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**MOORING MECHANICS
A COMPREHENSIVE COMPUTER STUDY**

Volume II

**Three Dimensional Dynamic Analysis of
Moored and Drifting Buoy Systems**

by
Narender K. Chhabra

December 1976

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The Charles Stark Draper Laboratory, Inc.

Cambridge, Massachusetts 02139

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ABSTRACT

Ocean currents and surface waves may induce serious errors in oceanographic measurements obtained from moored and drifting buoy systems. A general, computationally efficient solution for the dynamics of moored buoy systems, free drifting buoys, and drogued buoy systems in three-dimensional space is described in this report. Time-domain computer simulations of four specific configurations in various environments are presented. The mathematical model of one of these configurations, a subsurface mooring, was evaluated and improved using full scale ocean test data. The model of the surface mooring configuration studied will soon also be evaluated using recent test data.

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NOMENCLATURE

$\bar{A}$	Acceleration vector.
A_D	Area used in drag calculations.
$\bar{A}_{Ec}$	Acceleration of point c in the E frame.
A_N, A_T	Normal and Tangential areas for a cylindrical body.
$\bar{A}_R$	Relative acceleration vector.
a	Point of attachment between the surface buoy and the mooring line.
$\bar{AM}$	Added mass force vector.
c, x', y', z'	Body coordinate system, Frame B.
c	Mid point of the mooring line element.
{CF}	Array of constant nodal forces.
C_D	Drag coefficient.
C_N, C_T	Added mass coefficients, normal and tangential directions, surface buoy.
C_{DN}, C_{DT}	Normal and tangential drag coefficients for a cylindrical body.
C_{DP}	Pressure drag coefficient for the end plate of a spar buoy.
$CDIN, CDIT, CDIA$	Appropriate drag constants corresponding to DN , DT , and DA .
$\bar{DA}$	Viscous drag force on a body inserted in a mooring line.
$\bar{DF}$	Viscous drag force vector.
$\bar{DM}$	Moment vector due to viscous drag force.
$\bar{DN}, \bar{DT}$	Normal and tangential drag forces per unit stretched length of the mooring line.

$\overline{DN}_n, \overline{DT}_n, \overline{DA}_n$	Viscous drag forces acting on the n^{th} node of the mooring line.
dB	Mass of water displaced by a differential disk of the surface buoy, $\rho_0 dz'$.
dM	Mass of a differential disk of the surface buoy.
$\overline{F}$	Total force vector acting on the body.
$\{F_A\}$	Array of additional forces and moments.
$\{F_D\}$	Array of wave exciting forces and moments.
$\overline{FF}$	Froude Krylov exciting force vector.
$\{F_G\}$	Array of hydrostatic and gravitational forces and moments.
$\overline{FH}$	Hydrostatic pressure force acting on the mooring line, per unit stretched length.
g	Acceleration due to gravity.
H	Length of the surface buoy.
h_0	Instantaneous draft of the surface buoy, measured up to the mean free surface.
h_1	Spar buoy drum height.
h_2	Spar buoy mast height.
h_i	h_0 plus the wave elevation component.
$\overline{IF}$	Inertia force per unit stretched length of the mooring line.
$\overline{IF}_b$	Inertia force vector of the inserted body.
I_{xx}, I_{yy}	Moments of inertia of the spar buoy in roll and pitch about c .
$\hat{i}_x, \hat{i}_y, \hat{i}_z$	Unit vectors of o, x, y, z .
$\hat{i}_{x'}, \hat{i}_{y'}, \hat{i}_{z'}$	Unit vectors of c, x', y', z' .

$\bar{K}$	Wave number vector.
K_x	Wave number component, $ \bar{K} \cos \theta$.
K_y	Wave number component, $ \bar{K} \sin \theta$.
k_n	Stiffness coefficient of the n^{th} segment.
u_1, u_2, u_3	Integration limits as defined.
M	Total mass of the spar buoy.
$[M]$	Matrix of tensors of inertia and added inertia.
m_a	Added mass of the inserted body, shape other than cylindrical.
m	Mass of the inserted body.
m_{dn}	Mass of water displaced by the n^{th} mass.
m_n, m_t	Added mass components (normal and tangential) due to cylindrical bodies.
N	Number of nodes or segments of a mooring line.
$\overline{NB}$	Net bouyancy of the inserted body.
n	Arbitrary line segment, or lumped mass.
o, x, y, z	Fixed coordinate system, frame E.
P	Hydrostatic pressure of the fluid.
P_i	$= \int_{\ell}^u 2 \, dB(z'_p)^i$
p	Location of the centroid of the differential disk of a surface buoy.
Q_i	$= \int_{\ell}^u 2 \, dB e^{Kz_w} (z'_p)^i$
$\bar{R}$	Displacement vector.
$\bar{R}_{oc}$	Displacement vector from point o to point c.

r	Radius of the spar buoy or the reduced radius of the mooring line.
r_1, r_2	Radii of the tuned spar buoy.
$\vec{s}$	Spatial vector in the x-y (horizontal) plane, describing propagation of the surface wave.
S_0	Cross-sectional area of the surface buoy.
S_{01}, S_{02}	Cross-sectional areas of the tuned spar buoy.
s	spatial coordinate of the mooring line.
$\vec{T}$	The tension vector.
T_e	Effective tension variable along the mooring line.
$\overline{TM}$	Moment of the tension force $\vec{T}$.
$\overline{TF}$	Tension force vector on a mooring line node.
t	Time variable.
$\vec{U}$	Fluid velocity vector.
$\overline{UR}$	Relative fluid velocity vector.
$\overline{UNR}, \overline{UTR}$	Normal and tangential components of $\overline{UR}$.
$\vec{V}$	Velocity vector.
$\hat{e}$	Unit vector along the mooring line.
(V)	Array of translation and rotation rates.
V_{Bw}	Velocity of water particle w in frame B.
V_b	Tangential relative fluid velocity at bottom of the tuned spar.
V_{ox}, V_{oy}, V_{oz}	Components of surface current.
V_s	Tangential relative fluid velocity at step of the tuned spar, also static velocity of the drogued drifting buoy system.

$\bar{W}$	Gravitational force on a line element of mooring line, of angular velocity of the buoy.
W_a	Weight in air per unit length of line at the mooring line.
W_w	Weight in water, of the moored body or per unit length of mooring line.
$\bar{W}_n, \bar{W}_t$	Normal and tangential components of $\bar{W}$.
w	Location of the particle of water.
$\bar{Y}$	Displacement vector of the mooring line.
$\bar{Y}_n, \bar{Y}_t$	Normal and tangential components of the mooring-line element.
z_c	Distance between z and the bottom of the buoy.
ϕ	Velocity potential function.
ω	Wave frequency.
θ	Angle between wave direction and the x-axis.
ζ_0	Wave amplitude.
ζ	Wave surface elevation.
ψ	Roll angle of the surface buoy; also used for mooring line inclination.
θ	Pitch angle of the surface buoy.
γ	Yaw angle of the surface buoy.
ρ	Water density.
ΔL_n	Change in length of the mooring line.
α	Added mass coefficient for the case of the spar buoy.

1.0 INTRODUCTION

Oceanographic measurements from moored buoy systems are contaminated by mooring motion. Numerous articles have been published in the last three years pointing out quantitatively the errors introduced by the surface wave field. Large vertical excursions have been experienced at great depths by instruments located on the synthetic rope portions of surface-buoyed mooring lines (WUNSCH and DAHLEN, 1974). Observations during the POLYMODE experiment showed subsurface mooring lines to have undergone hundreds of meters of vertical excursions. For any operational moored instrument system we need two models whose accuracy can be established with known confidence: a mooring system model from which the motion environment of instruments can be predicted, and a model of the instrument motion response characteristics from which the measurement error can be estimated. Possession of these models is required for more effective mooring systems design and for optimum interpretation of oceanographic measurements obtained from moored systems. The first of these two models is the main subject of this report.

An alternative to the moored approach is the use of surface-trackable drogued drifting buoys. Such a

buoy system employs a high drag device (or drogue) at some depth, tethered to a trackable buoy at the surface. The major impediments in the widespread use of this approach have been inadequate component and system design. Both of these impediments can be removed by the development of dynamic modeling of drogued buoys. A part of this report deals with the dynamic modeling of drogued buoys.

Dynamic models of drogued buoys do not exist except in primitive form, while such models of moored buoys and free-drifting buoys are further along. This difference stems from the fact that drogued buoy systems are dynamically very complex, and they have only recently been considered essential to major programs. Even though many dynamic mathematical models for moored buoys and free-drifting buoys are available, not much has been done toward the evaluation of these models using full-scale ocean test data. It is the purpose of this report to present a general, computationally efficient approach for analyzing moored buoy systems, free drifting buoys, and drogued buoy systems and to simulate this analysis on the computer so that these models can then be readily evaluated with full scale ocean test data.

This report constitutes parts 3 and 4 of a four part report. Parts 1 and 2 were published in

volume 1 and dealt with the "Three Dimensional Static Analysis and Design of Single Point Taut and Slack Moored Buoy Systems" (CHHABRA, 1973).

1.1 Background

We at Charles Stark Draper Laboratory Inc. have evaluated one of the mathematical models presented in this report. The mathematical model of a subsurface mooring system (Section 2.2.1 and simulation 5.2 in this report) has been shown previously to predict the mooring motion forced by ocean currents of periods greater than 15 min. (CHHABRA, DAHLEN and FROIDEVAUX, 1974). The model was evaluated in a full scale ocean test that provided experimental data on mooring response and ocean current forces. The ocean test was conducted jointly with Woods Hole Oceanographic Institution on R. V. Chain cruise 107. We obtained a record of the motion of an acoustic transceiver near the top of the subsurface mooring line at the 500-m depth. The transceiver sent out a sound pulse every minute and recorded the four return times of replies from four near-bottom acoustic transponders at 5460m. By comparison with this and other data from precision pressure recorders, tensiometers, and inclinometers, the mooring model had been found to predict well the observed motions. The r.m.s. difference

between the experimental and predicted trajectory of the acoustic transceiver was 11.8m, about 10% of the mean excursion. This mathematical model was evaluated a second time by data from the central mooring (Mooring No. 1, Station 431) of the Mid-Ocean Dynamics Experiment (MODE). In that study (CHHABRA, 1976), the current record from the topmost vector-averaging current meter on the central mooring of the MODE experiment was corrected for the effects of mooring motion, and power spectra of the uncorrected and corrected signals were compared. The correction was $< 1 \text{ cm s}^{-1}$, for that mooring line (vertical excursion $< 12 \text{ m}$). Creep in the synthetic portion of the mooring line was also identified.

In addition, we are planning to evaluate a second mathematical model presented in this report. The mathematical model is of a tuned spar buoy (35 ft. long, 1 ft. dia. at W.L.) tethered to a subsurface mooring line by a stiff buoyant line. An instrument line is hanging from the base of the spar (Sections 2.1.1, 2.2.2, 2.3 and simulation 5.3). This configuration was recently tested in the ocean during the October 1976 ONR/NDBO Mooring Dynamics Experiment. A total of fifteen motion sensing instruments (4 Force Vector Recorders, 6 Temperature/Pressure Recorders, 1 POPMIP, and 4 Acoustic

Beacons) were attached along this mooring system. The evaluation and improvement of this mathematical model would be done by comparing measured responses with those computed by the computer simulation for the measured/observed environment.

The above mentioned evaluation and improvement is planned to be completed in CY77. If such an evaluation and improvement is done, the only mathematical model to be evaluated and improved from the analysis presented in this report would be the drifting drogued buoys. We plan to do that task in the near future.

1.2 Assumptions, Capabilities, and Limitations

Surface gravity waves treated in this report have a single frequency and amplitude, propagating in a single direction. It is assumed that the surface buoy is oscillating in the path of small (i.e. amplitude of wave train much less than its wavelength) incident surface waves which are long in relation to the body dimension in the direction of wave propagation. Wave direction has been generalized to include any arbitrary direction in the horizontal plane. For the three-dimensional analysis of surface buoys, we also assume small displacements of buoy when compared to vertical dimension of the buoy. Hence only the first powers of

small quantities are retained for the three-dimensional surface buoy analysis. Moments of inertia about all axes in a horizontal plane are taken equal. Even though moment of inertia about the longitudinal axis of the spar buoy is negligible, it is included for numerical computational purposes.

In all mathematical models an earth fixed frame is assumed to be the valid inertial frame. Viscous drag forces are computed based on the square drag law. For cylindrical shapes these forces are assumed to act in directions normal and tangential to the longitudinal axis; and for any other shape they act in the direction of the relative flow. For the case of a tuned spar viscous drag forces and 'added mass' due to the bottom base and the step are also included. The ideal fluid damping (wave-damping) as derived by Newman (1963) for a spar buoy was found to be negligible and hence is neglected.

All mooring lines are considered elastic.

In the continuous line formulation of mooring lines, nonlinear elasticity dependent on the prior loading history as explained in volume 1 (CHHABRA, 1973) is considered. For the lumped parameter formulation; a linear stress-strain curve is derived from the nonlinear curves for the range of stresses in study. Dynamic effects on these stress-

strain curves are neglected. Internal damping forces in the mooring lines are assumed negligible when compared with viscous drag and stiffness forces. The hydrostatic pressure forces are treated more rigorously than they are in the traditional method employed in most previous studies. As it turned out this new method changed only the stress distribution along the mooring system, while the configuration of the mooring system remained essentially unchanged. Instruments or buoyancy packages attached along the mooring lines are treated as concentrated forces which have length and give rise to forces as explained in Section 2.2. A time varying current profile of any nature and shape can be inputted to the computer programs. A list of current profiles used in various simulations is given in volume 1 (CHHABRA, 1973). In its present form no allowance for shrinkage or creep of the synthetic ropes is taken into account, but creep was identified in CHHABRA (1976). Elongations due to rotation of non-torque balanced cables is also not considered. In the mooring line models viscous drag forces due to the velocity field generated by the surface waves are included. Wave-damping forces are neglected. In the continuous line formulation of the mooring line, exciting forces exerted on the line by the wave system are neglected; whereas in the lumped

parameter formulation, exciting forces are taken equal to the 'Froude-Krylov' forces as it is assumed that the presence of the mooring line does not disturb the wave particle motion. For added mass purposes, it is assumed for the continuous line formulation that the body (continuous cylindrical line) motion accelerates fluid only in the direction normal to its longitudinal axis.

2.0 THEORETICAL ANALYSIS - MATHEMATICAL MODELS

This section derives all the equations of motion pertinent to the analysis presented in this report. Surface floats are analyzed in Subsection 2.1, mooring line in 2.2, their attachment in Section 2.3, and a window shade drogue in Section 2.4. The mooring line analysis includes all subsurface floats, instruments etc. attached to the mooring line.

2.1 Surface Buoys

As is well known, the analysis of the wave induced response of floating bodies is, in general, a most formidable task. Initially the action of the fluid must be decomposed into real (viscous) and ideal (inviscid) effects. Each effect then must be modeled as to its interaction with the floating body. Further, it is convenient to assume that the ocean waves are "gentle" enough so as to permit first order linear surface wave theory to serve as a foundation for the calculation of the wave exciting forces.

The ideal fluid problem is still so difficult in general that only the simplest body shapes are amenable to rigorous analytical solution (potential flow theory). This solution involves the determination of the (velocity) potential function $\phi(\vec{S}, t)$ which must

satisfy: 1) Laplace's equation, 2) the kinematic boundary condition on the moving body surface, 3) the free surface (and bottom) boundary condition(s) and 4) the radiation condition at great distance from the body.

Assuming this potential, $\phi(\vec{r}, t)$, has been determined by some means, the next step is to substitute it into the Bernoulli's expression for fluid pressure which in turn is integrated over the immersed portion of the body to yield the instantaneous force and moment vectors. In principle this is a straight-forward procedure. In practice it is hardly ever possible to find a tractable potential function which satisfies the above four conditions. In a few notable cases, however, for simple geometries and small body motions, rigorous solutions have been worked out. In particular Newman (1963) has derived the linearized equations of motion for a vertical, cylindrical air buoy responding to the influence of a unidirectional wave train. The axisymmetry of the buoy as well as its postulated slenderness was greatly exploited in the work to yield manageable results. In Newman's work as well as in others of comparable rigor, the computed potential is the result of an intricate distribution of singularities (sources, sinks, dipoles, etc.) within or over the wetted body surface.

From these studies, it turns out that the linearized

hydrodynamic forces and moments in general body geometries may be decomposed into constituents proportional to body acceleration, body velocity, displaced fluid acceleration, and the displaced fluid velocity. In general, rigorous integration of the fluid pressure, both hydrostatic and wave, over the body surface yields:

1) the 'Froude-Krylov' force from the pressure distribution due to the undisturbed wave system; 2) the "diffraction" force from disturbance of the waves by the presence of the body; 3) the force due to the motion of the body; and 4) the hydrostatic pressure force. The Froude-Krylov force and the diffraction force in combination are also known as the exciting forces exerted on the body by the wave system. The Froude-Krylov force equals in value to the product of the mass of the displaced fluid times the acceleration of the local undisturbed fluid particles. The diffraction force and the force due to the motion of the body both yield "added mass" coefficients proportional to accelerations and wave-damping coefficients proportional to velocities of the fluid and the body respectively. As shown in Chung (1976), the added mass coefficients are same (opposite signs) for both the fluid and body accelerations. Also the two wave-damping coefficients have the same (opposite signs) value.

Hence, these two forces may be combined to represent an "added mass" force proportional to the relative acceleration (fluid acceleration minus the body acceleration), and a wave-damping force proportional to the relative velocity. Other forces (apart from the fluid pressure forces) acting on the body are: 1) the weight of the body; and 2) the actual unbalanced force which accelerates the body. In the real (viscous) fluid another force, the viscous drag force, acts on the submerged portion of the body.

Analysis in this section includes all the above mentioned forces, as derived from the potential flow theory, except the following deviations: 1) the ideal fluid damping (wave-damping) is omitted; 2) the added mass force is treated slightly differently, and 3) the viscous drag forces are added on to the general equations of motion.

The ideal fluid damping is omitted here for expediency. Later analysis of specific problem addressed in this report showed the wave damping force, as derived by Newman (1963) for a spar buoy, to be negligible compared to the viscous drag force on the submerged body. The added mass force, which is proportional to the relative acceleration is separated into two terms; an added mass force proportional to the longitudinal component

of the relative acceleration, and an added mass force proportional to the transverse component of the relative acceleration. Each of these two terms is multiplied by a different constant coefficient, depending on the shape of the floating body. These coefficients and the coefficients used to derive the viscous drag force are experimentally determined hydrodynamic coefficients.

As will be shown below, our general equations of motion reduce to the ones given in Rudnick (1967) for a particular value of these coefficients. Rudnick in his analysis used the reasoning of Lamb (1945) who treats a uniform two-dimensional flow across a long circular cylinder.

Formulation of the General Equations of Motion:

Derivation of the equations of motion for surface piercing buoys in a train of regular harmonic ocean waves is given. In addition a surface current (not due to surface waves) is present. Surface buoys are considered as rigid bodies with six degrees of freedom. The problem under consideration is represented schematically in Figure 2.1. An earth fixed cartesian coordinate system (z positive upwards) is situated at the undisturbed level of the free surface. Call it the frame E with origin at o ;

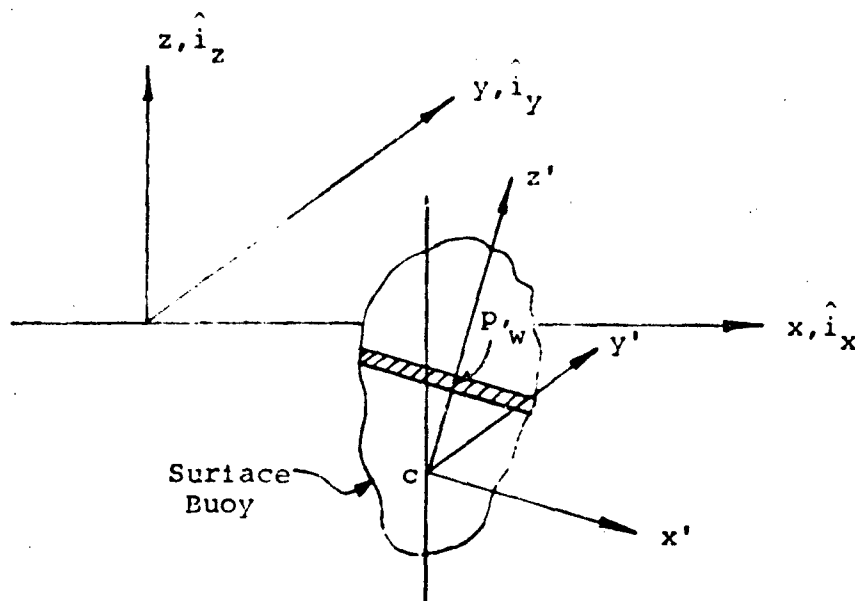


Figure 2.1 Coordinate System for Surface Piercing Buoys

and $\hat{i}_x, \hat{i}_y$, and $\hat{i}_z$ the unit vectors along x, y , and z respectively. Frame B is a body (surface buoy) fixed cartesian coordinate system with its origin at the centroid (c) of the surface buoy, and x', y', z' being parallel to x, y, z respectively when the buoy is in the upright and non-rotating position. In this analysis $\bar{R}$ represents a displacement vector; $\bar{V}$, a velocity vector, and $\bar{A}$, an acceleration vector. It is assumed that the surface buoy is oscillating in the path of small (i.e., amplitude of wave train much less than its wavelength) incident surface waves which are long in relation to the body dimensions in the direction of wave propagation. Point

p is the location of the centroid of a differential disk (height = dz') of the surface buoy (Figure 2.1).

First, the wave direction will be generalized to include any arbitrary direction in the x-y plane. To this end let the wavenumber vector $\bar{K}$ specify the direction of propagation (Figure 2.2) where:

$$\bar{K} = \frac{\omega^2}{g} (\cos \theta \hat{i}_x + \sin \theta \hat{i}_y)$$

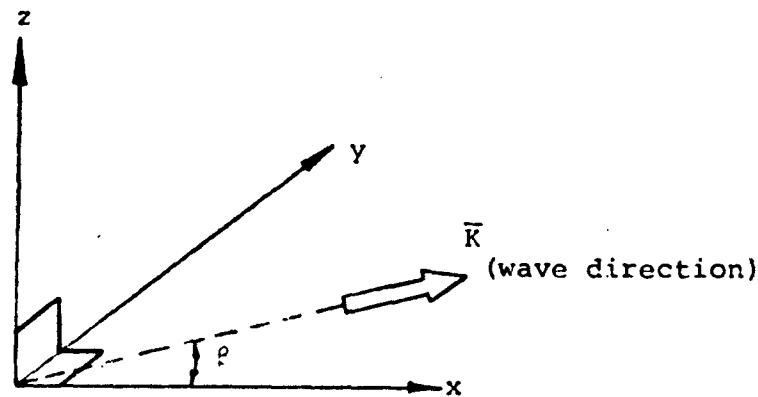


Figure 2.2 - Generalized Wave Direction

The generalized velocity potential field of the incident wave system may now be written:

$$\phi = \frac{g\xi_0}{\omega} e^{|\bar{K}|z} \cos (\bar{K} \cdot \bar{S} - \omega t)$$

where; $\bar{S} = \hat{i}_x x + \hat{i}_y y$; and the wave surface elevation ξ is

given by:

$$\xi = -\xi_0 \sin(\bar{K} \cdot \bar{r} - \omega t)$$

Here, g is the acceleration due to gravity, and ξ_0 is the amplitude of the incident wave of frequency ω . In the earth fixed frame (frame E), assumed to be a valid inertial frame for this problem, we define the fluid particle acceleration vector:

$$\bar{A}_{Ew} = \hat{i}_x \ddot{\phi}_x + \hat{i}_y \ddot{\phi}_y + \hat{i}_z \ddot{\phi}_z$$

where w is the particle of water next to the differential disk of centroid p . It is assumed that A_{Ew} is constant over the entire differential disk. Components of A_{Ew} at point p are given by:

$$\ddot{\phi}_x = \frac{d}{dt} \frac{\partial \phi}{\partial x} = g K_x \xi_0 e^{|\bar{K}| z_w} \cos(\bar{K} \cdot \bar{r} - \omega t)$$

$$\ddot{\phi}_y = \frac{d}{dt} \frac{\partial \phi}{\partial y} = g K_y \xi_0 e^{|\bar{K}| z_w} \cos(\bar{K} \cdot \bar{r} - \omega t)$$

and
$$\ddot{\phi}_z = \frac{d}{dt} \frac{\partial \phi}{\partial z} = g |\bar{K}| \xi_0 e^{|\bar{K}| z_w} \sin(\bar{K} \cdot \bar{r} - \omega t)$$

where; $K_x = |\bar{K}| \cos \beta$; $K_y = |\bar{K}| \sin \beta$, and the higher order terms in ϕ_z have been neglected.

Now; $\bar{R}_{op} = \bar{R}_{oc} + \bar{R}_{cp}$

$\therefore \bar{V}_{Ep} = [\dot{\bar{R}}_{op}]_E = [\dot{\bar{R}}_{oc}]_E + \bar{W}_{EB} \times \bar{R}_{cp}$

and $\bar{A}_{Ep} = [\ddot{\bar{R}}_{oc}]_E + [\dot{\bar{W}}_{EB}]_E \times \bar{R}_{cp} + \bar{W}_{EB} \times (\bar{W}_{EB} \times \bar{R}_{cp})$

where; $\bar{W}_{EB}$ is the angular velocity of the buoy.

Let us also define the relative acceleration vector of the fluid particles with respect to the buoy ($\bar{A}_R$), at the location of point p as:

$\bar{A}_R \equiv \bar{A}_{Ew} - \bar{A}_{Ep}$

also let, $\bar{A}_{RT} = (\bar{A}_R \cdot \hat{i}_z) \hat{i}_z$,

and $\bar{A}_{RN} = \bar{A}_R - \bar{A}_{RT}$

The vector force equations of motion may now be written as:

$$\begin{aligned} \int_{\ell}^{u_1} dM \bar{A}_{Ep} &= \int_{\ell}^{u_2} dB \bar{A}_{Ew} + C_N \int_{\ell}^{u_2} dB \bar{A}_{RN} + \\ &C_T \int_{\ell}^{u_2} dB \bar{A}_{RT} - \int_{\ell}^{u_3} dB \bar{g} + \int_{\ell}^{u_1} dM \bar{g} \end{aligned} \quad (2.1)$$

where the viscous drag forces and the tension forces (due to the attached mooring lines) are left out for later introduction.

Here; $l = -z_c$, $u_1 = H - z_c$, $u_2 = h_o - z_c$, and $u_3 = h_i - z_c$. z_c is the distance between c and the bottom of the buoy. H is the length of the buoy, h_o is the instantaneous draft of the buoy measured up to the mean free surface, and h_i is h_o plus the wave elevation component. Distances z_c , l , h_o and h_i are measured along $\hat{i}_z$. dM is the mass of the differential disk (Figure 2.1) of height dz' and δB is the mass of water displaced by this disk. C_N AND C_T are the appropriate hydrodynamic constants.

In equation (2.1) the integrals in order of their appearance represent: 1) the actual unbalanced force which accelerates the buoy; 2) the Froude-Krylov force; 3) the normal component of the added mass force; 4) the tangential component of the added mass force; 5) the hydrostatic pressure force; and 6) the weight of the buoy. This equation reduces to the equation in Rudnick (1967), which was derived for a spar buoy, for $C_N = 1.0$, and $C_T = 0.0$.

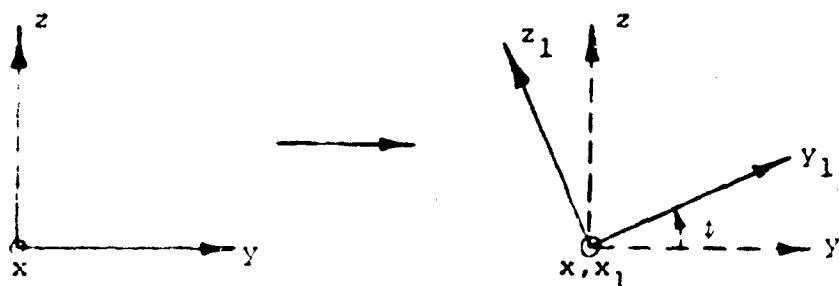
The vector moment equation can now be written similarly as: (moments about C.G.)

$$\begin{aligned}
 \int dM(\bar{R}_{cp} \times \bar{A}_{Ep}) &= \int dB(\bar{R}_{cp} \times \bar{A}_{Ew}) + \\
 &+ C_N \int dB(\bar{R}_{cp} \times \bar{A}_{R1}) + \\
 &+ C_T \int dB(\bar{R}_{cp} \times \bar{A}_{RT}) - \\
 &- \int dB(\bar{R}_{cp} \times \bar{g}) + \int dM(\bar{R}_{cp} \times \bar{g})
 \end{aligned}
 \tag{2.2}$$

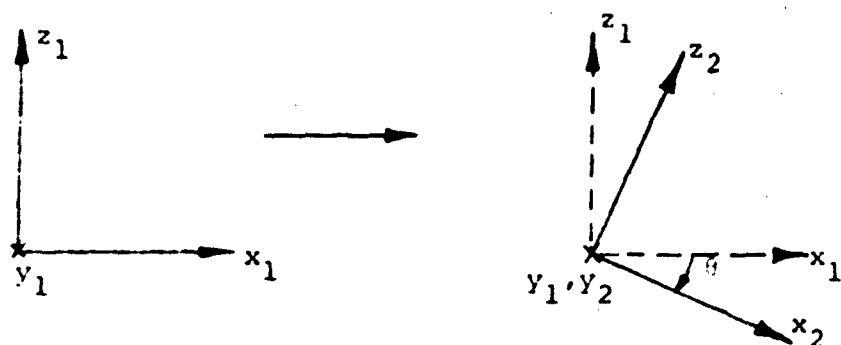
It is also necessary to determine an appropriate transformation scheme to relate vectors in the E frame to vectors in the B frame. A sequence of rotations (Figure 2.3) about the body x', y', z' axes (roll = ϕ , pitch = θ , and yaw = ψ) respectively yield the following transformation.

$$\begin{Bmatrix} \hat{i}_{x'} \\ \hat{i}_{y'} \\ \hat{i}_{z'} \end{Bmatrix} = \begin{bmatrix} \cos\psi \cos\theta & \sin\psi \cos\theta & \sin\psi \sin\theta \\ -\sin\psi \cos\theta & \cos\psi \cos\theta & \cos\psi \sin\theta \\ \sin\theta & -\cos\theta \sin\phi & \cos\theta \cos\phi \end{bmatrix} \begin{Bmatrix} \hat{i}_x \\ \hat{i}_y \\ \hat{i}_z \end{Bmatrix}$$

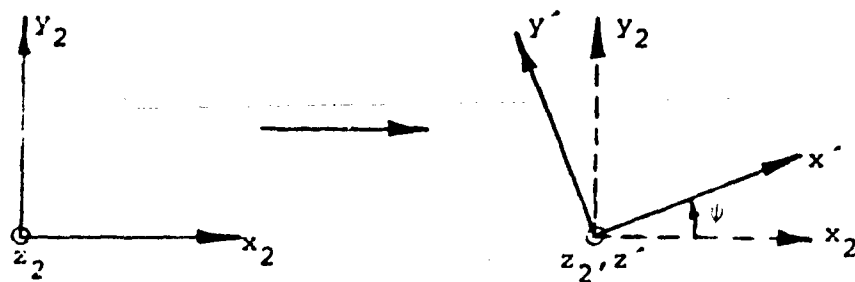
The inverse of this transformation matrix is given by its transpose. The force and moment equations (2.1) and (2.2) may be combined and written in matrix form as:



Step 1 - Roll (ϕ): Rotate y, z about x to obtain x_1, y_1, z_1



Step 2 - Pitch (θ): Rotate x_1, z_1 about y_1 to obtain x_2, y_2, z_2



Step 3 - Yaw (ψ): Rotate x_2, y_2 about z_2 to obtain x', y', z'

Figure 2.3 Transformation Matrix Rotations

$$[M] \cdot \frac{d}{dt} \{V\} = \{F_D\} + \{F_G\} + \{F_A\} \quad (2.3)$$

Here the matrix $[M]$ is composed of the tensors of inertia and added inertia. The $\{V\}$ column vector has three translation rates and three angular rates. The vector $\{F_D\}$ contains the wave exciting forces and moments. The vector $\{F_G\}$ comprises the hydrostatic pressure and weight restoring forces and moments (i.e. gravitational and buoyancy effects); and $\{F_A\}$ is any additional forces and moments including viscous drag forces and moments introduced next and tension forces/moments from attached mooring lines introduced in Section 2.3.

Viscous drag forces and moments. A square drag law, where viscous drag forces are proportional to the square of the relative fluid velocity is used. In Figure 2.1

$$\bar{R}_{ow} = \bar{R}_{oc} + \bar{R}_{cp} + \bar{R}_{pw}$$

$$\therefore \bar{V}_{Ew} = \bar{V}_{Ec} + \bar{W}_{EB} \times \bar{R}_{cp} + [\dot{\bar{R}}_{pw}]_E$$

If point p and w are overlapping then:

$$[\dot{\bar{R}}_{pw}]_E = [\dot{\bar{R}}_{pw}]_B = \bar{V}_{Bw} = \bar{V}_{Ew} - \bar{V}_{Ec} - \bar{W}_{EB} \times \bar{R}_{cp}$$

Here; $\vec{V}_{Ew} = \hat{i}_x (V_{ox} + \dot{x}) + \hat{i}_y (V_{oy} + \dot{y}) + \hat{i}_z (V_{oz} + \dot{z})$ and $\vec{V}_{Bw}$ is the relative velocity of water seen by the buoy at the local point p. Hence the viscous drag force on the submerged portion of the buoy is given by:

$$\overline{DF} = \frac{1}{2} \rho C_D \int_{\ell}^{u_2} dA_D |\vec{V}_{Bw}| \vec{V}_{Bw} \quad (2.4)$$

where, ρ is the water density, C_D the drag coefficient, and dA_D the appropriate differential area. The viscous drag moment can similarly be written as:

$$\overline{DM} = \frac{1}{2} \rho C_D \int_{\ell}^{u_2} dA_D |\vec{V}_{Bw}| (\vec{R}_{cp} \times \vec{V}_{Bw}) \quad (2.5)$$

Limits of integration for both (2.4) and (2.5) are the entire submerged depth of the buoy.

In the remainder of Section 2.1 the equations (2.1), (2.2), (2.4), and (2.5) presented above will be specialized to the specific problems at hand. The matrices of equation (2.3) will then be derived and presented for these specific problems.

2.1.1 Spar Buoy

A tuned spar buoy (Figure 2.4) will be analyzed in this section to obtain general spar buoy equations.

Note: Shown for zero rotation and translation.

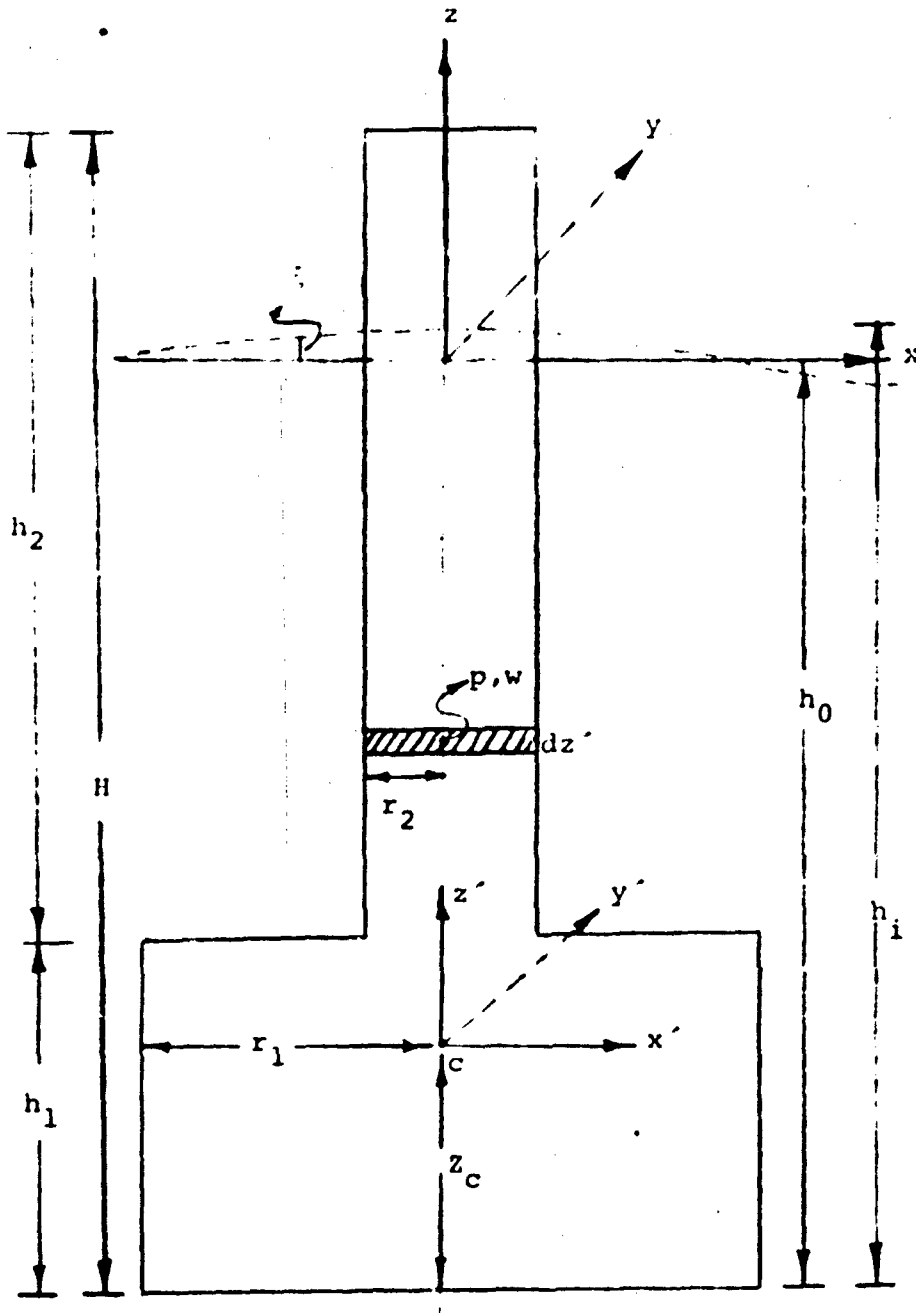


Figure 2.4 - A Tuned Spar Buoy

Cylindrical spar buoy equations can then be obtained as a special case of the tuned spar buoy. Both two-dimensional and three-dimensional analysis will be presented. We will find the equations of motion of its response for a train of surface gravity waves having a single frequency and amplitude, propagating in any single direction. Variation in wave acceleration and velocity over the horizontal buoy dimensions are neglected. In addition in the three-dimensional analysis, all rotational angles (roll, pitch, and yaw) are restricted to be small.

Two-Dimensional Analysis: In this analysis $\bar{K}$ is a scalar with $\tau = 0$; and $\bar{S}$ is replaced by the scalar x which is the direction of propagation. Also, $\bar{A}_{EW} = i_x i_x + i_z i_z$.

$$\bar{A}_{RN} = (\bar{A}_E \cdot \hat{i}_x) \hat{i}_x,$$

and the appropriate transformation with $\tau = 0$, and $\psi = 0$ becomes

$$\begin{Bmatrix} \hat{i}_x \\ \hat{i}_z \end{Bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{Bmatrix} \hat{i}_x \\ \hat{i}_z \end{Bmatrix}$$

Now let, $[\ddot{\mathbf{R}}_{oc}]_E = \hat{i}_x \ddot{x}_c + \hat{i}_z \ddot{z}_c$

$$\bar{\mathbf{W}}_{EB} = \hat{i}_y \dot{\theta}$$

and $\bar{\mathbf{R}}_{cp} = z'_p \hat{i}_z$

$$\begin{aligned} \therefore \bar{\mathbf{A}}_{Ep} &= \hat{i}_x [\ddot{x}_c + z'_p (\ddot{\theta} \cos\theta - \dot{\theta}^2 \sin\theta)] \\ &\quad + \hat{i}_z [\ddot{z}_c + z'_p (-\ddot{\theta} \sin\theta - \dot{\theta}^2 \cos\theta)] \end{aligned}$$

Also, $\bar{\mathbf{A}}_{Ew} = \epsilon_0^2 e^{Kz_w} [\hat{i}_x \cos(Kx_c - t) + \hat{i}_z \sin(Kx_c - t)]$

Define; $P_i = \int_1^{u_2} dB (z'_p)^i \quad i = 0, 1, 2$

and $Q_i = \int_1^{u_2} dB e^{Kz_w} (z'_p)^i \quad i = 0, 1$

Integrals of equation (2.1) can now be written as:

$$1. \int dM \bar{\mathbf{A}}_{Ep} = M[\hat{i}_x \ddot{x}_c + \hat{i}_z \ddot{z}_c]$$

Where M is the total mass of the buoy.

$$2. \int dB \bar{\mathbf{A}}_{Ew} = \epsilon_0^2 Q_0 [\hat{i}_x \cos(Kx_c - t) + \hat{i}_z \sin(Kx_c - t)]$$

$$\begin{aligned}
 3. \quad C_N \int dB \bar{A}_{RN} = C_N \hat{i}_x \{ \omega^2 \xi_0 Q_0 [\cos(Kx_C - \omega t) \cos^2 \theta - \\
 - \sin(Kx_C - \omega t) \sin \theta \cos \theta] + \\
 + P_0 (\ddot{z}_C \sin \theta \cos \theta - \ddot{x}_C \cos^2 \theta) - P_1 \cos \theta \ddot{\theta} \} + \\
 + C_N \hat{i}_z \{ \omega^2 \xi_0 Q_0 [\sin(Kx_C - \omega t) \sin^2 \theta - \cos(Kx_C - \omega t) \\
 \sin \theta \cos \theta] + P_0 (\ddot{x}_C \sin \theta \cos \theta - \ddot{z}_C \sin^2 \theta) + \\
 + P_1 \sin \theta \ddot{\theta} \}
 \end{aligned}$$

$$\begin{aligned}
 4. \quad C_T \int dB \bar{A}_{RT} = C_T \hat{i}_x \{ \omega^2 \xi_0 Q_0 [\cos(Kx_C - \omega t) \sin^2 \theta + \\
 + \sin(Kx_C - \omega t) \sin \theta \cos \theta] + \\
 + P_0 (-\ddot{x}_C \sin^2 \theta - \ddot{z}_C \sin \theta \cos \theta) + \\
 + P_1 \sin \theta \ddot{\theta}^2 \} + C_T \hat{i}_z \{ \omega^2 \xi_0 Q_0 [\cos(Kx_C - \omega t) \sin \theta \cos \theta + \\
 + \sin(Kx_C - \omega t) \cos^2 \theta] + P_0 (-\ddot{x}_C \sin \theta \cos \theta - \\
 - \ddot{z}_C \cos^2 \theta) + P_1 \cos \theta \ddot{\theta}^2 \}
 \end{aligned}$$

$$5. \quad - \int_l^{u_3} dB \bar{g} = g \int_l^{u_2} dB \hat{i}_z + g \int_0^{-\xi_0 \sin(Kx_C - \omega t) / \cos \theta} dB \hat{i}_z$$

Also; $dB = \rho S_0 dz'$, where S_0 is the cross-sectional area of the disk of height dz' . Then;

$$-\int_l^{u_3} dB \bar{g} = \hat{i}_z [P_O g - \rho S_{O2} g \xi_O \sin(Kx_C - \omega t) / \cos \theta]$$

where; $S_{O2} = \pi r_2^2$

$$6. \int dM \bar{g} = - Mg \hat{i}_z$$

Integrals of equation (2.2) can also be written

as:

$$1. \int dM (\bar{R}_{cp} \times \bar{A}_{Ep}) = \int dM z'^2 \ddot{\theta} \hat{i}_y = I_{yy} \ddot{\theta} \hat{i}_y$$

$$2. \int dB (\bar{R}_{cp} \times \bar{A}_{Ew}) = \hat{i}_y \{ \omega^2 \xi_O Q_1 [\cos(Kx_C - \omega t) \cos \theta - \sin(Kx_C - \omega t) \sin \theta] \}$$

$$3. C_N \int dB (\bar{R}_{cp} \times \bar{A}_{RN}) = C_N \hat{i}_y \{ \omega^2 \xi_O Q_1 [\cos(Kx_C - \omega t) \cos \theta - \sin(Kx_C - \omega t) \sin \theta] - P_1 \cos \theta \ddot{x}_C + P_1 \sin \theta \ddot{z}_C - P_2 \ddot{\theta} \}$$

$$4. C_T \int dB (\bar{R}_{cp} \times \bar{A}_{RT}) = 0$$

$$5. - \int_l^{u_3} dB (\bar{R}_{cp} \times \bar{g}) \cong - \int_l^{u_2} dB (\bar{R}_{cp} \times \bar{g}) = \hat{i}_y (-P_1 g \sin \theta)$$

$$6. \int dM (\bar{\mathbf{R}}_{cp} \times \bar{\mathbf{g}}) = 0$$

Viscous Drag Forces and Moments: For a tuned spar buoy the viscous drag forces are assumed to be acting in normal and tangential directions proportional to their respective drag coefficients and areas.

$$\begin{aligned} \mathbf{V}_{Bw} &= \hat{\mathbf{i}}_x [V_{Ox} + \phi_x - \dot{x}_c - z_p' \dot{\theta} \cos \theta] + \\ &\quad + \hat{\mathbf{i}}_z [V_{Oz} + \phi_z - \dot{z}_c + z_p' \dot{\theta} \sin \theta] \\ &= \hat{\mathbf{i}}_x [(V_{Ox} + \phi_x - \dot{x}_c) \cos \theta \\ &\quad - (V_{Oz} + \phi_z - \dot{z}_c) \sin \theta - z_p' \dot{\theta}] \\ &\quad + \hat{\mathbf{i}}_z [(V_{Ox} + \phi_x - \dot{x}_c) \sin \theta \\ &\quad + (V_{Oz} + \phi_z - \dot{z}_c) \cos \theta] \\ &= \hat{\mathbf{i}}_x V_{Bwx} + \hat{\mathbf{i}}_z V_{Bwz} \end{aligned}$$

Drag force components can now be written:

$$DF_{x'} = \rho C_{DN} \int r |V_{Bwx}| V_{Bwx} dz'$$

$$\begin{aligned} DF_{z'} &= \rho C_{DT} \pi \int r |V_{Bwz}| V_{Bwz} dz' \\ &\quad + \frac{1}{2} \rho C_{Dp} \pi \{ r_1^2 |V_b| V_b + (r_1^2 - r_2^2) |V_s| V_s \} \end{aligned}$$

Here; C_{DN} and C_{DT} are normal and tangential drag coefficients. C_{DP} is the pressure drag coefficient due to the bottom, and the step, of the tuned spar buoy.

$$V_s = V_{Bwz}, (z' = h_1 - z_c)$$

and $V_b = V_{Bwz}, (z' = -z_c)$

$$\text{Also; } DM = \rho C_{DN} \int r |V_{Bwx}| V_{Bwx} z'_p dz'$$

Using the transformation matrix:

$$\begin{Bmatrix} DF_x \\ DF_z \end{Bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{Bmatrix} DF_{x'} \\ DF_{z'} \end{Bmatrix}$$

Added Mass Force Due to the Step and Bottom Base of the Tuned Spar: In addition to the forces mentioned above, an added mass force proportional to the tangential body acceleration ($\bar{A}_{Et}$) and acting perpendicular to the step and the bottom end of the tuned spar is considered. Lamb (1945) in his analysis of a cylindrical spar moving in still water, gives an added mass coefficient equal to $4/3\rho r^3$. Following representation is used in this analysis for this force ($\bar{AM}$).

$$\overline{AM} = \frac{4}{3} \alpha \rho \left[r_1^3 \overline{A}_{Et} |_{z'=-z_c} + (r_1-r_2)^3 \overline{A}_{Et} |_{z'=h_1-z_c} \right] \quad (2.6)$$

where; $\overline{A}_{Et} = (\overline{A}_{Ep} \cdot \hat{i}_z) \hat{i}_z$, and α is a constant.

$\overline{AM}$ is added to the left hand side of equation (2.1).

$\overline{R}_{cp} \times \overline{AM}$ would be equal to zero.

Combining all these forces and moments; the matrix coefficients of equation (2.3) can be written as:

$$M_{11} = M + P_O (C_N \cos^2 \theta + C_T \sin^2 \theta) + \frac{4}{3} \alpha \rho [r_1^3 + (r_1-r_2)^3] \sin^2 \theta$$

$$M_{12} = P_O \sin \theta \cos \theta (C_T - C_N) + \frac{4}{3} \alpha \rho [r_1^3 + (r_1-r_2)^3] \sin \theta \cos \theta$$

$$M_{13} = C_N P_1 \cos \theta$$

$$M_{21} = P_O \sin \theta \cos \theta (C_T - C_N) + \frac{4}{3} \alpha \rho [r_1^3 + (r_1-r_2)^3] \sin \theta \cos \theta$$

$$M_{22} = M + P_O (C_N \sin^2 \theta + C_T \cos^2 \theta) + \frac{4}{3} \alpha \rho [r_1^3 + (r_1-r_2)^3] \cos^2 \theta$$

$$M_{23} = - C_N P_1 \sin \theta$$

$$M_{31} = C_N P_1 \cos \theta$$

$$M_{32} = - C_N P_1 \sin \theta$$

$$M_{33} = I_{yy} + C_N P_2$$

$$\{V\} = [\dot{x}_c, \dot{z}_c, \dot{\theta}]; \text{ which is a column vector.}$$

$$F_{D1} = \omega^2 \xi_o Q_o [\cos(Kx_c - \omega t) (1 + C_N \cos^2 \theta + C_T \sin^2 \theta) + \sin(Kx_c - \omega t) \sin \theta \cos \theta (C_T - C_N)]$$

$$F_{D2} = \omega^2 \xi_o Q_o [\sin(Kx_c - \omega t) (1 + C_N \sin^2 \theta + C_T \cos^2 \theta) + \cos(Kx_c - \omega t) \sin \theta \cos \theta (C_T - C_N)]$$

$$F_{D3} = \omega^2 \xi_o Q_1 [\cos(Kx_c - \omega t) \cos \theta - \sin(Kx_c - \omega t) \sin \theta] (1 + C_N)$$

$$F_{G1} = 0.$$

$$F_{G2} = P_o g - \rho S_{o2} g \xi_o \sin(Kx_c - \omega t) / \cos \theta - Mg$$

$$F_{G3} = - P_1 g \sin \theta$$

$$F_{A1} = C_T P_1 \sin \theta \dot{\theta}^2 + D F_x - \frac{4}{3} \alpha \rho [r_1^3 + (r_1 - r_2)^3] z_c \sin \theta \dot{\theta}^2$$

$$F_{A2} = C_T P_1 \cos \theta \dot{\theta}^2 + D F_z + \frac{4}{3} \alpha \rho [r_1^3 + (r_1 - r_2)^3]$$

$$(h_1 - z_c) \cos \theta \dot{\theta}^2$$

and $F_{A3} = DM$

Three-Dimensional Analysis: For small angles the appropriate transformation matrix reduces to:

$$\begin{pmatrix} \hat{i}_{x'} \\ \hat{i}_{y'} \\ \hat{i}_{z'} \end{pmatrix} = \begin{bmatrix} 1 & 0 & -\theta \\ -\psi & 1 & \phi \\ \theta & -\phi & 1 \end{bmatrix} \begin{pmatrix} \hat{i}_x \\ \hat{i}_y \\ \hat{i}_z \end{pmatrix}$$

In this analysis we let $C_N = 1.0$, $C_T = 0.0$, and define:

$$[\ddot{\mathbf{R}}_{oc}]_E = \hat{i}_x \ddot{x}_c + \hat{i}_y \ddot{y}_c + \hat{i}_z \ddot{z}_c$$

$$\bar{\mathbf{W}}_{EB} = \hat{i}_x \dot{\phi} + \hat{i}_y \dot{\theta} + \hat{i}_z \dot{\psi}$$

and $\bar{\mathbf{R}}_{cp} = z'_p \hat{i}_{z'}$

$$\therefore \bar{A}_{Ep} = \hat{i}_x (\ddot{x}_C + z_p'') + \hat{i}_y (\ddot{y}_C - z_p') + \hat{i}_z \ddot{z}_C$$

and;

$$\bar{A}_{Ew} = \omega^2 \epsilon_0 e^{i\vec{K} \cdot \vec{r}_w} [\hat{i}_x \cos \theta \cos(\vec{K} \cdot \vec{r} - t) + \hat{i}_y \sin \theta \cos(\vec{K} \cdot \vec{r} - t) + \hat{i}_z \sin(\vec{K} \cdot \vec{r} - t)]$$

Integrals of equation (2.1) can now be written

as:

$$1. \int dM \bar{A}_{Ep} = M(\hat{i}_x \ddot{x}_C + \hat{i}_y \ddot{y}_C + \hat{i}_z \ddot{z}_C)$$

$$2. \int dB \bar{A}_{Ew} = \omega^2 \epsilon_0 Q_0 [\hat{i}_x \cos \theta \cos(\vec{K} \cdot \vec{r} - t) + \hat{i}_y \sin \theta \cos(\vec{K} \cdot \vec{r} - t) + \hat{i}_z \sin(\vec{K} \cdot \vec{r} - t)]$$

also;

$$\bar{A}_{RN} = \hat{i}_x (\dot{x}_C - \ddot{x}_C - z_p' + \ddot{z}_C) + \hat{i}_y (\dot{y}_C - \ddot{y}_C + z_p' - \ddot{z}_C) + \hat{i}_z (\ddot{x}_C - \ddot{y}_C)$$

$$3. \int dB \bar{A}_{RN} = \hat{i}_x [\omega^2 \epsilon_0 \cos \theta Q_0 \cos(\vec{K} \cdot \vec{r} - t) - P_0 \ddot{x}_C - P_1 \ddot{y}_C + P_0 \ddot{z}_C + \hat{i}_y [\omega^2 \epsilon_0 \sin \theta Q_0 \cos(\vec{K} \cdot \vec{r} - t) - P_0 \ddot{y}_C + P_1 \ddot{x}_C - P_0 \ddot{z}_C] + \hat{i}_z [P_0 (\ddot{x}_C - \ddot{y}_C) - \omega^2 \epsilon_0 \sin(\vec{K} \cdot \vec{r} - t)]$$

$$4. - \int_0^u dB \bar{g} = \int_0^u dB g \hat{i}_z + \int_0^u dB g \hat{i}_z = \hat{i}_z [P_0 g - (S_0 2 g \epsilon_0 \sin(\vec{K} \cdot \vec{r} - t))]$$

$$5. \quad \int dM \bar{g} = - Mg \hat{i}_z$$

Similarly integrals of equation (2.2) can be written as:

$$1. \quad \int dM (\bar{R}_{cp} \times \bar{A}_{Ep}) = \hat{i}_x I_{xx} \ddot{\phi} + \hat{i}_y I_{yy} \ddot{\theta}$$

$$\text{where; } I_{xx} = I_{yy} = \int_0^H dM z'^2$$

$$2. \quad \int dB (\bar{R}_{cp} \times \bar{A}_{Ew}) = \hat{i}_x \{-\omega^2 \epsilon_0 Q_1 \sin \beta \cos (\bar{K} \cdot \bar{E} - \omega t)\} \\ + \hat{i}_y \{\omega^2 \epsilon_0 Q_1 \cos \beta \cos (\bar{K} \cdot \bar{E} - \omega t)\}$$

$$3. \quad \int dB (\bar{R}_{cp} \times \bar{A}_{RN}) = \hat{i}_x \{-\omega^2 \epsilon_0 Q_1 \sin \beta \cos (\bar{K} \cdot \bar{E} - \omega t) + P_1 \ddot{y}_c - P_2 \ddot{\phi} \\ + P_1 \ddot{\phi} z_c\} + \hat{i}_y \{\omega^2 \epsilon_0 Q_1 \cos \beta \cos (\bar{K} \cdot \bar{E} - \omega t) \\ - P_1 \ddot{x}_c - P_2 \ddot{\theta} + P_1 \ddot{\theta} z_c\} - \hat{i}_z P_1 \{\theta \ddot{y}_c + \phi \ddot{x}_c\}$$

$$4. \quad - \int_{\ell}^{u_3} dB (\bar{R}_{cp} \times \bar{g}) = - \int_{\ell}^{u_2} dB (\bar{R}_{cp} \times \bar{g}) \\ = \hat{i}_x (-P_1 \phi g) + \hat{i}_y (-P_1 \theta g)$$

$$5. \quad \int dM (\bar{R}_{cp} \times \bar{g}) = 0$$

Viscous drag Forces and Moments:

$$\begin{aligned}
 \vec{V}_{Bw} &= \hat{i}_x [v_{ox} + \phi_x - \dot{z}_c - z_p' \dot{\theta}] + \\
 &\quad \hat{i}_y [v_{oy} + \phi_y - \dot{y}_c + z_p' \dot{\phi}] + \\
 &\quad \hat{i}_z [v_{oz} + \phi_z - \dot{z}_c] \\
 &= \hat{i}_x [v_{ox} + \phi_x - \dot{z}_c - z_p' \dot{\theta} + v_{oy} \psi - \dot{y}_c \psi - v_{oz} \theta + \dot{z}_c \theta] \\
 &\quad + \hat{i}_y [-v_{ox} \psi + \dot{x}_c \psi + v_{oy} + \phi_y - \dot{y}_c + z_p' \dot{\phi} + v_{oz} \phi - \dot{z}_c \phi] \\
 &\quad + \hat{i}_z [v_{ox} \theta - \dot{x}_c \theta - v_{oy} \phi + \dot{y}_c \phi + v_{oz} + \phi_z - \dot{z}_c] \\
 &= \hat{i}_x v_{Bwx}' + \hat{i}_y v_{Bwy}' + \hat{i}_z v_{Bwz}'
 \end{aligned}$$

Hence; $DF_{x'} = \rho C_{DN} \int r \sqrt{v_{Bwx}'^2 + v_{Bwy}'^2} (v_{Bwx}') dz'$

$$DF_{y'} = \rho C_{DN} \int r \sqrt{v_{Bwx}'^2 + v_{Bwy}'^2} (v_{Bwy}') dz'$$

and, $DF_{z'} = \rho C_{DT} \pi \int r |v_{Bwz}'| v_{Bwz}' dz' +$

$$\frac{1}{2} \rho C_{DP} \pi \{ r_1^2 |v_b| v_b + (r_1^2 - r_2^2) |v_s| v_s \}$$

Using the transformation matrix:

$$\begin{Bmatrix} DF_x \\ DF_y \\ DF_z \end{Bmatrix} = \begin{bmatrix} 1 & -\psi & \theta \\ \psi & 1 & -\phi \\ -\theta & \phi & 1 \end{bmatrix} \begin{Bmatrix} DF_{x'} \\ DF_{y'} \\ DF_{z'} \end{Bmatrix}$$

Also $\overline{DM} = \int (\overline{R}_{cp} \times d\overline{DF})$

$$= \hat{i}_{x'} (DM_{x'}) + \hat{i}_{y'} (DM_{y'})$$

Therefore:

$$\begin{Bmatrix} DM_x \\ DM_y \\ DM_z \end{Bmatrix} = \begin{bmatrix} 1 & -\psi & \theta \\ \psi & 1 & -\phi \\ -\theta & \phi & 1 \end{bmatrix} \begin{Bmatrix} DM_{x'} \\ DM_{y'} \\ 0 \end{Bmatrix}$$

Combining all these forces and equation (2.6) of additional added mass, we can write the non-zero elements of matrices in equation (2.3) as:

$$M_{11} = M + P_0 = M_{22}$$

$$M_{13} = 4/3 \alpha \rho [r_1^3 + (r_1 - r_2)^3] \theta - P_c \theta = M_{31}$$

$$M_{15} = P_1 = M_{51}$$

$$M_{23} = P_0 \phi - 4/3 \alpha \rho [r_1^3 + (r_1 - r_2)^3] \phi = M_{32}$$

$$M_{24} = -P_1 = M_{42}$$

$$M_{33} = M + 4/3 \rho [r_1^3 + (r_1 - r_2)^3]$$

$$M_{43} = -P_1 \phi$$

$$M_{44} = I_{xx} + P_2$$

$$M_{53} = -P_1 \theta$$

$$M_{55} = I_{yy} + P_2$$

$$M_{61} = P_1 \phi \quad M_{62} = P_1 \theta$$

$\{V\} = [\dot{z}_c, \dot{y}_c, \dot{z}_c, \dot{\phi}, \dot{\theta}, \dot{\psi}]$ is a column vector.

$$F_{D1} = 2\omega^2 \xi_o Q_o \cos \beta \cos (\bar{K} \cdot \bar{S} - \omega t)$$

$$F_{D2} = 2\omega^2 \xi_o Q_o \sin \beta \cos (\bar{K} \cdot \bar{S} - \omega t)$$

$$F_{D3} = \omega^2 \xi_o Q_o \sin (\bar{K} \cdot \bar{S} - \omega t)$$

$$F_{D4} = -2\omega^2 \xi_o Q_1 \sin \beta \cos (\bar{K} \cdot \bar{S} - \omega t)$$

$$F_{D5} = 2\omega^2 \xi_o Q_1 \cos \beta \cos (\bar{K} \cdot \bar{S} - \omega t)$$

$$F_{G3} = P_0 g - \rho S_{O2} g \epsilon_0 \sin(\vec{K} \cdot \vec{C} - \omega t) - Mg$$

$$F_{G4} = - P_1 g$$

$$F_{G5} = - P_1 g$$

and $\{F_A\} = [DF_x, DF_y, DF_z, DM_x, DM_y, DM_z]$ is a column vector.

As can be seen from these matrix elements, the sixth degree of freedom corresponding to ψ does not drop out; but M_{66} equals zero. M_{66} is introduced in the equations for computational purposes. M_{66} is given by:

$$M_{66} = \frac{\rho \pi}{2} [r_1^4 h_1 + r_2^4 (H - h_1)]$$

which represents the moment of inertia of a tuned spar about its longitudinal axis.

2.1.2 Other Shapes

To obtain the velocity potential function for any shapes other than for simple geometries and small body motions is hardly ever possible. Hence to obtain any reasonable solution to this problem one has to depend on empirical approaches. Solution to these problems has thus been left out of this report. In order to use the

analysis and computer programs of this report, the readers will have to substitute their own elements for $[M]$, $\{F_D\}$, $\{F_G\}$, and $\{F_A\}$ matrices of equation 2.3; pertinent to the particular surface buoy in question.

2.2 Mooring Line

A mooring line connects a surface or a subsurface buoy to the anchor. In this report, a line attached to a buoy but not to an anchor is also considered a mooring line. Such a line could be connecting a drogue with the surface buoy or be an instrument line hanging from a moored surface buoy. In general a mooring line is made of any type or number of materials (steel, nylon, dacron, etc.) and has any type or number of instruments (including subsurface floats) inserted along its length.

The mathematical model of mooring line dynamics will be formulated in two different approaches. The "first" approach is called the "continuous line formulation". In this approach the mooring line differential equations, with respect to the spatial coordinate s , are integrated incrementally down the mooring line, to obtain its dynamic equilibrium at any instant of time. Velocities and accelerations of the mooring line differential elements and the instruments (including subsurface floats) inserted in the mooring line are computed by differentiating

positions found by the dynamic equilibriums. This approach was used to model the low frequency motion of a subsurface mooring system, as presented later in the report. Exciting forces exerted on the mooring line by the wave system are neglected and so are the wave-damping forces. Viscous drag effects due to the velocity field generated by the surface waves is included. This approach was found to be computationally inefficient, and hard to solve numerically (due to the differentiation of positions to find velocities and accelerations) for the high frequency motion of a surface moored system.

For the high frequency motion of a surface moored system a "second" approach called the "lumped parameter formulation" is presented. Here exciting forces exerted on the mooring line by the wave system are taken equal to the "Froude-Krylov" forces, as it is assumed that the presence of the mooring line does not disturb the wave particle motion. Again, viscous drag effects due to the velocity field generated by the surface waves are included, and the wave-damping forces are neglected. In both approaches, a velocity profile (could-be time varying) can be present along with the surface wave. Both formulations are presented in three-dimensions and can be reduced to two-dimensions when needed.

2.2.1 Continuous Line Formulation

On a differential element of a continuous mooring line the forces acting are: (a) the constant force due to gravitational attraction, (b) the variable tensile forces transmitted from the adjoining elements, and (c) the variable pressure (normal) and shear (tangential) forces applied by the fluid. The fluid forces can be broken down into (1) the hydrostatic pressure force, (2) the pressure force due to the acceleration of the fluid by the element (the so-called added mass force), and (3) the viscous drag forces due to fluid relative velocity, which have both pressure (normal) and shear (tangential) components. Internal damping forces are neglected in this analysis as these are assumed small compared to tensile and viscous drag forces. Exciting and damping forces due to the wave system are also neglected. From Newton's second law of mechanics the above forces should equal the mass of the differential element multiplied by its acceleration. By computing these forces the differential equations for the mooring line dynamic equilibrium are derived as follows:

The mooring line is considered to be a cylindrical slender body. A free body diagram of a differential element of length ds of the mooring line is shown in Figure 2.5.

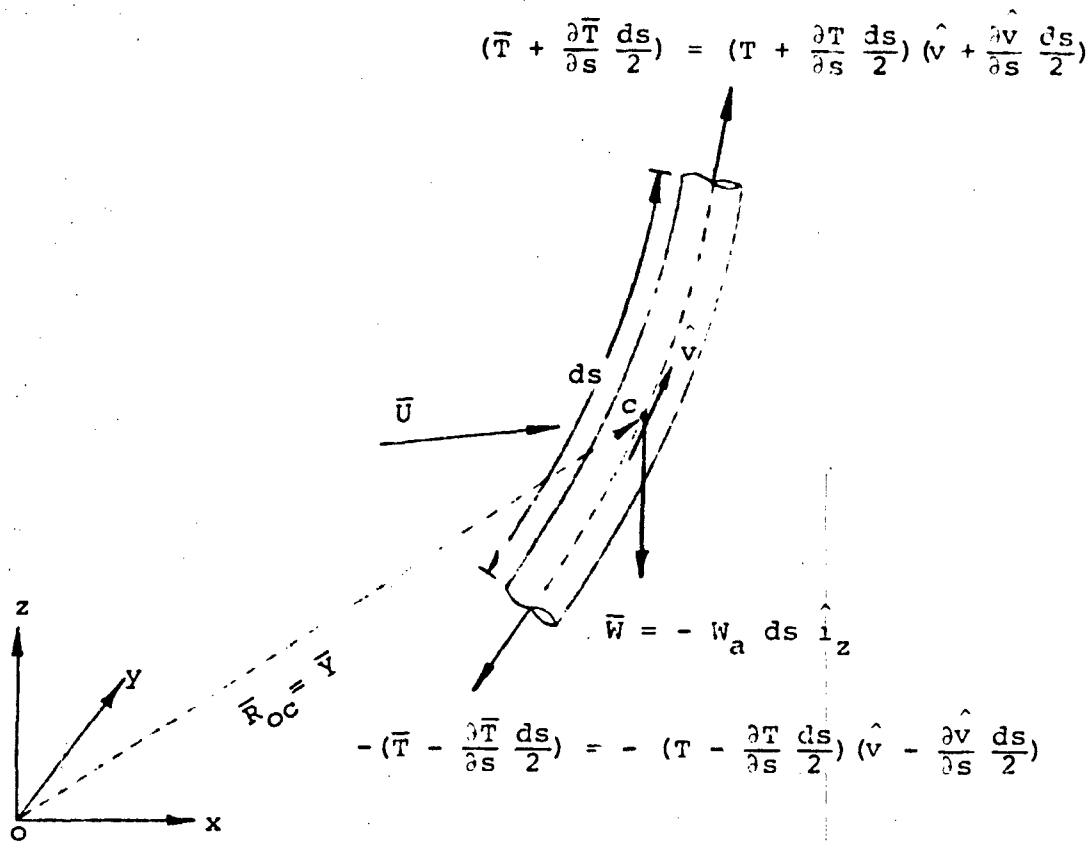


Figure 2.5 Mooring Line Differential Element

The internal tension and the inclination of the line segment change to keep all the above-mentioned forces in equilibrium. In Figure 2.5 $\hat{v}$ is a unit vector along the mooring line given by:

$$\hat{v} = \hat{i}_x \cos \phi_1 + \hat{i}_y \cos \phi_2 + \hat{i}_z \cos \phi_3$$

$\vec{T}$ is a tension vector, and $\vec{U}$ is a fluid velocity vector (obtained from the current profile and the velocity field generated by the surface wave). The relative velocity vector $\vec{UR}$ is given by:

$$\vec{UR} = \vec{U} - \dot{\vec{Y}}_c$$

where; $\vec{U} = \hat{i}_x U_x + \hat{i}_y U_y + \hat{i}_z U_z$

and, $\dot{\vec{Y}}_c = \hat{i}_x \dot{x}_c + \hat{i}_y \dot{y}_c + \hat{i}_z \dot{z}_c$ is the velocity of point c on the differential segment.

The viscous drag forces are computed according to the square drag law and are assumed to act in normal and tangential directions to the cylindrical line, proportional to their respective drag coefficients and areas.

$$\text{Let: } \vec{UR} = \vec{UTR} + \vec{UNR}$$

where; $\vec{UTR} = (\vec{UR} \cdot \hat{v}) \hat{v}$

and, $\vec{UNR} = \vec{UR} - \vec{UTR}$

From $\vec{UNR}$ and $\vec{UTR}$, normal ($\vec{DN}$) and tangential ($\vec{DT}$) drag forces can be calculated using the pertinent drag coefficients (C_{DN} and C_{DT}). Representative areas of the differential element 'ds' can be calculated using the

reduced diameter and stretched length. These are given by dA_N and dA_T . The standard formulation is given by:

$$\overline{DN}ds = \rho/2 (C_{DN}) (dA_N) |\overline{UNR}| \overline{UNR}$$

and
$$\overline{DT}ds = \rho/2 (C_{DT}) (dA_T) |\overline{UTR}| \overline{UTR}$$

The constant force due to gravitational attraction is $\overline{W} = -W_a ds \hat{i}_z$. Here, W_a is the weight in air per unit stretched length of the mooring line. $\overline{W}$ is resolved into normal ($\overline{WN}$) and tangential ($\overline{WT}$) components as:

$$\overline{WT} = (\overline{W} \cdot \hat{v}) \hat{v} = -W_a ds \cos \phi_3 \hat{v}$$

and
$$\begin{aligned} \overline{WN} &= \overline{W} - \overline{WT} \\ &= -W_a ds (\hat{i}_z - \hat{v} \cos \phi_3) \end{aligned}$$

The hydrostatic pressure force on the differential element by the surrounding fluid is given by the weight of the fluid displaced minus the hydrostatic pressure forces on the end cross-sections of the element. Or,

$$\begin{aligned} \overline{FH}ds &= (W_a - W_w) ds \hat{i}_z - \pi r^2 [\hat{v} (P_b - P_t) - \\ &\quad \frac{1}{2} \frac{\partial \hat{v}}{\partial s} ds (P_b + P_t)] \end{aligned}$$

where, W_w is the weight in water per unit stretched length, and r is the reduced radius due to stretch of the mooring line. P_b and P_t are the hydrostatic pressures at the bottom and top ends of the differential element. Assume;

$$(P_b - P_t) = \rho g ds \cos \phi_3$$

$$\frac{P_b + P_t}{2} = P_c$$

and $W_a - W_w = \rho g \pi r^2$

$$\therefore \overline{FH} ds = (W_a - W_w) ds \left\{ (\hat{i}_z - \hat{v} \cos \phi_3) + \frac{P_c}{\rho g} \frac{\partial \hat{v}}{\partial s} \right\}$$

Here, P_c is the hydrostatic pressure at the midpoint c of the differential element.

For a continuous cylindrical body it is also assumed that the body motion accelerates water only in the direction normal to its longitudinal axis. The inertia forces due to body's (differential element) own acceleration and the so called "added mass" term can now be written as:

$$\overline{IF} \, ds = - \frac{W_a}{g} \, ds \, \ddot{\overline{Y}} - \frac{(W_a - W_w)}{g} \, ds \, \ddot{\overline{YN}}$$

where; $\ddot{\overline{YN}} = \ddot{\overline{Y}} - \ddot{\overline{YT}}$

$\ddot{\overline{YT}} = (\ddot{\overline{Y}} \cdot \hat{v}) \hat{v}$; and the added mass of the differential element is assumed to be the mass of water displaced by this element.

Combining all these forces, the force equilibrium is written as:

$$\left(T + \frac{\partial T}{\partial s} \frac{ds}{2} \right) \left(\hat{v} + \frac{\partial \hat{v}}{\partial s} \frac{ds}{2} \right) - \left(T - \frac{\partial T}{\partial s} \frac{ds}{2} \right) \left(\hat{v} - \frac{\partial \hat{v}}{\partial s} \frac{ds}{2} \right) +$$

$$\overline{DN} \, ds + \overline{DT} \, ds - W_a \, ds \cos \phi_3 \hat{v} -$$

$$W_a \, ds (\hat{i}_z - \hat{v} \cos \phi_3) +$$

(2.7)

$$(W_a - W_w) ds (\hat{i}_z - \hat{v} \cos \phi_3) +$$

$$\left\{ \frac{P_c}{\rho g} \frac{\partial \hat{v}}{\partial s} \right\} - \frac{W_a}{g} \, ds (\ddot{\overline{YN}} + \ddot{\overline{YT}}) - \frac{(W_a - W_w)}{g} \, ds \, \ddot{\overline{YN}} = 0$$

Or, $\left(\frac{\partial T}{\partial s} \right) \hat{v} = (-DT + W_a \cos \phi_3 + \frac{W_a}{g} \dot{\overline{YT}}) \hat{v}$ (2.8)

and

$$\left\{ T + \frac{P_c (W_a - W_w)}{g} \right\} \frac{\partial \hat{v}}{\partial s} = - \overline{DN} + W_w (\hat{i}_z - \hat{v} \cos \phi_3) +$$

$$\left(\frac{2W_a - W_w}{g} \right) \frac{\partial \bar{Y}}{\partial s} \quad (2.9)$$

Also we can write:

$$\frac{\partial \bar{Y}}{\partial s} = \hat{v} \quad (2.10)$$

Here T , the tension, $\hat{v}$, the unit vector along the mooring line, and $\bar{Y}$, the geometric displacement vector are the dependent variables of interest. Equations (2.8) and (2.9) can be simplified if we define a new variable called the effective tension (T_e) similar to the one described in GOODMAN and BRESLIN (1976).

$$T_e = T + \frac{P_c (W_a - W_w)}{g} \quad (2.11)$$

Differentiating T_e with respect to s

$$\frac{\partial T_e}{\partial s} = \frac{\partial T}{\partial s} + \frac{W_a - W_w}{g} \frac{\partial P_c}{\partial s}$$

$$= \frac{\partial T}{\partial s} - (W_a - W_w) \cos \phi_3 \quad (2.12)$$

Substituting (2.11) and (2.12) in (2.8) and (2.9) we obtain:

$$\frac{\partial T_e}{\partial s} = -DT + W_w \cos \phi_3 + \frac{W_a}{g} \ddot{Y}T \quad (2.13)$$

and

$$T_e \frac{\partial \hat{v}}{\partial s} = -DN + W_w (\hat{i}_z - \hat{v} \cos \phi_3) + \left[\frac{2W_a - W_w}{g} \right] \ddot{Y}N \quad (2.14)$$

Equations (2.13) and (2.14) are derived for a line of an arbitrary stretched length and of consequent reduced diameter. Also we assume, as reasoned in GOODMAN and BRESLIN (1976) for materials obeying Hooke's law, that the effective tension and not the actual tension controls the extensibility of the mooring line. The extension of the mooring line is caused by (1) pulling on the line due to tension and (2) squeezing of the line due to hydrostatic pressure. Volume I (CHHABRA, 1973) plotted non-linear curves between tension and elongation of various mooring cables and ropes. Using the effective tension instead of the actual tension, mooring line stretch and the consequent reduced diameter from some initial reference state are found as discussed in Volume I.

The analysis also allows for an arbitrary number of intermediate bodies such as sensor packages and subsurface floats inserted in the mooring line. For this case equations (2.13) and (2.14) are replaced by:

$$\bar{T}_e|_{n+1} = \bar{T}_e|_n + \bar{F} \quad (2.15)$$

Here, $\bar{F}$ is the summation of gravitational attraction, fluid, and inertia forces of the body inserted between n , the differential element above the body, and $n+1$, below it.

For cylindrical packages, the viscous drag representation remains similar to the one for cylindrical mooring line. For a spherical or any other shaped package this viscous drag force is computed as:

$$\overline{DA} = \rho/2 C_{Db} A_{Db} |\overline{UR}| \overline{UR}$$

where; C_{Db} and A_{Db} are the appropriate drag coefficient and area.

The gravitational force and the hydrostatic pressure force are combined to give:

$$\overline{NB} = - W_{wb} \hat{i}_z$$

where; W_{wb} is the weight in water of the inserted body.

Inertia forces for cylindrical packages are changed slightly from that of a continuous cylindrical mooring line. Here the assumption of the body motion accelerating water only in the direction normal to its longitudinal axis is dropped. Instead two added mass terms; one normal and the other tangential to the longitudinal axis are used. For cylindrical packages inertia force is given by:

$$\overline{IF}_b = - (m_b \ddot{Y} + m_{nb} \ddot{Y}_N + m_{tb} \ddot{Y}_T)$$

and for any other shape this force is:

$$\overline{IF}_b = - (m_b + m_{ab}) \ddot{Y}$$

Here; m_b is the mass, m_{nb} and m_{tb} are the normal and tangential components of added mass for cylindrical bodies, and m_{ab} is the added mass for any other shaped body. Summation of gravitational attraction, inertia, and fluid forces gives $\overline{F}$. Integration of equations (2.10), (2.13), (2.14) and (2.15) along the spatial coordinate, s , of the mooring line gives the dynamic equilibrium of the

mooring system at discrete times. At any time these four equations can be solved for positions, $\bar{Y}(t)$, of the mooring system. Positions $\bar{Y}$ are calculated at short enough time intervals, Δt , so that velocities $\dot{\bar{Y}}$ and accelerations $\ddot{\bar{Y}}$ during these time intervals can be computed as piecewise constant. Accelerations are neglected at t_0 and t_1 , velocities are neglected at t_0 .

2.2.2 Lumped Parameter Formulation

In this formulation, the mooring line is reduced to a discretized dynamic system which is solved by a lumped parameter approach. All forces acting on the mooring line and the inserted packages are transferred to a fixed number of nodes (say N). If each node has q degrees of freedom then $N \times q$ simultaneous differential equations, q for each node, describing these nodes can be solved simultaneously by using any of the numerical integration techniques. Once again the forces transferred at each node (lumped mass) are the same as listed in 2.2.1, with the exception of Froude-Krylov forces which are not neglected here. The forces for the n^{th} mass for a N node system are derived next.

Consider the discretized dynamic system as shown in Figure 2.6, where an N -mass system is shown. In this figure $\hat{v}_n$ is a unit vector along the n^{th} segment

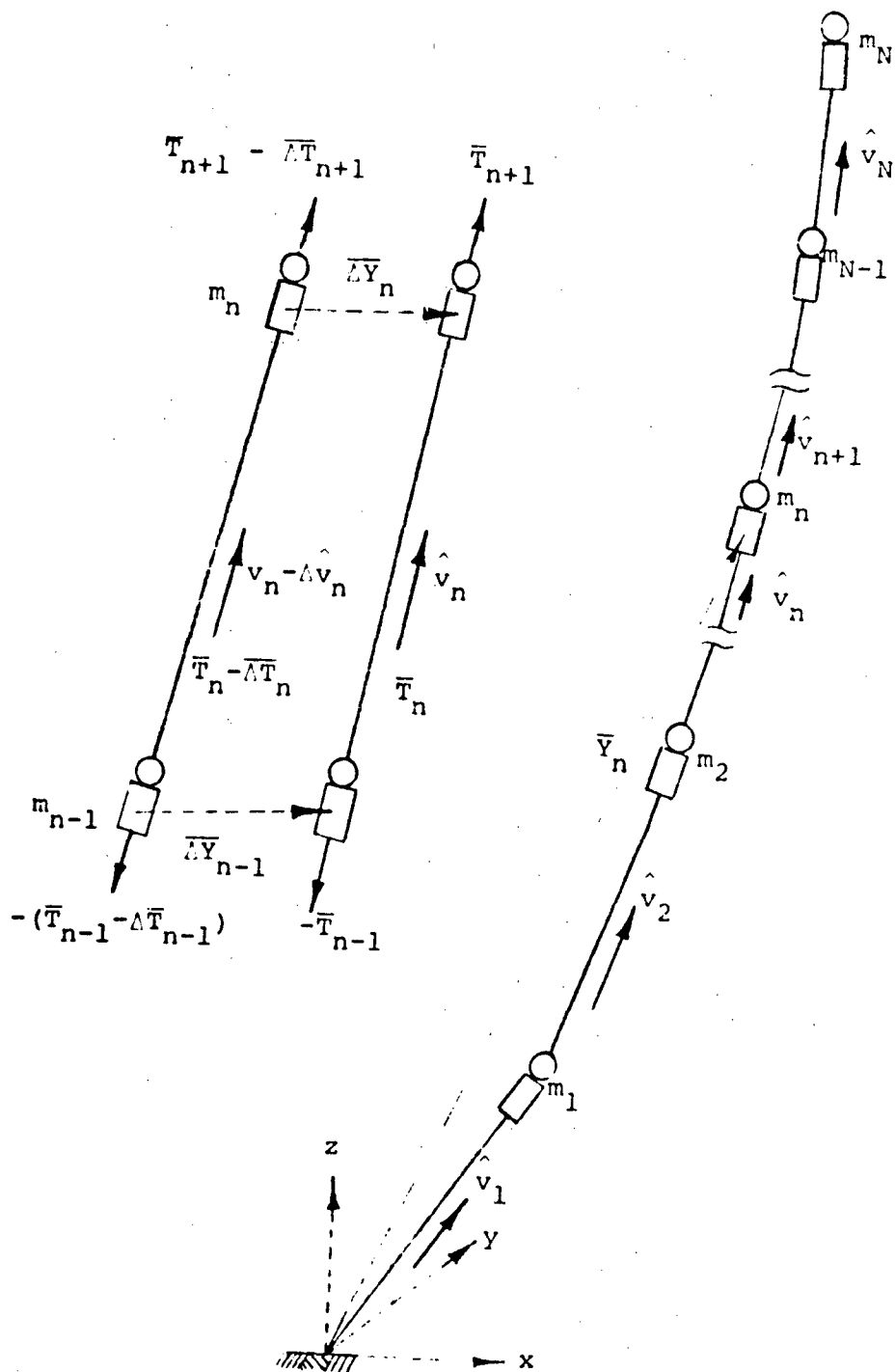


Figure 2.6 N-Mass System

(below the n^{th} mass). This can be written as:

$$\hat{v}_n = \hat{i}_x \cos\phi_{1n} + \hat{i}_y \cos\phi_{2n} + \hat{i}_z \cos\phi_{3n}$$

$\bar{T}_n, \bar{T}_{n+1}$ are the tension vectors, and $\bar{U}_n$ is the fluid velocity vector at the location of the n^{th} mass. A typical mass could consist of cylindrical components, spherical components, and any other shaped components. The transfer of all adjacent forces to a typical n^{th} mass is done as explained below.

Viscous drag forces acting on the mooring system have to be transferred at 'N' nodes. Any scheme devised for this transfer should take into account the different behavior of cylindrical and spherical components of the mooring system. Let $\overline{DN}_n$ be the drag force transferred to n^{th} node which is equivalent to contribution of normal drag forces on cylindrical components lumped at the n^{th} node. Similarly, $\overline{DT}_n$ is the drag force on n^{th} node equivalent to tangential drag forces on cylindrical components lumped at the n^{th} node. $\overline{DA}_n$ is the drag force acting on the n^{th} node in the direction of the relative velocity $\bar{U}_n$, and is a contribution from spherical or any other shaped components lumped at the n^{th} node. These forces are given by:

$$\overline{DN}_n = (CDIN)_n |\overline{UNR}_n| \overline{UNR}_n$$

$$\overline{DT}_n = (CDIT)_n |\overline{UTR}_n| \overline{UTR}_n$$

$$\overline{DA}_n = (CDIA)_n |\overline{UR}_n| \overline{UR}_n$$

$$\text{and } \overline{DF}_n = \overline{DN}_n + \overline{DT}_n + \overline{DA}_n \quad (2.16)$$

Here; $\overline{U}_n$ is computed from the current profile and the velocity of water due to surface waves at the location of the n^{th} mass which is exponentially attenuated with depth. CDIN, CDIT, and CDIA are the appropriate drag coefficients multiplied by respective areas.

The gravitational forces and the hydrostatic pressure forces are combined for all the transferred components to give:

$$\overline{NB}_n = - w_{wn} \hat{i}_z \quad (2.17)$$

Inertia forces for the components, in general, are given by:

$$\begin{aligned} \overline{IF}_n = & - (m_n + m_{an}) \ddot{\overline{Y}}_n - m_{nn} \ddot{\overline{YN}}_n \\ & - m_{tn} \ddot{\overline{YT}}_n \end{aligned} \quad (2.18)$$

This representation is similar to the one used for inserted bodies in the continuous line formulation.

The tensile forces acting on the n^{th} mass change in direction as well as magnitude with the motion of the nodes. The direction can be obtained by $\bar{Y}_n$ and $\bar{Y}_{n-1}$ (refer Figure 2.6); but as the present problem is one of large displacements the magnitude is determined at each integration step by updating its value at the beginning of the time step by the stiffness force due to the incremental deformation during this time step. Let the incremental change in length of the n^{th} segment $\Delta \bar{L}_n$ be:

$$\Delta \bar{L}_n = \bar{Y}_n - \bar{Y}_{n-1}$$

For small $\bar{Y}_n$ and $\bar{Y}_{n-1}$;

$$\Delta L_n \approx \Delta \bar{L}_n \cdot \hat{v}_n = \left[-\cos \phi_{1n} - \cos \phi_{2n} - \cos \phi_{3n} \cos \phi_{1n} \cos \phi_{2n} \cos \phi_{3n} \right] \begin{Bmatrix} \Delta \bar{Y}_{n-1} \\ \Delta \bar{Y}_n \end{Bmatrix}$$

If k_n (stiffness coefficient) is defined as the force required at node n for a unit displacement along $\hat{v}_n$; then the change in tension magnitude ΔT_n is:

$$\Delta T_n = k_n \Delta L_n$$

or; $\bar{T}_n = (T_n - \Delta T_n) \hat{v}_n + \Delta T_n \hat{v}_n$

Also, as tension vectors $\bar{T}_n$ and $\bar{T}_{n+1}$ are acting at the n^{th} mass, we have:

$$\bar{T}F_n = \bar{T}_{n+1} - \bar{T}_n \quad (2.19)$$

Froude-Krylov force on the submerged 'mass n' is assumed to be given by the product of mass of water displaced by the 'nth mass' and the acceleration of water at that location which is exponentially attenuated with depth. Or,

$$\begin{aligned} \bar{T}F_n = m_{dn} g \xi_0 e^{|\bar{K}| z_n} & \left[\hat{i}_x K_x \cos(\bar{K} \cdot \bar{S} - \omega t) + \right. \\ & \left. \hat{i}_y K_y \cos(\bar{K} \cdot \bar{S} - \omega t) + \hat{i}_z |\bar{K}| \sin(\bar{K} \cdot \bar{S} - \omega t) \right] \end{aligned} \quad (2.20)$$

where, m_{dn} is the mass of water displaced by the n^{th} mass.

Combining equations (2.16) through (2.20) for all nodes and taking into account all the boundary conditions, the force balance reduces in matrix notation to:

$$[M]\{\ddot{Y}\} = \{DF\} + \{CF\} + \{TF\} + [K]\{\Delta Y\} + \{FF\} \quad (2.21)$$

where; $[M]$ is a matrix of appropriate inertia terms.
 $[K]$ is a matrix of stiffness terms.
 $\{TF\}$ is an array of tension components at the nodes.
 $\{DF\}$ is an array of appropriate viscous drag terms.
 $\{CF\}$ is an array of constant nodal forces
 (gravitational and hydrostatic).
 $\{FF\}$ is an array of Froude-Krylov forces.
 $\{\ddot{Y}\}$ is an array of accelerations at the nodes.
 and $\{\Delta Y\}$ is an array of incremental deformations of
 the nodes.

Derivation of Matrices: In equation (2.21), the order of each matrix and array is equal to the number of nodes (N) multiplied by degrees of freedom ($q = 3$) per node. For simplicity the masses were assumed to be lumped at 'N' nodes, in which case the off-diagonal terms in the mass matrix are zero, i.e., force at node 'i' due to an acceleration at node 'j' equals zero. A consistent mass matrix analysis can be done to allow for distributed mass.

Mass Matrix - $[M]$

$$[M] = \begin{bmatrix} M_1 & & & & & \\ & M_2 & & & & \\ & & M_3 & & & \\ & & & \ddots & & \\ & & & & M_n & \\ & & & & & \ddots \\ & & & & & & M_{N-1} \\ & & & & & & & M_N \end{bmatrix}$$

where; $M_1, M_2, \dots, M_N$ are 3×3 matrices given by:

$$M_n = \begin{bmatrix} m_n + m_{an} + m_{nn} \sin^2 \phi_{1n} & (m_{tn} - m_{nn}) \cos \phi_{1n} \cos \phi_{2n} & (m_{tn} - m_{nn}) \cos \phi_{1n} \cos \phi_{3n} \\ +m_{tn} \cos^2 \phi_{1n} & & \\ (m_{tn} - m_{nn}) \cos \phi_{2n} \cos \phi_{1n} & m_n + m_{an} + m_{nn} \sin^2 \phi_{2n} & (m_{tn} - m_{nn}) \cos \phi_{2n} \cos \phi_{3n} \\ +m_{tn} \cos^2 \phi_{2n} & & \\ (m_{tn} - m_{nn}) \cos \phi_{3n} \cos \phi_{1n} & (m_{tn} - m_{nn}) \cos \phi_{3n} \cos \phi_{2n} & m_n + m_{an} + m_{nn} \sin^2 \phi_{3n} \\ +m_{tn} \cos^2 \phi_{3n} & & \end{bmatrix}$$

m_n = mass lumped at the n^{th} node.

m_{an} = added mass at n^{th} node due to non-cylindrical components

and m_{nn}, m_{tn} = Normal and tangential components of added mass at n^{th} node due to cylindrical components

Stiffness Matrix [K] - 4 Mass System

$$[K] = \begin{bmatrix} K^1 + K^2 & -K^2 & 0 & 0 \\ -K^2 & K^2 + K^3 & -K^3 & 0 \\ 0 & -K^3 & K^3 + K^4 & -K^4 \\ 0 & 0 & -K^4 & K^4 \end{bmatrix}$$

where; K^S are 3×3 matrices whose elements are given by

$$K_{ij}^S = -k_s cs_{i,s} cs_{j,s}$$

k_s = Stiffness coefficient for segment s .

and $cs_{i,s} = \cos \phi_i$ of segment s .

$\{TF\}$, $\{DF\}$, $\{CF\}$, $\{FF\}$, $\{\ddot{Y}\}$ and $\{\Delta Y\}$ are column matrices. These matrices can be written as

$$\{TF\} = \begin{Bmatrix} -(T_1 - \Delta T_1) CS_{1,1} + (T_2 - \Delta T_2) CS_{1,2} \\ -(T_1 - \Delta T_1) CS_{2,1} + (T_2 - \Delta T_2) CS_{2,2} \\ -(T_1 - \Delta T_1) CS_{3,1} + (T_2 - \Delta T_2) CS_{3,2} \\ \cdot \\ \cdot \\ \cdot \\ -(T_{N-1} - \Delta T_{N-1}) CS_{3,N-1} + (T_N - \Delta T_N) CS_{3,N} \\ \cdot \\ - (T_N - \Delta T_N) CS_{1,N} \\ \cdot \\ - (T_N - \Delta T_N) CS_{2,N} \\ \cdot \\ - (T_N - \Delta T_N) CS_{3,N} \end{Bmatrix} \quad \{FF\} = \begin{Bmatrix} FF_{1,1} \\ FF_{2,1} \\ FF_{3,1} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ FF_{1,N} \\ FF_{2,N} \\ FF_{3,N} \end{Bmatrix}$$

$$\{DF\} = \begin{Bmatrix} DF_{1,1} \\ DF_{2,1} \\ DF_{3,1} \\ . \\ . \\ DF_{1,N} \\ DF_{2,N} \\ DF_{3,N} \end{Bmatrix}; \{CF\} = \begin{Bmatrix} CF_{1,1} \\ CF_{2,1} \\ CF_{3,1} \\ . \\ . \\ CF_{1,N} \\ CF_{2,N} \\ CF_{3,N} \end{Bmatrix}; \{\ddot{Y}\} = \begin{Bmatrix} \ddot{Y}_{1,1} \\ \ddot{Y}_{2,1} \\ \ddot{Y}_{3,1} \\ . \\ . \\ \ddot{Y}_{1,N} \\ \ddot{Y}_{2,N} \\ \ddot{Y}_{3,N} \end{Bmatrix}; \{\Delta Y\} = \begin{Bmatrix} \Delta Y_{1,1} \\ \Delta Y_{2,1} \\ \Delta Y_{3,1} \\ . \\ . \\ \Delta Y_{1,N} \\ \Delta Y_{2,N} \\ \Delta Y_{3,N} \end{Bmatrix}$$

Equation (2.21) can be re-written as

$$\{\ddot{Y}\} = [M]^{-1} \{TF\} + [M]^{-1} [K] \{\Delta Y\} + [M]^{-1} \{DF\} + [M]^{-1} \{FF\} + [M]^{-1} \{CF\}$$

These 3N second-order differential equations can be reduced to a set of 6N first-order differential equations which can be solved by any of the numerical methods.

2.3 Attachment Between Surface Buoy and a Mooring Line

Figure 2.7 shows the attachment between a surface buoy and a mooring line. Let the attachment point be 'a', which is fixed rigidly to the buoy.

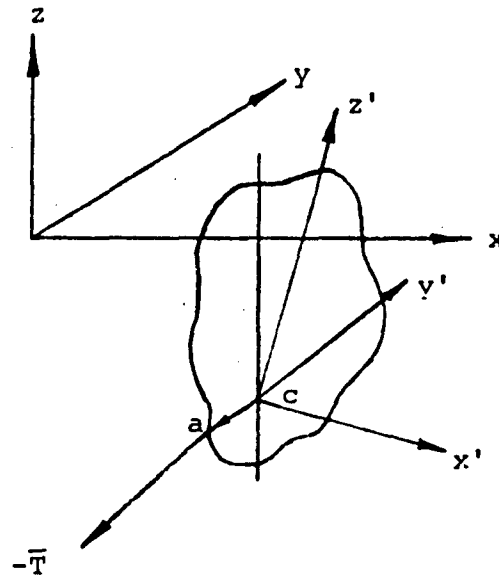


Figure 2.7 Attachment Between Surface Buoy and a Mooring Line

In addition to all the other forces acting on the surface buoy as outlined in Section 2.1; a tension vector $\bar{T}$ is acting at point a, which should be included in the force balance.

$$\bar{T} = T\hat{v} = T_x\hat{i}_x + T_y\hat{i}_y + T_z\hat{i}_z \quad (2.22)$$

also let vector $\bar{R}_{ca}$ be given by:

$$\bar{R}_{ca} = \hat{i}_x x_a' + \hat{i}_y y_a' + \hat{i}_z z_a'$$

Taking moment of $\bar{T}$ about point c we obtain:

$$\bar{TM} = - \bar{R}_{ca} \times \bar{T} \quad (2.23)$$

for inclusion in the moment balance.

Two Dimensional Analysis:

$$\begin{aligned} \bar{R}_{ca} &= \hat{i}_x x_a' + \hat{i}_z z_a' \\ &= \hat{i}_x (x_a' \cos \theta + z_a' \sin \theta) + \hat{i}_z (z_a' \cos \theta - x_a' \sin \theta) \end{aligned}$$

and; $\bar{TM} = \hat{i}_y [T_z (x_a' \cos \theta + z_a' \sin \theta) - T_x (z_a' \cos \theta - x_a' \sin \theta)]$

Three Dimensional Analysis:

$$\begin{aligned} \bar{R}_{ca} &= \hat{i}_x (x_a' - \psi y_a' + \theta z_a') + \hat{i}_y (\psi x_a' + y_a' - \phi z_a') + \\ &\quad \hat{i}_z (-\theta x_a' + \phi y_a' + z_a') \end{aligned}$$

And;
$$\begin{aligned} \overline{TM} = & \hat{i}_x [T_y (-\theta x'_a + \phi y'_a + z'_a) - T_z (\psi x'_a + y'_a - \phi z'_a)] + \\ & \hat{i}_y [T_z (x'_a - \psi y'_a + \theta z'_a) - T_x (-\theta x'_a + \phi y'_a + z'_a)] + \\ & \hat{i}_z [T_x (\psi x'_a + y'_a - \phi z'_a) - T_y (x'_a - \psi y'_a + \theta z'_a)] \end{aligned}$$

Change in mooring line tension magnitude due to buoy rotation
(coefficients in the stiffness matrix).

Two Dimensional Analysis:

Rotate $\overline{R}_{ca}$ through $\Delta\theta$ to obtain $\overline{R}_{ca_1}$

$$\overline{R}_{ca_1} = \begin{bmatrix} 1 & \Delta\theta \\ -\Delta\theta & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{Bmatrix} x'_a \\ z'_a \end{Bmatrix}$$

For small rotations change in tension magnitude ΔT is given by

$$\Delta T = k (\overline{R}_{ca_1} - \overline{R}_{ca}) \cdot \hat{v}$$

where k is the stiffness coefficient for the line segment attached to the surface buoy.

If
$$\hat{v} = \hat{i}_x \cos\phi + \hat{i}_z \sin\phi$$

$$\begin{aligned} \Delta T = & k [\cos\phi (z'_a \cos\theta - x'_a \sin\theta) + \sin\phi (-z'_a \sin\theta - \\ & x'_a \cos\theta)] \Delta\theta \end{aligned}$$

Three Dimensional Analysis:

$$\bar{R}_{ca_1} = \begin{bmatrix} 1 & -\Delta\psi & \Delta\theta \\ \Delta\psi & 1 & -\Delta\theta \\ -\Delta\theta & \Delta\psi & 1 \end{bmatrix} \begin{bmatrix} 1 & -\psi & \theta \\ \psi & 1 & -\theta \\ -\theta & \psi & 1 \end{bmatrix} \begin{Bmatrix} x'_a \\ y'_a \\ z'_a \end{Bmatrix}$$

and;

$$\Delta T = k \begin{bmatrix} \cos\theta_2 (\theta x'_a - y'_a - z'_a) + \cos\theta_1 (-\psi x'_a + \psi y'_a + z'_a) + \cos\theta_1 (-\psi x'_a - y'_a + \theta z'_a) \\ + \cos\theta_3 (\psi x'_a + y'_a - \theta z'_a) + \cos\theta_3 (-x'_a + \psi y'_a - \theta z'_a) + \cos\theta_2 (x'_a - \psi y'_a + \theta z'_a) \end{bmatrix}$$

$$\cdot \begin{Bmatrix} \Delta\theta \\ \Delta\psi \\ \Delta\psi \end{Bmatrix}$$

If more than one mooring lines are attached to the surface buoy, as is the case of simulation study in Section 5.3, force and moment of both these lines are to be included in the appropriate equations. It is important to note that within the definition of the coordinate frame (fixed at the anchor) if a mooring line hangs from the buoy (as is the case in 5.3), $-\bar{T}$ of this section would be replaced by $+\bar{T}$. Also in this case ΔT would be given by

$$\Delta T = -k(\bar{R}_{ca_1} - \bar{R}_{ca}) \cdot \hat{v}$$

2.4 Window Shade Drogue

Sometimes an alternative approach to mooring surface buoys is the use of surface trackable drogued drifting buoys. Such a buoy system employs a high drag device (or drogue) tethered to a trackable buoy at the surface. A drogue is subjected to the same forces (including the wave effects) as a differential element of the mooring line. A special case of drogues, the window shade drogue, as shown in Figure 2.3 is considered in this analysis. The drogue, modeled dynamically as a lumped parameter system (Figure 2.8), is reduced to strips. The mass, the acting forces, and the dynamic behavior of each strip is lumped at an assigned nodal point. Nodal masses are linked by suitably elastic lines.

The window shade drogue is a compliant sheet. Close observation of scale model drogues (Vachon, 1973) has revealed that it seems nearly locked to the local water mass insofar as motion normal to the sheet is concerned. However, in the tangential direction, it seems to slip through the local water mass rather easily. The forces were therefore described as follows: the wave forces (Froude-Krylov force; as it is assumed that there is no diffracted wave) acting on a material strip are estimated by calculating the force normal to the axis

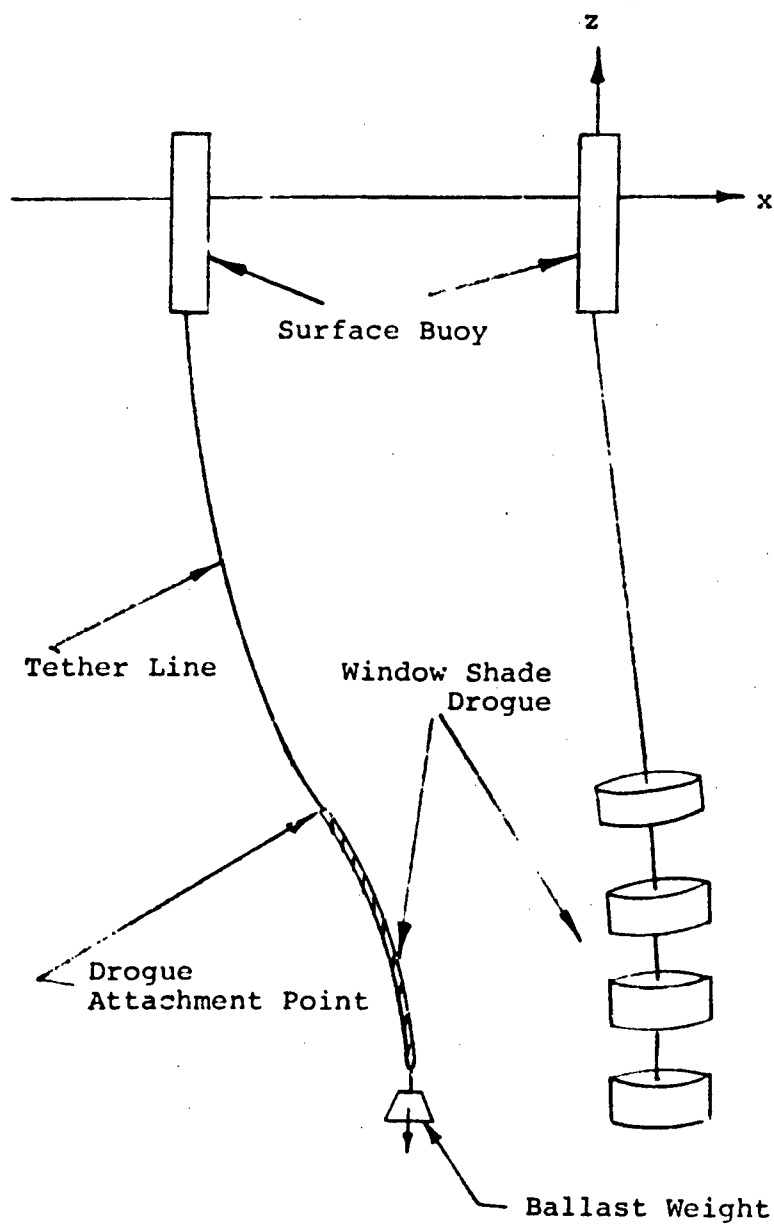


Figure 2.8 Drifting Drogued Buoy System

of revolution of a hypothetical cylindrical element, the diameter and length of which is equal to the breadth and length of the strip respectively. This force is given by the mass of the hypothetical water cylinder multiplied by the component of water acceleration normal to the cylinder's longitudinal axis. Or;

$$\overline{FF}_n = m_{dn} \cdot [\overline{A}_{Ew} - (\overline{A}_{Ew} \cdot \hat{v}_n) \hat{v}_n] \quad (2.24)$$

where w is the particle of water next to the n^{th} node, and m_{dn} is the mass of the hypothetical water cylinder. For inertia forces m_{an} and m_{tn} equal zero; whereas m_{nn} equals m_{dn} . The viscous drag of this strip in the normal direction is calculated using the frontal area of the strip and the drag coefficient measured in water-filled quarry tests (Vachon, 1975). The elemental strips of the drogue are assumed subject to tangential viscous drag calculated using the area of the strip and a tangential drag coefficient measured during drop tests in the ocean (Vachon, 1975).

Gravitational, hydrostatic pressure, and tensile forces on a drogue strip are similar to the ones outlined in Section 2.2 for a discretized dynamic system.

3.0 METHOD OF SOLUTION

Section 2 outlined all the necessary ingredients of the mathematical models used in this report. In this section some details of how to use these ingredients, to obtain a solution for a specific problem on hand, are given. For illustrative purposes a specific case study, of a spar buoy at the surface and a mooring line connecting this buoy with the anchor, will be discussed. Method of solution for both the two-dimensional and the three-dimensional analysis of the moored system will be outlined in

3.1. Section 3.2 outlines the approximate initial conditions for the steady state analysis of a spar buoy freely floating in a surface wave.

Another case study of a spar buoy at the surface and connected to a window shade drogue at a given depth is discussed in section 3.3.

3.1 Moored System Analysis

Two-Dimensional:

In this case, the current profile is in the same plane as the train of surface gravity waves having a single frequency and amplitude; and its direction parallel or anti-parallel to the direction of surface wave propagation.

Static Solution:

For the static solution, the analysis is started at the spar buoy. The pertinent static equations for the spar buoy (Section 2.1) with an attached mooring line (Section 2.3) are:

$$DF_x - T_x = 0 \quad (3.1)$$

$$P_o g - Mg + DF_z - T_z = 0 \quad (3.2)$$

and

$$\begin{aligned} -P_1 g \sin\theta + DM + T_z(x'_a \cos\theta + z'_a \sin\theta) - \\ - T_x(z'_a \cos\theta - x'_a \sin\theta) = 0 \end{aligned} \quad (3.3)$$

Here; we have three equations and four (T_x, T_z, θ, h_s) unknowns, where h_s is the static draft of the spar buoy. Now if we assume a value for h_s , the other three unknowns can be solved by equations (3.1) through (3.3). The solution for the three unknowns T_x , T_z , and θ is found by iterating on the pitch angle θ . If θ is known; DF_x , DF_z , and DM can be computed explicitly. P_o and P_1 can be calculated if h_s is known and thus T_x and T_z are given by equations (3.1) and (3.2) respectively. Moment balance can now be performed by using equation (3.3)

to obtain the error. This error is minimized by iterating on the pitch angle θ .

Having solved the buoy force and moment balances for a assumed h_g the mooring line can be analyzed next.

For the continuous line formulation, a numerical integration is carried down the mooring line by dividing it into a finite number of segments. In this representation equations (2.13), (2.14), and (2.10) for two-dimensions can be written as:

$$\Delta T_e = (-DT + W_w \sin\phi) \Delta S \quad (3.4)$$

$$T_e \Delta\phi = [DN + W_w \cos\phi] \Delta S \quad (3.5)$$

$$\text{and} \quad \Delta Y = \hat{i}_x \Delta S \cos\phi + \hat{i}_z \Delta S \sin\phi \quad (3.6)$$

Having found T_x and T_z (or T and ϕ) at the top of the mooring line equations (3.4) through (3.6) are used to find the configuration and the tension distribution along the mooring line. At the end of the mooring line the boundary condition of the ocean floor depth should be met. If not, the whole process is repeated with a new h_g ; thus computing the correct value of h_g for the given boundary condition. Whenever the mooring line is

discontinued to insert an intermediate body, the discontinuity is taken as a lumped mass and resolved as done below for the lumped parameter approach.

For the lumped parameter formulation, the configuration and the tension distribution along the mooring line can be found by (Section 2.2):

$$\bar{T}_n = \bar{T}_{n+1} - W_{wn} \hat{i}_z + \overline{DF}_n \quad (3.7)$$

where; $\bar{T}_n$ is the tension vector below the n^{th} mass and $\bar{T}_{n+1}$ above it.

Dynamic Solution:

Equations of Section 2.2 which are written for three-dimensional analysis can be easily reduced for the two dimensional problem. Using the lumped parameter approach for the mooring line; equations (2.21) are combined with equations (2.3), (2.22), and (2.23) to obtain a global matrix equation for the mooring/buoy system. Stiffness coefficients for the change in mooring line tension magnitude due to buoy rotation are obtained from Section 2.3. This matrix equation can be solved in the time domain using various numerical integration techniques. Integrals of P_i and Q_i , given in Section 2.1,

are computed exactly; whereas the integrals for viscous drag forces and moments DF_x , DF_z , and DM are computed numerically by dividing the spar buoy in disks of height dz' . The composite matrices are of order $2N + 3$. Matrices $[M]$, $\{TF\}$, $\{K\}$, $\{DF\}$, $\{FF\}$, $\{FD\}$, $\{FG\}$, and $\{FA\}$ are computed at each time step of the integration.

For the continuous line formulation; the matrices of equation (2.3) combined with equations (2.22) and (2.23) can be solved for $\bar{T}$ in equation (2.22) if the buoy draft h_0 is known. Then integration of equations (2.13), (2.14), (2.10), and (2.15) along the spatial coordinate, s , of the mooring line gives the dynamic equilibrium of the mooring system at discrete times. Iteration on the unknown h_0 may be necessary to meet the boundary condition of the ocean floor depth.

Three Dimensional:

Here; the train of surface gravity waves having a single frequency and amplitude, can propagate in a general direction. The current profile is also three-dimensional and can vary, in magnitude and direction, with depth.

Static Solution:

The pertinent static equations for the spar buoy (Section 2.1) with an attached mooring line (Section 2.3) are:

$$-\int dB\bar{g} + \int dM\bar{g} + \overline{DF} - \bar{T} = 0 \quad (3.8)$$

$$\text{and; } -\int dB(\bar{R}_{cp} \times \bar{g}) + \overline{DM} + \overline{TM} = 0 \quad (3.9)$$

Integrations for dB are over the static draft of the buoy, and integration for DM is over the length of the buoy.

Equations (3.8) and (3.9) can be written as:

$$P_O g \hat{i}_z - Mg \hat{i}_z + \overline{DF} - \bar{T} = 0 \quad (3.10)$$

$$\text{and } \begin{Bmatrix} \phi \\ \theta \\ \psi \end{Bmatrix} = [S]^{-1} \{F\} \quad (3.11)$$

where; $[S]$ is a 3 x 3 matrix with components given by:

$$S_{11} = P_O g l_1 + DF_z l_2 - T_y y'_a - T_z z'_a$$

$$S_{12} = T_y x'_a ; \quad S_{13} = T_z x'_a$$

$$S_{21} = T_x y'_a ; \quad S_{23} = T_z y'_a$$

$$S_{22} = P_O g \ell_1 + DF_z \ell_2 - T_x x'_a - T_z z'_a$$

$$S_{31} = - DF_x \ell_2 + T_x z'_a$$

$$S_{32} = - DF_y \ell_2 + T_y z'_a$$

$$S_{33} = - T_x x'_a - T_y y'_a$$

and $\{F\}$ is a 3×1 array

$$F_1 = - DF_y \ell_2 + T_y z'_a - T_z y'_a$$

$$F_2 = DF_x \ell_2 - T_x z'_a + T_z x'_a$$

$$F_3 = T_x y'_a - T_y x'_a$$

also $P_O = \int dB$; and ℓ_1, ℓ_2 are the distances between center of gravity and center of buoyancy and center of

drag respectively. For the static case we also assume (small angles):

$$DF_x \propto |V_{Bwx}| V_{Bwx}$$

$$DF_y \propto |V_{Bwy}| V_{Bwy}$$

and $DF_z \propto |V_{Bwz}| V_{Bwz}$

$\therefore \bar{T}$ and ϕ , θ , and ψ can be solved explicitly from equations (3.10) and (3.11) if h_s is known. h_s is determined by iteration on the boundary condition of the ocean floor depth.

Mooring line solution is similar to the one presented for the two-dimensional analysis.

Dynamic Solution:

This dynamic solution is similar to the one for two-dimensional analysis except that the order of matrices in this case is $3N + 6$.

3.2 Initial Conditions for the Steady-State Analysis of a Spar Buoy

Two-Dimensional Analysis:

Let $C_N = 1.0$, $C_T = 0.0$, and $x_c = 0.0$. Also assuming small pitch of the buoy and neglecting viscous

drag forces, the equations of motion for a spar buoy become:

$$2M\ddot{x}_C + P_1\ddot{\theta} = 2\omega^2 \xi_O Q_O \cos \omega t \quad (3.12)$$

$$\begin{aligned} M^*\ddot{z}_C + \rho S_{O2}gz &= -\omega^2 \xi_O Q_O \sin \omega t \\ &+ \rho S_{O2}g\xi_O \sin \omega t \end{aligned} \quad (3.13)$$

$$\text{and} \quad P_1\ddot{x}_C + (I_{yy} + P_2)\ddot{\theta} + P_1g\theta = 2\omega^2 \xi_O Q_1 \cos \omega t \quad (3.14)$$

$$\text{where,} \quad M^* = M + 4/3 \rho [r_1^3 + (r_1 - r_2)^3]$$

Surge Equation 3.12:

$$\text{Let } P_1\ddot{\theta} \ll 2M\ddot{x}_C \text{ and } x = C_1 \sin \omega t + C_2 \cos \omega t + C_3.$$

Now differentiating x twice and substituting in (3.12) we obtain

$$x = -\xi_O Q_O \cos \omega t / M + C_3$$

$$\text{and} \quad \dot{x} = \omega \xi_O Q_O \sin \omega t / M + C_3 t$$

$$\text{Hence} \quad \dot{x}(0) = 0$$

$$\text{and} \quad x(0) = -\xi_O Q_O / M$$

Heave equation (3.13)

$$\text{Let } z = C_1 \sin \omega t + C_2 \cos \omega t + C_3$$

Differentiating twice and substituting in equation (3.13) we obtain:

$$C_2 = C_3 = 0$$

and $C_1 = \xi_0 (\rho S_{O2} g - \omega^2 Q_0) / (\rho S_{O2} g - M \omega^2)$

Hence, $z(0) = 0$

$$\dot{z}(0) = \omega C_1$$

Pitch Equation 3.14:

Substituting $\ddot{x}_c$ from equation (3.12) into equation (3.14) we obtain:

$$A\ddot{\theta} + B\theta = C \cos \omega t \quad (3.15)$$

where, $A = (I_{yy} + P_2 - P_1^2/2M)$

$$B = P_1 g$$

and $C = (2\omega^2 \xi_0 Q_1 - P_1 \omega^2 \xi_0 Q_0 / M)$

Now let $\theta = C_1 \sin \omega t + C_2 \cos \omega t + C_3$

Differentiating θ twice and substituting in equation (3.15) gives:

$$C_1 = C_3 = 0$$

and $C_2 = C / (B - A\omega^2)$

Or, $\theta = \frac{C}{B - A\omega^2} \cos \omega t$

Hence,

$$\theta(0) = \frac{\omega^2 \xi_0 (2Q_1 - P_1 Q_0 / M)}{P_1 g - \omega^2 (I_{yy} + P_2 - P_1^2 / 2M)}$$

and $\dot{\theta}(0) = 0$

Three Dimensional Analysis:

Similarly; for the three dimensional case with wave direction at an angle β with the x-axis we have:

$$x(o) = - \xi_o \cos \beta Q_o / M$$

$$y(o) = - \xi_o \sin \beta Q_o / M$$

$$z(o) = 0.$$

$$\dot{x}(o) = \dot{y}(o) = 0$$

$$\dot{z}(o) = \frac{\omega \xi_o (\rho S_o 2g - \omega^2 Q_o)}{\rho S_o 2g - M^* \omega^2}$$

$$\phi(o) = - \frac{\omega^2 \xi_o \sin \beta (2Q_1 - P_1 Q_o / M)}{P_1 g - \omega^2 (I_{yy} + P_2 - P_1^2 / 2M)}$$

$$\theta(o) = \frac{\omega^2 \xi_o \cos \beta (2Q_1 - P_1 Q_o / M)}{P_1 g - \omega^2 (I_{yy} + P_2 - P_1^2 / 2M)}$$

$$\psi(o) = 0$$

$$\dot{\phi}(o) = \dot{\theta}(o) = \dot{\psi}(o) = 0.$$

3.3 Drifting Drogued Spar Buoy

In this analysis the drogue and the mooring line connecting it to the spar buoy are modeled dynamically as a lumped parameter system.

Two-Dimensional Analysis

Static Solution:

The pertinent static equations for the spar buoy (Section 2.1) with an attached mooring line (Section 2.3) are:

$$DF_x + T_x = 0 \quad (3.16)$$

$$P_O g - Mg + DF_z + T_z = 0 \quad (3.17)$$

and

$$\begin{aligned} -P_1 g \sin\theta + DM - T_z (x'_a \cos\theta + z'_a \sin\theta) \\ + T_x (z'_a \cos\theta - x'_a \sin\theta) = 0 \end{aligned} \quad (3.18)$$

Here; we have three equations and five (V_s , h_s , T_x , T_z , θ) unknowns where V_s is the static velocity of the drogued drifting system. These equations are solved by iterating on V_s and θ . Assuming a value for V_s , T_x and T_z can be calculated by known weights, buoyancies, viscous drag forces, and elastic characteristics of the system beneath the spar. Now if a value for θ is assumed equation (3.17) is solved for h_s and the error in equation (3.18) is found. This error is minimized by iterations on θ . Next V_s is iterated upon to satisfy equation (3.16).

Dynamic Solution

This solution is similar to the lumped parameter solution discussed in Section 3.1.

Three Dimensional Analysis

Static Solution:

Here the pertinent equations (static) remain the same as Section 3.1 expect for replacing $-\bar{T}$ by $+\bar{T}$ and $+\bar{T}\bar{M}$ by $-\bar{T}\bar{M}$.

Also; following similar reasoning of small angles $\bar{T}$, ϕ , θ , and ψ can be solved explicitly from equations (3.10) and (3.11) with new $\bar{T}$ and $\bar{T}\bar{M}$ if V_s is known. V_s is determined by iteration on the equation similar to (3.10).

Dynamic Solution

This solution is similar to the one in Section 3.1 .

4.0 COMPUTER PROGRAM DETAILS

This section will describe the various computer programs written for the theoretical analysis presented in Section 2 and using solution methods of Section 3. Description of the programs is general and specific details can be found in the program listings presented in Appendix A. All computer programs are written in FORTRAN IV and have been run for many case studies on the CSDL AMDAHL 470 V6 computer. Input data required for these programs is also explained in this section. Some of the simulations, along with the accompanying input data, are presented in Section 5.

Computer programs written for the surface moored and drifting systems are tabulated in Table 4.1. Programs for subsurface moored systems are tabulated in Table 4.2. Program listings for some of these computer programs are presented in Appendix A. Listings not presented are duplications of what is presented and can be obtained after minor modifications.

4.1 Surface Moored/Drifting Systems

Computer program SD3.FORT is coded for the three-dimensional analysis of surface moored and drifting systems presented in Section 2. Four case studies can be simulated by this program:

1. A freely floating spar buoy.

FILE NAME	MOORING SYSTEM	DEGREES OF FREEDOM	PHYSICAL FORCES	INPUT ENVIRONMENT (Forcing Function)
SD3.FORT	Cylindrical spar buoy analysis. Tuned spar buoy analysis. Cylindrical or tuned spar buoy anchored by a mooring line. Cylindrical or tuned spar buoy anchored by a mooring line and an instrument line hanging from the buoy. Drifting drogued buoy (cylindrical or tuned spar) analysis.	Surface buoy-six Mooring line-three	All applicable forces of Section 2.	Surface wave and constant absolute current profile.
SD2.FORT	Same as above	Surface buoy-three Mooring line-two	All applicable forces of Section 2.	Surface wave and constant absolute current profile.
BUOY.FORT	Cylindrical spar buoy analysis. Tuned spar buoy analysis.	Buoy-six	All applicable forces of Section 2.	Surface wave and constant absolute current profile.
MDE.FORT	Cylindrical or tuned spar buoy anchored by a mooring line and an instrument line hanging from the buoy.	Surface buoy-six Mooring line-three	All applicable forces of Section 2.	Surface wave and constant absolute current profile.
LDB.FORT	Drifting drogued buoy analysis (cylindrical or tuned spar)	Surface buoy-six Mooring line-three	All applicable forces of Section 2.	Surface wave and constant absolute current profile.

TABLE 4.1 Programs for Surface Moored/Drifting Systems

FILE NAME	MOORING SYSTEM	DEGREES OF FREEDOM	PHYSICAL FORCES	INPUT EQUIVARIANT (Forcing Function)
SSD3.FORT	Subsurface moored system.	Three	All applicable forces from Section 2.	Time varying relative current profile.
SSD31.FORT	Subsurface moored system.	Three	All applicable forces from Section 2.	Time varying absolute current profile.
SSS3.FORT	Subsurface moored system.	Three	All applicable forces from Section 2 except inertia forces.	Time varying relative current profile.

TABLE 4.2 Programs for Subsurface Moored Systems

2. A spar buoy anchored with a lumped parameter mooring line.
3. A spar buoy anchored with a lumped parameter mooring line and a lumped parameter mooring line hanging from the buoy.
4. A spar buoy attached to a window shade drogue.

SD2.FORT does the same simulations as SD3.FORT, but is coded for the two dimensional analysis of Section 2. Computer programs BUOY.FORT, MDE.FORT, and DDB.FORT are subsets of the computer program SD3.FORT. BUOY.FORT simulates case study 1, MDE.FORT simulates case study 3, and DDB.FORT simulates case study 4. All these programs use the lumped parameter formulation for the mooring line. Input data necessary for these programs is outlined below.

Input Data Required for Computer Program
SD3.FORT (All units in F.P.S.)

General data:

- NM: Number of nodes (masses). Equals
 one for case study 1.
- NB: Number of the surface buoy. Starting
 from the anchor, the number of the node
 (anchor is not counted as a node) where

the buoy is located. $NM = NB$ for case study 3. NB equals one for case studies 1 and 4.

ND: Number of the first node on the drogue.

NST: Number of steps. This is used for dividing the spar buoy in NST strips for integration of viscous drag forces and moments over the entire submerged buoy.

NC: Number of cycles, i.e. number of surface waves for which the simulations are to be done.

MOOR: Index for case study control. Equals zero for case study 1; one for case studies 2 and 3, and two for case study 4.

DEPTH: Ocean depth.

DT: Integration time step ΔT .

TMAX: Maximum time of simulation.

T2: Time interval for simulation printout.

Buoy Data:

RD1, RD2: Two radii (r_1 and r_2) of the tuned spar buoy. RD1 is for the drum and equals RD2 for a cylindrical spar.

ZCG: Distance ($Z_c > 0$) between centroid of the spar base and its C.G.

RGYR: Radius of gyration of the buoy about any axis in the horizontal plane.

HMAX: Length of the spar buoy (H).

CDL: Normal viscous drag coefficient (C_{DN}) used for the spar. πC_{DT} is found by multiplying $CDL \times 0.02$

HST: Length of the drum for the tuned spar buoy (h_1). For cylindrical spar HST is some arbitrary number between zero and the submerged depth of the spar.

CON: Added mass coefficient (C_N) for the buoy. (Used & read in SD2.FORT only).

COT: Added mass coefficient (C_T) for the buoy. (Used & read in SD2.FORT only).

CDP: Viscous drag coefficient for the base and step of the tuned spar (C_{DP}).

ALPHA: Added mass coefficient for the base and step of the tuned spar (α).

Mooring Line Data:

W(I): Weight in water (w_{wi}) of components lumped at the I^{th} node. Equals weight in air for the buoy (NB^{th} node)

CM(I): Mass of components lumped at the I^{th} node (m_i).

CMS(I): Added mass of spherical components lumped at the I^{th} node (m_{ai}).

CMD(I): Mass of water displaced by components lumped at the I^{th} node (m_{di}).

CMN(I),
CMT(I): Normal and tangential components of added mass of cylindrical components lumped at the I^{th} node (m_{ni} and m_{ti}).

SL(I): Slack length of the mooring line proceeding the I^{th} node (L_i).

EK(I): Elastic coefficient for SL(I) (k_i).

CDIN(I),
CDIT(I): Normal and tangential viscous drag constants ($1/2 \rho C_D A_D$) of cylindrical components lumped at the I^{th} node.

CDIA(I): Viscous drag constant of non-cylindrical components lumped at the I^{th} node.

DEP(I): Depth (less than zero) of the I^{th} node (approximate) (z_i).

Attachment Data:

XC1,YC1, Coordinates of the attachment point
ZC1: between spar and the anchoring line
in spar buoy coordinate frame (x' ,
 y' , z').

XC2,YC2, Coordinates of the attachment point
ZC2: between the spar and the instrument
line in x' , y' , z' frame.

Current Profile Data:

V(I,J,I): Absolute velocity of water in J^{th}
(x , y , and z) direction at the depth
of I^{th} node.

Surface Wave Data:

WE: Frequency (ω) of the surface wave.
AMP: Amplitude (ξ_0) of the surface wave.
BETA: Wave direction (β).

All of the input data are read in the main
program. The main program calls subroutines: (a) STATIC;
which calculates the mean configuration or steady state
(static) solution without any surface wave forcing.
(b) BUOYS; which calculates the mass matrix $[M]$ for the
spar buoy. (c) DBDRG, which calculates the viscous drag

forces/moments on the spar buoy. (d) MINVER, which inverts a matrix. (e) MATRIX, which calculates the wave exciting plus hydrostatic forces on the spar buoy, the stiffness matrix $[K]$, the remaining mass matrix for the mooring lines, viscous drag forces on the mooring lines, Froude Krylov forces on the mooring lines, and the tension force components at each node. It also integrates the differential equations of motion. (f) NEXT, which updates the geometry of the system.

Before finding the steady state dynamic response of a system to a sinusoidal surface wave, an equivalent current profile has to be determined to compute the mean configuration of the system. This current profile depends on the original current profile plus the contribution due to the surface wave. In effect it is a rectification of the time constant current profile by the oscillatory velocity field generated by the surface wave. From Section 2.1 we have:

$$\begin{aligned}\bar{V}_{EW} = & \hat{i}_x (V_{ox} + \phi_x) + \hat{i}_y (V_{oy} + \phi_y) \\ & + \hat{i}_z (V_{oz} + \phi_z)\end{aligned}$$

also;
$$|\bar{V}_{EW}|^2 = (V_{ox} + \phi_x)^2 + (V_{oy} + \phi_y)^2 + (V_{oz} + \phi_z)^2$$

And as viscous drag forces determine the moored or drifting system configurations for the static solution, and these are proportional to $|\bar{V}_{EW}| \bar{V}_{EW}$, the square root of mean $(|\bar{V}_{EW}| \bar{V}_{EW})$ over a wave period (T) would give the equivalent rectified current vector at this location.

$$\begin{aligned} \text{Let;} \quad \bar{E} &= \frac{1}{\omega T} \int_0^{\omega T} |\bar{V}_{EW}| (\bar{V}_{EW}) d(\omega t) \\ &= \hat{i}_x E_x + \hat{i}_y E_y + \hat{i}_z E_z \end{aligned}$$

Now the equivalent current vector can be written as:

$$\bar{V}_{eq} = \bar{E} / EE$$

$$\text{where:} \quad EE = \sqrt{|\bar{E}|}$$

This calculation to find $\bar{V}_{eq}$, before a static solution is performed, is done in the main program. The numerical integration of differential equations in the subroutine MATRIX was performed by the rectangular rule, which was found adequate for all case studies simulated.

4.2 Subsurface Moored Systems

Computer program SSD3.FORT is coded for the three dimensional analysis of subsurface moored systems

(continuous line formulation) given in Section 2. The program simulates the response of a subsurface moored system to the time varying current fields. SSD3.FORT reads in the relative (relative to the mooring system) velocity profile in the three orthogonal x, y, and z directions. From these three components the viscous drag forces can be computed directly. These components of the relative velocity could be the ones measured by current sensors on the mooring line. Alternatively, SSD31.FORT reads in the absolute (relative to the mean floor) velocity profile in the three orthogonal (x, y, and z) directions. Profile is read in as amplitudes of the sinusoidally varying currents. All components (varying with depth) are driven by a single frequency. In addition a surface wave of given amplitude, frequency, and direction generates velocity field exponentially attenuated in depth. The program computes the total absolute velocity and then subtracts the mooring line velocity from this absolute velocity to compute the viscous drag forces. SSS3.FORT is the same as SSD3.FORT but it neglects all inertia forces of the mooring system. All three programs use the continuous line formulation of the mooring line, given in Section 2, as the mathematical model. Input data, except

for the input forcing function (velocity profiles and surface wave) are the same for all three programs and are detailed below:

General Data:

NI: Number of intermediate bodies
(instrument clusters or inserted floats) to be included in the analysis.

NP: Number of mooring parts (each mooring part can have different properties).

NS: Number of segments the continuous line is to be divided in.

NPT: Number of dynamic equilibriums to be performed.

IKK: Number of locations the velocity profile changes along the ocean depth.

DEPTH: Depth of the ocean. (Meters)

DDT: Approximate depth of the mooring line top. (Meters)

ER: Used for iteration on the boundary condition of ocean depth. If DDT is known with reasonable accuracy, ER is not required. (Meters)

ER1: Used for error control in SSD31.FORT.

Instrument Data:

PI(J) Position of the J^{th} instrument in
 slack length distance from the
 top. (Meters)

SI(J): Length of the J^{th} instrument.
 Equals zero for non-cylindrical
 instruments. (Meters)

ZM(J): Mass of the J^{th} instrument (m_j). (Slugs)

ZMC(J): Normal added mass component (m_{nj})
 for cylindrical instruments. (Slugs)

ZMV(J): Added mass (m_{aj}) of the J^{th}
 instrument for a non-cylindrical
 instrument. Tangential added mass
 component (m_{tj}) for cylindrical
 instruments. (Slugs)

F(3,J): Weight in water (W_{wj}) of the J^{th}
 instrument. (Pounds)

CDIN(J), Normal and tangential viscous
CDIT(J): drag constants ($1/2 \rho C_D A_D$) of the
 instruments. CDIT is not used for
 spherical instruments. (F.P.S. units)

Mooring Line Data:

DIAL(I): Nominal diameter of the I^{th} mooring
 part. (Inches)

SLL(I): Slack length of the I^{th} mooring part. (Meters)

AWL(I): Weight in air per unit slack length of the I^{th} mooring part (W_a). (lbs/M)

WWL(I): Weight in water per unit slack length of the I^{th} mooring part (W_w). (lbs/M)

TPL(I): Transient peak load on the I^{th} mooring part. For materials which obey Hook's law this is substituted with the Young's modulus of elasticity. (Refer volume 1 CHHABRA, 1973). (lbs. or lbs/in^2)

COL(I),
CO2(I),
PO1(I),
PO2(I),
AO(I): Constants for stress-strain relationships of mooring line materials. (CHHABRA, 1973). Col is used for jacket diameter (in.) in jacketed wire rope.

CDN(I),
CDT(I): Normal and tangential drag coefficients of the I^{th} mooring part. (C_{DN} and C_{DT})

Current Profile Data:

D(I): Depths, greater than zero, where the current changes direction. I goes from 1 to IKK. (Meters)

V(I,J): For SSD3.FORT: Relative velocity in the I^{th} (x, y, and z) direction for the J^{th} zone. J goes from 1 to IKK + 1. (mm/sec)

For SSD31.FORT: Absolute velocity amplitude in the I^{th} direction for the J^{th} zone. Following data are used only in SSD31.FORT.

WE: Frequency of current profile amplitudes $V(I,J)$. (Rad/sec)
DT: Time step for calculation of the mooring system dynamic equilibrium.
T2: Time step for simulation output.
TMAX: Maximum time for simulation.
WS: Surface wave frequency (ω). (Rad/sec)
AMP: Surface wave amplitude (ζ_0). (mm)
BETA: Surface wave direction (θ). (Radians)

All of input data are read in the main program.

The main program calls subroutine MOTION which calculates the dynamic equilibrium of the subsurface mooring system at discrete times. Subroutine MOTION calls subroutine FORCES, whenever an instrument package or a subsurface float is encountered in the mooring line incremental integration scheme. Both MOTION and FORCES call subroutine SPEED to find the current for viscous drag calculations.

5.0 CASE STUDIES/SIMULATIONS

This section presents computer simulations of specific case studies; using the computer programs outlined in Section 4. All simulations are done in the time-domain. Simulations of the freely floating spar buoy, as presented in Section 5.1, were combined to obtain the response amplitude operator (RAO) vs K_h (wave number multiplied by the appropriate buoy draft). Section 5.2 presents the subsurface moored system simulations. Section 5.3 gives simulations of a surface moored system, and Section 5.4 presents simulations of a drifting drogued buoy.

5.1 Spar Buoy

Both the two-dimensional computer program (SD2.FORT) and the three-dimensional computer program (SD3.FORT) are used to simulate each of the two spar buoys; one cylindrical, and the other tuned presented in Sections 5.1.1 and 5.1.2 respectively. RAO vs K_h plots are presented for the two-dimensional simulations of cylindrical and tuned spar buoys.

5.1.1 Cylindrical Spar

The spar buoy (Figure 5.1) used in these simulations was tested in a series of wave tank tests to

derive empirical response data for comparison with the computer simulations. This comparison has not been presented in this report. The dimensions

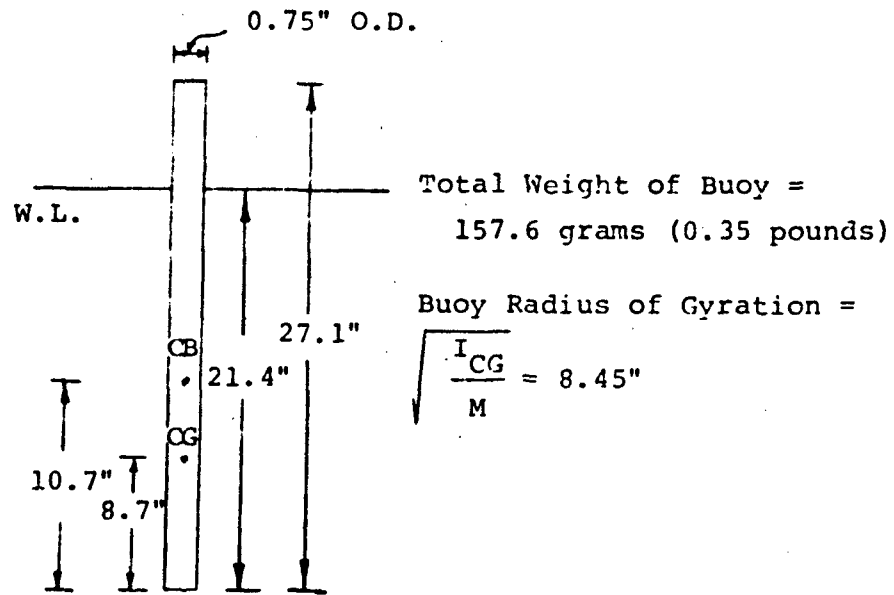


Figure 5.1 Scale Model Spar Buoy Description

and characteristics of this scale model spar buoy are shown in Figure 5.1.

Two Dimensional Simulations

Figures 5.2, 5.3, and 5.4 present the typical surge, heave, and pitch response of the buoy to surface waves of

Figure 5.2 Surge Motions of the Freely Floating Cylindrical Spar Buoy

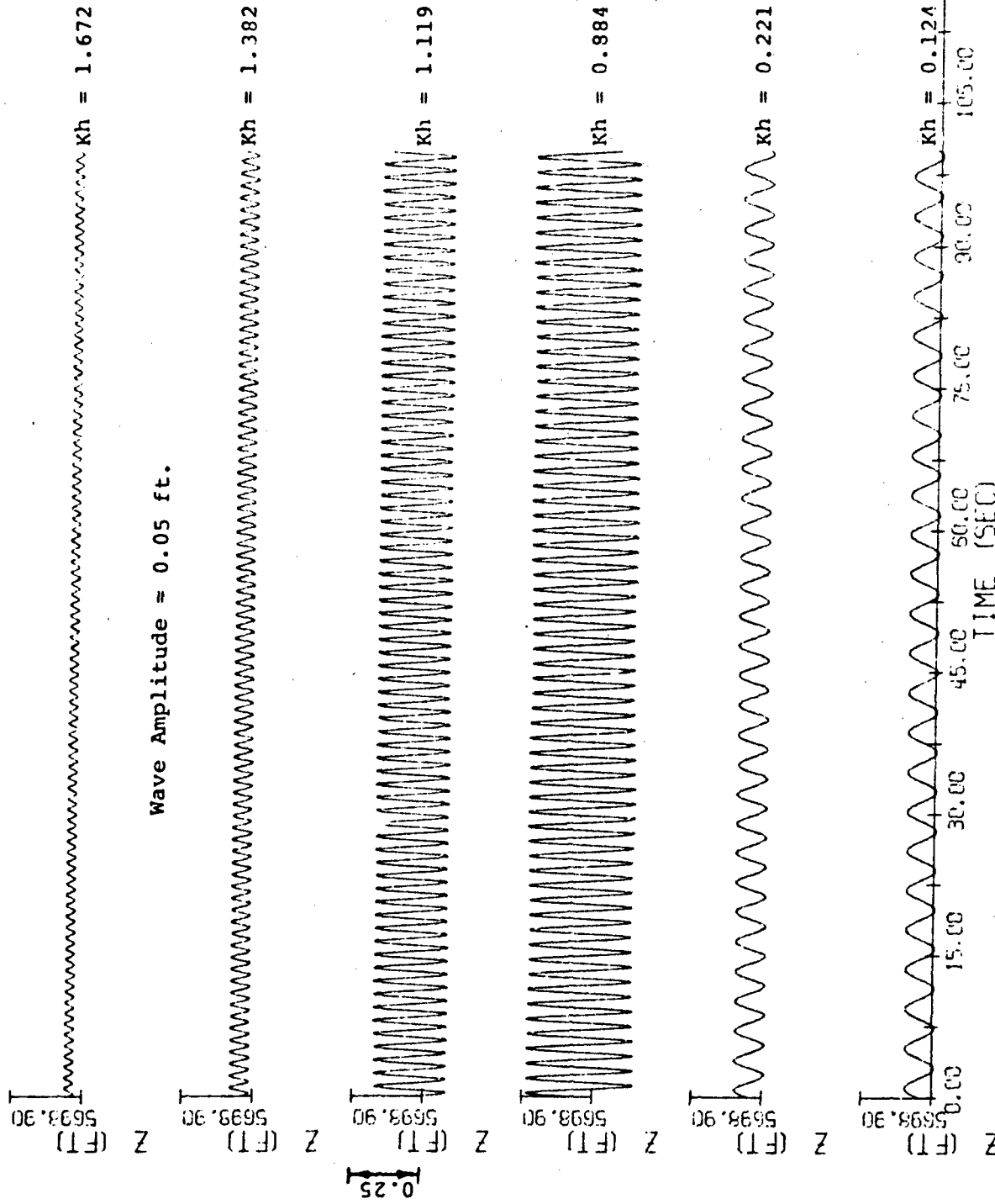


Figure 5.3 Heave Motions of the Freely Floating Cylindrical Spar Buoy

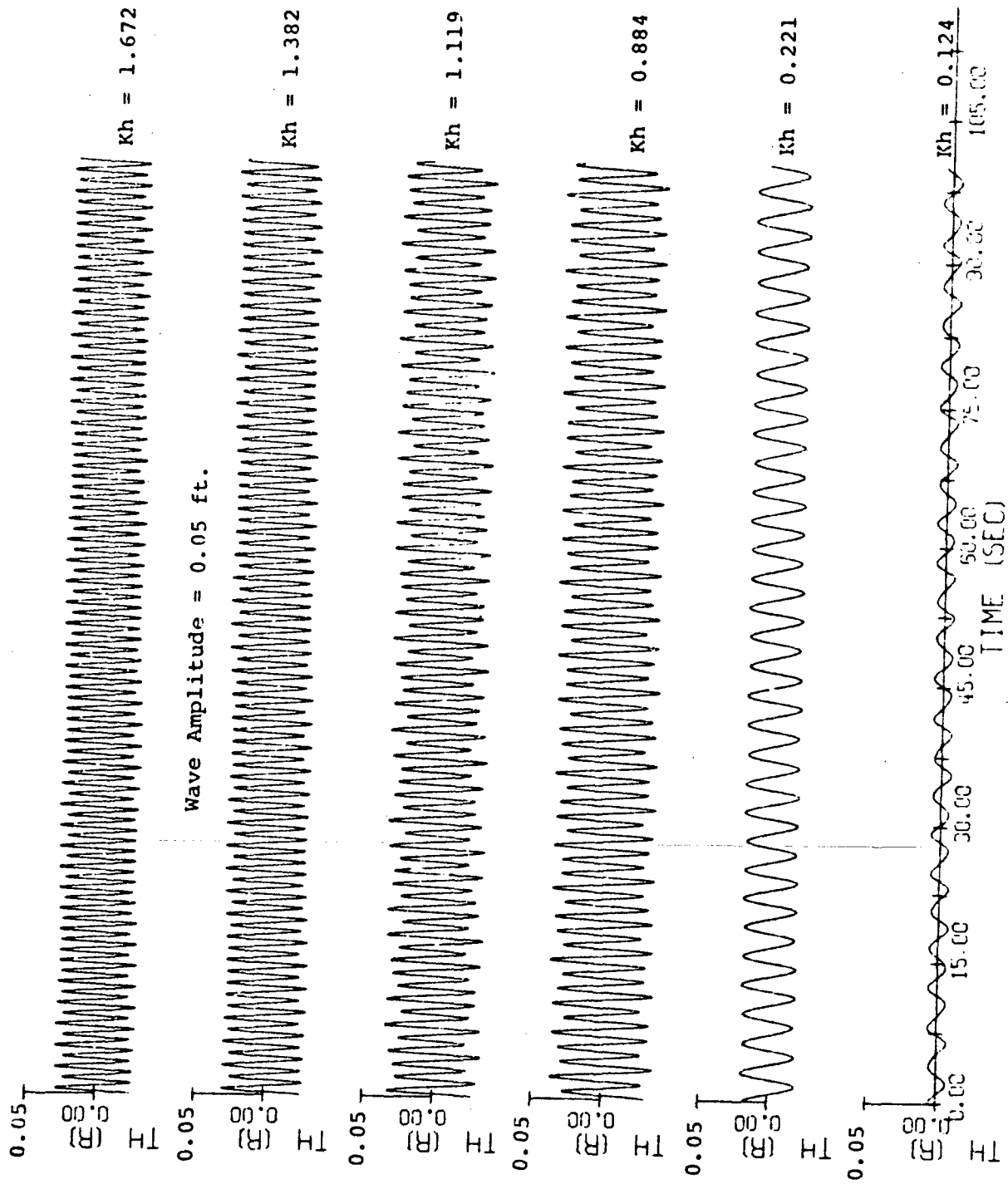


Figure 5.4 Pitch Motions of the Freely Floating Cylindrical Spar Buoy

amplitude 0.05 ft. and different frequencies. Table 5.1 is the input data used in conjunction with the computer program SD2.FORT to obtain these simulations. Noteworthy is the fact that the viscous (CDL) and pressure (CDP) drag coefficients in this study equal zero. Surge drift evident in Fig. 5.2 is due to the bias or rectification of wave exciting forces, which would be present for example if an oscillatory hydrodynamic force were to act on a heaving body whose motions were at varying with this force. Simulations for the same data as Table 5.1 and viscous and pressure drag coefficients equal to 1.0 are shown in Figures 5.5, 5.6, and 5.7. Comparison between the surge motions of Figures 5.2 and 5.5 shows the effect of viscous drag on surge drift. Surge drift due to viscous drag occurs as the submerged area of the spar and its inclination vary over a wave period, and is a function of their phase relationships. This drift is most predominant near resonance. In this case heave resonance occurs at $K_h = 1.0$, and pitch resonance occurs at $K_h = 0.4$. Heave motions (Figures 5.3 and 5.6) are not effected much by viscous drag because the tangential drag due to CDL and CDP is comparatively small. Comparison of Figures 5.4 and 5.7 shows that the system frequency present in the top four curves of Figure 5.4, due to approximate initial conditions, is damped out in Figure 5.7 due to viscous drag damping. Otherwise, the amplitudes do not show much change.

SD2C.DATA		1	0	25	6	0	5700.	0.020	100.0	0.1
00010	1	0.03125	0.0	0.725	0.704	1.0	0.020	100.0	0.1	
00020	0.03125	0.0	0.0	0.0	0.0	0.0	2.258	0.0	1.1	
00021	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
00055	0.35	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
00090	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
00105	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
00110	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
00120	1.50	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	
00130	2.00	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	
00140	4.00	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	
00150	4.50	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	
00160	5.00	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	
00170	5.50	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	
READY										

Table 5.1 Input Data for the 2-D Analysis of the Cylindrical Spar Buoy

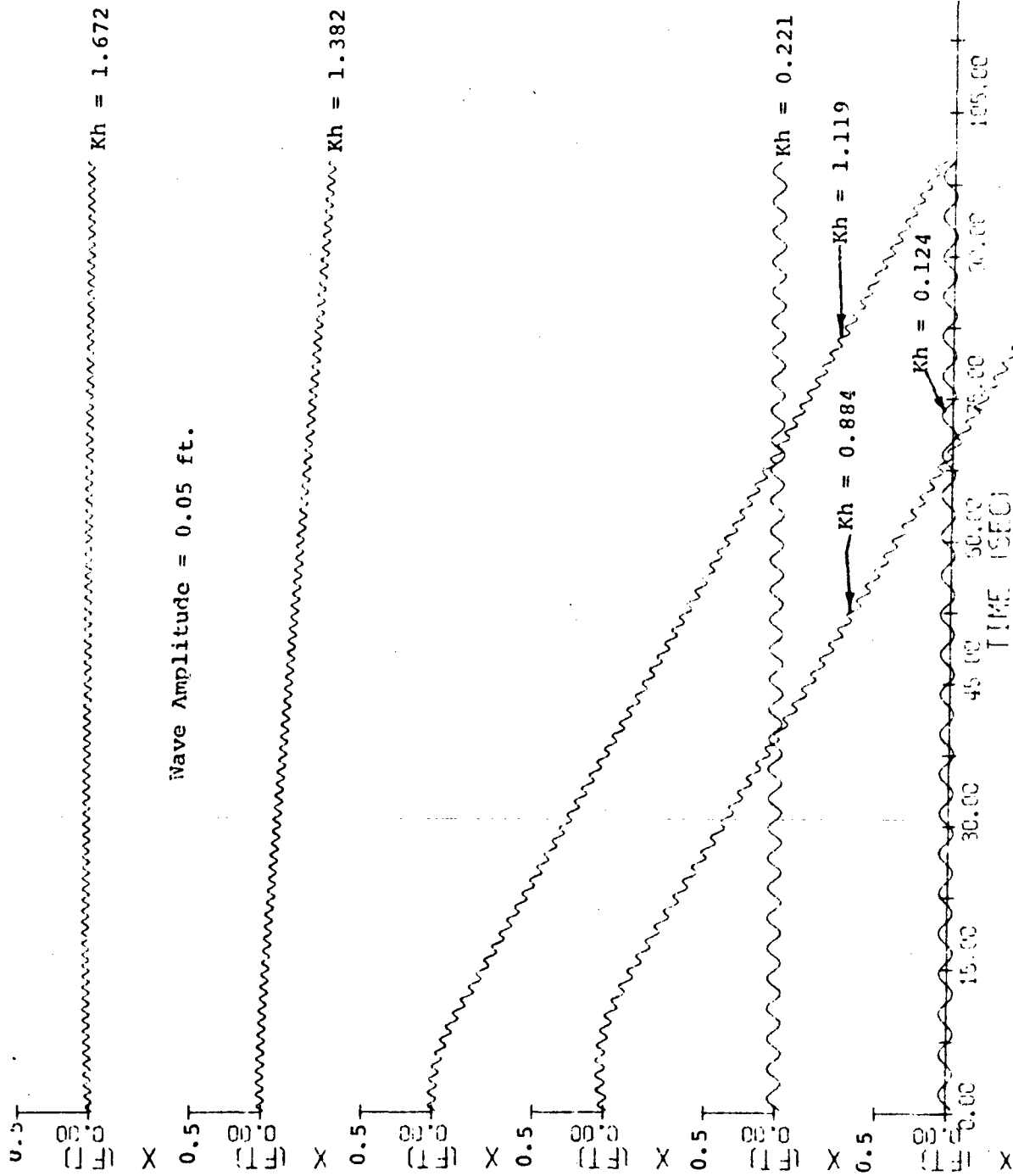
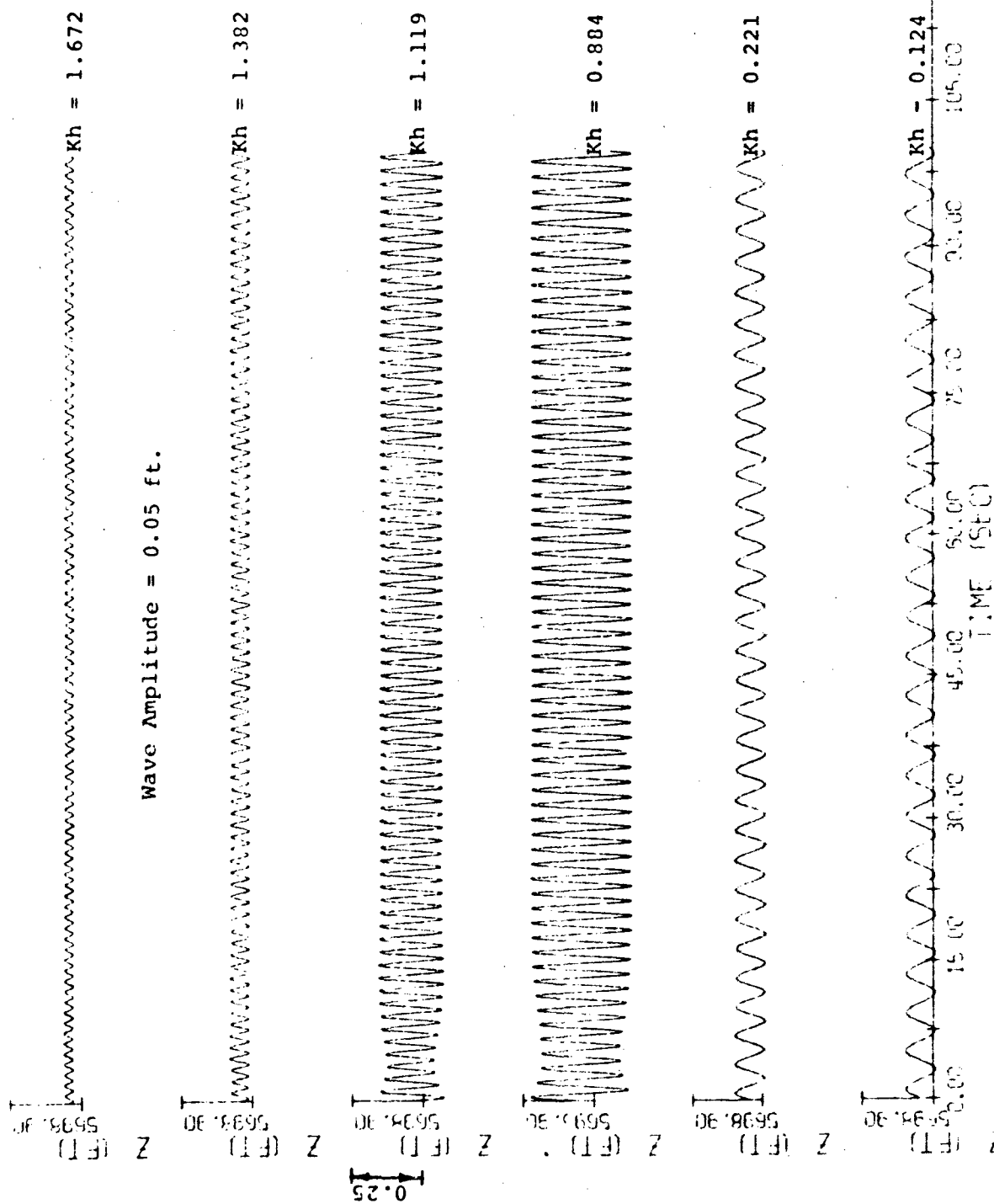


Figure 5.5 Surge Motions of the Freely Floating Cylindrical Spar Buoy



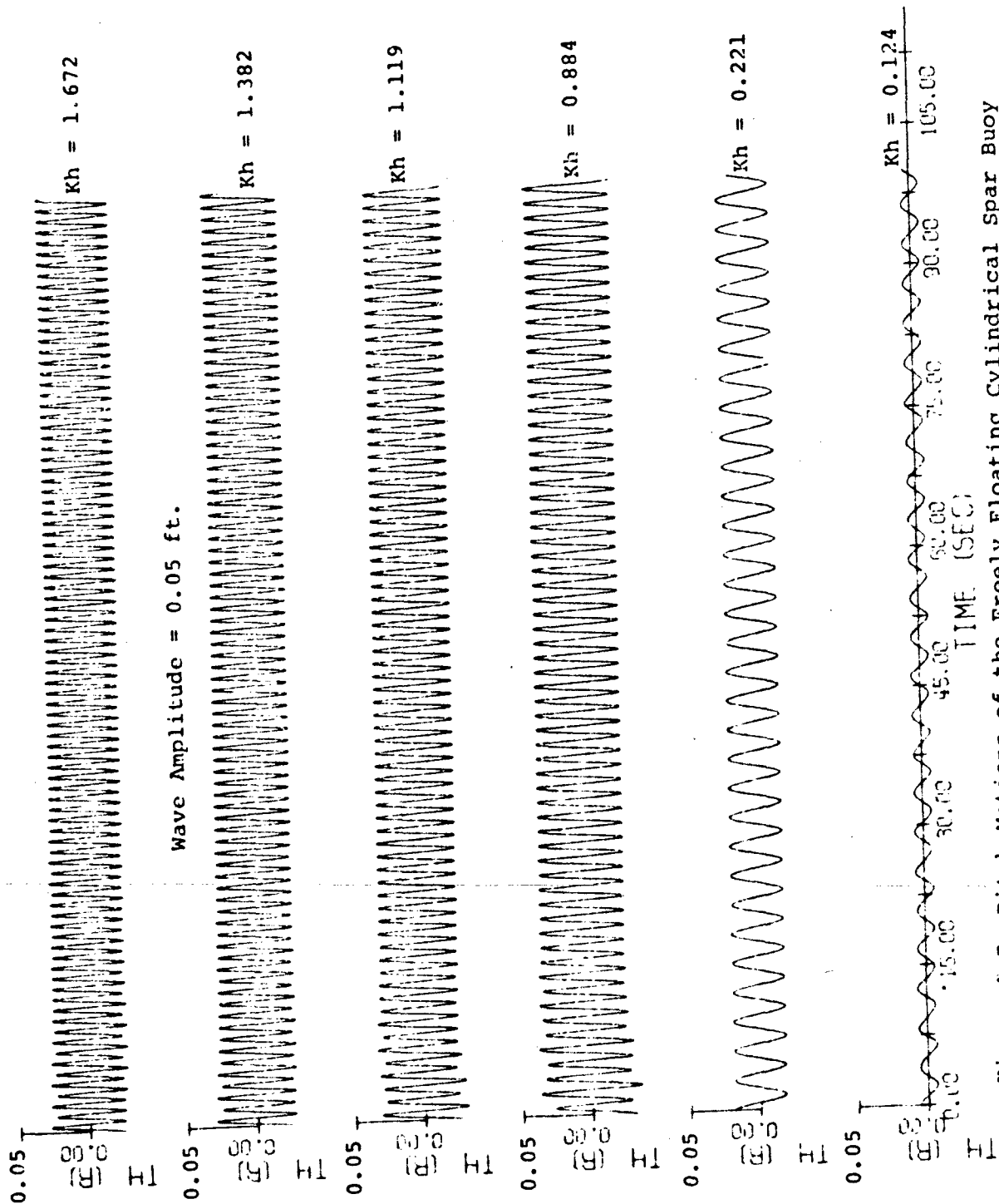


Figure 5.7 Pitch Motions of the Freely Floating Cylindrical Spar Buoy

Response amplitudes of surge, heave, and pitch; and surge drift as calculated from the steady state portions of the times histories presented in Figures 5.2 through 5.7, and other similar simulations are tabulated in Table 5.2. In this table column 1 tabulates the frequencies (ω) of waves studied. From these frequencies Kh is calculated and tabulated in column 2. Columns 3 through 6 are four columns of surge amplitude divided by ξ_0 . The first of these four columns presents the results of the undamped linear analytical solution as derived in Section 3.2. The second presents the computer simulation results with ξ_0 equal to 0.05 ft. and no viscous drag. Third and fourth present the computer simulation results with viscous drag and ξ_0 equal to 0.05 ft. and 0.25 ft. respectively. The simulated plots are shown for two wave amplitudes to emphasize the nonlinear response obtained by the nonlinear theory. Similarly columns 7 through 10 tabulate four columns of heave amplitude divided by ξ_0 , and columns 11 through 14 tabulate four columns of pitch amplitude divided by $K\xi_0$. $K\xi_0$ is the maximum wave slope. Analytical solution for surge drift was not found hence only three columns 15 through 17 are given for surge drift divided by $K\omega\xi_0^2$. $K\omega\xi_0^2$ is the mean "stokes drift" velocity at the mean free surface. Data points of Table 5.2 are plotted in Figures 5.8 through 5.11

ω	Kh	Surge Amplitude/ ξ_o				Heave Amplitude/ ξ_o				Pitch Amplitude/ ξ_o				Surge Drift/ $K\omega^2 \xi_o$			
		1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
1.50	0.12	-0.94	0.95	0.95	0.95	1.01	1.01	1.01	1.01	1.71	1.79	1.72	1.72	2.91	2.91	0.91	0.94
2.52	0.35	-0.84		0.86	0.75	1.09		1.13	1.23	18.19		7.94	5.42			-6.37	-1.10
2.69	0.40	-0.82		0.72		1.12		1.16		-26.52		8.22				-6.31	
2.85	0.45	-0.81		0.68	0.70	1.17		1.17	1.22	-7.97		6.96	4.01			-4.11	-0.48
3.30	0.60	-0.75		0.67		1.39		1.39		-2.67		2.65				-0.61	
3.56	0.70	-0.72		0.66	0.65	1.69		1.68	1.62	-1.91		1.89	1.85			-0.43	0.13
4.00	0.88	-0.66	0.61	0.64	0.85	3.79	3.75	3.53	1.97	-1.28	1.26	1.22		2.36		-7.91	0.19
4.10	0.93	-0.65		0.53		6.18		4.23		-1.19		1.01				-11.88	
4.20	0.98	-0.64		0.53		21.35		4.30		-1.11		0.35				-12.45	
	1.00																
4.30	1.02	-0.63		0.51		-12.13		3.76		-1.04		0.86				-10.53	
4.40	1.07	-0.61		0.53		-4.39		2.96		-0.98		0.90				-7.58	
4.50	1.12	-0.60	0.54	0.57	0.62	-2.56	2.57	2.15	1.34	-0.92	0.86	0.89	0.45	0.20		-5.63	-0.38
4.67	1.21	-0.58		0.52		-1.40		1.35		-0.83		0.82				-2.47	
5.00	1.38	-0.54	0.50	0.43	0.50	-0.64	0.64	0.64	0.60	-0.70	0.70	0.70	0.66	0.47		-0.47	-0.07
5.50	1.67	-0.49	0.44	0.44	0.45	-0.27	0.27	0.28	0.28	-0.55	0.56	0.55	0.55	0.49		0.02	0.10
6.72	2.50	-0.37		0.32	0.33	-0.05		0.05	0.06	-0.34		0.34	0.35			9.15	0.16

TABLE 5.2 Dynamic Response (Surge, Heave and Pitch Amplitude; and Surge Drift) of a Cylindrical Spar Buoy to Waves

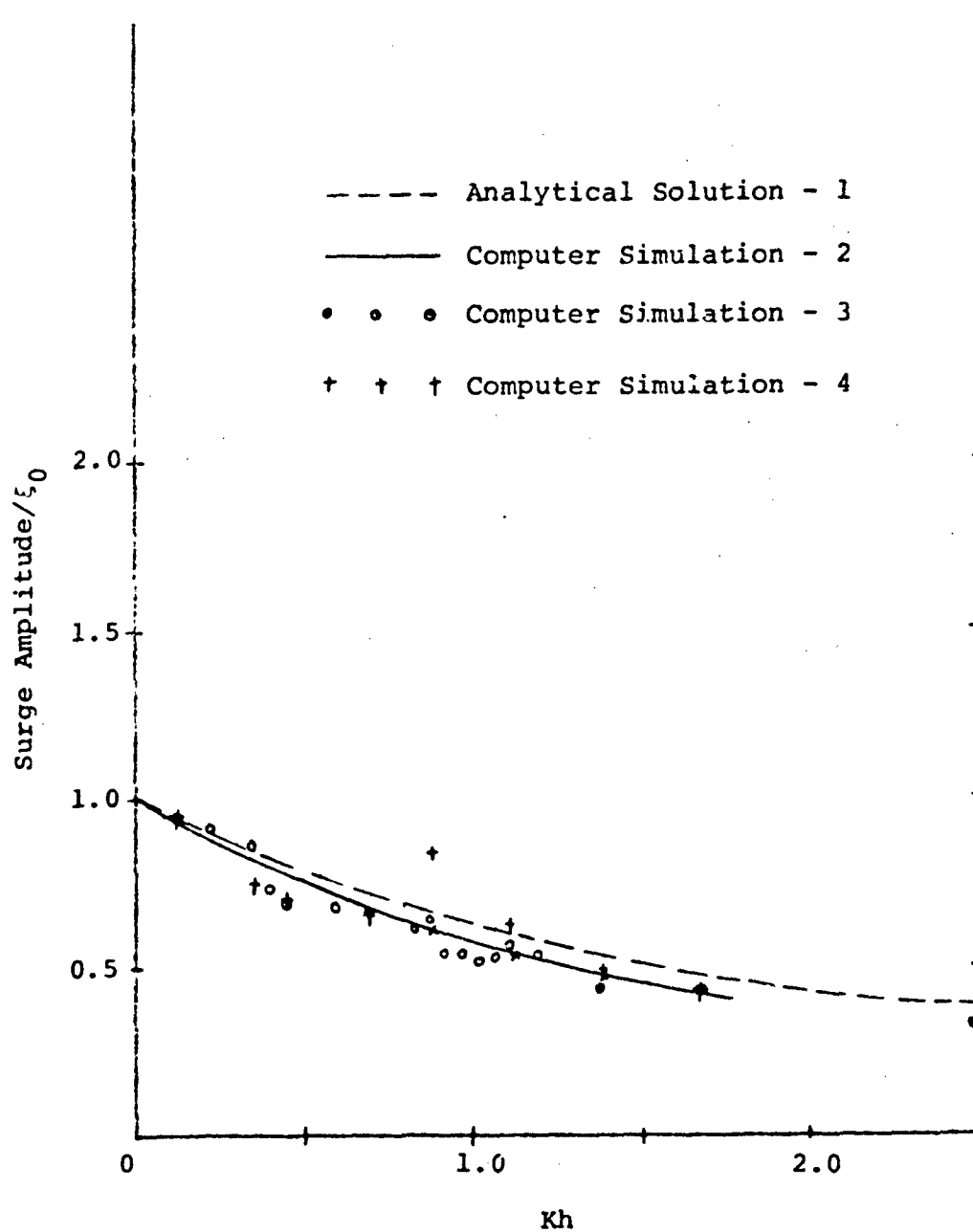


Figure 5.8 Surge Magnification Vs Kh

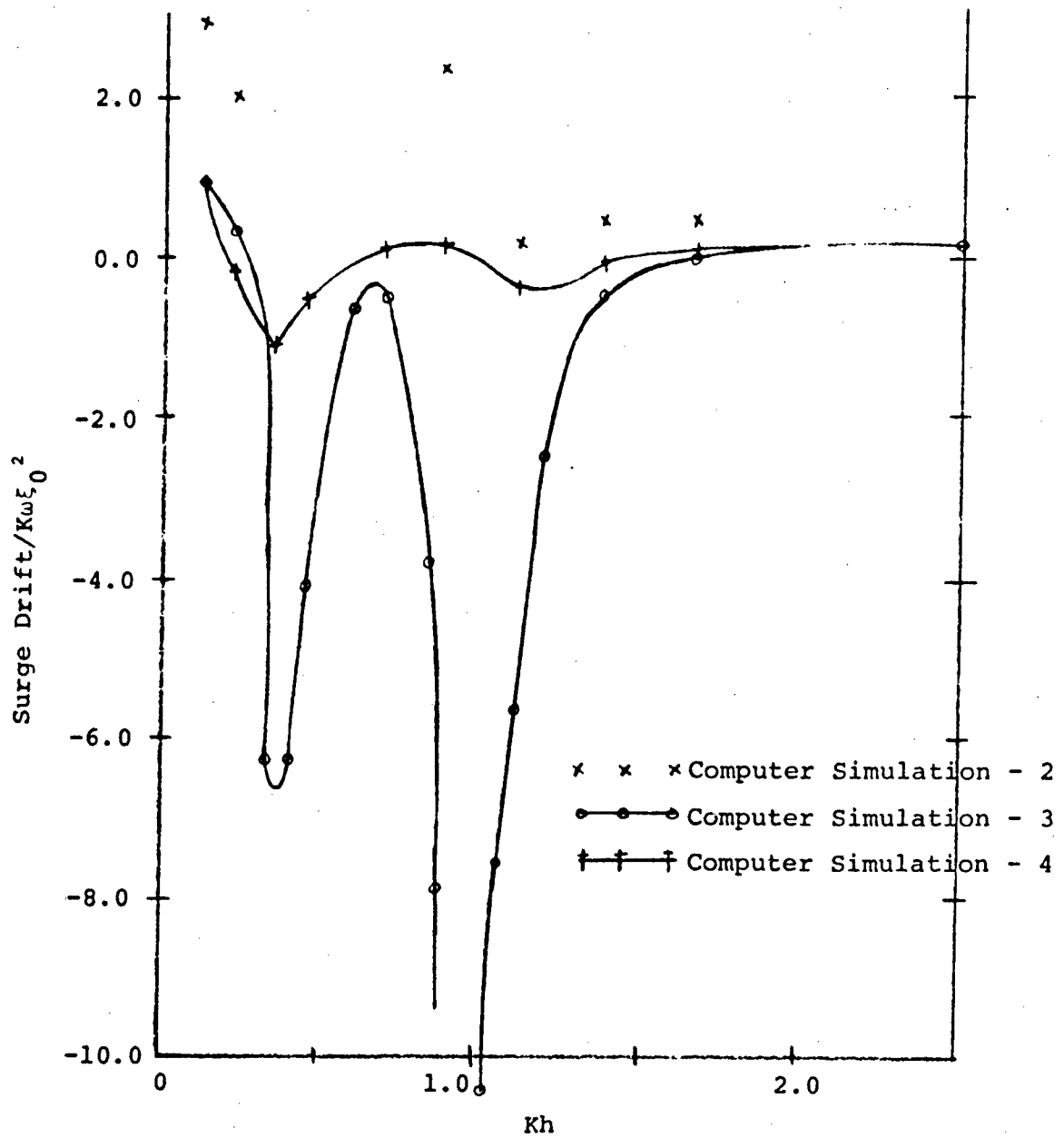


Figure 5.9 Surge Drift Vs Kh

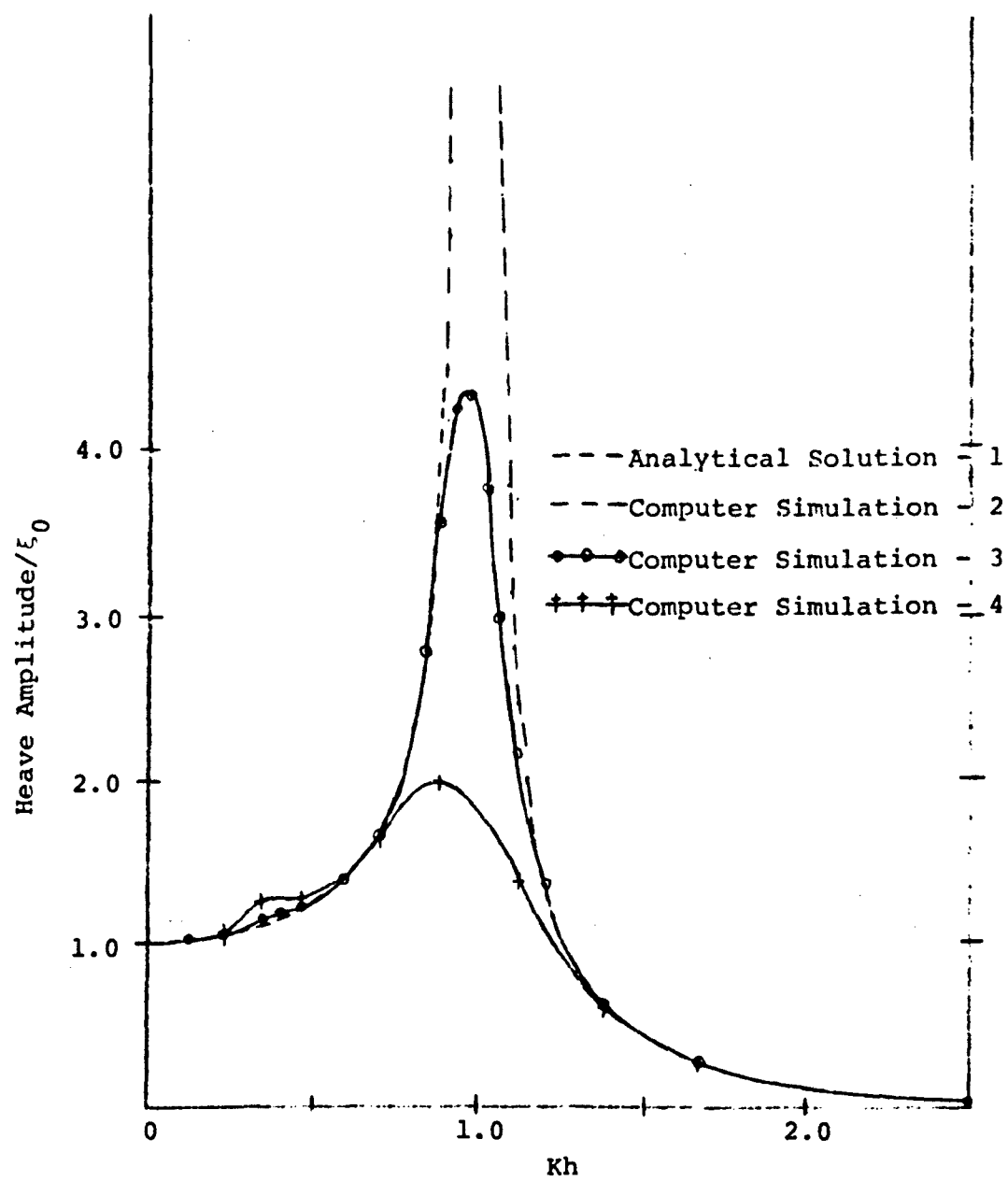


Figure 5.10 Heave Magnification Vs Kh

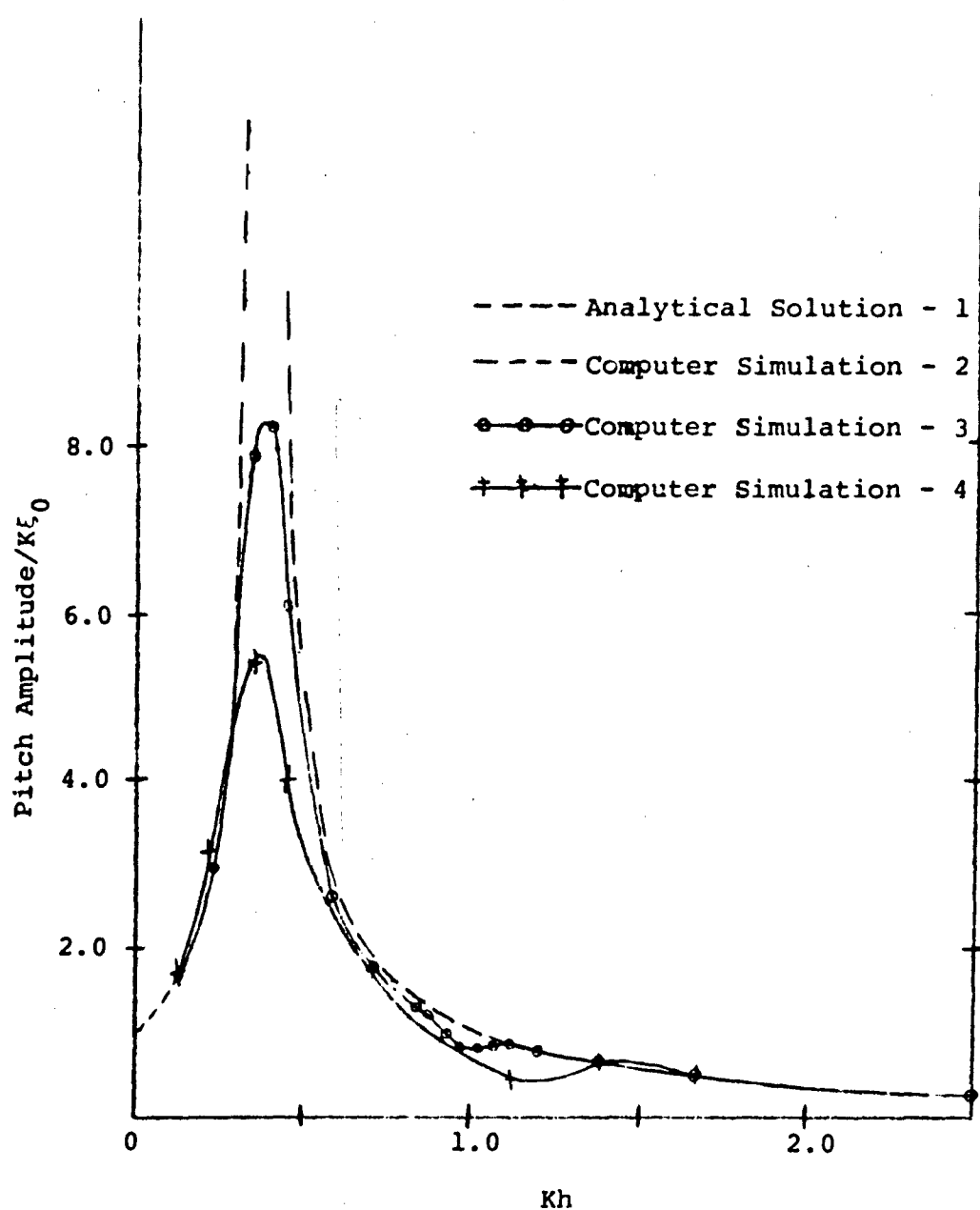


Figure 5.11 Pitch Magnification Vs Kh

to present plots of RAO vs Kh . Figure 5.8 shows the buoy surge per foot of wave amplitude as a function of Kh . For zero frequency this ratio is one and for increasing frequencies it decreases to zero. Figure 5.9 shows the buoy drift (in surge direction) per $K\omega\xi_0^2$ as a function of Kh . Figure 5.10 shows the buoy heave per foot of wave amplitude as a function of Kh . This ratio is one for zero frequency and exhibits resonance at Kh of approximately 1.0. The ratio decreases to zero with increasing frequencies. Figure 5.11 shows the buoy pitch per $K\xi_0$ as a function of Kh . This ratio is one at zero frequency and, after exhibiting resonance phenomenon at around 0.4 Kh , the ratio goes to zero with increasing frequencies. Figures 5.8 through 5.11 are further discussed in Section 6.

Three Dimensional Simulations

Figures 5.12 through 5.17 present the surge, sway, heave, roll, pitch and yaw responses of the buoy to surface waves of the same amplitude and frequencies as in Figures 5.5 and 5.7. Table 5.3 is the input data used in conjunction with the computer program SD3.FORT to obtain these simulations. Here the angle β , giving the wave propagation direction is zero. These simulations should give the same response as Figures 5.5 through 5.7. Comparing Figures 5.5 and 5.12, we see a lot of difference in the surge drifts. Figure 5.12 looks more like Figure 5.2 which

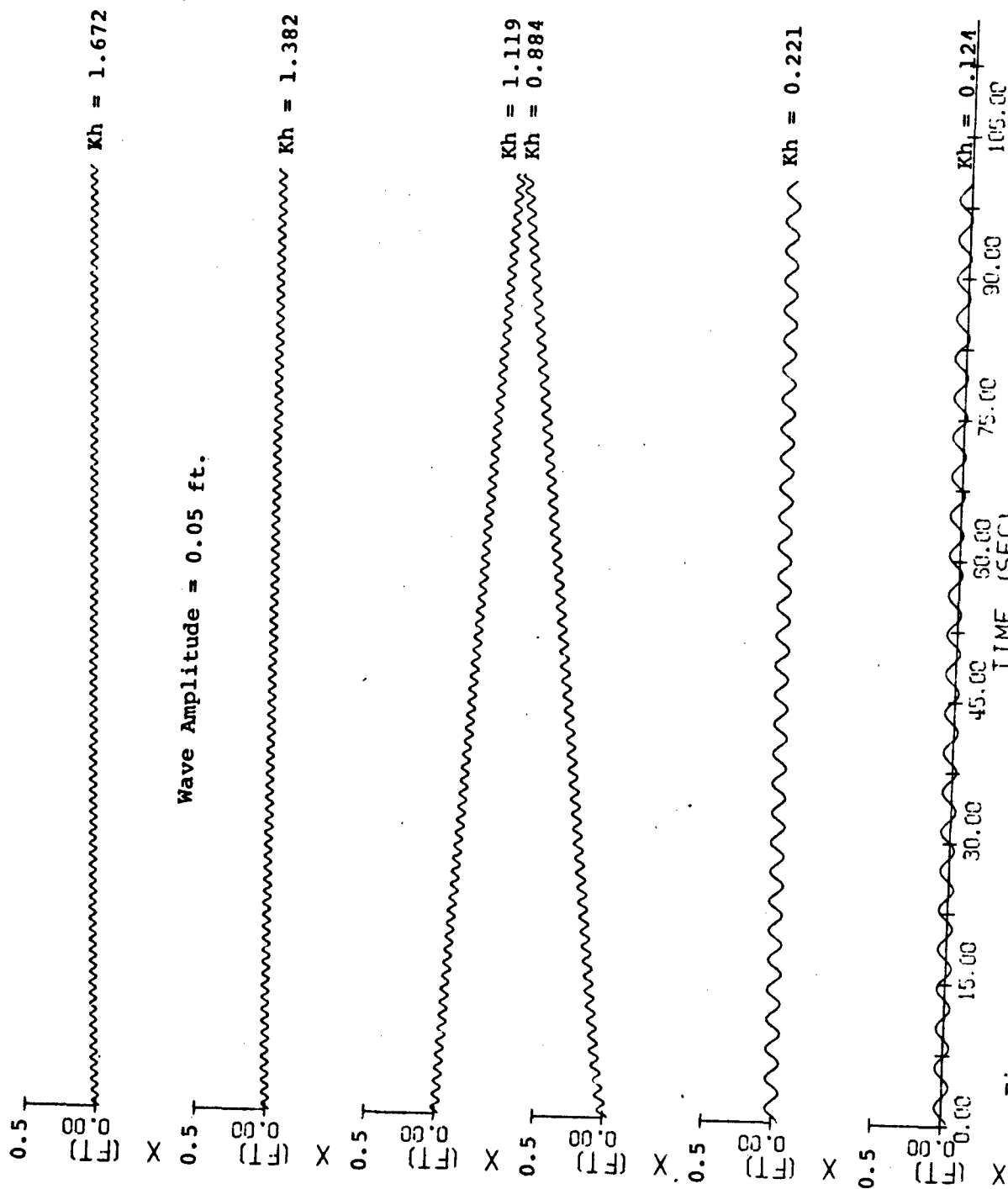


Figure 5.12 Surge Motions of the Freely Floating Cylindrical Spar Buoy

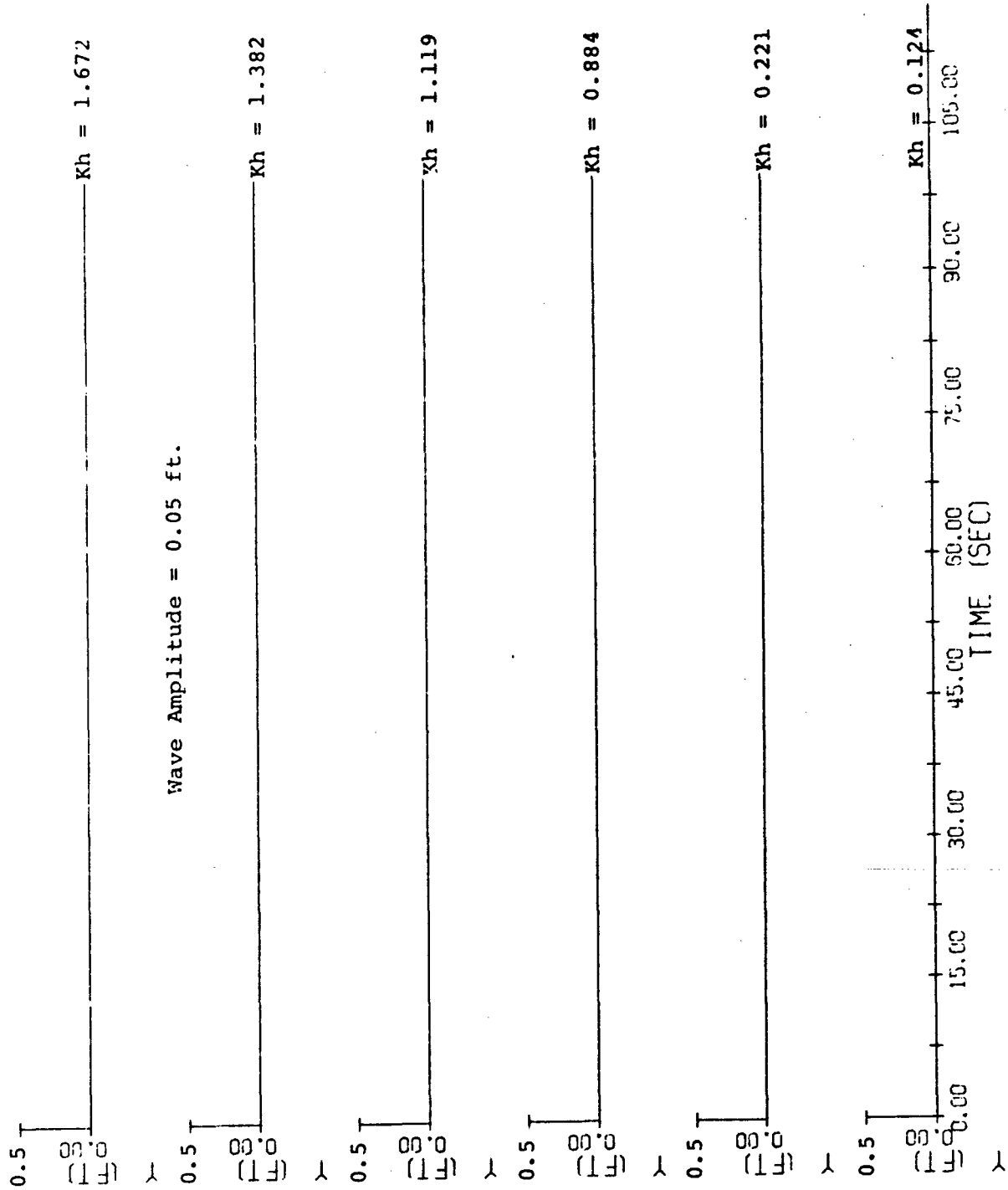


Figure 5.13 Sway Motions of the Freely Floating Cylindrical Spar Buoy

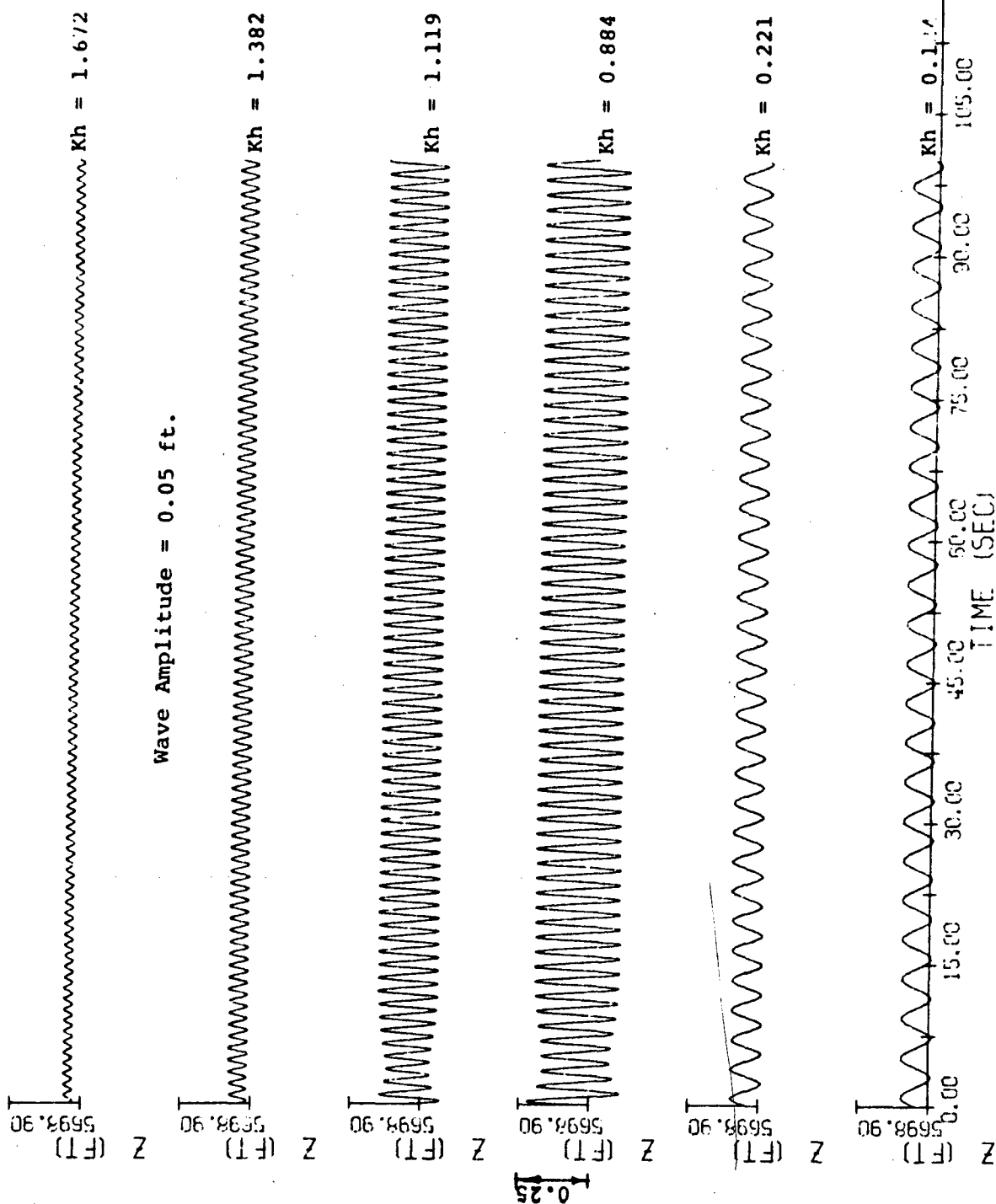


Figure 5.14 Heave Motions of the Freely Floating Cylindrical Spar Buoy

Figure 5.15 Roll Motions of the Freely Floating Cylindrical Spar Buoy

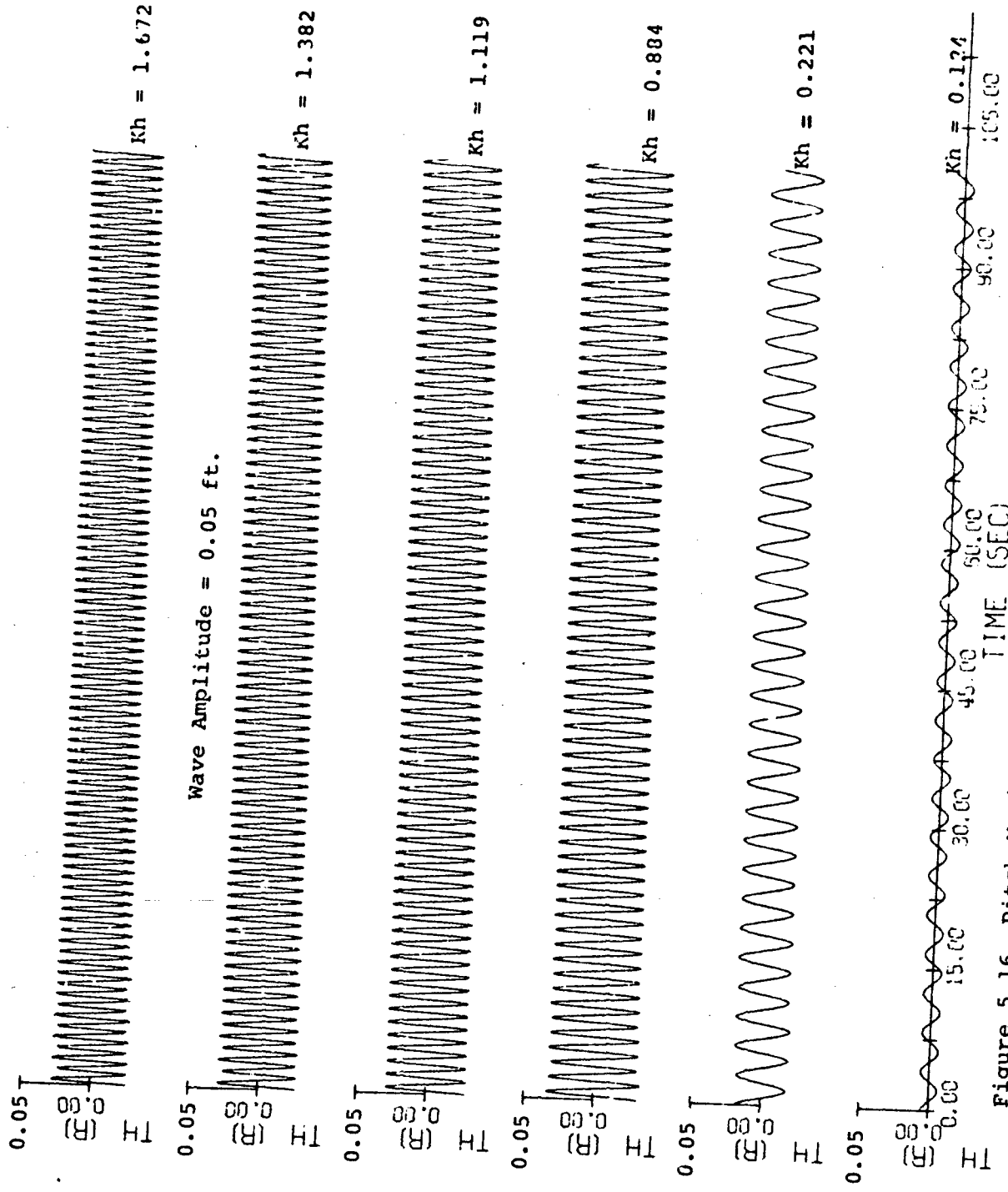


Figure 5.16 Pitch Motions of the Freely Floating Cylindrical Spar Buoy

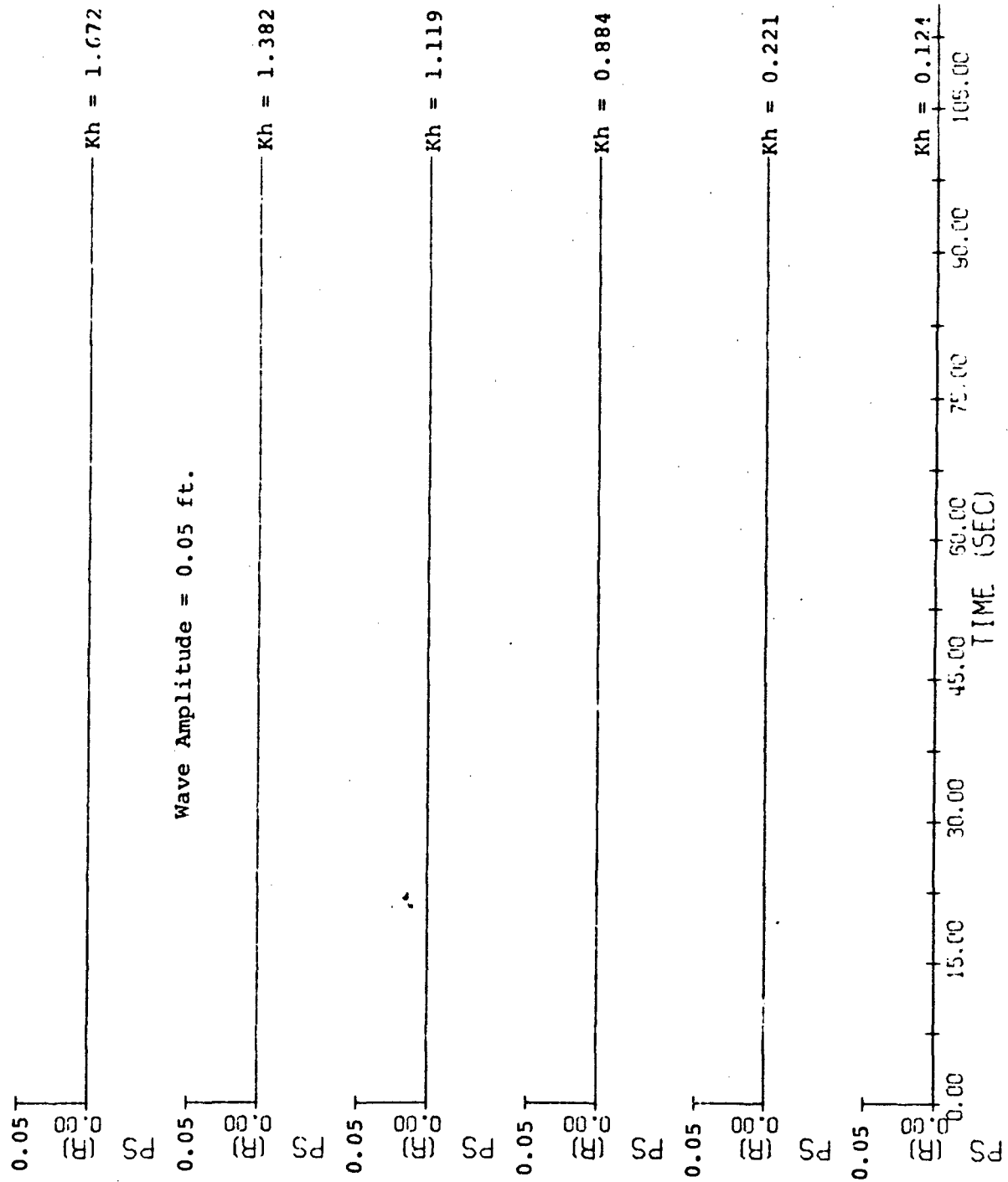


Figure 5.17 Yaw Motions of the Freely Floating Cylindrical Spar Buoy

SCBUOY.DATA									
	1	0	25	6	0	0.5700.	0.020	100.0	0.1
00010	1	0.03125	0.725	0.704	0.020	2.258	1.0	0.1	1.1
00020	0.03125	1.0	0.	0.	0.	0.	0.	0.	0.
00021	1.	0.	0.	0.	0.	0.	0.	0.	0.
00055	0.35	0.	0.	0.	0.	0.	0.	0.	0.
00090	0.	0.	0.	0.	0.	0.	0.	0.	0.
00105	0.	0.	0.	0.	0.	0.	0.	0.	0.
00110	0.	0.	0.	0.	0.	0.	0.	0.	0.
00120	1.50	0.05	0.0	0.0	0.0	0.0	0.0	0.0	0.0
00130	2.00	0.05	0.0	0.0	0.0	0.0	0.0	0.0	0.0
00140	4.00	0.05	0.0	0.0	0.0	0.0	0.0	0.0	0.0
00150	4.50	0.05	0.0	0.0	0.0	0.0	0.0	0.0	0.0
00160	5.00	0.05	0.0	0.0	0.0	0.0	0.0	0.0	0.0
00170	5.50	0.05	0.0	0.0	0.0	0.0	0.0	0.0	0.0
READY									

Table 5.3 Input Data for the 3-D Analysis of the Cylindrical Spar Buoy

was obtained by neglecting viscous drag. Hence we see that the viscous drag effects on surge drift which show up in the two-dimensional non-linear simulations (Figure 5.5), do not show up in the three-dimensional simulation (Figure 5.12) where assumptions of small motions are made. This assumption may be in error the most near the resonance frequency (Heave resonance; $K_h = 1.0$. Pitch resonance; $K_h = 0.4$), where the two Figures 5.5 and 5.12 differ the most. Comparison of Figures 5.14 and 5.16 to Figures 5.6 and 5.7 shows only slight differences. Figures 5.13, 5.15, and 5.17 show no response as expected. Simulations for the same data as Table 5.3 but $\beta = \pi/4$ are presented in Figures 5.18 through 5.23. These figures show the resolution of surge in Figure 5.12 to surge and sway in Figures 5.18 and 5.19; and the resolution of pitch in Figure 5.16 to roll and pitch in Figures 5.21 and 5.22. Yaw response remains the same as expected.

5.1.2 Tuned Spar

The spar buoy used in these simulations along with its dimensions and characteristics is shown in Figure 5.24.

Two-Dimensional Simulations

Figures 5.25, 5.26, and 5.27 show the typical surge, heave, and pitch response of the buoy to surface waves of amplitude 0.5 ft. and different frequencies. Table 5.4 is the input data used in conjunction with the computer program SD2.FORT to obtain these simulations. Here again

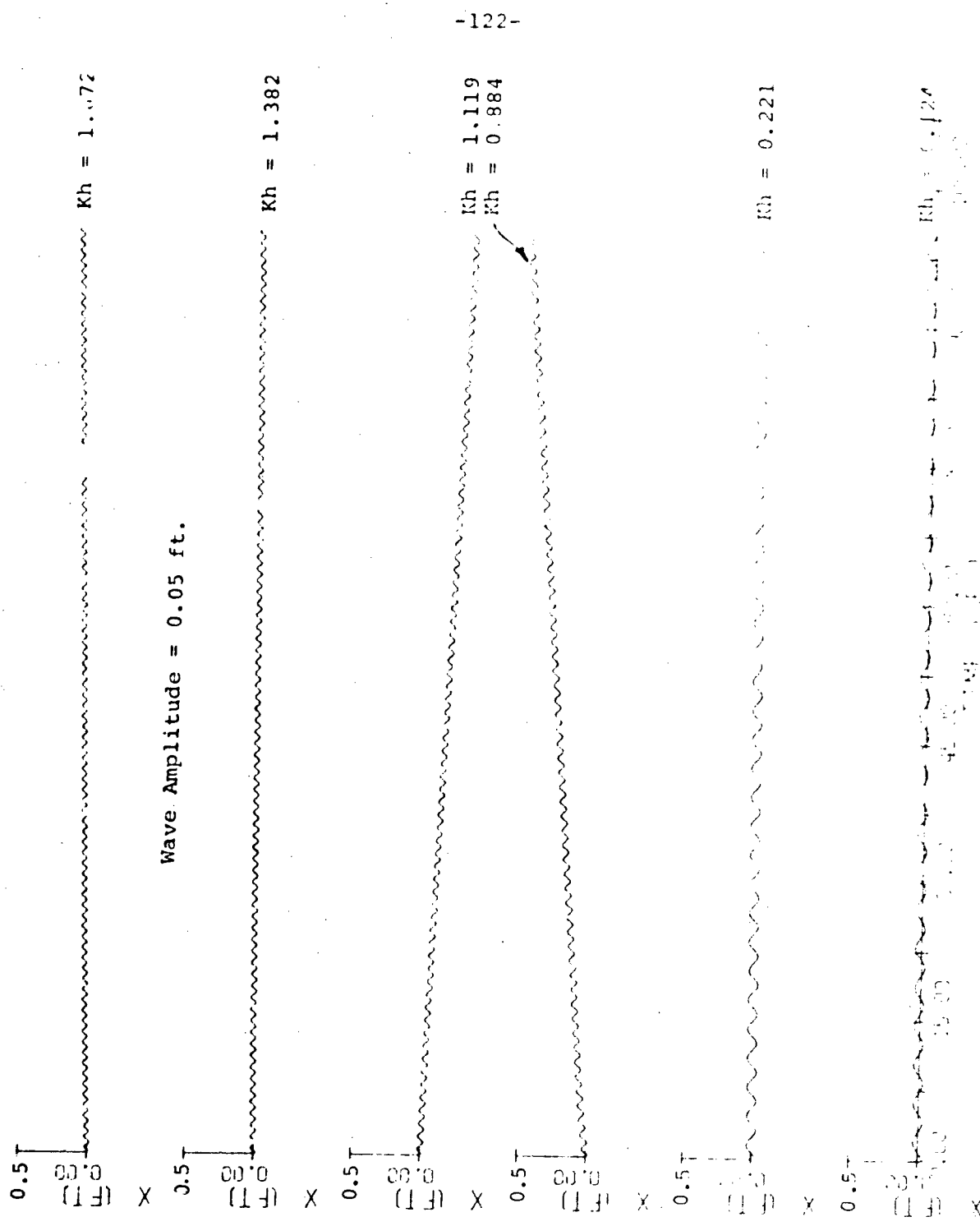


Figure 5.13 Surge Motions of the Freely Floating Cylindrical Spar Buoy

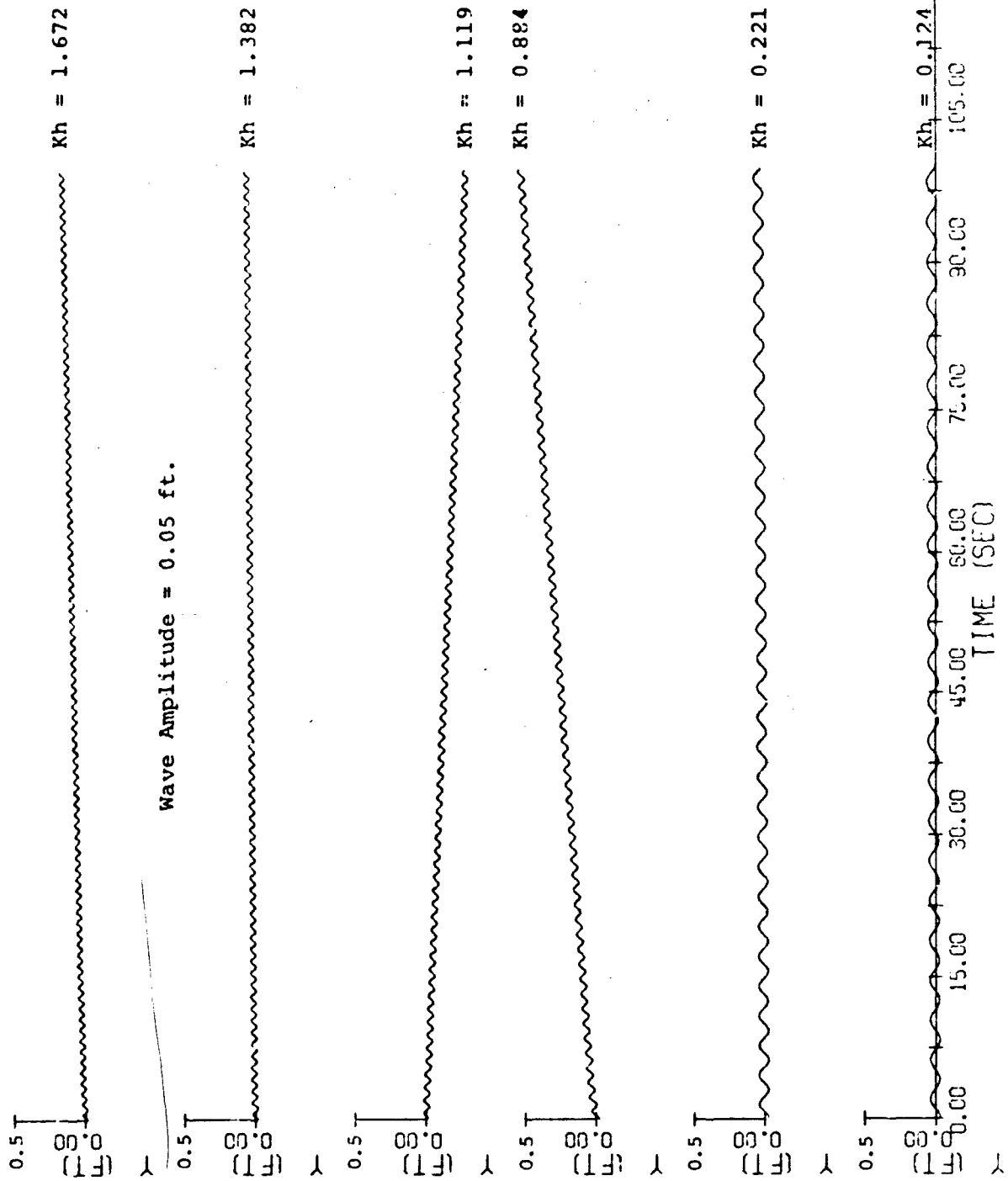


Figure 5.19 Sway Motions of the Freely Floating Cylindrical Spar Buoy

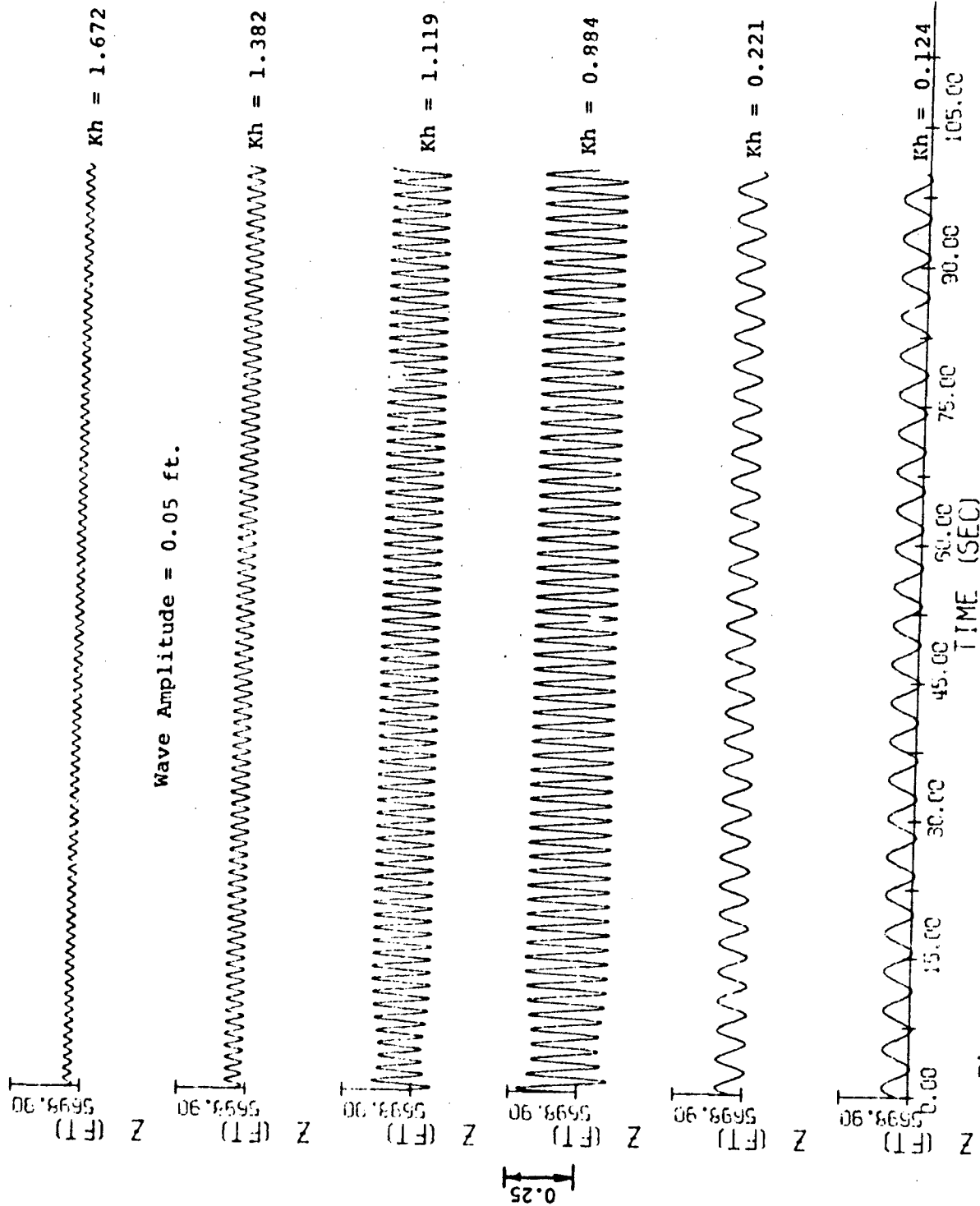


Figure 5.20 Heave Motions of the Freely Floating Cylindrical Spar Buoy

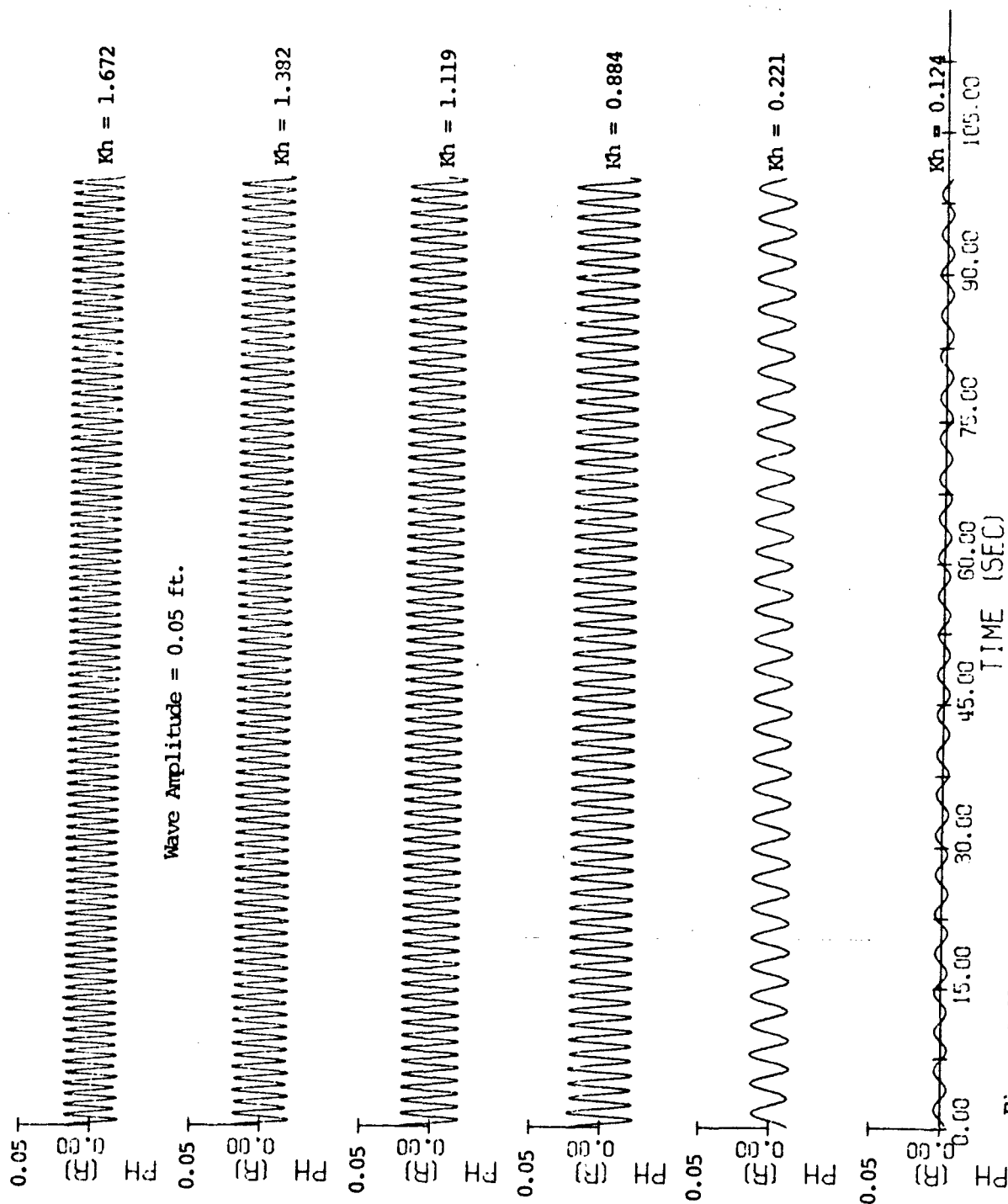


Figure 5.21 Roll Motions of the Freely Floating Cylindrical Spar Buoy

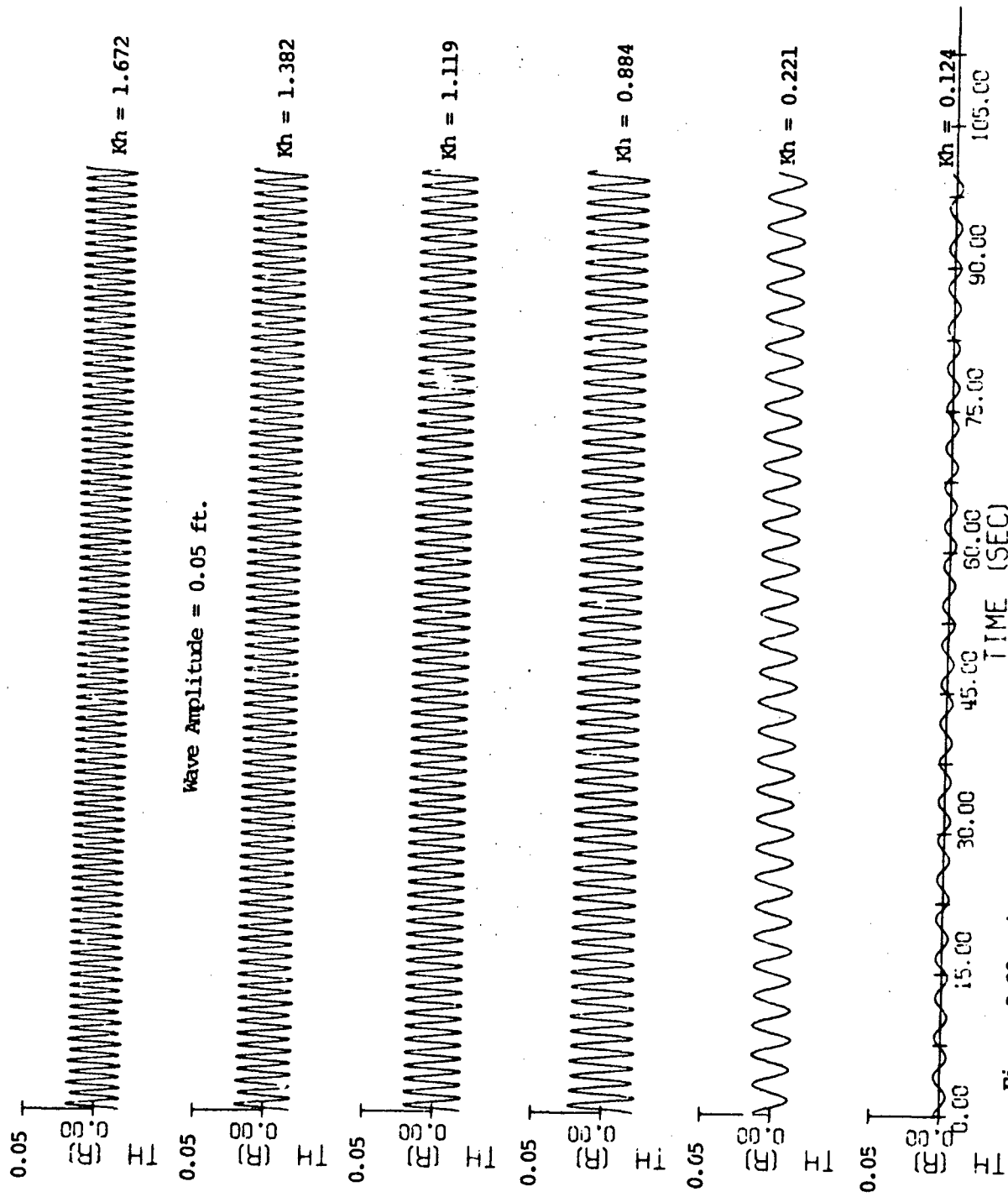


Figure 5.22 Pitch Motions of the Freely Floating Cylindrical Spar Buoy

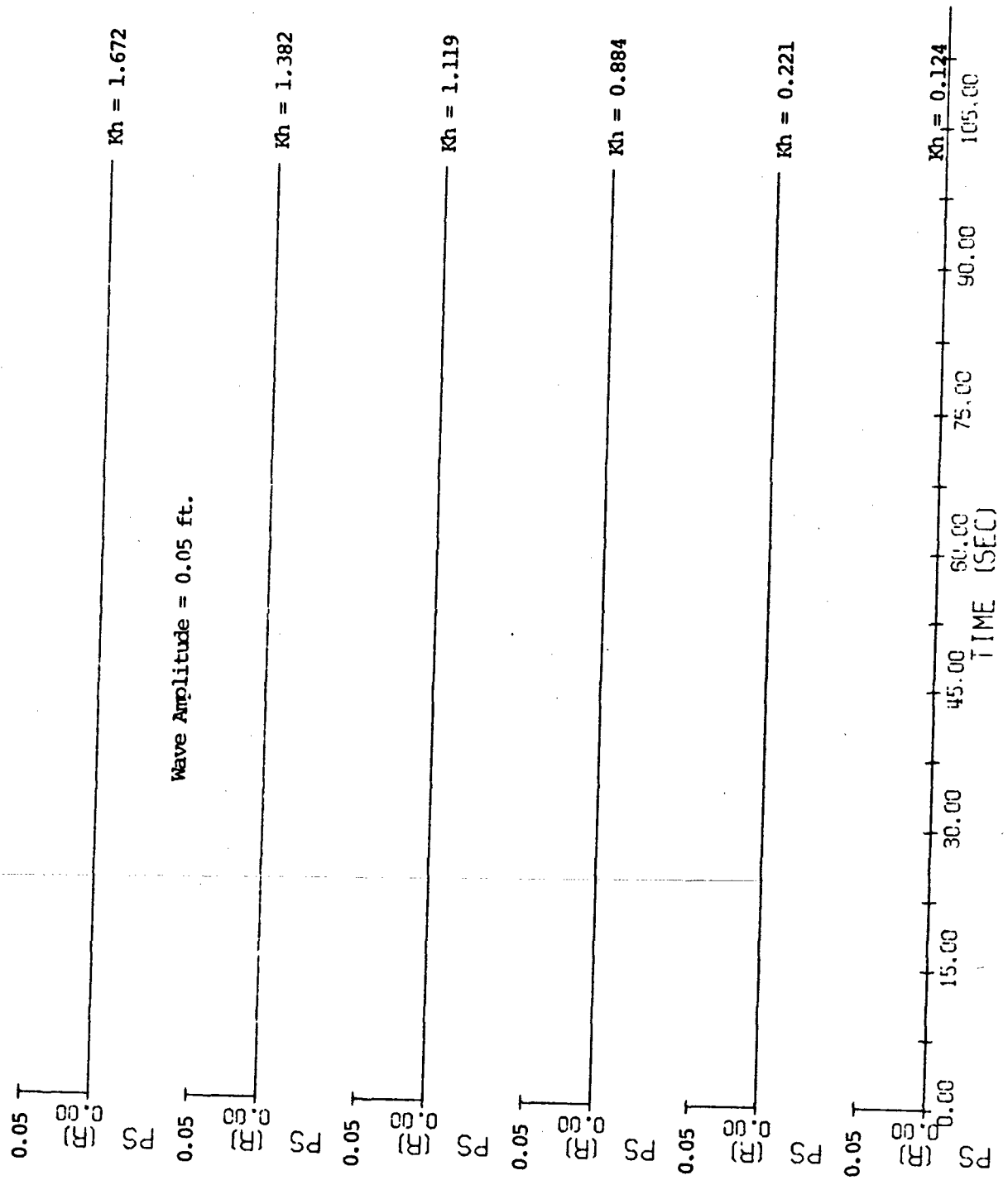
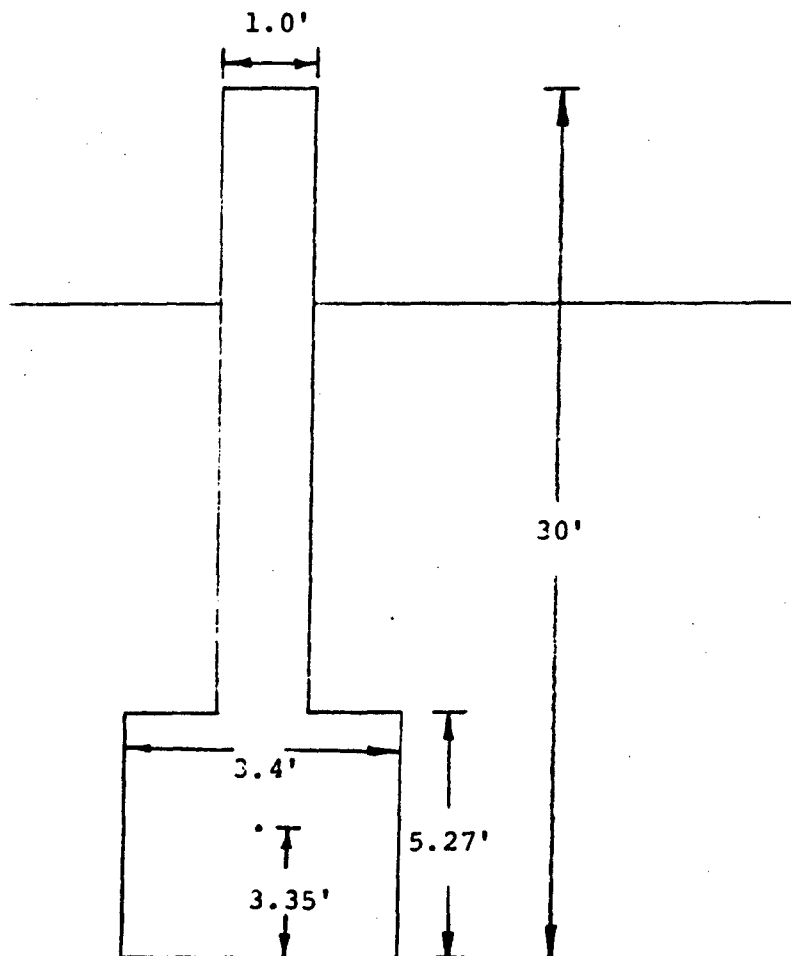


Figure 5.23 Yaw Motions of the Freely Floating Cylindrical Spar Buoy



Buoy Radius of Gyration = 5.11'

Total Weight of the Buoy = 3745.62 pounds

Figure 5.24 Tuned Spar Buoy Description

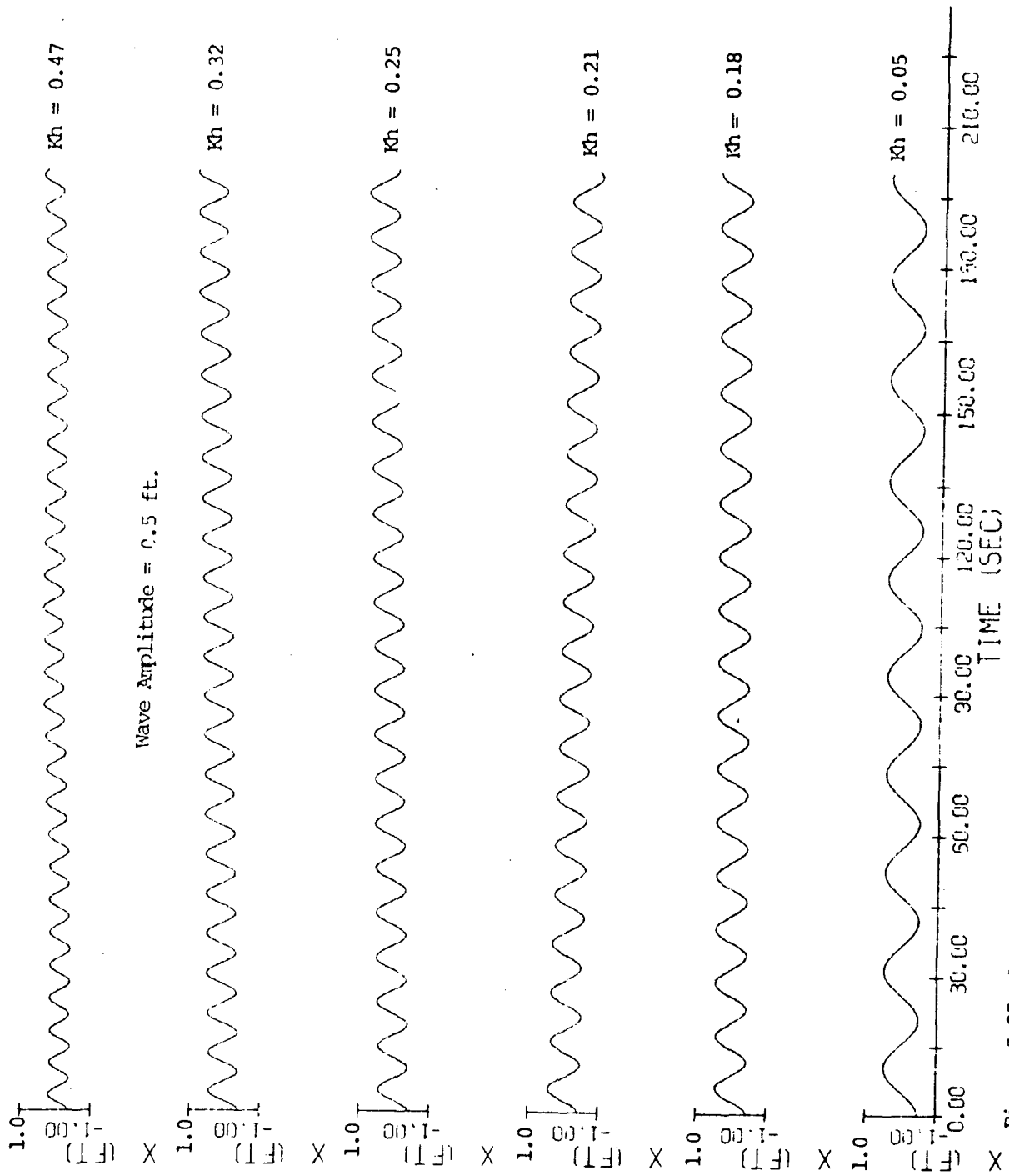


Figure 5.25 Surge Motions of the Freely Floating Tuned Spar Buoy

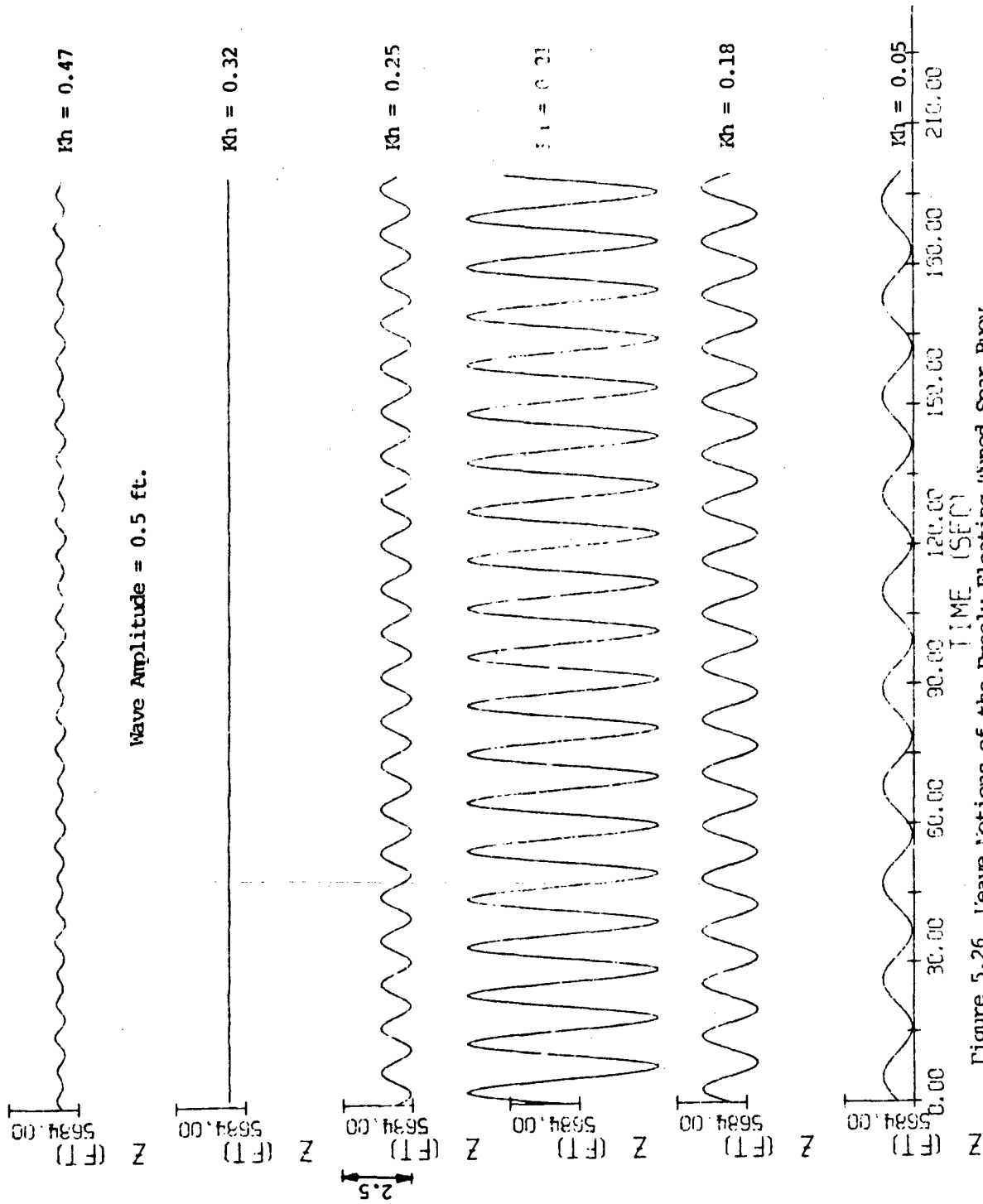


Figure 5.26 Wave Motions of the Freely Floating Tuned Spar Buoy

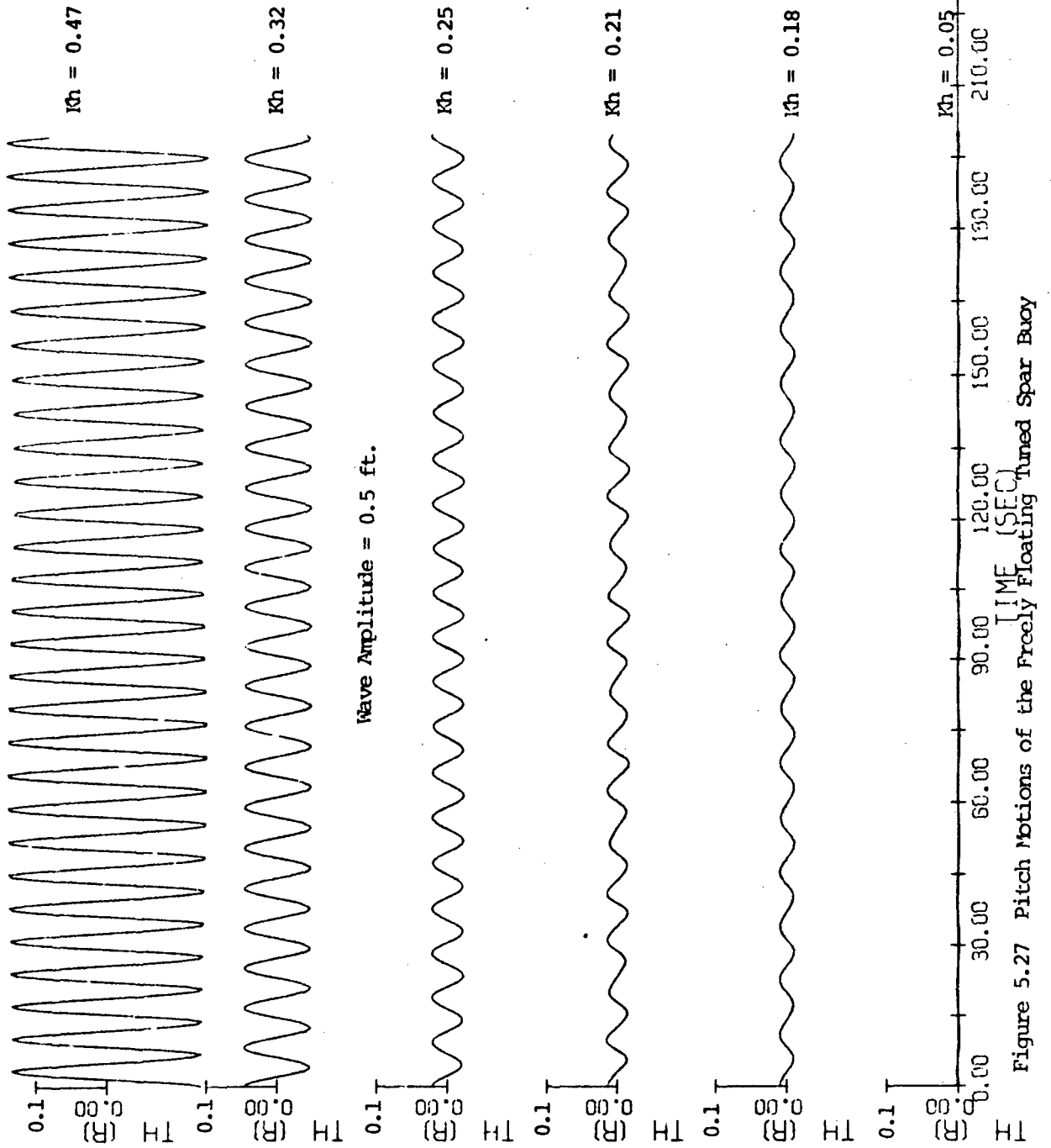


Figure 5.27 Pitch Motions of the Freely Floating Tuned Spar Buoy

SD2T.DATA										
	1	1	0	25	6	0	5700.	0.020	200.0	0.5
00010	1.7	0.5	0.5	3.35	0.0	5.11	30.	0.0	0.0	5.27
00020	1.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	
00021	3745.62	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
00055	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
00090	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-18.	
00105	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0		
00110	0.0									
00120	0.30		0.5							
00130	0.55		0.5							
00140	0.60		0.5							
00150	0.66		0.5							
00160	0.74		0.5							
00170	0.90		0.5							
READY										

Table 5.4 Input Data for the 2-D Analysis of the Tuned Spar Buoy

viscous and pressure drag coefficients equal zero. Figure 5.28 through 5.30 use the same data as Table 5.4 except that viscous and pressure drag coefficients are equal to 1.0. Once again comparisons between the two sets of figures (5.25 through 5.30) show similar differences as explained for the cylindrical spar buoy. In this case the heave resonance occurs at $K_h = 0.22$ and the pitch resonance at $K_h = 0.4$. Also in this case the viscous damping in heave and pitch responses is more pronounced.

Response amplitudes of surge, heave, and pitch; and surge drift as calculated from the steady-state portions of the time histories presented in Figures 5.25 through 5.30, and other similar simulations are tabulated in Table 5.5 which is similar to Table 5.2. In this case the wave amplitudes ξ_0 studied are 0.5 ft. and 2.5 ft. respectively. Data points of Table 5.5 are plotted in Figures 5.31 through 5.34 to presents plots of RAO vs K_h . These plots are similar to Figures 5.8 through 5.11. Figure 5.33b is an expanded view of Figure 5.33a, near the heave resonance frequency. The ratio between buoy heave and the wave amplitude is one at zero frequency and exhibits resoance at K_h of approximately 0.2. The ratio decreases to zero at K_h equal to 0.32, after which it increases a little before decreasing to zero with increasing frequencies. Such behavior, where the heave response is minimized at a given frequency is the main function of a tuned (as opposed to cylindrical) spar buoy.

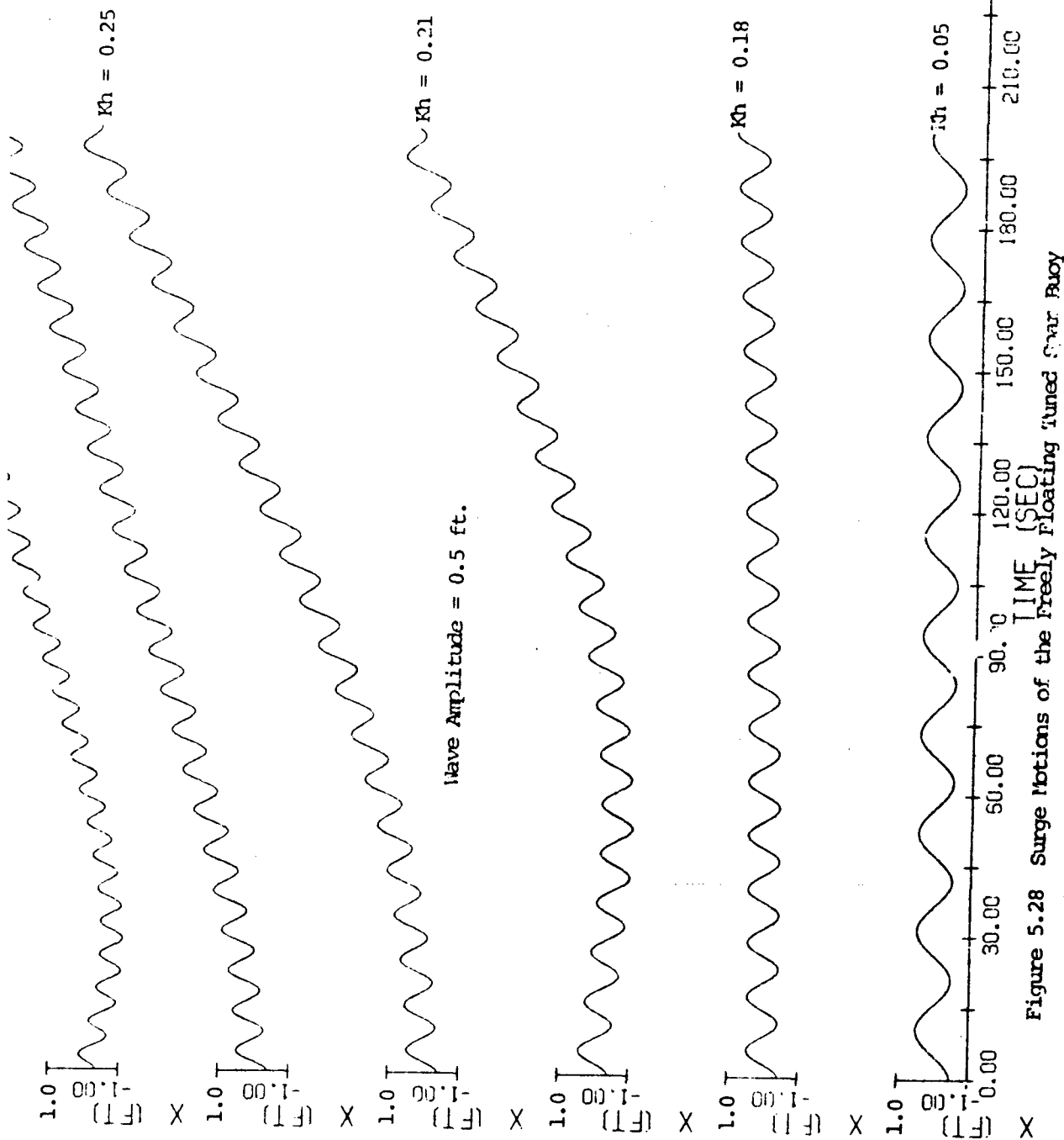


Figure 5.28 Surge Motions of the Freely Floating Tuned Spar Buoy

Figure 5.29 Heave Motion of the Freely Floating Tuned Spar Buoy

Figure 5.30 Pitch: Rotations of the Freely Floating Tuned Spar Buoy

ω	Kh	Surge Amplitude/ ξ_o				Heave Amplitude/ ϵ_o				Pitch Amplitude/ k_o				Surge Drift/ kw_o^2			
		1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
0.30	0.05	-0.96	0.96	0.96	0.96	1.05	1.05	1.05	1.05	1.21	1.23	1.23	1.36	1.38	1.38	1.41	0.48
0.55	0.18	-0.87	0.88	0.88	0.88	1.99	1.99	1.99	1.94	2.04	2.16	2.04	2.02	0.51	0.51	4.97	2.80
0.57	0.19	-0.87		0.87	0.88	2.58			2.27	1.51	2.18	2.17	2.17			14.90	4.13
0.59	0.20	-0.86		0.85		4.20			2.53		2.34	2.33				33.40	
0.60	0.21	-0.85	0.86	0.82	0.85	6.87	6.87		2.50	1.46	2.43	2.77	2.44	-1.08	-1.08	41.66	5.31
0.63	0.23	-0.84		0.78		-4.15			1.87		2.75	2.77				48.93	
0.66	0.25	-0.82	0.84	0.81	0.81	-1.07	1.07		1.01	0.86	3.10	3.26	2.94	1.49	1.49	31.62	5.50
0.70	0.29	-0.81		0.82		-0.29			0.34		4.03	3.96				17.24	
0.74	0.32	-0.79	0.82	0.81	0.77	-0.02	0.02		0.11	0.40	5.51	5.51	3.52	1.04	1.04	14.36	3.96
0.83	0.40	-0.74		0.67		0.21			0.24		35.79	7.81				15.61	
0.90	0.47	-0.70	0.55	0.60	0.60	0.27	0.36		0.28	0.31	-10.62	11.34	3.20	0.97	0.97	7.90	1.73
1.01	0.59	-0.64		0.58		0.30			0.30		-3.43	3.26				1.82	
1.17	0.80	-0.55		0.51	0.51	0.28			0.28	0.20	-1.69	1.67	1.55			0.50	0.42
1.61	1.50	-0.34		0.30	0.30	0.17			0.17	0.17	-0.62	0.62	0.65			0.20	0.17

Table 5.5 Dynamic Response (Surge, Heave and Pitch Amplitude; and Surge Drift) of a Tuned Spar Buoy to Waves

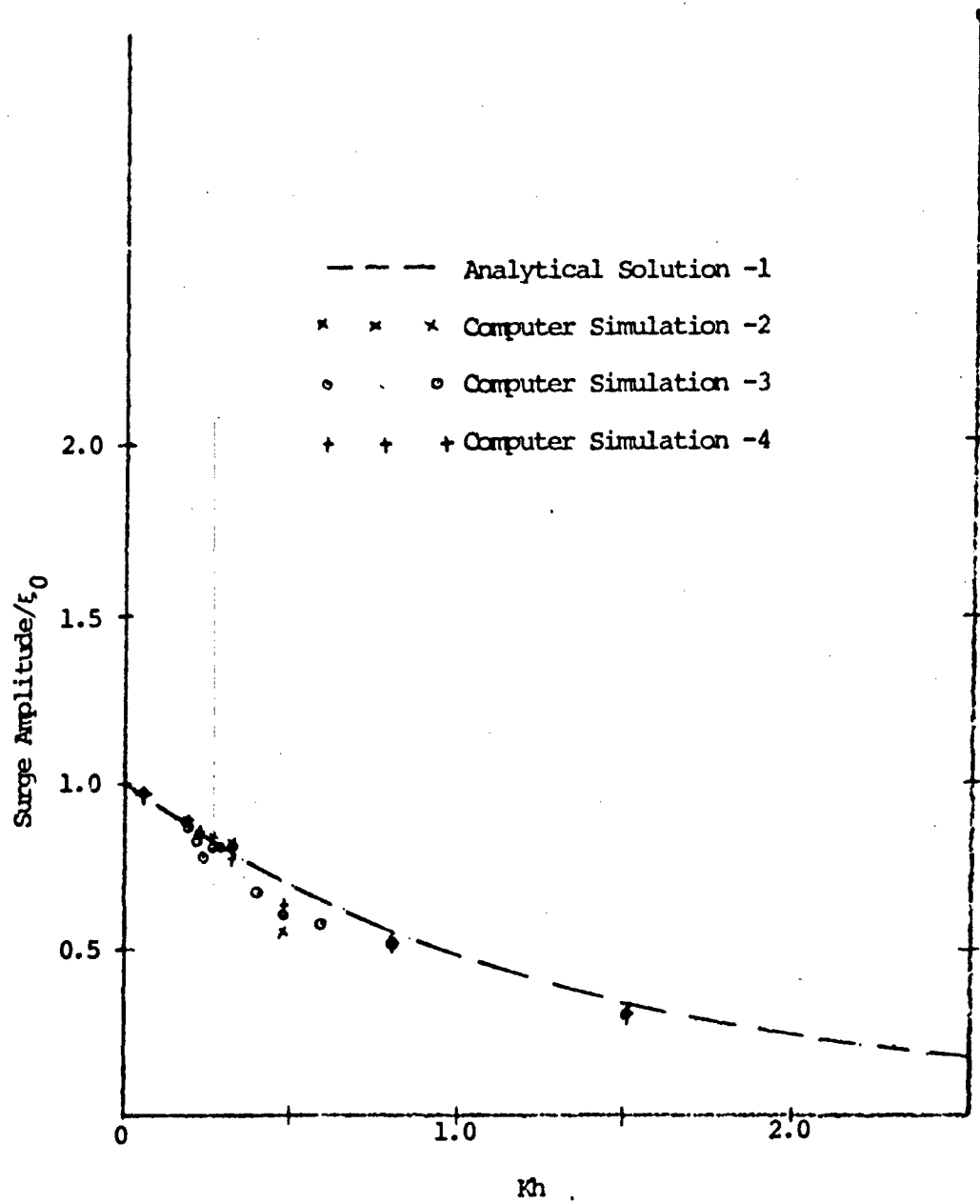


Figure 5.31 Surge Magnification Vs Kh

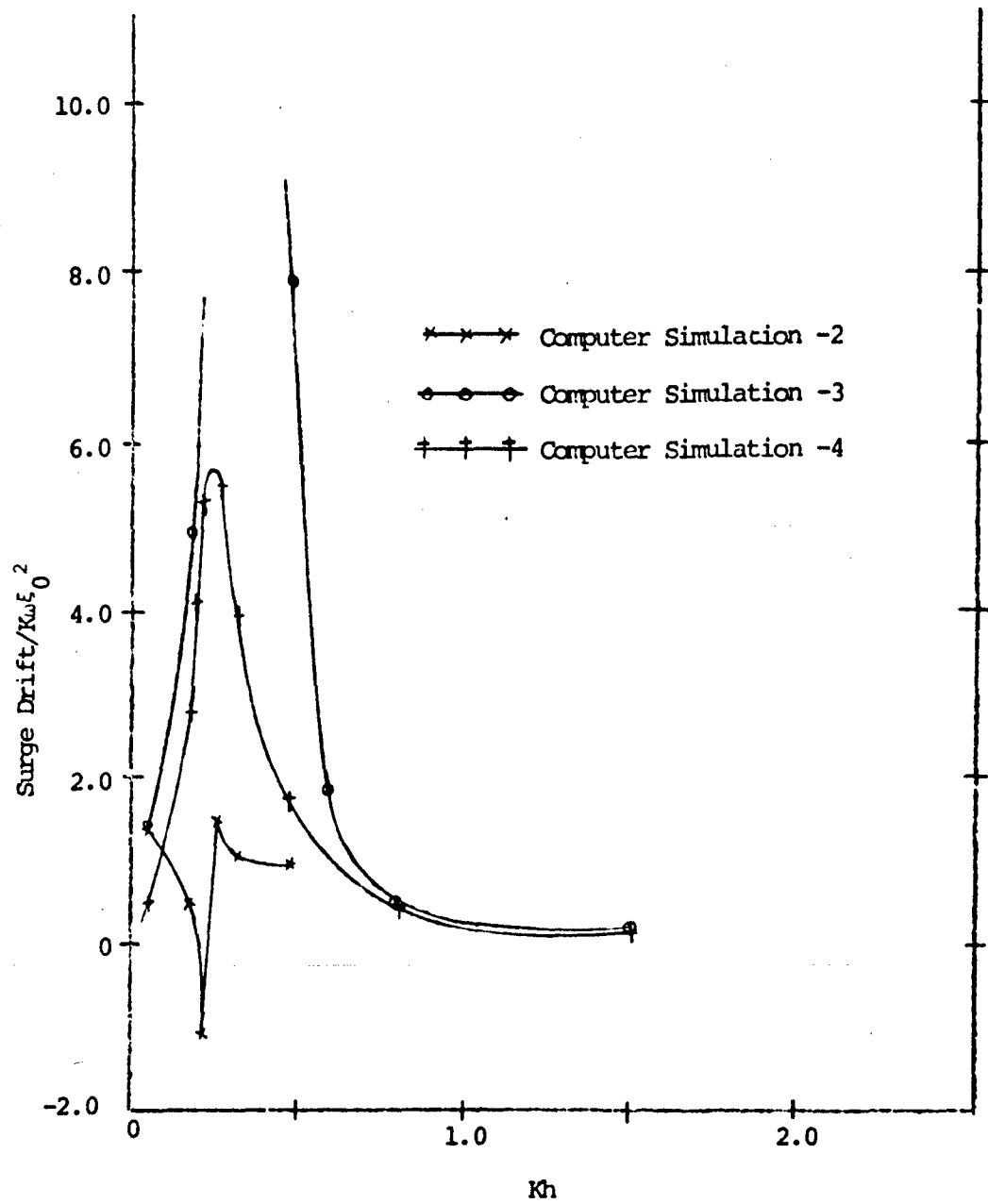


Figure 5.32 Surge Drift Vs Kh

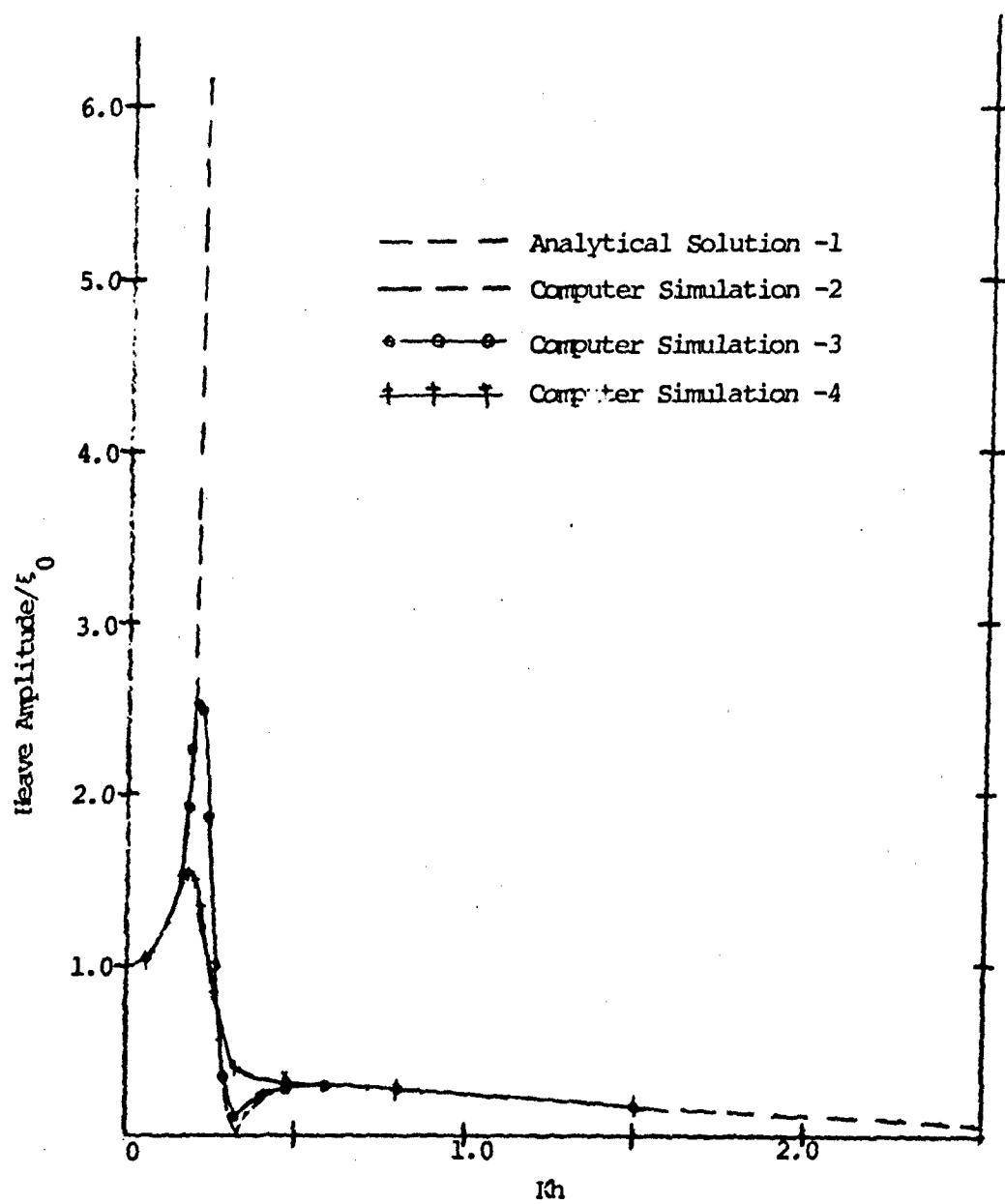


Figure 5.33a Heave Magnification Vs Kh

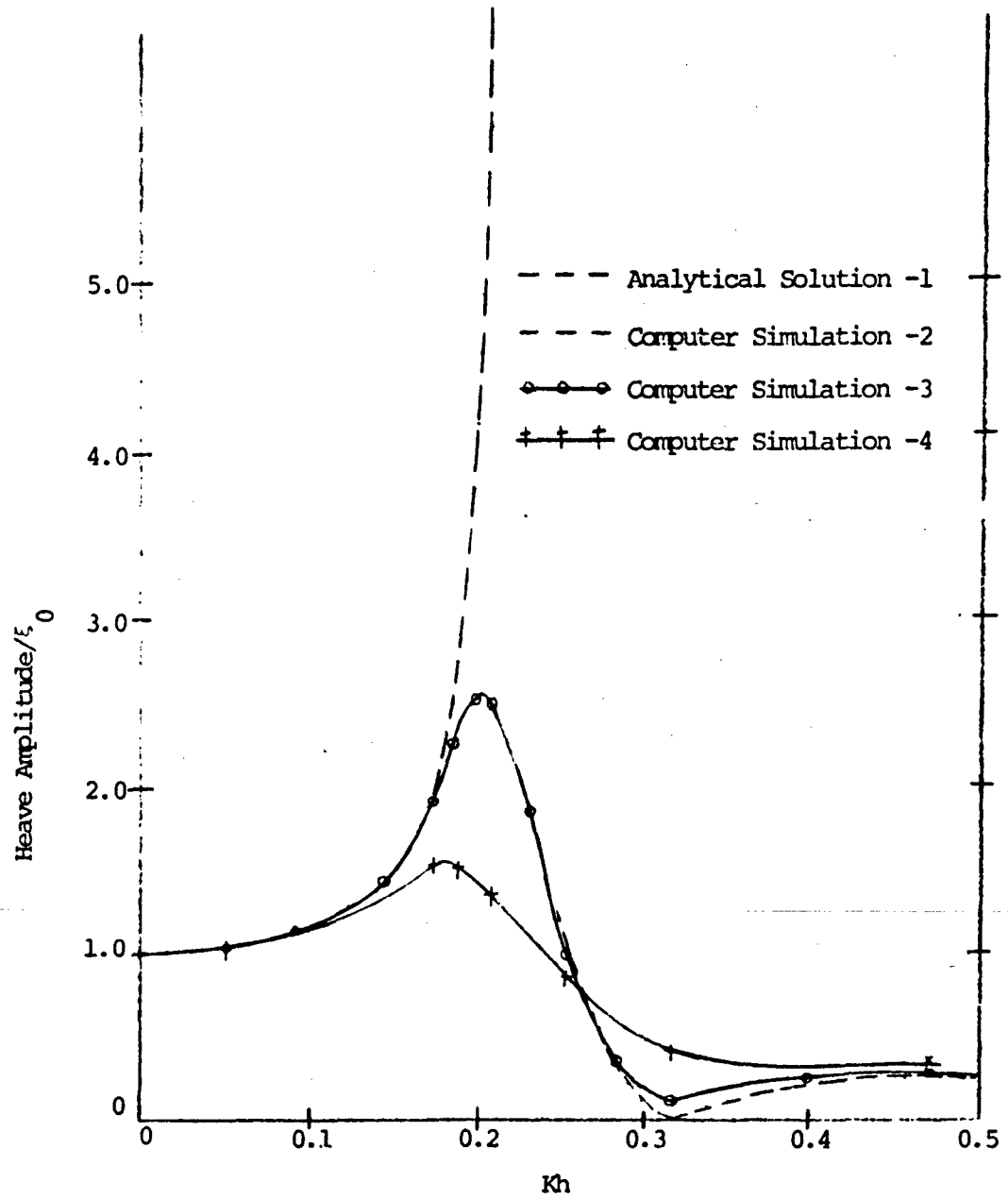


Figure 5.33b Expanded View of Figure 5.33a

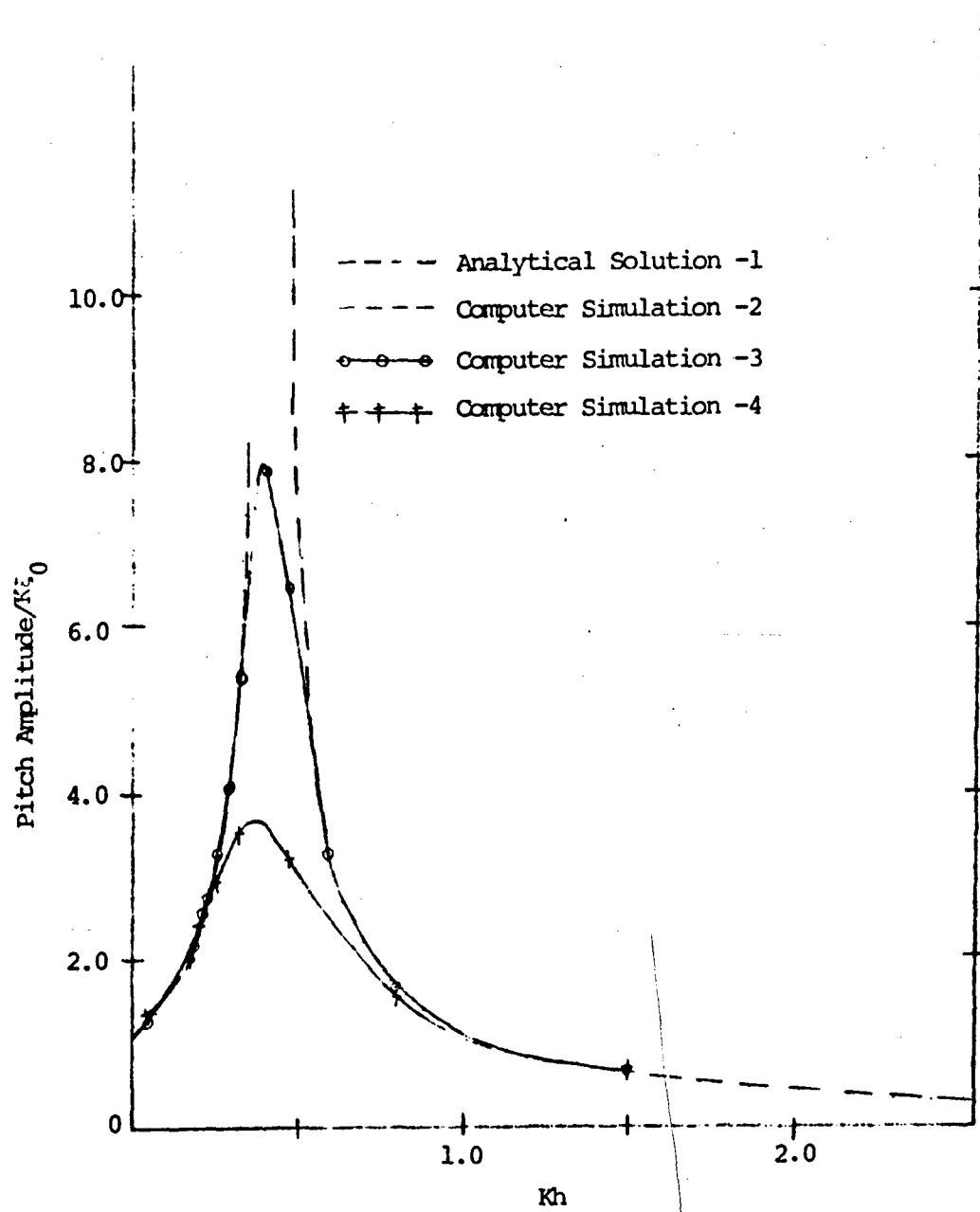


Figure 5.34 Pitch Magnification Vs Kh

Three-Dimensional Simulations

Figures 5.35 through 5.40 present the surge, sway, heave, roll, pitch, and yaw responses of the buoy to surface waves of the same amplitude and frequencies as in Figures 5.28 to 5.30. Table 5.6 is the input data used in conjunction with the computer program SD3.FORT to obtain these simulations. Here β equals zero. Differences between Figure 5.28 and 5.35 can be attributed directly to the assumptions of three-dimensional analysis. Simulations for the same data as Table 5.6 and $\beta = \pi/4$ are presented in Figures 5.41 through 5.46.

5.2 Subsurface Moored System

Three dimensional computer programs (S3S3.FORT, SSD3.FORT and SSD31.FORT) described in Section 4.2 are used for these simulations. Mooring system used as a case study for these simulations is shown in Figure 5.47. This mooring system was actually deployed during the MODE experiment and was called mooring No. 1 (station 481). Figures 5.48 and 5.49 display the three coordinates (x, y, z) of the top of this mooring line in response to the relative velocity data displayed in Figures 5.50 and 5.51. This simulation is a part of the study (CHHABRA, 1976), where the current record from the topmost vector-averaging

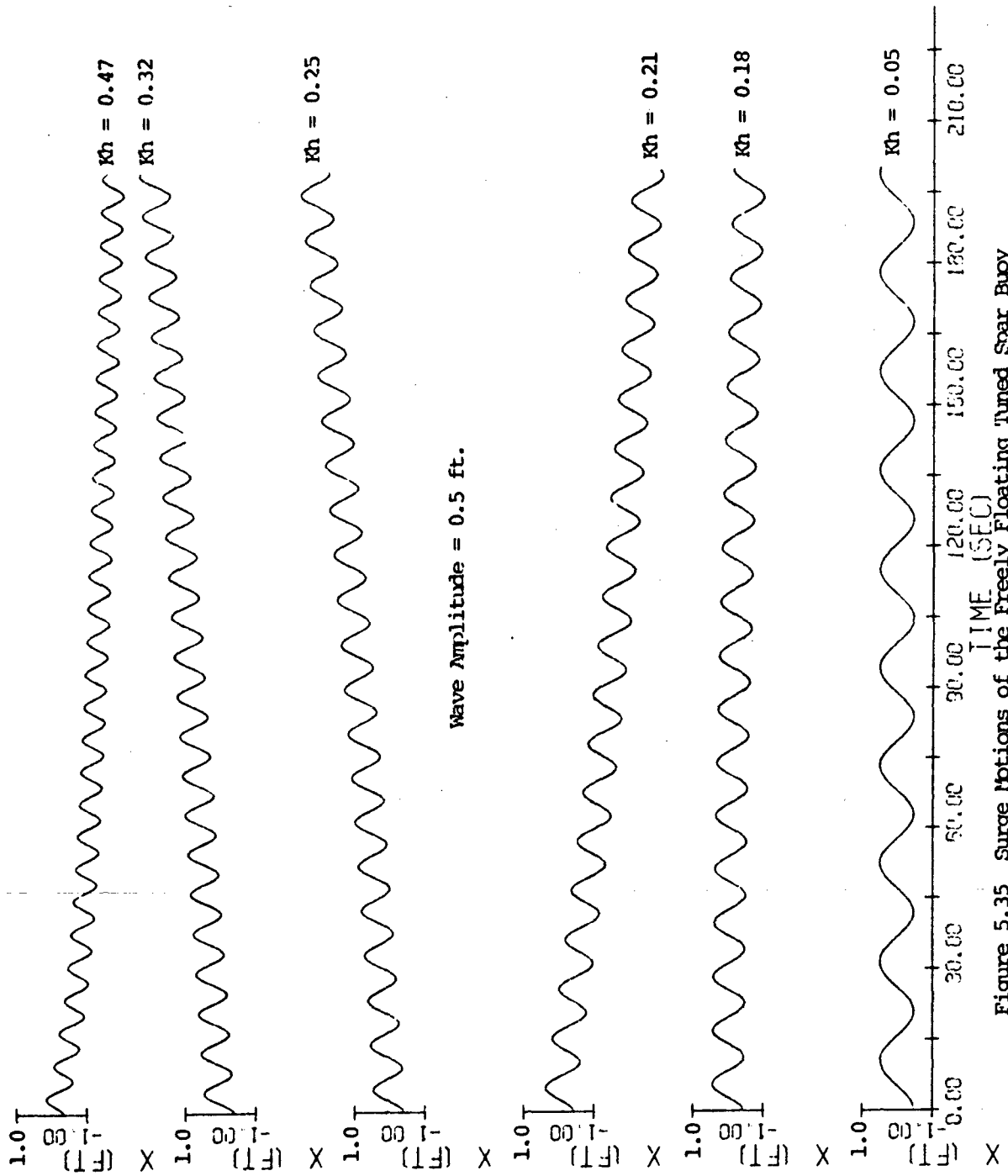


Figure 5.35 Surge Motions of the Freely Floating Tuned Spar Buoy

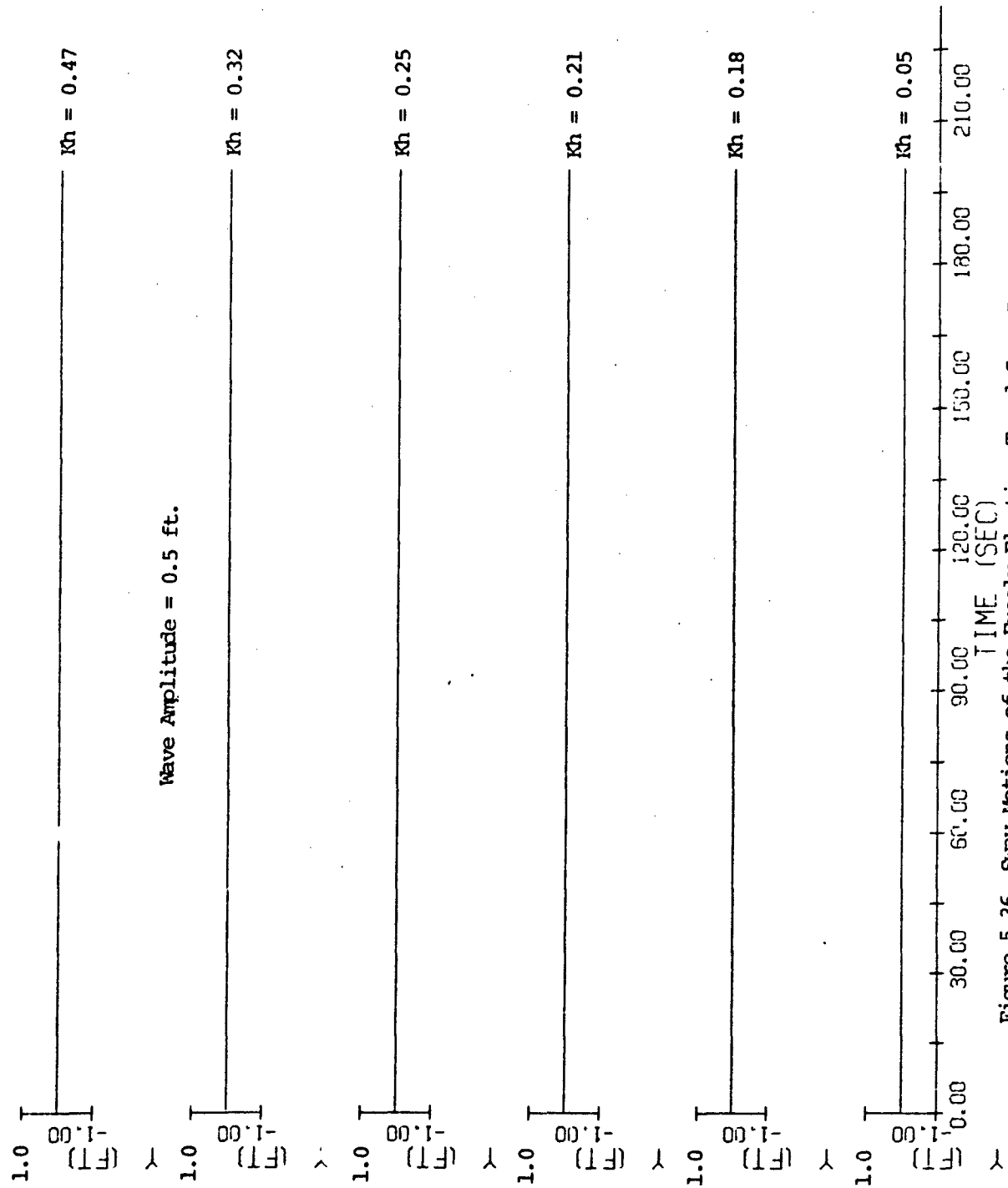


Figure 5.36 Sway Motions of the Freely Floating Tuned Spar Buoy

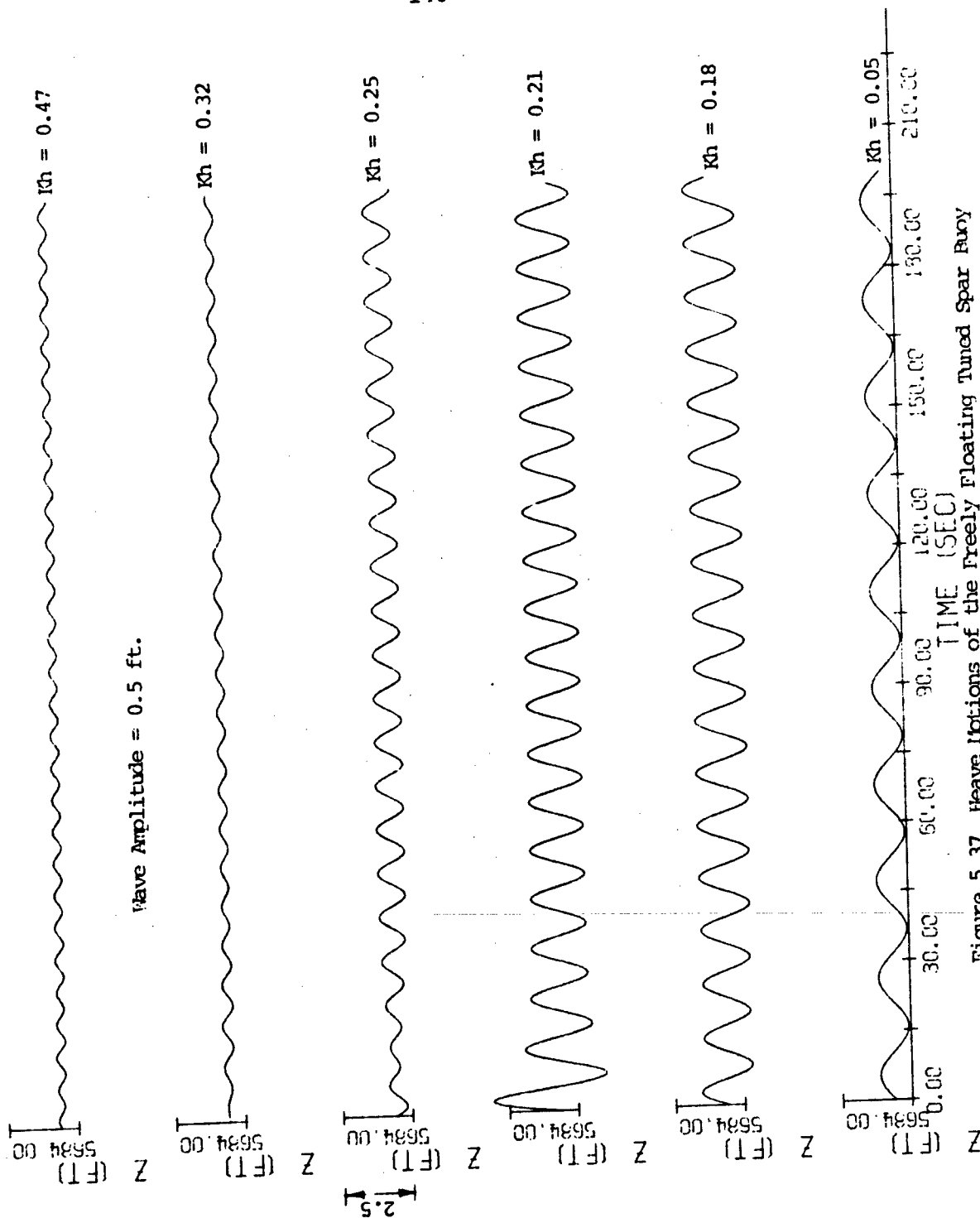


Figure 5.37 Heave Motions of the Freely Floating Tuned Spar Buoy

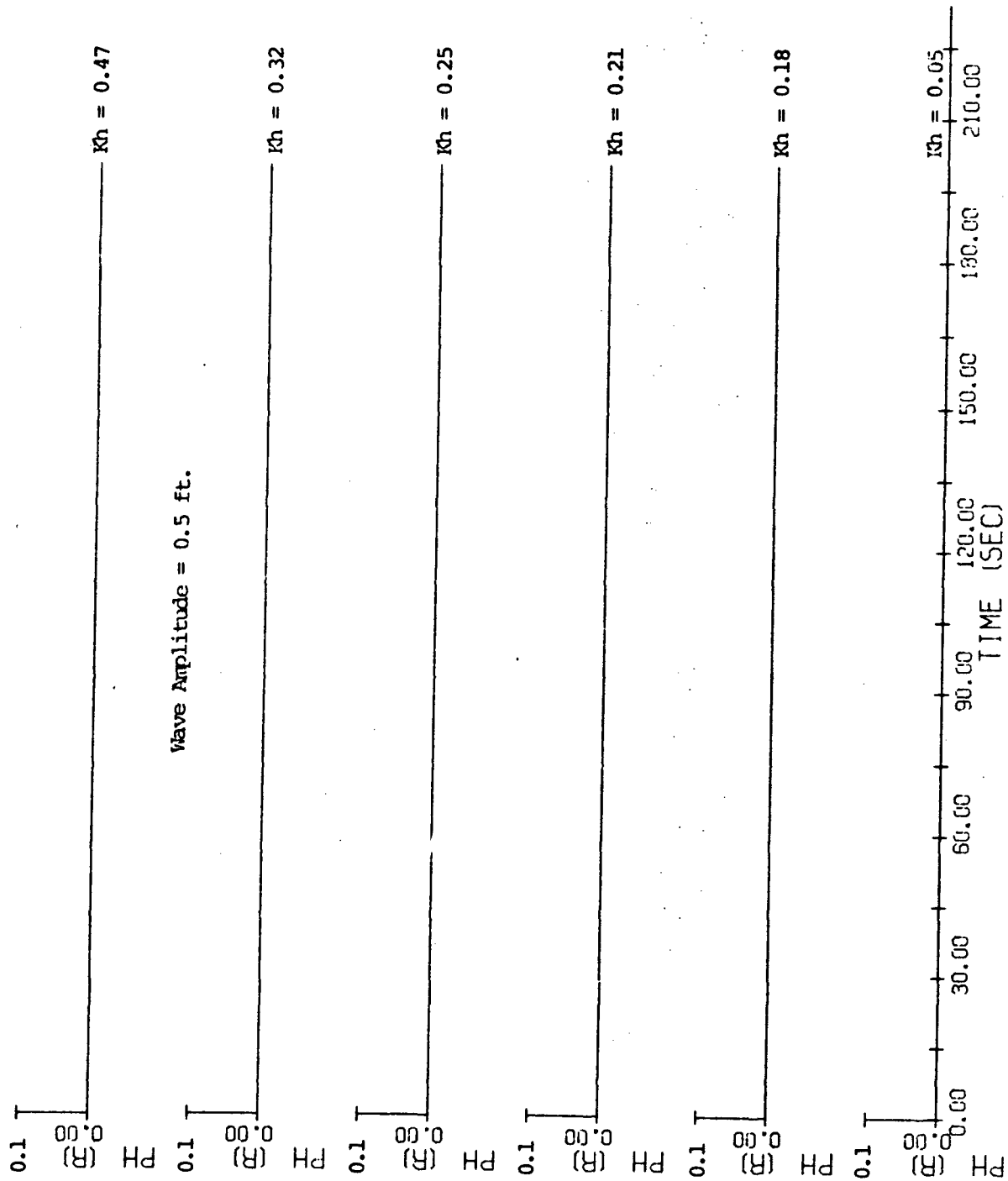
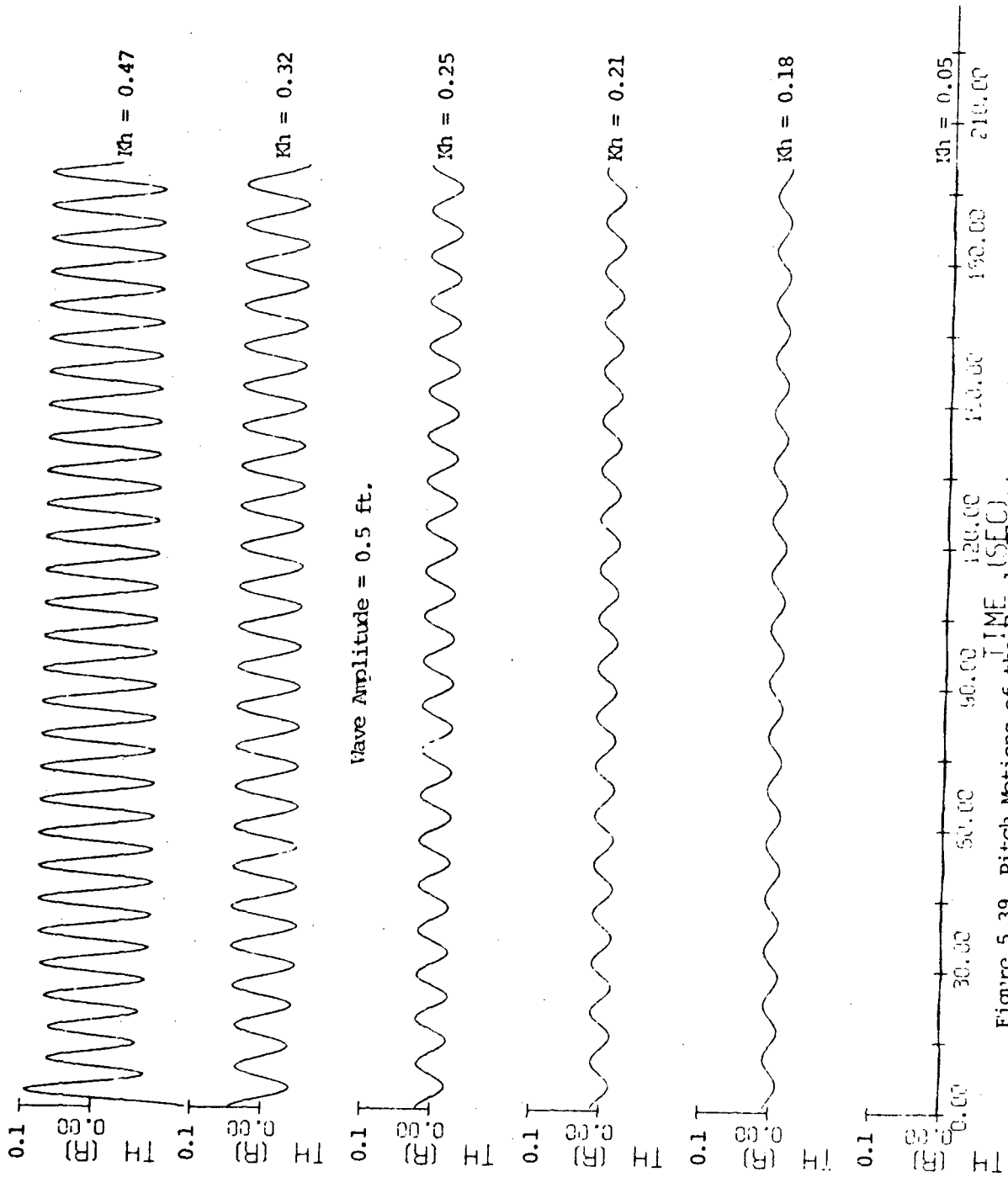


Figure 5.38 Roll Motions of the Freely Floating Tuned Spar Buoy



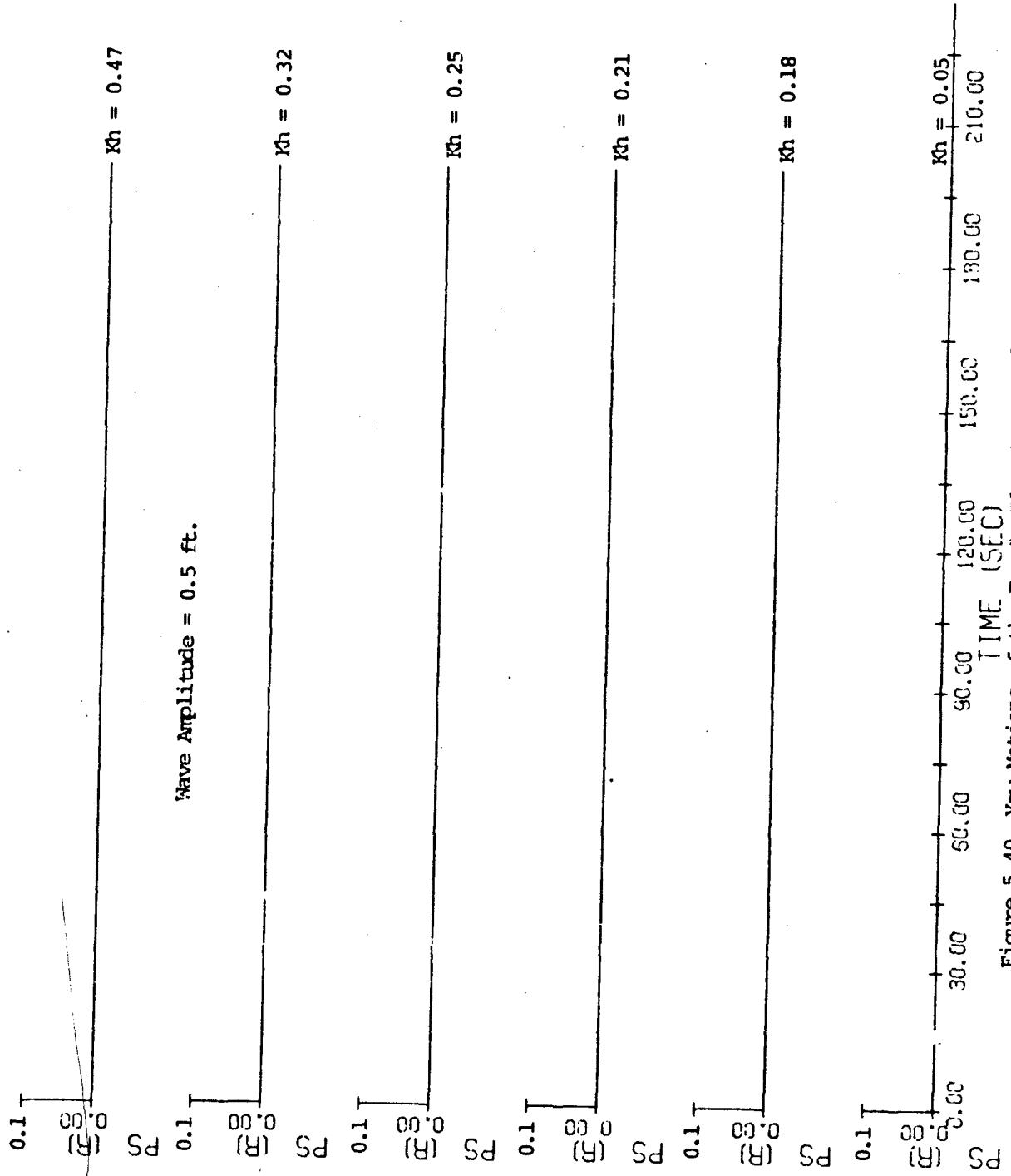


Figure 5.40 Yaw Motions of the Freely Floating Tuned Spar Buoy

STBUOY.DATA										
	1	1	0	25	6	0	5700.	0.020	200.0	0.5
00010	1.7	0.5		3.35			5.11	30.	1.0	5.27
00020	1.0	1.0								
00021	3745.62	0.		0.			0.	0.	0.	
00055	0.	0.		0.			0.	0.	0.	
00090	0.	0.		0.			0.	0.	0.	
00105	0.	0.		0.			0.	0.	0.	
00110	0.	0.		0.			0.	0.	0.	
00120	0.30	0.5		0.0						
00130	0.55	0.5		0.0						
00140	0.60	0.5		0.0						
00150	0.66	0.5		0.0						
00160	0.74	0.5		0.0						
00170	0.90	0.5		0.0						
READY										

Table 5.6 Input Data for the 3-D Analysis of the Tuned Spar Buoy

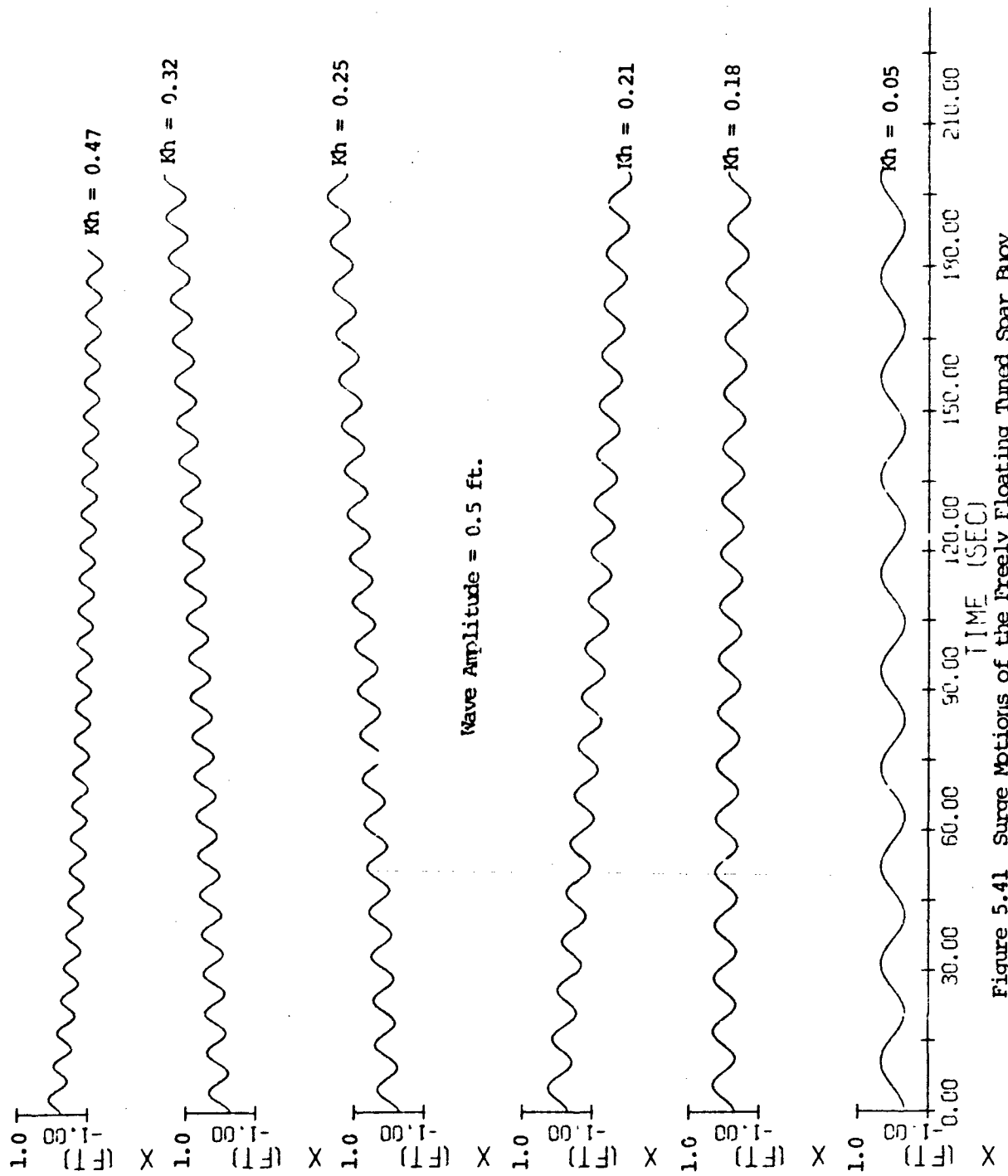


Figure 5.41 Surge Motions of the Freely Floating Tuned Spar Buoy

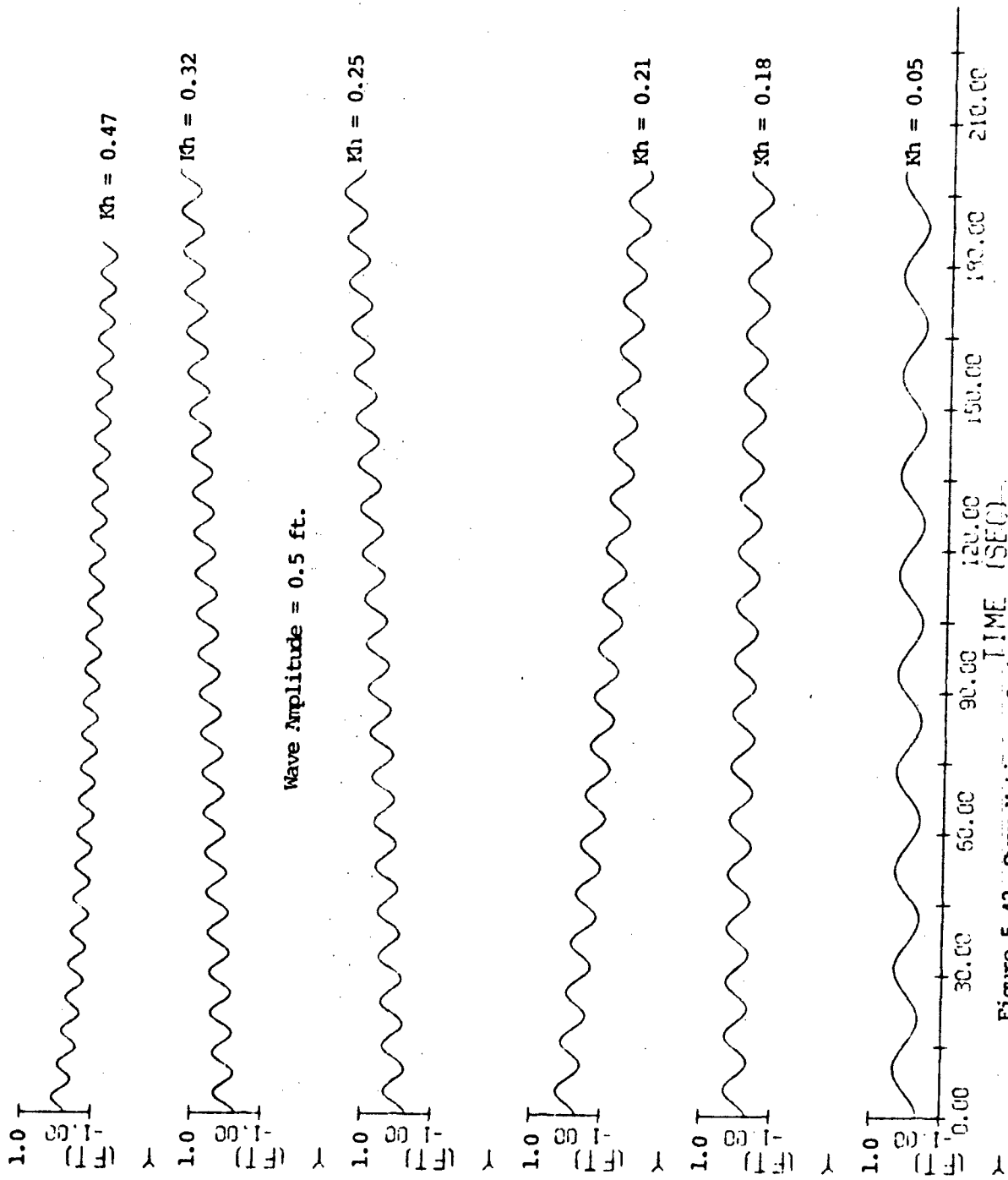


Figure 5.42 Sway motions of the Freely Floating Tuned Spar Buoy

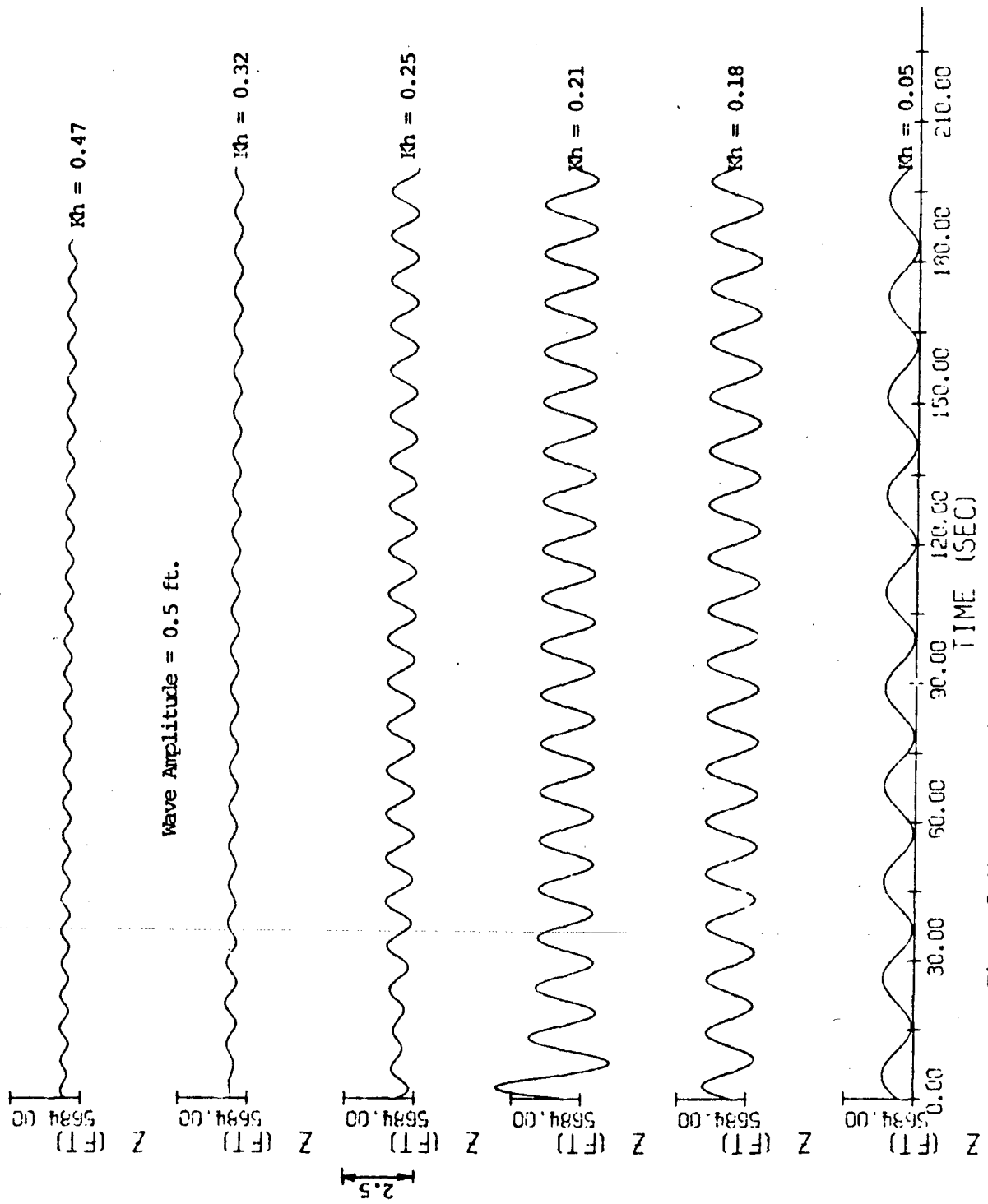


Figure 5.43 Heave motions of the Freely Floating Tuned Spar Buoy

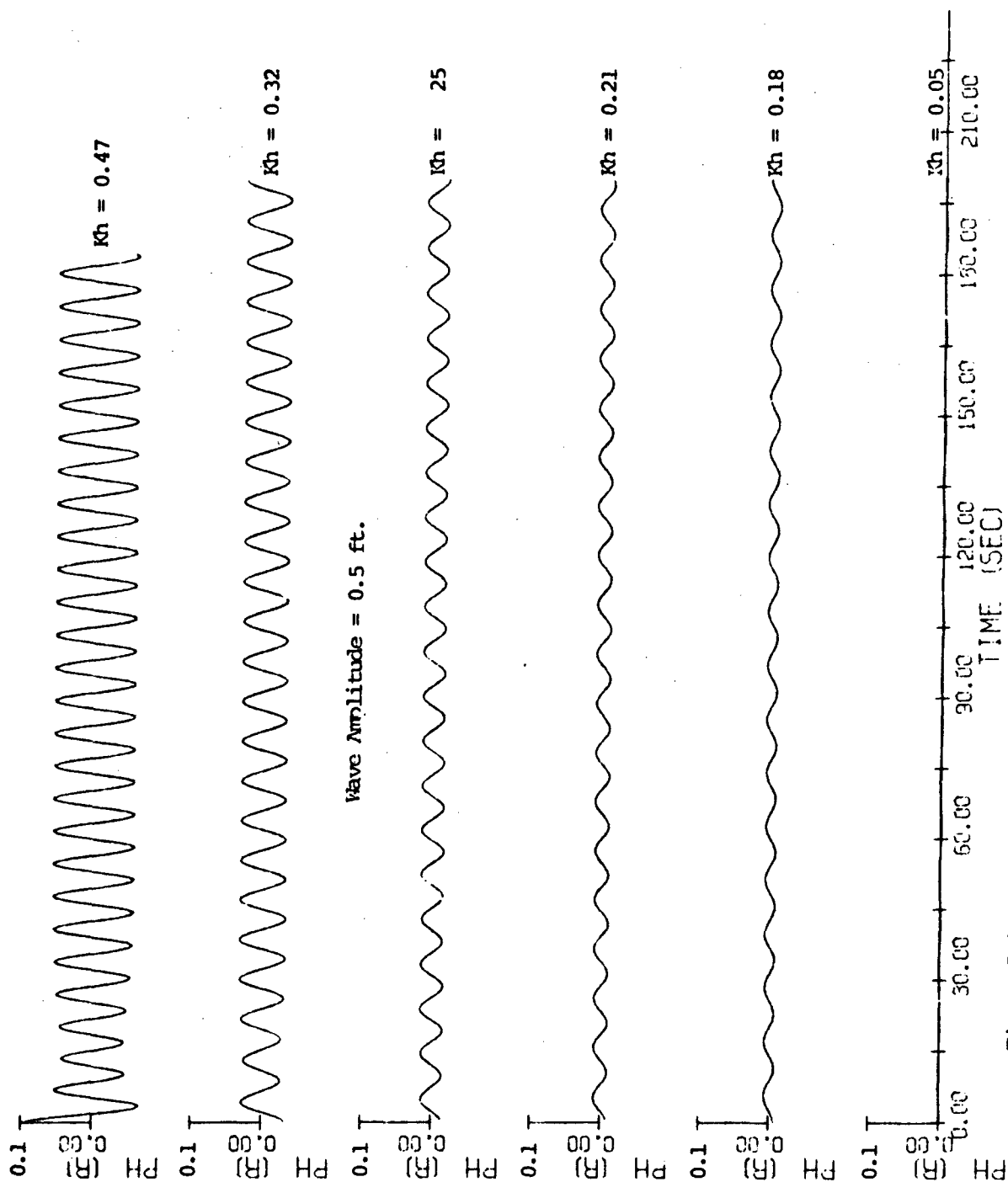


Figure 5.44 Roll motions of the Freely Floating Tuned Spar Buoy

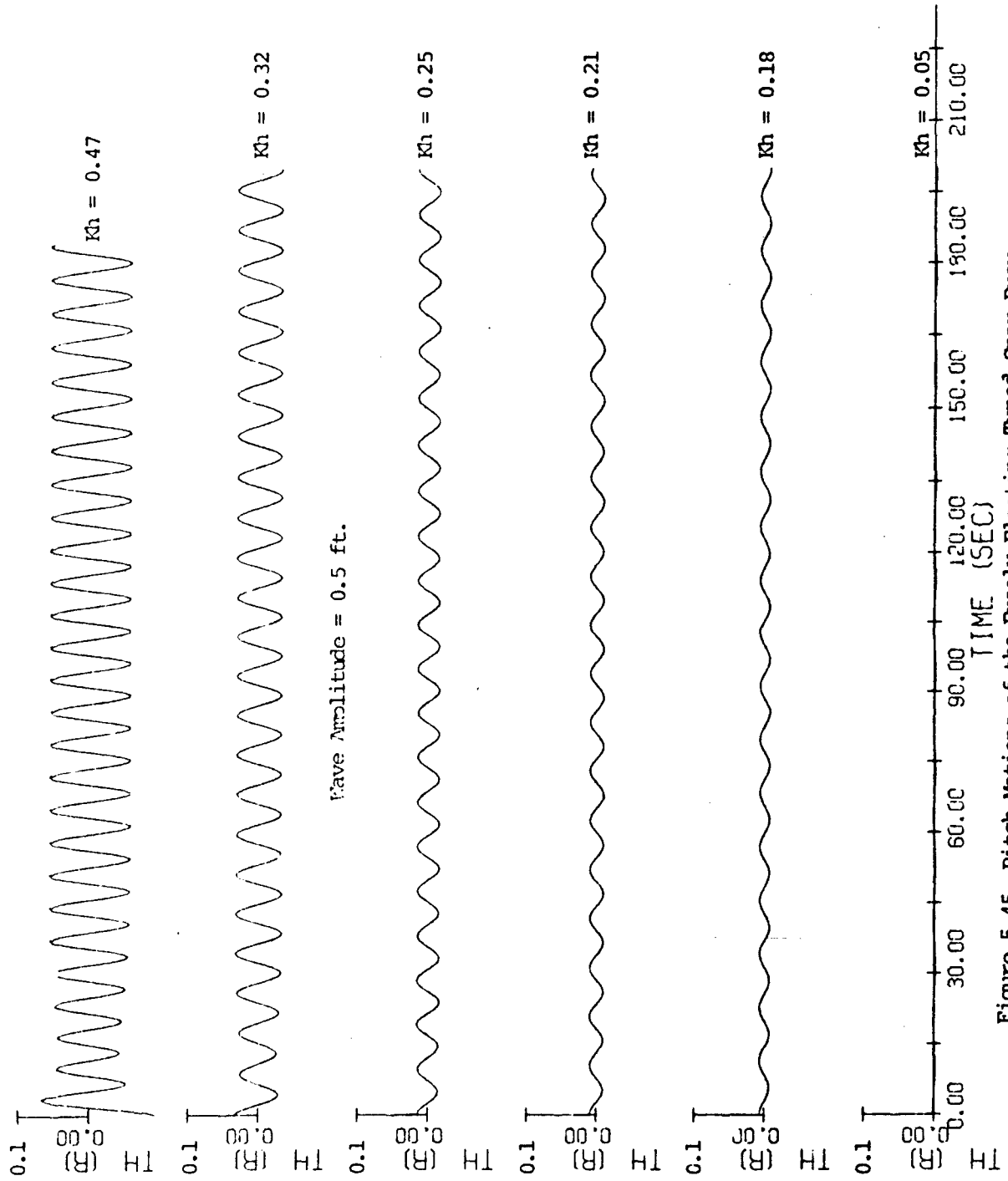


Figure 5.45 Pitch motions of the Freely Floating Tuned Spar Bury

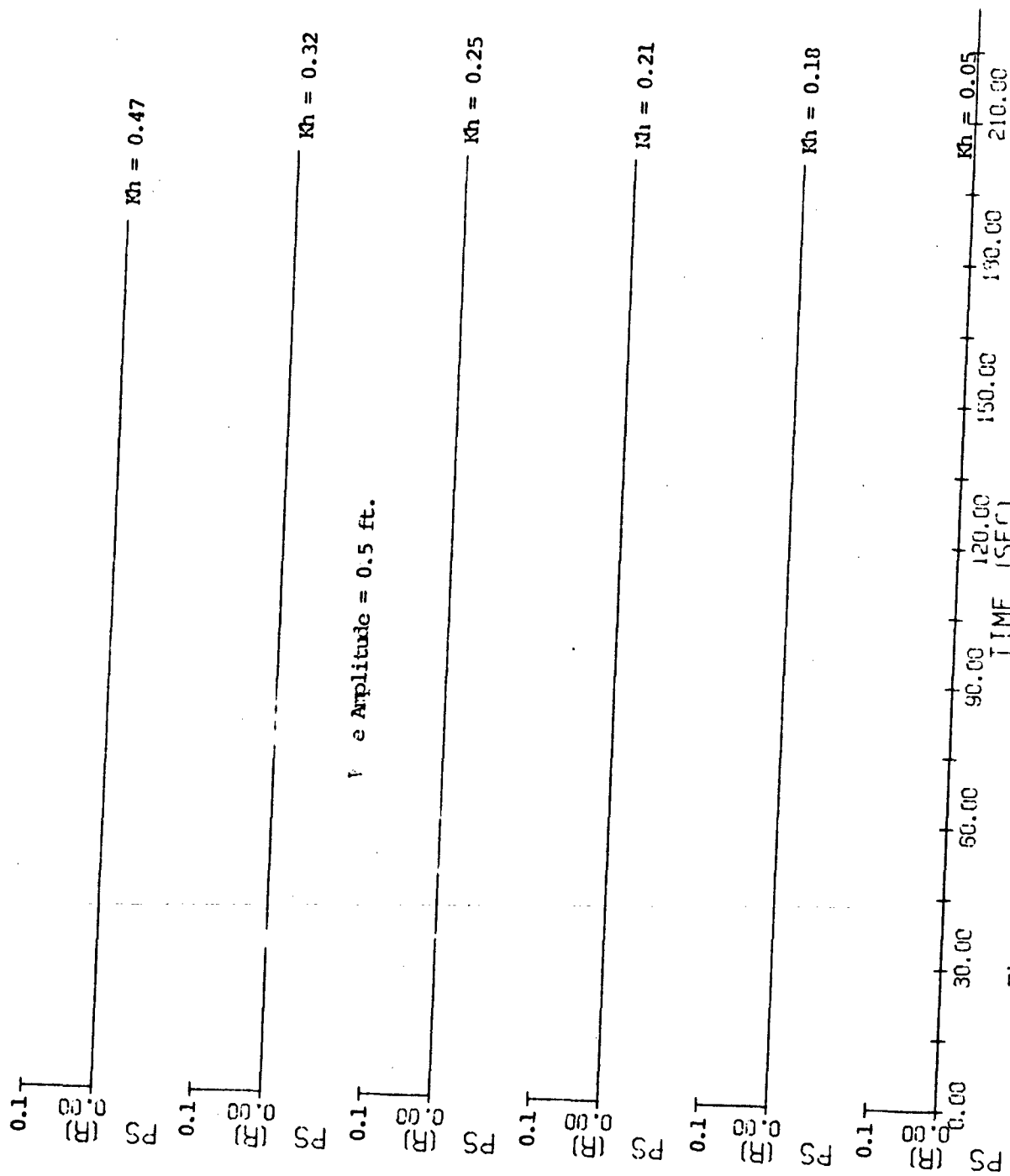


Figure 5.46 Yaw Motions of the Freely Floating Tuned Spar Buoy

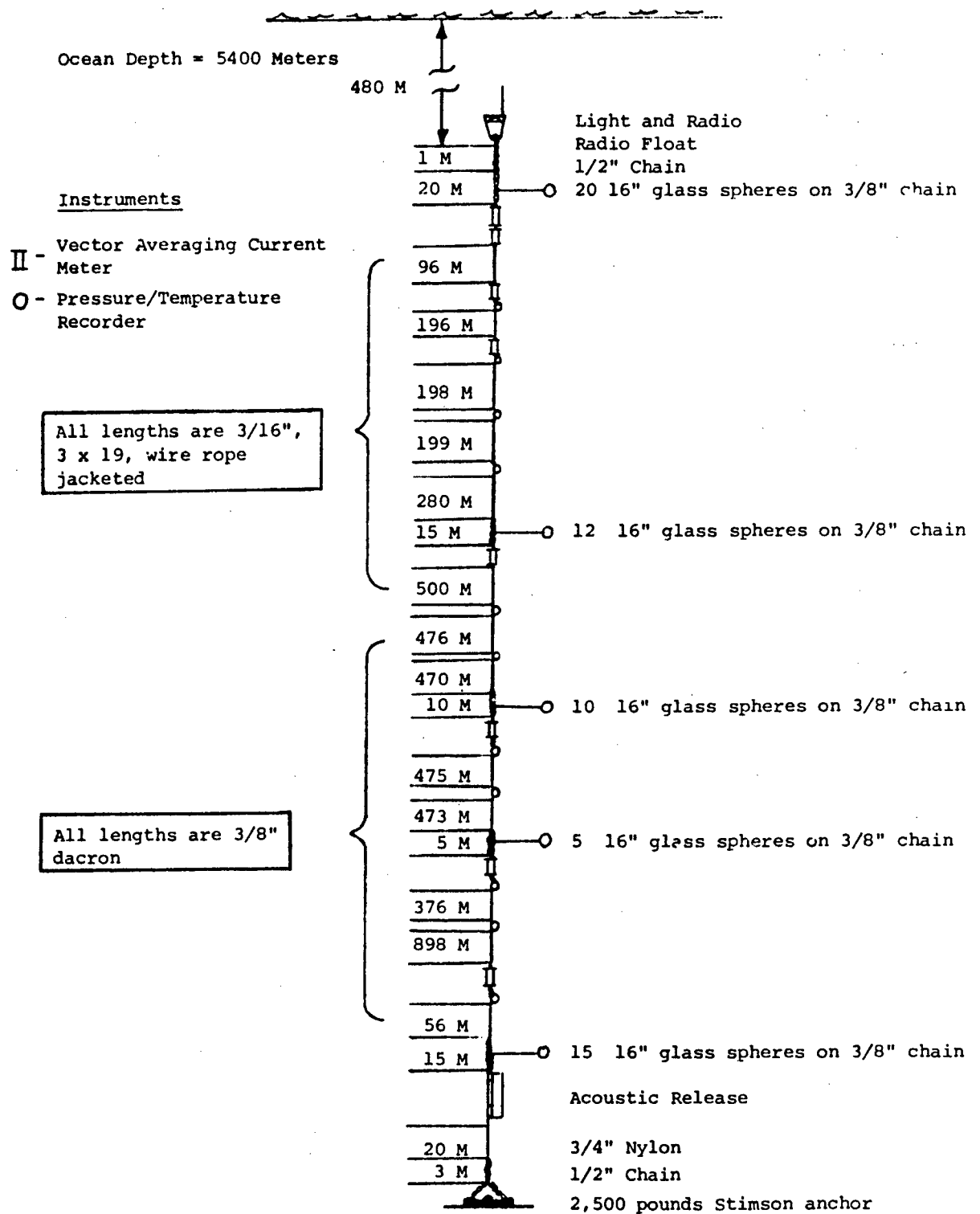


Figure 5.47 Subsurface Moored System

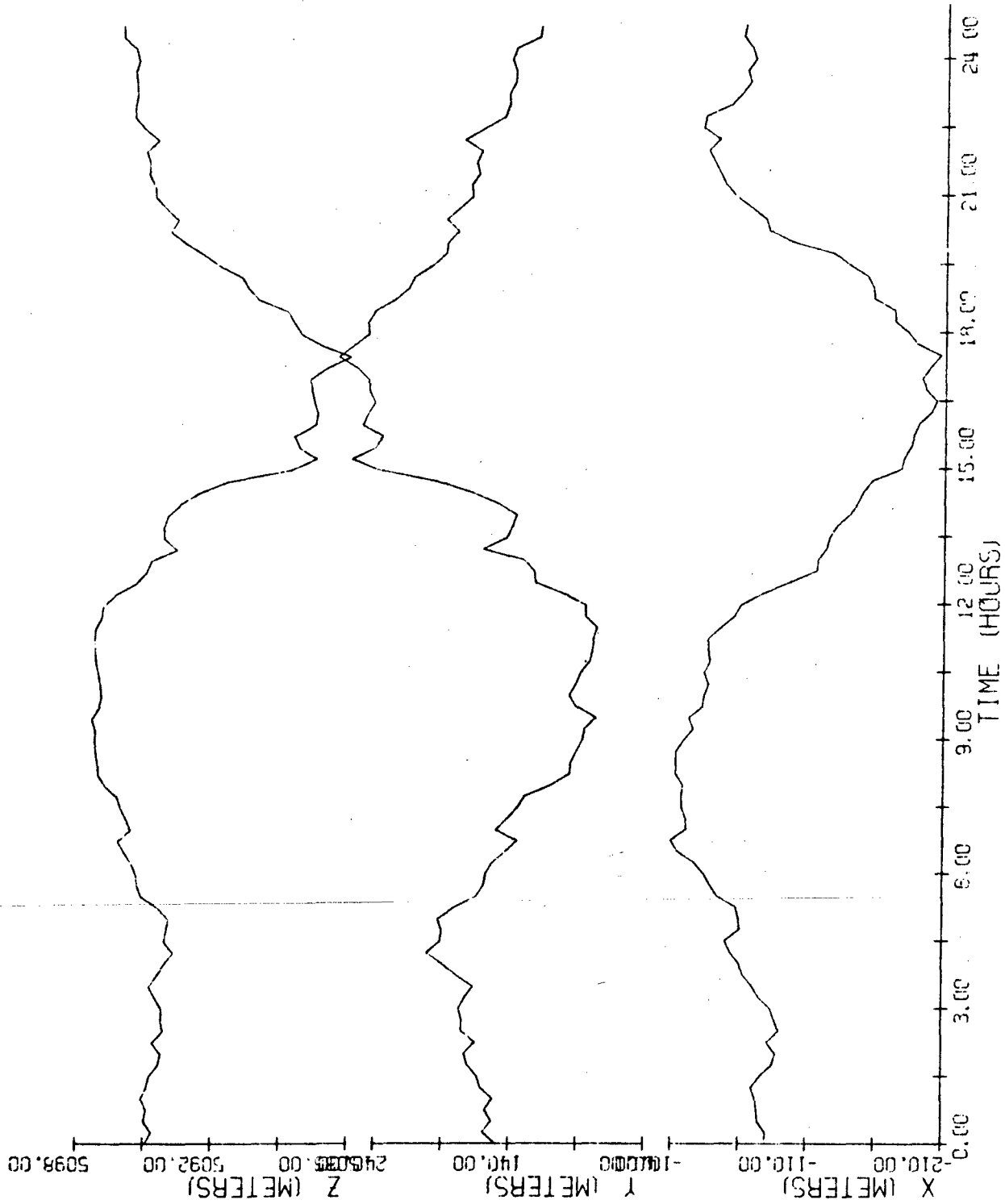


Figure 5.48 Motion of the Subsurface Moored System Top

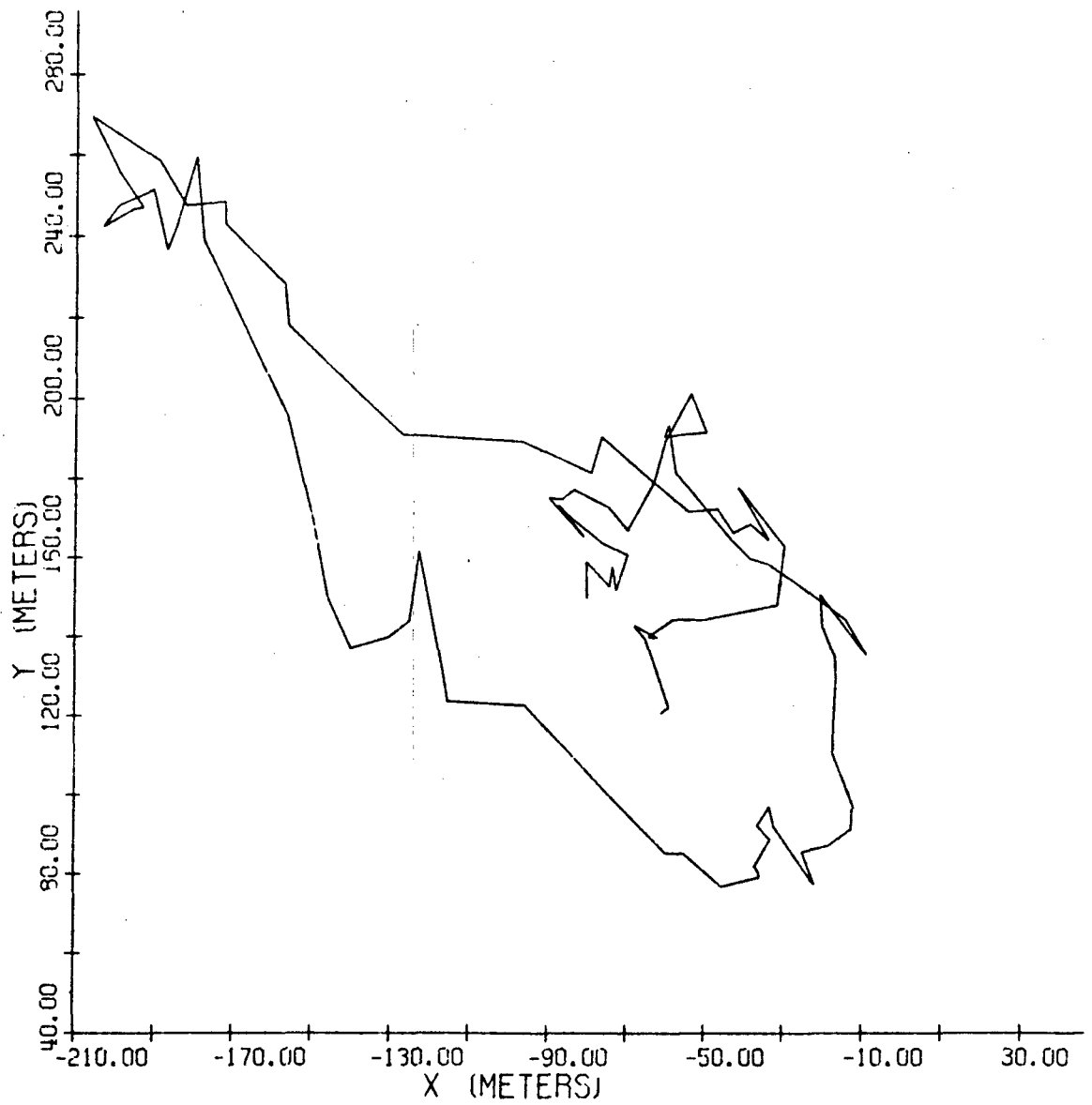


Figure 5.49 Trajectory of the Subsurface Moored System Top

VACM-EAST COMPONENTS

-160-

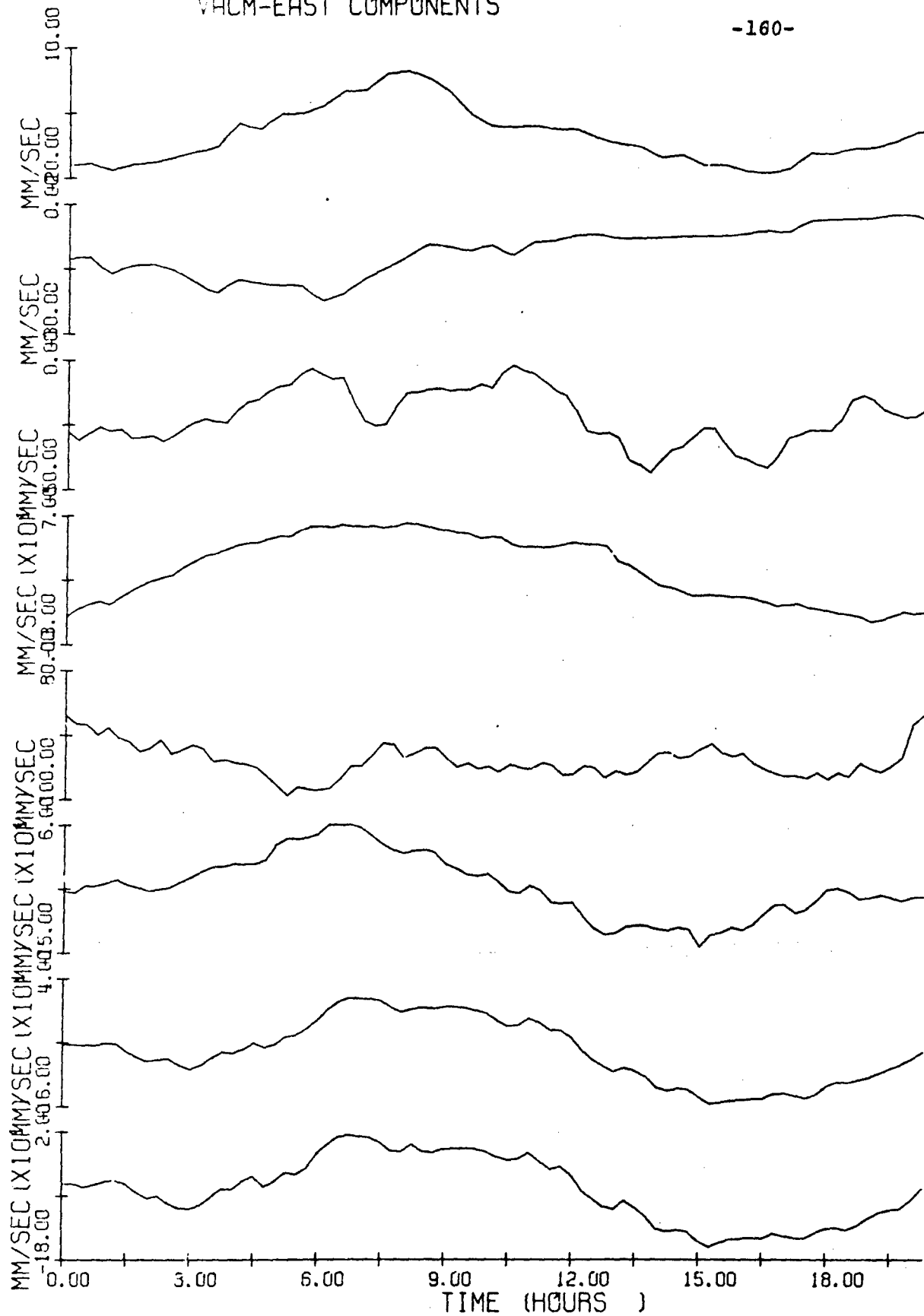


Figure 5.50 Relative Velocity Data for Subsurface Moored System

VACM-NORTH COMPONENTS

-161-

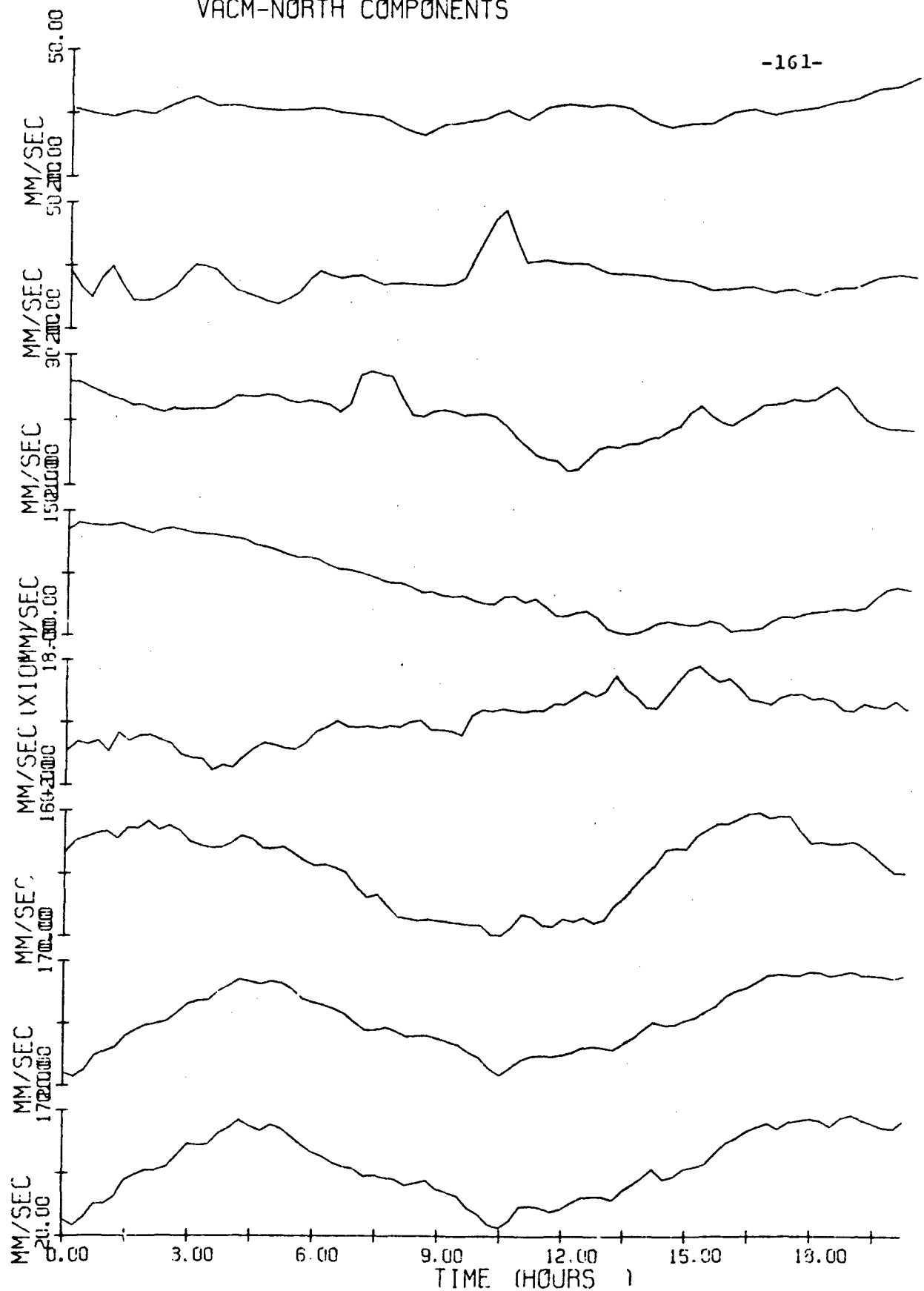


Figure 5.51 Relative Velocity Data for the Subsurface Moored System

current meter of this mooring was corrected for the effects of mooring motion, and power spectra of the uncorrected and corrected signals were compared. The relative velocity data as shown in Figures 5.50 and 5.51 was sampled every fifteen minutes at eight locations on this mooring line. Computer program SSS3.FORT was used for this simulation and no surface wave was assumed present. Figures 5.52 and 5.53 show this same response with the computer program SSD3.FORT, which calculates and includes inertia forces. Overlay of the two responses shows no noticeable difference. Input data used with these two computer programs, describing the system are shown in Tables 5.7 and 5.8.

Response of the top of this mooring line to hypothetical absolute velocity data (Figures 5.54 and 5.55), is shown in Figures 5.56 and 5.57. This simulation was done with computer program SSD31.FORT and the input data shown in Table 5.9. Surface wave was again assumed to be absent in this simulation.

5.3 Surface Moored System

The three-dimensional computer program SD3.FORT is used for this simulation. The surface moored system simulated is shown in Figure 5.58. This system of spar

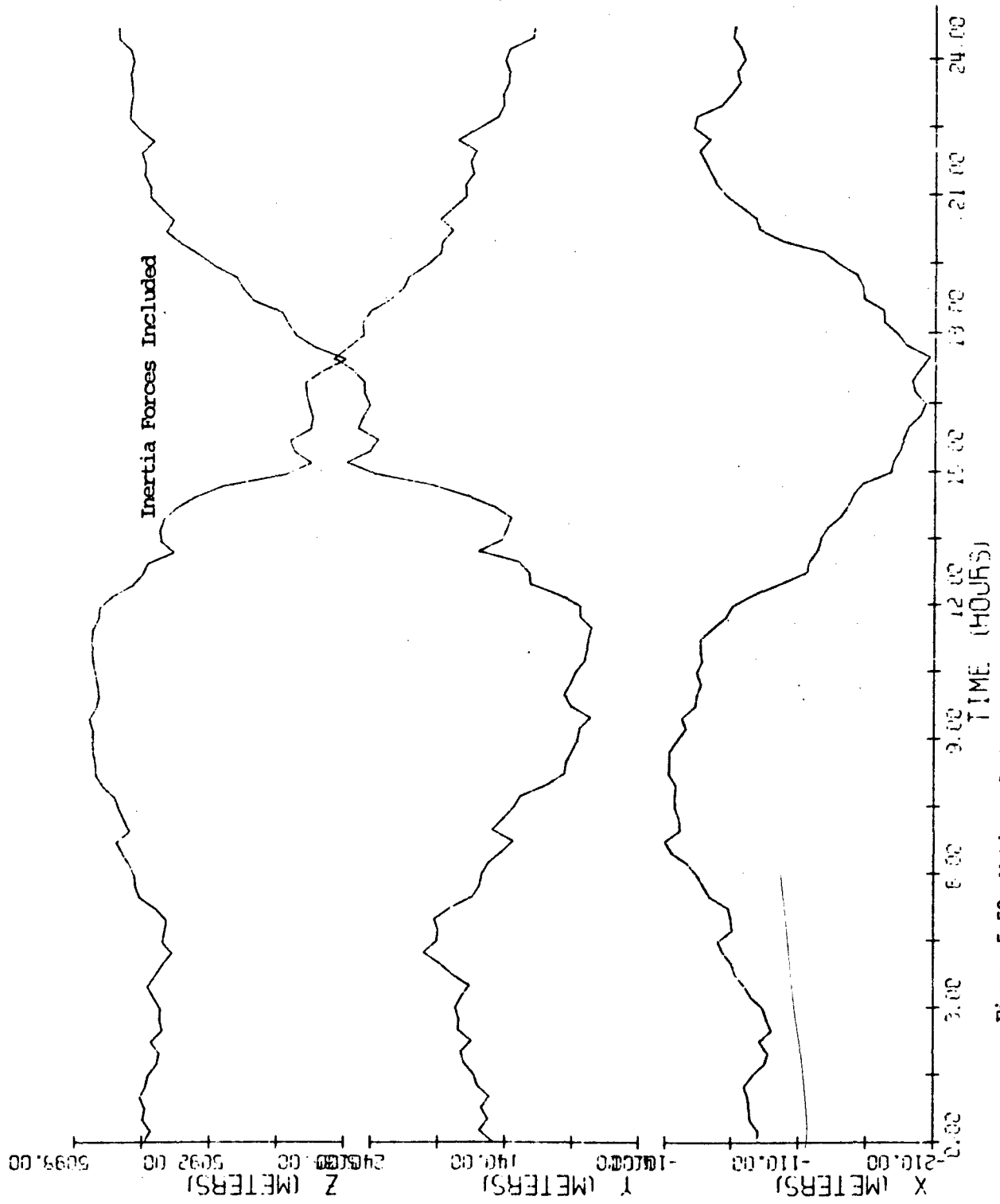


Figure 5.52 Motion of the Subsurface Moored System Top

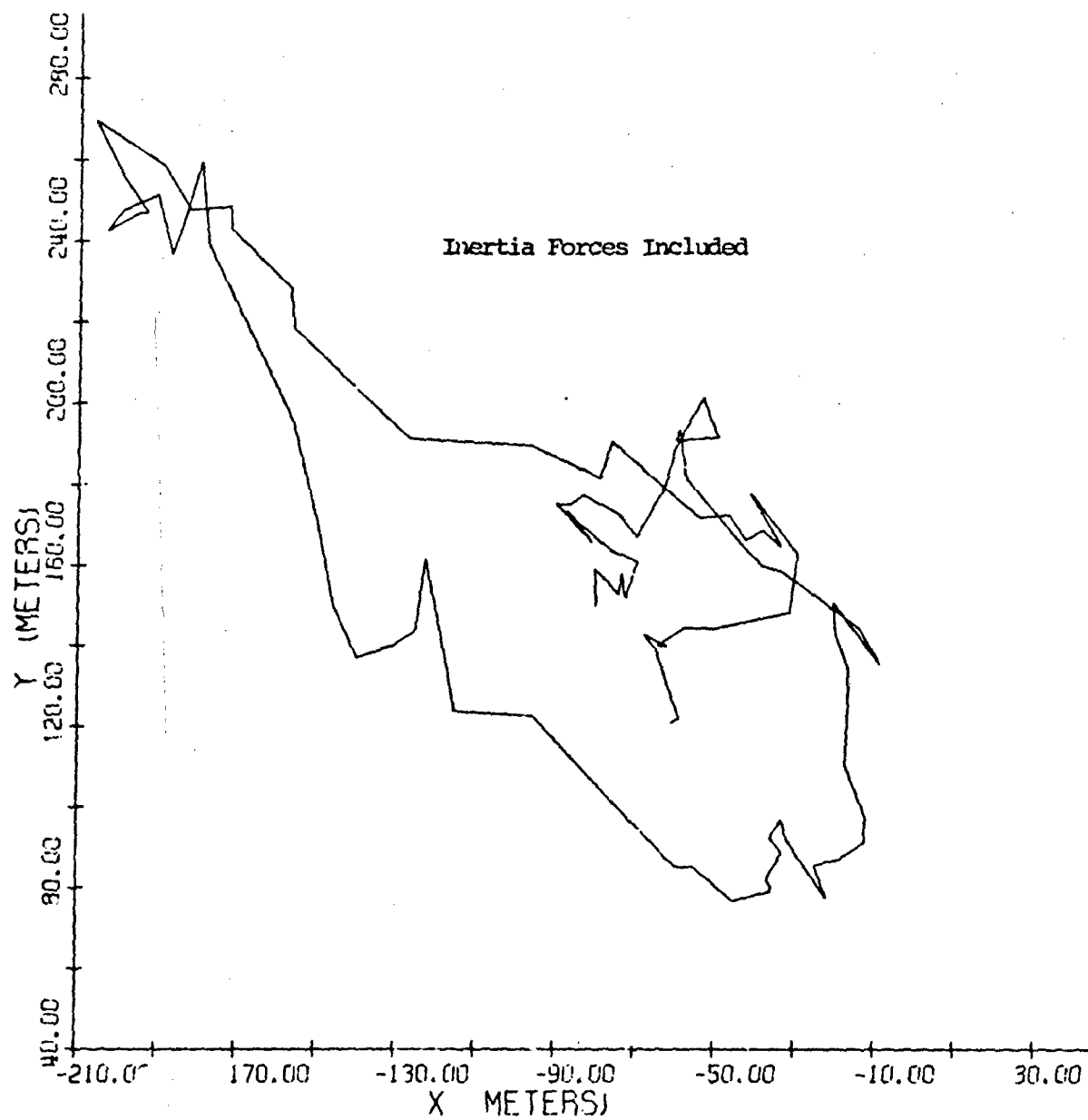


Figure 5.53 Trajectory of the Subsurface Moored System Top

SSS3. DATA	27	17	50	100	7	5462.	365.	3.
00010	0.	0.	0.	0.	0.	45.	1.75	0.
00020	0.	0.	0.	0.	0.	976.	14.0	0.
00030	0.0	22.0	0.	0.	0.	-107.00	4.62	0.13
00040	0.0	5.0	0.	0.	0.	-160.	14.4	0.50
00050	22.0	5.50	0.	0.	0.	-90.0	7.66	0.26
00060	123.0	0.	0.	0.	0.	-24.	0.4	0.
00070	123.0	5.50	0.	0.	0.	-90.0	7.66	0.26
00080	328.9	0.	0.	0.	0.	-24.	0.4	0.
00090	328.9	0.	0.	0.	0.	-24.	0.4	0.
00100	533.4	0.	0.	0.	0.	-24.	0.4	0.
00110	730.4	0.	0.	0.	0.	585.6	8.4	0.
00120	1011.4	0.	0.	0.	0.	-155.	10.65	0.34
00130	1011.4	18.	0.	0.	0.	-24.	0.4	0.
00140	1529.9	0.	0.	0.	0.	-24.	0.4	0.
00150	2009.9	0.	0.	0.	0.	488.	7.0	0.
00160	2493.4	0.	0.	0.	0.	-131.	9.5	0.31
00170	2493.4	13.	0.	0.	0.	-24.	0.4	0.
00180	2508.4	0.	0.	0.	0.	-20.	0.4	0.
00190	2993.9	0.	0.	0.	0.	244.	3.5	0.
00200	3498.9	0.	0.	0.	0.	-79.	5.76	0.15
00210	3498.9	8.	0.	0.	0.	-24.	0.41	0.
00220	3508.9	0.	0.	0.	0.	-24.	0.4	0.
00230	3885.9	0.	0.	0.	0.	-53.	4.6	0.12
00240	4791.9	2.5	0.	0.	0.	-24.	0.4	0.
00250	4796.4	0.	0.	0.	0.	732.	10.5	0.
00260	4860.9	0.	0.	0.	0.	-152.	7.15	0.2
00270	4860.9	18.	0.	0.	0.	-30.	0.93	0.03
00280	4898.9	4.5	0.	0.	0.	0.154	1000000.	
00290	0.269	96.	0.239	0.239	0.239	0.154	1000000.	
00300	0.269	200.40	0.239	0.239	0.239	0.154	1000000.	
00310	0.269	199.00	0.239	0.239	0.239	0.154	1000000.	

Table 5.7 Input Data for the Subsurface Moored System

00320	0.269	197.00	0.239	0.154	10000000.		
00330	0.269	281.	0.239	0.154	10000000.		
00340	0.269	500.50	0.239	0.154	10000000.		
00350	0.375	480.00	0.148	0.0375	1123.		
00360	0.375	483.50	0.148	0.0375	1110.		
00370	0.5	2.0	0.262	0.0667	1578.		
00380	0.375	485.50	0.148	0.0375	1558.		
00390	0.375	505.00	0.148	0.0375	1544.		
00400	0.50	2.0	0.262	0.0667	1765.		
00410	0.375	377.0	0.148	0.0375	1749.		
00420	0.375	906.00	0.148	0.0375	1736.		
00430	0.5	2.0	0.262	0.0667	1690.		
00440	0.375	64.5	0.148	0.0375	1672.		
00450	0.75	20.	0.476	0.0457	2407.		
00460	0.064	0.	0.	0.	1.0	1.4	0.028
00470	0.064	0.	0.	0.	1.0	1.4	0.028
00480	0.064	0.	0.	0.	1.0	1.4	0.028
00490	0.064	0.	0.	0.	1.0	1.4	0.028
00500	0.064	0.	0.	0.	1.0	1.4	0.028
00510	0.064	0.	0.	0.	1.0	1.4	0.028
00520	1250.	1.8	1350.	1.6	1.0	1.4	0.028
00530	1250.	1.8	1350.	1.6	1.0	1.4	0.028
00540	1250.	1.8	1350.	1.6	1.0	1.4	0.028
00550	1250.	1.8	1350.	1.6	1.0	1.4	0.028
00560	1250.	1.8	1350.	1.6	1.0	1.4	0.028
00570	1250.	1.8	1350.	1.6	1.0	1.4	0.028
00580	1250.	1.8	1350.	1.6	1.0	1.4	0.028
00590	1250.	1.8	1350.	1.6	1.0	1.4	0.028
00600	1250.	1.8	1350.	1.6	1.0	1.4	0.028
00610	1250.	1.8	1350.	1.6	1.0	1.4	0.028
00620	88.	1.94	74.8	1.87	1.0	1.4	0.028
00630	392.75	443.5	596.5	1048.5	1400.	3450.5	4664.

Table 5.7 (cont.) Input Data for the Subsurface Moored System

SSD3.DATA									
00010	27	17	50	100	7	5462.	365.	3.	
00020	0.	0.	2.02	1.01	45.	1.75	0.		
00030	0.0	0.	18.76	9.38	976.	14.0	0.		
00040	0.0	22.0	3.70	3.70	-107.00	4.62	0.13		
00050	22.0	5.0	10.25	10.25	-160.	14.4	0.50		
00060	123.0	5.50	5.43	5.43	-90.0	7.66	0.26		
00070	123.0	0.	2.02	1.01	-24.	0.4	0.		
00080	328.9	5.50	5.43	5.43	-90.0	7.66	0.26		
00090	328.9	0.	2.02	1.01	-24.	0.4	0.		
00100	533.4	0.	2.02	1.01	-24.	0.4	0.		
00110	730.4	0.	2.02	1.01	-24.	0.4	0.		
00120	1011.4	0.	11.25	5.63	585.6	8.4	0.		
00130	1011.4	18.	7.61	7.61	-155.	10.65	0.34		
00140	1529.9	0.	2.02	1.01	-24.	0.4	0.		
00150	2009.9	0.	2.02	1.01	-24.	0.4	0.		
00160	2493.4	0.	9.38	4.69	488.	7.0	0.		
00170	2493.4	13.	6.83	6.83	-131.	9.5	0.31		
00180	2508.4	0.	2.02	1.01	-24.	0.4	0.		
00190	2993.9	0.	2.02	1.01	-20.	0.4	0.		
00200	3498.9	0.	4.69	2.35	244.	3.5	0.		
00210	3498.9	8.	5.06	5.06	-79.	5.76	0.15		
00220	3508.9	0.	2.02	1.01	-24.	0.41	0.		
00230	3885.9	0.	2.02	1.01	-24.	0.4	0.		
00240	4791.9	2.5	3.50	3.50	-53.	4.6	0.12		
00250	4796.4	0.	2.02	1.01	-24.	0.4	0.		
00260	4860.9	0.	14.07	7.03	732.	10.5	0.		
00270	4860.9	18.	8.54	8.54	-152.	7.15	0.2		
00280	4898.9	4.5	1.60	1.60	-30.	0.93	0.03		
00290	0.269	96.	0.239	0.154	0.154	10000000.			
00300	0.269	200.40	0.239	0.154	0.154	10000000.			
00310	0.269	199.00	0.239	0.154	0.154	10000000.			

Table 5.8 Input Data for the Subsurface Moored System

00320	0.269	197.00	0.239	0.154	10000000.	1.4	0.028
00330	0.269	281.	0.239	0.154	10000000.	1.4	0.028
00340	0.269	500.50	0.239	0.154	10000000.	1.4	0.028
00350	0.375	480.00	0.148	0.0375	1123.	1.4	0.028
00360	0.375	483.50	0.148	0.0375	1110.	1.4	0.028
00370	0.5	2.0	0.262	0.0667	1578.	1.4	0.028
00380	0.375	485.50	0.148	0.0375	1558.	1.4	0.028
00390	0.375	505.00	0.148	0.0375	1544.	1.4	0.028
00400	0.50	2.0	0.262	0.0667	1765.	1.4	0.028
00410	0.375	377.0	0.148	0.0375	1749.	1.4	0.028
00420	0.375	906.00	0.148	0.0375	1736.	1.4	0.028
00430	0.5	2.0	0.262	0.0667	1690.	1.4	0.028
00440	0.375	64.5	0.148	0.0375	1672.	1.4	0.028
00450	0.75	20.	0.476	0.0457	2407.	1.4	0.028
00460	0.064	0.	0.	0.	1.0	1.4	0.028
00470	0.064	0.	0.	0.	1.0	1.4	0.028
00480	0.064	0.	0.	0.	1.0	1.4	0.028
00490	0.064	0.	0.	0.	1.0	1.4	0.028
00500	0.064	0.	0.	0.	1.0	1.4	0.028
00510	0.064	0.	0.	0.	1.0	1.4	0.028
00520	1250.	1.8	1350.	1.6	1.0	1.4	0.028
00530	1250.	1.8	1350.	1.6	1.0	1.4	0.028
00540	1250.	1.8	1350.	1.6	1.0	1.4	0.028
00550	1250.	1.8	1350.	1.6	1.0	1.4	0.028
00560	1250.	1.8	1350.	1.6	1.0	1.4	0.028
00570	1250.	1.8	1350.	1.6	1.0	1.4	0.028
00580	1250.	1.8	1350.	1.6	1.0	1.4	0.028
00590	1250.	1.8	1350.	1.6	1.0	1.4	0.028
00600	1250.	1.8	1350.	1.6	1.0	1.4	0.028
00610	1250.	1.8	1350.	1.6	1.0	1.4	0.028
00620	88.	1.94	74.8	1.87	1.0	1.4	0.028
00630	392.75	443.5	596.5	1048.5	1400.	3450.5	4664.

Table 5.8 (cont.) Input Data for the Subsurface Moored System

VACM-EAST COMPONENTS

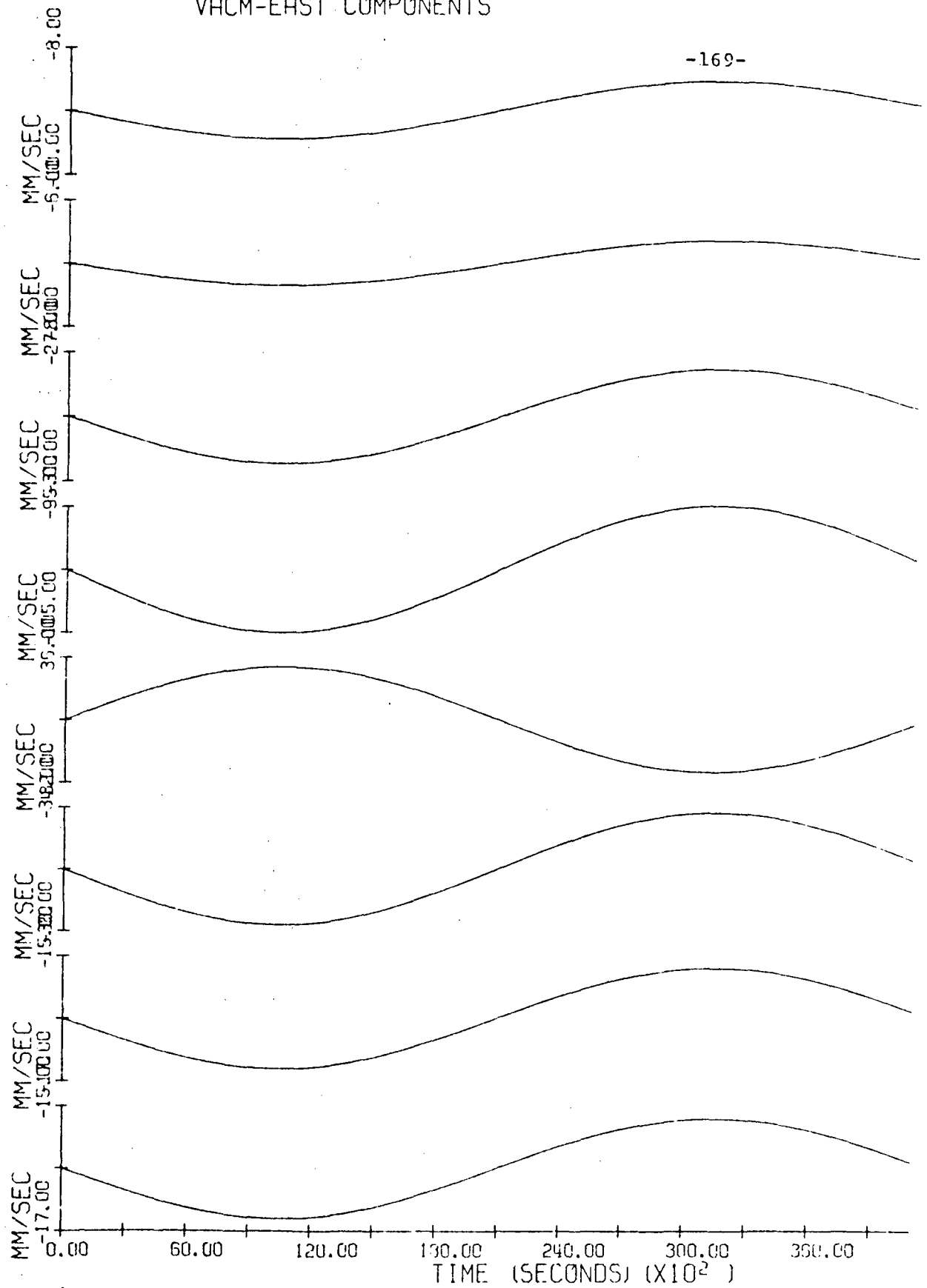


Figure 5.54 Absolute Velocity Data for the Subsurface Moored System

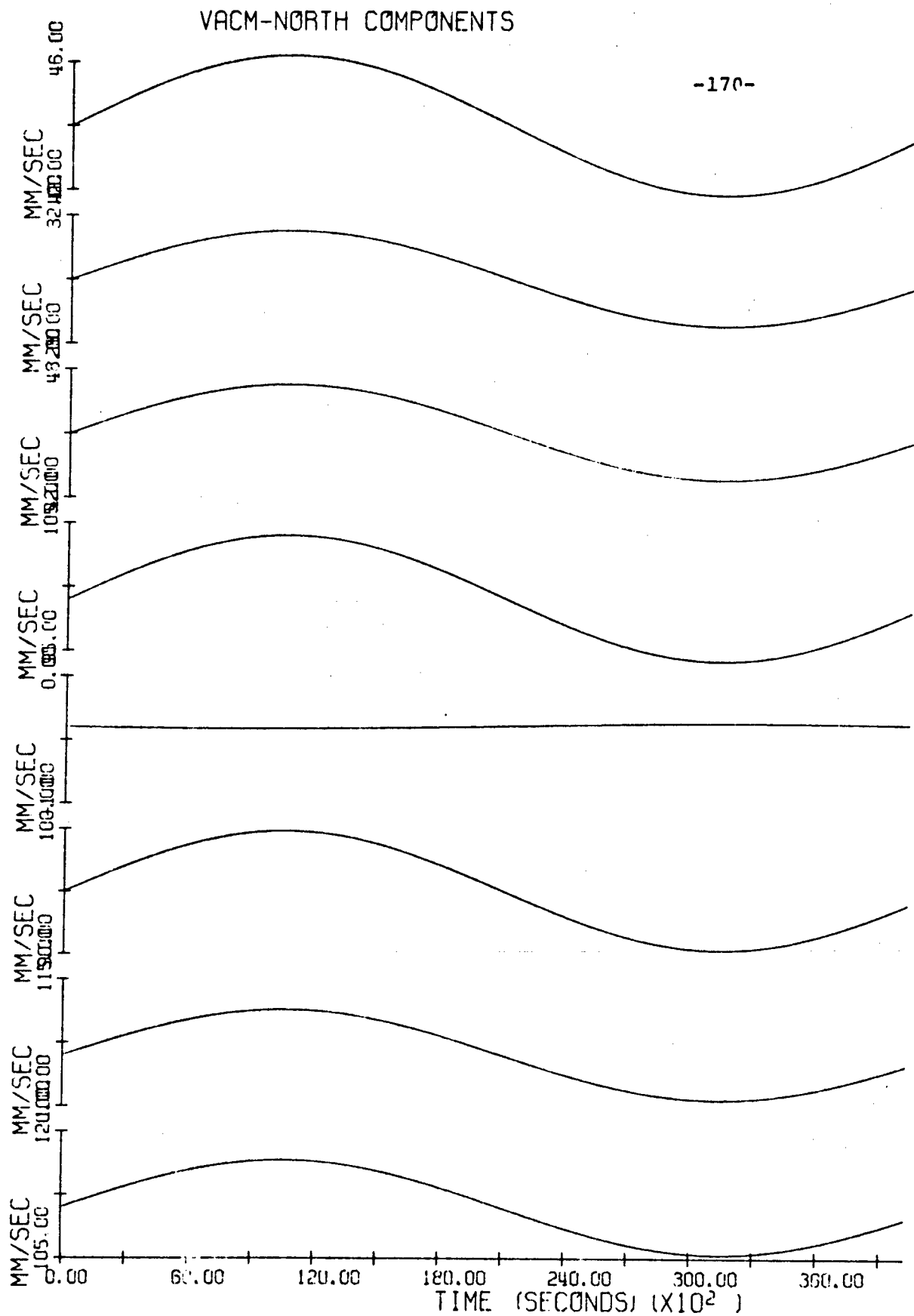


Figure 5.55 Absolute Velocity Data for the Subsurface Moored System

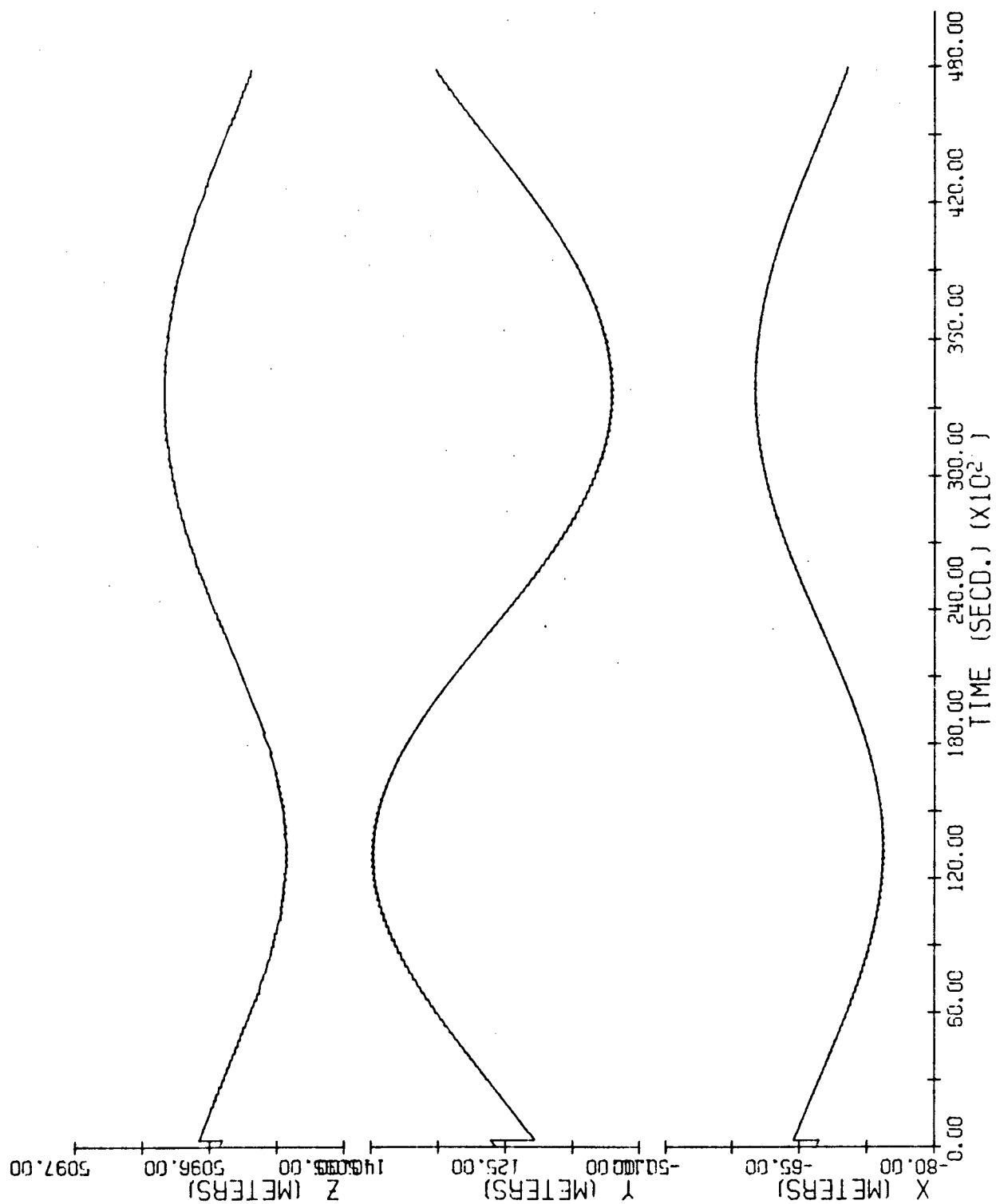


Figure 5.56 Motion of the Subsurface Moored System Top

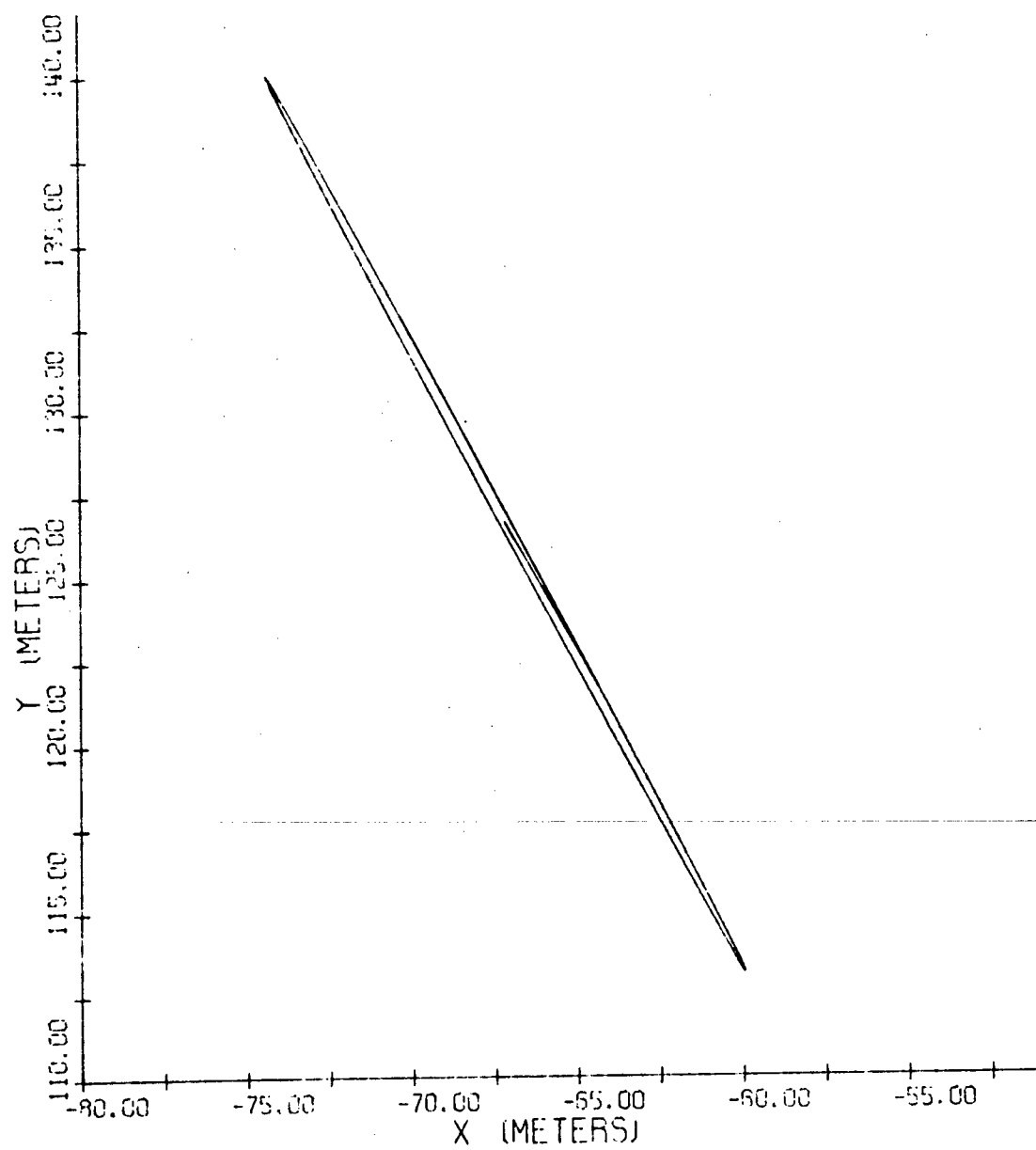


Figure 5.57 Trajectory of the Subsurface Moored System Top

SSD31.DATA									
00010	27	17	125	7	5462.	365.	3.	0.1	
00020	0.	0.	2.02	1.01	45.	45.	1.75	0.	
00030	0.0	0.	18.76	9.38	976.	976.	14.0	0.	
00040	0.0	22.0	3.70	3.70	-107.00	-107.00	4.62	0.13	
00050	22.0	5.0	10.25	10.25	-160.	-160.	14.4	0.50	
00060	123.0	5.50	5.43	5.43	-90.0	-90.0	7.66	0.26	
00070	123.0	0.	2.02	1.01	-24.	-24.	0.4	0.	
00080	328.9	5.50	5.43	5.43	-90.0	-90.0	7.66	0.26	
00090	328.9	0.	2.02	1.01	-24.	-24.	0.4	0.	
00100	533.4	0.	2.02	1.01	-24.	-24.	0.4	0.	
00110	730.4	0.	2.02	1.01	-24.	-24.	0.4	0.	
00120	1011.4	0.	11.25	5.63	585.6	585.6	8.4	0.	
00130	1011.4	18.	7.61	7.61	-155.	-155.	10.65	0.34	
00140	1529.9	0.	2.02	1.01	-24.	-24.	0.4	0.	
00150	2009.9	0.	2.02	1.01	-24.	-24.	0.4	0.	
00160	2493.4	0.	9.38	4.69	488.	488.	7.0	0.	
00170	2493.4	13.	6.83	6.83	-131.	-131.	9.5	0.31	
00180	2508.4	0.	2.02	1.01	-24.	-24.	0.4	0.	
00190	2993.9	0.	2.02	1.01	-20.	-20.	0.4	0.	
00200	3496.9	0.	4.69	2.35	244.	244.	3.5	0.	
00210	3498.9	8.	5.06	5.06	-79.	-79.	5.76	0.15	
00220	3508.9	0.	2.02	1.01	-24.	-24.	0.41	0.	
00230	3885.9	0.	2.02	1.01	-24.	-24.	0.4	0.	
00240	4791.9	2.5	3.50	3.50	-53.	-53.	4.6	0.12	
00250	4796.4	0.	2.02	1.01	-24.	-24.	0.4	0.	
00260	4860.9	0.	14.07	7.03	732.	732.	10.5	0.	
00270	4860.9	18.	8.54	8.54	-152.	-152.	7.15	0.2	
00280	4898.9	4.5	1.60	1.60	-30.	-30.	0.93	0.03	
00290	0.269	96.	0.239	0.239	0.154	0.154	10000000.		
00300	0.269	200.40	0.239	0.239	0.154	0.154	10000000.		
00310	0.269	199.00	0.239	0.239	0.154	0.154	10000000.		
00320	0.269	197.00	0.239	0.239	0.154	0.154	10000000.		
00330	0.269	281.	0.239	0.239	0.154	0.154	10000000.		
00340	0.269	500.50	0.239	0.239	0.154	0.154	10000000.		

Table 5.9 Input Data for the Subsurface Moored System

00350	0.375	480.00	0.148	0.0375	1123.	1.4	0.028
00360	0.375	483.50	0.148	0.0375	1110.	1.4	0.028
00370	0.5	2.0	0.262	0.0667	1578.	1.4	0.028
00380	0.375	485.50	0.148	0.0375	1558.	1.4	0.028
00390	0.375	505.00	0.148	0.0375	1544.	1.4	0.028
00400	0.50	2.0	0.262	0.0667	1765.	1.4	0.028
00410	0.375	377.0	0.148	0.0375	1749.	1.4	0.028
00420	0.375	906.00	0.148	0.0375	1736.	1.4	0.028
00430	0.5	2.0	0.262	0.0667	1690.	1.4	0.028
00440	0.375	64.5	0.148	0.0375	1672.	1.4	0.028
00450	0.75	20.	0.476	0.0457	2407.	1.4	0.028
00460	0.064	0.	0.	0.	1.0	1.4	0.028
00470	0.064	0.	0.	0.	1.0	1.4	0.028
00480	0.064	0.	0.	0.	1.0	1.4	0.028
00490	0.064	0.	0.	0.	1.0	1.4	0.028
00500	0.064	0.	0.	0.	1.0	1.4	0.028
00510	0.064	0.	0.	0.	1.0	1.4	0.028
00520	1250.	1.8	1350.	1.6	1.0	1.4	0.028
00530	1250.	1.8	1350.	1.6	1.0	1.4	0.028
00540	1250.	1.8	1350.	1.6	1.0	1.4	0.028
00550	1250.	1.8	1350.	1.6	1.0	1.4	0.028
00560	1250.	1.8	1350.	1.6	1.0	1.4	0.028
00570	1250.	1.8	1350.	1.6	1.0	1.4	0.028
00580	1250.	1.8	1350.	1.6	1.0	1.4	0.028
00590	1250.	1.8	1350.	1.6	1.0	1.4	0.028
00600	1250.	1.8	1350.	1.6	1.0	1.4	0.028
00610	1250.	1.8	1350.	1.6	1.0	1.4	0.028
00620	88.	1.94	74.8	1.87	1.0	1.4	0.028
00630	392.75	443.5	596.5	1048.5	1400.	3450.5	4564.
00640	-15.	111.	0.	-16.	106.	0.	-36.
00650	95.	0.	34.	-0.4	0.	-100.	99.
00660	0.	-29.	45.	0.	-7.	30.	0.
00670	-9.	44.	0.				
00680	0.000150	300.	300.	50000.			
00690	1.0	0.0	0.0				

Table 5.9 (cont.) Input Data for the Subsurface Moored System

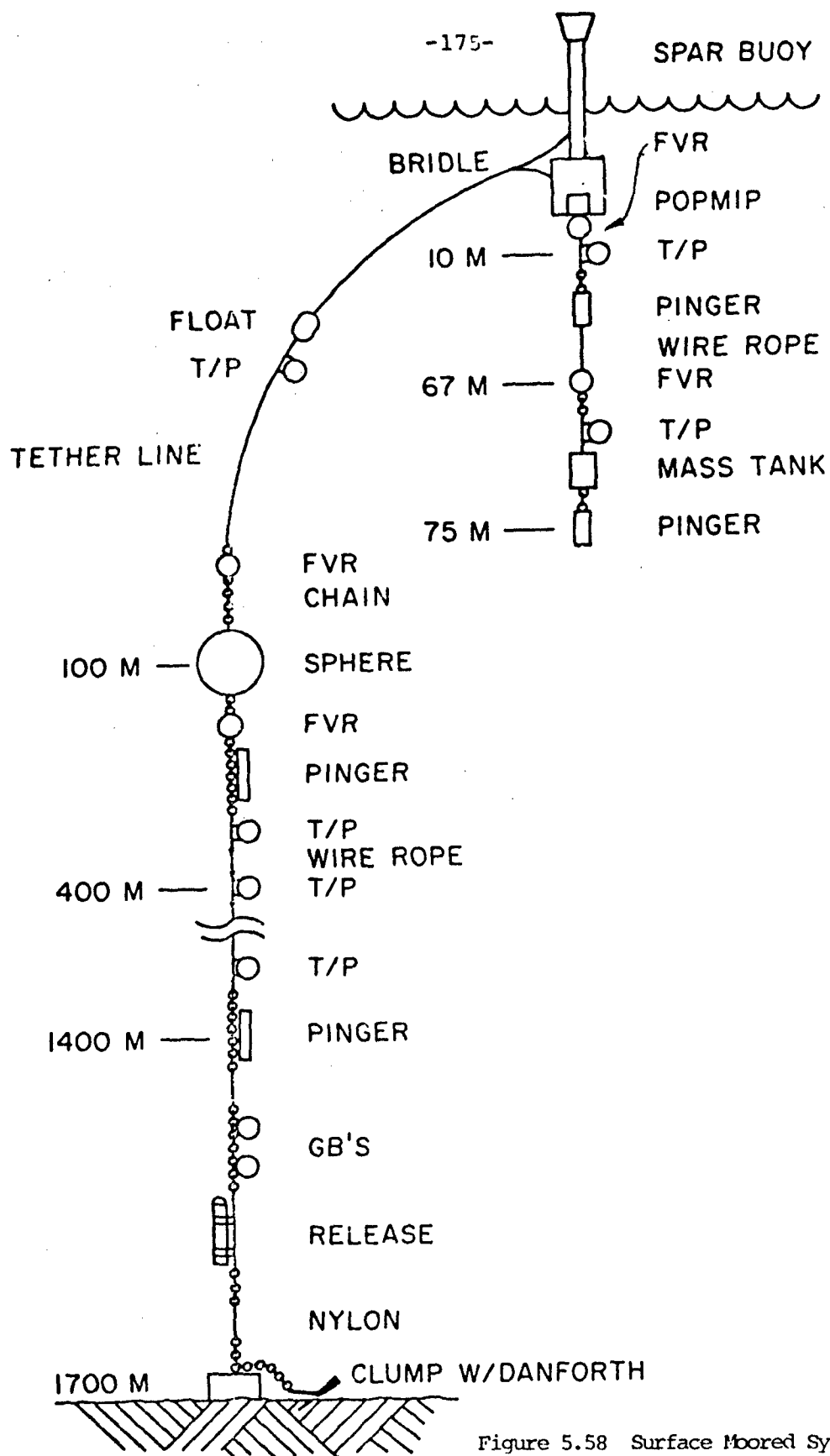


Figure 5.58 Surface Moored System

buoy/buoyant tether/subsurface mooring line with an instrument line hanging beneath the spar was deployed as part of the ONR/NDBO Mooring Dynamics Experiment during October 1976. The system is reduced to seven nodes for this simulation. These nodes are shown in Figure 5.59. Figure 5.59 shows the mean configuration of these nodes in the x-z plane. This configuration is the result of the static solution calculated by SD3.FORT using the equivalent velocity profile described in Section 4.1. The equivalent velocity profile and the actual velocity profile in the x-z plane is also shown in Figure 5.59. Figures 5.60 through 5.64 display time responses of these seven nodes to the input data presented in Table 5.10 which describes the moored system and the environment forcing this system. Forcing the system is a constant current profile and a surface wave of amplitude 1 foot, a frequency of 1 radian per second, and having a wave direction inclined at 0.1 radians with the x-axis of the earth fixed frame. Figures 5.60 through 5.62 are the plots of surge, sway and heave responses of the seven nodes. Figure 5.63 displays the roll, pitch and yaw of the tuned spar buoy (node 6), and Figure 5.64 displays tensions in the seven segments of mooring lines preceding the seven nodes.

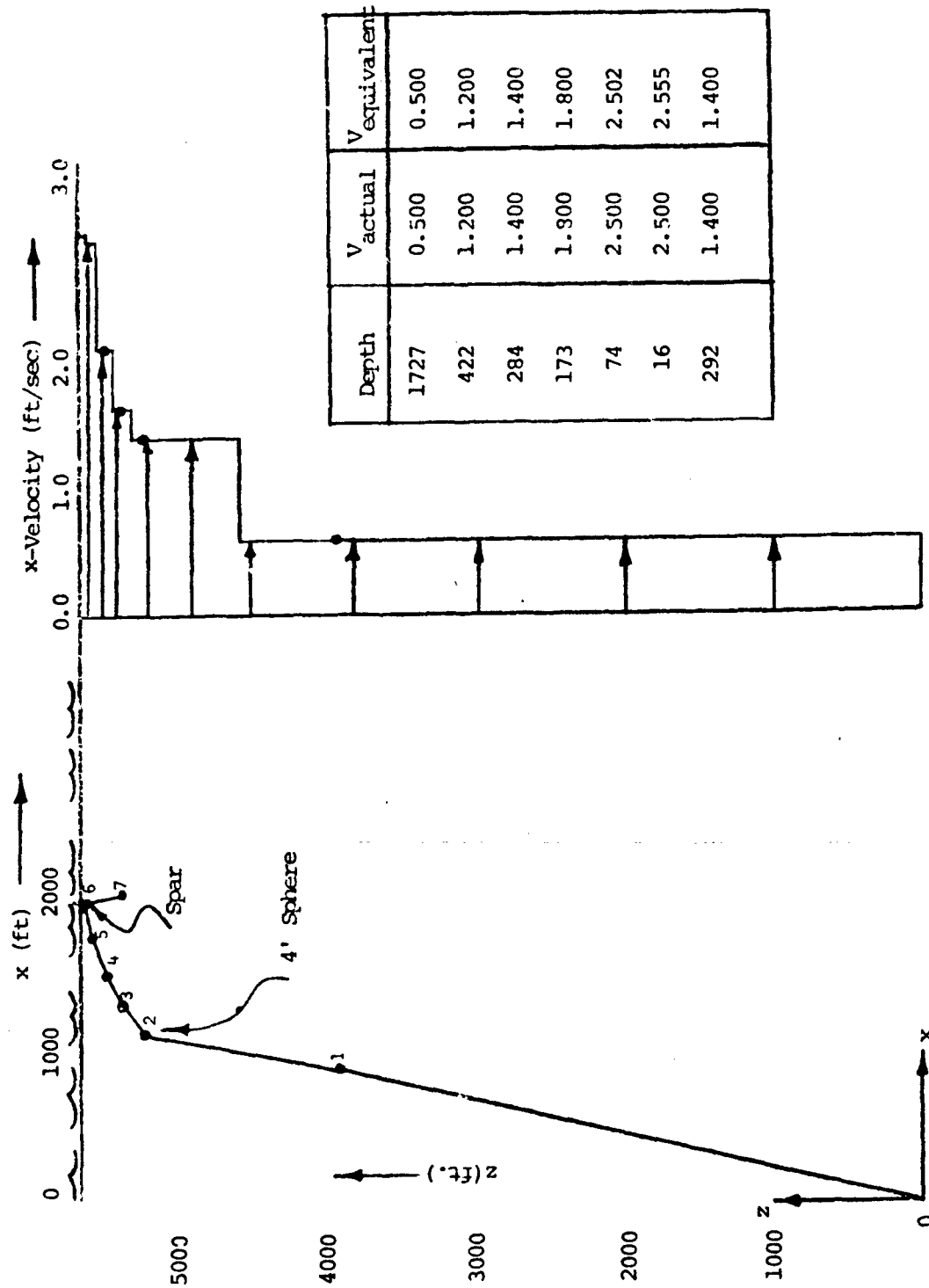


Figure 5.59 Static (Mean) Configuration of Surface Moored System

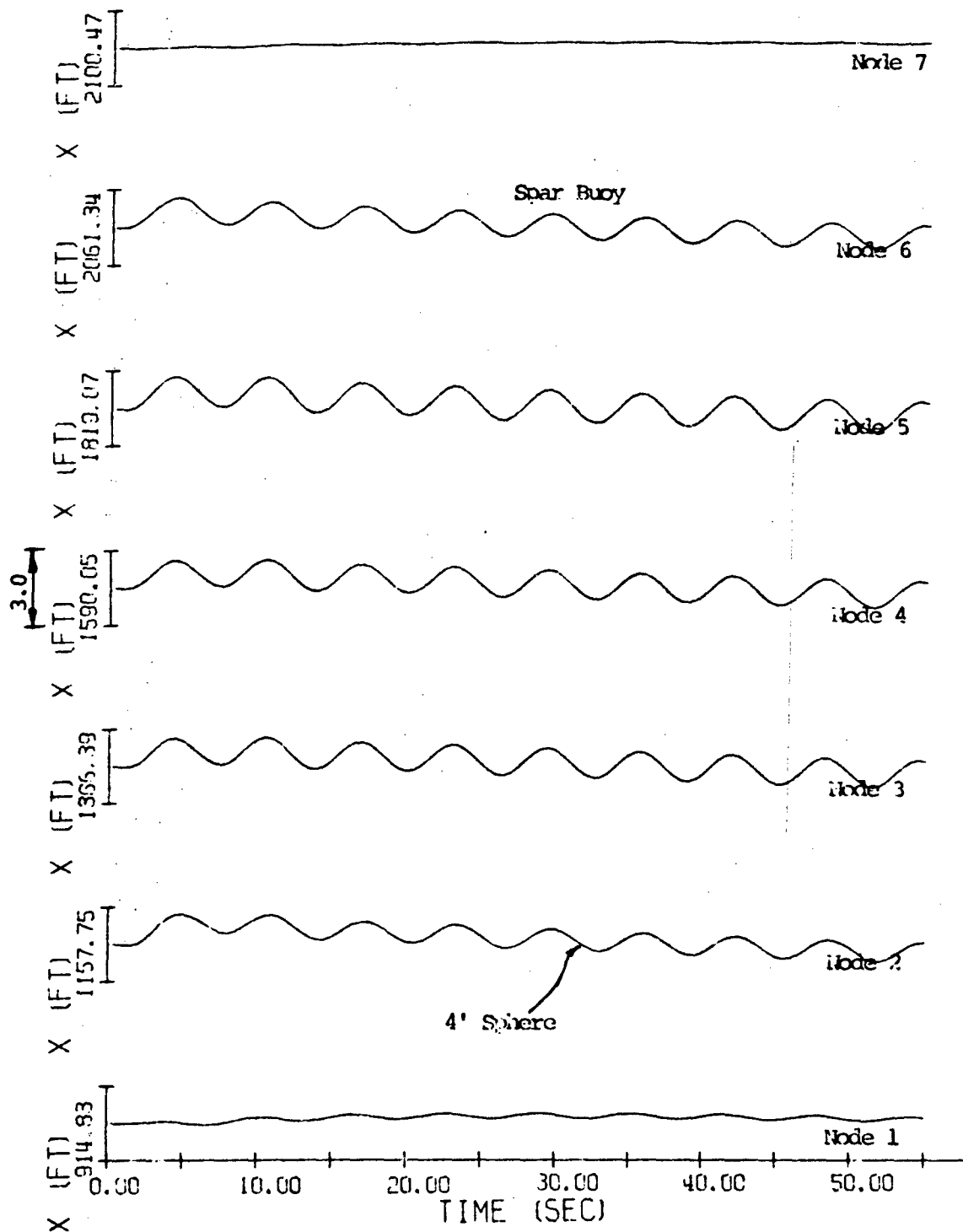


Figure 5.60 Surge Motions of a Surface Moored System

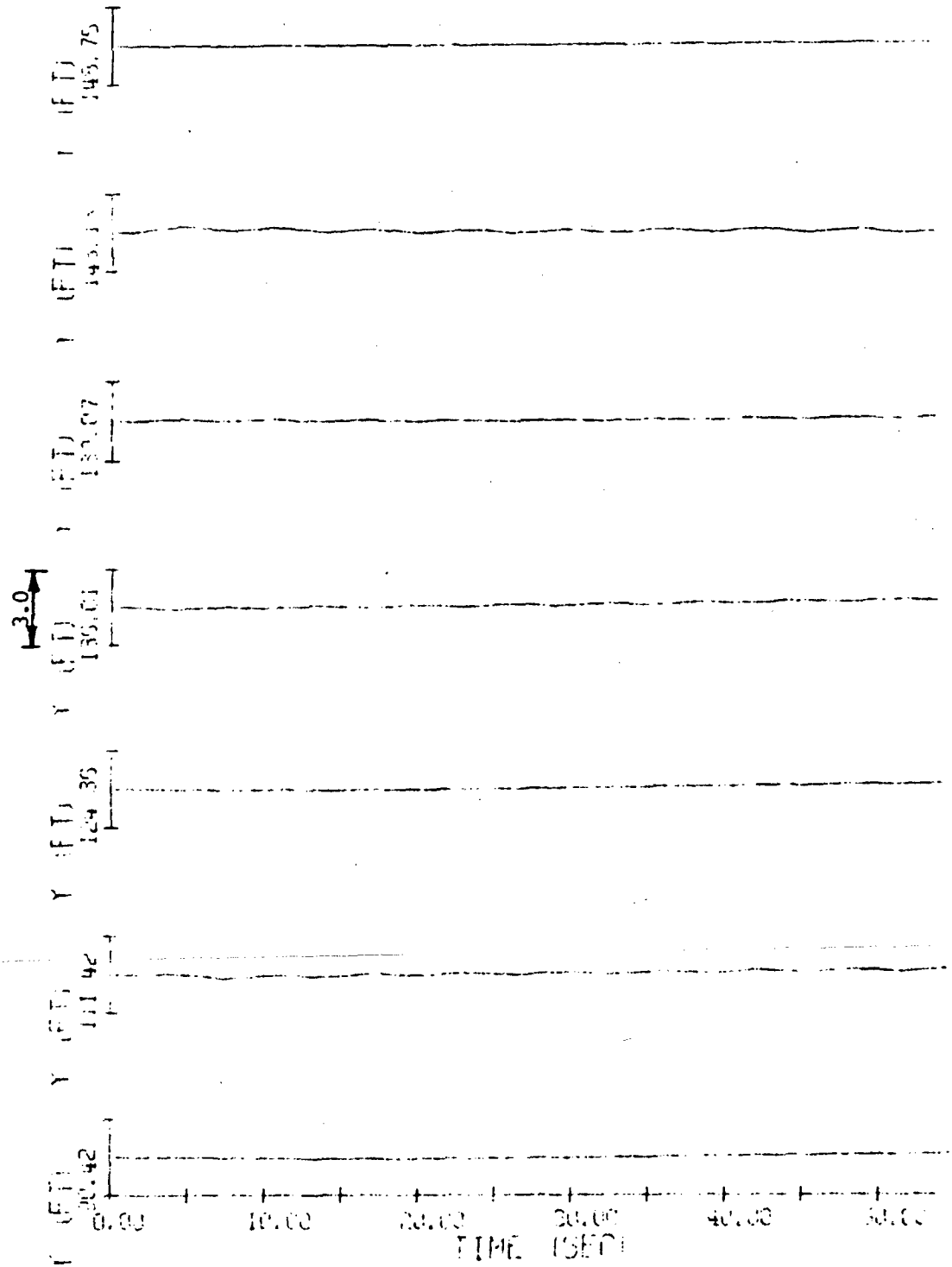


Figure 5.61 Sway Motions of a Surface Moored System

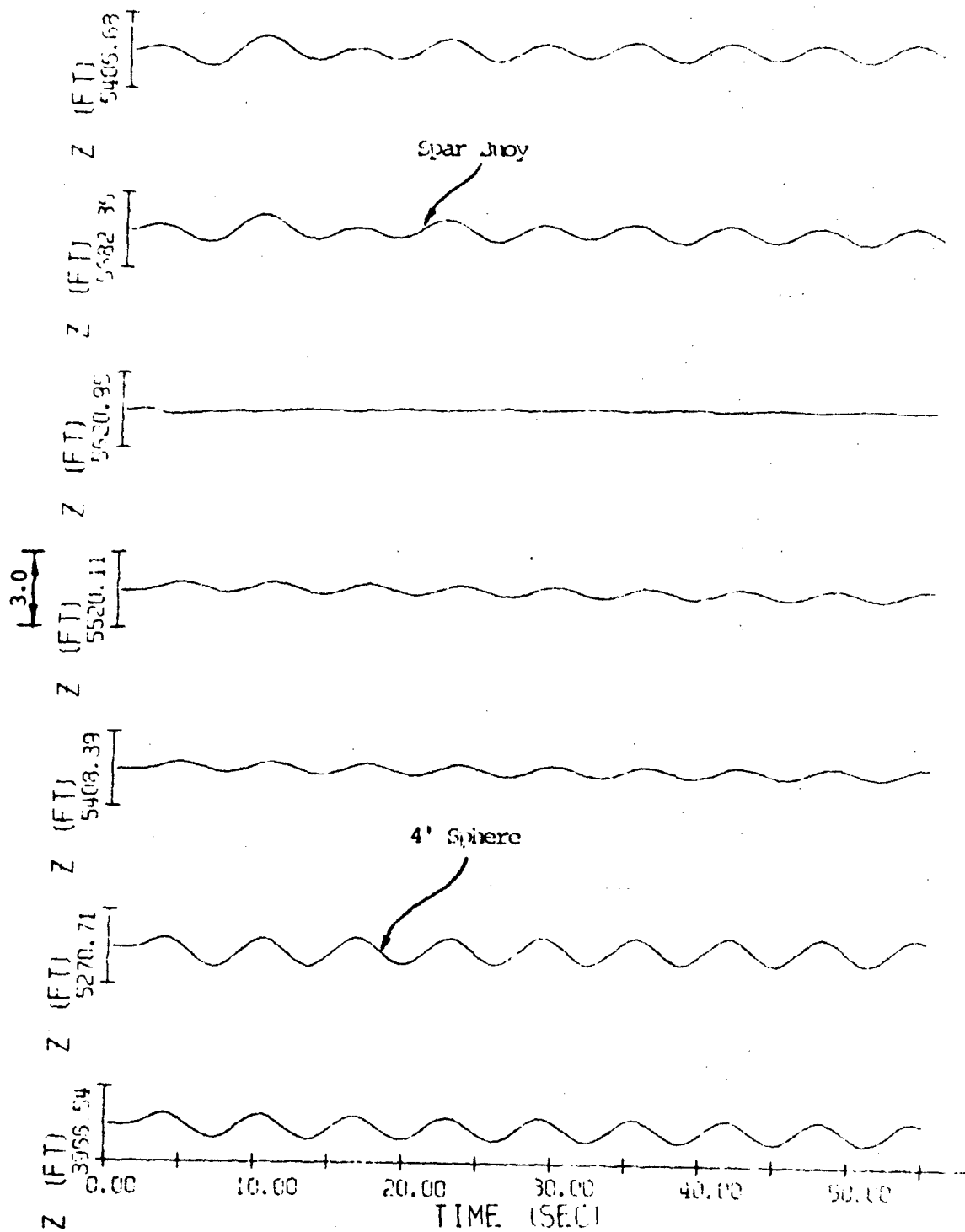


Figure 5.62 Heave Motions of a Surface Moored System

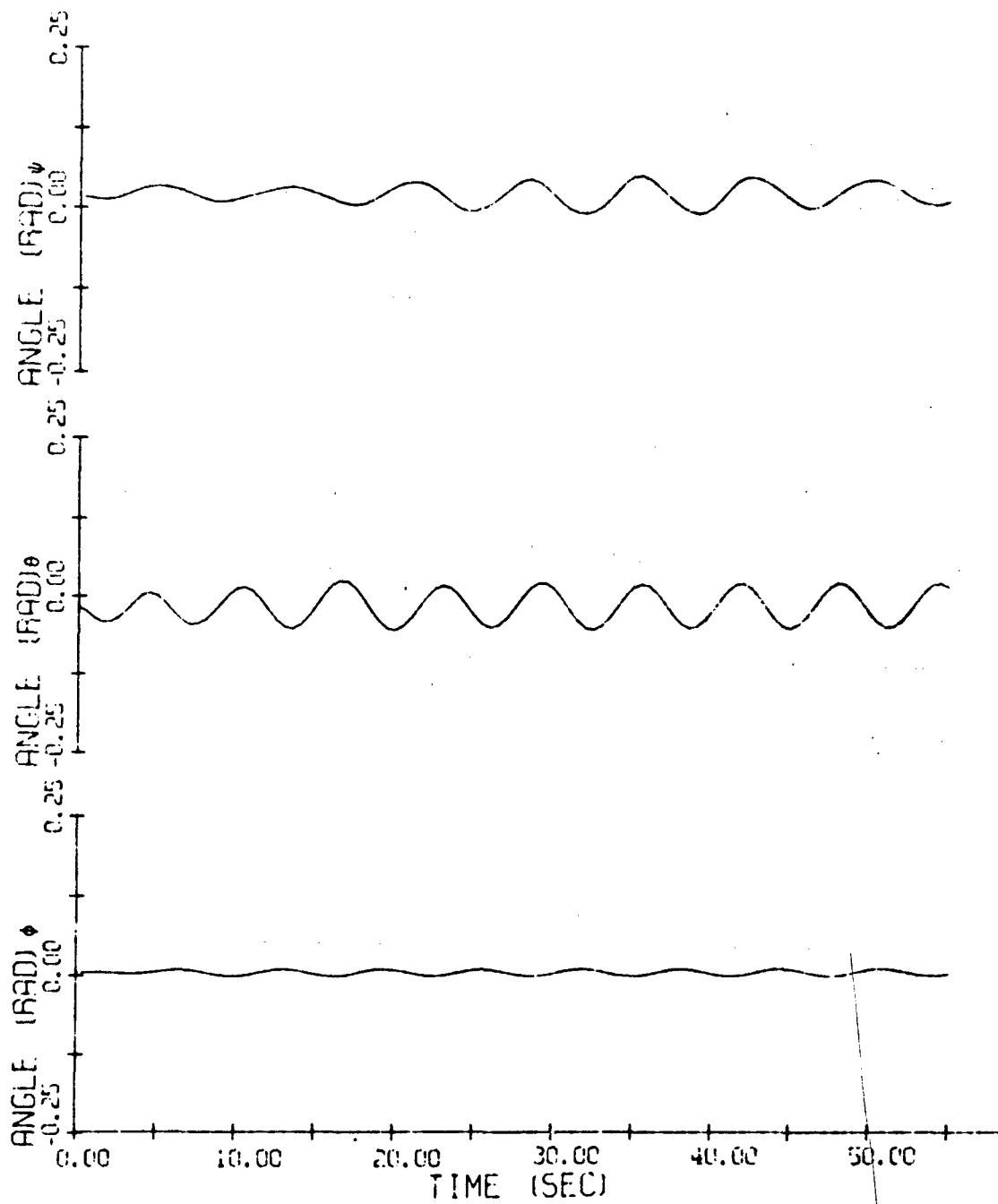


Figure 5.63 Roll, Pitch, and Yaw Motion of the Moored Spar Buoy

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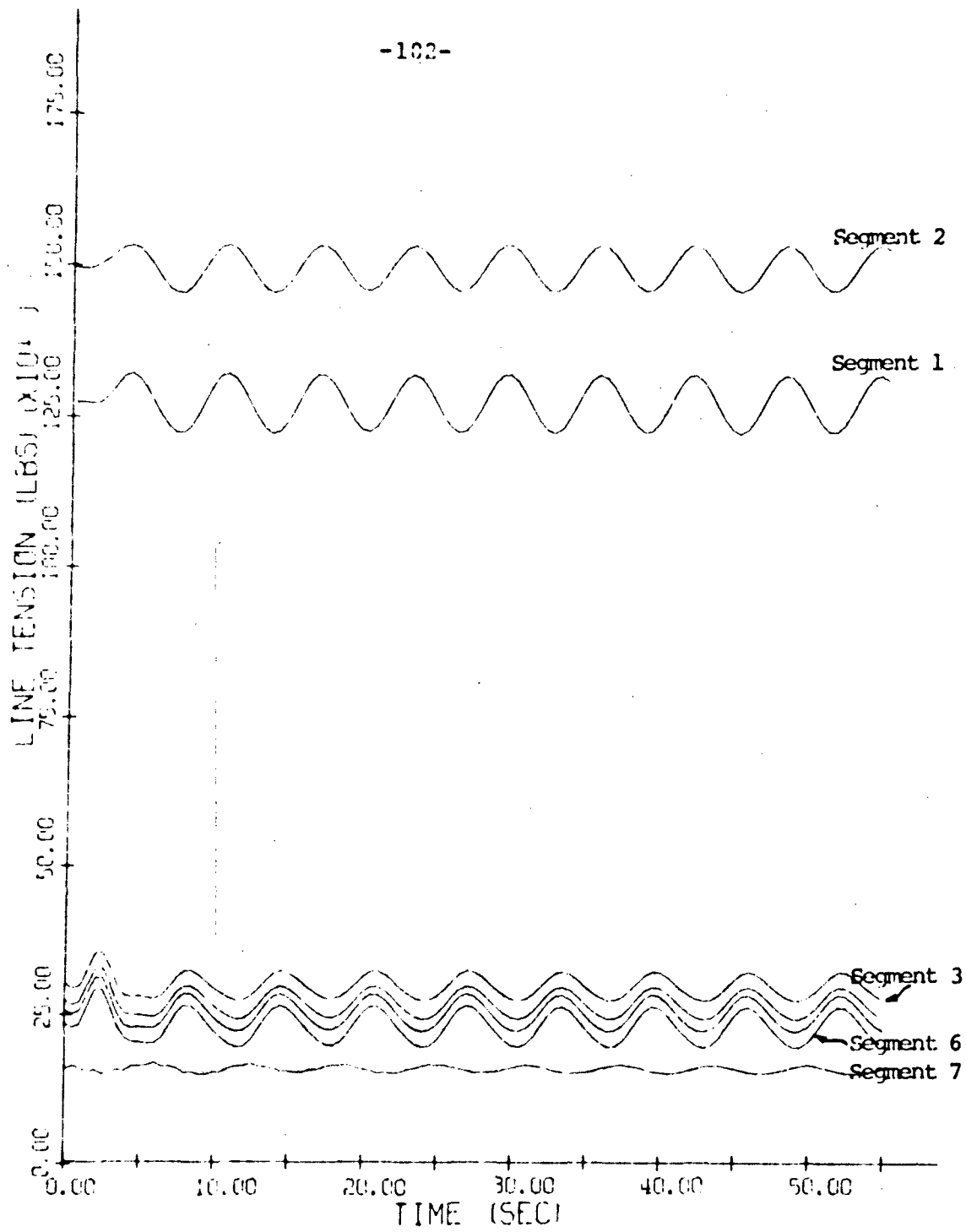


Figure 5.64 Tension Magnitudes in Different Segments of the Surface Moored System

SMDE. DATA	7	6	0	25	1	1	5700.	0.02	55.0	0.5
00010	1.7	0.5	0.5	3.35	1	5.11	30.	1.0	1.0	5.27
00020	1.0	1.0	1.0	2.93	2.93	12.86	6.99	1.19	1.19	
00021	230.	16.71	34.2	36.27	36.27	78.22	5.64	0.46	0.46	
00025	-1317.61	4.2	5.75	0.52	0.52	6.1	6.1	0.	0.	
00030	-54.99	4.2	4.2	0.	0.	7.14	6.1	0.	0.	
00035	-34.99	4.2	4.2	0.	0.	6.1	6.1	0.	0.	
00040	-54.99	4.2	4.2	0.	0.	6.1	6.1	0.	0.	
00050	3545.62	14.12	102.	2.93	2.93	15.73	9.86	1.95	1.95	
00055	162.24	102.	315.	46.36	46.36	1.40	1.48	-1727.	-1727.	
00060	4061.	315.	1000.	28.51	28.51	0.85	7.78	-422.	-422.	
00070	1322.	1000.	1000.	31.25	31.25	0.94	0.	-284.	-284.	
00075	250.0	1000.	1000.	31.25	31.25	0.94	0.	-173.	-173.	
00080	250.0	1000.	1000.	31.25	31.25	0.94	0.	-74.	-74.	
00085	250.0	1000.	1000.	0.	0.	0.31	1.48	-16.	-16.	
00090	250.0	1000.	1000.	10.25	10.25	0.0	0.0	-292.	-292.	
00100	276.	0.0	0.2	3.00	3.00	1.2	0.3	-3.35	-3.35	
00105	-0.5	0.2	0.	0.	0.	0.4	0.	0.	0.	1.4
00110	0.5	0.	2.5	1.8	1.8	0.	1.4	2.5	2.5	0.0
00111	0.2	1.	1.	0.0	0.0	0.	0.	0.2	0.2	0.
00112	0.	1.	1.	0.1	0.1	0.	0.	0.	0.	
00120	1.0	1.	1.	0.1	0.1	0.	0.	0.	0.	
READY										

Table 5.10 Input Data for the Surface Moored Buoy System

5.4 Drifting Drogued Buoy System

A submerged window shade drogue attached to a cylindrical spar buoy by means of an elastic nylon line is simulated on the computer by the three dimensional computer program SD3.FORT. The system is subjected to simple harmonic surface waves without any current profile. A 47 feet long 3/8" nylon line is attached to the center of the bottom of the buoy. The spar buoy is 30 feet long and 0.66 foot in diameter. Its static draft with no lines attached is 19.43 feet and weighs 426 pounds in air. The distance between the center of gravity and the bottom end of the buoy is 8.69 feet. The window shade drogue is made of nylon cloth ($9 \frac{1}{4}$ oz/yd²) and has the dimensions of 7.5' x 32'. Its approximate thickness is 0.02". The drogue has two metallic bars, one at each end. The bottom bar also acts as the dead weight, keeping the drogue in tension. For computer simulation the drogue was divided into four strips and the parameters of each strip lumped at a node. Tensions in three links between these four nodes and the tension in the nylon tether line were simulated on the computer. Figure 5.65 shows the system. Figures 5.66 through 5.70 show the time histories of the response of the system in a surface wave of amplitude 1 foot and circular frequency of 1 radian per second, and having a wave direction inclined at 0.25 radians with the x-axis. No current profile is present. Figures 5.66

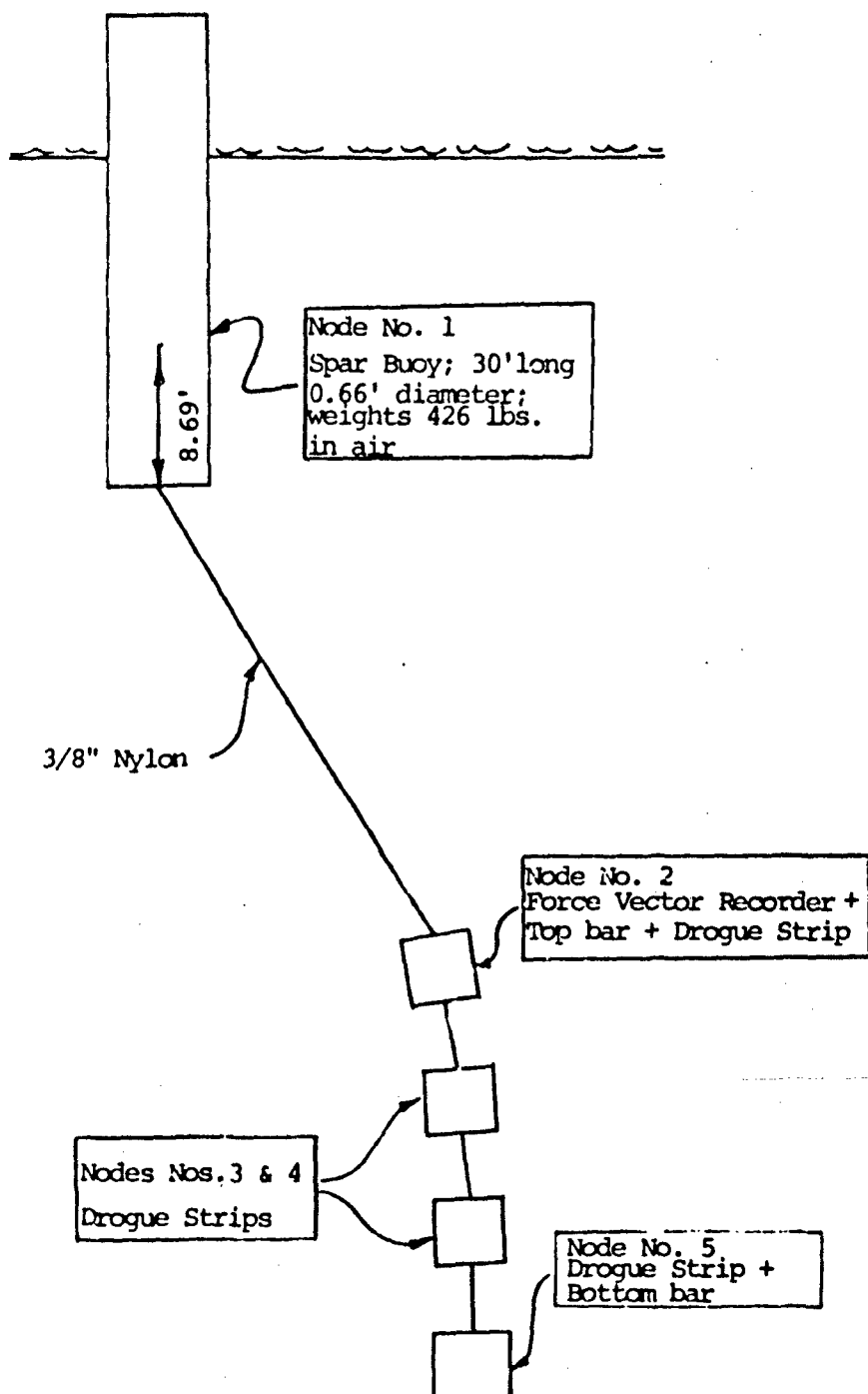


Figure 5.65 Drifting Drogued Buoy System

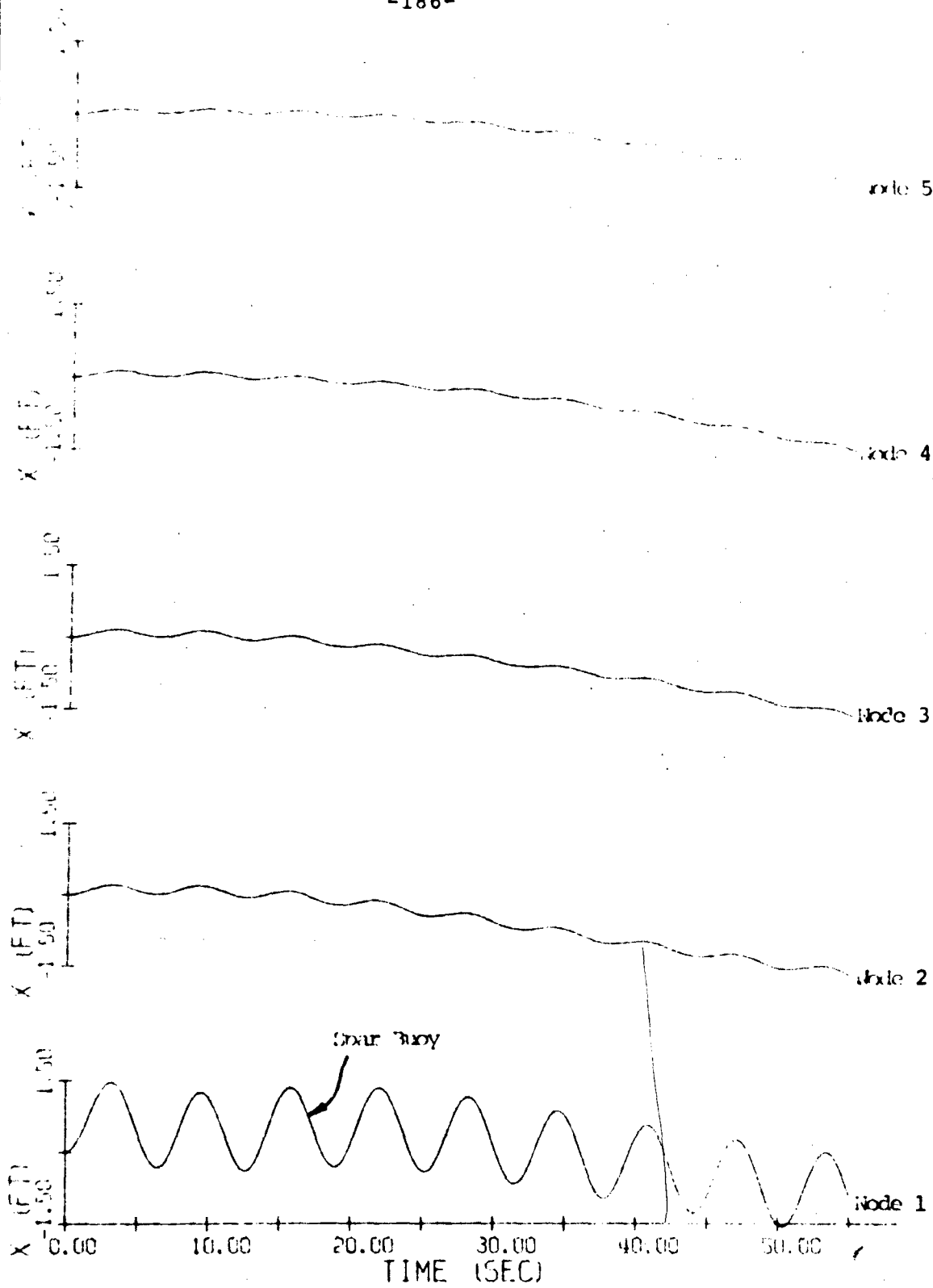


Figure 5.66 Surge Motions of the Drifting Moored System

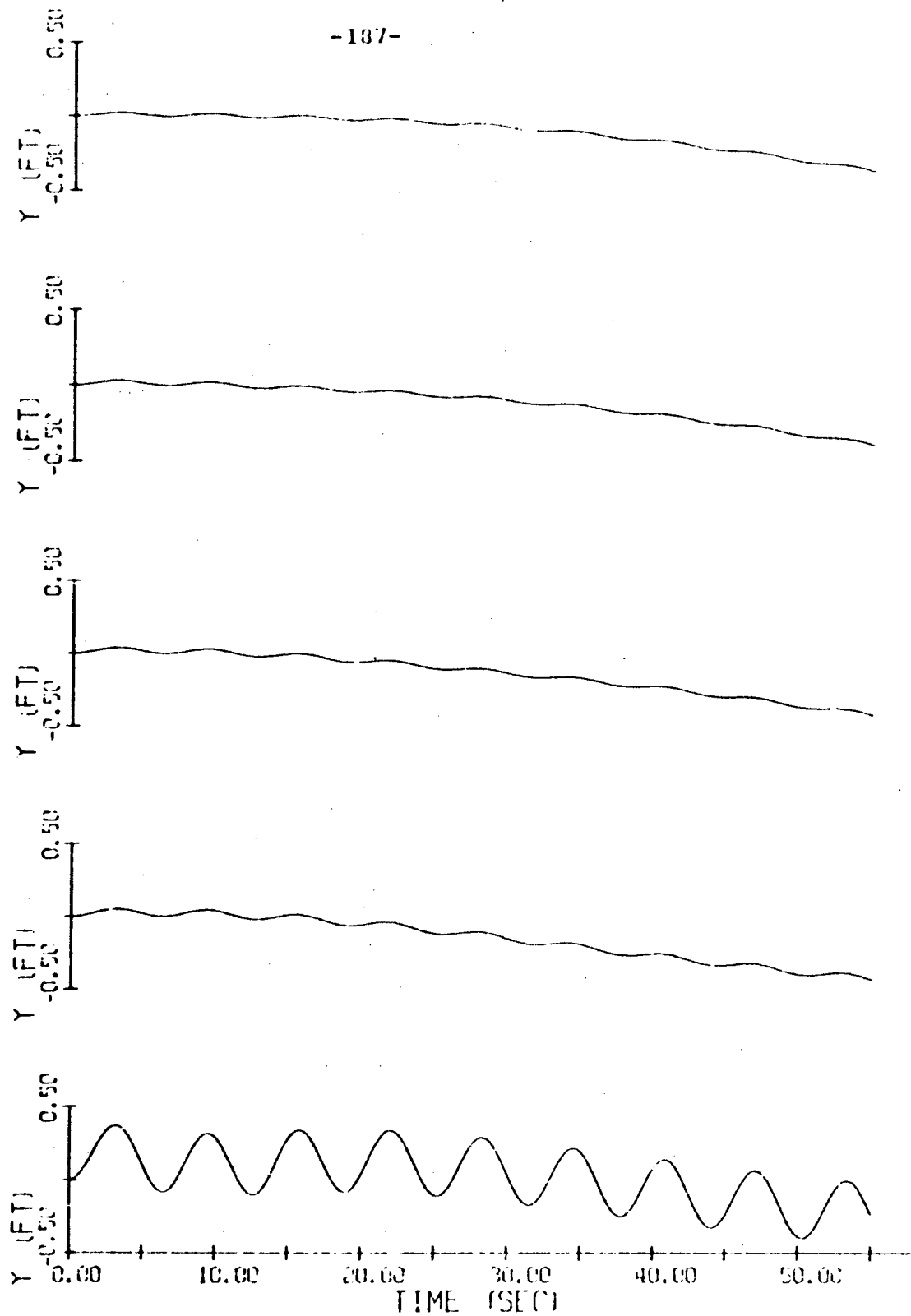


Figure 5.67 Sway Motions of the Drifting Drogued System

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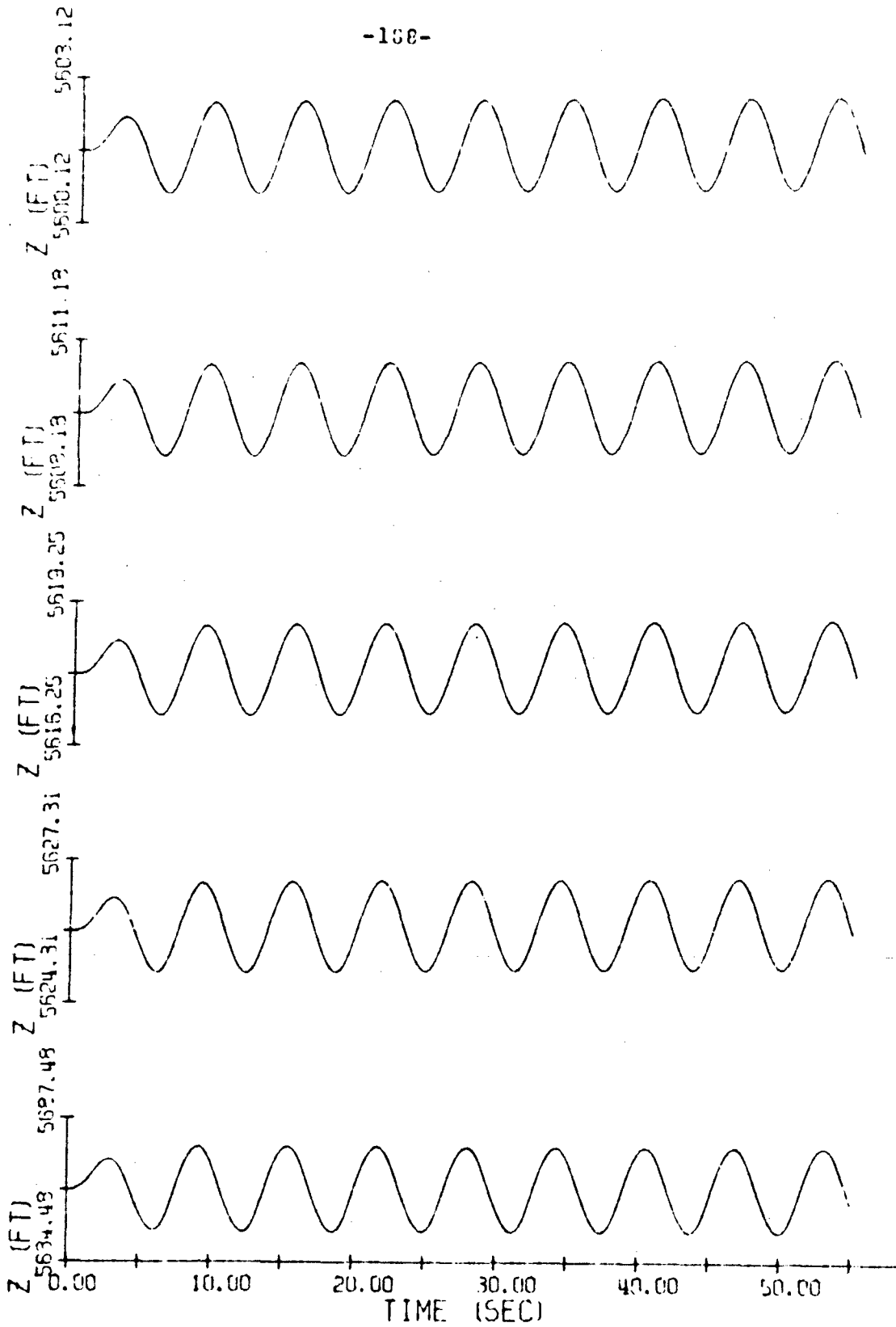


Figure 5.68 Heave Motions of the Drifting Moored System

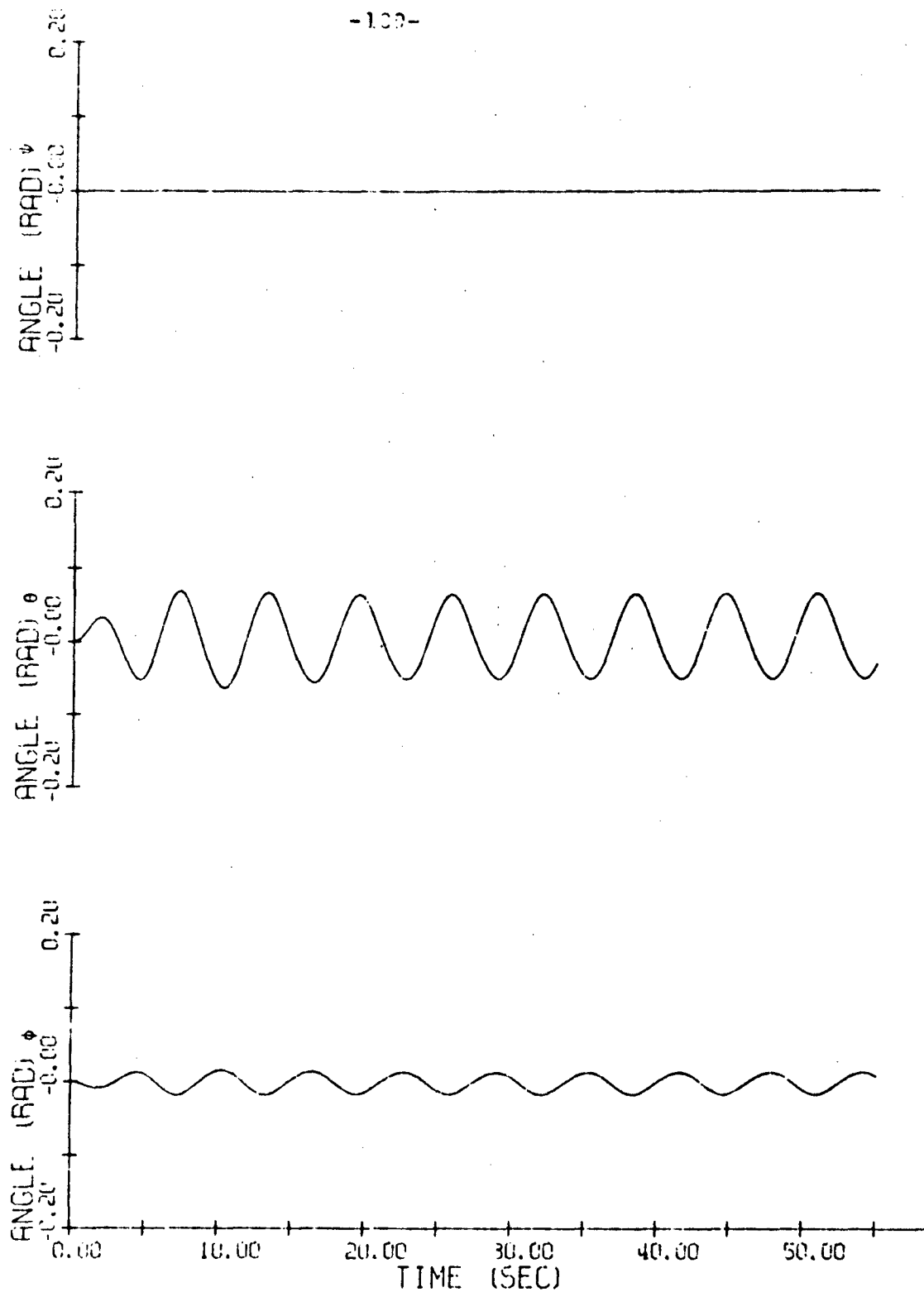


Figure 5.09 Roll, Pitch, and Yaw Motion of the Drifting Drogued Spar Buoy

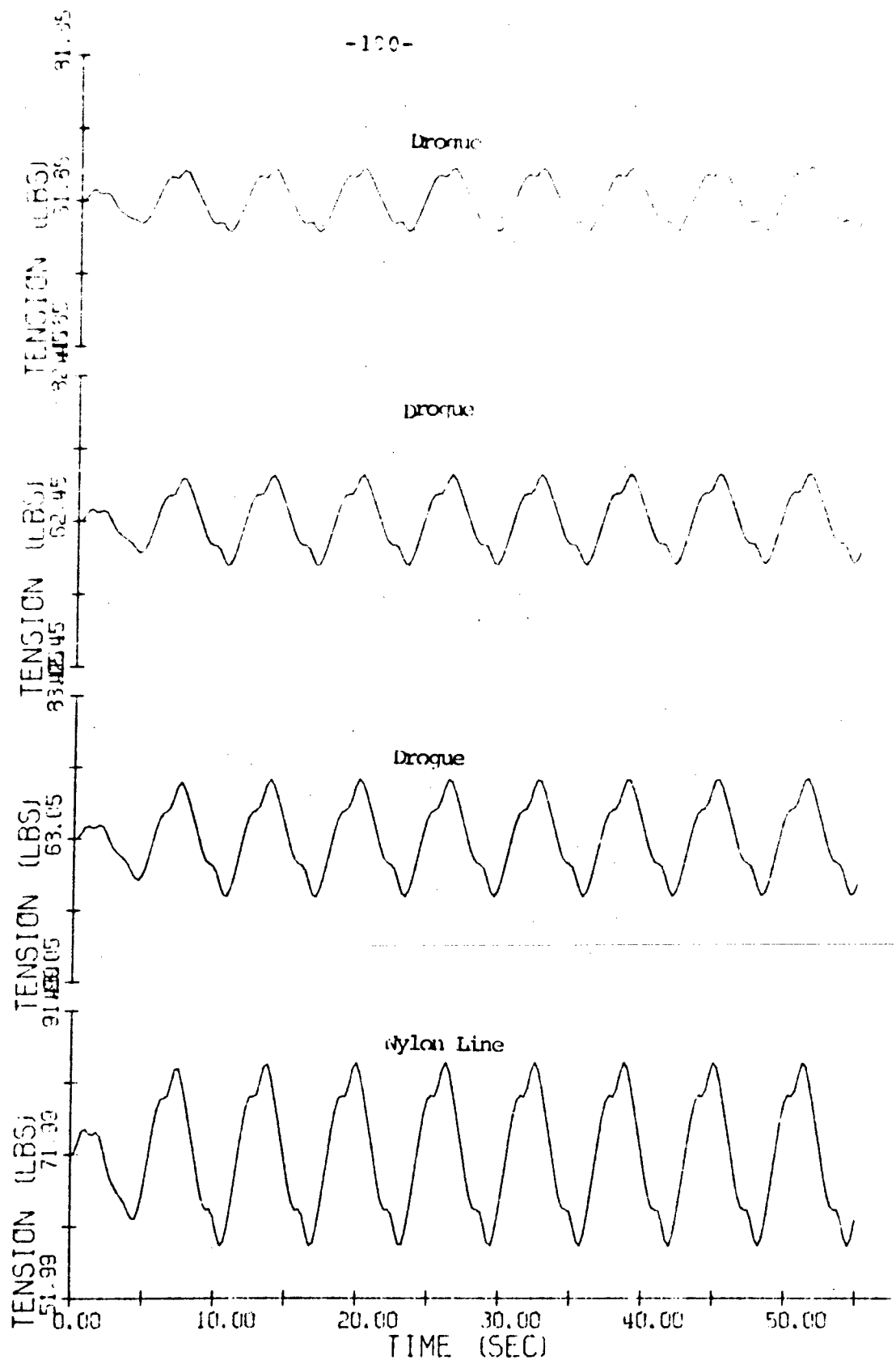


Figure 5.70 Tension Magnitude in Different Segments of the Drifting Drogue Buoy System

through 5.68 show the surge, sway and heave coordinates of the spar buoy C.G. and the four nodes on the drogue. Figure 5.69 shows the roll, pitch, and yaw angles of the buoy and Figure 5.70 shows the tension magnitudes in the nylon line below the buoy and the three segments of the drogue. The input data used for this simulation is presented in Table 5.11.

SDPB.DATA										
	5	1	2	25	1	2	5700.	0.010	55.0	0.25
00010	0.33		0.33	8.69			6.45	30.	1.0	5.27
00020	1.0		1.0							
00021	426.									
00030	8.94		3.0	1.1			705.1	704.	0.	
00031	0.6		0.1	0.			704.	704.	0.	
00032	0.6		0.1	0.			704.	704.	0.	
00033	61.85		2.6	0.5			704.5	704.	0.	
00034	5500.		0.0						-14.	
00060	51.		150.	114.5			2.3	2.	-74.	
00062	8.		1000.0	114.5			2.3	0.	-82.	
00064	8.		1000.0	114.5			2.3	0.	-90.	
00066	8.		1000.0	114.5			2.3	1.3	-98.	
00100	0.0		0.0	0.0			0.0	0.0	-8.69	
00105	0.0		0.0	0.0			0.00	0.0	0.0	0.00
00112	0.0		0.0	0.0			0.00	0.0	0.0	0.0
00113	0.0		0.0	0.00			0.0	0.0	0.00	0.0
00114	0.0									
00120	1.		1.0	0.25						
READY										

Table 5.11 Input Data for the Drifting Drogued Buoy System

6.0 SUMMARY

Dynamics of moored and drifting buoy systems in a three-dimensional space has been presented in this report. Formulation of mathematical models, which are programmed on the inhouse computer AMDAHL 470 V6, along with time-domain computer simulations of these systems are presented. Four case studies (freely floating spar buoy; a single point subsurface moored system; a spar buoy plus instrument line/buoyant tether/subsurface mooring; a freely drifting spar buoy attached to a window shade drogue) are simulated with forcing being supplied by a velocity profile and a fully developed surface wave field. One (a single point subsurface moored system) of these four case studies was evaluated with full scale ocean test data (CHHABRA, DAHLEN and FROIDEVAUX, 1974; CHHABRA, 1976). Simulations of other three case studies can now be readily evaluated with full scale ocean test data.

For the two-dimensional analysis of a freely floating spar buoy, where the assumption of small motions is not made, three sets of computer simulations are compared with the undamped linear analytical solution. This comparison is presented in Figures 5.8 through 5.11 for a cylindrical spar and Figures 5.31 through 5.34 for a tuned spar buoy. This comparison shows the effects of viscous drag and varying wave amplitude on the response

of a freely floating spar buoy. Plots for the undamped linear analytical solution and the computer simulation where wave amplitude is small and viscous drag is neglected are almost identical. For the case of a cylindrical spar, when viscous drag is added, the surge magnification vs Kh plots show a scatter near the heave and pitch natural frequencies (Figure 5.8). This emphasizes the coupling effects. Surge drift vs. Kh plots in Figure 5.9 show a markedly different responses near these same frequencies and viscous damping is evident in heave and pitch magnification plots (Figures 5.10 and 5.11). The non-linearity in the response with respect to wave amplitude is also evident. These plots also show a heave-pitch coupling near their resonance frequencies. Similar observations can be made for the case of the tuned spar.

For the three-dimensional analysis of a freely floating spar buoy, comparison is made with the two-dimensional analysis for the same input data. The only appreciable difference is observed in the surge drift, which can be attributed directly to the small motion assumption of the three-dimensional analysis.

Simulation of the spar buoy plus instrument line tethered to a subsurface mooring line shows a noteworthy transmittal of motion from the surface buoy to the subsurface line, by the tether. The tether line seems to

move mainly in the tangential direction due to the high viscous drag force in the transverse direction. As a result the combined surge, sway, and heave motions of the spar are transmitted to the subsurface line. Hence, the surge/sway motions of the spar (even if heave was negligible) could be transmitted to the subsurface line as heave. Therefore some modifications of the system may be necessary to minimize longitudinal motions of the subsurface line.

Simulation of the freely drifting spar buoy attached to a window shade drogue shows drift of the buoy and the drogue even though no current (except surface wave) is present. This drift is due to the same reasons as postulated for the freely floating spar buoy.

Full scale ocean test data for the spar buoy plus instrument line/buoyant tether/subsurface mooring is now available (October 1976 ONR/NDBO Mooring Dynamics Experiment). Using this and other data which might be available in the future; the next logical step is the evaluation of the mathematical models of the remaining three case studies.

APPENDIX A

Computer Program Listings	Page
SD3.FORT	197
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SSS3.FORT	255
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```

SD3.FORT
00010 C
00020 C
00030 C
00040 C
00050 C
00060 C
00070 C
00080 C
00090 C
00100 C
00110 C
00120 C
00130 C
00140 C
00150 C
00160 C
00170 C
00180 C*****START OF THE MAIN PROGRAM*****
00190 IMPLICIT REAL*8(A-H,O-Z)*****
00200 COMMON W(8),CM(8),CMS(8),CMD(8),CMN(8),CMT(8),SL(8),EK(8),CDIN(8),
00210 1CDIT(8),CDIA(8),DEP(8),TN(8),TNN(8),TL(8),YDD(27),YD(27),Y(27),X(2
00220 17),F(27),DD(27),TC(30),CS(30),VN(3),V(3,8),VV(3,8),GK(8,27),GM(27,
00230 127),WWD1,WWD2,HMAX,H0,HI,HST,HOD,XC1,YC1,ZC1,XC2,YC2,ZC2,RRH01,RRH
00240 102,XJ1,YJ1,ZJ1,XJ2,YJ2,ZJ2,RD1,RD2,S01,S02,BB1,BB2,CSB,SNB,THASS,Y
00250 1NERT1,YNERT2,VV0,ZCG,SNK,CSK,DEPTH,CDL,CDP,PI,RHO,GR,OEXP,AW,AWX,A
00260 1WY,AMP,WE,T,DT,P1,P2,Q0,Q1,NST,NM,NM1,NM2,NDF,NB,NB1,NB2,NN,MOOR,N
00270 1D
00280 100 FORMAT(12F10.2)
00290 101 FORMAT(7F10.0)
00300 102 FORMAT(6I5,4F10.0)
00310 103 FORMAT(6F10.0)

3 D DYNAMIC ANALYSIS OF A SPAR BUOY, A SURFACE MOORED SYSTEM,
AND A DRIFTING DROGUED SPAR BUOY.
-----
THE UNITS USED IN THIS ANALYSIS ARE FEET/POUNDS/SECONDS.
-----
CASE STUDY 1--MOOR=0--SPAR BUOY FREELY FLOATING IN A SURFACE WAVE.
CASE STUDY 2--MOOR=1--SPAR BUOY ANCHORED WITH A LUMPED MASS
MOORING LINE; NM=NB.
CASE STUDY 3--MOOR=1--SPAR BUOY ANCHORED WITH A LUMPED MASS
MOORING LINE AND A LUMPED MASS INSTRUMENT LINE HANGING FROM THE
BUOY; NM NOT EQUAL TO NB.THIS CASE STUDY SIMULATES THE MOORING
DYNAMICS EXPERIMENT ALTERNATE MOORING SYSTEM CONFIGURATION.
CASE STUDY 4--MOOR=2--SPAR BUOY ATTACHED TO A WINDOW SHADE DROGUE
NB=1.
-----

```

```

00320 C*****GENERAL DATA*****
00330 READ(5,102)NM,NB,ND,NST,NC,MOOR,DEPTH,DT,TMAX,T2
00340 C*****BUOY DATA*****
00350 READ(5,101)RD1,SD2,ZCG,RGYR,HMAX,CDL,HST,CDF,ALPHA
00360 C*****MOORING LINE DATA*****
00370 READ(5,103)(W(I),CM(I),CMS(I),CMD(I),CMN(I),CMT(I),I=1,NM)
00380 READ(5,103)(SL(I),EK(I),CDIN(I),CDIT(I),CDIA(I),DEP(I),I=1,NM)
00390 C*****ATTACHMENT DATA*****
00400 READ(5,101)XC1,YC1,ZC1,XC2,YC2,ZC2
00410 C*****CURRENT PROFILE DATA*****
00420 READ(5,101)((V(IJ,I),IJ=1,3),I=1,NM)
00430 NM1=NM-NB
00440 NN=3*NB
00450 NB1=NB-1
00460 NB2=NB+1
00470 NM2=NM+1
00480 NDF=3*NM+3
00490 C*****INITIALIZE PHYSICAL CONSTANTS*****
00500 PI=4.*DATAN(1.D0)
00510 S01=PI*RD1*RD1
00520 S02=PI*RD2*RD2
00530 RHO=1.9905
00540 GR=32.2
00550 TMASS=W(NB)/GR
00560 YNERT1=TMASS*RGYR*RGYR
00570 RRH01=RHO*CDL*RD1*HST
00580 RRH02=RHO*CDL*RD2
00590 BB1=RHO*S01
00600 BB2=RHO*S02
00610 YNERT2=(BB1*RD1*RD1*HST+BB2*RD2*RD2*(HMAX-HST))/2.
00620 WWD1=BB1*GR*HST
00630 WWD2=BB2*GR

```

```

00640      VV0=1.33*RHO*ALPHA*(RD1**3+(RD1-RD2)**3)
00650 C*****START LOOP FOR DYNAMIC RESPONSE TO DIFFERENT SURFACE WAVES*****
00660      DO 12 ICC=1,NC
00670 C*****SURFACE WAVE DATA*****
00680      READ(5,101)WE,AMP,BETA
00690      CSB=DCOS(BETA)
00700      SNB=DSIN(BETA)
00710 C*****INITIALIZE DYNAMIC VARIABLES*****
00720      DO 1 I=1,NDF
00730      F(I)=0.
00740      YD(I)=0.
00750      Y(I)=0.
00760      TC(I)=0.
00770      DD(I)=0.
00780      DO 1 J=1,NM
00790      1 GK(J,I)=0.
00800      AW=WE*WE/GR
00810      AWX=AW*CSB
00820      AWY=AW*SNB
00830      T3=0.
00840 C*****FIND THE CURRENT PROFILE FOR STATIC SOLUTION*****
00850      DO 4 I=1,NM
00860      DO 2 IS=1,3
00870      2 VV(IS,I)=0.
00880      ZX1=WE*AMP*DEXP(AW*DEP(I))
00890      DO 3 J=1,12
00900      VN(1)=V(1,I)+ZX1*CSB*DSIN(PI*J/6.)
00910      VN(2)=V(2,I)+ZX1*SNB*DSIN(PI*J/6.)
00920      VN(3)=V(3,I)+ZX1*DCOS(PI*J/6.)
00930      VEM=DSQRT(VN(1)**2+VN(2)**2+VN(3)**2)
00940      DO 3 IJK=1,3
00950      3 VV(IJK,I)=VV(IJK,I)+VEM*VN(IJK)

```

```

00960 VEM1=(VV(1,I)**2+VV(2,I)**2+VV(3,I)**2)**0.25
00970 DO 4 IKJ=1,3
00980 VV(IKJ,I)=VV(IKJ,I)/(VEM1*12.**0.5)
00990 4 IF(DABS(VV(IKJ,I)).LE.0.000001)VV(IKJ,I)=0.
01000 WRITE(6,100)((VV(IJ,I),IJ=1,3),I=1,NM)
01010 TC(NDF+1)=0.
01020 TC(NDF+2)=0.
01030 TC(NDF+3)=0.
01040 IF(NM.EQ.NB)GO TO 8
01050 CS(NDF+1)=0.
01060 CS(NDF+2)=0.
01070 CS(NDF+3)=-1.
01080 IF(MOOR.EQ.)GO TO 8
01090 C*****STATIC SOLUTION OF THE LUMPED MASSES (INSTRUMENT LINE)*****
01100 DO 7 I=1,NM1
01110 J=NM2-I
01120 K=J+1
01130 II=3*K+1
01140 VN(1)=VV(1,J)*(1.-CS(II)*CS(II))-VV(2,J)*CS(II)*CS(II+1)
01150 1-VV(3,J)*CS(II)*CS(II+2)
01160 VN(2)=VV(2,J)*(1.-CS(II+1)*CS(II+1))-VV(3,J)*CS(II+1)*CS(II+2)
01170 1-VV(1,J)*CS(II+1)*CS(II)
01180 VN(3)=VV(3,J)*(1.-CS(II+2)*CS(II+2))-VV(1,J)*CS(II+2)*CS(II)
01190 1-VV(2,J)*CS(II+2)*CS(II+1)
01200 VT=VV(1,J)*CS(II)+VV(2,J)*CS(II+1)+VV(3,J)*CS(II+2)
01210 VNM=DSQRT(VN(1)*VN(1)+VN(2)*VN(2)+VN(3)*VN(3))
01220 VNM1=DSQRT(VV(1,J)*VV(1,J)+VV(2,J)*VV(2,J)+VV(3,J)*VV(3,J))
01230 DDTT=CDIT(J)*VT*DABS(VT)
01240 DO 5 IC=1,3
01250 KK=II+IC-4
01260 TC(KK)=TC(NK+3)+CDIA(J)*VNM1*VV(IC,J)+DDTT*CS(KK+3)+CDIN(J)*VNM*VN
01270 1(IC)

```

```

01280 5 CONTINUE
01290 TC(II-1)=TC(II-1)-W(J)
01300 TNN(J)=DSORT(TC(II-3)**2+TC(II-2)**2+TC(II-1)**2)
01310 DO 6 IC=1,3
01320 KK=II-4+IC
01330 CS(NK)=TC(KK)/TNN(J)
01340 7 TL(J)=SL(J)+TNN(J)/EK(J)
01350 8 CALL STATIC
01360 IF(MOOR.GT.0)GO TO 9
01370 CALL BUOYS
01380 CALL MINVER(NDF,GM)
01390 9 T=0.
01400 10 SS=AWX*X(NN-2)+AWY*X(NN-1)-WE*T
01410 SNK=DSIN(SS)
01420 CSK=DCOS(SS)
01430 OEXP=DEXP(-HO*AW)
01440 HI=HO-AMP*SNK
01450 IF(CDL.NE.0.0)CALL DEDRG
01460 IF(MOOR.GT.0)CALL BUOYS
01470 CALL MATRIX
01480 IF(T.LT.T3)GO TO 11
01490 WRITE(6,100)T
01500 WRITE(6,100)(X(I),I=1,NDF)
01510 IF(MOOR.EQ.1)WRITE(6,100)(TN(K),K=1,NH)
01520 IF(MOOR.EQ.2)WRITE(6,100)(TN(K),K=2,NH)
01530 IF(T.GT.TMAX)GO TO 12
01540 T3=T3+T2
01550 11 T=T+DT
01560 IF(MOOR.GT.0)CALL NEXT
01570 GO TO 10
01580 12 CONTINUE
01590 CALL EXIT

```

```

01600      END
01610      IMPLICIT REAL*8(A-H,O-Z)
01620      SUBROUTINE STATIC
01630 C      THIS SUBROUTINE CALCULATES THE STATIC SOLUTION TO THE FOUR CASE
01640 C      STUDIES OUTLINED IN THE MAIN PROGRAM.
01650 C      -----
01660 C
01670      COMMON W(8),CM(8),CMS(8),CMT(8),CMN(8),CMT(8),SL(8),EK(8),CDIN(8),
01680      1CDIT(8),CDIA(8),DEP(8),TN(8),TNN(8),TL(8),YDD(27),YD(27),Y(27),X(2
01690      17),F(27),DD(27),TC(30),CS(30),VN(3),V(3,8),VV(3,8),GK(8,27),GM(27,
01700      127),WWD1,WWD2,HMAX,H0,HI,HST,HOU,XC1,YC1,ZC1,XC2,YC2,ZC2,RRH01,RRH
01710      102,XJ1,YJ1,ZJ1,XJ2,YJ2,ZJ2,RD1,RD2,S01,S02,RB1,RB2,CSB,SNB,TMASS,Y
01720      1NERT1,YNERT2,UVO,ZCG,SNK,CSK,DEPTH,CDL,CDP,PI,RHD,GR,OEXF,AW,AWX,A
01730      1WY,AMP,WE,T,DT,F1,P2,Q0,Q1,NST,NM,NM1,N*2,NDF,NB,NB1,NB2,NN,MOOR,N
01740      1D
01750      100 FORMAT(12F10.2)
01760      102 FORMAT (///10X,65HSTATIC CALCULATION TERMINATED -- RUDY HAS EXCEED
01770      1ED MAXIMUM DRAFT )
01780      103 FORMAT (///10X,100HSTATIC CALCULATION TERMINATED -- NUMBER OF ITER
01790      1ATIONS TO FIND SYSTEM SOLUTION HAS EXCEEDED MAXIMUM ( I3,1H) )
01800      IF(MOOR.EQ.0)GO TO 26
01810      ITMAX=100
01820      IT=0
01830      IF(MOOR.EQ.1)GO TO 7
01840 C
01850 C*****CASE STUDY 4 *****
01860 C*****INITIALIZE VARIABLES*****
01870 C*****
01880      IS2=0
01890      DC3=0.
01900      VE=0.
01910      FXE=0.02*HMAX*RD2*V(1,1)*V(1,1)+0.01

```

```

01920 DUE=DABS(V(1,NM))/2.
01930 C***START STATIC VELOCITY ITERATION LOOP*****
01940 1 IT=IT+1
01950 IF(IT.GT.ITMAX)GO TO 27
01960 DO 3 I=1,NM1
01970 J=NM2-I
01980 K=J+1
01990 II=3*K+1
02000 U1=VV(1,J)-VE
02010 U2=VV(2,J)
02020 U3=VV(3,J)
02030 VN(1)=U1*(1.-CS(II)*CS(II))-U2*CS(II)*CS(II+1)-U3*CS(II)*CS(II+2)
02040 VN(2)=U2*(1.-CS(II+1)*CS(II+1))-U3*CS(II+1)*CS(II+2)-U1*CS(II+1)*C
02050 1S(JI)
02060 VN(3)=U3*(1.-CS(II+2)*CS(II+2))-U1*CS(II+2)*CS(II)-U2*CS(II+2)*CS(
02070 1II+1)
02080 VT=U1*CS(II)+U2*CS(II+1)+U3*CS(II+2)
02090 VNM=DSQRT(VN(1)*VN(1)+VN(2)*VN(2)+VN(3)*VN(3))
02100 VNM1=DSQRT(U1*U1+U2*U2+U3*U3)
02110 DDTT=CDIT(J)*VT*DABS(VT)
02120 TC(II-3)=TC(II)+CDIA(J)*VNM1*U1+DDTT*CS(II)+CDIN(J)*VNM*VN(1)
02130 TC(II-2)=TC(II+1)+CDIA(J)*VNM1*U2+DDTT*CS(II+1)+CDIN(J)*VNM*VN(2)
02140 TC(II-1)=TC(II+2)-W(J)+CDIA(J)*VNM1*U3+DDTT*CS(II+2)+CDIN(J)*VNM*V
02150 1N(3)
02160 TNN(J)=DSQRT(TC(II-3)**2+TC(II-2)**2+TC(II-1)**2)
02170 DO 2 IC=1,3
02180 KK=II-4+IC
02190 2 CS(KK)=TC(KK)/TNN(J)
02200 3 TL(J)=SL(J)+TNN(J)/EK(J)
02210 HO=HST+(W(1)-TC(9)-DC3-WWD1)/WWD2
02220 IF(HO.GT.0.99*HMAX)GO TO 28
02230 HOD=HO-HST

```

```

02240 VNM1=DSQRT((VV(1,1)-VE)**2+VV(2,1)**2)
02250 RRHO=RRHO1+RRHO2*HOD
02260 DC1=RRHO*VNM1*(VV(1,1)-VE)
02270 DC2=RRHO*VNM1*VV(2,1)
02280 DC3=(0.02*RRHO+0.5*RRHO*CDP*(2.*S01-S02))*DABS(VV(3,1))*VV(3,1)
02290 FX=DC1+TC(7)
02300 WRITE(6,100)VE,H0,FX
02310 IF(DABS(FX).LE.FXE)GO TO 4
02320 IS3=DSIGN(1.0D0,FX)
02330 IF(IS2+IS3.EQ.0)DVE=DVE/2.
02340 VE=VE+JS3*DVE
02350 IS2=IS3
02360 GO TO 1
02370 4 YD(1)=VE
02380 DO 5 I=2,NM
02390 II=3*I+1
02400 5 YD(II)=VE
02410 X(1)=0.
02420 X(2)=0.
02430 X(3)=ZCG-H0+DEPTH
02440 B=WWD1+WWD2*HOD
02450 TNN(1)=0.
02460 DO 6 IC=1,3
02470 6 CS(IC)=0.
02480 GO TO 18
02490 C
02500 C*****CASE STUDIES 2 AND 3 *****
02510 C
02520 7 INDEX=1
02530 NBB=NB-1
02540 H0=HST+(W(NB)-TC(NN+6)-WWD1)/WWD2
02550 DH=0.

```

```

02560      WADD=W(NBB)/10.
02570      VNM1=DSQRT(VV(1,NB)*VV(1,NB)+VV(2,NB)*VV(2,NB))
02580      C*****INITIALIZE ERROR TOLERANCE*****
02590      ER= 0.0010*DEPTH
02600      C*****START CABLE (BUOY DRAFT) ITERATION LOOP*****
02610      8  IT=IT+1
02620      DH=DH/2.
02630      IF (IT.GT.ITMAX) GO TO 27
02640      IF (HO.GT.0.99*HMAX)GO TO 28
02650      HOD=HO-HST
02660      B=WWD1+WWD2*HOD
02670      RRHO=KRHO1+RRHO2*HOD
02680      DC1=RRHO*VNM1*VV(1,NB)
02690      DC2=RRHO*VNM1*VV(2,NB)
02700      DC3=(0.02*RRHO+0.5*RRHO*CDP*(2.*S01-S02))*DABS(VV(3,NB))*VV(3,NB)
02710      TC(NN-2)=DC1+TC(NN+4)
02720      TC(NN-1)=DC2+TC(NN+5)
02730      TC(NN)=DC3+B-W(NB)+TC(NN+6)
02740      TNN(NB)=DSQRT(TC(NN-2)*TC(NN-2)+TC(NN-1)+TC(NN)*TC(NN))
02750      TL(NB)=SL(NB)+TNN(NB)/EK(NB)
02760      DO 9 IC=1,3
02770      KK=NN+IC-3
02780      CS(KK)=TC(KK)/TNN(NB)
02790      IF (NR.EQ.1)GO TO 14
02800      10  DO 13 I=1,NB1
02810      J=NB-I
02820      K=J+1
02830      II=J*3
02840      VN(1)=VV(1,J)*(1.-CS(II+1)*CS(II+1))-VV(2,J)*CS(II+1)*CS(II+2)
02850      1-VV(3,J)*CS(II+1)*CS(II+3)
02860      VN(2)=VV(2,J)*(1.-CS(II+2)*CS(II+2))-VV(3,J)*CS(II+2)*CS(II+3)
02870      1-VV(1,J)*CS(II+2)*CS(II+1)

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02880 VN(3)=VV(3,J)*(1.-CS(II+3)*CS(II+3))-VV(1,J)*CS(II+3)*CS(II+1)
02890 1-VV(2,J)*CS(II+3)*CS(II+2)
02900 VT=VV(1,J)*CS(II+1)+VV(2,J)*CS(II+2)+VV(3,J)*CS(II+3)
02910 VNM=DSQRT(VN(1)*VN(1)+VN(2)*VN(2)+VN(3)*VN(3))
02920 VNM2=DSQRT(VV(1,J)*VV(1,J)+VV(2,J)*VV(2,J)+VV(3,J)*VV(3,J))
02930 DDIT=CDIT(J)*VT*DABS(VT)
02940 DO 11 IC=1,3
02950 KK=II+IC-3
02960 TC(KK)=TC(KK+3)+CDIA(J)*VNM2*VV(IC,J)+DDIT*CS(KK+3)+CDIN(J)*VNM*VN
02970 1(IC)
02980 11 CONTINUE
02990 TC(II)=TC(II)-W(J)
03000 TNN(J)=DSQRT(TC(II)**2+TC(II-2)**2+TC(II-1)**2)
03010 DO 12 IC=1,3
03020 KK=II-3+IC
03030 CS(KK)=TC(KK)/TNN(J)
03040 TL(J)=SL(J)+TNN(J)/EK(J)
03050 ZJ1=C.
03060 DO 13 I=1,NB
03070 II=I*3
03080 ZJ1=ZJ1+TL(I)*CS(II)
03090 DEPTH1=ZJ1+(HO-ZCG-ZC1)
03100 EE=DEPTH-DEPTH1
03110 WRITE(6,100)HO,EE,(W(NCC),NCC=1,NB)
03120 EEE=DABS(EE)
03130 IF(EEE.LE.ER)GO TO 17
03140 IF(INDEX.EQ.1.AND.EE.LT.0.)GO TO 25
03150 IF(INDEX.EQ.2)GO TO 16
03160 INDEX=2
03170 DH=(HMAX-HO)/2.
03180 C*****ITERATION ON BUOY DRAFT*****
03190 HO=HO+DH

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```

03200      GO TO 8
03210      16  HO=HO+EEEE*DH/EE
03220      GO TO 8
03230      17  DEPTH=DEPTH1
03240      WRITE(6,100)DEPTH
03250      C*****STATIC SOLUTION HAS CONVERGED--COMPUTE ALL ANGLES AND COORDINATES
03260      18  AL1=(S01*HST*(HST/2.-ZCG)+S02*HOD*(H0+HST)/2.-ZCG)/(S01*HST+S02*
03270      1HOD)
03280      AL2=(HST/2.-ZCG)+RD2*HOD*H0/(2.*(RD1*HST+RD2*HOD))
03290      GM(1,1)=B*AL1+DC3*AL2-TC(NN-1)*YC1-TC(NN)*ZC1+TC(NN+5)*YC2+TC(NN+6
03300      1)*ZC2
03310      GM(1,2)=TC(NN-1)*XC1-TC(NN+5)*XC2
03320      GM(1,3)=TC(NN)*XC1-TC(NN+6)*XC2
03330      GM(2,1)=TC(NN-2)*YC1-TC(NN+4)*YC2
03340      GM(2,2)=B*AL1+DC3*AL2-TC(NN-2)*XC1-TC(NN)*ZC1+TC(NN+4)*XC2
03350      1+TC(NN+6)*ZC2
03360      GM(2,3)=TC(NN)*YC1-TC(NN+6)*YC2
03370      GM(3,1)=-DC1*AL2+TC(NN-2)*ZC1-TC(NN+4)*ZC2
03380      GM(3,2)=-DC2*AL2+TC(NN-1)*ZC1-TC(NN+5)*ZC2
03390      GM(3,3)=-TC(NN-2)*XC1-TC(NN-1)*YC1+TC(NN+4)*XC2+TC(NN+5)*YC2
03400      IF(GM(3,3).EQ.0.)GM(3,3)=0.00001
03410      VN(1)=-DC2*AL2+TC(NN-1)*ZC1-TC(NN)*YC1-TC(NN+5)*ZC2+TC(NN+6)*YC2
03420      VN(2)=DC1*AL2-TC(NN-2)*ZC1+TC(NN)*XC1+TC(NN+4)*ZC2-TC(NN+6)*XC2
03430      VN(3)=TC(NN-2)*YC1-TC(NN-1)*XC1-TC(NN+4)*YC2+TC(NN+5)*XC2
03440      CALL MINVER(3,GM)
03450      DO 19 IC=1,3
03460      X(NN+IC)=0.
03470      DO 19 IC=1,3
03480      19  X(NN+IC)=X(NN+IC)+GM(IC,IC1)*VN(IC1)
03490      IF(MOOR.EQ.2)GO TO 22
03500      X(1)=TL(1)*CS(1)
03510      X(2)=TL(1)*CS(2)

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```
03520 X(3)=TL(1)*CS(3)
03530 IF(NB.EQ.1)GO TO 21
03540 DO 20 I=2,NB
03550 II=3*I
03560 X(II-2)=X(II-5)+TL(I)*CS(II-2)
03570 X(II-1)=X(II-4)+TL(I)*CS(II-1)
03580 X(II)=X(II-3)+TL(I)*CS(II)
03590 20 XJ1=X(NN-2)
03600 21 X(NN-2)=XJ1-XC1
03610 YJ1=X(NN-1)
03620 X(NN-1)=YJ1-YC1
03630 X(NN)=ZJ1-ZC1
03640 IF(NH.EQ.NB)GO TO 24
03650 22 XJ2=X(NN-2)+XC2
03660 YJ2=X(NN-1)+YC2
03670 ZJ2=X(NN)+ZC2
03680 X(NN+4)=XJ2+TL(NB2)*CS(NN+4)
03690 X(NN+5)=YJ2+TL(NB2)*CS(NN+5)
03700 X(NN+6)=ZJ2+TL(NB2)*CS(NN+6)
03710 IF(NB2.EQ.NH)GO TO 24
03720 NB3=NB2+1
03730 DO 23 I=NB3,NH
03740 II=3*I+1
03750 X(II)=X(II-3)+TL(I)*CS(II)
03760 X(II+1)=X(II-2)+TL(I)*CS(II+1)
03770 X(II+2)=X(II-1)+TL(I)*CS(II+2)
03780 23 WRITE(6,100)HO,X(NN+1),X(NN+2),X(NN+3)
03790 24 WRITE(6,100)(X(I),I=1,NDF)
03800 RETURN
03810 C*****IF PART OF THE MOORING LINE IS FLOATING ON THE SURFACE FIND IT****
03820 25 W(NBB)=W(NBB)-WADD
03830 IF(W(NBB).LT.0.)GO TO 10
```

```
03840 W(NBB)=0.
03850 NBB=NBB-1
03860 GO TO 25
03870 C
03880 C*****CASE STUDY 1 *****
03890 C
03900 26 HO=HST+(W(1)-WWD1)/WWD2
03910 WAW=AW*HO
03920 OEXP=DEXP(-WAW)
03930 HOD=HO-HST
03940 BK1=(BB1-BB2)*HST
03950 BK2=BB2*HO
03960 P1=BK2*(0.5*HO-ZCG)+BK1*(0.5*HST-ZCG)
03970 P2=BK2*(HO*HO/3.-HO*ZCG+ZCG*ZCG)+BK1*(HST*HST/3.-HST*ZCG+ZCG*ZCG)
03980 Q02=DEXP(-HOD*AW)
03990 Q0=(BB1*(Q02-OEXP)+BB2*(1.-Q02))/AW
04000 Q1=(BB1*(Q02*(HST-1./AW-ZCG)+OEXP*(1./AW+ZCG))+BB2*(HO-1./AW-ZCG-Q
04010 1Q2*(HST-1./AW-ZCG)))/AW
04020 X(1)=-AMP*CSB*Q0/TMASS
04030 X(2)=-AMP*SNB*Q0/TMASS
04040 X(3)=ZCG-HO+DEPTH
04050 X(4)=-WE*WE*AMP*SNB*(2.*Q1-P1*Q0/TMASS)/(P1*GR-WE*WE*(YNERT1+P2-P1
04060 1*P1/(2.*TMASS)))
04070 X(5)=WE*WE*AMP*CSB*(2.*Q1-P1*Q0/TMASS)/(P1*GR-WE*WE*(YNERT1+P2-P1*
04080 1P1/(2.*TMASS)))
04090 X(6)=0.
04100 YD(1)=VV(1+1)
04110 YD(2)=VV(2+1)
04120 YD(3)=WE*AMP*(RHO*S02*GR-WE*WE*Q0)/(RHO*S02*GR-WE*WE*(TMASS+VV0))
04130 XAW=X(1)/AMP
04140 YAW=X(2)/AMP
04150 ZAW=YD(3)/(WE*AMP)
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04160 PAW=X(5)/(AW*AMP)
04170 RAW=X(4)/(AW*AMP)
04180 WRITE(6,100)WAW,XAW,YAW,ZAW,RAW,PAW
04190 RETURN
04200 WRITE(6,103)ITMAX
04210 CALL EXIT
04220 WRITE(6,102)
04230 CALL EXIT
04240 END
04250 IMPLICIT REAL*8(A-H,O-Z)
04260 SUBROUTINE DBDRG
04270 C
04280 C THIS SUBROUTINE CALCULATES THE VISCOUS DRAG FORCE ON THE
04290 C SPAR BUOY DURING ITS DYNAMIC MOTIONS. INTEGRATION OF FORCES
04300 C IS DONE ALONG THE AXIS OF THE SPAR.
04310 C -----
04320 COMMON W(8),CM(8),CMS(8),CMD(8),CMN(8),CMT(8),SL(8),EK(8),CDIN(8),
04330 1CDIT(8),CDIA(8),DEP(8),TN(8),TNN(8),TL(8),YDD(27),YD(27),X(2
04340 17),F(27),DD(27),TC(30),CS(30),VN(3),V(3,8),VV(3,8),GK(8,27),GM(27,
04350 127),WWD1,WWD2,HMAX,H0,HI,HST,HOD,XC1,YC1,ZC1,XC2,YC2,ZC2,RRH01,RRH
04360 102,XJ1,YJ1,ZJ1,XJ2,YJ2,ZJ2,RD1,RD2,S01,S02,BB1,BB2,CSB,SNB,TMASS,Y
04370 1NERT1,YNERT2,VVO,ZCG,SNK,CSK,DEPTH,CDL,PDF,PI,RHO,GR,OEXP,AW,AWX,A
04380 1WY,AMP,WE,T,DT,F1,F2,Q0,Q1,NST,NM,NM1,NM2,NDF,NB,NB1,NB2,NN,MOOR,N
04390 1D
04400 C*****COEFFICIENTS OF THE VELOCITY EQUATION ARE CALCULATED BELOW***
04410 V1=V(1,NB)-YD(NN-2)+X(NN+3)*(V(2,NB)-YD(NN-1))-X(NN+2)*(V(3,NB)-YD
04420 1(NN))
04430 V2=WE*AMP*CSB*SNK
04440 V3=-X(NN+3)*(V(1,NB)-YD(NN-2))+V(2,NB)-YD(NN-1)+X(NN+1)
04450 1*(V(3,NB)-YD(NN))
04460 V4=WE*AMP*SNB*SNK
04470 V5=X(NN+2)*(V(1,NB)-YD(NN-2))-X(NN+1)*(V(2,NB)-YD(NN-1))

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04480 1+V(3,NB)-YD(NN)
04490 V6=-WE*AMP*CSK
04500 VTB=V5-V6*OEXP
04510 VTS=V5-V6*OEXP*DEXP(AW*HST)
04520 C*****PRESSURE DRAG DUE TO BOTTOM END AND STEP OF THE SPAR IS CALCULATED
04530 DP=0.5*RHO*CDP*(S01*DABS(VTB)*VTB+(S01-S02)*DABS(VTS)*VTS)
04540 DNT1=0.
04550 DNT2=0.
04560 DDTT=0.
04570 DM1=0.
04580 DM2=0.
04590 C*****THE VISCOUS DRAG FORCE AND MOMENT ON THE SPAR IS CALCULATED*****
04600 RD=RD1
04610 NDS1=NST*HST/HI
04620 NDS2=NST-NDS1
04630 NDS=NDS1
04640 DZ=HST/NDS
04650 ZZ=DZ/2.-ZCG
04660 RHO=RHO*CDL
04670 DO 2 INZ=1,2
04680 DO 1 IZ=1,NDS
04690 XXE=DEXP(AW*(ZCG+ZZ))
04700 UN1=V1-V2*OEXP*XXE-ZZ*YD(NN+2)
04710 UN2=V3-V4*OEXP*XXE+ZZ*YD(NN+1)
04720 VNM=DSORT(VN1*VN1+VN2*VN2)
04730 DN1=VNM*VN1*RD*DZ
04740 DN2=VNM*VN2*RD*DZ
04750 DNT1=DNT1+DN1
04760 DNT2=DNT2+DN2
04770 VTT=V5-V6*OEXP*XXE
04780 DDTT=DDTT+VTT*DABS(VTT)*RD*DZ
04790 DM1=DM1-ZZ*DN2

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04800 DM2=DM2+ZZ*DN1
04810 ZZ=ZZ+DZ
04820 RD=RD2
04830 NDS=NDS2
04840 DZ=(HI-HST)/NDS
04850 ZZ=HST+DZ/2.-ZCG
04860 2 CONTINUE
04870 DD(NN-2)=RH00*(DNT1-X(NN+3)*DNT2+X(NN+2)*(DDTT*0.02+DP/RH00))
04880 DD(NN-1)=RH00*(DNT1*X(NN+3)+DNT2-X(NN+1)*(DDTT*0.02+DP/RH00))
04890 DD(NN)=RH00*(-DNT1*X(NN+2)+DNT2*X(NN+1)+DUTT*0.02+DP/RH00)
04900 DD(NN+1)=RH00*(DM1-DM2*X(NN+3))
04910 DD(NN+2)=RH00*(DM1*X(NN+3)+DM2)
04920 DD(NN+3)=RH00*(-DM1*X(NN+2)+DM2*X(NN+1))
04930 RETURN
04940 END
04950 IMPLICIT REAL*8(A-H,O-Z)
04960 SUBROUTINE BUOYS
04970 C THIS SUBROUTINE CALCULATES THE MASS MATRIX FOR THE SPAR BUOY
04980 C
04990 C
05000 COMMON W(8),CM(8),CMS(8),CMD(8),CMN(8),CMT(8),SL(8),EK(8),CDIN(8),
05010 1CDIT(8),CDIA(8),DEP(8),TN(8),TNN(8),TL(8),YDD(27),YD(27),Y(27),X(2
05020 17),F(27),DD(27),TC(30),CS(30),UN(3),V(3,8),VV(3,8),GK(8,27),GM(27,
05030 127),WWD1,WWD2,HMAX,H0,HI,HST,HOD,XC1,YC1,ZC1,XC2,YC2,ZC2,RRH01,RRH
05040 102,XJ1,YJ1,ZJ1,XJ2,YJ2,ZJ2,RD1,RD2,S01,S02,BB1,BB2,CSB,SNB,THASS,Y
05050 1NERT1,YNERT2,VV0,ZCG,SNK,CSK,DEPTH,COL,CDP,PI,RHD,GR,DEXP,AW,AWX,A
05060 1WY,AMP,WE,T,DT,P1,P2,Q0,Q1,NST,NM,NM1,NM2,NDF,NB,NB1,NB2,NN,MOOR,N
05070 1D
05080 DO 1 I=1,NDF
05090 DO 1 J=1,NDF
05100 1 GM(I,J)=0.
05110 BK1=(BB1-BB2)*HST

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05120 BK2=BB2*HQ
05130 PO=BB1*HST+BB2*(HO-HST,
05140 P1=BK2*(0.5*HO-ZCG)+BK1*(0.5*HST-ZCG)
05150 P2=BK2*(HO*HO/3.-HO*ZCG+ZCG*ZCG)+BK1*(HST*HST/3.-HST*ZCG+ZCG*ZCG)
05160 QQ2=DEXP(-HOD*AW)
05170 QO=(BB1*(QQ2-DEXP)+BB2*(1.-QQ2))/AW
05180 Q1=(BB1*(QQ2*(HST-1./AW-ZCG)+DEXP*(1./AW+ZCG))+BB2*(HO-1./AW-ZCG-Q
05190 1Q2*(HST-1./AW-ZCG))/AW
05200 C*****COEFFICIENTS IN EQUATIONS OF BUOY MOTION*****
05210 GM(NN-2,NN-2)=TMASS+FO
05220 GM(NN-2,NN)=X(NN+2)*(VVD-PO)
05230 GM(NN-2,NN+2)=P1
05240 GM(NN-1,NN-1)=GM(NN-2,NN-2)
05250 GM(NN-1,NN)=X(NN+1)*(FO-VVD)
05260 GM(NN-1,NN+1)=-GM(NN-2,NN+2)
05270 GM(NN,NN-2)=GM(NN-2,NN)
05280 GM(NN,NN-1)=GM(NN-1,NN)
05290 GM(NN,NN)=TMASS+VVD
05300 GM(NN+1,NN)=-P1*X(NN+1)
05310 GM(NN+1,NN-1)=GM(NN-1,NN+1)
05320 GM(NN+1,NN+1)=YNERT1+P2
05330 GM(NN+2,NN-2)=GM(NN-2,NN+2)
05340 GM(NN+2,NN)=-P1*X(NN+2)
05350 GM(NN+2,NN+2)=GM(NN+1,NN+1)
05360 GM(NN+3,NN-2)=-GM(NN+1,NN)
05370 GM(NN+3,NN-1)=-GM(NN+2,NN)
05380 GM(NN+3,NN+3)=YNERT2
05390 RETURN
05400 END
05410 IMPLICIT REAL*(A-H,O-Z)
05420 SUBROUTINE NEXT
05430 C THIS SUBROUTINE UPDATES THE GEOMETRY OF THE SYSTEM DURING

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ITS DYNAMIC MOTIONS.

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05440 C
05450 C
05460 C
05470
05480 COMMON W(8),CH(8),CME(8),CMD(8),CMN(8),CMT(8),SL(8),EK(8),CDIN(8),
05490 1CDIT(8),CDIA(8),DEP(8),TN(8),TNN(8),TL(8),YDD(27),YD(27),Y(27),X(2
05500 17),F(27),DD(27),TC(30),CS(30),VN(3),V(3,8),VV(3,8),GK(8,27),GM(27,
05510 127),WWD1,WWD2,HMAX,H0,HI,HST,HOD,XC1,YC1,ZC1,XC2,YC2,ZC2,RRH01,RRH
05520 102,XJ1,YJ1,ZJ1,XJ2,YJ2,ZJ2,RD1,RD2,S01,S02,BB1,BB2,CS3,SNB,TMASS,Y
05530 1NERT1,YNERT2,VVO,ZCG,SHK,CSK,DEPTH,CDL,CDP,PI,RHO,GR,DEXP,AW,AWX,A
05540 1WY,AMP,WE,T,DT,P1,P2,Q0,Q1,NST,NM,NM1,NM2,NDF,NB,NB1,NB2,NN,MOOR,N
1D
05550 IF(MOOR.EQ.2)GO TO 3
05560 XJ1=X(NN-2)+XC1
05570 YJ1=X(NN-1)+YC1
05580 ZJ1=X(NN)+ZC1
05590 IF(NB.EQ.1)GO TO 1
05600 TL(1)=DSQRT(X(1)**2+X(2)**2+X(3)**2)
05610 CS(1)=X(1)/TL(1)
05620 CS(2)=X(2)/TL(1)
05630 CS(3)=X(3)/TL(1)
05640 TL(NB)=DSQRT((XJ1-X(NN-5))**2+(YJ1-X(NN-4))**2+(ZJ1-X(NN-3))**2)
05650 CS(NN-2)=(XJ1-X(NN-5))/TL(NB)
05660 CS(NN-1)=(YJ1-X(NN-4))/TL(NB)
05670 CS(NN)=(ZJ1-X(NN-3))/TL(NB)
05680 GO TO 2
05690 1 TL(1)=DSQRT(XJ1**2+YJ1**2+ZJ1**2)
05700 CS(1)=XJ1/TL(1)
05710 CS(2)=YJ1/TL(1)
05720 CS(3)=ZJ1/TL(1)
05730 2 IF(NM.EQ.NB)GO TO 4
05740 3 XJ2=X(NN-2)+XC2
05750 YJ2=X(NN-1)+YC2

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05760 ZJ2=X(NN)+ZC2
05770 TL(NB2)=DSQRT((X(NN+4)-XJ2)**2+(X(NN+5)-YJ2)**2+(X(NN+6)-ZJ2)**2)
05780 CS(NN+4)=(X(NN+4)-XJ2)/TL(NB2)
05790 CS(NN+5)=(X(NN+5)-YJ2)/TL(NB2)
05800 CS(NN+6)=(X(NN+6)-ZJ2)/TL(NB2)
05810 DO 5 I=2,NN
05820 IF(I.EQ.NB.OR.I.EQ.NB2)GO TO 5
05830 II=3*II+1
05840 IF(I.LT.NB)II=II-3
05850 TL(I)=DSQRT((X(II)-X(II-3))**2+(X(II+1)-X(II-2))**2+(X(II+2)-X(II-
05860 11)**2)
05870 CS(II)=(X(II)-X(II-3))/TL(I)
05880 CS(II+1)=(X(II+1)-X(II-2))/TL(I)
05890 CS(II+2)=(X(II+2)-X(II-1))/TL(I)
05900 5 CONTINUE
05910 RETURN
05920 END
05930 IMPLICIT REAL*8(A-H,O-Z)
05940 SUBROUTINE MINVER(N,C)
05950 C THIS SUBROUTINE CALCULATES THE INVERSE OF A MATRIX 'C'
05960 C OF ORDER 'N'
05970 C -----
05980 C DIMENSION C(27,27),H(27),G(27)
05990 NN=N-1
06000 C(1,1)=1.0/C(1,1)
06010 DO 6 M=1,NN
06020 K=M+1
06030 DO 1 I=1,M
06040 G(I)=0.
06050 DO 1 J=1,M
06060 1 G(I)=G(I)+C(I,J)*C(J,K)
06070

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06080      D=0.
06090      DO 2 I=1,M
06100      D=D+C(K,I)*G(I)
06110      E=C(K,K)-D
06120      C(K,K)=1.0/E
06130      DO 3 I=1,M
06140      C(I,K)=-G(I)*C(K,K)
06150      DO 4 J=1,M
06160      H(J)=0.
06170      DO 4 I=1,M
06180      H(J)=H(J)+C(K,I)*C(I,J)
06190      DO 5 J=1,M
06200      C(K,J)=-H(J)*C(K,K)
06210      DO 6 I=1,M
06220      DO 6 J=1,M
06230      C(I,J)=C(I,J)-G(I)*C(K,J)
06240      RETURN
06250      END
06260      IMPLICIT REAL*8(A-H,O-Z)
06270      SUBROUTINE MATRIX
06280      C
06290      C THIS SUBROUTINE CALCULATES THE REMAINING ELEMENTS OF THE GLOBAL
06300      C MATRICES AND INTEGRATES THE 'NDF' SIMULTANEOUS DIFFERENTIAL
06310      C EQUATIONS OF MOTION BY THE METHOD OF RECTANGULAR RULE.
06320      C -----
06330      COMMON W(8),CM(8),CMS(8),CMD(8),CMN(8),CMT(8),SL(8),EK(8),CDIN(8),
06340      1CDIT(8),CDIA(8),DEF(8),TN(8),TNN(8),TL(8),YDD(27),YD(27),X(2
06350      17),F(27),DD(27),IC(30),CS(30),VN(3),V(3,8),VV(3,8),GK(8,27),GM(27,
06360      127),WWD1,WWD2,NMAX,H0,HI,HST,HOD,XC1,YC1,ZC1,XC2,YC2,ZC2,RRH01,RRH
06370      102,XJ1,YJ1,ZJ1,XJ2,YJ2,ZJ2,RD1,RD2,S01,S02,BB1,BB2,CSB,SNB,TMASS,Y
06380      1NERT1,YNERT2,VVO,ZCG,SNK,CSK,DEPTH,CDL,CDP,PI,RHO,GR,DEXP,AW,AWX,A
06390      1WY,AMP,WE,T,DT,P1,P2,Q0,Q1,NST,NM,NM1,NM2,NDF,NB,NB1,NB2,NN,MODR,N

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06400 1D
06410 C*****WAVE EXCITING PLUS HYDROSTATIC FORCES ON THE SPAR BUOY*****
06420 F(NN-2)=2.*WE*WE*AMP*CSB*QO*CSK
06430 F(NN-1)=2.*WE*WE*AMP*SNB*QO*CSK
06440 F(NN)=WE*WE*AMP*QO*SNK+GR*(BB1*HST+BB2*(HI-HST)-THASS)
06450 F(NN+1)=-WE*WE*2.*AMP*SNB*Q1*CSK-GR*P1*X(NN+1)
06460 F(NN+2)=WE*WE*2.*AMP*CSB*Q1*CSK-GR*P1*X(NN+2)
06470 IF(MOOR.EQ.0)GO TO 11
06480 C*****STIFFNESS MATRIX FOR THE MOORING LINES*****
06490 AA1=XC1-YC1*X(NN+3)+ZC1*X(NN+2)
06500 AA2=XC2-YC2*X(NN+3)+ZC2*X(NN+2)
06510 AB1=XC1*X(NN+3)+YC1-ZC1*X(NN+1)
06520 AB2=XC2*X(NN+3)+YC2-ZC2*X(NN+1)
06530 AC1=-XC1*X(NN+2)+YC1*X(NN+1)+ZC1
06540 AC2=-XC2*X(NN+2)+YC2*X(NN+1)+ZC2
06550 GK(1,1)=EK(1)*CS(1)
06560 GK(1,2)=EK(1)*CS(2)
06570 GK(1,3)=EK(1)*CS(3)
06580 GK(NB,NN+1)=EK(NB)*(-AC1*CS(NN-1)+AB1*CS(NN))
06590 GK(NB,NN+2)=EK(NB)*(-AC1*CS(NN-2)+AA1*CS(NN))
06600 GK(NB,NN+3)=EK(NB)*(-AB1*CS(NN-2)+AA1*CS(NN-1))
06610 IF(NM.EQ.1)GO TO 5
06620 IF(NM.EQ.NB)GO TO 1
06630 GK(NB2,NN-2)=-EK(NB2)*CS(NN+4)
06640 GK(NB2,NN-1)=-EK(NB2)*CS(NN+5)
06650 GK(NB2,NN)=-EK(NB2)*CS(NN+6)
06660 GK(NB2,NN+1)=-EK(NB2)*(-AC2*CS(NN+5)+AB2*CS(NN+6))
06670 GK(NB2,NN+2)=-EK(NB2)*(-AC2*CS(NN+4)+AA2*CS(NN+6))
06680 GK(NB2,NN+3)=-EK(NB2)*(-AB2*CS(NN+4)+AA2*CS(NN+5))
06690 GK(NB2,NN+4)=-GK(NB2,NN-2)
06700 GK(NB2,NN+5)=-GK(NB2,NN-1)
06710 GK(NB2,NN+6)=-GK(NB2,NN)

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```

06720 1 DO 2 I=2,NM
06730 IF(I.EQ.NB2)GO TO 2
06740 II=3*I
06750 IF(I.LT.NB)II=II+3
06760 GK(I,II-5)=-EK(I)*CS(II-2)
06770 GK(I,II-4)=-EK(I)*CS(II-1)
06780 GK(I,II-3)=-EK(I)*CS(II)
06790 GK(I,II)=-GK(I,II-3)
06800 GK(I,II-1)=-GK(I,II-4)
06810 GK(I,II-2)=-GK(I,II-5)
06820 2 CONTINUE
06830 C*****MASS MATRIX FOR THE MOORING LINES*****
06840 DO 4 I=1,NM
06850 IF(I.EQ.NB)GO TO 4
06860 II=3*I+1
06870 IF(I.LT.NB)II=II-3
06880 GM(II,II)=CM(I)*CMS(I)+CMN(I)*(1.-CS(II+3)**2)+CMT(I)*CS(II+3)**2
06890 GM(II+1,II+1)=CM(I)+CMS(I)+CMN(I)*(1.-CS(II+4)**2)+CMT(I)*CS(II+4)
06900 1**2
06910 GM(II+2,II+2)=CM(I)+CMS(I)+CMN(I)*(1.-CS(II+5)**2)+CMT(I)*CS(II+5)
06920 1**2
06930 GM(II,II+1)=(CMT(I)-CMN(I))*CS(II+4)*CS(II+3)
06940 GM(II+1,II)=GM(II,II+1)
06950 GM(II,II+2)=(CMT(I)-CMN(I))*CS(II+5)*CS(II+3)
06960 GM(II+1,II+2)=(CMT(I)-CMN(I))*CS(II+5)*CS(II+4)
06970 GM(II+2,II)=GM(II,II+2)
06980 GM(II+2,II+1)=GM(II+1,II+2)
06990 UWE=GR*AMP*DEXP(AU*(X(II+2)-DEPTH))
07000 SUE=AU*X*(II)+AU*Y*(II+1)-WE*T
07010 SNK1=DSIN(SUE)
07020 CSK1=DCOS(SUE)
07030 PHX=UWE*AU*X*CSK1

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07040 PHY=WWE*AWY*CSK1
07050 PHZ=WWE*AW*SNK1
07060 C*****VISCOUS DRAG FORCE ARRAY FOR THE MOORING LINES*****
07070 U1=V(1,I)-YD(II)-AWX*SNK1*WWE/WE
07080 U2=V(2,I)-YD(II+1)-AWY*SNK1*WWE/WE
07090 U3=V(3,I)-YD(II+2)+AW*CSK1*WWE/WE
07100 UT1=U1*CS(II+3)+U2*CS(II+4)+U3*CS(II+5)
07110 VN(1)=U1*(1.-CS(II+3)*CS(II+3))-U2*CS(II+4)-U3*CS(II+3)*
07120 1CS(II+5)
07130 VN(2)=U2*(1.-CS(II+4)*CS(II+4))-U3*CS(II+4)*CS(II+5)-U1*CS(II+4)*
07140 1CS(II+3)
07150 VN(3)=U3*(1.-CS(II+5)*CS(II+5))-U1*CS(II+5)*CS(II+3)-U2*CS(II+5)*
07160 1CS(II+4)
07170 DDTT=CDIT(I)*UT1*.ABS(UT1)
07180 VNM=DSQRT(VN(1)*VN(1)+VN(2)*VN(2)+VN(3)*VN(3))
07190 VNM1=DSQRT(U1*U1+U2*U2+U3*U3)*CDIA(I)
07200 DD(II)=CDIN(I)*VNM*VN(1)+DDTT*CS(II+3)+VNM1*U1
07210 DD(II+1)=CDIN(I)*VNM*VN(2)+DDTT*CS(II+4)+VNM1*U2
07220 DD(II+2)=CDIN(I)*VNM*VN(3)+DDTT*CS(II+5)+VNM1*U3
07230 IF(MOOR.EQ.2.AND.I.GE.ND)GO TO 3
07240 C*****FROUDE KRYLOV FORCE ARRAY FOR THE MOORING LINES*****
07250 F(II)=CMD(I)*PHX
07260 F(II+1)=CMD(I)*PHY
07270 F(II+2)=-W(I)+CMD(I)*PHZ
07280 GO TO 4
07290 3 F(II)=CMD(I)*(PHX*(1.-CS(II+3)*CS(II+3))-PHY*CS(II+3)*CS(II+4)-PHZ
07300 1*CS(II+3)*CS(II+5))
07310 F(II+1)=CMD(I)*(PHY*(1.-CS(II+4)*CS(II+4))-PHZ*CS(II+4)*CS(II+5)-P
07320 1HX*CS(II+4)*CS(II+3))
07330 F(II+2)=CMD(I)*(PHZ*(1.-CS(II+5)*CS(II+5))-PHX*CS(II+5)*CS(II+3)-P
07340 1HY*CS(II+5)*CS(II+4))-W(I)
07350 4 CONTINUE

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07360 C*****TENSION FORCE ARRAY FOR THE MOORING LINES*****
07370 5 DO 7 K=1,NM
07380 TG=0.
07390 DO 6 IK=1,NDF
07400 TG=.3+GK(K,IK)*Y(IK)
07410 TNN(K)=TG+TNN(K)
07420 TN(K)=TNN(K)
07430 IF (TN(K).LE.0.)TN(K)=0.
07440 KK=3*KK+1
07450 IF (K.LE.NB)KK=KK-3
07460 TC(KK)=TN(K)*CS(KK)
07470 TC(KK+2)=TN(K)*CS(KK+2)
07480 TC(KK+1)=TN(K)*CS(KK+1)
07490 TC(NN+1)=-AC1*TC(NN-1)+AB1*TC(NN)+AC2*TC(NN+5)-AB2*TC(NN+6)
07500 TC(NN+2)=AC1*TC(NN-2)-AA1*TC(NN)-AC2*TC(NN+4)+AA2*TC(NN+6)
07510 TC(NN+3)=-AB1*TC(NN-2)+AA1*TC(NN-1)+AB2*TC(NN+4)-AA2*TC(NN+5)
07520 IF (NM.EQ.1)GO TO 11
07530 IF (MOOR.EQ.2)GO TO 9
07540 NZ=NN-3
07550 DO 8 N=1,NZ
07560 TC(N)=TC(M)-TC(M+3)
07570 TC(NN-2)=TC(NN-2)-TC(NN+4)
07580 TC(NN-1)=TC(NN-1)-TC(NN+5)
07590 TC(NN)=TC(NN)-TC(NN+6)
07600 IF (NM.EQ.NB)GO TO 11
07610 NZ=NN+4
07620 DO 10 M=NZ,NDF
07630 TC(M)=TC(M)-TC(M+3)
07640 C*****SOLUTION OF THE DIFFERENTIAL EQUATIONS*****
07650 CALL MINVER(NDF,GM)
07660 11 DO 13 M=1,NDF
07670 YDD(M)=0.

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```
07680 DO 12 K=1,NDF
07690   YDD(M)=YDD(M)+GM(M,K)*(DD(K)-TC(K)+F(K))
07700   YD(M)=YD(M)+YDD(M)*DT
07710   Y(M)=YD(M)*DT
07720   X(M)=X(M)+Y(M)
07730   HO=HO-Y(NN)
07740   HOD=HO-HST
07750   RETURN
07760   END
READY
```

```

SD2.FORT
00010 C
00020 C
00030 C
00040 C
00050 C
00060 C
00070 C
00080 C
00090 C
00100 C
00110 C
00120 C
00130 C
00140 C
00150 C
00160 C
00170 C
00180 C
00190 C
00200 C
00210 C
00220 C
00230 C
00240 C
00250 C
00260 C
00270 C
00280 C
00290 C
00300 C
00310 C

2 D DYNAMIC ANALYSIS OF A SPAR BUOY, A SURFACE MOORED SYSTEM,
AND A DRIFTING DROGUE SPAR BUOY.

-----
THE UNITS USED IN THIS ANALYSIS ARE FEET/POUNDS/SECONDS.
-----
CASE STUDY 1--MOOR=0--SPAR BUOY FREELY FLOATING IN A SURFACE WAVE.
CASE STUDY 2--MOOR=1--SPAR BUOY ANCHORED WITH A LUMPED MASS
MOORING LINE; NM=NB.
CASE STUDY 3--MOOR=1--SPAR BUOY ANCHORED WITH A LUMPED MASS
MOORING LINE AND A LUMPED MASS INSTRUMENT LINE HANGING FROM
THE BUOY; NM NOT EQUAL TO NB.
CASE STUDY 4--MOOR=2--SPAR BUOY ATTACHED TO A WINDOW SHADE
DROGUE; NB=1.
-----

*****START OF THE MAIN PROGRAM*****
IMPLICIT REAL*8(A-H,O-Z)
COMMON W(8),CM(8),CMS(8),CMD(8),CMN(8),CHT(8),SL(8),EK(8),CDIN(8),
1CDIT(8),CDIA(8),DEP(8),V(8),VU(8),TN(8),TNN(8),TL(8),CS(9),SN(9),Y
1DD(17),YD(17),Y(17),X(17),F(17),DD(17),TC(19),GK(8,17),GM(17,17),D
1C1,DC2,DC3,XC1,ZC1,XC2,ZC2,WWD1,WWD2,S01,S02,CS1,SN1,RD1,RD2,XJ1,Y
1J1,XJ2,YJ2,BB1,BB2,HMAX,H0,HST,HOD,RRH01,RRH02,TMASS,YNERT,ZCG,HI,
1THO,CDP,CDL,CON,COT,ALPHA,PI,RHO,GR,AW,I,DT,AMP,WE,SNK,CSK,DEPTH,O
1K,VUV(8),OEXP,VVO,VU1,XW,ZW,EMW,MOOR,NST,NM,NM1,NM2,NDF,NB,NB1,NB2
1,NN,ND
100 FORMAT(12F10.2)
101 FORMAT(7F10.0)
102 FORMAT(6I5,4F10.0)
103 FORMAT(6F10.0)
*****GENERAL DATA*****

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00320 READ(5,102)NM,NR,ND,NST,NC,MOOR,DEPTH,DT,TMAX,T2
00330 C*****BUOY DATA*****
00340 READ(5,101)RD1,RD2,ZCG,RGYR,HMAX,CDL,HST,CON,COT,CDF,ALPHA
00350 C*****MOORING LINE DATA*****
00360 READ(5,103)(W(I),CM(I),CMS(I),CMD(I),CMN(I),CMT(I),I=1,NM)
00370 READ(5,103)(SL(I),EK(I),CDIN(I),CDIT(I),CDIA(I),DEF(I),I=1,NM)
00380 C*****ATTACHMENT DATA*****
00390 READ(5,101)XC1,ZC1,XC2,ZC2
00400 C*****CURRENT PROFILE DATA*****
00410 READ(5,101)(V(I),I=1,NM)
00420 NM1=NM-NB
00430 NN=NB+NB
00440 NB1=NB-1
00450 NB2=NB+1
00460 NM2=NM+1
00470 NDF=2*NM+1
00480 C*****INITIALIZE PHYSICAL CONSTANTS*****
00490 PI=4.*DATAN(1.DO)
00500 S01=PI*RD1*RD1
00510 S02=PI*RD2*RD2
00520 RH0=1.9905
00530 GR=32.2
00540 TMASS=W(NB)/GR
00550 YNERT=TMASS*RGYR*RGYR
00560 RRH01=RH0*CDL*RD1*HST
00570 RRH02=RH0*CDL*RD2
00580 BB1=RH0*S01
00590 BB2=RH0*S02
00600 WWD1=BB1*GR*HST
00610 WWD2=BB2*GR
00620 VVO=1.33*RH0*ALPHA*RD1**3
00630 VV1=1.33*RH0*ALPHA*(RD1-RD2)**3

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00640 C*****START LOOP FOR DYNAMIC RESPONSE TO DIFFERENT SURFACE WAVES*****
00650 DO 8 ICC=1,NC
00660 READ(5,101)WE,AMP
00670 C*****INITIALIZE DYNAMIC VARIABLES*****
00680 DC1=0.
00690 DC2=0.
00700 DC3=0.
00710 DO 1 I=1,NDF
00720 YD(I)=0.
00730 TC(I)=0.
00740 Y(I)=0.
00750 DO 1 J=1,NM
00760 GK(J,I)=0.
00770 AW=WE*WE/GR
00780 T3=0.
00790 C*****FIND THE CURRENT PROFILE FOR STATIC SOLUTION*****
00800 DO 3 I=1,NM
00810 VV(I)=0.
00820 ZX4=0.
00830 ZX1=WE*AMP*DEXP(AW*DEP(I))
00840 DO 2 J=1,12
00850 ZX3=ZX1*DCOS(PI*J/6.)
00860 ZX2=V(I)+ZX1*DSIN(PI*J/6.)
00870 VEM=DSQRT(ZX2**2+ZX3**2)
00880 ZX4=ZX4+ZX3*VEM
00890 VV(I)=VV(I)+ZX2*VEM
00900 VEM1=(ZX4**2+VV(I)**2)**0.25
00910 VV(I)=VV(I)/(VEM1*12.**0.5)
00920 IF(DABS(VV(I)).LE.0.000001)VV(I)=0.
00930 WRITE(6,100)(VV(I),I=1,NM)
00940 TC(NDF+1)=0.
00950 TC(NDF+2)=0.

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00960 IF(NM.EQ.NR)GO TO 5
00970 CS(NM2)=0.
00980 SN(NM2)=-1.
00990 IF(MOOR.EQ.2)GO TO 5
01000 C*****STATIC SOLUTION OF THE LUMPED MASSES (INSTRUMENT LINE)*****
01010 DO 4 I=1,NM1
01020 J=NM2-I
01030 K=J+1
01040 II=J+J
01050 UN=VV(J)*SN(K)
01060 VT=VV(J)*CS(K)
01070 DI=CDIA(J)*VV(J)*DABS(VV(J))
01080 DDTT=CDIT(J)*VT*DABS(VT)
01090 DN=CDIN(J)*VN*DABS(VN)
01100 TC(II)=TC(II+2)+DI+DDTT*CS(K)+DN*SN(K)
01110 TC(II+1)=TC(II+3)+DDTT*SN(K)-DN*CS(K)-W(J)
01120 TNN(J)=DSQRT(TC(II)**2+TC(II+1)**2)
01130 CS(J)=TC(II)/TNN(J)
01140 SN(J)=TC(II+1)/TNN(J)
01150 TL(J)=SL(J)+TNN(J)/EK(J)
01160 4 CALL STATIC
01170 T=0.
01180 6 SS=AW*X(NN-1)-WE*T
01190 SNK=DSIN(SS)
01200 CSK=DCOS(SS)
01210 OK=AW*CS1
01220 OEXP=DEXP(-HO*CK)
01230 HI=HO-AMP*SNK/CS1
01240 IF(CDL.NE.0.0)CALL DBDRG
01250 CALL BUOYS
01260 CALL MATRIX
01270 IF(T.LT.T3)GO TO 7

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01280 WRITE(6,100)T
01290 WRITE(6,100)(X(I),I=1,NDF)
01300 IF(MOOR.EQ.1)WRITE(6,100)(TN(K),K=1,NM)
01310 IF(MOOR.EQ.2)WRITE(6,100)(TN(K),K=2,NM)
01320 IF(T.GT.TMAX)GO TO 8
01330 T3=T3+T2
01340 T=T+DT
01350 IF(MOOR.GT.0)CALL NEXT
01360 GO TO 6
01370 8 CONTINUE
01380 CALL EXIT
01390 END
01400 IMPLICIT REAL*8(A-H,D-Z)
01410 SUBROUTINE STATIC
01420 C THIS SUBROUTINE CALCULATES THE STATIC SOLUTION OF THE FOUR CASE
01430 C STUDIES OUTLINED IN THE MAIN PROGRAM
01440 C -----
01450 C COMMON W(8),CM(8),CMS(8),CMD(8),CMN(8),CMT(8),SL(8),EK(8),CDIN(8),
01460 1CDIT(8),CDIA(8),DEP(8),V(8),VV(8),TN(8),TNN(8),TL(8),CS(9),SN(9),Y
01470 1DD(17),YD(17),Y(17),X(17),F(17),DD(17),TC(19),GK(8,17),GM(17,17),D
01480 1C1,DC2,DC3,XC1,ZC1,XC2,ZC2,WWD1,WWD2,SQ1,SQ2,CS1,SN1,RD1,RD2,XJ1,Y
01490 1J1,XJ2,YJ2,BB1,BB2,HMAX,HD,HST,HOD,RRHD1,RRHD2,TMASS,YNERT,ZCG,HI,
01500 1THO,CDP,COL,CON,COT,ALPHA,PI,RHQ,GR,AW,T,DT,AMP,WE,SNK,CSK,DEPTH,O
01510 1K,VUV(8),OEXP,VVD,VV1,XW,ZW,EMW,MOOR,NST,NM,NM1,NM2,NDF,NB,NB1,NB2
01520 1,NN,ND
01530 100 FORMAT(9F12.3)
01540 101 FORMAT(///9X11HSTATIC CALCULATION TERMINATED -- NUMBER OF ITERATI
01550 101 ONS TO DETERMINE BUOY EQUILIBRIUM CONDITION EXCEEDS MAXIMUM ( 13,
01560 1 1H) )
01570 102 FORMAT (///10X,6SHSTATIC CALCULATION TERMINATED -- BUOY HAS EXCEED
01580 1ED MAXIMUM DRAFT )
01590

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01600 103 FORMAT (///10X,100HSTATIC CALCULATION TERMINATED -- NUMBER OF ITER
01610 ATIONS TO FIND STATIC SOLUTION HAS EXCEEDED MAXIMUM ( I3,1H))
01620 IF(MOOR.EQ.0)GO TO 33
01630 DATA ITMAX,IT1MAX/100,100/
01640 IT1=0
01650 INDEX=1
01660 EMAX=.01*W(NB)*(RD1+RD2)
01670 IF(MOOR.EQ.1)GO TO 8
01680 C
01690 C*****CASE STUDY 4 *****
01700 C
01710 C*****INITIALIZE VARIABLES*****
01720 IS2=0
01730 DC2=0.
01740 VE=0.
01750 FXE=0.02*HMAX*RD2*V(1)*V(1)+0.01
01760 DVE=DABS(V(NM))/2.
01770 X(3)=0.
01780 CS1=1.
01790 SN1=0.
01800 DTHI=0.05
01810 C*****START STATIC VELOCITY ITERATION LOOP*****
01820 1 INDEX=INDEX+1
01830 2 IT1=IT1+1
01840 IF(IT1.GT.IT1MAX)GO TO 30
01850 DTH=DTHI
01860 IT=0
01870 IS=0
01880 DO 3 IC=1,NM
01890 VV(V(IC))=VV(IC)-VE
01900 DO 4 IC=2,NM
01910 K=NM-IC+3

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01920 J=K-1
01930 II=J+J
01940 VN=VVV(J)*SN(K)
01950 VT=VVV(J)*CS(K)
01960 DN=CDIN(J)*VN*DABS(VN)
01970 DDT=CDIT(J)*VT*DABS(VT)
01980 DI=CDIA(J)*VVV(J)*DABS(VVV(J))
01990 TC(II)=TC(II+2)+DI+DDTT*CS(K)+DN*SN(K)
02000 TC(II+1)=TC(II+3)+DDTT*SN(K)-DN*CS(K)-W(J)
02010 TNN(J)=DSQRT(TC(II)*TC(II)+TC(II+1)*TC(II+1))
02020 CS(J)=TC(II)/TNN(J)
02030 SN(J)=TC(II+1)/TNN(J)
02040 TL(J)=SL(J)+TNN(J)/EK(J)
02050 IF(INDEX.EQ.3)GO TO 11
02060 HO=HST+W(1)-TC(5)-DC2-WWD1)/WWD2
02070 RRHO=RRH01+RRH02*(HO-HST)
02080 VN=VVV(1)*CS1
02090 VT=VVV(1)*SN1
02100 DDT=(0.02*RRHO+0.5*RRHO*CDP*(2.*S01-S02))*DABS(VT)*VT
02110 DN=RRHO*VN*DABS(VN)
02120 DC1=DN*CS1+DDTT*SN1
02130 FX=DC1+TC(4)
02140 WRITE(6,100)VE,HO,FX
02150 IF(DABS(FX).LE.FXE)GO TO 6
02160 IS3=DSIGN(1.0D0,FX)
02170 IF(IS2+IS3.EQ.0)DVE=DVE/2.
02180 VE=VE+IS3*DVE
02190 IS2=IS3
02200 GO TO 2
02210 IF(INDEX.EQ.2)GO TO 1
02220 WRITE(6,100)VE,HO,E
02230 YD(1)=VE

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4

5

6

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02240 DO 7 I=2,NM
02250 II=I+I
02260 7 YD(II)=VE
02270 X(1)=0.
02280 X(2)=(ZCG-HO)*CS1+DEPTH
02290 TNN(1)=0.
02300 CS(1)=0.
02310 SN(1)=0.
02320 GO TO 25
02330 C
02340 C*****CASE STUDIES 2 AND 3 *****
02350 C
02360 8 NBB=NB-1
02370 UVV(NB)=VV(NB)
02380 HO=HST+(W(NB)-TC(NN+3)-WWD1)/WWD2
02390 DH=0.
02400 WADD=W(NBB)/10.
02410 C*****INITIALIZE ERROR TOLERANCES*****
02420 ER= 0.0010*DEPTH
02430 9 HO=HO+DH
02440 X(NN+1)=0.0
02450 DTHI=0.05
02460 IT1=0
02470 C*****START CABLE (BUOY DRAFT) ITERATION LOOP*****
02480 10 IT1=IT1+1
02490 IF(IT1.GT.IT1MAX)GO TO 30
02500 DTH=DTHI
02510 IT=0
02520 IS=0
02530 11 E=1.D25
02540 C*****START BUOY ITERATION*****
02550 12 IF(MOOR.EQ.2)HO=HST+(W(1)-TC(S)-DC2-WWD1)/WWD2

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02560 IF(HO.GT.0.99*HMAX)GO TO 31
02570 HOD=HO-HST
02580 B=WWD1+WWD2*HOD
02590 E1=E
02600 IT=IT+1
02610 IF(IT.GT.ITMAX)GO TO 32
02620 RRHO=RRHO1+RRHO2*HOD
02630 CS1=DCOS(X(NN+1))
02640 SN1=DSIN(X(NN+1))
02650 VN=VVV(NB)*CS1
02660 VT=VVV(NB)*SN1
02670 DDTT=(.02*RRHO+.5*RRHO*CDP*(2.*SO1-SO2))*DABS(VT)*VT
02680 DN=RRHO*VN*DABS(VN)
02690 DC1=DN*CS1+DDTT*SN1
02700 DC2=DDTT*CS1-DN*SN1
02710 DC3=DN*(HST/2.-ZCG)+RRHO2*HO*HOD*VN*DABS(VN)/2.
02720 IF(MOOR.EQ.2)GO TO 13
02730 TC(NN-1)=DC1+TC(NN+2)
02740 TC(NN)=DC2+B-W(NB)+TC(NN+3)
02750 13 ARMX1=ZC1*SN1+XC1*CS1
02760 ARMZ1=ZC1*CS1-XC1*SN1
02770 ARMX2=ZC2*SN1+XC2*CS1
02780 ARMZ2=ZC2*CS1-XC2*SN1
02790 C*****CALCULATE UNBALANCED MOMENT ON RUDDY*****
02800 BGZ=SN1*(WWD1*(HST/2.-ZCG)+WWD2*HST*(HO+HST)/2.-ZCG))
02810 E=DC3-BGZ-TC(NN-1)*ARMZ1+TC(NN)*ARMX1+TC(NN+2)*ARMZ2-TC(NN+3)*ARMX
02820 12
02830 IF(MOOR.EQ.1)WRITE(6,100)HO,X(NN+1),DH,E
02840 IF(MOOR.EQ.2)WRITE(6,100)VE,HO,X(3),E
02850 C*****TEST FOR CONVERGENCE*****
02860 IF (DABS(E).LT.EMAX) GO TO 16
02870 IS1=DSIGN(1.D0,E)

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02880 C*****TEST FOR CHANGE IN SIGN OF UNBALANCED MOMENT (ERROR)*****
02890 IF((IS1+IS).EQ.0) GO TO 14
02900 IF(DABS(E).GT.DABS(E1)) GO TO 15
02910 C*****NO CHANGE IN SIGN - CONTINUE IN SAME DIRECTION*****
02920 X(NN+1)=X(NN+1)+IS1*DTH
02930 IS=IS1
02940 GO TO 12
02950 C*****CHANGE IN SIGN -- GO BACK HALF A STEP*****
02960 14 DTH=DTH/2.
02970 X(NN+1)=X(NN+1)+IS1*DTH
02980 IS=IS1
02990 GO TO 12
03000 C*****ERROR IS GROWING - ITERATE IN OPPOSITE DIRECTION*****
03010 15 DTH=-DTH
03020 X(NN+1)=X(NN+1)+IS1*DTH*2.
03030 IS=IS1
03040 GO TO 12
03050 16 IF(MOOR.EQ.2)GO TO 5
03060 C*****BUOY-CABLE SOLUTION HAS CONVERGED - START INTEGRATION DOWN CABLE
03070 DTH=DTH/2.
03080 DH=DH/2.
03090 TNN(NB)=DSQRT(TC(NN-1)*TC(NN-1)+TC(NN)*TC(NN))
03100 TL(NB)=SL(NB)+TNN(NB)/EK(NB)
03110 CS(NB)=TC(NN-1)/TNN(NB)
03120 SN(NB)=TC(NN)/TNN(NB)
03130 IF(NB.EQ.1)GO TO 19
03140 17 DO 18 I=1,NB1
03150 J=NB-I
03160 K=J+1
03170 II=J+J
03180 VN=VV(J)*SN(K)
03190 VT=VV(J)*CS(K)

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03200  UZ=CDIA(J)*VU(J)*DABS(VU(J))
03210  DDTT=CDIT(J)*VT*DABS(VT)
03220  DN=CDIN(J)*VN*DABS(VN)
03230  TC(II-1)=TC(II+1)+DI+DDTT*CS(K)+DN*SN(K)
03240  TC(II)=TC(II+2)+DDTT*SN(K)-DN*CS(K)-W(J)
03250  TNN(J)=DSQRT(TC(II-1)**2+TC(II)**2)
03260  CS(J)=TC(II-1)/TNN(J)
03270  SN(J)=TC(II)/TNN(J)
03280  TL(J)=SL(J)+TNN(J)/EK(J)
03290  YJ1=0.
03300  DO 20 I=1,NB
03310  YJ1=YJ1+TL(I)*SN(I)
03320  DEPTH1=YJ1+(HO-ZCG-ZC1)*CS1+XC1*SN1
03330  EE=DEPTH-DEPTH1
03340  WRITE(6,100)HO,X(NN+1),EE,(W(NCC),NCC=1,NB)
03350  EEE=DABS(EE)
03360  C*****TEST FOR CONVERGENCE*****
03370  IF(EEE.LE.ER)GO TO 22
03380  IF(INDEX.EQ.1.AND.EE.LT.0.)GO TO 29
03390  IF(INDEX.EQ.2)GO TO 21
03400  INDEX=2
03410  DH=(HMAX-HO)/2.
03420  GO TO 9
03430  21 HO=HO+EEE*Dn/EE
03440  GO TO 10
03450  C*****STATIC SOLUTION HAS CONVERGED- COMPUTE ALL COORDINATES*****
03460  22 DEPTH=DEPTH1
03470  WRITE(6,100)DEPTH
03480  X(1)=TL(1)*CS(1)
03490  X(2)=TL(1)*SN(1)
03500  IF(NB.EQ.1)GO TO 24
03510  DO 23 I=2,NB

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03520      II=I+I
03530      X(II-1)=X(II-3)+TL(I)*CS(I)
03540      X(II)=X(II-2)+TL(I)*SN(I)
03550      XJ1=X(NN-1)
03560      X(NN-1)=XJ1-XC1*CS1-ZC1*SN1
03570      X(NN)=YJ1-ZC1*CS1+XC1*SN1
03580      IF(RH.EQ.NB)GO TO 27
03590      XJ2=X(NN-1)+XC2*CS1+ZC2*SN1
03600      YJ2=X(NN)+ZC2*CS1-XC2*SN1
03610      X(NN+2)=XJ2+TL(NB2)*CS(NB2)
03620      X(NN+3)=YJ2+TL(NB2)*SN(NB2)
03630      IF(NB2.EQ.NH)GO TO 27
03640      NB3=NB2+1
03650      DO 26 I=NB3,NH
03660      II=I+I
03670      X(II)=X(II-2)+TL(I)*CS(I)
03680      X(II+1)=X(II-1)+TL(I)*SN(I)
03690      WRITE(6,100)HO,X(NN+1),TC(NN-1),TC(NN),E
03700      WRITE(6,100)(X(I),I=1,NDF)
03710      RETURN
03720 C*****PART OF THE MOORING LINE IS FLOATING ON THE SURFACE(FIND II)*****
03730      W(NBR)=W(NBB)-WADD
03740      IF(W(NBB).LT.0.)GO TO 17
03750      W(NBB)=0.
03760      NBB=NBB-1
03770      GO TO 29
03780      WRITE(6,103)IT1MAX
03790      CALL EXIT
03800      WRITE(6,102)
03810      CALL EXIT
03820      WRITE(6,101)ITMAX
03830      CALL EXIT
```

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03840 C
03850 C***CASE STUDY 1 *****
03860 C
03870 33 HO=HST+(W(1)-WWD1)/WWD2
03880 WAW=AW*HO
03890 OEXP=DEXP(-WAW)
03900 HOD=HO-HST
03910 BK1=(BB1-BB2)*HST
03920 BK2=BB2*HO
03930 P1=BK2*(0.5*HO-ZCG)+BK1*(0.5*HST-ZCG)
03940 P2=BK2*(HO*HO/3.-HO*ZCG+ZCG*ZCG)+BK1*(HST*HST/3.-HST*ZCG+ZCG*ZCG)
03950 QQ2=DEXP(-HOD*AW)
03960 Q0=(BB1*(QQ2-OEXP)+BB2*(1.-QQ2))/AW
03970 Q1=(BB1*(QQ2*(HST-1./AW-ZCG)+OEXP*(1./AW+ZCG))+BB2*(HO-1./AW-ZCG-Q
03980 1Q2*(HST-1./AW-ZCG)))/AW
03990 YD(1)=VV(1)
04000 YD(2)=WE*AMP*(RHO*S02*GR-WE*WE*Q0)/(RHO*S02*GR-WE*WE*(TMASS+VV0+VV
04010 11))
04020 X(1)=-AMP*Q0/TMASS
04030 X(2)=ZCG-HO+DEPTH
04040 X(3)=WE*WE*AMP*(2.*Q1-P1*Q0/TMASS)/(P1*GR-WE*WE*(YNERT+P2-P1*P1/(2
04050 1.*TMASS)))
04060 XAW=X(1)/AMP
04070 ZAW=YD(2)/(WE*AMP)
04080 TAW=X(3)/(AW*AMP)
04090 SN1=DSIN(X(3))
04100 CS1=DCOS(X(3))
04110 WRITE(6,100)WAW,XAW,ZAW,TAW
04120 RETURN
04130 END
04140 IMPLICIT REAL*8(A-H,O-Z)
04150 SUBROUTINE DBDRG

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04160 C      THIS SUBROUTINE CALCULATES THE VISCOUS DRAG FORCE ON THE
04170 C      SPAR BUOY DURING ITS DYNAMIC MOTIONS.  INTEGRATION OF FORCES
04180 C      IS DONE ALONG THE AXIS OF THE SPAR.
04190 C      -----
04200 C
04210 C      COMMON W(8),CM(8),CMS(8),CMD(8),CMN(8),CMT(8),SL(8),EK(8),CDIN(8),
04220 C      1CDIT(8),CDIA(8),DEP(8),V(8),VV(8),TN(8),TNN(8),TL(8),CS(9),SN(9),Y
04230 C      1DD(17),YD(17),Y(17),X(17),F(17),DD(17),TC(19),GK(8,17),GM(17,17),D
04240 C      1C1,DC2,DC3,XC1,ZC1,XC2,ZC2,WWD1,WWD2,S01,S02,CS1,SN1,RD1,RD2,XJ1,Y
04250 C      1J1,XJ2,YJ2,BB1,BB2,HMAX,H0,HST,HOD,RRH01,RRH02,TMASS,YNERT,ZCG,HI,
04260 C      1TH0,CDP,CDL,CON,COT,ALPHA,PI,RHO,GR,AW,I,DT,AMP,WE,SNK,CSK,DEPTH,O
04270 C      1K,VVV(8),OEXP,VV0,VV1,XW,ZW,EMW,MOOR,NST,NM,NM1,NM2,NDF,NB,NB1,NB2
04280 C      1,NN,ND
04290 C      *****COEFFICIENTS OF THE VELOCITY EQUATION ARE CALCULATED BELOW*****
04300 C      V1=(V(NB)-YD(NN-1))*CS1+YD(NN)*SN1
04310 C      V2=WE*AMP*(SNK*CS1+CSK*SN1)
04320 C      V3=(V(NB)-YD(NN-1))*SN1-YD(NN)*CS1
04330 C      V4=WE*AMP*(SNK*SN1-CSK*CS1)
04340 C      VTB=V3-V4*OEXP
04350 C      VTS=V3-V4*OEXP*DEXP(OK*HST)
04360 C      *****PRESSURE DRAG DUE TO BOTTOM END AND STEP OF THE SPAR IS CALCULATED
04370 C      DP=0.5*RHO*CDP*(S01*DABS(VTB)*VTB+(S01-S02)*DABS(VTS)*VTS)
04380 C      DN=0.
04390 C      DDTT=0.
04400 C      DM=0.
04410 C      *****THE VISCOUS DRAG FORCE AND MOMENT ON THE SPAR BUOY ARE CALCULATED
04420 C      RD=RD1
04430 C      NDS1=NST*HST/HI
04440 C      NDS2=NST-NDS1
04450 C      NDS=NDS1
04460 C      DZ=HST/NDS
04470 C      ZZ=DZ/2.-ZCG

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04480 RH00=RH0*CDL
04490 DO 2 INZ=1,2
04500 DO 1 IZ=1,NDS
04510 XXE=DEXP(OK*(ZZ+ZCG))
04520 VNN=V1-V2*OEXP*XXE-ZZ*YD(NN+1)
04530 DNN=VNN*DABS(VNN)*RD*DZ
04540 DN=DN+DNN
04550 VTT=V3-V4*OEXP*XXE
04560 DDTT=DDTT+VTT*DABS(VTT)*RD*DZ
04570 DM=DM+DNN*ZZ
04580 1 ZZ=ZZ+DZ
04590 RD=RD2
04600 NDS=NDS2
04610 DZ=(HI-HST)/NDS
04620 ZZ=HST+DZ/2.-ZCG
04630 2 CONTINUE
04640 DN=RH00*DN
04650 DDTT=0.02*RH00*DDTT
04660 DC3=RH00*DM
04670 C****DC1&DC2 ARE THE VISCOUS DRAG FORCES IN SURGE AND HEAVE DIRECTIONS*
04680 DC1=DN*CS1+(DDTT+DP)*SN1
04690 DC2=(DDTT+DP)*CS1-DN*SN1
04700 RETURN
04710 END
04720 IMPLICIT REAL*8(A-H,O-Z)
04730 SUBROUTINE BUOYS
04740 C THIS SUBROUTINE CALCULATES THE MASS MATRIX FOR THE SPAR BUOY,
04750 C PART(EXCEPT VISCOUS) OF THE (FA) MATRIX AS DD, AND (FD+FG)
04760 C ARRAYS AS XW,ZW,AND EMW.
04770 C -----
04780 C COMMON W(8),CM(8),CMS(8),CMD(8),CMN(8),CMT(8),SL(8),EK(8),CDIN(8),
04790

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04800 1CDIT(8),CDIA(8),DEP(8),VV(8),TN(8),TNN(8),TL(8),CS(9),SN(9),Y
04810 1DD(17),YD(17),Y(17),X(17),F(17),DD(17),JC(19),GK(8,17),GM(17,17),D
04820 1C1,DC2,DC3,XC1,ZC1,XC2,ZC2,WWD1,WWD2,S01,S02,CS1,SN1,RD1,RD2,XJ1,Y
04830 1J1,XJ2,YJ2,BB1,BB2,HMAX,H0,HST,HOD,RRH01,RRH02,TMASS,YNERT,ZCG,HI,
04840 1TH0,CDP,CDL,CON,COT,ALPHA,PI,RHO,GR,AW,I,DT,AMP,WE,SNK,CSK,DEPTI,0
04850 1K,VVV(8),OEXP,VVO,VV1,XW,ZW,EMW,MOOR,NST,NM,NM1,NM2,NDF,NB,NB1,NB2
04860 1,N,N,N
04870 DO 1 I=1,NDF
04880 DO 1 J=1,NDF
04890 1 GM(I,J)=0.
04900 BK1=(BB1-BB2)*HST
04910 BK2=BB2*H0
04920 P0=BB1*HST+BR2*(H0-HST)
04930 P1=BK2*(0.5*H0-ZCG)+BK1*(0.5*HST-ZCG)
04940 P2=BK2*(H0*H0/3.-H0*ZCG+ZCG*ZCG)+BK1*(HST*HST/3.-HST*ZCG+ZCG*ZCG)
04950 QQ2=DEXP(-H0D*OK)
04960 Q0=(BB1*(QQ2-OEXP))+BB2*(1.-QQ2))/OK
04970 Q1=(BB1*(QQ2*(HST-1./OK-ZCG)+OEXP*(1./OK+ZCG))+BB2*(H0-1./OK-
04980 1ZCG-QQ2*(HST-1./OK-ZCG)))/OK
04990 C*****COEFFICIENTS IN EQUATIONS OF BUOY MOTION(MASS MATRIX)*****
05000 GM(NN-1,NN-1)=TMASS+P0*(CON*CS1*CS1+COT*SN1*SN1)+(VVO+VV1)*SN1*SN
05010 11
05020 GM(NN-1,NN)=P0*SN1*CS1*(COT-CON)+(VVO+VV1)*SN1*CS1
05030 GM(NN-1,NN+1)=P1*CS1*CON
05040 GM(NN,NN-1)=GM(NN-1,NN)
05050 GM(NN,NN)=TMASS+P0*(CON*SN1*SN1+COT*CS1*CS1)+(VVO+VV1)*CS1*CS1
05060 GM(NN,NN+1)=-P1*SN1*CON
05070 GM(NN+1,NN-1)=GM(NN-1,NN+1)
05080 GM(NN+1,NN)=GM(NN,NN+1)
05090 GM(NN+1,NN+1)=YNERT+P2*CON
05100 C*****PART OF THE (FA) ARRAY*****
05110 DD(NN-1)=(COT*P1*SN1-VVO*SN1*ZCG+VV1*SN1*(HST-ZCG))*YD(NN+1)*YD(NN

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05120      1+1)
05130      DD(NN)=(COT*P1*CS1-VVO*CS1*ZCG+VV1*CS1*(HST-ZCG))*YD(NN+1)*YD(NN+1
05140      1)
05150      DD(NN+1)=0.
05160      C*****WAVE EXCITATION AND HYDROSTATICS FORCES ON THE SPAR*****
05170      XW=WE*WE*AMP*Q0*(CSK*(1.+CON*CS1*CS1+COT*SN1*SN1)+SNK*SN1*CS1*(COT
05180      1-CON))
05190      ZW=WE*WE*AMP*Q0*(SNK*(1.+CON*SN1*SN1+COT*CS1*CS1)+CSK*SN1*CS1*(COT
05200      1-CON))+GR*(BB1*HST+BB2*(HI-HST))-TMASS)
05210      EMW=WE*WE*AMP*Q1*(1.+CON)*(CSK*CS1-SNK*SN1)-GR*P1*SN1
05220      RETURN
05230      END
05240      IMPLICIT REAL*8(A-H,O-Z)
05250      SUBROUTINE NEXT
05260      C      THIS SUBROUTINE UPDATES THE GEOMETRY OF THE MOORING SYSTEM
05270      C      DURING ITS DYNAMIC MOTIONS.
05280      C      -----
05290      C
05300      COMMON W(8),CM(8),CMS(8),CMD(8),CMN(8),CMT(8),SL(8),EK(8),CDIN(8),
05310      1CDIT(8),CDIA(8),DEP(8),V(8),VV(8),TN(8),TNN(8),TL(8),CS(9),SN(9),Y
05320      1DD(17),YD(17),Y(17),X(17),F(17),DD(17),TC(19),GK(8,17),GM(17,17),D
05330      1C1,DC2,DC3,XC1,ZC1,XC2,ZC2,WWD1,WWD2,S01,S02,CS1,SN1,RD1,RD2,XJ1,Y
05340      1J1,XJ2,YJ2,BB1,BB2,HMAX,H0,HST,HOD,RRH01,RRH02,TMASS,YNERT,ZCG,HI,
05350      1THO,CDP,CRL,CON,COT,ALPHA,FI,RHO,GR,AW,T,DT,AMP,WE,SNK,CSK,DEPTH,O
05360      1K,VVV(8),OEXP,VVO,VV1,XW,ZW,EMW,MOOR,NST,NM,NM1,NM2,NDF,NB,NB1,NB2
05370      1,NN,ND
05380      IF(MOOR.EQ.2)GO TO 3
05390      XJ1=X(NN-1)+ZC1*SN1+XC1*CS1
05400      YJ1=X(NN)+ZC1*CS1-XC1*SN1
05410      IF(NB.EQ.1)GO TO 1
05420      TL(1)=DSQRT(X(1)**2+X(2)**2)
05430      CS(1)=X(1)/TL(1)

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05440 SN(1)=X(2)/TL(1)
05450 TL(NB)=DSQRT((XJ1-X(NN-3))**2+(YJ1-X(NN-2))**2)
05460 CS(NB)=(XJ1-X(NN-3))/TL(NB)
05470 SN(NB)=(YJ1-X(NN-2))/TL(NB)
05480 GO TO 2
05490 1 TL(1)=DSQRT(XJ1**2+YJ1**2)
05500 CS(1)=XJ1/TL(1)
05510 SN(1)=YJ1/TL(1)
05520 2 IF(NM.EQ.NB)GO TO 4
05530 3 XJ2=X(NN-1)+ZC2*SN1+XC2*CS1
05540 YJ2=X(NN)+ZC2*CS1-XC2*SN1
05550 TL(NB2)=DSQRT((X(NN+2)-XJ2)**2+(X(NN+3)-YJ2)**2)
05560 CS(NB2)=(X(NN+2)-XJ2)/TL(NB2)
05570 SN(NB2)=(X(NN+3)-YJ2)/TL(NB2)
05580 4 DO 5 I=2,NM
05590 IF(I.EQ.NB.OR.I.EQ.NB2)GO TO 5
05600 II=I+1
05610 IF(I.LT.NB)II=II-1
05620 TL(I)=DSQRT((X(II)-X(II-2))**2+(X(II+1)-X(II-1))**2)
05630 CS(I)=(X(II)-X(II-2))/TL(I)
05640 SN(I)=(X(II+1)-X(II-1))/TL(I)
05650 5 CONTINUE
05660 DO 6 I=1,NM
05670 IF(SN(I).LT.-1.)SN(I)=-1.0
05680 RETURN
05690 END
05700 IMPLICIT REAL*8(A-H,O-Z)
05710 SUBROUTINE MINVER(N,C)
05720 C THIS SUBROUTINE CALCULATES THE INVERSE OF MATRIX 'C' OF ORDER 'N'
05730 C -----
05740 C
05750 DIMENSION C(17,17),H(17),G(17)

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05760 NN=N-1
05770 C(1,1)=1.0/C(1,1)
05780 DO 6 M=1,NN
05790 K=M+1
05800 DO 1 I=1,M
05810 G(I)=0.
05820 DO 1 J=1,M
05830 1 G(I)=G(I)+C(I,J)*C(J,K)
05840 D=0.
05850 DO 2 I=1,M
05860 2 D=D+C(K,I)*G(I)
05870 E=C(K,K)-D
05880 C(K,K)=1.0/E
05890 DO 3 I=1,M
05900 3 C(I,K)=-G(I)*C(K,K)
05910 DO 4 J=1,M
05920 H(J)=0.
05930 DO 4 I=1,M
05940 4 H(J)=H(J)+C(K,I)*C(I,J)
05950 DO 5 J=1,M
05960 5 C(K,J)=-H(J)*C(K,K)
05970 DO 6 I=1,M
05980 DO 6 J=1,M
05990 6 C(I,J)=C(I,J)-G(I)*C(K,J)
06000 RETURN
06010 END
06020 IMPLICIT REAL*8(A-H,O-Z)
06030 SUBROUTINE MATRIX
06040 C
06050 C THIS SUBROUTINE CALCULATES ALL THE REMAINING ELEMENTS OF THE
06060 C GLOBAL MATRICES AND INTEGRATES THE 'NDF' SIMULTANEOUS DIFFERENTIAL
06070 C EQUATIONS OF MOTION BY THE METHOD OF RECTANGULAR RULE.
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06080 C
06090 COMMON W(8),CM(8),CMS(8),CMD(8),CMN(8),CMT(8),SL(8),EK(8),CDIN(8),
06100 1CDIT(8),CDIA(8),DEP(8),V(8),VV(8),TN(8),TNN(8),IL(8),CS(9),SN(9),Y
06110 1DD(17),YD(17),Y(17),X(17),F(17),DD(17),IC(19),GK(8,17),GM(17,17),D
06120 1C1,DC2,DC3,XC1,ZC1,XC2,ZC2,WWD1,WWD2,SC1,SC2,SG1,SG2,RD1,RD2,XC1,
06130 1J1,XJ2,YJ2,BB1,BB2,HMAX,H0,HST,HOD,RRHG1,RRHG2,TMASS,YNERT,ZCG,HT
06140 1THO,CDP,CBL,CON,COT,ALPHA,PI,RHO,GR,AW,T,DT,AMP,WE,SNM,CSK,DEPTH,L
06150 1K,VVV(8),DEXP,VVO,VV1,XW,ZW,EMW,MOOR,NST,NM,NM1,NM2,NDF,NB,NB1,NB2
06160 1,NN,ND
06170 C*****WAVE EXCITING + HYDROSTATIC + VISCOUS DRAG FORCES ON THE SPAR*****
06180 F(NN-1)=DC1+XW
06190 F(NN)=ZW+DC2
06200 F(NN+1)=DC3+EMW
06210 IF(MOOR.EQ.0)GO TO 11
06220 C*****STIFFNESS MATRIX FOR THE MOORING LINES*****
06230 AA1=XC1*CS1+ZC1*SN1
06240 AA2=XC2*CS1+ZC2*SN1
06250 AB1=-XC1*SN1+ZC1*CS1
06260 AB2=-XC2*SN1+ZC2*CS1
06270 GK(1,1)=EK(1)*CS(1)
06280 GK(1,2)=EK(1)*SN(1)
06290 GK(NB,NN+1)=EK(NB)*(AB1*CS(NB)-AA1*SN(NB))
06300 IF(NM.EQ.1)GO TO 5
06310 IF(NM.EQ.NB)GO TO 1
06320 GK(NB2,NN-1)=-EK(NB2)*CS(NB2)
06330 GK(NB2,NN)=-EK(NB2)*SN(NB2)
06340 GK(NB2,NN+1)=-EK(NB2)*(AB2*CS(NB2)-AA2*SN(NB2))
06350 GK(NB2,NN+2)=-GK(NB2,NN-1)
06360 GK(NB2,NN+3)=-GK(NB2,NN)
06370 1 DO 2 I=2,NN
06380 IF(I.EQ.NB2)GO TO 2
06390 II=I+1

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06400 IF(I.GT.NB)II=II+1
06410 GK(I,II-3)=-EK(I)*CS(I)
06420 GK(I,II-2)=-EK(I)*SN(I)
06430 GK(I,II)=-GK(I,II-2)
06440 GK(I,II-1)=-GK(I,II-3)
06450 2 CONTINUE
06460 C*****MASS MATRIX FOR THE MOORING LINES*****
06470 DO 4 I=1,NM
06480 IF(I.EQ.NB)GO TO 4
06490 II=I+1
06500 IF(I.LT.NB)II=II-1
06510 GM(II,II)=CM(I)+CMS(I)+CMN(I)*SN(I)**2+CMT(I)*CS(I)**2
06520 GM(II+1,II+1)=CM(I)+CMS(I)+CMN(I)*CS(I)**2+CMT(I)*SN(I)**2
06530 GM(II,II+1)=(CMT(I)-CMN(I))*SN(I)*CS(I)
06540 GM(II+1,II)=GM(II,II)
06550 WWE=WE*AMP*DEXP(AW*(X(II+1)-DEPTH))
06560 SWE=AW*X(II)-WWE*
06570 SNK1=DSIN(SWE)
06580 CSK1=DCOS(SWE)
06590 C*****VISCOUS DRAG FORCES ON THE MOORING LINES*****
06600 U1=V(I)-YD(II)-WWE*SNK1
06610 U2=-YD(II+1)+WWE*CSK1
06620 UT1=U1*CS(I+1)+U2*SN(I+1)
06630 UN1=U1*SN(I+1)-U2*CS(I+1)
06640 DN=CDIN(I)*UN1*DABS(UN1)
06650 DDT1=CDIT(I)*UT1*DABS(UT1)
06660 U3=DSQRT(U1**2+U2**2)*CDIA(I)
06670 DO(II)=DN*SN(I+1)+DDT1*CS(I+1)+U3*U1
06680 DD(II+1)=DDT1*SN(I+1)-DN*CS(I+1)+U3*U2
06690 IF(MOOR.EQ.2.AND.I.GE.ND)GO TO 3
06700 F(II)=CMD(I)*WWE*WWE*CSK1
06710 F(II+1)=CMD(I)*WWE*WWE*SNK1-W(I)

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06720 GO TO 4
06730 F(II)=WE*WWE*CMD(I)*SN(I)*(CSK1*SN(I)-SNK1*CS(I))
06740 F(II+1)=WE*WWE*CMP(I)*CS(I)*(SNK1*CS(I)-CSK1*SN(I))-W(I)
06750 4 CONTINUE
06760 5 DO 7 K=1,NM
06770 TG=0.
06780 DO 6 IK=1,NDF
06790 TG=TG+GK(K,IK)*Y(IK)
06800 TNN(K)=TG+TNN(K)
06810 TN(K)=TNN(K)
06820 IF(TN(K).LE.0.)TN(K)=0.
06830 KK=K+K
06840 IF(K.LE.NB)KK=KK-1
06850 TC(KK)=TN(K)*CS(K)
06860 TC(KK+1)=TN(K)*SN(K)
06870 TC(NN+1)=-TC(NN-1)*(-ZC1*CS1+XC1*SN1)+TC(NN)*(-XC1*CS1-ZC1*
06880 1SN1)+TC(NN+2)*(-ZC2*CS1+XC2*SN1)-TC(NN+3)*(-XC2*CS1-ZC2*SN1))
06890 IF(NM.EQ.1)GO TO 11
06900 IF(MOOR.EQ.2)GO TO 9
06910 NZ=NN-2
06920 DO 8 M=1,NZ
06930 TC(M)=TC(M)-TC(M+2)
06940 TC(NN-1)=TC(NN-1)-TC(NN+2)
06950 TC(NN)=TC(NN)-TC(NN+3)
06960 IF(NM.EQ.NB)GO TO 11
06970 NZ=NN+2
06980 DO 10 M=NZ,NDF
06990 TC(M)=TC(M)-TC(M+2)
07000 11 CALL MINVER(NDF,GM)
07010 DO 13 M=1,NDF
07020 YDD(M)=0.
07030 DO 12 K=1,NDF

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```
07040 12 YDD(M)=YDD(M)+GM(M,K)*(DD(K)-TC(K))+F(K))
07050 YD(M)=YD(M)+YDD(M)*DT
07060 Y(M)=YD(M)*DT
07070 13 X(M)=X(M)+Y(M)
07080 CS1=DCOS(X(NN+1))
07090 SN1=DSIN(X(NN+1))
07100 HO=ZCG+(DEPTH-X(NN))/CS1
07110 HOD=HO-HST
07120 RETURN
07130 END
READY
```

[illegible]

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00320 READ(5,102)(PI(J),SI(J),ZM(J),ZMV(J),F(3,J),CDIN(J),CDIT(J)
00330 1,J=1,NI)
00340 DO 1 I1=1,2
00350 DO 1 I2=1,NI
00360 1 F(I1,I2)=0.
00370 C*****READ IN THE PARAMETERS FOR DIFFERENT MOORING PARTS*****
00380 C DIAL IS IN INCHES; SLL IN METERS; AWL AND WWL IN LBS/M; TPL IN
00390 C POUNDS OR LBS/(SQ.IN.); C01 IS IN INCHES FOR JACKETED WIRE ROPE
00400 READ(5,103)(DIAL(I),SLL(I),AWL(I),WWL(I),TPL(I),I=1,NP)
00410 READ(5,105)(C01(I),P01(I),C02(I),P02(I),A0(I),CDN(I),CDT(I)
00420 1,I=1,NP)
00430 C*****READ THE FORCING CURRENT PROFILE*****
00440 C D ARE IN METERS; V IN MM/SEC; AND WE IN RAD/SEC.
00450 READ(5,105)(D(I),I=1,IKK)
00460 READ(5,105)((V(I,J),I=1,3),J=1,8)
00470 READ(5,105)WE,DT,T2,TMAX
00480 C*****READ IN THE SINUSOIDAL SURFACE WAVE.*****
00490 C WS IS IN RAD/SEC; AMP IN MM.
00500 READ(5,105)WS,AMP,BETA
00510 CSB=DCCS(BETA)
00520 SNB=DSIN(BETA)
00530 PII=4.*DATAN(1.DO)
00540 RHO=1.9905
00550 GR=32.2
00560 CONSI=RHO*GR*PII*0.0057
00570 AW=WS*WS/GR
00580 AWX=AW*CSB
00590 AWY=AW*SNB
00600 SLT=0.
00610 DO 2 I=1,NP
00620 IF(P01(I).EQ.0.0)GO TO 2
00630 C*****PERMANENT STRETCH FOR SYNTHETIC ROPES.*****

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00640 PSTRA(I)=((TPL(I)/(DIAL(I)**2)-200.0)/CO1(I))**(1.0/PO1(I))
00650 2 SLT=SLT+SL(I)
00660 C*****SEGMENTATION OF THE MOORING LINE IN NPTS SEGMENTS*****
00670 DO 3 I=1,NP
00680 NM(I)=(SLL(I)/SLT)*NS+0.5
00690 IF(NM(I).EQ.0)NM(I)=1
00700 SL(I)=SLL(I)/NM(I)
00710 SWA(I)=AWL(I)*SL(I)
00720 SWW(I)=WWL(I)*SL(I)
00730 SWD(I)=SWA(I)-SWW(I)
00740 CDIA(I)=SL(I)*(DIAL(I)**2)
00750 3 CONS(I)=CDIA(I)*CONS1
00760 NS1=NS+NP
00770 C*****INITIALIZE DYNAMIC VARIABLES*****
00780 DO 5 I1=1,3
00790 DO 4 I2=1,NS1
00800 YDDL(I2,I1)=0.
00810 DO 4 I3=1,2
00820 YDL(I3,I2,I1)=0.
00830 DO 5 I4=1,30
00840 YDDI(I4,I1)=0.
00850 DO 5 I5=1,2
00860 YDI(I5,I4,I1)=0.
00870 DO 5 I6=1,2
00880 YI(I6,I5,I4,I1)=0.
00890 CALL MOTION
00900 CALL EXIT
00910 END
00920 IMPLICIT REAL*8(A-H,O-Z)
00930 SUBROUTINE MOTION
00940 C THIS SUBROUTINE CALCULATES THE DYNAMIC EQUILIBRIUM OF THE MOORING
00950 C LINE FOR TMAX SECONDS.

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00960 C
00970 C
00980
00990 COMMON PI(30),SI(30),F(3,30),CDIN(30),CDIT(30),DIAL(20),SLL(20),AW
01000 1L(20),CDN(20),CDT(20),TPL(20),CN(20),C02(20),PD1(20),PD2(20),CONS
01010 1(20),V(3,10),NM(20),SL(20),SWA(20),CIA(20),U(3),X(3),THETA(3),AWX
01020 1,DN(3),WN(3),CS(3),SN(3),DB(3),UN(3),IT(3),PSTRA(20),AD(20),D(10),
01030 1MWL(20),SWW(20),SWD(20),YDBL(150,3),YDL(2,150,3),YL(2,150,3),YDDI(
01040 130,3),YDI(2,30,3),YI(2,30,3),YDDT(3),YDDN(3),ZM(30),ZMV(30),DEPTH,
01050 1VS(3),S2,TE,H2,IK,NI,J,NC,WS,AMP,NS,NP,DDT,E,ER,PII,T,SINT,AW,AWY,
01060 1DIA,ERI,XOLD(3),GR,WE,DT,T2,TMAX,INK,I
01070 101 FORMAT(5F10.2)
01080 102 FORMAT(1X,' SOLUTION IS TERMINATING AS THE ERROR IN MOORING LINE V
01090 1ELOCITY AND ACCELERATION CALCULATIONS IS EXCESSIVE'/1X,' ERI IS EX
01100 1CEEDED BY=',F5.3)
01110 1T=0.
01120 1T3=0.
01130 1VTRY=GR*AMP/WS
01140 1SINT=DSIN(WE*T)
01150 1SINTW=DSIN(WS*T)
01160 1VS(1)=-VTRY*AWX*SINTW
01170 1VS(2)=-VTRY*AWY*SINTW
01180 1VS(3)=VTRY*AW*DCOS(WS*T)
01190 1INDEX=0
01200 2 INDEX=INDEX+1
01210 2S2=0.0
01220 2X(3)=DDT
01230 2K=1
01240 2X(1)=0.
01250 2X(2)=0.
01260 2J=1
01270 2TE=0.0

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01280 THETA(1)=PII/2.
01290 THETA(2)=THETA(1)
01300 THETA(3)=0.
01310 DO 13 I=1,NP
01320 NK=NM(I)+K-1
01330 DO 12 N=K,NK
01340 C*****CHECK FOR ANY INSERTED INSTRUMENTS OR FLOATS*****
01350 IF(J.GT.NI)GO TO 4
01360 3 IF((S2+SL(I)*0.5).GE.PI(J))CALL FORCES
01370 IF(J.GT.NI)GO TO 4
01380 IF((S2+SL(I)*0.5).GE.PI(J))GO TO 3
01390 C*****STRESS-STRAIN CALCULATIONS FOR A MOORING SEGMENT*****
01400 4 IF(FO1(I).EQ.0.0)GO TO 6
01410 C*****SYNTHETIC ROPES*****
01420 STR=TE/(DIAL(I)**2)
01430 IF(STR.LE.200.0)GO TO 5
01440 TSTRA=PSTRA(I)+((STR-200.0)/CO2(I))**((1.0/PO2(I))
01450 S1=A0(I)*SL(I)*(1.0+TSTRA/100.0)
01460 GO TO 7
01470 5 S1=SL(I)
01480 GO TO 7
01490 C*****WIRE ROPES-TPL IS ELASTIC MODULUS AND CO1 IS JACKET DIA*****
01500 6 TSTRA=TE/((.7854*PI(I)*(DIAL(I)-CO1(I))**2)
01510 S1=SL(I)*(1.+TSTRA)
01520 C*****BEGINNING OF THE NUMERICAL INTEGRATION PROCEDURE*****
01530 7 DIA=DSQRT(CDIA(I)/S1)
01540 S2=S2+SL(I)
01550 DO 8 IC=1,3
01560 SN(IC)=DSIN(THETA(IC))
01570 CS(IC)=DCOS(THETA(IC))
01580 X(IC)=X(IC)+S1*CS(IC)
01590 8 YL(2,N,IC)=X(IC)

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01600 H2=X(3)-S1*CS(3)/2.
01610 CALL SPEED
01620 DO 9 IC=1,3
01630 U(IC)=U(IC)/304.8-YDL(2,N,IC)
01640 UT(IC)=U(IC)*CS(IC)
01650 YDDT(IC)=YDDL(N,IC)*CS(IC)
01660 WN(IC)=SWW(I)*CS(IC)*CS(3)
01670 WN(3)=WN(3)-SWW(I)
01680 UTT=UT(1)+UT(2)+UT(3)
01690 YDDTT=YDDT(1)+YDDT(2)+YDDT(3)
01700 DTT=3.28*CDT(I)*S1*DIA*DABS(UTT)*UTT/12.
01710 TE=TE-SWW(I)*CS(3)+DTT-SWA(I)*YDDTT/GR
01720 TN=TE-H2*SWD(I)/S1
01730 DO 10 IC=1,3
01740 YDDN(IC)=YDDL(N,IC)-YDDTT*CS(IC)
01750 UN(IC)=U(IC)-UTT*CS(IC)
01760 VNM=DSQRT(UN(1)**2+UN(2)**2+UN(3)**2)
01770 DO 11 IC=1,3
01780 DN(IC)=3.28*CDN(I)*S1*DIA*VNM*UN(IC)/12.+WN(IC)-((2.*SWA(I)-SWW(I)
01790 1)/GR)*YDDN(IC)
01800 IF(SN(3).EQ.0.)GO TO 11
01810 THETA(IC)=THETA(IC)-DN(IC)/(TE*SN(IC))
01820 CONTINUE
01830 CONTINUE
01840 K=NK+1
01850 CONTINUE
01860 C*****CHECK FOR ANY INSERTED INSTRUMENTS OR FLOATS BEFORE THE ANCHOR***
01870 14 IF(J.GT.NI)GO TO 15
01880 CALL FORCES
01890 GO TO 14
01900 15 E=DEPTH-X(3)
01910 IF(T.EQ.0.)GO TO 21

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01920 IF(INDEX,EQ,1)GO TO 17
01930 IF(T,EQ,DT)GO TO 21
01940 DO 16 IE=1,2
01950 TRY=DARS((X(IE)-XOLD(IE))/X(IE))
01960 IF(TRY,GT,ER1)GO TO 25
01970 16 CONTINUE
01980 GO TO 21
01990 DO 18 IC=1,3
02000 18 XOLD(IC)=X(IC)
02010 DO 20 IC=1,3
02020 DO 19 I1=1,N
02030 YDL(2,I1,IC)=(X(IC)-YI(2,I1,IC)-YI(1,I1,IC))*3.28/DT
02040 YDDL(I1,IC)=(YDL(2,I1,IC)-YDL(1,I1,IC))/DT
02050 DO 20 I2=1,NI
02060 YDI(2,I2,IC)=(X(IC)-YI(2,I2,IC)-YI(1,I2,IC))*3.28/DT
02070 YDDI(I2,IC)=(YDI(2,I2,IC)-YDI(1,I2,IC))/DT
02080 WRITE(6,101)T,(X(KJ),KJ=1,3),E
02090 GO TO 2
02100 DO 23 IC=1,3
02110 DO 22 I1=1,N
02120 YL(1,I1,IC)=X(IC)-YL(2,I1,IC)
02130 YDL(1,I1,IC)=YDL(2,I1,IC)
02140 DO 23 I2=1,NI
02150 YI(1,I2,IC)=X(IC)-YI(2,I2,IC)
02160 YDI(1,I2,IC)=YDI(2,I2,IC)
02170 IF(T,LT,T3)GO TO 24
02180 X(3)=X(3)-DDT
02190 WRITE(6,101)T,(X(KJ),KJ=1,3),E
02200 IF(T,GT,TMAX)GO TO 26
02210 T3=T3+T2
02220 T=T+DT
02230 24 GO TO 1

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02240      EE1=TRY-ER1
02250      WRITE(6,102)EE1
02260      RETURN
02270      END
02280      IMPLICIT REAL*8(A-H,O-Z)
02290      SUBROUTINE FORCES
02300      C      THIS SUBROUTINE CALCULATES THE FORCE EQUILIBRIUM FOR INSERTED
02310      C      INSTRUMENTS OR FLOATS.
02320      C      -----
02330      C
02340      COMMON PI(30),SI(30),F(3,30),CDIN(30),CDIT(30),DIAL(20),SLL(20),AW
02350      1L(20),CDN(20),CDT(20),TPL(20),C01(20),C02(20),F01(20),F02(20),CONS
02360      1(20),V(3,10),NM(20),SL(20),SWA(20),CDIA(20),U(3),X(3),THETA(3),AWX
02370      1,DN(3),WN(3),CS(3),SN(3),DD(3),UN(3),UT(3),PSTRA(20),AD(20),D(10),
02380      1WWL(20),SWW(20),SWD(20),YDDL(150,3),YDL(2,150,3),YL(2,150,3),YDDI(
02390      130,3),YDI(2,30,3),YI(2,30,3),YDDT(3),YDDN(3),ZM(30),ZMV(30),DEPTH,
02400      1VS(3),S2,TE,H2,IK,NI,J,NC,WS,AMP,NS,NP,DDT,E,ER,PII,T,SINT,AW,AWY,
02410      1DIA,ERI,XOLE(3),GR,WE,DT,T2,TMAX,IKK,I
02420      C*****NUMERICAL INTEGRATION PROCEDURE*****
02430      S2=S2+SI(J)
02440      DO 1 IC=1,3
02450      SN(IC)=DSIN(THETA(IC))
02460      CS(IC)=DCOS(THETA(IC))
02470      X(IC)=X(IC)+SI(J)*CS(IC)
02480      1 YI(2,J,IC)=X(IC)
02490      H2=X(3)-SI(J)*CS(3)/2.
02500      CALL SPEED
02510      DO 2 IC=1,3
02520      U(IC)=U(IC)/304.8-YDI(2,J,IC)
02530      2 CONTINUE
02540      IF(SI(J).NE.0.)GO TO 4
02550      VNM=DSQRT(U(1)**2+U(2)**2+U(3)**2)

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02560      DO 3 IC=1,3
02570      DD(IC)=TE*CS(IC)+CDIN(J)*VNM*U(IC)+F(IC,J)-(ZM(J)+ZMV(J))
02580      1*YDDI(J,IC)
02590      GO TO 8
02600
02610      DO 5 IC=1,3
02620      YDDT(IC)=YDDI(J,IC)*CS(IC)
02630      UT(IC)=U(IC)*CS(IC)
02640      UTT=UT(1)+UT(2)+UT(3)
02650      YDDT=YDDT(1)+YDDT(2)+YDDT(3)
02660      DTT=CDIT(J)*DABS(UTT)*UTT
02670      DO 6 IC=1,3
02680      YDDN(IC)=YDDI(J,IC)-\DDTT*CS(IC)
02690      UN(IC)=U(IC)-UTT*CS(IC)
02700      VNM=DSQRT(UN(1)**2+UN(2)**2+UN(3)**2)
02710      DO 7 IC=1,3
02720      DD(IC)=(TE+DTT)*CS(IC)+CDIN(J)*VNM*UN(IC)+F(IC,J)
02730      1-ZM(J)*YDDI(J,IC)-ZMV(J)*YDDN(IC)
02740      7 CONTINUE
02750      TE=DSQRT(DD(1)**2+DD(2)**2+DD(3)**2)
02760      DO 9 IC=1,3
02770      THETA(IC)=DARCOS(DD(IC)/TE)
02780      J=J+1
02790      RETURN
02800      END
02810      IMPLICIT REAL*8(A-H,O-Z)
02820      SUBROUTINE SPEED
02830      THIS SUBROUTINE CALCULATES THE TOTAL (PROFILE+SURFACE WAVE VEL.)
02840      ABSOLUTE VELOCITY FIELD; GIVEN VELOCITY AT FEW DEPTHS AND A SIN.
02850      SURFACE WAVE. STEPPED PROFILE (#2 OF VOLUME 1 CHHABRA(1973)) IS
02860      USED.
02870

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02880 COMMON PI(30),SI(30),F(3,30),CDIN(30),CDIT(30),DIAL(20),SLL(20),AW
02890 1L(20),CDN(20),CDT(20),TPL(20),C01(20),C02(20),P01(20),P02(20),CONS
02900 1(20),V(3,10),NM(20),SL(20),SWA(20),CDIA(20),U(3),X(3),THETA(3),AWX
02910 1,DN(3),WN(3),CS(3),SN(3),DD(3),UN(3),UT(3),PSTRA(20),AD(20),D(10),
02920 1WWL(20),SWW(20),SWD(20),YDDL(150,3),YDL(2,150,3),YL(2,150,3),YDDI(
02930 130,3),YDI(2,30,3),YI(2,30,3),YDDT(3),YDDN(3),ZM(30),ZMV(30),DEPTH,
02940 1VS(3),S2,TE,H2,IK,NI,J,NC,WS,AMP,NS,NP,DDT,E,ER,PII,T,SINT,AW,AWY,
02950 1DIA,ERI,XOLD(3),GR,WE,DT,T2,THAX,IKK,I
02960 OEXP=DEXP(-AW#H2)
02970 M=1
02980 DO 5 JN=1,3
02990 IF(H2.LE.D(1))GO TO 3
03000 IF(H2.GE.D(IKK))GO TO 4
03010 1 IF(H2.GT.D(M),AND.H2.LE.D(M+1))GO TO 2
03020 M=M+1
03030 GO TO 1
03040 2 U(JN)=V(JN,M+1)*(1.+0.05*SINT)+VS(JN)*OEXP
03050 GO TO 5
03060 3 U(JN)=U(JN,1)*(1.+0.05*SINT)+VS(JN)*OEXP
03070 GO TO 5
03080 4 U(JN)=V(JN,IKK+1)*(1.+0.05*SINT)+VS(JN)*OEXP
03090 5 CONTINUE
03100 RETURN
03110 END
READY

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SSS3.FORT
00010 C
00020 C
00030 C
00040 C
00050 C
00060 C
00070 C
00080 C*****BEGINNING OF THE MAIN PROGRAM*****
00090 COMMON PI(30),SI(30),F(3,30),CDIN(30),CDIT(30),DIAL(20),SLL(20),AW
00100 1L(20),CDN(20),CDT(20),TFL(20),C01(20),C02(20),F01(20),F02(20),CONS
00110 1(20),V(850,2,10),NM(20),SL(20),SWA(20),CDIA(20),U(3),X(3),THETA(3)
00120 1,DN(3),WN(3),CS(3),SN(3),DD(3),UN(3),UT(3),PSTRA(20),AO(20),D(10),
00130 1,WL(20),SWW(20),SWD(20),DEPTH,TSTRA,I,S2,TE,H2,IK,NI,J,NC,INDEX,JJ
00140 1,NPT,NS,NP,DDT,E,ER,PII,CONS1,DIA,IKK
00150 C*****BEGINNING OF FORMAT STATEMENTS*****
00160 101 FORMAT(5I5,5F10.0)
00170 102 FORMAT(7F8.0)
00180 103 FORMAT(5F10.0)
00190 104 FORMAT(10F7.0)
00200 105 FORMAT(7F10.0)
00210 106 FORMAT(16F7.2)
00220 107 FORMAT(10F8.2)
00230 C*****GENERAL DATA*****
00240 C DEPTH,DDT,AND ER ARE IN METERS.
00250 READ(5,101)NI,NP,NS,NPT,IKK,DEPTH,DDT,ER
00260 C*****READ IN THE INSERTED INSTRUMENT OR FLOAT SPECIFICATIONS*****
00270 C PI,SI ARE IN METERS; F IN POUNDS; AND CDIN,CDIT IN F.P.S. UNITS.
00280 READ(5,102)(PI(J),SI(J),F(1,J),F(2,J),F(3,J),CDIN(J),CDIT(J)
00290 1,J=1,NI)
00300 C*****READ IN THE PARAMETERS FOR DIFFERENT MOORING PARTS*****
00310 C DIAL IN INCHES, SLL IN METERS, AWL AND WWL IN POUNDS/M; TPL IN

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00320 C      POUNDS/(SQ.IN.) OR POUNDS.CO1 IS JACKET DIA.(IN.) FOR WIRE ROPE.
00330 READ(5,103)(DIAL(I),SLL(I),AWL(I),WWL(I),JPL(I),I=1,NF)
00340 READ(5,105)(CO1(I),FO1(I),CO2(I),FO2(I),AO(I),CON(I),CDT(I),I=1,NF
00350 1)
00360 C****READ THE FORCING CURRENT PROFILE*****
00370 C      D ARE IN METERS, AND V IN MM/SEC.
00380 READ(5,105)(D(I),I=1,INK)
00390 H(3)=0.
00400 READ(10,106)((V(L,I,J),I=1,2),J=1,8),L=1,NFT)
00410 PII=4.*ATAN(1.)
00420 RHO=1.9905
00430 GR=32.2
00440 CONSI=RHO*GR*PII*.0057
00450 SLT=0.
00460 DO 1 I=1,NF
00470 IF(FO1(I).EQ.0.0)GO TO 1
00480 C****PERMANENT STRETCH FOR SYNTHETIC LINES*****
00490 PSTRA(I)=((TPL(I)/(DIAL(I)**2)-200.0)/CO1(I))**(1.0/FO1(I))
00500 1 SLT=SLT+SLI(I)
00510 C****SEGMENTATION OF THE MOORING LINE IN NS SEGMENTS*****
00520 DO 2 I=1,NF
00530 NM(I)=(SLL(I)/SLT)*NS+0.5
00540 IF(NM(I).EQ.0)NM(I)=1
00550 SL(I)=SLL(I)/NM(I)
00560 SWA(I)=AWL(I)*SL(I)
00570 SWW(I)=WWL(I)*SL(I)
00580 SWD(I)=SWA(I)-SWW(I)
00590 CDIA(I)=SL(I)*(DIAL(I)**2)
00600 2 CONSI=CDIA(I)*CONSI
00610 CALL MOTION
00620 CALL EXIT
00630 END

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00640 SUBROUTINE MOTION
00650 C THIS SUBROUTINE CALCULATES THE DYNAMIC EQUILIBRIUM OF THE MOORING
00660 C LINE AT NPT INSTANTS OF TIMES.
00670 C -----
00680
00690 COMMON PI(30),SI(30),F(3,30),CDIN(30),CDIT(30),DIAL(20),SLL(20),AW
00700 1L(20),CDN(20),CDT(20),TPL(20),CO1(20),CO2(20),FO1(20),FO2(20),CONS
00710 1(20),V(850,2,10),NM(20),SL(20),SWA(20),CDIA(20),U(3),X(3),THETA(3)
00720 1,DN(3),WN(3),CS(3),SN(3),DD(3),UN(3),UT(3),PSTRA(20),AO(20),D(10),
00730 1WWL(20),SWW(20),SWD(20),DEPTH,TSTRA,I,S2,TE,H2,IK,NI,J,NC,INDEX,JJ
00740 1,NFT,NS,NP,DDT,E,ER,PII,CONS1,DIA,INK
00750 101 FORMAT(3F8.2)
00760 C*****BEGINNING OF MOORING LINE CALCULATIONS*****
00770 DO 14 IK=1,NPT
00780 S2=0.0
00790 X(3)=DDT
00800 K=1
00810 X(1)=0.
00820 X(2)=0.
00830 J=1
00840 TE=0.0
00850 DIA=0.
00860 THETA(1)=PII/2.
00870 THETA(2)=THETA(1)
00880 THETA(3)=0.
00890 DO 11 I=1,NP
00900 NK=NM(I)+K-1
00910 DO 10 N=K,NK
00920 INDEX=1
00930 C*****CHECK FOR ANY INSERTED INSTRUMENT OR FLOAT*****
00940 IF(J.GT.NI)GO TO 2
00950 1 IF((S2+SL(I)*0.5).GE.FI(J))CALL FORCES

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00960 IF(J.GT.NI)GO TO 2
00970 IF((S2+SL(I)*0.5).GE.PI(J))GO TO 1
00980 2 CONTINUE
00990 C***STRESS-STRAIN CALCULATIONS FOR MOORING SEGMENT*****
01000 IF(P01(I).EQ.0.0)GO TO 4
01010 C***SYNTHETIC ROPES*****
01020 STR=TE/(DIAL(I)**2)
01030 IF(STR.LE.200.0)GO TO 3
01040 TSTRA=PSTRA(I)+((STR-200.0)/CO2(I))**(1.0/P02(I))
01050 S1=A0(I)*SL(I)*(1.0+TSTRA/100.0)
01060 GO TO 5
01070 3 S1=SL(I)
01080 GO TO 5
01090 C***WIRE ROPES-IPL IS ELASTIC MODULUS AND CO1 IS JACKET DIA*****
01100 4 TSTRA=TE/(-.7854*TFL(I)*(DIAL(I)-CO1(I))**2)
01110 S1=SL(I)*(1.+TSTRA)
01120 C***BEGINNING OF THE NUMERICAL INTEGRATION PROCEDURE*****
01130 5 DIA=SQRT(CDIA(I)/S1)
01140 S2=S2+SL(I)
01150 DO 6 IC=1,3
01160 SN(IC)=SIN(THETA(IC))
01170 CS(IC)=COS(THETA(IC))
01180 X(IC)=X(IC)+S1*CS(IC)
01190 IF(SN(3).EQ.0.)SN(3)=0.000001
01200 H2=X(3)-S1*CS(3)/2.
01210 CALL SPEED
01220 DO 7 IC=1,3
01230 UT(IC)=U(IC)*CS(IC)
01240 WN(IC)=SWW(I)*CS(IC)*CS(3)
01250 WN(3)=WN(3)-SWW(I)
01260 UTT=UT(1)+UT(2)+UT(3)
01270 DTT=(2.94E-6)*CDT(I)*S1*DIA*ABS(UTT)*UTT

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01280 TE=TE-SWW(I)*CS(3)+DTT
01290 T=TE-H2*SWD(I)/S1
01300 DO 8 IC=1,3
01310 8 UN(IC)=U(IC)-UTT*CS(IC)
01320 VNM=SQRT(UN(1)**2+UN(2)**2+UN(3)**2)
01330 DO 9 IC=1,3
01340 DN(IC)=(2.94E-6)*CDN(I)*SI*DIA*VNM*UN(IC)+WN(IC)
01350 9 THETA(IC)=THETA(IC)-DN(IC)/(TE*SN(IC))
01360 10 CONTINUE
01370 K=NK+1
01380 11 CONTINUE
01390 C*****CHECK FOR ANY INSTRUMENT OR FLOAT BEFORE ANCHOR*****
01400 INDEX=1
01410 12 IF(J.GT.NI)GO TO 13
01420 CALL FORCES
01430 GO TO 12
01440 13 E=DEPTH-X(3)
01450 X(3)=X(3)-DDT
01460 WRITE(6,101)(X(KJ),KJ=1,3)
01470 14 CONTINUE
01480 RETURN
01490 END
01500 SUBROUTINE FORCES
01510 C THIS SUBROUTINE CALCULATES THE FORCE EQUILIBRIUM FOR INSERTED
01520 C INSTRUMENTS OR FLOATS.
01530 C -----
01540 C COMMON PI(30),SI(30),F(3,30),CDIN(30),CDIT(30),DIAL(20),SLL(20),AW
01550 1L(20),CDN(20),CDT(20),IPL(20),CO1(20),CO2(20),FO1(20),FO2(20),CONS
01560 1(20),V(850,2,10),NM(20),SL(20),SWA(20),CDIA(20),U(3),X(3),THETA(3)
01570 1,DN(3),WN(3),CS(3),CH(3),DD(3),UN(3),UT(3),PSTRA(20),AD(20),D(10),
01580 1IWL(20),SWW(20),SWD(20),DEPTH,TSTRA,I,S2,TE,H2,IK,NI,J,NC,INDEX,JJ

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01600      1,NPT,NS,NP,DDT,E,ER,PII,CONSI,DIA,IKK
01610      C*****NUMERICAL INTEGRATION PROCEDURE*****
01620      S2=S2+SI(J)
01630      DO 1 IC=1,3
01640      SN(IC)=SIN(THETA(IC))
01650      CS(IC)=COS(THETA(IC))
01660      1 X(IC)=X(IC)+SI(J)*CS(IC)
01670      IF(SN(3).EQ.0.)SN(3)=0.000001
01680      H2=X(3)-SI(J)*CS(3)/2.
01690      CALL SPEED
01700      IF(SI(J).NE.0.)GO TO 3
01710      VNM=SQRT(U(1)**2+U(2)**2+U(3)**2)
01720      DO 2 IC=1,3
01730      DD(IC)=TE*CS(IC)+CDIN(J)*VNM*U(IC)/92903.+F(IC,J)
01740      GO TO 7
01750      3 DO 4 IC=1,3
01760      4 UT(IC)=U(IC)*CS(IC)
01770      UTT=UT(1)+UT(2)+UT(3)
01780      DT=CDIT(J)*ABS(UTT)*UTT/92903.
01790      DO 5 IC=1,3
01800      UN(IC)=U(IC)-UTT*CS(IC)
01810      VNM=SQRT(UN(1)**2+UN(2)**2+UN(3)**2)
01820      DO 6 IC=1,3
01830      DD(IC)=(TE+DT)*CS(IC)+CDIN(J)*VNM*UN(IC)/92903.+F(IC,J)
01840      7 TE=SQRT(DD(1)**2+DD(2)**2+DD(3)**2)
01850      DO 8 IC=1,3
01860      8 THETA(IC)=ARCOS(DD(IC)/TE)
01870      J=J+1
01880      INDEX=2
01890      RETURN
01900      END
01910      SUBROUTINE SPEED

```

THIS SUBROUTINE CALCULATES THE VELOCITY PROFILE GIVEN THE VELOCITY
AT A FEW DEPTHS. HERE THE STEPPED PROFILE (#2 IN VOLUME 1
(CHHABRA,1973)), IS USED.

```

01920 C
01930 C
01940 C
01950 C
01960 C
01970
01980
01990
02000
02010
02020
02030
02040
02050
02060
02070
02080
02090
02100
02110
02120
02130
02140
02150
02160
02170
READY

COMMON PI(30),SI(30),F(3,30),CDIN(30),CDIT(30),DIAL(20),SLL(20),AW
1L(20),CDN(20),CDT(20),TFL(20),C01(20),C02(20),P01(20),P02(20),CONS
1(20),V(850,2,10),NM(20),SL(20),SWA(20),CDIA(20),U(3),X(3),THETA(3)
1,DN(3),WN(3),CS(3),SN(3),I(3),UN(3),UT(3),PSTRA(20),AD(20),D(10),
1,WL(20),SWW(20),SWD(20),DEPTH,TSTRA,I,S2,TE,H2,IK,NI,J,NC,INDEX,JJ
1,NPT,NS,NP,DDT,E,ER,PII,CONS1,DIA,IKK
M=1
DO 5 JN=1,2
IF(H2.LE.D(1))GO TO 3
IF(H2.GE.D(IKK))GO TO 4
1 IF(H2.GT.D(M).AND.H2.LE.D(M+1))GO TO 2
M=M+1
GO TO 1
2 U(JN)=V(IK,JN,M+1)
GO TO 5
3 U(JN)=V(IK,JN,1)
GO TO 5
4 U(JN)=V(IK,JN,IKK+1)
5 CONTINUE
RETURN
END

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SSD3,FORT
00010 C
00020 C
00030 C
00040 C
00050 C
00060 C
00070 C
00080 C
00090 C
00100 C
00110 C
00120 C
00130 C
00140 C
00150 C
00160 C
00170 C
00180 C
00190 C
00200 C
00210 C
00220 C
00230 C
00240 C
00250 C
00260 C
00270 C
00280 C
00290 C
00300 C
00310 C

PROGRAM TO COMPUTE LOW FREQ. MOTION OF SUBSURFACE MOORED SYSTEMS.
THIS ANALYSIS IS DONE IN THREE DIMENSIONS, AND INCLUDES INERTIA
FORCES. INPUT VELOCITY PROFILE RELATIVE TO THE MOORING SYSTEM IS
READ AT EACH TIME STEP.
-----
*****BEGINNING OF THE MAIN PROGRAM*****
COMMON PI(30),SI(30),F(3,30),CDIN(30),CDIT(30),DIAL(20),SLL(20),AW
1L(20),CDN(20),CDT(20),TPL(20),C01(20),C02(20),F01(20),F02(20),CONS
1(20),V(850,2,10),NM(20),SL(20),SWA(20),CDIA(20),U(3),X(3),THETA(3)
1,DN(3),WN(3),CS(3),SN(3),DD(3),UN(3),UT(3),PSTRA(20),AD(20),D(10),
1WWL(20),SWW(20),SWD(20),YDDL(150,3),YDDI(2,150,3),YDL(2,150,3),YDDI(
130,3),YDI(2,30,3),YI(2,30,3),YDDT(3),YDDN(3),ZM(30),ZMV(30),DEPTH,
1TSTRA,I,S2,TE,H2,IK,NI,J,NPT,NS,NP,DDI,E,ER,P11,CONS1,DIA,GR,IKK
*****BEGINNING OF FORMAT STATEMENTS*****
101 FORMAT(5I5,5F10.0)
102 FORMAT(7F8.0)
103 FORMAT(5F10.0)
104 FORMAT(10F7.0)
105 FORMAT(7F10.0)
106 FORMAT(16F7.2)
107 FORMAT(10F8.2)
*****GENERAL DATA*****
DEPTH, DDT, AND ER ARE IN METERS.
READ(5,101)NI,NP,NS,NPT,IKK,DEPTH,DDT,ER
*****READ IN THE INSERTED INSTRUMENT OR FLOAT SPECIFICATIONS*****
PI, SI ARE IN METERS; ZM, ZMV IN SLUGS; F IN POUNDS; AND CDIN,
CDIT IN F.P.S. UNITS.
READ(5,102)(PI(J),SI(J),ZM(J),ZMV(J),F(3,J),CDIN(J),CDIT(J)
1,J=1,NI)

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```

00320 DO 1 I1=1,2
00330 DO 1 I2=1,N1
00340 1 F(I1,I2)=0.
00350 C*****READ IN THE PARAMETERS FOR DIFFERENT MOORING PARTS*****
00360 C DIAL IN INCHES, SLL IN METERS; AWL AND WWL IN LBS/M; AND TPL IN
00370 C POUNDS OR POUNDS/(SQ.IN.). C01 IS IN INCHES FOR JACKETED WIRE ROPE
00380 C READ(5,103)(DIAL(I),SLL(I),AWL(I),WWL(I),TPL(I),I=1,NP)
00390 C READ(5,105)(C01(I),P01(I),C02(I),P02(I),A0(I),CDN(I),I=1,NP
00400 C 1)
00410 C*****READ THE FORCING CURRENT PROFILE*****
00420 C D ARE IN METERS; AND V ARE IN MM/SEC.
00430 C READ(5,105)(D(I),I=1,IKK)
00440 C U(3)=0.
00450 C READ(10,106)((V(L,I,J),I=1,2),J=1,8),L=1,NPT)
00460 C PII=4.*ATAN(1.)
00470 C RHO=1.9905
00480 C GR=32.2
00490 C CONS1=RHO*GR*PII*.0057
00500 C SLT=0.
00510 DO 2 I=1,NP
00520 IF(P01(I).EQ.0.0)GO TO 2
00530 C*****PERMANENT STRETCH FOR SYNTHETIC LINES.*****
00540 C PSTRA(I)=((TPL(I)/(DIAL(I)**2)-200.0)/C01(I))**(1.0/P01(I))
00550 C 2 SLT=SLT+SLL(I)
00560 C*****SEGMENTATION OF THE MOORING LINE IN NS SEGMENTS*****
00570 DO 3 I=1,NP
00580 NM(I)=(SLL(I)/SLT)*NS+0.5
00590 IF(NM(I).EQ.0)NM(I)=1
00600 SL(I)=SLL(I)/NM(I)
00610 SWA(I)=AWL(I)*SL(I)
00620 SWW(I)=WWL(I)*SL(I)
00630 SWD(I)=SWA(I)-SWW(I)

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00640      CDIA(I)=SL(I)*(DIAL(I)**2)
00650      3  CONS(I)=CDIA(I)*CONSI
00660      NS1=NS+NP
00670      C*****INITIALIZE ALL DYNAMIC VARIABLES.*****
00680      DO 5 I1=1,3
00690      DO 4 I2=1,NS1
00700      YDDL(I2,I1)=0.
00710      DO 4 I3=1,2
00720      YDL(I3,I2,I1)=0.
00730      4  YL(I3,I2,I1)=0.
00740      DO 5 I4=1,30
00750      YDDI(I4,I1)=0.
00760      DO 5 I5=1,2
00770      YDI(I5,I4,I1)=0.
00780      5  YI(I5,I4,I1)=0.
00790      CALL MOTION
00800      CALL EXIT
00810      END
00820      SUBROUTINE MOTION
00830      C  THIS SUBROUTINE CALCULATES THE DYNAMIC EQUILIBRIUM OF THE MOORING
00840      C  LINE AT NPT INSTANTS OF TIMES.
00850      C  -----
00860      C
00870      COMMON PI(30),SI(30),F(3,30),CDIN(30),CDIT(30),DIAL(20),SLL(20),AW
00880      1L(20),CDN(20),CDT(20),TPL(20),CO1(20),CO2(20),PO1(20),PO2(20),CONS
00890      1(20),V(850,2,10),NM(20),SL(20),SWA(20),CDIA(20),U(3),X(3),THETA(3)
00900      1,DN(3),WN(3),CS(3),SN(3),DD(3),UN(3),UT(3),FSTRA(20),AO(20),B(10),
00910      1,WL(20),SWW(20),SWD(20),YDDL(150,3),YDL(2,150,3),YL(2,150,3),YDDI(
00920      130,3),YDI(2,30,3),YI(2,30,3),YDDT(3),YDDN(3),ZM(30),ZMV(30),DEPTH,
00930      1,TSIRA,1,S2,IE,H2,IK,N1,J,NPT,NS,NP,DDT,E,ER,PII,CONSI,DIA,GR,IKK
00940      101  FORMAT(3F8.2)
00950      DO 16 IK=1,NPT

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00960 C*****BEGINNING OF MOORING LINE CALCULATIONS*****
00970 S2=0.0
00980 X(3)=DDT
00990 K=1
01000 X(1)=0.
01010 X(2)=0.
01020 J=1
01030 TE=0.0
01040 THETA(1)=PII/2.
01050 THETA(2)=THETA(1)
01060 THETA(3)=0.
01070 DO 11 I=1,NF
01080 NK=NM(I)+K-1
01090 DO 10 N=N,NK
01100 C*****CHECK FOR ANY INSERTED INSTRUMENT OR FLOAT *****
01110 IF(J.GT.NI)GO TO 2
01120 1 IF((S2+SL(I)*0.5).GE.PI(J))CALL FORCES
01130 IF(J.GT.NI)GO TO 2
01140 IF((S2+SL(I)*0.5).GE.PI(J))GO TO 1
01150 C*****STRESS-STRAIN CALCULATIONS FOR MOORING SEGMENT*****
01160 2 IF(F01(I).EQ.0.0)GO TO 4
01170 C*****SYNTHETIC ROPES*****
01180 STR=TE/(DIAL(I)**2)
01190 IF(STR.LE.200.0)GO TO 3
01200 TSTRA=PSTRA(I)+((STR-200.0)/CO2(I))**((1.0/F02(I))
01210 S1=AD(I)*SL(I)*(1.0+TSTRA/100.0)
01220 GO TO 5
01230 3 S1=SL(I)
01240 GO TO 5
01250 C*****WIRE ROPES-IPL IS ELASTIC MODULUS AND CO1 IS JACKET DIA*****
01260 4 TSTRA=TE/((.7854*IPL(I)*(DIAL(I)-CO1(I))**2)
01270 S1=SL(I)*(1.+TSTRA)

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01280 C*****BEGINING OF THE NUMERICAL INTEGRATION PROCEDURE*****
01290 5 DIA=SQRT(CDIA(I)/S1)
01300 S2=S2+SL(I)
01310 DO 6 IC=1,3
01320 SN(IC)=SIN(THETA(IC))
01330 CS(IC)=COS(THETA(IC))
01340 X(IC)=X(IC)+S1*CS(IC)
01350 YOL(1,N,IC)=X(IC)
01360 IF(SN(3).EQ.0.)SN(3)=0.000001
01370 H2=X(3)-S1*CS(3)/2.
01380 CALL SPEED
01390 DO 7 IC=1,3
01400 UT(IC)=U(IC)*CS(IC)
01410 YDOT(IC)=YDOT(N,IC)*CS(IC)
01420 WN(IC)=SWW(I)*CS(IC)*CS(3)
01430 WN(3)=WN(3)-SWW(I)
01440 UTT=UT(1)+UT(2)+UT(3)
01450 YDOTT=YDOT(1)+YDOT(2)+YDOT(3)
01460 DTT=(2.94E-6)*CDT(I)*S1*DIA*ABS(UTT)*UTT
01470 TE=TE-SWW(I)*CS(3)+DTT-SWA(I)*YDOTT/GR
01480 T=TE-H2*SWD(I)/S1
01490 DO 8 IC=1,3
01500 YDDN(IC)=YDDL(N,IC)-YDOTT*CS(IC)
01510 UN(IC)=U(IC)-UTT*CS(IC)
01520 VNM=SQRT(UN(1)**2+UN(2)**2+UN(3)**2)
01530 DO 9 IC=1,3
01540 DN(IC)=(2.94E-6)*CDN(I)*S1*DIA*VNM*UN(IC)+WN(IC)-((2.*SWA(I)-SWW(I)
01550 1))/GR)*YDDN(IC)
01560 9 THETA(IC)=THETA(IC)-DN(IC)/(TE*SN(IC))
01570 10 CONTINUE
01580 K=NK+1
01590 11 CONTINUE

```

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01600 *****CHECK FOR ANY INSERTED INSTRUMENT OR FLOAT BEFORE ANCHOR*****
01610 12 IF(J.GT.NI)GO TO 13
01620 CALL FORCES
01630 GO TO 12
01640 13 E=DEPTH-X(3)
01650 X(3)=X(3)-DDT
01660 WRITE(6,101)(X(KJ),KJ=1,3)
01670 *****CALCULATIONS FOR VELOCITIES AND ACCELERATIONS OF THE MOORING.
01680 DO 15 IC=1,3
01690 DO 14 I1=1,N
01700 YL(1,I1,IC)=YL(2,I1,IC)
01710 YL(2,I1,IC)=X(IC)-YDL(1,I1,IC)
01720 YDL(1,I1,IC)=YDL(2,I1,IC)
01730 YDL(2,I1,IC)=(YL(2,I1,IC)-YL(1,I1,IC))*0.003645
01740 YDDL(1,I1,IC)=(YDL(2,I1,IC)-YDL(1,I1,IC))/900.
01750 14 IF(IK.LE.2)YDDL(I1,IC)=0.
01760 DO 15 I2=1,NI
01770 YI(1,I2,IC)=YI(2,I2,IC)
01780 YI(2,I2,IC)=X(IC)-YDI(1,I2,IC)
01790 YDI(1,I2,IC)=YDI(2,I2,IC)
01800 YDI(2,I2,IC)=(YI(2,I2,IC)-YI(1,I2,IC))*0.003645
01810 YDDI(1,I2,IC)=(YDI(2,I2,IC)-YDI(1,I2,IC))/900.
01820 15 IF(IK.LE.2)YDDI(I2,IC)=0.
01830 16 CONTINUE
01840 RETURN
01850 END
01860 SUBROUTINE FORCES
01870 C THIS SUBROUTINE CALCULATES THE FORCE EQUILIBRIUM FOR INSERTED
01880 C INSTRUMENTS OR FLOATS.
01890 C -----
01900 C COMMON PI(30),SI(30),F(3,30),CDIN(30),CDIT(30),DIAL(20),SLL(20),AW
01910

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01920 1L(20),CDN(20),CDI(20),TFL(20),C01(20),C02(20),P01(20),P02(20),CONS
01930 1(20),V(850,2,10),NM(20),SL(20),SWA(20),CDIA(20),U(3),X(3),THETA(3)
01940 1,DN(3),WN(3),CS(3),SN(3),DD(3),UN(3),UT(3),PSTRA(20),AO(20),D(10),
01950 1WWL(20),SWW(20),SWD(20),YDDL(150,3),YDL(2,150,3),YL(2,150,3),YDDI(
01960 130,3),YDI(2,30,3),YI(2,30,3),YDDT(3),YDDN(3),ZM(30),ZMV(30),DEPTH,
01970 1TSTRA,I,S2,TE,H2,IK,NI,J,NPT,NS,NF,DDT,E,ER,PII,CONSI,DIA,GR,IKK
01980 C***NUMERICAL INTEGRATION PROCEDURE*****
01990 S2=S2+SI(J)
02000 DO 1 IC=1,3
02010 SN(IC)=SIN(THETA(IC))
02020 CS(IC)=COS(THETA(IC))
02030 X(IC)=X(IC)+SI(J)*CS(IC)
02040 YDI(1,J,IC)=X(IC)
02050 IF(SN(3).EQ.0.)SN(3)=0.000001
02060 H2=X(3)-SI(J)*CS(3)/2.
02070 CALL SPEED
02080 IF(SI(J).NE.0.)GO TO 3
02090 VNM=SQRT(U(1)**2+U(2)**2+U(3)**2)
02100 DO 2 IC=1,3
02110 DD(IC)=TE*CS(IC)+CDIN(J)*VNM*U(IC)/92903.+F(IC,J)-(ZM(J)+ZMV(J))*Y
02120 1DDI(J,IC)
02130 GO TO 7
02140 DO 4 IC=1,3
02150 YDDT(IC)=YDDI(J,IC)*CS(IC)
02160 UT(IC)=U(IC)*CS(IC)
02170 UTT=UT(1)+UT(2)+UT(3)
02180 YDDTT=YDDT(1)+YDDT(2)+YDDT(3)
02190 DTT=CDIT(J)*ABS(UTT)*UTT/92903.
02200 DO 5 IC=1,3
02210 YDDN(IC)=YDDI(J,IC)-YDDTT*CS(IC)
02220 UN(IC)=U(IC)-UTT*CS(IC)
02230 VNM=SQRT(UN(1)**2+UN(2)**2+UN(3)**2)

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02240      DO 6 IC=1,3
02250      DD(IC)=(TE+DTT)*CS(IC)+CDIN(J)*UNM*UN(IC)/92903.+F(IC,J)
02260      1-ZM(J)*YDDI(J,IC)-ZMV(J)*YDDN(IC)
02270      7 TE=SQRT(DD(1)**2+DD(2)**2+DD(3)**2)
02280      DO 8 IC=1,3
02290      8 THETA(IC)=ARCOS(DD(IC)/TE)
02300      J=J+1
02310      RETURN
02320      END
02330      SUBROUTINE SPEED
02340      C THIS SUBROUTINE CALCULATES THE VELOCITY PROFILE, GIVEN THE
02350      C VELOCITY AT A FEW DEPTHS. HERE THE STEPPED PROFILE (#2 IN
02360      C VOLUME 1 CHHABRA(1973)) IS USED.
02370      C -----
02380      C
02390      COMMON PI(30),SI(30),F(3,30),CDIN(30),CDIT(30),DIAL(20),SLL(20),AW
02400      1L(20),CDN(20),CDT(20),TFL(20),CO1(20),CO2(20),FO1(20),FO2(20),CONS
02410      1(20),V(850,2,10),NM(20),SL(20),SWA(20),CDIA(20),U(3),X(3),THETA(3)
02420      1,DN(3),WN(3),CS(3),SN(3),DD(3),UN(3),UT(3),FSTRA(20),AO(20),D(10),
02430      1WWL(20),SWW(20),SWD(20),YDDL(150,3),YDL(2,150,3),YL(2,150,3),YDDI(
02440      130,3),YDI(2,30,3),YI(2,30,3),YDDT(3),YDDN(3),ZM(30),ZMV(30),DEPTH,
02450      1TSTRA,I,S2,TE,H2,IK,NI,J,NPT,NS,NP,DDT,E,ER,PII,CONS1,DIA,GR,INK
02460      M=1
02470      DO 5 JN=1,2
02480      IF(H2.LE.D(1))GO TO 3
02490      IF(H2.GE.D(INK))GO TO 4
02500      1 IF(H2.GT.D(M).AND.H2.LE.D(M+1))GO TO 2
02510      M=M+1
02520      GO TO 1
02530      2 U(JN)=V(IK,JN,M+1)
02540      GO TO 5
02550      3 U(JN)=V(IK,JN,1)

```

GO TO 5
4 U(JN)=V(IN,JN,INK+1)
5 CONTINUE
RETURN
END

02560
02570
02580
02590
02600
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