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(6) THE J INTEGRAL. A REVIEW OF THEORY AND A SUGGESTED METHOD FOR J_c DETERMINATION

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ABSTRACT

Following a review of the theoretical justification for the use of J_c as an elastic-plastic fracture criterion, the report suggests a new method for its laboratory determination. Advantages of the new method include: unambiguous continuity with K_c; full compatibility with the existing standard for COD testing; and an ability to deal with slow crack growth and the effects of lateral constraint. The report concludes with an outline of future work and detailed instructions for the determination of J_c in a 3pt bend specimen with a crack length to width ratio of between 0.25 and 0.6.

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NOMENCLATURE

A	in-plane area of a body (used in formulation of J)
a	crack length
B	material thickness
C_n	constraint factor used to describe effective elevation of yield stress in 3pt bending
E	Youngs modulus
E^1	$E/(1-\nu^2)$ plane strain, = E plane stress)
$f_{ij}(\theta)$, $g_{ij}(\theta)$, $h_{ij}(\theta)$,	dimensionless functions used in describing crack tip stress-strain distributions
F	ratio of load point displacement q, to mouth opening displacement V
G	available energy release rate in a unit thickness, linear elastic, cracked body
$h(\frac{a}{w})$	polynomial used to describe mouth opening compliance
J	J integral, or rate of change in absorbed energy per unit thickness with crack length for specimens loaded monotonically to a given applied displacement
J_c	value of J at unstable fracture
J_e, J_p	elastic and plastic components of J
K	linear elastic stress intensity factor
K_{Ic}	critical value of K in plane strain
K_f	value of K at final load
k	plastic stress intensity factor
n	work hardening exponent
P	potential energy
Q	load
Q_L	limit load or net section yielding load
Q_f	final load
q	load point displacement
r	constant used to describe centre of rotation during plastic deformation
r,θ	crack tip polar co-ordinate system
S	distance along perimeter of cracked body
s	distance along a contour path (used in formulation of J integral)
T	external boundary forces
U	area under the load/load point displacement curve, or total absorbed energy
U_{crack}	component of U due to the presence of the crack

U_e, U_p	elastic and plastic components of U
V	mouth opening displacement
V_g	displacement of knife edges positioned at crack mouth
W	width of specimen
w	strain energy density
x,y	cartesian co-ordinate system
Y	geometry factor used in calculating K
z	height of knife edges above specimen surface

Greek Symbols

α	constant used in work hardening relationship
β	geometry dependent constant used in slow crack growth correction
Γ	crack tip integration path (used in formulation of J)
δ	crack opening displacement (COD)
δ_c	critical value of δ (at unstable fracture)
δ_p	plastic component of δ
ϵ	strain
ϵ^p	plastic strain
θ	angular co-ordinate
η_e, η_p	geometry dependent constants used in linking J to absorbed energy
σ	stress
σ_y	yield stress
ν	poisson's ratio

Abbreviations

COD	crack opening displacement
CTS	compact tension specimen
LEFM	linear elastic fracture mechanics

THE J INTEGRAL. A REVIEW OF THEORY AND A SUGGESTED METHOD FOR J_c DETERMINATION

by

J D G Sumpter

INTRODUCTION

1. The objective of fracture mechanics analysis is to predict the behaviour of defects in a large complex structure from the results of a small, easily performed, laboratory test. Most attempts to do this are based on the concept of a fracture criterion, the critical value of which is postulated to be the same in both test piece and structure. The two most widely used fracture-safe design procedures are those based on linear elastic fracture mechanics (LEFM), fracture criterion K_{1c}, and on the crack opening displacement (COD) concept, fracture criterion δ_c.

2. Strict application of LEFM requires that crack tip plasticity be negligible in both test piece and structure. This requirement is met in practice by stipulating that (1)*

$$a,\ B,\ W\text{-}a > 2.5\left(\frac{K_{Ic}}{\sigma_y}\right)^2 \qquad [1]$$

where a is crack length, B is material thickness, W is specimen width and σ_y is the material yield strength. The ductility of most engineering materials is such as to make this condition an extremely restrictive one, and it is seldom possible to obtain a valid K_{Ic} at the thickness and temperature of interest. Most fracture analyses must consequently be based on a parameter which can be measured in a small test piece irrespective of the extent of plasticity at fracture.

3. The COD approach fulfils this requirement. The basic assumption of the method is that a crack deforms with a well defined opening at its tip and that fracture occurs when this discrete opening reaches a critical value δ_c. A standard method for determination of δ_c is laid down in (2). The test piece is conventionally a 3pt bend specimen, span to width ratio, $S/W = 4$, crack length to width ratio, $0.25 < a/w < 0.6$, and thickness to width ratio $B/W = 1.0$ or $B/W = 0.5$. COD is inferred from a plot of load against displacement, the latter being monitored by a clip gauge mounted across the mouth of the crack.

4. Although determination of δ_c from a laboratory test piece is reasonably straightforward, the use of COD in fracture-safe design is less satisfactorily established. The main difficulties arise from an inability to provide accurate

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*( ) = References on page 14-15

COD solutions when geometry is complex and stress gradient significant; and from the uncertain relationship between δ and K for near elastic behaviour. Suggested procedures for the use of COD in critical defect size analysis (3, 4) overcome these difficulties by the use of a universal design curve. This inevitably contains a degree of conservatism in some cases and there are advantages to be gained from using a fracture criterion which can provide explicit solutions in terms of crack size and stress distribution at any structural location. The J integral fulfils this requirement and has the added advantage of providing unambiguous compatibility with K for near elastic behaviour (paragraph 8). The use of J for fracture design is further discussed in (5, 6, 7).

5. No generally agreed procedure yet exists for determination of J_c although there is considerable interest in its use, especially in the US, where the E24 committee is formulating a draft standard. This report describes a method for obtaining J_c using the instrumentation and specimen geometry already standardised for COD testing. COD practice is followed further in defining J_c as the point of unstable fracture (which may, or may not, be preceded by stable crack growth) in a test piece having the same thickness as the structure of interst.

THEORETICAL BASIS FOR THE USE OF J AS A FRACTURE CHARACTERISING PARAMETER

6. It is reasonable to assume that fracture initiation from a sharp crack is governed by the very high stresses and strains which exist ahead of that crack. An ability to quantify this stress-strain field in terms of overall geometry and loading should consequently lead to an ability to predict the conditions under which fracture will occur. It is well known, that having made the assumption of perfectly linear elastic behaviour, it is possible, on a continuum mechanics scale at least, to provide just such a complete description of the near crack tip region.

$$\sigma_{ij}, \epsilon_{ij} = \frac{K}{\sqrt{2\pi r}} f_{ij}(\theta) \qquad [2]$$

where σ_{ij}, ϵ_{ij} are the cartesian co-ordinates of stress and strain at the crack tip; r, θ is a polar co-ordinate system centred at the crack tip; and K is the linear elastic stress intensity factor. Since $f_{ij}(\theta)$ is unique for $r \to 0$ (8), only the magnitude, K, of the $1/\sqrt{r}$ singularity is dependent on geometry and loading conditions. K consequently provides a single parameter description of the crack tip environment, and its critical value may be expected to provide a geometry invariant fracture criterion, as long as the assumption of linear elastic conditions at the crack tip is a valid one. As noted in paragraph 2, K has in fact been shown to have a geometry invariant critical value, K_{1c}, provided the size constraints of [1] are met. Outside these constraints the size of the crack tip plastic zone is such as to invalidate [2] and the critical value of K is no longer a material property.

7. In principle, it should be possible to extend the continuum mechanics description of the crack tip region to account for plastic deformation; but, in practice, elastic-plastic solutions are difficult to obtain and complete solutions are available only for some very specialised cases. For the important case of a crack under a remotely applied tensile stress field only an approximate solution exists (9, 10, 11). This gives the near tip ($r \to 0$) stress distribution well inside the plastic zone of a work hardening material following a deformation plasticity flow rule with no unloading (essentially non-linear elasticity in that stress and strain have a unique relationship). No limitation is placed on the size of the plastic zone, but finite boundary conditions are not explicitly accounted for. The important point to emerge is that, once again, as for the linear elastic case, there is a unique distribution of singular terms at the crack tip. For a material following a stress-strain behaviour of the form

$$\frac{\epsilon^P}{\epsilon_y} = \alpha\left(\frac{\sigma}{\sigma_y}\right)^n \qquad [3]$$

the near tip stress-strain distribution is of the form (10, 11)

$$\frac{\sigma_{ij}}{\sigma_y} = k\left(\frac{r}{a}\right)^{\frac{-1}{(n+1)}} g_{ij}(\theta)$$

$$\frac{\epsilon^P_{ij}}{\epsilon_y} = \alpha k^n \left(\frac{r}{a}\right)^{\frac{-n}{(n+1)}} h_{ij}(\theta) \qquad [4]$$

where ϵ^P is plastic strain, n is the work hardening exponent, k is a plastic stress intensity factor, and $g_{ij}(\theta)$ and $h_{ij}(\theta)$ are dimensionless functions detailed in (10, 11). Although correlations for k have been provided in some configurations (12) using specially formulated finite elements, it is more convenient to have the strength of the dominant singularity expressed in terms of a parameter more directly obtainable from the overall load deflection behaviour of the cracked body. The J integral provides a particularly convenient method of doing this.

8. For a body of unit thickness, composed of a linear or non-linear elastic material, the available energy release rate or crack extension force G is defined

$$G = \frac{-dP}{da} = \frac{d}{da}\left[\int_S T_i du_i dS - \int_A dw dA\right] \qquad [5]$$

where P is the potential energy, T_i and u_i are the forces and displacements on the body perimeter S, A is the area, and w is the strain energy density.

Suppose the body is described by a co-ordinate system x,y; by a transformation of co-ordinates, most easily seen for the steady state case where

$$\frac{-dP}{da} = \frac{\partial P}{\partial x}\bigg|_{y,a} \qquad [6]$$

together with application of Green's theorem to the term $\int_A \partial w dA$, the RHS of [5] can be expressed as a path independent line integral, designated J by Rice (13),

$$G = \frac{-dP}{da} = \int_{\Gamma} \left(w dy - \frac{T_i \partial u_i ds}{\partial x} \right) = J \qquad [7]$$

where s is the distance along any arbitrary contour path, Γ, surrounding the crack tip. It follows, that for linear elasticity, the identity between G and K (8) holds also between J and K.

$$J = \frac{K^2}{E^1} \qquad [8]$$

where E^1 = E plane stress

$$= \frac{E}{(1-\nu^2)} \text{ plane strain}$$

ν is Poisson's ratio.

9. The potential energy decrease which occurs during crack extension, Δa, may be conveniently visualised as the area between the overall load deflection curves of a body containing a crack length a, and that of a body containing a crack length (a+Δa). It can be shown that this area is the same irrespective of whether load or displacement are held constant during crack extension (13). This being so, [5] can be simplified by taking the energy change under fixed boundary displacements. The change in external work $\int_s T du_i ds$ is then zero, and only the stored energy, $U = \int_A w dA$, changes (figure 1). For the general case of a body thickness B

$$J = \frac{1}{B} \left(\frac{\partial U}{\partial a} \right)_q \qquad [9]$$

where q is the load point displacement. For a plastic material, where U is dissipated in irreversible plastic work, J can no longer be identified with the elastic energy release rate G (14), but it will still be the rate of decrease of area under the load/load point deflection curve with crack length. This makes it a relatively easy parameter to determine experimentally (paragraph 15).

10. The J integral with w and u representing total (elastic plus plastic) strain energy density and displacement will also retain path independence for a material following a deformation plasticity flow rule. This allows it to be regarded as a measure of the crack tip singularity revealed in (9, 10, 11) Evaluation of J as $r \to 0$ may be achieved by substitution of [4] into [7]. It follows that

$$\frac{\sigma_{ij}}{\sigma_y} = \left(\frac{JE}{I\sigma_y^2 a}\right)^{\frac{1}{(n+1)}} \left(\frac{r}{a}\right)^{\frac{-1}{n+1}} g_{ij}(\theta)$$

$$\frac{\epsilon_{ij}^P}{\epsilon_y} = \alpha\left(\frac{JE}{I\sigma_y^2 a}\right)^{\frac{n}{n+1}} \left(\frac{r}{a}\right)^{\frac{-n}{n+1}} h_{ij}(\theta) \qquad [10]$$

where I is a dimensionless constant (10, 11). [10] provides the basis for an elastic-plastic fracture mechanics approach analogous in form to LEFM. The hypothesis is: that there is a unique stress-strain distribution at the crack tip; that J characterises this stress-strain distribution; and that J_c might consequently be expected to provide a geometry invariant fracture criterion.

11. It should be emphasised that the above hypothesis can be criticised on a number of grounds. Firstly, the derivation of [10] is based on deformation flow rule plasticity. Real materials are better represented by an incremental flow rule (where strain is history dependent, not uniquely defined by the current value of stress). Differences between the two theories are only large when significant rotation of the principal stress system or unloading occurs, but this cannot be ruled out near the crack tip, especially if slow tearing precedes final fracture. Secondly, available slip line field solutions for different cracked body geometries would seem to indicate that a single parameter description of the crack tip may no longer be possible in the presence of different patterns of plastic deformation (15). It has been argued (16, 17) that work hardening will overide these differences and create a unique environment, or automonous zone, at the crack tip. Elastic-plastic finite element analysis, in so far as it can be expected to provide accurate answers in the near tip region, does not seem to support this view (18, 19).

12. A number of questions thus surround the theoretical basis for using J as a crack tip characterising parameter and ultimately, its usefulness as a fracture design tool is likely to be decided by experimental evidence. Before this can be done there is an obvious need to fix on a generally agreed procedure for J_c testing. There is moreover a requirement that, in order to encourage its widespread use, any proposed test method should be reasonably easy to perform and preferably fully compatible with existing standards for K_{1c} and δ_c testing (1, 2). The remainder of this report is devoted to suggesting such a method.

DETERMINATION OF J_c

Use of Finite Element Based Correlations

13. The contour integral definition of J [7] is well adapted to evaluation from elastic-plastic finite element analysis (20) and a large number of studies have been performed on test piece geometries. Such computations provide

correlations between J and applied displacement which should be directly applicable to fracture toughness testing. Figure 2 shows plane strain (21) and plane stress (22) results for J against mouth opening displacement V in the 3pt bend geometry as an example. Unfortunately, these correlations are rather inflexible since each one refers only to a specific geometry and does not in itself give any indication of how the value of J will be affected by minor variations away from this configuration. Crack length in particular cannot be specified a-priori in standard fracture toughness tests. It can moreover be shown that the various estimation procedures described below (paragraph 14) all show close agreement with computed results for J (23). This together with their greater flexibility means that they are more suitable than finite element based correlations for determination of J_c in standard test piece geometries. Numerical analysis still has a major role to play in the analysis of non-standard test pieces such as those containing very shallow notches, and for investigation of unusual structural configurations (23).

Correlations Between J and Area under the Load Deflection Curve

14. The most convenient basis for estimation of J in small test pieces is that provided by [9], where for an elastic-plastic material U is the total absorbed energy, or area under the load/load point deflection curve. Early attempts to establish J_c (16, 24) used a series of specimens with different initial notch depth to evaluate J directly from [9]; the possibility of obtaining J_c from a single specimen load deflection curve using an estimation procedure based on elastic compliance and slip line field considerations was later pointed out (25), and more recently (26) attention has focussed on the use of correlations between J and the total absorbed energy. The first of these (27), for specimens in which the uncracked ligament is subjected to bending, took the form

$$J = \frac{2\,U_{crack}}{(W-a)\,B} \qquad [11]$$

where U_{crack} denotes the component of energy absorbtion due to the presence of the crack. For deeply cracked ($a/w > 0.6$) 3pt bend and compact tension specimens, where the energy absorbtion component of the uncracked body is small it was suggested (26) that [11] be simplified to

$$J = \frac{2U}{(W-a)B} \qquad [12]$$

where U is once again the total area under the load/load point deflection curve.

15. A more general version of [11], which does not involve subtraction of the uncracked body energy and is applicable to any geometry for which the elastic compliance and limit load are known, can be written in the form (23)

$$J = J_e + J_p = \frac{\eta_e U_e}{(W-a)B} + \frac{\eta_p U_p}{(W-a)B} \qquad [13]$$

J and U are here divided into elastic and plastic components (figure 3). In contrast with [11], U_e here includes the energy of the uncracked body. η_e and η_p are geometry dependent constants. η_e is based on the stress intensity factor, which, in turn, can be used to calculate J_e from [8], and U_e from load point compliance relationships (28). Table 1 shows η_e values for the 3pt bend and compact tension geometries based on stress intensity and compliance relationships given in (25) and (29) respectively. η_p may be calculated from a knowledge of the relationship between crack length and limit load Q_L. For example, for the 3pt bend specimen, provided that plasticity is confined to the uncracked ligament, the net section yielding (or limit) load is a function of ligament length squared

$$Q_L = C_n \sigma_y \frac{B}{S} (w-a)^2 \qquad [14]$$

where C_n is a constant. For a non-work hardening material

$$J_p = \frac{-1}{B}\left(\frac{\partial U_p}{\partial a}\right) \qquad [15]$$

$$U_p = Q_L q_p \qquad [16]$$

whence

$$J_p = 2C_n \sigma_y \frac{(W-a)}{S} q_p \qquad [17]$$

$$\text{or } J_p = \frac{2U_p}{(W-a)B} \qquad [18]$$

It follows that $\eta_p = 2$ for the 3pt bend geometry at all crack to width ratios greater than about $a/W = 0.2$. Limit load for the CTS is less well defined, but η_p might be expected to approach 2 for deeply cracked specimens where the deformation is predominantly bending.

TABLE 1

VALUES OF η_e FOR [13]

a/w	0.2	0.3	0.4	0.5	0.6	0.7
3pt bend S/W = 4	1.4	1.7	1.9	2.0	2.0	1.9
CTS H/W = .6	3.7	2.7	2.4	2.3	2.2	2.2

16. For the CTS (a/w > 0.6), and the 3pt bend geometry (0.4 < a/w < 0.7) where $\eta_e = \eta_p = 2$, [13] reduces to [12]. It should be re-emphasised, however, that the values of η_e in Table 1 do not involve subtraction of an uncracked body energy. There will therefore be a discrepancy between [13] and [11] when the uncracked body energy is significant, as it is in the 3pt bend geometry (30% of the total elastic at a/w = 0.5, 70% at a/w = 0.3). It has recently been suggested (30) that the source of this discrepancy is an error in the formulation of [11], but, at the time of writing the question has still to be completely resolved. The draft US standard for J testing (31) seems likely to adopt the specimen design and procedure suggested in (26), namely a deeply notched, (a/w > 0.6) CTS with the mouth of the notch machined away to allow for insertion of the clip gauge directly under the loading pins. This type of specimen is not always satisfactory, especially when testing weldments (32) and it remains to be seen whether the proposed specimen design is revised in the light of (30).

17. Application of [13], or its simplified version [12], requires measurement of load point displacement. This is not a quantity routinely measured in fracture toughness testing. The standard procedure (1, 2) is to measure crack mouth opening V. As noted above, it is possible for the CTS geometry, to relocate the clip gauge directly under the loading pins, thereby measuring load point displacement directly. This option is not available for the 3pt bend geometry. Rather than attempt a direct measurement of load point displacement in this specimen it is probably preferable to convert V to load point displacement q, by assuming a centre of rotation a fixed distance r(W-a) below the crack tip. So that, before application of [13], the area under a load/mouth opening deflection curve should be factored by F, where

$$F = \frac{q}{V} = \frac{W}{[a+r(W-a)]} \qquad [19]$$

It is well known from studies in support of the COD concept (33) that r is both load and geometry dependent. After limit load, however, r does achieve a fixed value for increments in q and V in a given geometry. Finite element results (22) for the 3pt bend geometry in plane strain indicate that this fixed value is 0.45 at a/W = 0.3, and 0.4 at a/W = 0.5. The plastic component of F can be calculated from [19] using these values. Modifying [19] for use with areas under a load/mouth opening displacement curve by employing separate values of F for elastic and plastic behaviour results in

$$J = \frac{F_e \eta_e U_{ve}}{(W-a)B} + \frac{F_p \eta_p U_{vp}}{(W-a)B} \qquad [20]$$

where U_{ve} and U_{vp} are the elastic and plastic areas under the load/mouth opening deflection curve. Relevant values for the 3pt bend specimen a/w = 0.3 and 0.5, and the CTS a/w = 0.5 are summarised in Table 2. F_e for the 3pt bend geometry comes from boundary collocation results for elastic mouth opening given in (34); F_e for the CTS comes from (35); and F_p for the CTS is calculated on the basis r = 0.4 (33).

TABLE 2

GEOMETRY FACTORS FOR USE WITH [20]

Specimen	3pt bend a/W = 0.3	3pt bend a/W = 0.5	CTS a/W = 0.5
F_e	2.45	1.65	0.70
$F_e \eta_e$	4.20	3.25	1.60
F_p	1.65	1.45	0.75
$F_p \eta_p$	3.25	2.85	1.50

A New Method for J_c Determination, based on Load, and Clip Gauge Plastic Displacement

18. Although [20] provides a reasonably satisfactory method of determining J it is worth questioning whether measurement of absorbed energy really provides the best basis for a new standard test method. The two fracture toughness tests at present in most widespread use are based on load (K_{1c}) and clip gauge measured mouth opening (δ_c). There is much to be said for providing as close a compatibility as possible with these established test procedures. At the linear elastic end of the range particularly, it is

essential that there should be continuity between J_c and K_c. Discrepancies between experimental and theoretically predicted elastic compliances can easily upset this agreement if K_c is determined in the usual manner from load whilst J_c is determined from area under the curve. There seems to be a strong case for forcing compatibility between K and the elastic component of J by use of [8] . Suppose the load at failure (figure 3) is Q_f. Then

$$J_e = \frac{K_f^2}{E^1} \qquad [21]$$

$$K_f = \frac{Q_f Y}{BW^{\frac{1}{2}}} \qquad [22]$$

where Y is a function of a/w (1).

19. Since J_e is to be evaluated from load in accordance with (1) it seems consistent to follow COD testing practice (2) and determine J_p from mouth opening displacement. This can be done by a straightforward substitution of q_p from [19] in [17].

$$J_p = 2\, C_n \sigma_y \frac{(W-a)}{S} F_p V_p \qquad [23]$$

Evaluation of C_n could be based on slip line field theory. For 3pt bending the value of this constant is about 1.5 (36, 37), but a better alternative, which does not involve the assumption that the analytical and experimental limit loads will coincide is to substitute for C_n from [14]

$$C_n = \frac{Q_L S}{B\sigma_y (W-a)^2} \qquad [24]$$

whence

$$J_p = \frac{2 Q_L F_p V_p}{(W-a)\ B} \qquad [25]$$

20. Strictly speaking, [25] applies only to a non-work hardening material, where Q_L is both the load at net section yielding and the maximum load reached. In this paper Q_L is used to signify the net section yielding (not the maximum) load in a work hardening material. In most geometries and materials this is denoted by an easily definable inflexion in the experimental load deflection curve. [13] indicates that J at a given deflection will be higher in a work hardening, than in a non-work hardening material, whilst [25], unless it includes a slight rise in net section yielding load, will not give this trend. Evidence on the effect of work hardening on J is not conclusive. The J estimation procedures for cracked test pieces in tension presented in (27) all predict an increase in J with work hardening, but finite element analysis on this

geometry (23) does not seem to exhibit this trend. Finite element results for 3pt bending show a similarly small effect (22). It would, of course, be fairly simple to replace Q_L in [25] by $(Q_L + Q_f)/2$, or even Q_f, but, at present, it seems more sensible to follow the usual lower bound philosophy of fracture toughness testing and retain [25].

21. It is noted in (26) that [12] over-estimates J when slow crack growth occurs. This point is illustrated in figure 4 in which a_i denotes the crack length at the start of the test, and a_f the crack length after slow growth Δa. In so far as J can be said to have any significance to crack tip conditions after slow growth, the absorbed energy U in [12] should properly be that appropriate to a crack length a_f monotonically loaded to a given deformation (shaded area in figure 4), not the area under the experimental load deflection curve, which represents a crack length a_i growing to length a_f during the course of the test. From [23], the value of J after slow growth is

$$J_p = 2C_n \sigma_y \frac{(W-a_f)}{S} F_p V_p \qquad [26]$$

The constant in [26] can be evaluated as before from [24], but, assuming no slow crack growth before limit load, the relevant crack length is a_i not a_f

$$C_n = \frac{Q_L S}{B\sigma_y (W-a_i)^2} \qquad [27]$$

$$J_p = \frac{2Q_L (W-a_f) F_p V_p}{B(W-a_i)^2} \qquad [28]$$

If there is no slow growth [28] reduces to [25]. If slow growth does take place [28] does not over-estimate J in the way that [12] does. In small specimens, where slow growth causes a significant change in elastic compliance, the measured value of V_p should be adjusted appropriately. This procedure is described more fully in the Appendix. F_p ahd J_e should, of course, be evaluated on the basis of a_f not a_i.

22. As shown in figure 2, there is, for 3pt bending, a significant difference between the plane stress and plane strain values of J. This difference is less pronounced in some other geometries, notably centre cracked tension (23), but in general, use of a finite element correlation, or of an estimation procedure of the type suggested in (25), requires a knowledge of whether the test

piece is in plane stress or, plane strain, or some intermediate condition. The use of [13] or [28] circumvents this problem since the lower value of J in plane stress automatically follows from the lower experimental limit load. The only assumption required is that the limit load varies as the square of the ligament length in bending (or directly with the ligament length for pure tension). In passing, it should be noted that finite element analysis (23) indicates that an equal value of J in plane stress or plane strain by no means ensures identical crack tip conditions.

RELATIONSHIP BETWEEN J AND COD IN 3pt BENDING

23. There are two recommended methods for evaluation of COD from mouth opening displacement (2). The first assumes a fixed centre of rotation a distance r(w-a) below the crack tip throughout the loading range

$$\delta = \left[\frac{r(w-a)}{a + r(w-a)}\right]V \qquad [29]$$

r is put equal to 0.33 on the basis of experimental investigations (33) to give lower bound values of δ. The second method due to Wells (38) uses an estimation procedure very similar to that proposed for J in (25). For elastic behaviour a relationship $\delta = G/2.1\sigma_y$ is assumed, G being linked to V through elastic compliance; at an assumed limit load, derived from slip line field theory, the relationship is changed to [29], but with r chosen as 0.45, again on the basis of slip line field theory.

24. There is an obvious similarity between COD defined by [29] and J_p defined by any of the mouth opening displacement related equations suggested above, in that both assume a centre of rotation a fixed distance r(w-a) below the crack tip. Considering for the moment the plastic component of COD, δ_p, [25] and [29] together give

$$\frac{J_p}{\delta_p} = \frac{2Q_L W}{rB\,(w-a)^2} \qquad [30]$$

or, assuming a specific value of r = 0.4 and Cn = 1.5, [23] and [29] give

$$J_p = 2\sigma_y \delta_p \qquad [31]$$

This relationship has been widely noted in the literature from finite element and other analysis on the 3pt bend geometry (21, 39, 40). The existence of a similar relationship between J and δ, for plane strain small scale yielding has been demonstrated (40, 41) and has also been suggested on a more intuitive basis by Wells (42). The assumption of $\delta = G/2.1\sigma_y$ in the Wells COD formula follows from this latter reference and means that near compatibility should exist between total values of J and δ defined in this way. There are, however, a number of factors, such as the non-agreement of the experimental limit load with the Wells predicted limit load which may upset this agreement in practice.

The relationship between total values of J and COD, defined with r = 1/3 in the elastic and plastic regimes, will in general be load dependent, varying from $J = 0.5\sigma_y\delta$ just outside the ASTM limit, to $J > 2\sigma_y\delta$ well after net section yield (23).

SUMMARY AND DISCUSSION

25. The proposed method for determination of J_c for a 3pt bend specimen is summarised in the Appendix. Few of the ideas used to derive this procedure are completely new and can be found in various forms in previous literature on the subject (25, 27). It is considered, however, that the method proposed here is an especially appealing one, both in its simplicity for general laboratory use and because of its compatibility and similarity with existing fracture toughness testing standards (1, 2). This similarity is an obvious advantage in gaining acceptance for a new testing method. Especially in the UK, where experience and confidence has been gained in the use of COD, there is an understandable reluctance to make radical changes in procedure. As shown here, J_c may be obtained as easily as δ_c, using identical test pieces and instrumentation. The two major reasons for favouring the use of J over δ are its unambiguous continuity with K for near elastic behaviour, and its greater flexibility and ease of definition in fracture-safe design procedures (5, 6, 7).

FUTURE WORK

26. A fairly extensive experimental program is needed to fully evaluate the proposed method. Such a program would need to cover the effect of specimen size and geometry on J_c, and slow crack growth prior to J_c. The main point of comparison would be with the US proposed method (31) based on [12], but comparisons with COD would also be made to establish a link with UK practice.

27. One problem which has already come to light is the very early onset of slow crack growth in weld metals. In large specimens (B = 50 mm), slow tearing apparently begins well before the limit load (43), and the resulting non-linearity obscures the well defined inflection which would otherwise occur in the load deflection curve at Q_L. This makes selection of an appropriate limit load value for use in [28] a difficult procedure. It is not yet clear how much this uncertainty affects the final calculated value of J_c, but this, and possible methods of circumventing the problem, will be the subject of future investigation.

28. Both the proposed method of J_c determination, and the Wells formula for δ_c, although quite amenable to calculation by hand, involve a fair amount of arithmetic, which can be tedious when a large number of specimens is involved. A small computer program is being prepared to calculate all relevant fracture parameters from the basic input data specified in the Appendix.

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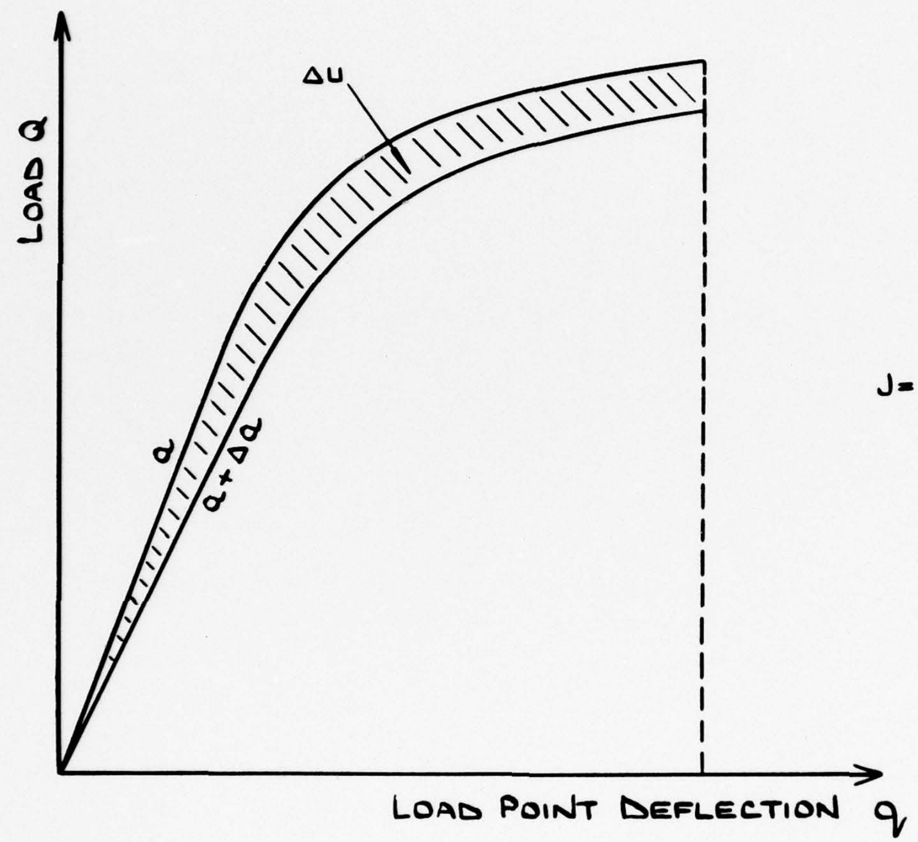
<u>GRAPHICAL REPRESENTATION OF J AS THE RATE OF DECREASE IN AREA UNDER THE LOAD DEFLECTION CURVE WITH INCREASING CRACK LENGTH</u>



$$J = -\frac{1}{B}\left(\frac{\Delta U}{\Delta a}\right)$$

FIGURE 1

FINITE ELEMENT CALCULATED VALUES OF J FOR A 3PT. BEND SPECIMEN $a/W=0{\cdot}5$, $S/W=4$.

PLANE STRESS & PLANE STRAIN

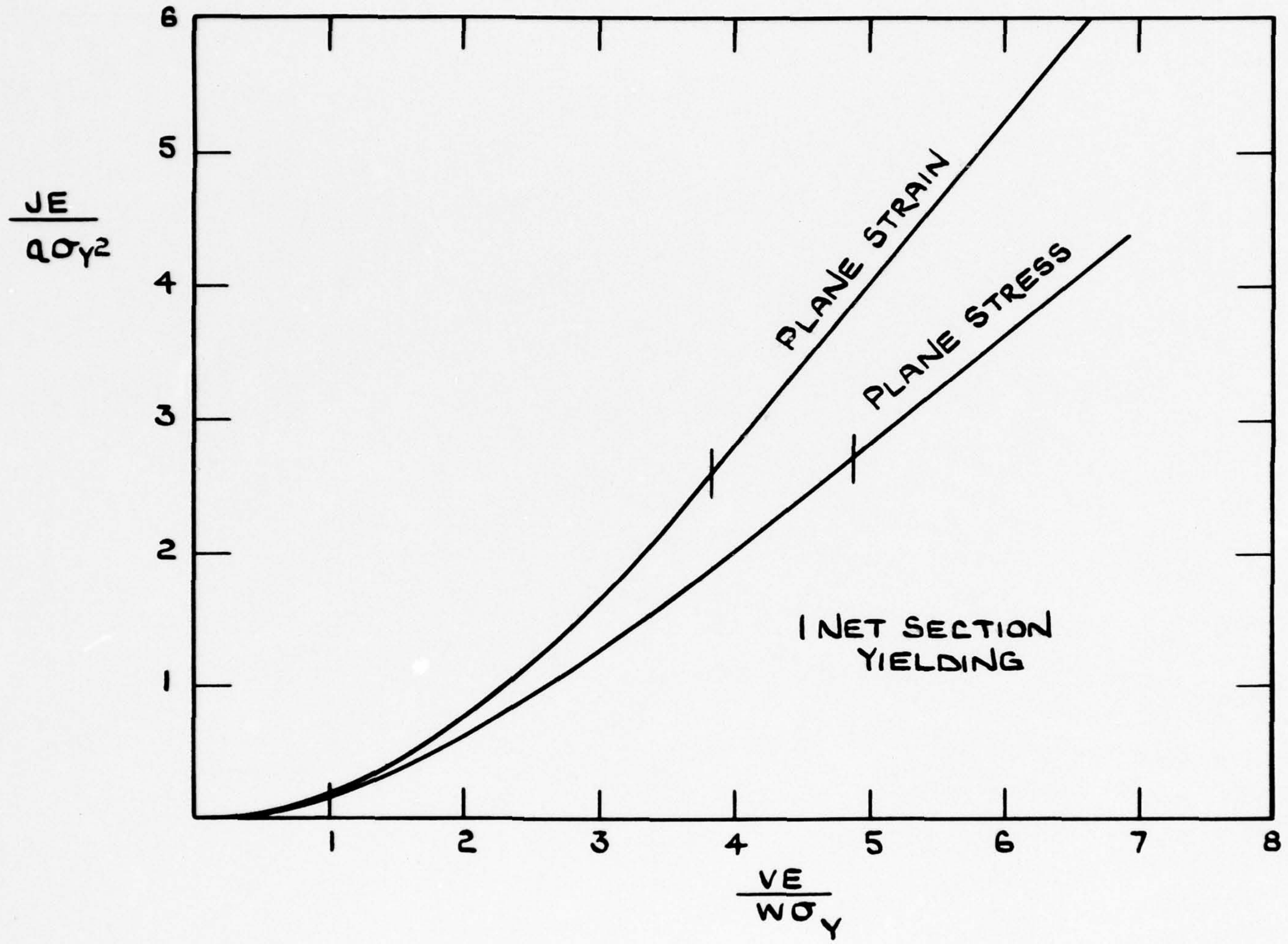


FIGURE 2

DIVISION OF THE LOAD DEFLECTION CURVE INTO ELASTIC & PLASTIC ENERGIES

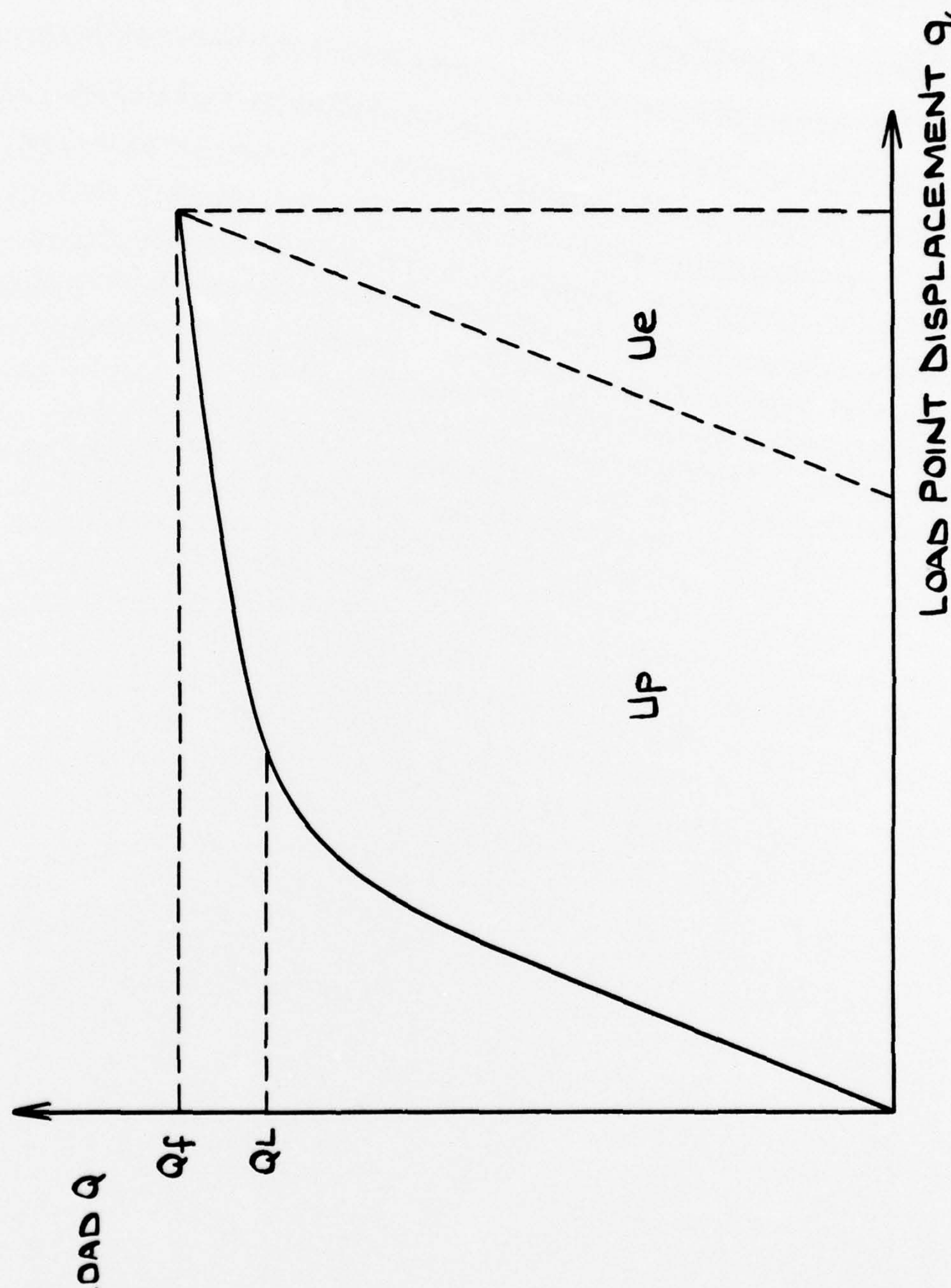


FIGURE 3

EFFECT OF SLOW CRACK GROWTH ON THE CORRELATION BETWEEN J & ABSORBED ENERGY.

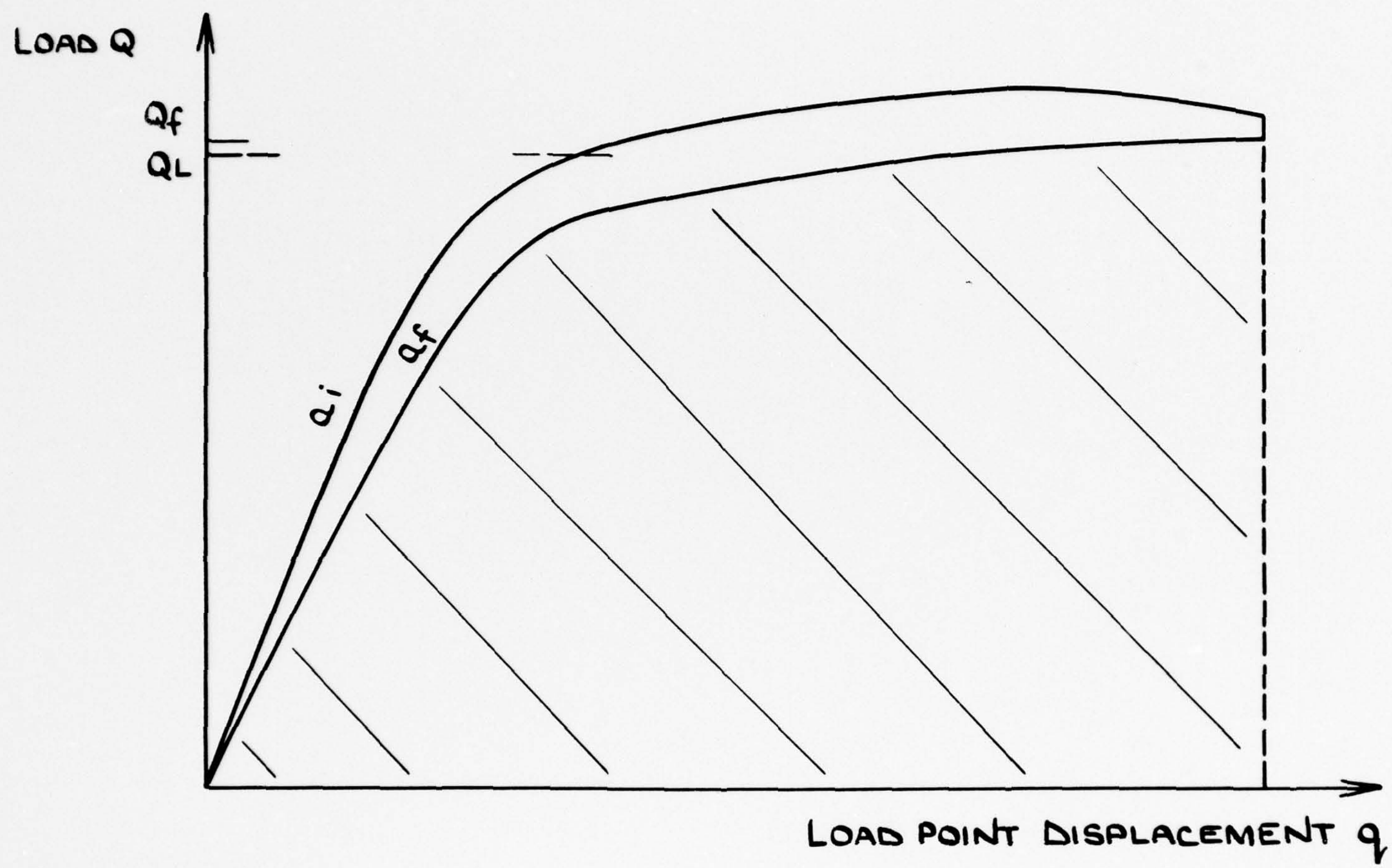


FIGURE 4

LOAD DEFLECTION CURVE MEASUREMENTS FOR CALCULATION OF J

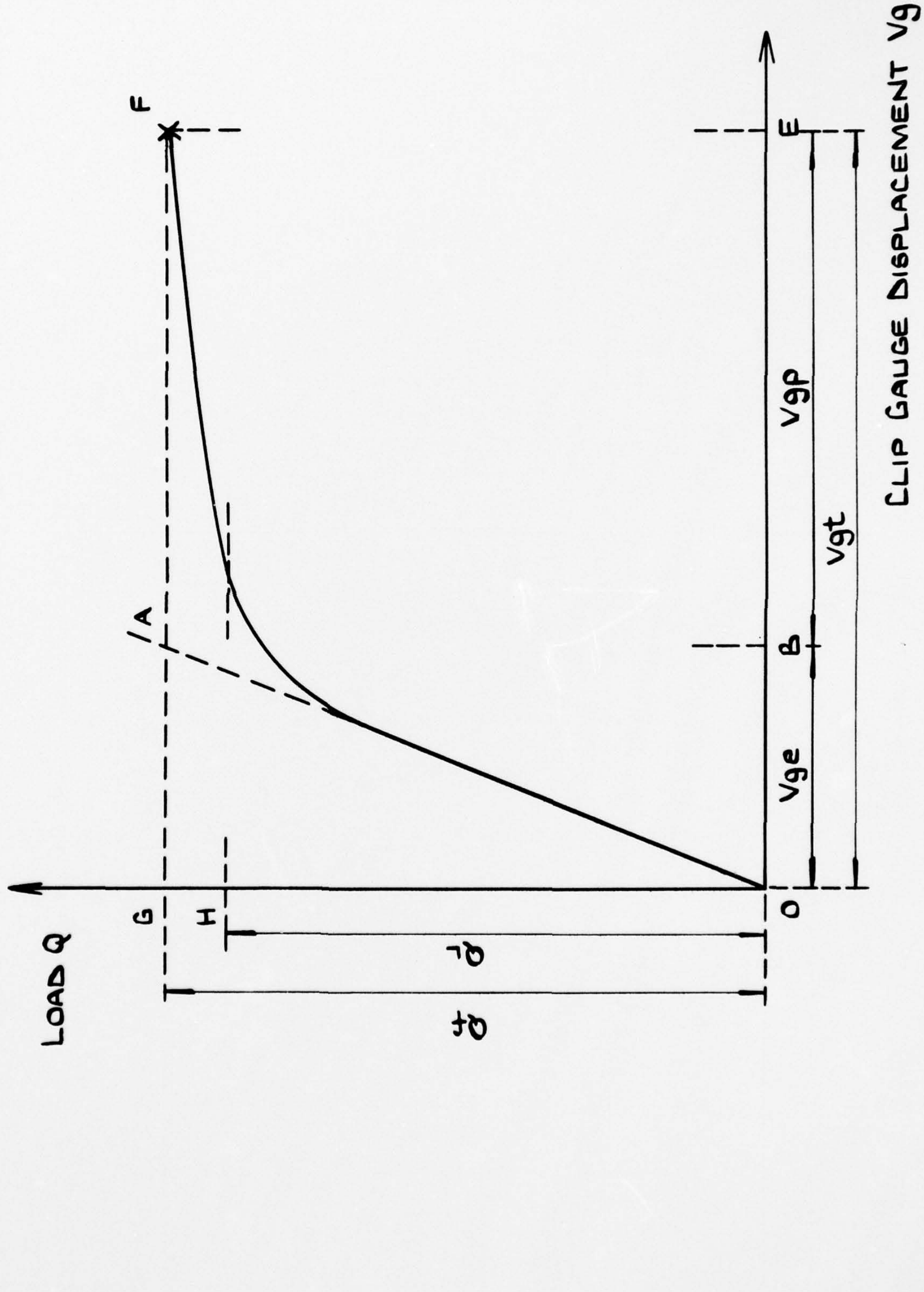


FIGURE 5

APPENDIX

Proposed Method for Calculating J from the Load/Clip Gauge Displacement Curve of a 3pt Bend Specimen $0.25 < a/w < 0.60$

1. From the specimen measure:

   W,B,S – overall specimen dimensions

   a_i – initial crack length (to end of fatigue crack)

   a_f – final crack length (to end of slow tearing)

2. Draw a line OA through the elastic part of the load/clip gauge deflection (Q/V_g) curve (figure 5). Draw a horizontal GF at load Q_f. Record:

   Q_F(OG) – final load (may be after maximum load)

   Q_L(OH) – limit (net section yielding) load

   V_{ge}(OB) – elastic clip gauge displacement (assuming no slow crack growth)

   V_{gt}(OE) – total clip gauge displacement

3. Calculate:

$$J_e = \frac{K_f^2}{E'} \qquad [A1]$$

where

$$K_f = \frac{Q_f S}{BW^{3/2}} \cdot Y \qquad [A2]$$

$$Y = \left(\frac{a_f}{W}\right)^{\frac{1}{2}} \left[2.9 - 4.6\left(\frac{a_f}{W}\right) + 21.8\left(\frac{a_f}{W}\right)^2 - 37.6\left(\frac{a_f}{W}\right)^3 + 38.7\left(\frac{a_f}{W}\right)^4\right] \qquad [A3]$$

4. Calculate:

$$J_p = \frac{2Q_L(W - a_f)}{B(W - a_i)^2} \left[\frac{W}{a_f + r(W - a_f) + z}\right] \left[V_{gt} - \beta\, V_{ge}\right] \qquad [A4]$$

   where z is the height of the clip gauge above the specimen surface

$$\beta = \frac{h\left(\frac{a_f}{W}\right)}{h\left(\frac{a_i}{W}\right)} \qquad [A5]$$

$$h\left(\frac{a}{W}\right) = -43.0 + 403.0\,\frac{a}{W} - 1073.2\left(\frac{a}{W}\right)^2 + 1162.8\left(\frac{a}{W}\right)^3 \qquad [A6]$$

$$r = 0.45\,,\ a/W < 0.45 \qquad [A7]$$
$$\quad 0.4\,,\ a/W > 0.45$$

5. The value of J at any given clip gauge displacement is

$$J = J_e + J_p \qquad [A8]$$

6. Notes

a. It is necessary to decide whether to use the plane strain or plane stress value of E' in [A1]. A possible basis for choice might be the criterion for determining J_{1c} validity suggested in (26)

$$\text{plane stress } \frac{B\sigma_Y}{25} < J_c < \frac{B\sigma_Y}{25} \text{ plane strain} \qquad [A9]$$

b. [A4] is not applicable if significant slow crack growth occurs before net section yielding. J_p can be calculated before net section yielding in the absence of slow crack growth by substituting Q_f for Q_L in [A4].

c. [A6] is a polynomial fit, over the range, $0.25 < a/W < 0.6$, to elastic compliance data given in (34). Its use outside this range is not recommended. A slight error might be introduced by applying this to the clip gauge as opposed to the mouth opening displacement, but this is likely to be small since β is a ratio not an absolute quantity.

d. The values of r given in [A7] are based on finite element analysis. Alternative values may be used if experimental evidence dictates.

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J Integral
Fracture Mechanics
Fracture Toughness Testing

Abstract

Following a review of the theoretical justification for the use of J_c as an elastic-plastic fracture criterion, the report suggests a new method for its laboratory determination. Advantages of the new method include: unambiguous continuity with K_c; full compatibility with the existing standard for COD testing; and an ability to deal with slow crack growth and the effects of lateral constraint. The report concludes with an outline of future work and detailed instructions for the determination of J_c in a 3pt bend specimen with a crack length to width ratio of between 0.25 and 0.6.

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by J D G Sumpter 1976

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