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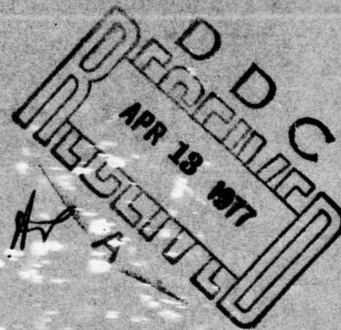
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SMALL-SIGNAL THIRD-ORDER DISTORTION ANALYSIS OF TRANSISTOR AMPLIFIERS

J. DANIEL COBB



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SMALL-SIGNAL THIRD-ORDER DISTORTION ANALYSIS OF TRANSISTOR AMPLIFIERS

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J. Daniel Cobb

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OF TRANSISTOR AMPLIFIERS

BY

J. DANIEL COBB

B.S., Illinois Institute of Technology, 1975

THESIS

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1. INTRODUCTION

1.1. Third Order Distortion--Causes and Effects

When a sinusoidal signal is applied to a nonlinear system, the signals which result internally and at the output of the system can, in the steady state, be decomposed into a sum of sinusoids whose frequencies are various integer multiples of the frequency of the input signal. That component which has frequency three times the frequency of the input signal is commonly referred to as third order distortion.

Appreciable amounts of third order distortion can exist in almost any circuit configuration which contains nonlinear elements. However there are some examples in which third order distortion does not play a major role. One such case is a network containing only linear elements and field effect transistors. Since FET characteristics can be closely approximated by quadratic equations, third order distortion does not appear to a great extent in such networks. For this reason bipolar junction transistors will be considered as the only nonlinear elements in subsequent analyses.

Since intermodulation distortion and crossmodulation distortion are third order effects, it is of great importance to know how to deal with third order distortion in general.

1.2. Elimination of Distortion

It has been observed in many instances that third order distortion in bipolar transistor amplifiers can be nearly eliminated by biasing the various transistors properly. Two distinct explanations for this phenomenon have been suggested in the literature.

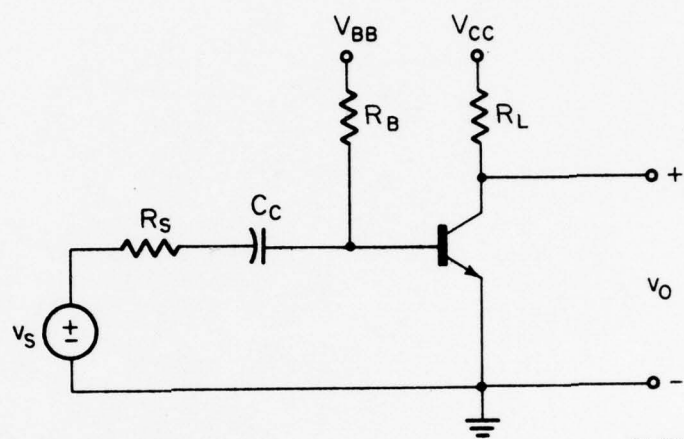
The simpler of the two explanations [1] involves the nonlinear effects of the exponential characteristic of the base-emitter junction. The reference given above only briefly suggests that interactions can occur within this base-emitter nonlinearity itself resulting in a net reduction in distortion.

The other explanation [2,3] involves the interaction of both the base-emitter nonlinearity and the avalanche breakdown phenomenon of the collector-base junction. This interaction has been dealt with in greater detail. Both types of interactions will be analyzed in great depth and it will become apparent that there is an intimate relationship between the two phenomena.

1.3. Test Circuit

All analyses will be performed on the circuit shown in Figure 1. A standard common emitter amplifier was chosen because of its simplicity and because of the relative ease with which one can make measurements on a physical network of this type. Two power supplies are needed in order that one can set the operating point arbitrarily. The input coupling capacitor and the bias resistor R_B are assumed to be large enough to be ignored in a small signal analysis.

Many results obtained from this circuit configuration can be easily generalized to more complex situations. Others may require a more detailed analysis. However, the techniques used to analyze the given test circuit can be readily applied to other configurations.



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Figure 1. Test Circuit

1.4. Notational Preliminaries

A consistent notation will be used throughout. To denote a signal which contains in general a constant component and a time varying component, a lower case letter with an upper case subscript will be used such as i_B . A lower case letter with a lower case subscript such as i_b will be used to denote the small signal variation of the signal about its dc value.

Upper case letters such as I_B will often be used to denote the dc value of a signal. Occasionally an upper case letter will be used to denote the amplitude of a sinusoid, either a peak value or a root mean square value. Also a superscript will sometimes be used to indicate that a number is the amplitude of some component of a nonsinusoidal periodic signal (e.g. $A^{(n)}$ will denote the amplitude of the nth harmonic of a waveform). It will be made clear by the context of the material exactly what is meant in each case.

1.5. Review of Volterra Series

The Volterra series method [4] of obtaining an approximation to the response of a nonlinear network to a sinusoidal input will be employed in the analyses. A brief review of the fundamental concepts characterizing the method will now be given.

If the input to a nonlinear system is $v_s(t)$ then the output can be expressed as the infinite series

$$v_o(t) = \sum_{n=1}^{\infty} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} h_n(\tau_1, \dots, \tau_n) \prod_{k=1}^n x(t-\tau_k) d\tau_1 \dots d\tau_n. \quad (1)$$

The n th order kernel $h_n(t_1, \dots, t_n)$ characterizes the system. The n th order transfer function can be defined as

$$H_n(s_1, \dots, s_n) \triangleq \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} h_n(\tau_1, \dots, \tau_n) e^{-(s_1 \tau_1 + \dots + s_n \tau_n)} d\tau_1 \dots d\tau_n. \quad (2)$$

If the system is memoryless as in the case of the common emitter test circuit (neglecting frequency effects inside the transistor), then H_n is frequency independent.

If

$$v_s(t) = A \cos \omega t \quad (3)$$

and the system is memoryless, then the output can be expressed as a Fourier cosine series. Let the root mean square amplitude of the n th term in the series be $v_o^{(n)}$.

1.6. Network Response to Sinusoidal Inputs

If the input to a memoryless network is given by (3), then

$$v_o^{(1)} = \frac{A}{\sqrt{2}} |H_1| \quad (4)$$

and

$$v_o^{(3)} = \left| \sum_{\substack{n=3 \\ n \text{ odd}}}^{\infty} \frac{A^n H_n}{2^{n-\frac{1}{2}} \left(\frac{n-3}{2}\right)! \left(\frac{n+3}{2}\right)!} \right|. \quad (5)$$

Assuming that A is small enough, the first term in the series (5) dominates and the series reduces to

$$v_o^{(3)} = \frac{A^3}{24\sqrt{2}} |H_3|. \quad (6)$$

Throughout all analyses the preceding approximation will be assumed valid. All experimental data was taken with an input small enough so that the small signal approximation holds.

1.7. Application to Circuit Analysis

The Volterra series method can be of great use when applied to electric network problems. It is a natural tool to employ because of its obvious similarity to conventional Laplace transform techniques.

One method of obtaining the transfer functions of a circuit is the exponential input method. If the input to a network is

$$v_s(t) = e^{s_1 t} \quad (7)$$

then the output is in general an infinite sum of exponential terms whose exponents are integer multiples of $s_1 t$.

$$v_o(t) = \sum_{n=1}^{\infty} c_n(s_1) e^{ns_1 t} \quad (8)$$

It turns out that

$$H_1(s_1) = C_1(s_1). \quad (9)$$

If the input to a network is

$$v_s(t) = e^{s_1 t} + e^{s_2 t} \quad (10)$$

then the output is of the form

$$v_o(t) = C_{10}(s_1) e^{s_1 t} + \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} C_{nm}(s_1, s_2) e^{(ns_1 + ms_2)t}. \quad (11)$$

In this case

$$H_2(s_1, s_2) = C_{11}(s_1, s_2). \quad (12)$$

It is apparent that a network with input

$$v_s(t) = \sum_{k=1}^n l^{s_k t} \quad (13)$$

has a term at its output

$$C_{1\dots 1}(s_1, \dots, s_n) l^{(s_1 + \dots + s_n)t} = H_n(s_1, \dots, s_n) l^{(s_1 + \dots + s_n)t}. \quad (14)$$

Therefore it is only necessary to calculate the coefficient of the exponential term at frequency $s_1 + \dots + s_n$ in order to obtain the n th order transfer function of the system.

Further discussion of these techniques will be postponed until it is necessary to analyze a specific circuit. At that point, the actual details of the derivation of the n th order transfer function will be shown by example.

2. ANALYSIS OF BASE-EMITTER NONLINEAR MODEL

2.1. Model Description

Consider the simple model in Figure 2 for the npn transistor biased in the active region. Only the base-emitter nonlinearity is considered. The motivation behind the use of a model of such relative simplicity is not the complete characterization of all sources of distortion, but the isolation and investigation of one effect that accounts for observed phenomena.

The exponential characteristic of the base-emitter junction is given by the equation

$$v_{BE} = V_T \ln\left(\frac{\beta i_B}{I_S} + 1\right). \quad (15)$$

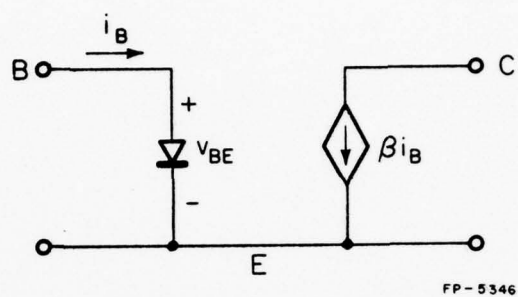
β is the transistor current gain and V_T and I_S are the usual diode parameters, the voltage equivalent of temperature and the reverse saturation current respectively. β is assumed constant, however it will be shown that subsequent results are independent of β . Equation (15) can be simplified if one assumes that the collector current is much larger than the reverse saturation current.

$$v_{BE} \simeq V_T \ln \frac{\beta i_B}{I_S} \quad (16)$$

2.2. Small Signal Equivalent Circuit

Expression (16) can be expanded in a Taylor series about the operating point.

$$v_{BE} = \sum_{n=0}^{\infty} b_n (i_B - I_B)^n \quad (17)$$



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Figure 2. Low Frequency Model With Base-Emitter Nonlinearity Only

$$b_n = \begin{cases} V_T \ln \frac{\beta I_B}{I_S}, & n=0 \\ (-1)^{n+1} \frac{V_T}{n I_B^n}, & n > 0 \end{cases} \quad (18)$$

I_B is the DC base current. One can define the small signal base current and the small signal base-emitter voltage as

$$i_b \triangleq i_B - I_B \quad (19)$$

$$v_{bc} \triangleq v_{BE} - V_{BE} \quad (20)$$

where V_{BE} is the DC base-emitter voltage. Clearly

$$b_0 = V_{BE} \quad (21)$$

and the Taylor expansion (17) reduces to

$$v_{be} = \sum_{n=1}^{\infty} b_n i_b^n \quad (22)$$

The series (22) converges if

$$|i_b| < I_B \quad (23)$$

Inequality (23) indicates merely that the small signal base current should be smaller in magnitude than the DC base current. The above equations are only valid in the active region so that the transistor should not be driven into the cutoff region nor the saturation region. Grouping terms in (22) yields

$$v_{be} = b_o i_b + v_e \quad (24)$$

where

$$v_e = \sum_{n=2}^{\infty} b_n i_b^n \quad (25)$$

Equation (24) suggests the small signal model shown in Figure 3. Combining this model with the small signal equivalent of the test circuit of Figure 1 yields the circuit of Figure 4 upon which all analyses of this section will be performed.

2.3. Analysis of Small Signal Model

Consider the circuit of Figure 4. It is desired to calculate H_1 and H_3 for this network. If an input given by (7) is applied then the output is given by (8). Since the voltage generator v_e is the only non-linear element and since it will not produce terms which are proportional to $e^{s_1 t}$, the generator can be short circuited without affecting the value of H_1 . Therefore H_1 is the gain of the amplifier with the nonlinear voltage generator equal to zero.

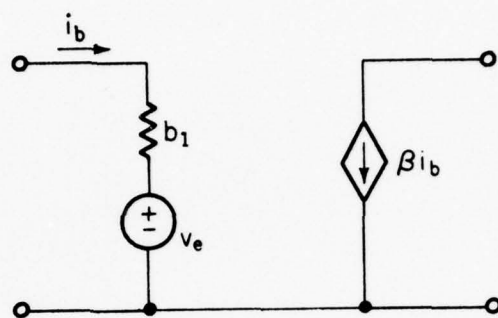
$$H_1 = - \frac{\beta I_B R_L}{I_B (R_S + b_1)} = \frac{-\beta I_B R_L}{I_B R_S + V_T} \quad (26)$$

In order to calculate H_3 let the input be given by

$$v_S(t) = e^{s_1 t} + e^{s_2 t} + e^{s_3 t} \quad (27)$$

Then the first order base current is

$$i_b^{(1)} = \frac{1}{R_S + b_1} (e^{s_1 t} + e^{s_2 t} + e^{s_3 t}) \quad (28)$$



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Figure 3. Small-Signal Model of Transistor

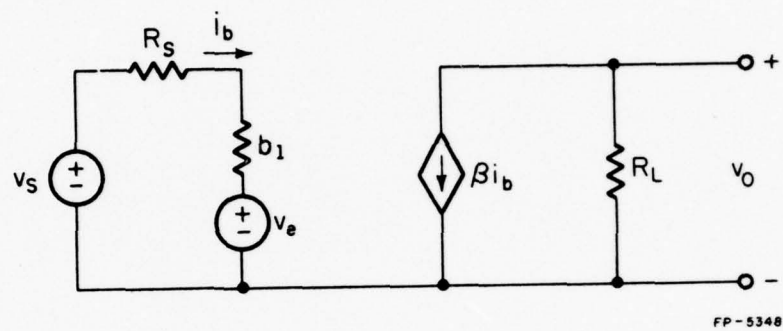


Figure 4. Small-Signal Equivalent of Test Circuit

where $v_e = 0$ since it does not contribute any first order terms.

Next it is necessary to calculate the second order base current $i_b^{(2)}$. In order to do this the second order component of the error voltage generator $v_e^{(2)}$ must be first calculated. Since second order terms result only from the squaring of first order terms in (25),

$$v_e^{(2)} = \frac{2b_2}{(R_S + b_1)^2} [e^{(S_1 + S_2)t} + e^{(S_2 + S_3)t} + e^{(S_1 + S_3)t}] . \quad (29)$$

It follows that

$$i_b^{(2)} = - \frac{2b_2}{(R_S + b_1)^3} [e^{(S_1 + S_2)t} + e^{(S_2 + S_3)t} + e^{(S_1 + S_3)t}] . \quad (30)$$

Applying (18) yields

$$i_b^{(2)} = \frac{V_T I_B}{(I_B R_S + V_T)^3} [e^{(S_1 + S_2)t} + e^{(S_2 + S_3)t} + e^{(S_1 + S_3)t}] . \quad (31)$$

Third order terms result from two distinct types of interaction.

They appear due to the cubing in (25) of the first order terms. Also the square of the base current i_b in (25) contains third order terms since i_b is composed in part of both $i_b^{(1)}$ and $i_b^{(2)}$ whose cross product consists of third order terms. The third order error voltage $v_{e1}^{(3)}$ due to the cubing of $i_b^{(1)}$ in (25) is

$$v_{e1}^{(3)} = \frac{6b_3}{(R_S + b_1)^3} e^{(S_1 + S_2 + S_3)t} . \quad (32)$$

$$v_{e1}^{(3)} = \frac{2V_T}{(I_B R_S + V_T)^3} e^{(S_1 + S_2 + S_3)t} \quad (33)$$

The third order error voltage $v_{e2}^{(3)}$ due to the cross product between $i_b^{(1)}$ and $i_b^{(2)}$ is

$$v_{e2}^{(3)} = - \frac{12b_2^2}{(R_S + b_1)^4} e^{(s_1 + s_2 + s_3)t} \quad (34)$$

The total third order component of the error voltage is then

$$v_e^{(3)} = v_{e1}^{(3)} + v_{e2}^{(3)} = \left[\frac{6b_3}{(R_S + b_1)^3} - \frac{12b_2^2}{(R_S + b_1)^4} \right] e^{(s_1 + s_2 + s_3)t} \quad (35)$$

It follows that the third order base current is

$$i_b^{(3)} = - \frac{1}{R_S + b_1} \left[\frac{6b_3}{(R_S + b_1)^3} - \frac{12b_2^2}{(R_S + b_1)^4} \right] e^{(s_1 + s_2 + s_3)t} \quad (36)$$

Simplifying and using (18) yields

$$i_b^{(3)} = \frac{V_T I_B (V_T - 2I_B R_S)}{(I_B R_S + V_T)^5} \quad (37)$$

It readily follows that

$$H_3 = - \frac{\beta I_B V_T R_L (V_T - 2I_B R_S)}{(I_B R_S + V_T)^5} \quad (38)$$

Later it will be convenient to consider the ratio $\frac{v_o^{(3)}}{v_o^{(1)}}$ which is proportional to $\left| \frac{H_3}{H_1} \right|$ by (4) and (6) and can be considered the normalized third order distortion. Normalizing with respect to $v_o^{(1)}$ allows comparisons to be made among the amplifiers with different gains. In the case at hand

$$\left| \frac{H_3}{H_1} \right| = \frac{V_T |V_T - 2I_B R_S|}{(I_B R_S + V_T)^4} \quad (39)$$

2.4. Existence of Third Order Null

From (38) it is apparent that if

$$I_B = \frac{V_T}{2R_S} \quad (40)$$

then

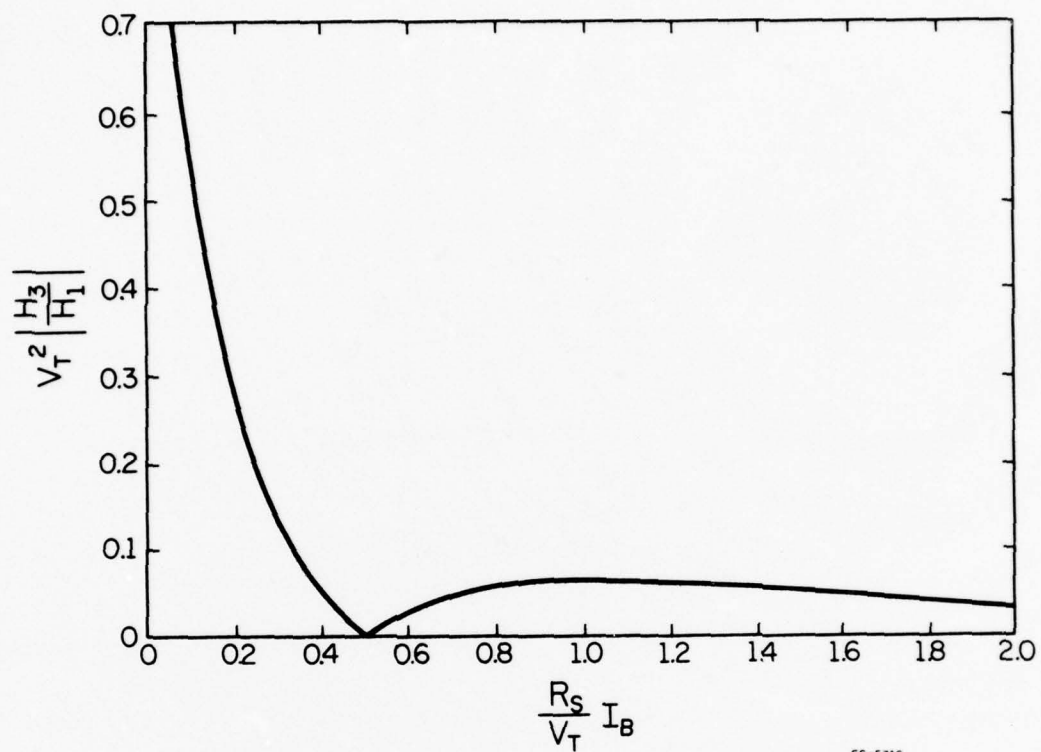
$$H_3 = 0. \quad (41)$$

Because third order distortion is approximately given by (6), biasing the transistor with the DC base current (40) results in a nearly total elimination of third order distortion generated by the base-emitter nonlinearity. Equation (39) is plotted versus DC base current in Figure 5.

2.5. Measured Third Order Distortion

Data was taken over a wide range of collector currents for various values of load and source resistance. The collector-base voltage was held at a constant ten volts DC. The collector-base breakdown voltage of the transistor is approximately forty volts so that the avalanche breakdown effect can be ignored. A sinusoidal oscillator was used to drive the circuit at a frequency of fifty kilohertz.

The normalized level of third order distortion is plotted versus DC collector current in Figure 6 for a source resistance of fifty ohms and for three values of load resistance. By measuring the base currents corresponding to several values of collector current it became possible to



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Figure 5. Third order distortion versus base current.

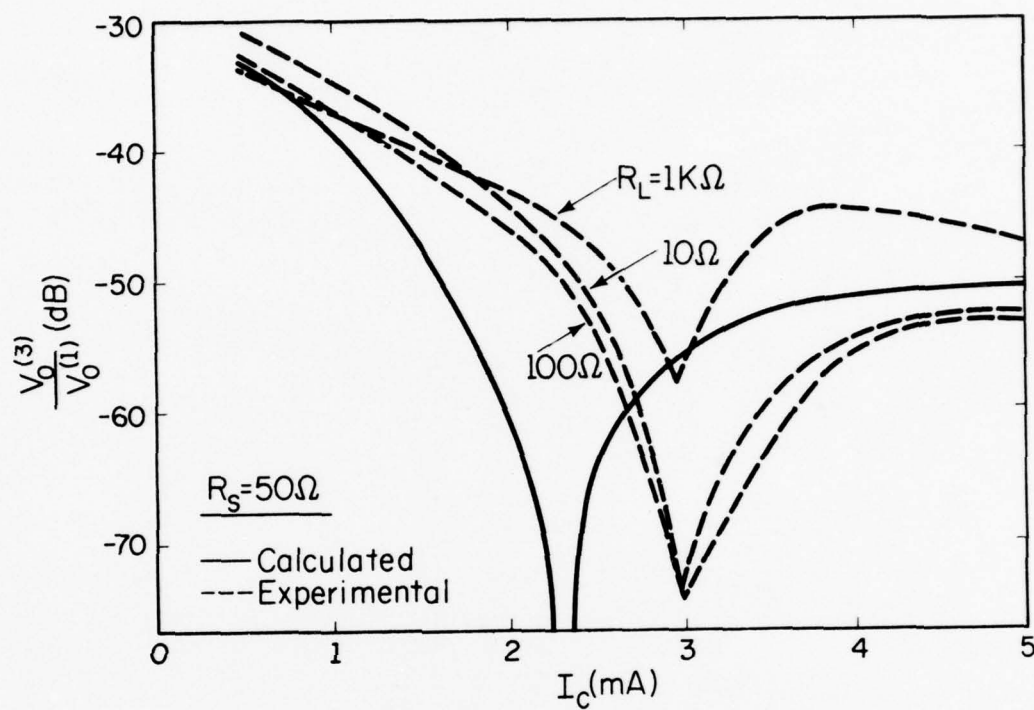


Figure 6. Measured and theoretical distortion for $R_S = 50\Omega$

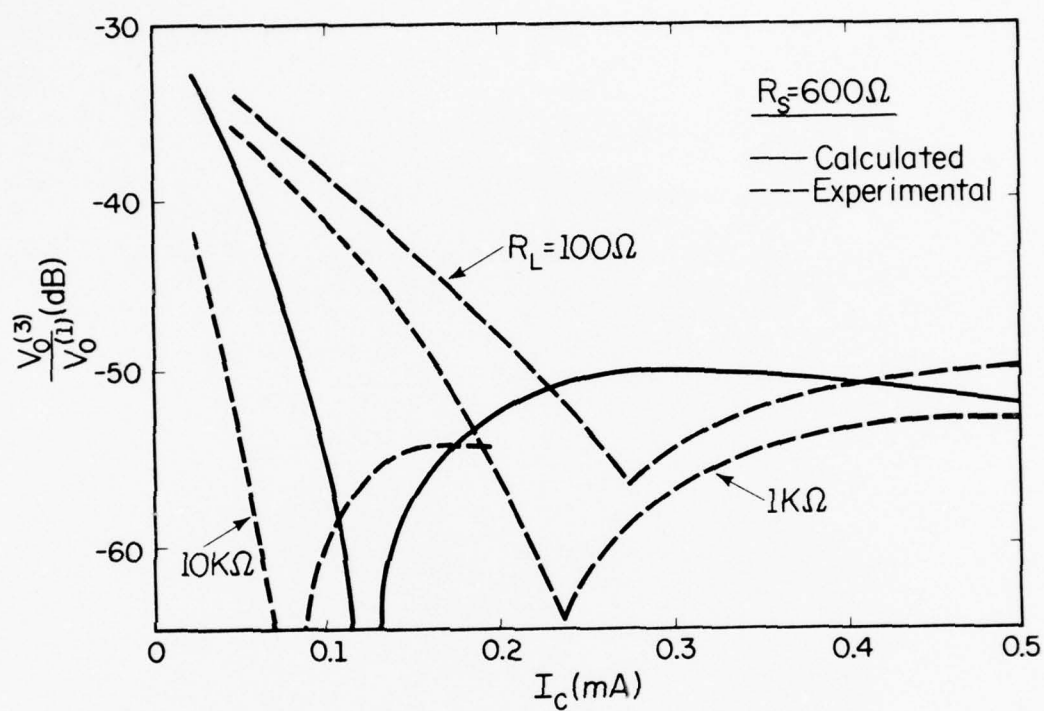
plot equation (39) in the same figure. One should note the characteristic drop in distortion near a collector current of 2.30 mA which corresponds to the predicted base current (40).

Figure 7 shows a similar set of curves for a source resistance of six hundred ohms. Here the results are somewhat more erratic, however the same sort of minimization of distortion is apparent. Again the theoretical, load-independent curve is plotted for purposes of comparison.

It is important to note that measured levels of distortion are very nearly equal to levels that were predicted while considering only the base-emitter nonlinearity. This fact indicates that the base-emitter nonlinearity is the dominant source of distortion at low frequencies in the active region and away from the breakdown region. This result will play an important role when the avalanche nonlinearity is considered in the next section.

2.6. Discrepancies Between Experimental and Theoretical Results

There are a multitude of possible explanations for the obvious differences between calculated and measured results in Figures 6 and 7. It should be realized first that these discrepancies were expected and are not necessarily of great importance. After all, the transistor model that is used is of an extremely simple nature and is not intended to take into account all sources of distortion. Even so, the experimental results are remarkably close to those predicted. The purpose of the model is to isolate the nonlinear effect that accounts for the sharp drop in distortion. Clearly, this objective has been accomplished.



FS-5319

Figure 7. Measured and theoretical distortion for $R_S = 600\Omega$.

There were many possible origins of distortion in the experimental set up other than the transistor itself. Probably most significant was the source generator which of course did not produce a perfectly pure sinusoid. This and other effects may account for the observed dependence on load resistance.

The actual operating temperature of the transistor was not monitored closely. This accounts in part for the shift in the position of the null from the predicted collector current. The position of the null (40) is proportional to temperature since V_T is proportional to temperature. The theoretical curves in Figures 6 and 7 correspond to an operating temperature of 25°C .

3. ANALYSIS OF AVALANCHE NONLINEAR MODEL

3.1. Model Description

It was seen in the previous section that it is possible to bring about a significant reduction in third order distortion by properly biasing the base-emitter junction of a transistor. It seems natural then to ask how the biasing of the collector-base junction might affect the distortion characteristics of a transistor circuit. More specifically, it has been proposed that distortion might be reduced through the interaction of the base-emitter and avalanche breakdown nonlinearities.

Consider the transistor model shown in Figure 8. Once again the exponential characteristic of the base-emitter junction is included in the model and is given by (16). Also included is the avalanche breakdown phenomenon described by

$$i_C = \frac{\beta i_B}{1 - \left(\frac{V_{CB}}{V_{CBO}}\right)^4} \quad (42)$$

where V_{CBO} is the collector-base breakdown voltage of the transistor. As noted previously, the base-emitter nonlinearity is normally the dominant source of third order distortion in the active region. However, it was also shown that one can reduce this dominance by properly biasing the base-emitter junction. In fact, it will be seen that the distortion due to the base-emitter nonlinearity can be reduced to the point where its level is on the same order of magnitude as that of the avalanche distortion. It will also be seen that it is possible to bias the transistor in such a way that a

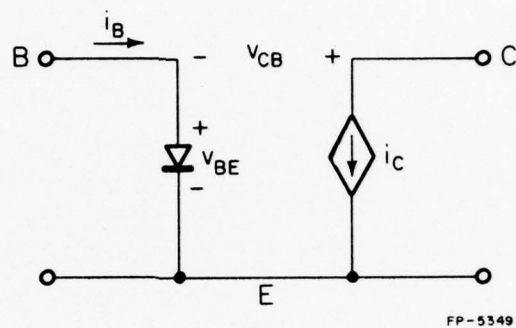


Figure 8. Low Frequency model with avalanche effect included.

complete cancellation of third order distortion between the two nonlinear effects can occur.

3.2. Small Signal Equipment Circuit

As in the previous section, it is desirable to develop a small signal model that includes the two nonlinear effects. Equation (42) can be expanded in a Taylor series about the operating point.

$$i_C = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} a_{nm} (i_B - I_B)^n (v_{CB} - V_{CB})^m \quad (43)$$

V_{CB} is the DC collector-base voltage. Here the derivation of the Taylor coefficients is considerably more complex than before. For this reason only those coefficients relevant to the third order analysis are given.

$$a_{00} = \frac{\beta I_B}{1 - \left(\frac{V_{CB}}{V_{CBO}}\right)^4} \quad (44)$$

$$a_{10} = \frac{\beta}{1 - \left(\frac{V_{CB}}{V_{CBO}}\right)^4} \quad (45)$$

$$a_{n0} = 0, \quad n \geq 2 \quad (46)$$

$$a_{01} = \frac{\frac{4\beta I_B}{V_{CBO}} \left(\frac{V_{CB}}{V_{CBO}}\right)^3}{\left[1 - \left(\frac{V_{CB}}{V_{CBO}}\right)^4\right]^2} \quad (47)$$

$$a_{11} = \frac{\frac{4\beta}{V_{CBO}} \left(\frac{V_{CB}}{V_{VBO}}\right)^3}{\left[1 - \left(\frac{V_{CB}}{V_{CBO}}\right)^4\right]^2} \quad (48)$$

$$a_{02} = \frac{\frac{2\beta I_B}{V_{CBO}^2} \left(\frac{V_{CB}}{V_{CBO}}\right)^2 \left[3 + 5\left(\frac{V_{CB}}{V_{CBO}}\right)^4\right]}{\left[1 - \left(\frac{V_{CB}}{V_{CBO}}\right)^4\right]^3} \quad (49)$$

$$a_{12} = \frac{\frac{2\beta}{V_{CBO}^2} \left(\frac{V_{CB}}{V_{CBO}}\right)^2 \left[3 + 5\left(\frac{V_{CB}}{V_{CBO}}\right)^4\right]}{\left[1 - \left(\frac{V_{CB}}{V_{CBO}}\right)^4\right]^3} \quad (50)$$

$$a_{03} = \frac{\frac{4\beta I_B}{V_{CBO}^3} \left(\frac{V_{CB}}{V_{CBO}}\right) \left\{ \left[1 + \left(\frac{V_{CB}}{V_{CBO}}\right)^4\right]^2 - \frac{4}{5} \right\}}{\left[1 - \left(\frac{V_{CB}}{V_{CBO}}\right)^4\right]^4} \quad (51)$$

Clearly,

$$a_{00} = I_C \quad (52)$$

One can define the small-signal perturbations

$$i_c \triangleq i_C - I_C \quad (53)$$

$$v_{cb} \triangleq v_{CB} - V_{CB} \quad (54)$$

It follows from (43), (46), (52), (53), and (54) that

$$i_c = \sum_{n=1}^{\infty} a_{on} v_{cb}^n + \sum_{n=0}^{\infty} a_{ln} i_b v_{cb}^n . \quad (55)$$

After grouping terms and defining the nonlinear effects in terms of a current generator of the form

$$i_e \triangleq \sum_{n=2}^{\infty} a_{on} v_{cb}^n + \sum_{n=0}^{\infty} a_{ln} i_b v_{cb}^n \quad (56)$$

(56) can be rewritten

$$i_c = a_{10} i_b + a_{01} v_{cb} + i_e . \quad (57)$$

This suggests the small signal model shown in Figure 9. Combining this model with the small-signal equivalent for the test circuit of Figure 1 yields the small-signal network of Figure 10 which will be analyzed throughout this section.

3.3. Analysis of Small Signal Model

In order to calculate third order distortion terms in the circuit of Figure 9, first order terms must be calculated. Since the distortion generators v_e and i_e contribute only second and higher order terms they can be ignored in the first order analysis.

It is convenient to introduce several transfer functions of the linearized network. Define

$$Y_{12} \triangleq \frac{1}{R_S + b_1} \quad (58)$$

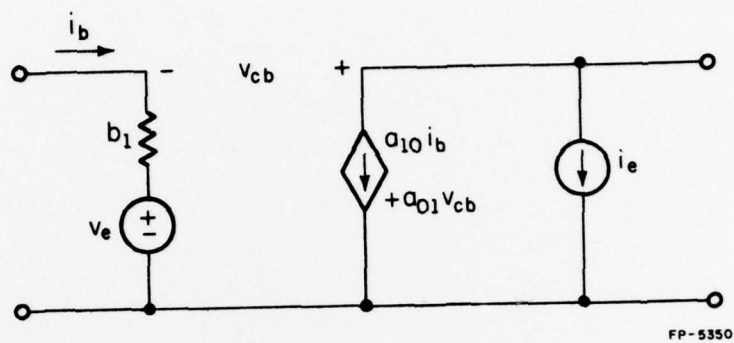


Figure 9. Small-signal model of transistor.

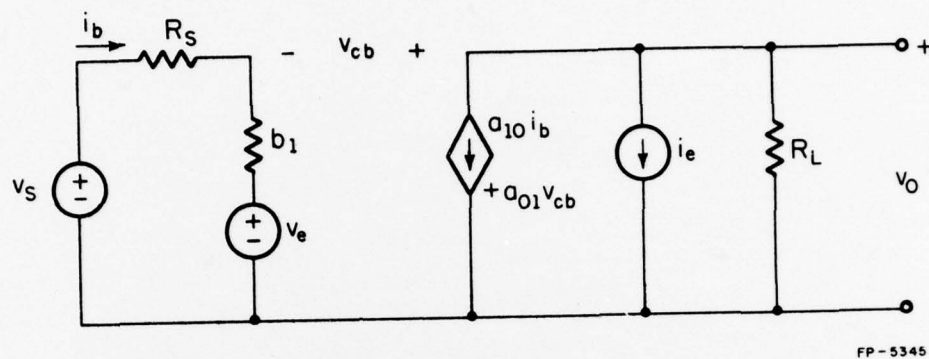


Figure 10. Small-signal equivalent of test circuit.

$$Y_{22} \triangleq - \frac{1}{R_S + b_1} \quad (59)$$

$$\mu_{13} \triangleq - \frac{a_{10} + b_1 G_L}{(a_{01} + G_L)(R_S + b_1)} \quad (60)$$

$$\mu_{15} \triangleq - \frac{a_{10} - b_1 a_{01}}{(a_{01} + G_L)(R_S + b_1)} \quad (61)$$

$$\mu_{23} \triangleq \frac{a_{10} - R_S G_L}{(a_{01} + G_L)(R_S + b_1)} \quad (62)$$

$$\mu_{25} \triangleq \frac{a_{01} R_S + a_{10}}{(a_{01} + G_L)(R_S + b_1)} \quad (63)$$

$$Z_{43} \triangleq - \frac{1}{a_{01} + G_L} \quad (64)$$

$$Z_{45} \triangleq - \frac{1}{a_{01} + G_L} \quad (65)$$

where

$$G_L \triangleq \frac{1}{R_L} . \quad (66)$$

The letter μ signifies a voltage transfer ratio, Y stands for a transductance, and Z is a transresistance. The numerical subscripts denote which ports a transfer function describes. For example, the first port is the input port and the fifth is the output port. Therefore μ_{15} is the voltage transfer ratio between input and output. The other ports are numbered from left to right. The second port corresponds to the base-emitter junction,

the third is the collector-base junction and the fourth is the dependent current source in parallel with the load.

After performing some elementary linear circuit analysis it is clear that if

$$v_s = e^{s_1 t} + e^{s_2 t} + e^{s_3 t} \quad (67)$$

then

$$i_b^{(1)} = Y_{12} (e^{s_1 t} + e^{s_2 t} + e^{s_3 t}) \quad (68)$$

$$v_{cb}^{(1)} = \mu_{13} (e^{s_1 t} + e^{s_2 t} + e^{s_3 t}) \quad (69)$$

and

$$v_o^{(1)} = \mu_{15} (e^{s_1 t} + e^{s_2 t} + e^{s_3 t}). \quad (70)$$

Clearly,

$$H_1 = \mu_{15}. \quad (71)$$

Second order terms can be calculated readily. The second order component of the error voltage generator $v_e^{(2)}$ is

$$v_e^{(2)} = 2b_2 Y_{12}^2 [e^{(s_1+s_2)t} + e^{(s_2+s_3)t} + e^{(s_1+s_3)t}]. \quad (72)$$

This expression is identical to that obtained for $v_e^{(2)}$ in the previous section. The second order component of the base current $i_b^{(2)}$ is also identical to that calculated in the previous section.

$$i_b^{(2)} = 2b_2 Y_{12}^2 Y_{22} [e^{(s_1+s_2)t} + e^{(s_2+s_3)t} + e^{(s_1+s_3)t}] \quad (73)$$

In fact, all results concerning the left loop of the circuit are identical to the corresponding results of the previous section since the alternation of the dependent current source in the right half of the circuit has no effect on the left.

The second order component of the error current generator $i_e^{(2)}$ results from the squaring of $v_{cb}^{(1)}$ and the multiplication of $i_b^{(1)}$ and $v_{cb}^{(1)}$ in (56).

$$i_e^{(2)} = (2a_{02}\mu_{13}^2 + 2a_{11}Y_{12}\mu_{13})[e^{(s_1+s_2)t} + e^{(s_2+s_3)t} + e^{(s_1+s_3)t}] \quad (74)$$

Both error sources contribute to the second order collector-base voltage $v_{cb}^{(2)}$.

$$\begin{aligned} v_{cb}^{(2)} &= v_e^{(2)}\mu_{23} + i_e^{(2)}Z_{43} \\ &= [2b_2Y_{12}^2\mu_{23} + (2a_{02}\mu_{13}^2 + 2a_{11}Y_{12}\mu_{13})Z_{43}][e^{(s_1+s_2)t} + e^{(s_2+s_3)t} + e^{(s_1+s_3)t}] \end{aligned} \quad (75)$$

It remains to calculate third order terms. Unfortunately, even in a simple circuit such as this, there are many different interactions which take place complicating the analysis. However, with perserverance one can obtain the desired result.

Since the left loop of the circuit is not affected by the right, the expression for $v_e^{(3)}$ can be taken from the previous section.

$$v_e^{(3)} = (6b_3Y_{12}^3 + 12b_2Y_{12}^2Y_{22})e^{(s_1+s_2+s_3)t} \quad (76)$$

The cubing of $v_{cb}^{(1)}$ in (56) gives one part of the third order component of the error current $i_e^{(3)}$.

$$i_{e1}^{(3)} = 6a_{03}\mu_{13}^3 e^{(s_1+s_2+s_3)t} \quad (77)$$

Another part of $i_e^{(3)}$ is obtained from the multiplication of the square of $v_{cb}^{(1)}$ with $i_b^{(1)}$.

$$i_{e2}^{(3)} = 6a_{12}\mu_{13}^2 Y_{12} e^{(s_1+s_2+s_3)t} \quad (78)$$

The multiplication of $i_b^{(2)}$ with $v_{cb}^{(1)}$ yields

$$i_{e3}^{(3)} = 12b_2 a_{11} Y_{12}^2 Y_{22} \mu_{13} e^{(s_1+s_2+s_3)t} \quad (79)$$

The product of $i_b^{(1)}$ and $v_{cb}^{(2)}$ gives

$$i_{e4}^{(3)} = 12a_{11} Y_{12} (b_2 Y_{12}^2 \mu_{23} + a_{02} \mu_{13}^2 Z_{43} + a_{11} Y_{12} \mu_{13} Z_{43}) e^{(s_1+s_2+s_3)t} \quad (80)$$

Finally, the square of v_{cb} yields a cross product between $v_{cb}^{(1)}$ and $v_{cb}^{(2)}$ which is the last third order term.

$$i_{e5}^{(3)} = 12a_{02} \mu_{13} (b_2 Y_{12}^2 \mu_{23} + a_{02} \mu_{13} Z_{43} + a_{11} Y_{12} \mu_{13} Z_{43}) e^{(s_1+s_2+s_3)t} \quad (81)$$

One part of the third order component of the output voltage $v_o^{(3)}$ is given by

$$v_{01}^{(3)} = v_e^{(3)} \mu_{25} \quad (82)$$

The other part is given by

$$v_{02}^{(3)} = i_e^{(3)} Z_{45} \quad (83)$$

where

$$i_e^{(3)} = \sum_{k=1}^5 i_{ek}^{(3)}. \quad (84)$$

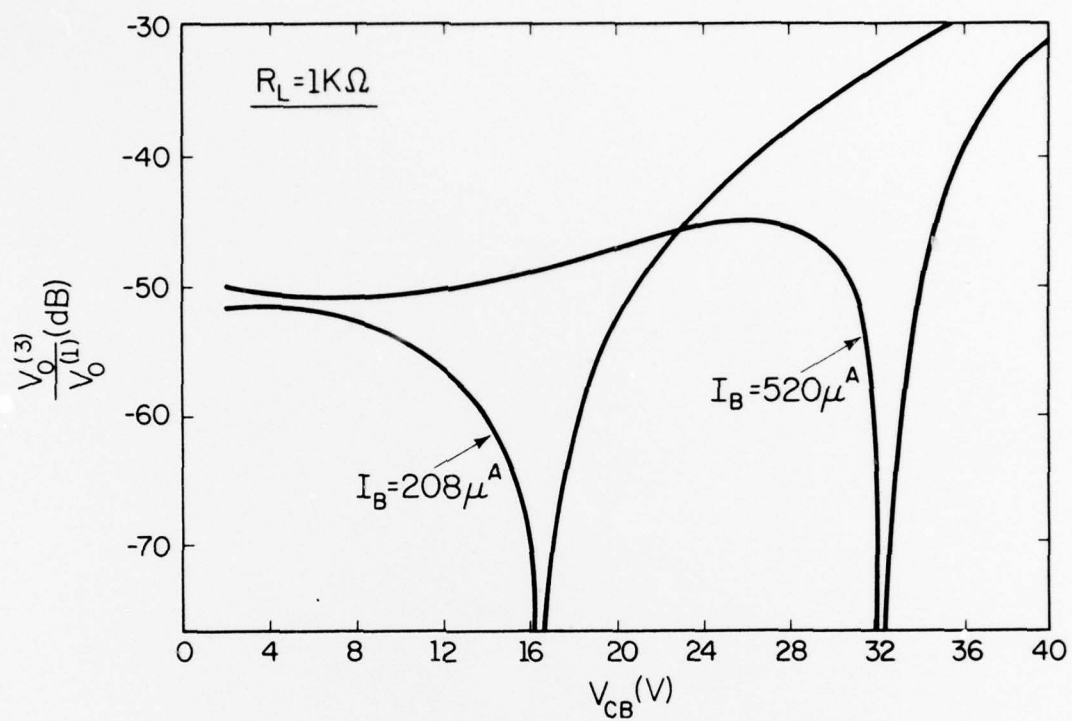
The total third order output is then

$$\begin{aligned} v_o^{(3)} = v_{01}^{(3)} + v_{02}^{(3)} = H_3 e^{(s_1 + s_2 + s_3)t} = & \{ (6b_3 Y_{12}^3 + 12b_2^2 Y_{12}^2 Y_{22}) \mu_{25} \\ & + [6a_{03} \mu_{13}^3 + 6a_{12} \mu_{13}^2 Y_{12} + 12b_2 a_{11} Y_{12}^2 Y_{22} \mu_{13} \\ & + 12a_{11} Y_{12} (b_2 Y_{12}^2 \mu_{23} + a_{02} \mu_{13}^2 Z_{43} + a_{11} Y_{12} \mu_{13} Z_{43}) \\ & + 12a_{02} \mu_{13} (b_2 Y_{12}^2 \mu_{23} + a_{02} \mu_{13}^2 Z_{43} + a_{11} Y_{12} \mu_{13} Z_{43})] Z_{45} \} e^{(s_1 + s_2 + s_3)t}. \quad (85) \end{aligned}$$

3.4. Existence of Third Order Nulls

Obviously the expression for H_3 given by (85) is unsuitable for anything but a computer analysis. The value of $\left| \frac{H_3}{H_1} \right|$ was calculated for a wide range of values of R_L , I_B , and V_{CB} . A typical set of data will now be presented.

The graph plotted in Figure 11 shows a typical pair of curves giving third order distortion as a function of collector-base voltage at a constant base current. It should be observed that a complete cancellation of distortion occurs at voltages of about 16V and 32V for the curves corresponding to base currents of 208 μ A and 520 μ A respectively. The collector-base breakdown voltage of the transistor is about 40V. Similarly, nulls exist for a large variety of loads and bias points.



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Figure 11. Third order distortion versus collector-base voltage.

3.5. Characteristics of Nulls

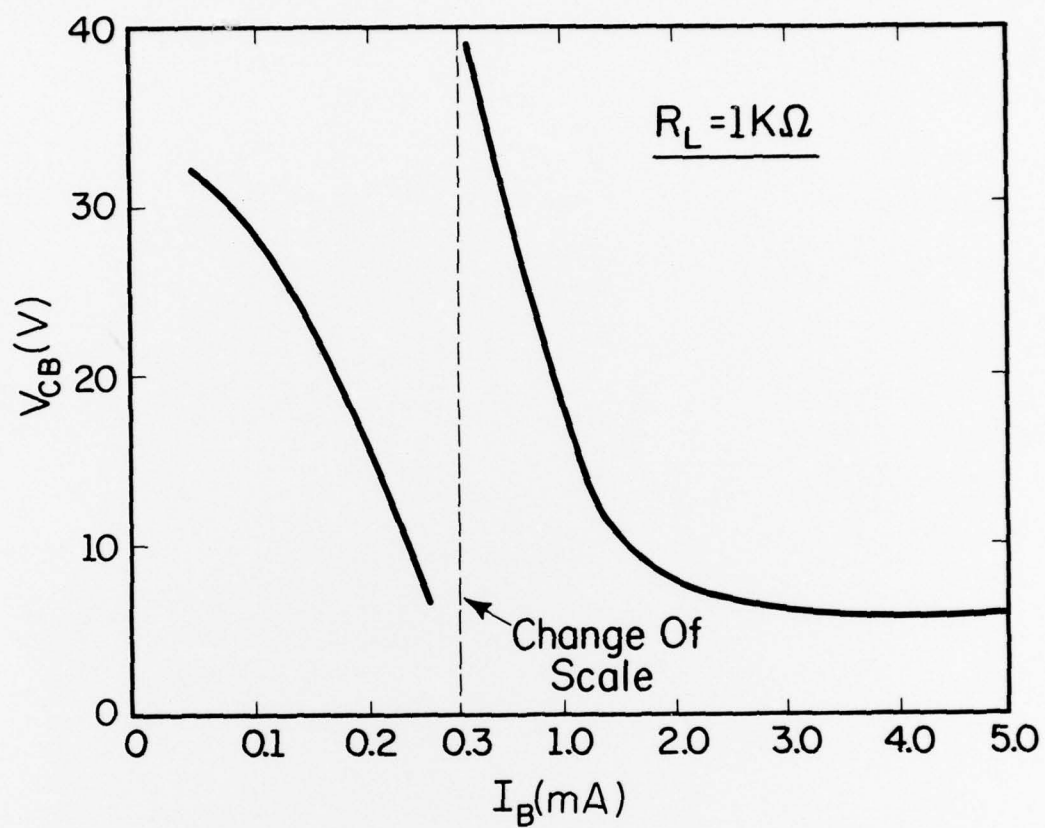
The value of the collector-base voltage at which a null exists depends on the base current. Figure 12 is a plot of the position of nulls versus base current for a load resistance of $1\text{K}\Omega$ and a source resistance of 50Ω . It should be noted that there are two regions in which nulls exist. This result is true regardless of the values of source and load resistances. In fact, by appropriately adjusting the two resistances the ranges of base current in which nulls exist can be positioned almost anywhere. They can even be made to overlap yielding two nulls at the same base current.

It should also be noted that the slope of each curve is everywhere negative. This also is true in general. From this result it is clear that an increase in base current causes a decrease in the value of collector-base voltage at which the null exists.

3.6. Relationship Between Models.

It is somewhat surprising that the avalanche effect can produce enough third order distortion to completely cancel the distortion produced by the normally more dominant base-emitter nonlinearity. This is especially true when the null exists at a collector-base voltage significantly less than the breakdown voltage. Here the avalanche effect produces practically no distortion.

The key to explaining this phenomenon is in the observation that nulls exist in the two regions corresponding exactly to those regions in which base-emitter distortion is small. A comparison of Figures 5



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Figure 12. Position of null versus base current.

and 12 shows that this is true. One region of Figure 12 in which nulls exist is in the vicinity of the critical base current (40). The other corresponds to relatively large values of base current.

The two models can be considered jointly to yield an interesting interpretation. In order to reduce third order distortion the base emitter junction can be biased correctly to produce a decrease in the dominance of the base-emitter nonlinearity. This follows from the analysis of the first model. The collector-base voltage can then be adjusted to further reduce the level of distortion. This can only be done if the base-emitter junction has been biased in such a way that the two nonlinearities can interact to bring about significant cancellation. The analyses of the two models show how base-emitter distortion can be reduced and how the interaction of the two nonlinearities depends upon its reduction.

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