AD-A038 155 NAVAL SURFACE WEAPONS CENTER DAHLGREN LAB VA F/G 4/1

A GRAVITATIONAL POTENTIAL FOR ATMOSPHERIC EARTH TIDES CAUSED BY--ETC(U)

NOV 76 R MANRIQUE, W GROEGER

UNCLASSIFIED NSWC/DL-TR-3638 NL

1 OF 1

ADA038155

END

DATE FILMED

4-77

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER TR-[redacted] 3638	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) A GRAVITATIONAL POTENTIAL FOR ATMOSPHERIC EARTH TIDES CAUSED BY THE MOON.		5. TYPE OF REPORT & PERIOD COVERED Final rept., 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) R. Manrique W. Groeger 26p.		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Surface Weapons Center Dahlgren Laboratory (Code DK-12) Dahlgren, VA 22448		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Defense Mapping Agency, Headquarters Naval Observatory Building 56 Washington, DC 20360		12. REPORT DATE November 1976 13. NUMBER OF PAGES 26
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) NSWC/DL-TR-3638		15. SECURITY CLASS. (of this report) UNCLASSIFIED 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Lunar air tide
Satellite geodesy
Force equation for satellite motion

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A potential function is presented for the force by which the atmospheric tide bulge acts on the orbits of artificial earth satellites. The tidal bulge is assumed to result from the fact that the earth rotates within the field of lunar mass attraction, the latter being inhomogeneous across the terrestrial globe. Only the main term of the semidiurnal tide is considered in this report.

The perturbing acceleration associated with the tide potential is also specified. The latter is intended for use with computer programs for satellite geodesy.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

S/N 0102-LF-014-6601

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

391598

LB

FOREWORD

This report documents one of the perturbing terms appearing in the force equations for satellite motion in the TERRA system of satellite geodesy computer programs. Equations are given for the lunar air tide, via the gravitational action which the tidal bulge exerts on the satellite orbit. An earlier report gave the equations for the air tides caused by solar gravitational and thermal effects.

The work was done in the Astronautics and Geodesy Division in support of astronautics computer program development.

Released by:

R. A. NIEMANN, Head
Warfare Analysis Department

CONTENTS

INTRODUCTION

The atmospheric tides of the earth are caused by a combination of the gravitational pull of the moon and the sun's thermal action. To establish a mathematical model of the tidal effects on satellite motion, it is both convenient and customary[1] to represent the tidal mass redistribution by a surface density layer and to subsequently obtain the potential, exterior to the earth, by integration.

One of the authors has recently derived the solar component of the atmospheric tide[2] from a surface pressure pattern described by Haurwitz.[1] This was done to produce a perturbing term, corresponding to the solar air tide, for the equations of motion which govern the orbits of the earth satellites in the TERRA[3] system of computer programs for satellite geodesy. For the same purpose, the following text introduces a model which accounts for the lunar tide in the terrestrial atmosphere. Specifically, the new tide model contains a potential function for the force by which the atmospheric tide bulge acts on the satellite orbits. The tidal bulge is assumed to result from the fact that the earth rotates within the field of lunar mass attraction, the latter being inhomogeneous across the terrestrial globe. Only the main term of the semidiurnal tide is considered. The perturbing acceleration associated with the tide potential is also specified for use with TERRA and with similar computer programs which involve force equations from which the orbits of geodetic satellites are being calculated by direct, numerical integration.

The text below augments Reference 2. Readers ought to have perused that reference first, especially the introduction. Much has been included there to facilitate the understanding of the more general aspects of the matter, which we shall not repeat here

THE TIDE POTENTIAL*

A suitable description of the variation in air pressure on the ground, caused by the semidiurnal lunar tide, was found in Reference 4 (pages 70, 73, 93, and 94):

$$L_2 = 55.2 \mu\text{bar} \cos^3 \theta \sin(2\tau + 75°) \qquad (1)$$

*Additional comments on the tide potential plus the necessary details of its derivation appear in Appendix A.

In this equation, L_2 is the surface pressure in microbars, θ is the geocentric latitude, and τ is the local, lunar mean time. Considering that the local surface pressure results from the weight of the atmospheric column, and also taking into account that geodetic satellites generally operate far above that altitude below which most of the atmosphere lies, the surface pressure was converted into a gravitating mass layer, the action of which on the satellite would resemble that of the actual tidal air bulge:

$$\sigma_2 = A_2 \cos^3 \theta \cos 2\alpha \tag{2}$$

$$A_2 = 0.0564 \text{ gr/cm}^2 \tag{3}$$

$$2\alpha = 2\lambda + [2(t^* - \nu) - 15°] \tag{4}$$

In these equations, λ is longitude East, t^* is Universal time, and ν is the difference between the mean mean longitudes of moon and sun.

Subsequent application of the Poisson integral produced the lunar tide potential, exterior to the main portion of the atmosphere,

$$U = a\frac{R^3}{r^3} P_2^2(\sin\theta)\cos 2\alpha - b\frac{R^5}{r^5} P_4^2(\sin\theta)\cos 2\alpha \tag{5}$$

$$a = A_2 GR\frac{5\pi^2}{64} \tag{6}$$

$$b = A_2 GR\frac{5\pi^2}{3072} = \frac{a}{48} \tag{7}$$

G is the gravitational constant, R is a suitably defined mean radius of the earth, and r is the geocentric distance of the satellite. The $P_i^j(x)$ are associated Legendre functions.

It should be noted that L_2 above is a yearly, worldwide average. Also, all equations appearing here and in the text below are presented in a form which renders them independent of the choice of any particular units. A system of compatible units must, however, be chosen prior to translating the present analysis into a computer algorithm.

THE MEAN LONGITUDES OF SUN AND MOON*

Explicitly and implicitly, the local, lunar mean time τ is the measure of time in the equations of the previous chapter. When converted into Universal time, it gave rise to the presence of the quantity ν in the combined azimuth/time arguments of surface density and potential.

ν is the difference between the mean mean longitude of the moon, s, and the mean mean longitude of the sun, h:

$$\nu = s - h \tag{8}$$

To be precise, s is the angle from the mean equinox of date, along the mean ecliptic of date, to the mean ascending node of the moon's orbit, plus the angle from that latter point, along the moon's mean orbit, to the mean perigee, plus the instantaneous value of the mean mean anomaly corresponding to the position of the moon. Correspondingly, h is the angle from the mean equinox of date, along the mean ecliptic of date, to the mean solar perigee, plus the instantaneous value of the mean mean anomaly corresponding to the position of the sun.

The terms, s and h, are connected to Universal time t* by the following algorithm. Let J be the number of the calendar year (1975 or 1976 or 1977), M be the number of the individual month in the calendar year, and D be the number of the particular day in the month. Also, observe

$$\delta(n, m) = \begin{cases} 1 & \\ & \text{if} \\ 0 & \end{cases} \begin{matrix} n = m \\ \\ n \neq m \end{matrix} \tag{9}$$

*See also Appendix B.

Then

$$N_1 = D + 31\delta(M,2) + 59\delta(M,3) + 90\delta(M,4) + 120\delta(M,5) + 151\delta(M,6) + 181\delta(M,7) + 212\delta(M,8) + 243\delta(M,9) + 273\delta(M,10) + 304\delta(M,11) + 334\delta(M,12) \quad (10)$$

$$N_2 = D + 31\delta(M,2) + 60\delta(M,3) + 91\delta(M,4) + 121\delta(M,5) + 152\delta(M,6) + 182\delta(M,7) + 213\delta(M,8) + 244\delta(M,9) + 274\delta(M,10) + 305\delta(M,11) + 335\delta(M,12) \quad (11)$$

$$N = N_1\,\delta(J,1975) + (365 + N_2)\delta(J,1976) + (731 + N_1)\delta(J,1977) \quad (12)$$

$$\Delta T = 5.28 \times 10^{-4} + 3.56 \times 10^{-8} \times N \quad (13)$$

$$d = 27392.5 + N + \Delta T + \frac{t^*}{24} \quad (14)$$

(Into this equation, t^* should be entered in decimal fractions of hours.)

$$T = \frac{d}{36525} \quad (15)$$

$$s = 270.434358 + 481267.883141T - 0.001133T^2 + 0.000002T^3 \quad (16)$$

$$h = 279.69668 + 36000.768930T + 0.000303T^2 \quad (17)$$

(s and h will result in decimal fractions of degrees.)

It is obvious how this algorithm may be generalized to time spans which extend beyond 1977 or to altogether different time spans. Also, one may desire to input the number of the day in the calendar year, N_1 or N_2, directly. That is easily accomplished by deleting the equations for N_1 and N_2. N_1 and N_2 would then be algorithm input, to be obtained from an external calendar. N_1 would then mean the number of the calendar day in the year, if J = 1975 or J = 1977. N_2 would be the number of the calendar day, if J = 1976.

DISTURBING ACCELERATION*

From the potential U (Equation 5), the disturbing acceleration $\overline{T}$ can now readily be obtained. A positive gradient convention will be observed,

$$\overline{T} = + \text{grad } U \tag{18}$$

Let y_1, y_2, y_3 be the earth-fixed, Cartesian coordinates of the TERRA system. Also, let r, λ, θ be the above used, earth-fixed, spherical coordinates (geocentric distance, longitude East, and geocentric latitude). Further, observe the geometrical relations,

$$r = \sqrt{y_1^2 + y_2^2 + y_3^2} \tag{19}$$

$$\tan \lambda = \frac{y_2}{y_1} \tag{20}$$

$$\cos \lambda = \frac{y_1}{\sqrt{y_1^2 + y_2^2}} \tag{21}$$

$$\sin \lambda = \frac{y_2}{\sqrt{y_1^2 + y_2^2}} \tag{22}$$

$$\tan \theta = \frac{y_3}{\sqrt{y_1^2 + y_2^2}} \tag{23}$$

*See also Appendix C.

$$\cos\theta = \frac{1}{r}\sqrt{y_1^2 + y_2^2} \tag{24}$$

$$\sin\theta = y_3/r \tag{25}$$

$$\frac{\partial r}{\partial y_i} = \frac{y_i}{r}, \qquad i = 1,2,3 \tag{26}$$

$$\frac{\partial \lambda}{\partial y_1} = -\frac{y_2}{y_1^2 + y_2^2} \tag{27}$$

$$\frac{\partial \lambda}{\partial y_2} = +\frac{y_1}{y_1^2 + y_2^2} \tag{28}$$

$$\frac{\partial \lambda}{\partial y_3} = 0 \tag{29}$$

$$\frac{\partial \theta}{\partial y_1} = \frac{-y_1 y_3}{r^2\sqrt{y_1^2 + y_2^2}} \tag{30}$$

$$\frac{\partial \theta}{\partial y_2} = \frac{-y_2 y_3}{r^2\sqrt{y_1^2 + y_2^2}} \tag{31}$$

$$\frac{\partial \theta}{\partial y_3} = \frac{\sqrt{y_1^2 + y_2^2}}{r^2} \tag{32}$$

Also observe the physical relations,

$$\frac{\partial U}{\partial r} = -a\frac{3R^3}{r^4} P_2^2(\sin\theta)\cos 2\alpha + b\frac{2R^5}{r^6} P_4^2(\sin\theta)\cos 2\alpha \tag{33}$$

$$\frac{\partial U}{\partial \lambda} = -a\frac{2R^3}{r^3} P_2^2(\sin\theta)\sin 2\alpha + b\frac{2R^5}{r^5} P_4^2(\sin\theta)\sin 2\alpha \tag{34}$$

$$\frac{\partial U}{\partial \theta} = -a\frac{3R^3}{r^3} \sin 2\theta \cos 2\alpha + b\,\frac{15R^5}{r^5}\,(7 \sin^2 \theta - 4) \sin 2\theta \cos 2\alpha \qquad (35)$$

$$P_2^2(x) = 3(1 - x^2) \qquad (36)$$

$$P_4^2(x) = \frac{15}{2}(1 - x^2)(7x^2 - 1) \qquad (37)$$

$$x = \sin \theta = \frac{y_3}{r} \qquad (38)$$

$$\sin 2\theta = \frac{2y_3\sqrt{y_1^2 + y_2^2}}{r^2} \qquad (39)$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha \qquad (40)$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha \qquad (41)$$

$$\alpha = \lambda + \alpha^* \qquad (42)$$

$$\alpha^* = t^* - \nu - 7.5° \qquad (43)$$

$$\nu = s - h \qquad (8)$$

$$\cos \alpha = \cos \alpha^* \cos \lambda - \sin \alpha^* \sin \lambda \qquad (44)$$

$$\sin \alpha = \sin \alpha^* \cos \lambda + \cos \alpha^* \sin \lambda \qquad (45)$$

$$\cos \lambda = \frac{y_1}{\sqrt{y_1^2 + y_2^2}} \qquad (21)$$

$$\sin \lambda = \frac{y_2}{\sqrt{y_1^2 + y_2^2}} \tag{22}$$

Then, the Cartesian components of the disturbing acceleration are

$$T_{y_1} = \frac{\partial U}{\partial r}\frac{\partial r}{\partial y_1} + \frac{\partial U}{\partial \lambda}\frac{\partial \lambda}{\partial y_1} + \frac{\partial U}{\partial \theta}\frac{\partial \theta}{\partial y_1} \tag{46a}$$

$$T_{y_2} = \frac{\partial U}{\partial r}\frac{\partial r}{\partial y_2} + \frac{\partial U}{\partial \lambda}\frac{\partial \lambda}{\partial y_2} + \frac{\partial U}{\partial \theta}\frac{\partial \theta}{\partial y_2} \tag{46b}$$

$$T_{y_3} = \frac{\partial U}{\partial r}\frac{\partial r}{\partial y_3} + \frac{\partial U}{\partial \lambda}\frac{\partial \lambda}{\partial y_3} + \frac{\partial U}{\partial \theta}\frac{\partial \theta}{\partial y_3} \tag{46c}$$

To be suitable for use in the satellite orbit integration, $\overline{T}$ will now have to be rotated into the inertial coordinate frame in terms of which TERRA executes the orbit integration. The necessary algorithm shall not be discussed here. It is available for implementation on the computer now and will appear in a future report which will document the complete TERRA equations of motion, strictly under an algorithm aspect.

Terms like "perturbing force" and "perturbing" or "disturbing potential" are frequently associated with General Theory. To prevent any misunderstanding, it should be remembered that TERRA, like its forerunners, generates the necessary trajectories by directly integrating a set of differential equations which relate the orbital acceleration to a variety of force terms, which in turn are rather complicated functions of satellite position, velocity, attitude, position relative to the earth's shadow, etc. This approach enables TERRA to readily cope with force models which reflect the state of the art and which are mathematically complex to the point where they might exceed the capabilities of General Theory. That is why we were able to formulate the algorithm in the simple-minded shape in which it appears, and this is also the reason why it lacks the elegance which is so characteristic of the analytical approach.

SUMMARY

The above text presents the essential features of only one from amongst four tidal force terms which the TERRA equations of motion contain. The analytical work has been completed for all these terms. Also, the necessary computer algorithms have been formulated. But only two perturbing terms have thus far been described formally. This includes the solar air tide which is covered by Reference 2 and the present attempt to elaborate that component of the perturbing acceleration which arises because of the presence of the lunar tide in the atmosphere. Two more reports, namely one on the tide of the solid earth as well as another covering the ocean tide, are in preparation.

The TERRA equations of motion are intended to serve as a basis for numerical orbit integration. While developing the individual force terms, we never gave any thought to General Theory. In keeping with this approach and for other, entirely pragmatic reasons as well, it was decided to make the tidal terms part of TERRA without first conducting a study of the relative magnitudes of the various force components and their consequences for the satellite motion. That might have involved us in orbital studies based on General Theory, which was considered undesirable in view of both the funding situation and the time frame of the TERRA development. Rather, it was felt that we already had reliable indications from experience as well as from the literature as to which effects could be expected to be relevant and which might safely be omitted. For that reason and also because it appeared to be more in keeping with the rest of the TERRA effort, we decided that all the selected forces should appear in the equations of motion exactly as they resulted from the respective physical models and that their existence should be finally justified by means of numerical studies which would have to be conducted anyway, as part of the TERRA trials on the computer.

REFERENCES

1. B. Haurwitz, "Atmospheric Tides," ***Science***, Vol. 144, No. 3625, (19 June 1964).

2. W. Groeger, ***A Gravitational Potential for Atmospheric Earth Tides Caused by the Sun***, Naval Surface Weapons Center/Dahlgren Laboratory, Dahlgren, Virginia, Technical Report NSWC/DL TR-3485, October 1976.

3. M. D. Harkins, "A New Dahlgren Model for Deriving the Earth Gravity Field," ***Proceedings of the 10th Anniversary Meeting of the Society of Engineering Science***, Raleigh, North Carolina, November 1973.

4. S. Chapman and R. S. Lindzen, ***Atmospheric Tides, Thermal and Gravitational***, New York: Gordon & Breach, 1970.

5. E. Jahnke and F. Emde, ***Tables of Functions***, New York: Dover Publications, 1945.

6. ***Explanatory Supplement to the Astronomical Ephemeris and the American Ephemeris and Nautical Almanac***, London, H. M. Stationary Office, 1961.

APPENDIX A

NOTES ON THE TIDE POTENTIAL

NOTES ON THE TIDE POTENTIAL

Our work on the lunar air tide is based on a tidal pressure wave (Equation 1) which is symmetric to the equator. Also, time and latitude are altogether decoupled from each other. Time is associated with the azimuth coordinate only. That is, of course, a simplification. Being caused by the moon, the orbit of which is inclined against the equator, the actual tide may be expected to depend on the lunar phase. That excludes strict, equatorial symmetry. But the earth's topography appears to complicate the tidal pressure wave. Also, data are still scarce over much of the globe. After having reviewed the recent literature, we were quite content with having available at last something tangible, namely, the tidal pressure averages represented by Equation 1.

The surface pressure wave we used can be imagined to be caused by a moon which moves in the celestial equator rather than in the actual lunar orbit. Figure A-1 indicates how the various azimuth coordinates appearing above in the main part of the report, and time, are related to each other.

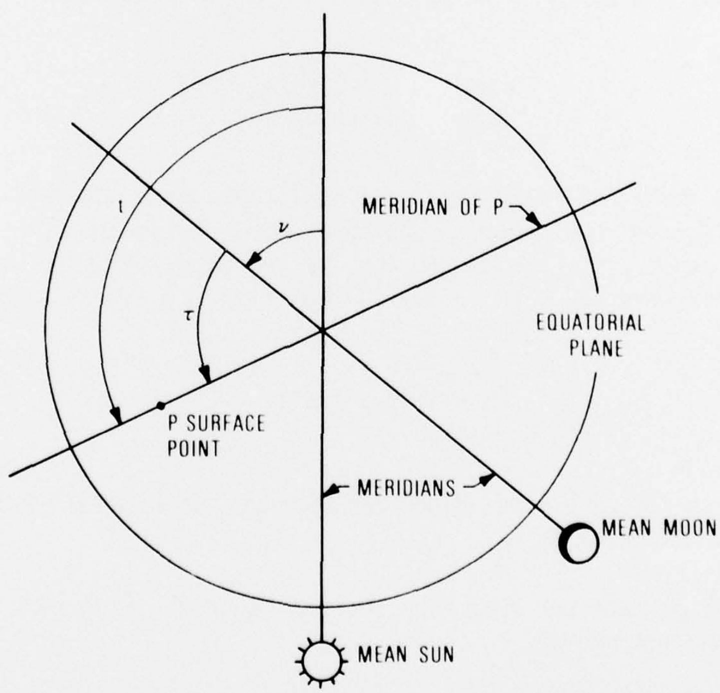

Figure A-1. Relations Amongst the Azimuth Variables and Time

t is local mean solar time. τ is local mean lunar time. Also, consider the familiar relationship between Universal time and t,

$$t^* = t - \lambda \tag{47}$$

Then,

$$\tau = t - \nu = t^* + \lambda - \nu \tag{48}$$

which leads directly to Equation 4 for the time/azimuth argument of the tidal mass layer.

From Equation 1 for the surface pressure follows the corresponding surface mass density σ_2 by simply replacing the unit of pressure by the area density related to the atmospheric column. The potential aloft follows by integration:

$$U(P) = \iint \frac{G\sigma_2(Q)}{|\bar{r} - \bar{\rho}|} ds \tag{49}$$

where P and Q denote the field point and the source point, respectively. $\bar{r}$ is the position vector for the field point, $\bar{\rho}$ indicates the source point. When used in the following, the field point coordinates will be unprimed while the source point coordinates will be primed. γ is the angle between $\bar{r}$ and $\bar{\rho}$. The area element ds is

$$ds = R^2 \cos\theta' d\theta' d\alpha' \tag{50}$$

There will appear below several expansions involving Legendre polynomials. These were taken from Reference 5.

To actually perform the integration, the inverse of $|\bar{r} - \bar{\rho}|$ is first expanded in terms of Legendre polynomials:

$$|\bar{r} - \bar{\rho}| = +\sqrt{r^2 + \rho^2 - 2r\rho\cos\gamma} \tag{51}$$

$$\cos\gamma = \sin\theta\sin\theta' + \cos\theta\cos\theta'\cos(\alpha - \alpha') \tag{52}$$

$$\frac{1}{|\bar{r} - \bar{\rho}|} = \sum_{n=0}^{\infty} \frac{R^n}{r^{n+1}} P_n(\cos\gamma) \tag{53}$$

Application of the Addition Theorem for the Legendre functions (Reference 5, Chapter VII, Paragraph 6) results then in

$$U = A_2 GR \sum_{n=0}^{\infty} \frac{R^{n+1}}{r^{n+1}} I_n \tag{54}$$

$$I_n = \int_0^{2\pi} \int_{-\pi/2}^{\pi/2} \cos^4\theta' \cos 2\alpha' \left\{\cdots\cdots\right\} d\theta' d\alpha' \tag{55}$$

$$\left\{\cdots\cdots\right\} = P_n(\sin\theta)P_n(\sin\theta') + 2\sum_{m=1}^{n} \frac{(n-m)!}{(n+m)!} P_n^m(\sin\theta)P_n^m(\sin\theta')\cos m(\alpha - \alpha') \tag{56}$$

If one now integrates over the azimuth coordinate first, it will at once be realized that I_n is a fairly simple expression:

$$I_n = 2\int_{-\pi/2}^{+\pi/2} d\theta' \cos^4\theta' \sum_{m=1}^{n} \frac{(n-m)!}{(n+m)!} K_m P_n^m(\sin\theta)P_n^m(\sin\theta') \tag{57}$$

$$K_m = \int_0^{2\pi} \cos m(\alpha - \alpha')\cos 2\alpha' d\alpha' \tag{58}$$

Inspection of K_m will show that only K_2 is non-zero.

$$K_m = K_2\delta(m, 2) \tag{59}$$

$$K_2 = \pi\cos 2\alpha \tag{60}$$

I_1 we shall be able to ignore for reasons indicated on Page A-5 of Reference 2. Thus, there remains

$$I_n = 2K_2 \frac{(n-2)!}{(n+2)!} P_n^2(\sin\theta) \int_{-\pi/2}^{+\pi/2} P_n^2(\sin\theta')\cos^4\theta' d\theta' \qquad (61)$$

$n \geqslant 2$

Execution of the latter integral and search for the non-zero I_n shows that I_3 is zero while I_2 and I_4 result in expressions which, upon entering into Equation 54, lead to the potential represented by Equations 5, 6, and 7.

APPENDIX B

NOTES ON THE MEAN LONGITUDE ALGORITHM

NOTES ON THE MEAN LONGITUDE ALGORITHM

Appendix A indicated how the perturbing potential was derived from the tidal pressure wave. It did not, however, clarify a problem hidden within the azimuth coordinate, namely that of how to reference the moon's position to that of the sun. That question is solved by the algorithm for the mean longitudes of sun and moon, specified above in the main part of the report. We shall now discuss some details which concern the computation of the solar and lunar mean longitudes.

In essence, the algorithm is based on Equations 16 and 17, which represent the mean mean longitudes of sun and moon as functions of time. These equations are from Pages 98 and 107 of Reference 6. They are part of the body of astronomical relationships on which the Nautical Almanac bases its fundamental ephemerides of sun and moon. The mean mean longitudes s and h are polynomials of T. T is time in Julian centuries. These polynomials are valid for the present century. d as defined by Equation 14 is time in days; 1900 January 0.5 ET (Ephemeris time) is the epoch. N is the number of days elapsed since the beginning of the calendar year 1975. ΔT is the Reduction of Time which is the difference between Ephemeris time and Universal time. Our Equation 13 actually is only an approximation to that difference, chosen to be valid for any time interval during which we anticipate that it may be required.

As already mentioned, epoch for counting time is the beginning of our century or, to be precise, the start of the calendar year 1900. Although the concept of the Julian day is simple, its exact connection with other time units such as the calendar day may likely confuse those who only occasionally encounter that particular unit. We add thus three illustrations, namely Figures B-1, B-2, and B-3. The additional Figure B-4 illustrates the role of the Reduction of Time in the relationship between Ephemeris time and Universal time.

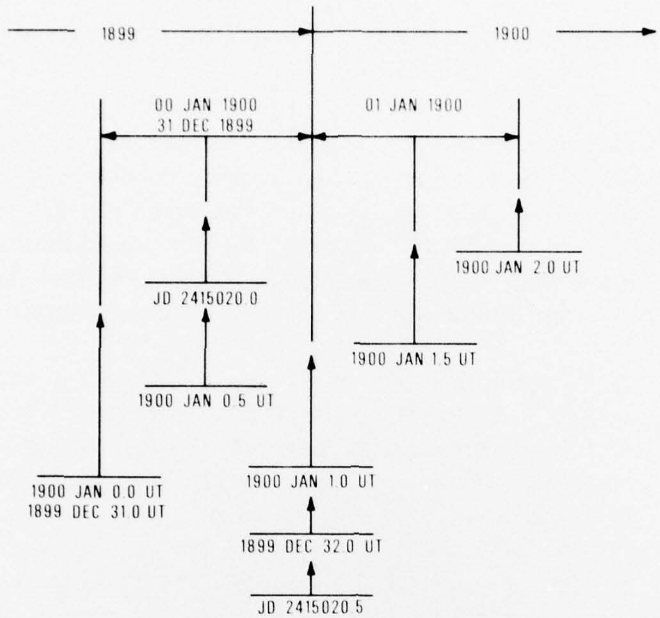

Figure B-1. Julian Date and Universal Time at Epoch

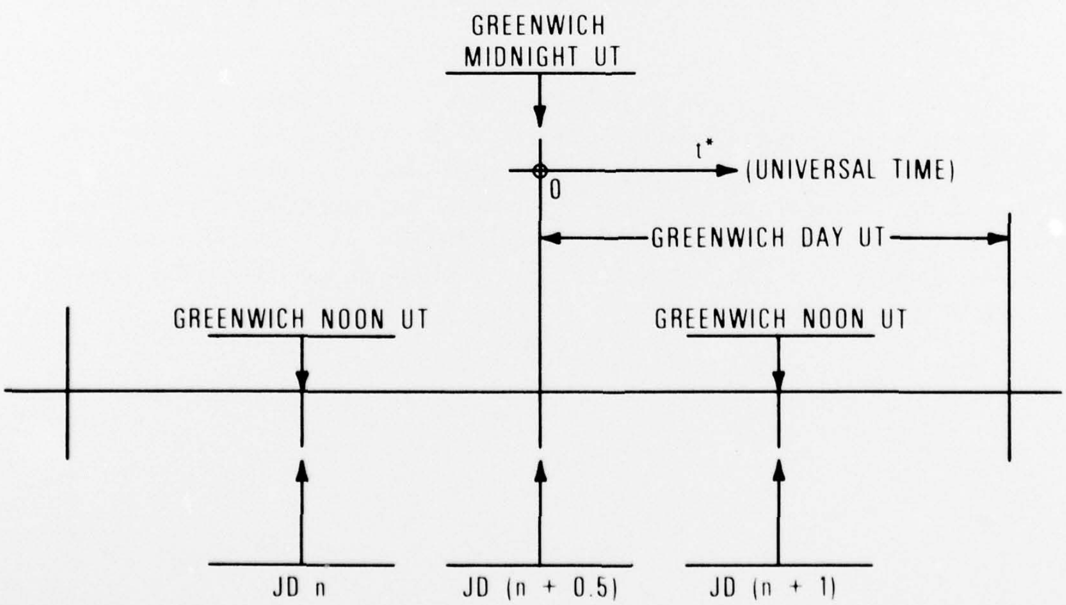

Figure B-2. Julian Date and Calendar Day

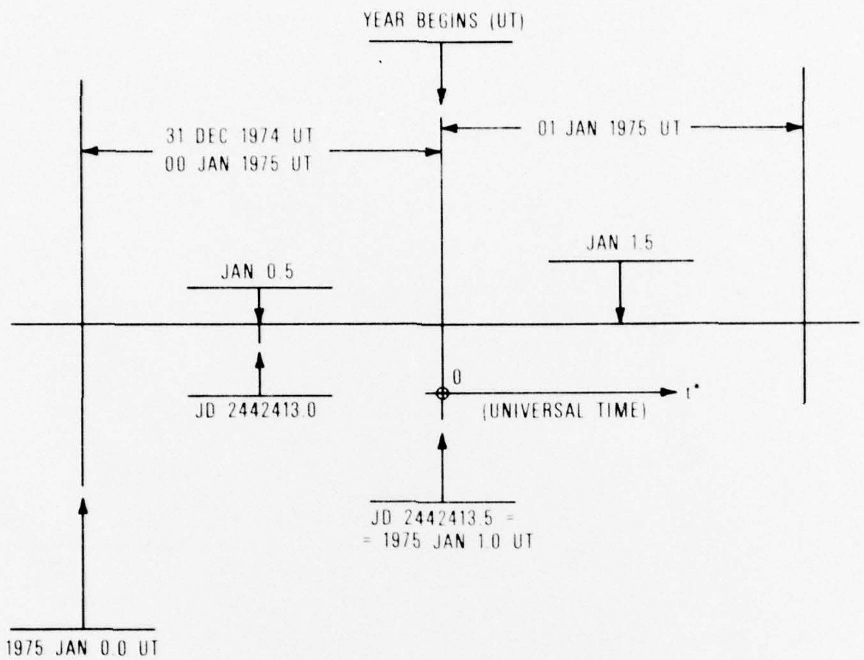

Figure B-3. Julian Date and Universal Time at Beginning of 1975

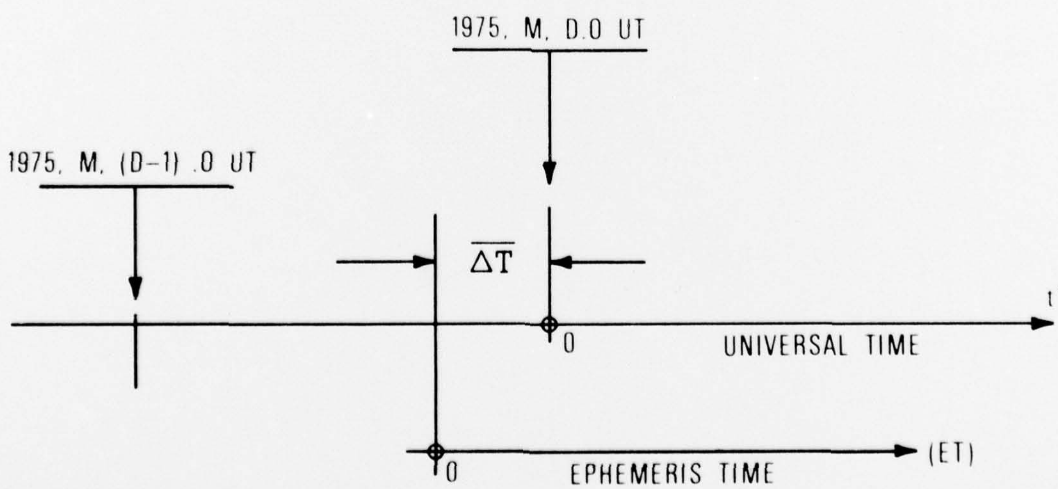

Figure B-4. Reduction of Time Scale
($\overline{\Delta T}$ is average Reduction of Time (of date):
$\overline{\Delta T} \approx 46^s$ for time spans considered)

APPENDIX C

NOTES CONCERNING THE GRADIENT

NOTES CONCERNING THE GRADIENT

The perturbing acceleration, as discussed above in the main portion of the report, will not require much additional elaboration. But we probably should mention that the integrations which resulted in the tide potential as well as the gradient formation itself were done by both authors, independently of each other, with great care. Further confidence in the results was derived from a check on the sign of the gradient. This was to assure that, for a low orbit, the radial gradient component associated with any individual potential term actually does point down (negative sign) towards the surface whenever the satellite happens to be positioned over a region of the surface where the corresponding atmospheric density term is positive and thus attracts rather than repels the satellite.

DISTRIBUTION

Defense Mapping Agency, Headquarters
Naval Observatory Building 56
Washington, DC 20360

Defense Mapping Agency
Aerospace Center
St. Louis, MO 63118
ATTN: Richard L. Ealum

Defense Mapping Agency
Hydrographic Center
Washington, DC 20390

Defense Mapping Agency
Topographic Center
6500 Brooks Lane
Washington, DC 20315

Defense Documentation Center
Cameron Station
Alexandria, VA 22314 (12)

Chief of Naval Research
Department of the Navy
Arlington, VA 22217

Department of the Air Force
HQ Space & Missile Test Center (AFSC)
Vandenberg Air Force Base, CA 93437
ATTN: Sven C. Ernberg

Naval Research Laboratory
Washington, DC 20360
ATTN: Code 4130

U. S. Navy Astronautics Group
Point Mugu, CA 93041
ATTN: Code AG33
Code SPMOO
Mr. Campbell

Commanding Officer
Naval Space Surveillance System
Dahlgren, VA 22448

Library of Congress
Washington, DC 20540
ATTN: Gift and Exchange Division (4)

Defense Printing Service
Washington Navy Yard
Washington, DC 20374

National Oceanic & Atmospheric Administration
National Ocean Survey
Rockville, MD 20850

National Aeronautics & Space Administration
Washington, DC 20546
ATTN: KSS-10

Goddard Space Flight Center
Geodynamics Branch
Greenbelt, MD 20771

Aerospace Corporation
2350 East El Segundo Boulevard
El Segundo, CA 90245
ATTN: Library

Applied Physics Laboratory
The Johns Hopkins University
8621 Georgia Avenue
Silver Spring, MD 20910

Local:

DK-01
DK-10 (35)
DK-11 (Groeger) (20)
DK-12 (Manrique) (20)

DK-50	(5)
DK-70	(3)
DX-21	(2)
DX-222	(6)
DX-40	
DX-43 (Elliott)	