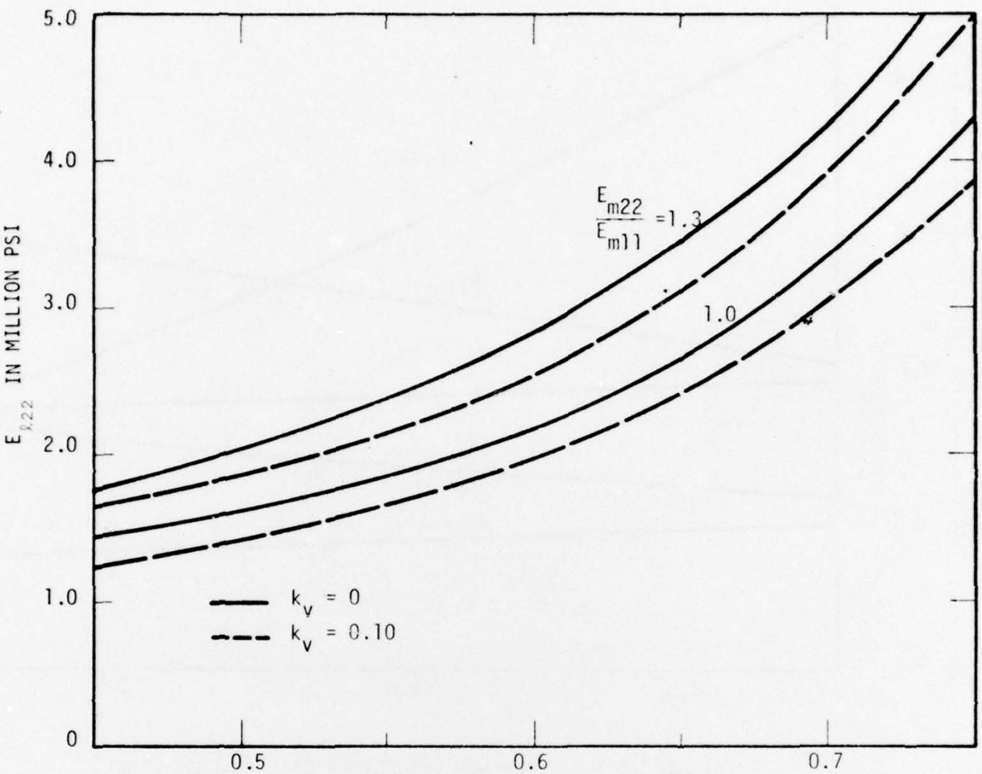


FIGURE 18. MATRIX ORTHOTROPICITY AND VOID EFFECTS (BORON-EPOXY)

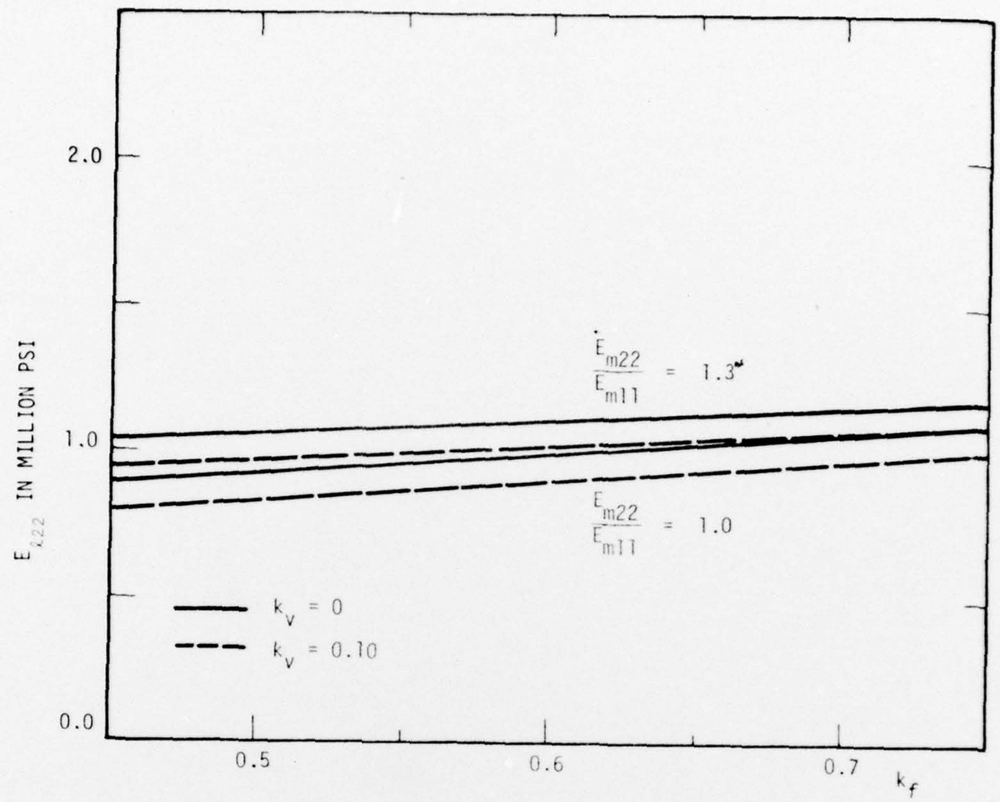


FIGURE 19. MATRIX ORTHOTROPICITY AND VOID EFFECTS (THORNEL-40-EPOXY)