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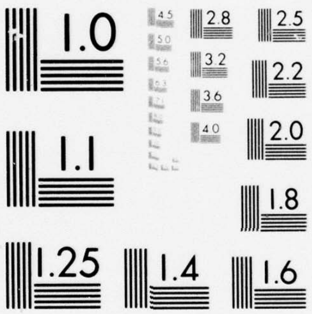

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INPUT-OUTPUT ANALYSIS IN NAVY MANPOWER PLANNING

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In large-scale organizations having a variety of outputs or products, many units within the organization perform primarily a support function. Changes in outputs or activity levels of individual units cause second and higher order effects throughout the system, resulting in a substantial impact on aggregate resource requirements. This report focuses on the issues involved in using intra-organization input-output models as a way of evaluating these influences.

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Cont

A simplified model of the Eleventh Naval District using 28 sectors was developed. Because of the size and complexity of the Navy support systems, problems of decomposition and aggregation take on special importance. The use of impact matrices as a basis for decomposition is illustrated. Analysis indicates that the district can be decomposed geographically. The implications of this result for data collection and the development of solution algorithms are discussed.

Finally, a number of possible applications of the I/O model are described. These include determining the impact of changes in output or final activity levels, determining the effect of changes in the mix or the location of outputs, and using the model to determine feasible solutions to a budget constrained problem.

FOREWORD

The analysis of fleet and shore demands described in this report was conducted in support of the Fleet Impact on Shore Requirements project (formerly known as the Manpower Requirements and Resources Control System (MARRCS)), which is a subproject under Advanced Development Z0109-PN (Manpower Requirements Development System).

The overall objective of this subproject is to test and evaluate methods for determining the impact of fleet and shore demands on major shore activities in the Eleventh Naval District. The development of an input-output model of the relationships between fleet demands and shore support represents the first phase of development.

J. J. CLARKIN
Commanding Officer

SUMMARY

Problem

The United States Navy is a complex economic system consisting of such diverse activities as hospitals, shipyards, airfields, research laboratories, supply activities, construction units, housing facilities, schools, and aircraft maintenance plants. All of these activities are designed to serve the operating forces, either directly or indirectly, in the performance of their missions. To plan the effective use of resources, including manpower, and to perform its mission within a limited budget, the Navy's resource managers must be able to understand and forecast the workload interactions of the activities that they seek to control. The impact of these interactions on manpower is especially important, since personnel costs consume over 50 percent of the Navy's budget.

Purpose

The purpose of this effort was to develop a procedure for studying these interactions.

Approach

A simplified input-output (I/O) model of the Eleventh Naval District using 28 sectors was developed. Because of the size and complexity of the Navy support system, problems of decomposition and aggregation take on special importance. The use of impact matrices as a basis for decomposition was illustrated.

A number of possible applications were described. These include determining the impact of changes in output or final activity levels, determining the effect of changes in the mix or the location of outputs, and using the model to determine feasible solutions to a budget constrained problem.

Findings

I/O analysis is applicable to the problem of linking the fleet structure to shore support to determine manpower requirements at the activity level. Special difficulties exist in data availability for an activity level model and in structuring the model for activities with multiple outputs. Also, although a Navy-wide activity level model will be very large, some decompositions should be possible.

Conclusions

I/O analysis is applicable to Navy manpower planning where the shore support must be related to fleet size. An I/O model permits consideration of a variety of planning problems. The model is flexible enough in structure that unique data-gathering problems and Navy organizational difficulties can be incorporated with only minor modifications.

Recommendations

The I/O approach to linking the shore support to fleet demands should be explored further. Emphasis should be placed on locating and developing data sources. Two models should be considered:

1. A more detailed model of the Eleventh Naval District should be developed by utilizing data available within individual activities. The model should place particular emphasis on organizational problems such as multiple outputs from an activity.

2. A model of the larger Navy activities should be developed using a standardized data source. A possible candidate for the data source is the Navy's financial management system, including Navy Industrial Funding data.

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INTRODUCTION

Problem

The United States Navy is a complex economic system consisting of such diverse activities as hospitals, shipyards, airfields, research laboratories, supply activities, construction units, housing facilities, schools and universities, and aircraft maintenance plants. All of these activities are designed to serve the operating forces, either directly or indirectly, in the performance of their missions. To plan the effective use of resources, including manpower, and to perform its mission within a limited budget, the Navy's resource managers must be able to understand and forecast the interactions of the activities which they seek to control. The impact of these interactions on manpower is especially important since personnel costs consume over 50 percent of the Navy's budget.

The specification of manpower requirements in the Navy is not only important for budget planning but is also an important input to other plans concerned with personnel acquisition and reassignment, as well as job and organization design. In large-scale organizations having a variety of outputs or products, many units within the organization perform primarily a support function. Changes in outputs or activity levels of individual units cause second and higher order effects throughout the system, resulting in a substantial impact on aggregate resource requirements.

Within the U.S. Navy, there are more than 1000 shore activities which support the ships and aircraft of the operating forces, as well as each other. Because the mission of the Navy is to protect the sea lanes and provide a strategic deterrent, the size of support activities should relate primarily to the size of the fleet. However, it will also be affected by the magnitude of the other activities supported.

Objective

The objective of this effort was to develop a procedure for studying these interactions by organizing Navy activities into an input-output (I/O) matrix or model, which relates the fleet to individual shore activities and these activities to each other. Using the matrix, a variety of management questions can then be considered:

1. Based on the size of the operating forces and current support policies, what should be the size of the supporting shore activities?

2. If the structure of the operating forces is changed by transferring ships from one homeport to another, how will the shore activities be affected?

3. What is the overall impact on the Navy budget of an investment in labor-saving equipment?

4. Which Navy activities are relatively independent of changes in the configuration or deployment of operating forces?

Background

Input-output analysis has been used to determine requirements in several models of manpower systems. Bezdek (1974) has applied national I/O tables in conjunction with econometric equations to forecast demand on industrial sectors. A manpower model converts the demands on each sector into required job skills, which are then aggregated to obtain national requirements. Charnes, Cooper, and Niehaus (1975) have projected the use of an I/O model to forecast manpower requirements for individual sectors. In their approach, the model is coupled with personnel flow models in a goal programming format to provide strategies for hiring, separation, and promotions within individual job skills.

Three steps in manpower planning are suggested by these two approaches. First, total demand on an activity is forecast using an I/O model. Second, a manpower model converts the aggregate demand into the job skills which are required to satisfy the demand. Third, the current work force is reconciled with the required work force by specifying strategies for recruitment, promotion, or separation.

While all three steps are relevant to Navy manpower planning, the model in this report will be concerned primarily with the first of the three steps, namely, the forecasting of demands on the activities. The second step will be considered only to the extent that manpower is treated as separate sectors in the input-output model. An ongoing Navy program, Shore Requirements, Standards and Manpower Planning System (SHORSTAMPS), is currently collecting data to determine the manpower required to perform the various functions within Navy activities. The SHORSTAMPS program may provide the data necessary for the kind of manpower model suggested in the second step.

In practice, input-output models for organizational planning have not been as successful as might be anticipated. Gols (1975) discusses some of the difficulties in implementation and concludes that the basic problem is that the I/O analyst has not come to grips with the detailed problems faced by the corporate planner. This report is directed toward increasing the practical applicability of I/O models by considering an organizational model in a Navy context.

The mathematics of I/O analysis are well known and are thoroughly described by Leontief (1951) and Chenery and Clark (1959). However, because modifications to the basic model are necessary for analysis of the Navy's manpower problems, a short summary of I/O analysis is provided in the appendix.

APPROACH

A simplified input-output (I/O) model of the Eleventh Naval District using 28 sectors was developed as a basis for relating the fleet to individual shore activities and these activities to each other. The data required by the I/O model specifies the relationship between inputs and outputs for each sector. The usual assumption in such models is that of constant returns to scale, which leads to the use of simple ratios. In fact, the underlying production or transfer functions for many activities are much more complex and this assumption is often inappropriate. Thus, first and higher order differences were considered as a way of more closely approximating the appropriate transfer function.

Because of the size and complexity of the Navy support system, problems of decomposition and aggregation take on special importance. For this reason, the use of impact matrices as a basis for decomposition was illustrated and analysis indicated that the district can be decomposed geographically. The implications of this result for data collection and the development of solution algorithms were discussed.

Finally, a number of possible applications were described. These include determining the impact of changes in output on final activity levels, determining the effect of changes in the mix or the location of outputs, and using the model to determine feasible solutions to a budget-constrained problem.

RESULTS

Input-Output Model of Eleventh Naval District

The sectors in the input-output model of the Eleventh Naval District (11ND) are listed in Table 1. The 20 shore activities (Nos. 4-13, 16-25) included in the model employed about 38,600 civilian employees at the end of fiscal year 1972, which comprises about 83 percent of the total civilian employees in the 11ND. The total number of military personnel included in the model is 65,650, including 54,970 on board ships based at Long Beach and San Diego.

Table 1

Sectors Included in Input-Output Model

No.	Name	Units	Abbrev.
1	Fleet, Long Beach	Ships	FLTL
2	Fleet, San Diego	Ships	FLTS
3	Retired + Their Dependents	Persons	RET
4	Weapons Center, China Lake	$ Millions	WCCL
5	Missile Center, Pt Mugu	$ Millions	MISL
6	Ship Weap Systems Center, Pt Hueneme	$ Millions	SWS
7	Shipyard, Long Beach	Man-Days, Millions	YARD
8	Pac Missile Range, Pt Mugu	Man-Hours, Millions	PMR
9	Air Station, Pt Mugu	Gallons/Month, Millions	NASP
10	Hospital, Long Beach	Admissions	HSPL
11	CBC, Port Hueneme	$ Millions	CBC
12	Station, Long Beach	Gallons/Month, Millions	STAL
13	Supply Center, Long Beach	$ Millions	NSCL
14	Military Manpower, Long Beach	Personnel	MILL
15	Dependents, Long Beach	Persons	DEPL
16	Training Center, San Diego	Students	NTC
17	Electronics Laboratory Center, San Diego	$ Millions	NELC
18	Undersea Center, San Diego	$ Millions	NUC
19	Air Rework Facility, San Diego	Man-Hours, Millions	NARF
20	Air Station, North Island	$ Millions	NASN
21	Air Station, Miramar	$ Millions	NASM
22	Hospital, San Diego	Admissions	HSPS
23	Station, San Diego	Personnel Supported	STAS
24	Supply Center, San Diego	$ Millions	NSCS
25	Public Works, San Diego	Plant Value, $ Millions	PWC
26	Military Manpower, San Diego	Personnel	MILS
27	Dependents, San Diego	Persons	DEPS
28	Civilian Manpower	Personnel	CIV

The 20 shore activities were selected on the basis of size and the availability of data from the Logistic Support Requirement (LSR) system.[1] For each activity, a measure of the output was selected. For research activities, the measure was millions of dollars of contracts. Other output measures, such as millions of man-days (i.e., Long Beach Shipyard) and millions of man-hours (i.e., Pacific Missile Range and Naval Air Rework Facility), came from each activity's workload measurement. Since the Public Works Center (PWC) does construction work and handles maintenance for other activities, the output of the PWC to each activity was assumed to be proportional to the plant account of the activity. For the supply centers, the naval air stations, the Construction Battalion Center, and the Ship Weapons Systems Center, a convenient output measure was dollars. For the Naval Training Center, the number of students appeared to be a reasonable output measure. For hospitals, the measurement chosen was admissions rather than the average number of patients occupying beds or outpatient visits.

When a single measure of output must be chosen and other functions ignored, certain aspects of an activity's workload may not be adequately represented. It is possible to use a vector approach, but this increases the size and complexity of the model and was not considered here. For example, the two naval stations and the air station at Pt. Mugu have so many outputs (e.g., landing/take-offs, ships berthed, squadrons based, tenant activities) that no single measure was adequate. The measure chosen in each case was based on making as much use as possible of the information available in the LSR system.

The fleets homeported at Long Beach and San Diego are incorporated in this model as pure consumers and are measured in total number of ships. Aggregation of all ship types is, of course, an oversimplification since different classes of ships place different amounts of demand on shore activities. However, in order to treat different ship classes as separate sectors, much more information is required.

In practice, an activity level model will require a detailed analysis of the fleet structure. If a ship is homeported in San Diego, it will have less impact upon activities in the 11ND when it is deployed to the Western Pacific than when it is operating off the Continental United States. There will be some impacts; for example, dependents of military personnel assigned to a ship in the Western Pacific will still be using the hospital and other facilities in the San Diego area. More detailed information is also required in squadron assignment. Aircraft squadrons are attached to an aircraft carrier during deployment but, when the ship is in its homeport, the squadrons disperse to nearby naval air stations. In the model developed, the details of fleet organization were ignored and a ship's impact was considered to be on its homeport only. Later extensions will consider the effects of ship deployments.

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[1] The LSR contains workload and customer data for each activity, but the quality and quantity of data are uneven. Because of this, it was necessary to include data from different fiscal years in the model. Thus, while the data permits illustrations of the applications of the model, at this point it is not adequate for actual planning.

The impacts of ships and squadrons on repair activities, such as shipyards and naval air rework facilities, also need to be explored. These activities are centrally loaded and their customers are the entire Navy rather than ships and squadrons in a particular geographic region. This suggests that it may be necessary to construct the model for all Navy activities and not just those of one Naval District. The magnitude of such a model requires some consideration of possible techniques of decomposition. One approach may be to treat activities outside the Naval District as elements of final demand.

An important feature of the model is the separate manpower sectors. They were disaggregated by homeport so that the effects of various ship combinations at Long Beach and San Diego could be studied. The data was arranged in an input-output matrix such that hospitals would support military manpower (and dependents), which in turn would support the fleet and shore activities by providing labor. This illustrates an advantage of input-output analysis; namely, the ability to study secondary effects (such as that of the fleet on hospitals), even while using data on direct impacts alone.

The results obtained by organizing the transactions data on which the model is based are shown in Table 2. Reading across a row gives the output from a given sector to all other sectors. Reading down a column gives the input to a given sector from all 28 sectors. The last column in the table provides the total output--including that satisfying final demand--for each sector. The research activities, fleets, and several specialized functions are pure consumers.

As indicated previously, the principal difficulty with any input-output model is the large requirement for data. Bezdek (1974) points out that even the most recent national table will always be several years out-of-date. Consequently, a major effort in the use of national tables is the projection of changes in the coefficients between compilations of successive tables. The I/O model for the 11ND posed fewer difficulties since the data appears to be available in the system for an activity level model. However, the data is scattered among the individual activities so the collection effort may preclude annual model revisions unless a standardized reporting system is developed. Also, many relationships which we would expect the model to reveal are not included here because of the limitations in the LSR system. For example, the Naval Supply Center in San Diego should have output to the Naval Air Rework Facility, the Hospital, and the Naval Station in San Diego. Qualitative information indicates that these relations exist, but no quantitative information was available. This deficiency would be corrected with an improved LSR or with a better data source.

From the transactions data in Table 2 the I/O coefficients, matrix A, and the inverse matrix, $(I-A)^{-1}$, can be calculated.

Table 2

Matrix of Data on Flows from
One Activity (Row) to Another (Column)

Sector	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	Total Output[a]
1 FLTL																													83.0
2 FLTS																													99.0
3 RET																													25000.
4 WCCL																													201.9
5 MISL																													52.6
6 SWS																													25.4
7 YARD																													1.0
8 PMR	0.2				0.4	0.1																							1.1
9 NASP					0.3			0.3	0.1																				1.7
10 HSPL			1320.											4019.	3075.														8496.
11 CBC						1.0					2.5																		3.5
12 STAL	3.3																												6.3
13 NSCL	71.2						15.8					2.5																	98.6
14 MILL	22932.			217.	292.	94.	53.	193.	445.	782.	254.	538.	21.																25821.
15 DEPL														34817.															34817.
16 NTC																													13460.
17 NELC																													36.9
18 NLC																													56.9
19 NARF																													7.8
20 NASN		204.2																	262.6										583.5
21 NASM		71.6																			95.9								188.8
22 HSPS			3726.																							9852.	13200.		28751.
23 STAS		21603.	8200.																				3687.				37500.		73500.
24 NSCS		100.2														14.8	0.9								0.6				153.9
25 PWC																94.2	26.2	21.1				40.0	104.4	53.4		196.9			687.5
26 MILS		32036.														2907.	118.	305.	43.	1133.	865.	1841.	536.	28.	18.				39830.
27 DEPS																										53717.			53717.
28 CIV				4519.	1696.	1669.	7300.	1609.	1254.	264.	1674.	719.	688.			878.	1461.	1657.	7171	1912.	565.	777.	369.	815.	1606.				53042.

[a] Total includes output from a given sector to all other sectors, plus that satisfying final demand.

Applications of the Input-Output Model

Final Demand

Several applications of the I/O model were made, testing the effects of changes in Fleet size and location, as well as the effects of a major base closure. Figure 1 shows how a 10 percent cut in the fleet would affect activities in the San Diego and Long Beach areas. The wide range of decreases in the other activities and in the manpower sector is of particular interest. While all shore activities serve the fleet directly (e.g., naval stations) or indirectly (e.g., laboratories), the economic effects on the activities (and hence the manpower effects) are not directly proportional to changes in the fleet.

Figure 2 illustrates the effects that relocating 41 ships (about 50 percent) from homeport in Long Beach to San Diego would have on supporting activities. The military personnel in the Long Beach area would be decreased about 40 percent, and those in the San Diego area, increased about 38 percent. Similarly, support required from the Long Beach Naval Hospital would be decreased about 38 percent, while that required from the San Diego Naval Hospital would be increased by about 30 percent. Again, the non-proportional changes are evident indicating the inappropriateness of fair share reductions in planning.

Finally, Figure 3 shows the estimated decreases in support required if the Naval Training Center at San Diego was closed. This figure includes both direct (solid bar) and indirect (hatched bar) effects.

Standard Costs

Many Navy manpower problems involve the calculation of costs as part of an effort to reduce total personnel costs. Since the costs of many items are indirect, an accurate figure can only be obtained only if the entire system is considered. For instance, the cost of military personnel includes not only pay and allowances, but also medical expenses (both for the personnel and their dependents), retirement benefits, training, recreation, and commissary privileges.

Using the I/O model, manpower costs can be calculated. The cost elements (in fiscal year 1972) are civilian salaries ($11,350 average) and military pay ($6,193 average). On this basis, the manpower cost of a ship homeported at Long Beach is $1,981,000 per annum. This cost includes $1,783,000 in direct manpower; $69,130 in indirect manpower from the Naval Supply Center, Long Beach; $74,910 from the Naval Station, Long Beach; and $53,600 from the Pacific Missile Range. These figures reflect the manpower cost at each activity necessary to service the ship.

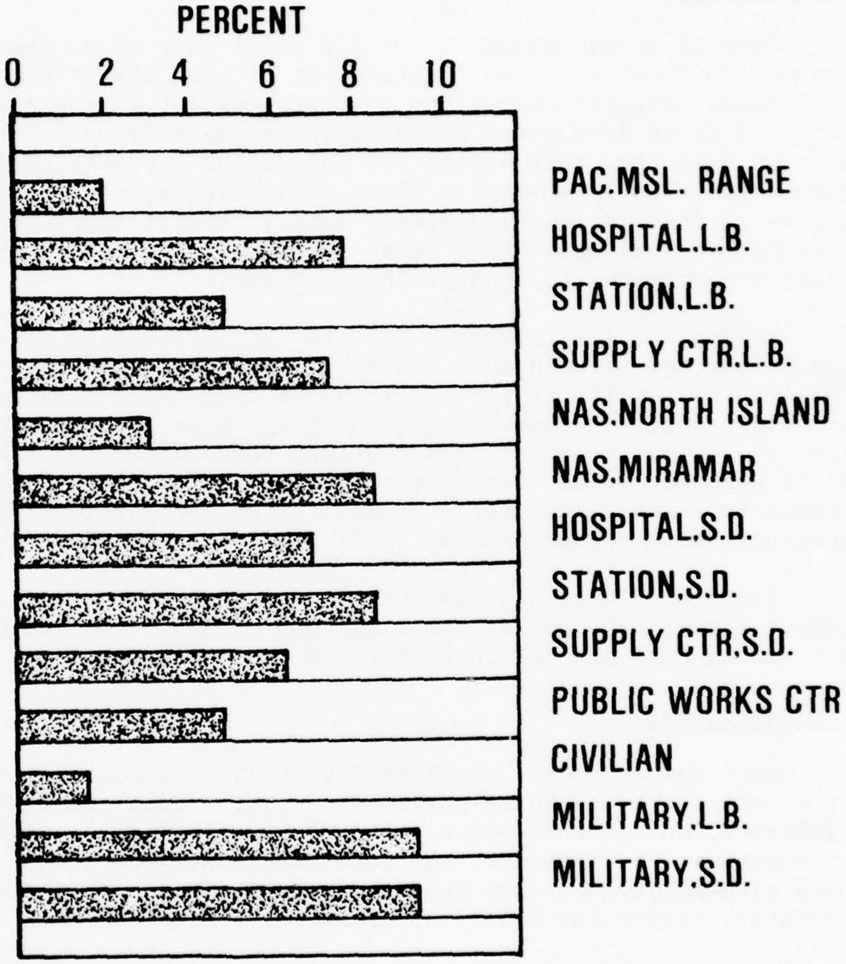

Figure 1. Effects of cutting the fleet by ten percent.

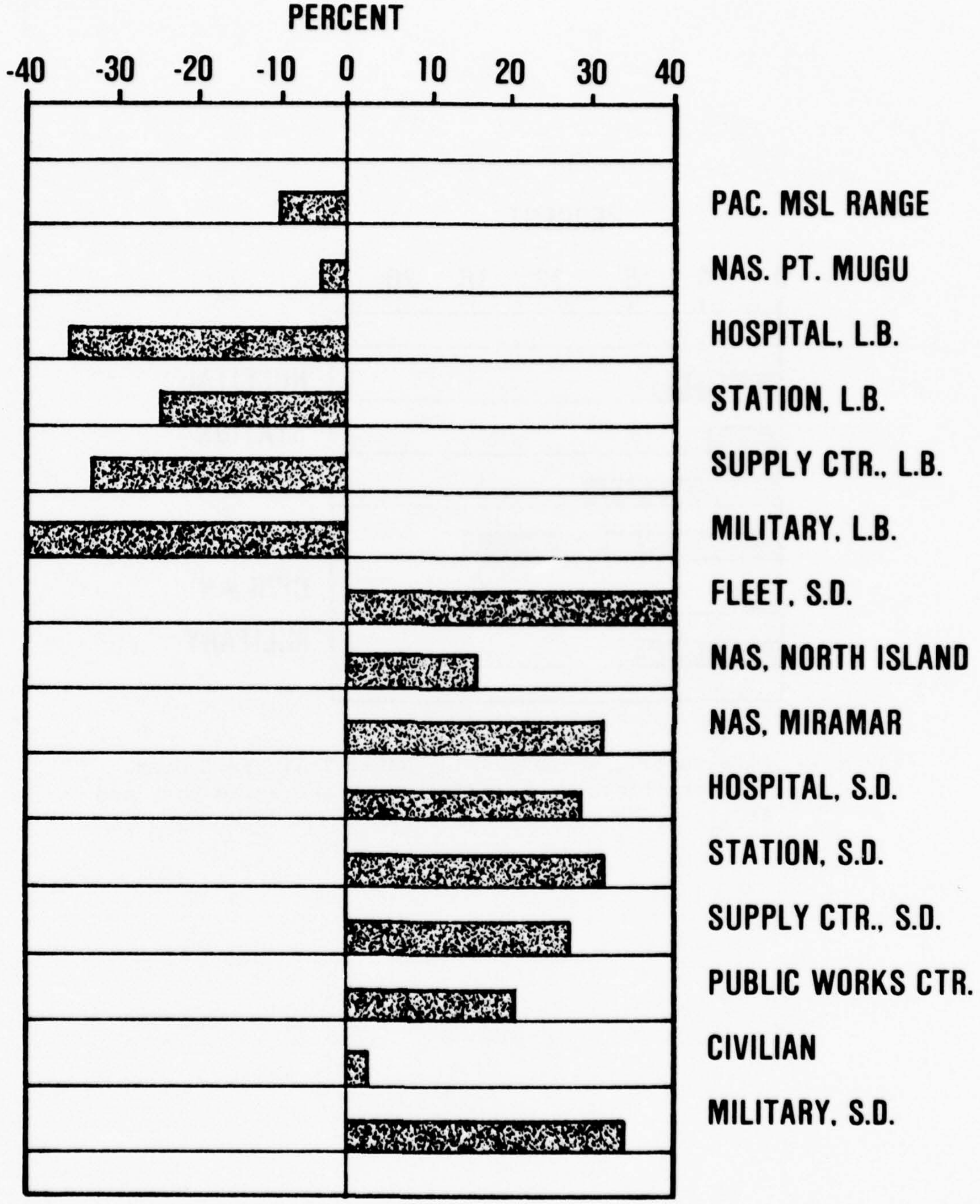

Figure 2. Effects of transferring 41 ships from Long Beach to San Diego.

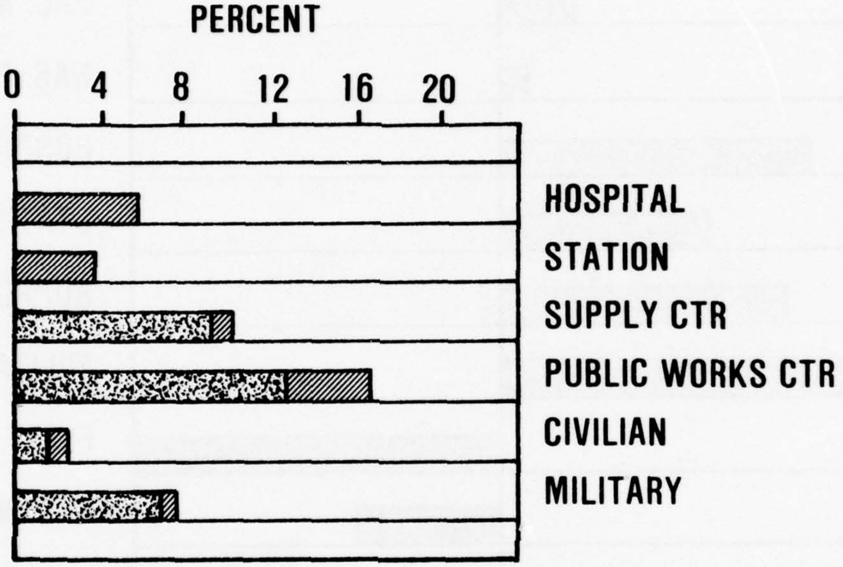

Figure 3. Effects of disestablishing Naval Training Center. (Direct effects are indicated by the solid bar, and indirect, by the hatched bar.)

Budget Constraints and Goal Programming

The Navy planning process goes through two main phases. In the first, operating force structures are set to fulfill the Navy's missions. From the operating force structure, the size and budget of each Navy shore activity are determined. The examples in the preceeding sections illustrate the use of an I/O model for determining the demands upon each activity and hence its size and budget.

The second phase of the Navy planning process begins after the Navy budget has been submitted for approval to the President and to Congress, where the budget is often modified. Using the new budget, several questions must be considered such as (1) whether the Navy can fulfill the original missions that were set, and (2) what missions the Navy can fulfill under the new budget, if the answer to the first question is "No." The answers to these questions require an understanding of the interactions between the activities in the Navy system.

One approach to this problem is that of goal programming. Under this formulation, the original I/O relations shown in the appendix are modified to

$$(I-A)\underline{x} = \underline{y} - \underline{d} \tag{1}$$

where $\underline{d}$ is a vector of the deviations from final demands. The goal of the program becomes the minimization of a function of $\underline{d}$, such as a weighted norm or a rank ordering on the importance of the components of $\underline{d}$ (enabling one sector to preempt the others). The goal program also includes budget constraints or constraints on the variables which are dictated by the structure of the problem. The final formulation is

$$\begin{aligned} &\text{Min } F(\underline{d}) \\ \text{s.t.}\quad &(I-A)\underline{x} = \underline{y} - \underline{d} \\ &B\underline{x} \leq \underline{b} \\ &\underline{x} \geq 0. \end{aligned} \tag{2}$$

Table 3 gives an illustration of the application of the I/O information with a budget constraint and a single optimizing goal. Final demands and total demands were related by the I/O equations. The total demands were constrained so that no sector could be cut by more than 15 percent. No constraints were placed on final demands. The number of ships homeported at San Diego was required to be 16 more than the number homeported at Long Beach. The budget constraint on wages was $800,000,000 for a wage rate of $6,193 for average military pay and $11,350 for average civilian pay. The goal of the program was to maximize the total number of ships in the two fleets. The program was solved for the final demands, including the number of ships, and these were input into the model to obtain Table 3.

Table 3

Results of Applying Input-Output Information With a Budget Constraint and a Single Optimizing Goal

Sector	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	Total Output [a]
1 FLTL																													80.0
2 FLTS																													96.0
3 RET																													25000.
4 WCCL																													171.6
5 MISL																													44.7
6 SWS																													21.6
7 YARD																													0.9
8 PMR	0.2				0.4	0.1																							1.0
9 NASP					0.3			0.2	0.1																				1.5
10 HSPL			1320.											3835.	2934.														8089.
11 CBC						0.9					2.1																		3.0
12 STAL	3.2																												5.3
13 NSCL	68.7						13.4					2.2																	84.2
14 MILL	22103.			184.	248.	60.	45.	164.	379.	745.	216.	456.	18.																24638.
15 DEPL														33222.															33222.
16 NTC																													11441.
17 NELC																													31.4
18 NUC																													48.3
19 NARF																													6.6
20 NASN		198.0																	223.2										495.2
21 NASM		69.5																			82.1								159.9
22 HSPS			3726.																							9350.	12528.		25604.
23 STAS		20948.	8200.																				3419.				35590.		68158.
24 NSCS		97.2														12.5	0.7								0.5				130.4
25 PWC																80.1	22.3	17.9				35.7	96.8	45.3		186.9			585.9
26 MILS		31065.														2471.	100.	259.	37.	951.	732.	1639.	497.	24.	15.				37802.
27 DEPS																										50982.			50982.
28 CIV				3841.	1442.	1419.	6205.	1370.	1067.	251.	1423.	609.	588.			746.	1242.	1408.	6055.	1622.	478.	692.	342.	690.	1369.				45077.

[a] Total includes output from a given sector to all other sectors, plus that satisfying final demands.

Most of the information obtained in the solution was expected. Sectors not directly supporting the fleets were decreased as much as possible so that the fleets and their supporting elements could be maximized. With this arrangement, 80 ships were assigned to Long Beach and 96 ships were assigned to San Diego. The final demand for civilian labor in Table 3 was 12,180 personnel. The total demand for civilian labor was at the lower constraint: 85 percent of base demands.

In this problem, the effectiveness of the fleet was assumed to be directly proportional to size. This may not be an adequate assumption in practice, but it is suitable for illustrative purposes. Of course, the 15 percent reduction constraint could be varied from one activity to another and constraints could be placed on final demands. Also, other goals besides maximization of the size of the fleet are possible. For example, the problem could have been structured to minimize the deviations from the original budget request for each sector.

Resource Trade-offs at the Activity Level

Accurate manpower cost allocation among activities is critical for the Navy since military manpower is paid from a central appropriation rather than by individual activities. Under this arrangement, military manpower and other expensive assets are often regarded as free goods in planning at the activity level. An action that appears cost-optimal at the activity level may not be cost-efficient at the total Navy level where manpower is over 50 percent of the budget. The I/O model developed in this effort can be used to illustrate the Navy-wide effects of a possible trade-off between capital and labor (see Table 4).

Table 4

Cost of Labor and Capital Under Varying Wage Rates

% Change in Wage Rate from Base	Manpower Cost ($Civ, $Mil)	Labor Total Cost ($000,000)	Revised Capital & Labor Total Cost ($000,000)	Savings ($000,000)
0	(11,350,6193)	1,008.44	1,008.33	.11
10	(12,480,6812)	1,109.29	1,108.59	.70
20	(13,620,7432)	1,210.13	1,208.84	1.29

Column 3 of Table 4 gives the total cost of the existing system when the added costs of civilian and military manpower are changed 0, 10, and 20 percent from the base figures $11,350 and $6,193. Now, assume that the three naval air stations (sectors 9, 20, and 21 in the model) invest in capital equipment which allows a 10 percent manpower reduction per unit of output. The total equipment costs per year for the three sectors are assumed to be \$1,708,000, \$2,885,000, and \$1,186,000 respectively. These equipment costs are chosen to be identical with the respective reductions in manpower costs when the salaries are at the base figures.

The 10 percent manpower reduction affects the structure of the system beyond the three sectors directly impacted. The change in the production functions of the air stations requires backward modifications through all sectors which supply the air stations either directly or indirectly (supply centers, dependents, hospitals, etc.). After these structural changes have been made to the model, the total Navy cost of the new system, based on labor plus capital investment, can be calculated. These figures are given in the fourth column of Table 4. The total savings from the investment in capital equipment are given in Column 5.

Consequently, even though the equipment costs were chosen to match the manpower-reduction saving at naval air stations for the first line, there is still a saving of \$110,000 for the total system due to the structural changes throughout the system. As manpower costs rise, the savings from capital goods investment increase substantially.

Impact Matrices and Decomposition

An alternative interpretation of the solution to the input-output model can be developed by noting that the inverse matrix $(I-A)^{-1}$ can be expanded as a geometric series.

$$(I-A)^{-1} = I + A + A^2 + A^3 + \ldots \qquad (3)$$

For a given final demand $\underline{y}$, the total output $\underline{x}$ can be written as

$$\underline{x} = \underline{y} + A\underline{y} + A^2\underline{y} + A^3\underline{y} + \ldots \qquad (4)$$

Total demand consists of final demand together with the derived demand for inputs needed to produce $\underline{y}$, that is, $A\underline{y}$, the second order derived demand $A^2\underline{y}$, the third order demand $A^3\underline{y}$, and so on. Assuming $(I-A)^{-1}$ exists, this series will be convergent and the successive terms will, in general, be progressively smaller.[2]

Although we are primarily interested in the static or long-run solution to the model, it should be recognized that the successive stages of derived demand do not all occur simultaneously. For this reason, it is sometimes useful to trace the sequence of components of total output.

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[2] This need not be true initially where some elements of the A^n matrix may first increase or oscillate before finally approaching zero. Sufficiency conditions for the convergence of the sum can be developed. See for example, Karlin (1959).

In many cases we may not be interested in the magnitudes of these components but only in their existence. That is, we are interested in questions such as: At what stage does activity j first have an impact on activity i, or does activity j ever have a direct or indirect impact on activity k? This can conveniently be studied by replacing the matrix A with a corresponding impact matrix J, formed by replacing all nonzero elements of A with ones. The impact matrix for the model considered in this report is shown in Table 5.

The existence of second andab higher order impacts can be determined by creating the successive Boolean products or powers of the J matrix, J^2, J^3, J^4, ... The matrix J^m will then give the m^{th} impact of A. When for some integer N, $J^{N+1} = J^N$, it can easily be seen that for all $n > N$, $J^n = J^N$ also. The accumulated impact matrix can be obtained by the Boolean sum of J^1 through J^N and will be equivalent to the impact matrix for total derived demand $A + A^2 + A^3 + \dots$. If all the elements of the accumulated impact matrix equal one, then every activity impacts on every other activity and no decomposition or computational simplification of the corresponding input-output model is possible. If zero entries exist, however, the rows and columns of the matrix can be interchanged to bring the matrix into a more convenient form.

Example of Impact Matrix

As an example of the use of the impact matrix, consider a simplified system consisting of the following Long Beach sectors: Navy Shipyard (7), Naval Hospital (10), Naval Supply Center (13), Military Manpower (14), and Dependents (15). The corresponding impact matrix, which was abstracted from Table 5, is shown in Table 6. It should be noted that sector 15, Dependents, is not in any sense an input into the Navy. On the other hand, its impact characteristic is the same as that of an actual input or support activity. As the number of military personnel increases, the number of dependents increases and thus the derived demand on the hospital facilities.

By looking at the sequence of impact matrices J^m in Table 6, it can be seen that the Shipyard has a first-order impact on the Supply Center but no higher level impacts; it has no first-order impact on the Hospital or on the number of Dependents, although higher-order impacts exist. The accumulated impact matrix stabilizes for $n \geq 5$. From this accumulated matrix, three different types of activities can be distinguished; the Shipyard (7), which is solely a consumer; the Supply Center (13), which supports only the final consumers; and the remaining activities which form the general support sector. It can also be seen that only Military Manpower and Dependents impact on the Hospital and conversely, after the second impact, only the Hospital impacts on the Military Manpower.

Table 5

Impact Matrix for Input-Output Model

From Activity		To Activity 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
1	FLTL																												
2	FLTS																												
3	RET																												
4	WCCL																												
5	MISL																												
6	SWS																												
7	YARD																												
8	PMR	1			1	1																							
9	NASP				1				1	1																			
10	HSPL			1											1	1													
11	CBC						1					1																	
12	STAL	1																											
13	NSCL	1						1					1																
14	MILL	1			1	1	1	1	1	1	1	1	1	1															
15	DEPL														1														
16	NTC																												
17	NELC																												
18	NUC																												
19	NARF																												
20	NASN		1																	1									
21	NASM		1																			1							
22	HSPS			1																							1	1	
23	STAS		1	1																				1				1	
24	NSCS		1														1	1								1			
25	PWC																1	1	1				1	1	1		1		
26	MILS		1														1	1	1	1	1	1	1	1	1	1			
27	DEPS																										1		
28	CIV				1	1	1	1	1	1	1	1	1	1			1	1	1	1	1	1	1	1	1	1			

<u>Note</u>: All zero entries in this matrix have been supressed to more clearly show the pattern of nonzero entries.

Table 6

Impact Matrices for the Simplified Example

$$J$$

$$\begin{array}{c|ccccc|}
 & \multicolumn{5}{c}{\underline{\text{Activity}}} \\
\text{Activity} & 7 & 10 & 13 & 14 & 15 \\
\hline
7 & 0 & 0 & 0 & 0 & 0 \\
10 & 0 & 0 & 0 & 1 & 1 \\
13 & 1 & 0 & 0 & 0 & 0 \\
14 & 1 & 1 & 1 & 0 & 0 \\
15 & 0 & 0 & 0 & 1 & 0
\end{array}$$

$$J^2 = \begin{vmatrix}
0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 1 \\
1 & 1 & 1 & 0 & 0
\end{vmatrix}$$

$$J^3 = \begin{vmatrix}
0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 0 \\
1 & 0 & 0 & 1 & 1
\end{vmatrix}$$

$$J^4 = \begin{vmatrix}
0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 0
\end{vmatrix}$$

$$J^5 = \begin{vmatrix}
0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1
\end{vmatrix}$$

$$\text{Accumulated Matrix} = \begin{vmatrix}
0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 \\
1 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1
\end{vmatrix}$$

Accumulated Impact Matrix and Decomposition

The accumulated impact matrix for the 28-activity model is shown in Table 7. In this table, rows and columns have been interchanged to more clearly identify the different types of activities. With the exception of two activities, Retired Personnel (3) and Civilian Personnel (28), the model can be completely decomposed into two separate models, one for Long Beach and one for San Diego. These two activities might be decomposed, if possible, for some analyses. Within the Long Beach model, three types of activities can be identified; final consumers (1, 4, 5, 6, 7), direct support (8, 12), and general support (9, 10, 11, 13, 14, 15). These last can be further divided into those activities which are impacted by all activities (10, 14, 15) and those others which give more specialized support. A similar classification can be made of the San Diego activities.

This partitioning of activities has the advantage of simplifying the solution of the inverse of (I-A). From Table 7, the I/O equations can be written as

$$\begin{vmatrix} I & 0 & 0 & 0 \\ -A_{21} & I & 0 & 0 \\ -A_{31} & -A_{32} & I-A_{33} & 0 \\ -A_{41} & -A_{42} & -A_{43} & I-A_{44} \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{vmatrix} = \begin{vmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{vmatrix}, \qquad (5)$$

where the subscripts refer to the four partitioned groups in the table. These equations can be solved sequentially as

$$\begin{aligned} X_1 &= y_1 \\ x_2 &= y_2 + A_{21}x_1 \\ x_3 &= (I-A_{33})^{-1}[y_3 + A_{31}x_1 + A_{32}x_2] \\ x_4 &= (I-A_{44})^{-1}[y_4 + A_{41}x_1 + A_{42}x_2 + A_{43}x_3]. \end{aligned} \qquad (6)$$

This effects a substantial reduction in the computations involved in calculating the inverse.

Table 7

Accumulated Impact Matrix for Input-Output Model

From		To Activity																										
Activity	3	1	4	5	6	7	8	12	9	11	13	10	14	15	2	16	17	18	19	20	21	22	23	24	25	26	27	28
3 RET																												
1 FLTL																												
4 WCCL																												
5 MISL																												
6 SWS																												
7 YARD																												
8 PMR		1		1	1																							
12 STAL		1																										
9 NASP		1		1	1		1		1																			
11 CBC					1					1																		
13 NSCL		1				1		1																				
10 HSPL	1	1	1	1	1	1	1	1	1	1	1	1	1	1														
14 MILL	1	1	1	1	1	1	1	1	1	1	1	1	1	1														
15 DEPL	1	1	1	1	1	1	1	1	1	1	1	1	1	1														
2 FLTS																												
16 NTC																												
17 NELC																												
18 NUC																												
19 NARF																												
20 NASN															1				1									
21 NASM															1						1							
22 HSPS	1														1	1	1	1	1	1	1	1	1	1	1	1	1	
23 STAS	1														1	1	1	1	1	1	1	1	1	1	1	1	1	
24 NSCS	1														1	1	1	1	1	1	1	1	1	1	1	1	1	
25 PWC	1														1	1	1	1	1	1	1	1	1	1	1	1	1	
26 MILS	1														1	1	1	1	1	1	1	1	1	1	1	1	1	
27 DEPS	1														1	1	1	1	1	1	1	1	1	1	1	1	1	
28 CIV	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	

Note: All zero entries in this matrix have been suppressed to more clearly show the pattern of nonzero entries.

DISCUSSION

This report has dealt with some of the issues in the development of an input-output model for manpower management based upon the treatment of Navy activities as an economic system. The implications of using linear or first difference planning factors for I/O models was discussed in the Appendix. The importance of decomposition to the successful application of I/O theory to the Navy required the development of impact matricies. Three areas still to be considered are data availability, levels of aggregation of sectors, and the impact of conflicting goals.

If a sector has a vector of outputs, then the model requires information on the internal flows between the parts of the sector and how much of the total input is received by each function. Sectors such as naval air stations have multiple outputs, including take-offs and landings, maintenance, berthing of aircraft carriers, servicing squadrons which are detached from their ships, and servicing permanent squadrons. The use of a single output measure in such cases may result in misallocation of resources. This occurs when the single chosen output changes while other functions remain constant or vice versa.

Usually, only a few functions in a given sector change greatly over time, and these functions could be analyzed by periodic studies. For example, the workload at a Bachelor Officers' Quarters might only need to be analyzed every 3 to 5 years. Between these special studies, the activity level of the Officers' Quarters could be assumed to have a constant relationship to the activity level of the sector in which it is included.

For the illustrative model in this report, the data was taken from the Logistic Support Requirement (LSR) system. Currently, the quality of LSR data is not adequate, although a substantially revised LSR system might be able to provide the necessary quantitative inputs and outputs for each activity. Current work with several Navy activities indicates that more adequate data exists within the records of many activities, but will require special data gathering methods.

Questions involving aggregation and disaggregation arise in two forms. First, the I/O model may need to be decomposed for purposes of solution. The impact matrices provide a way to examine structural properties of the system. A related problem is the aggregation of information for decision making at the appropriate level. Both problems depend upon the data in the model and may require empirical and theoretical development.

A major area for exploration involves the goals of the Navy system. It is axiomatic that the goal of the shore establishment is to support the fleet and the goal of the fleet is to keep the sea lanes open. The realization of these goals, however, is more complex. Increasingly, both budgetary and manpower restrictions are becoming tighter, and this limitation in resources is expected to continue. Consequently, the simple goals quickly become conflicting needs. Goal programming formulations may permit the trade-offs between goals to be made explicit.

DISCUSSION

[illegible] some of the changes in the development of [illegible] depends upon the [illegible] system. [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

CONCLUSIONS

Input-output analysis is applicable to Navy manpower planning where the shore support must be related to fleet size. An I/O model permits consideration of a variety of planning problems. The model is flexible enough in structure that unique data-gathering problems and Navy organizational difficulties can be incorporated with only minor modifications.

RECOMMENDATIONS

The input-output approach to linking the shore support to fleet demands should be further explored. Emphasis should be placed on locating and developing data sources. Two models should be considered:

1. A model of the Eleventh Naval District should be developed by utilizing data available within individual activities. The model should place particular emphasis on particular organizational problems such as multiple outputs from an activity.

2. A model of the larger Navy activities should be developed using a standardized data source. A possible candidate is Navy financial data, including the Navy Industrial Fund.

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APPENDIX

DESCRIPTION OF INPUT-OUTPUT ANALYSIS

DESCRIPTION OF INPUT-OUTPUT ANALYSIS

Basis

For purposes of input-output analysis, the system under consideration is divided into n disjoint sectors. Each sector is assumed to have a single, measurable output. If a sector has several outputs that can be expressed in a common unit, such as dollars, the common unit can be taken as the single output. For a sector k, k = 1,2,...n, the output b_{kj} is sent to sector j, j = 1,2,...n, and the total output c_k are obtained. The matrix $B = (b_{kj})$ is called the data matrix of the system and the vector $\underline{c} = (c_k)$ is the total output data vector. The difference,

$$y_k = c_k - \sum_{j=1}^{n} b_{kj},$$

is the final demand for sector k, and the vector $\underline{y} = (y_k)$ is the final demand data vector. The final demand from a sector is the amount of its output that is not consumed by other sectors and that, therefore, leaves the system.

The inputs to sector j are the b_{kj} for k = 1,2,...n. Since the total output c_j for sector j required an input b_{kj} from each sector k, it is assumed that an *output of one unit* from sector j will require a proportional input,

$$a_{kj} = \frac{b_{kj}}{c_j}.$$

This proportionality assumption is the basis of I/O analysis. The matrix $A = (a_{kj})$ is called the coefficients matrix of the system.

For a given final demand vector $\underline{y}$, the model is structured to determine the total output vector $\underline{x}$, that will produce $\underline{y}$. For each sector k, the total output x_k is equal to the final demand y_k, plus the sum of the inputs to each sector j of the economy. Since the input to each sector j is the amount required to produce the output from sector j, the input must be $a_{kj}x_j$. Thus, for each sector k

$$x_k = \sum_{j=1}^{n} a_{kj}x_j + y_k.$$

The corresponding matrix equation is

$$\underline{x} = A\underline{x} + \underline{y}.$$

Rewriting, we obtain

$$(I-A)\underline{x} = \underline{y}.$$

If the matrix I-A is nonsingular, then

$$\underline{x} = (I-A)^{-1}\underline{y}.$$

The matrix $(I-A)^{-1}$ is called the <u>inverse matrix</u>. For any final demand vector $\underline{y}$, the total output vector $\underline{x}$ for the system can be found. After $\underline{x}$ has been computed, the requirements for each sector j from sectors $k = 1,2,\ldots n$ can be computed by $a_{kj}x_j$.

To calculate the a_{ij}'s, it is necessary to have data on sector outputs in terms of their sector destinations. In practice, this is usually derived from data for a single specific time period. For a large system, data collection at this level of detail is very difficult. Consequently, national economic models are usually computed at intervals of 10 to 15 years, and, over this time, the structure of the economy may change considerably. For the type of intraorganizational analysis considered in this report, data collection should be easier and the model, therefore, can be updated more frequently.

An important characteristic of I/O models is their wide applicability in planning at different levels of aggregation. The model described in this report is constructed at the activity level and is based on logistic data for a related group of activities within the Eleventh Naval District. However, the same basic structure could be applied in a more aggregated form at the Systems Command level or a lower level, such as that of functions within activities.

Linear and First Difference Models

Input-output analysis is usually based on the assumption that the ratios of input to output are constant for all activities, both over time and as the output level changes. This is the proportionality assumption. In practice, all that can usually be said is that the production or transfer functions relating inputs to outputs are, at any point in time, monotonically nondecreasing functions of output. However, over limited ranges of output changes, it can usually be assumed that the function can be approximated by a linear model of the form:

$$\text{Input to activity } j \text{ from activity } i \text{ for time period } t = \beta_{ij} + \alpha_{ij}x_{jt}, \qquad (1)$$

where x_{jt} is the output of activity j, and α_{ij} and β_{ij} are constants.

Total output of activity i is then given by

$$x_{it} = y_{it} + \sum_{j=1}^{n} (\beta_{ij} + \alpha_{ij} x_{jt}), \tag{2}$$

where y_{it} is the final demand for i.

In addition, x_{it} will normally be subject to some form of random or accidental error. This may be due to measurement errors, to nonlinearity in the production functions, to slack resources so that actual inputs are greater than the minimum necessary inputs, or, finally, to the possibility that changes in inputs may affect output quality rather than quantity. In these cases the model becomes

$$x_{it} = y_{it} + \sum_{j=1}^{n} (\beta_{ij} + \alpha_{ij} x_{it} + \varepsilon_{ijt}). \tag{3}$$

Using this model, it is now possible to explore some of the alternative approaches to estimating the input-output coefficients.

<u>*Simple Ratio*</u>

The usual procedure for estimating the coefficient α_{ij}, assuming constant proportions, is to divide the input from i to j in some period t by the total output from j, x_{jt}, for the same period:

$$\begin{aligned} \hat{\alpha}_{ij} &= \frac{\beta_{ij} + \alpha_{ij} x_{jt} + \varepsilon_{ijt}}{x_{jt}} \\ &= \alpha_{ij} + \frac{\beta_{ij}}{x_{jt}} + \frac{\varepsilon_{ijt}}{x_{jt}}. \end{aligned} \tag{4}$$

Since ε_{ijt} is a random variable, the mean and variance of $\hat{\alpha}$ are

$$\begin{aligned} E(\hat{\alpha}_{ij}) &= \alpha_{ij} + \frac{\beta_{ij}}{x_{jt}} + \frac{1}{x_{jt}} E(\varepsilon) \\ V(\hat{\alpha}_{ij}) &= \frac{1}{x_{jt}^{2}} V(\varepsilon). \end{aligned} \tag{5}$$

$\hat{\alpha}$ is a biased estimator of α, with the bias depending on the ratio β to x. If x is large, this may be negligible. On the other hand, if β is large compared to x, it will result in an overestimate of α. Since error on the side of a large change in resource allocation may be less desirable and more unsettling than an insufficient change, this bias should be reduced or removed if at all possible.

An additional source of bias lies in the term $E(\varepsilon)/x$. If, in fact, β is not a constant but is shifting over time[1], $E(\varepsilon) \neq 0$. Again, if x is large, this may be negligible.

First Differences

One possible approach to removing the bias due to β is to base the model on first differences. Let $\omega_{jt} = x_{jt} - x_{j,t-1}$. The change in input from i to j is then

$$\begin{aligned}(\beta_{ij} + \alpha_{ij}x_{jt} + \varepsilon_{ijt}) - (\beta_{ij} + \alpha_{ij}x_{j,t-1} + \varepsilon_{ij,t-1}) \\ = \alpha_{ij}\omega_{jt} + (\varepsilon_{ijt} - \varepsilon_{ij,t-1}).\end{aligned} \tag{6}$$

If α is again estimated by a simple ratio, then

$$\begin{aligned}\hat{\alpha}_{ij} &= \frac{\alpha_{ij}\omega_{jt} + (\varepsilon_{ijt} - \varepsilon_{ij,t-1})}{\omega_{jt}} \\ &= \alpha_{ij} + \frac{1}{\omega_{jt}}(\varepsilon_{ijt} - \varepsilon_{ij,t-1}).\end{aligned} \tag{7}$$

While this removes the bias due to β, it does so at the expense of greatly increased instability in the estimator $\hat{\alpha}$. The mean and variance of $\hat{\alpha}$ are now

$$\begin{aligned}E(\hat{\alpha}_{ij}) &= \alpha_{ij} + \frac{1}{\omega_{jt}}[E(\varepsilon_{ijt}) - E(\varepsilon_{ij,t-1})] \text{ and} \\ V(\hat{\alpha}_{ij}) &= \frac{1}{\omega_{jt}^2}[V(\varepsilon_{ijt}) + V(\varepsilon_{ij,t-1}) - 2 \text{ cov } (\varepsilon_{ijt}\varepsilon_{ij,t-1})].\end{aligned} \tag{8}$$

Assume that $V(\varepsilon) = \sigma^2$, a constant over time, then

$$V(\hat{\alpha}_{ij}) = \frac{1}{\omega jt^2}[2\sigma^2(1-\rho)], \tag{9}$$

---

[1] There is some empirical evidence that this may be true in Navy manpower planning (Sorensen, S. W., & Willis, R. E. Aggregate Models for Manpower Planning (Working Paper)). San Diego: Navy Personnel Research and Development Center, 1976.

where ρ is the correlation coefficient between ε_t and ε_{t-1}. Depending on ρ, which is between -1 and +1,

$$0 \leq V(\hat{\alpha}_{ij}) \leq 4\sigma^2/\omega_{jt}^{\;2}. \tag{10}$$

More important, the magnitude of ω_{jt} will be much less than that of x_{jt}, which, potentially, can greatly increase the variance of the estimator and thus its instability. Empirical analysis (Sorensen & Willis, 1976) suggests that this instability can be a major problem.

Regression and Averaging

When working from a limited data base, the estimator given by equation (4) may be the most conservative despite the potential bias. On the other hand, an I/O model of Navy activities can use a regularly updated data base so that the error variance can be reduced by appropriate averaging or smoothing techniques. One such approach would be the use of simple regression procedures to estimate both α and β. Alternatively, the historical time series data could be examined for other, possibly more complex patterns.

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