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ON THE INTERPRETATION OF SHADOW-MOIRE FRINGES,(U)

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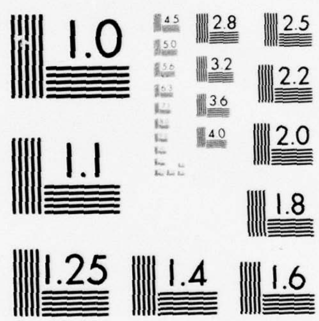
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ON THE INTERPRETATION OF SHADOW-MOIRÉ FRINGES

BY

J. BUITRAGO AND A. J. DURELLI

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ON

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REPORT NO. 42

SCHOOL OF ENGINEERING
OAKLAND UNIVERSITY
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ABSTRACT

Shadow-moiré is presented as one of several means of determining loci of constant height in curved surfaces (isotathmics). A general expression to interpret the fringes is derived taking into consideration possible rotations and translation of the grating. A discussion of the influence of such rotations and translation on the sensitivity of the response is also presented. Furthermore, simplified equations are obtained for particular cases of practical interest. An example of application to the case of an inverted perforated tube is shown.

Previous Technical Reports to the Office of Naval Research

1. A. J. Durelli, "Development of Experimental Stress Analysis Methods to Determine Stresses and Strains in Solid Propellant Grains"--June 1962. Developments in the manufacturing of grain-propellant models are reported. Two methods are given: a) cementing routed layers and b) casting.

2. A. J. Durelli and V. J. Parks, "New Method to Determine Restrained Shrinkage Stresses in Propellant Grain Models"--October 1962. The birefringence exhibited in the curing process of a partially restrained polyurethane rubber is used to determine the stress associated with restrained shrinkage in models of solid propellant grains partially bonded to the case.

3. A. J. Durelli, "Recent Advances in the Application of Photoelasticity in the Missile Industry"--October 1962. Two- and three-dimensional photoelastic analysis of grains loaded by pressure and by temperature are presented. Some applications to the optimization of fillet contours and to the redesign of case joints are also included.

4. A. J. Durelli and V. J. Parks, "Experimental Solution of Some Mixed Boundary Value Problems"--April 1964. Means of applying known displacements and known stresses to the boundaries of models used in experimental stress analysis are given. The application of some of these methods to the analysis of stresses in the field of solid propellant grains is illustrated. The presence of the "pinching effect" is discussed.

5. A. J. Durelli, "Brief Review of the State of the Art and Expected Advance in Experimental Stress and Strain Analysis of Solid Propellant Grains"--April 1964. A brief review is made of the state of the experimental stress and strain analysis of solid propellant grains. A discussion of the prospects for the next fifteen years is added.

6. A. J. Durelli, "Experimental Strain and Stress Analysis of Solid Propellant Rocket Motors"--March 1965. A review is made of the experimental methods used to strain-analyze solid propellant rocket motor shells and grains when subjected to different loading conditions. Methods directed at the determination of strains in actual rockets are included.

7. L. Ferrer, V. J. Parks and A. J. Durelli, "An Experimental Method to Analyze Gravitational Stresses in Two-Dimensional Problems"--October 1965. Photoelasticity and moiré methods are used to solve two-dimensional problems in which gravity-stresses are present.

8. A. J. Durelli, V. J. Parks and C. J. del Rio, "Stresses in a Square Slab Bonded on One Face to a Rigid Plate and Shrunk"--November 1965.
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9. A. J. Durelli, V. J. Parks and C. J. del Rio, "Experimental Determination of Stresses and Displacements in Thick-Wall Cylinders of Complicated Shape"--April 1966.
Photoelasticity and moiré are used to analyze a three-dimensional rocket shape with a star shaped core subjected to internal pressure.

10. V. J. Parks, A. J. Durelli and L. Ferrer, "Gravitational Stresses Determined Using Immersion Techniques"--July 1966.
The methods presented in Technical Report No. 7 above are extended to three-dimensions. Immersion is used to increase response.

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12. A. J. Durelli, V. J. Parks, H. C. Feng and F. Chiang, "Strains and Stresses in Matrices with Inserts,"-- May 1967.
Stresses and strains along the interfaces, and near the fiber ends, for different fiber end configurations, are studied in detail.

13. A. J. Durelli, V. J. Parks and S. Uribe, "Optimization of a Slot End Configuration in a Finite Plate Subjected to Uniformly Distributed Load,"--June 1967.
Two-dimensional photoelasticity was used to study various elliptical ends to a slot, and determine which would give the lowest stress concentration for a load normal to the slot length.

14. A. J. Durelli, V. J. Parks and Han-Chow Lee, "Stresses in a Split Cylinder Bonded to a Case and Subjected to Restrained Shrinkage,"--January 1968.
A three-dimensional photoelastic study that describes a method and shows results for the stresses on the free boundaries and at the bonded interface of a solid propellant rocket.

15. A. J. Durelli, "Experimental Stress Analysis Activities in Selected European Laboratories"--August 1968.
This report has been written following a trip conducted by the author through several European countries. A list is given of many of the laboratories doing important experimental stress analysis work and of the people interested in this kind of work. An attempt has been made to abstract the main characteristics of the methods used in some of the countries visited.

16. V. J. Parks, A. J. Durelli and L. Ferrer, "Constant Acceleration Stresses in a Composite Body"--October 1968.
Use of the immersion analogy to determine gravitational stresses in two-dimensional bodies made of materials with different properties.

17. A. J. Durelli, J. A. Clark and A. Kochev, "Experimental Analysis of High Frequency Stress Waves in a Ring"--October 1968.
A method for the complete experimental determination of dynamic stress distributions in a ring is demonstrated. Photoelastic data is supplemented by measurements with a capacitance gage used as a dynamic lateral extensometer.

18. J. A. Clark and A. J. Durelli, "A Modified Method of Holographic Interferometry for Static and Dynamic Photoelasticity"--April 1968.
A simplified absolute retardation approach to photoelastic analysis is described. Dynamic isopachics are presented.

19. J. A. Clark and A. J. Durelli, "Photoelastic Analysis of Flexural Waves in a Bar"--May 1969.
A complete direct, full-field optical determination of dynamic stress distribution is illustrated. The method is applied to the study of flexural waves propagating in a urethane rubber bar. Results are compared with approximate theories of flexural waves.

20. J. A. Clark and A. J. Durelli, "Optical Analysis of Vibrations in Continuous Media"--June 1969.
Optical methods of vibration analysis are described which are independent of assumptions associated with theories of wave propagation. Methods are illustrated with studies of transverse waves in prestressed bars, snap loading of bars and motion of a fluid surrounding a vibrating bar.

21. V. J. Parks, A. J. Durelli, K. Chandrashekhara and T. L. Chen, "Stress Distribution Around a Circular Bar, with Flat and Spherical Ends, Embedded in a Matrix in a Triaxial Stress Field"--July 1969.
A Three-dimensional photoelastic method to determine stresses in composite materials is applied to this basic shape. The analyses of models with different loads are combined to obtain stresses for the triaxial cases.

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The method described in Report No. 10 above is applied to two specific problems. An approach is suggested to extend the solutions to a class of surface traction problems.

23. J. A. Clark and A. J. Durelli, "Separation of Additive and Subtractive Moiré Patterns"--December 1969.
A spatial filtering technique for adding and subtracting images of several gratings is described and employed to determine the whole field of Cartesian shears and rigid rotations.

24. R. J. Sanford and A. J. Durelli, "Interpretation of Fringes in Stress-Holo-Interferometry"--July 1970.
Errors associated with interpreting stress-holo-interferometry patterns as the superposition of isopachics (with half order fringe shifts) and isochromatics are analyzed theoretically and illustrated with computer generated holographic interference patterns.

25. J. A. Clark, A. J. Durelli and P. A. Laura, "On the Effect of Initial Stress on the Propagation of Flexural Waves in Elastic Rectangular Bars"--December 1970.
Experimental analysis of the propagation of flexural waves in prismatic, elastic bars with and without prestressing. The effects of prestressing by axial tension, axial compression and pure bending are illustrated.

26. A. J. Durelli and J. A. Clark, "Experimental Analysis of Stresses in a Buoy-Cable System Using a Birefringent Fluid"--February 1971.
An extension of the method of photoviscous analysis is presented which permits quantitative studies of strains associated with steady state vibrations of immersed structures. The method is applied in an investigation of one form of behavior of buoy-cable systems loaded by the action of surface waves.

27. A. J. Durelli and T. L. Chen, "Displacements and Finite-Strain Fields in a Sphere Subjected to Large Deformations"--February 1972.
Displacements and strains (ranging from 0.001 to 0.50) are determined in a polyurethane sphere subjected to several levels of diametral compression. A 500 lines-per-inch grating was embedded in a meridian plane of the sphere and moiré effect produced with a non-deformed master. The maximum applied vertical displacement reduced the diameter of the sphere by 27 per cent.

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A transparent material with variable modulus of elasticity has been manufactured that exhibits good photoelastic properties and can also be strain analyzed by moiré. The results obtained suggests that the stress distribution in the homogeneous disk. It also indicates that the strain fields in both cases are very different, but that it is possible, approximately, to obtain the stress field from the strain field using the value of E at every point, and Hooke's law.

29. A. J. Durelli and J. Buitrago, "State of Stress and Strain in A Rectangular Belt Pulled Over a Cylindrical Pulley"--June 1972.
Two- and three-dimensional photoelasticity as well as electrical strain gages, dial gages and micrometers are used to determine the stress distribution in a belt-pulley system. Contact and tangential stress for various contact angles and friction coefficients are given.

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Strain fields obtained in a sphere subjected to large diametral compressions from a previous paper were converted into stress fields using two approaches. First, the concept of strain-energy function for an isotropic elastic body was used. Then the stress field was determined with the Hookean type natural stress-natural strain relation. The results so obtained were also compared.

31. A. J. Durelli, V. J. Parks and H. M. Hasseem, "Helices Under Load"--July 1973.
Previous solutions for the case of close coiled helical springs and for helices made of thin bars are extended. The complete solution is presented in graphs for the use of designers. The theoretical development is correlated with experiments.

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The same methods described in No. 27, were applied to a hollow sphere with an inner diameter one half the outer diameter. The hollow sphere was loaded up to a strain of 30 per cent on the meridian plane and a reduction of the diameter by 20 per cent.

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34. J. Wolak and V. J. Parks, "Evaluation of Large Strains in Industrial Applications"--April 1974.
It was shown that Mohr's circle permits the transformation of strain from one axis of reference to another, irrespective of the magnitude of the strain, and leads to the evaluation of the principal strain components from the measurement of direct strain in three directions.

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Continuation of Report No. 15 after a visit to Belgium, Holland, Germany, France, Turkey, England and Scotland.

36. A. J. Durelli, V. J. Parks and J. O. Bühler-Vidal, "Linear and Non-linear Elastic and Plastic Strains in a Plate with a Big Hole Loaded Axially in its Plane"--July 1975.
Strain analysis of the ligament of a plate with a big hole indicates that both geometric and material non-linearity may take place. The strain concentration factor was found to vary from 1 to 2 depending on the level of deformation.

37. A. J. Durelli, V. Pavlin, J. O. Bühler-Vidal and G. Ome, "Elastostatics of a Cubic Box Subjected to Concentrated Loads"--August 1975.
Analysis of experimental strain, stress and deflection of a cubic box subjected to concentrated loads applied at the center of two opposite faces. The ratio between the inside span and the wall thickness was varied between approximately 5 and 121.

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Experimental analysis of strain, stress and deflections in a cubic box subjected to either internal or external pressure. Inside span-to-wall thickness ratio varied from 5 to 14.

39. Y. Y. Hung, J. D. Hovanesian and A. J. Durelli, "New Optical Method to Determine Vibration-Induced Strains with Variable Sensitivity After Recording"--November 1976.
A steady state vibrating object is illuminated with coherent light and its image slightly misfocused. The resulting specklegram is "time-integrated" as when Fourier filtered gives derivatives of the vibrational amplitude.

40. Y. Y. Hung, C. Y. Liang, J. D. Hovanesian and A. J. Durelli, "Cyclic Stress Studies by Time-Averaged Photoelasticity"--November 1976.
"Time-averaged isochromatics" are formed when the photographic film is exposed for more than one period. Fringes represent amplitudes of the oscillating stress according to the zeroth order Bessel function.

41. Y. Y. Hung, C. Y. Liang, J. D. Hovanesian and A. J. Durelli, "Time-Averaged Shadow Moiré Method for Studying Vibrations"--November 1976.
Time-averaged shadow moiré permits the determination of the amplitude distribution of the deflection of a steady vibrating plate.

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Introduction

The solution of the problem of determining topographical contours of an arbitrary surface can be approached using optical methods. Different techniques can be used depending on the actual depth of the surface, its size and desired sensitivity. Newton rings have a sensitivity of the order on one tenth of the wave length used, but only for a very small range of depth. Holographic interferometry is useful when the depths to record are also very small, is subjected to strict requirements of stability of the instrumentation and is usually expensive.

The simplest method seems to be the projection-grating moiré whereby one projects a grating onto the surface of a plate before deformation and then again after deformation[1][2][3]. The superposition of the deformed and undeformed grating images, when the projection is a parallel one, yields moiré fringes of equal deflection (isothetics w). The method can be applied to surfaces which are originally curved and may then give the isotathmics or loci of equal depth (or height) with respect to a plane of reference. Projection moiré is suitable for relatively large deflections (0.1 mm and up).

Shadow-moiré fringes are produced when the master placed in front of the model interferes with its shadow on the surface of the model. Only one grating is required instead of the two used with the projection grating method and the surface of the model should be matte in order to receive the shadow of the projected grating.

Shadow-moiré, like the projection-grating method, does not require coherent light and is also of simple implementation. The shadow-moiré patterns may also be isothetics w, (for the case of bent plates for instance) or isostathmics when the observed surface is not originally plane, the fringe pattern giving then the map of the height of points of the surface with respect to the reference plane.

Weller and Shepard[4] were the first to suggest the use of shadow-moiré to determine loci of depth on surfaces. Later Theocaris[5] used the method to measure deflections of flexed plates. Theocaris[6] also generated isopachics by shadow-moiré that in combination with isochromatics were used to separate principal stresses. Successful applications of shadow-moiré techniques to the determination of living bodies topography have been reported by Takasaki[7,8]. Recently, Chiang[9] proposed the uses of a composite grating with two different pitches and Marasco[10] obtained displacements in a hyperboloid-parabolic shell with a non-plane grating.

Interpretation of Shadow-Moiré Fringes

Shadow-moiré fringes can be mathematically interpreted by using communication theory[11,12] although geometric optics suffices to completely analyze the phenomenon.

A geometric approach to the interpretation of shadow-moiré fringes has been published by Pirodda[13]. However, the author did not consider the effect of rotations of the grating. In what follows a completely general analysis of the shadow-moiré fringes is given including consideration of positions of the grating that require rotations and translation from the normal position defined as the position at which the grating is in contact with the body and is normal to axis of the camera. The influence of these motions on the sensitivity of the system is also considered.

General Equation: Defining the 3-D space by a Cartesian system of coordinates x, y and z, consider any plane x-y where a grating of pitch p has been translated from the model a distance c and then rotated an angle θ, (Fig. 1). The fringe order at an arbitrary point P(x) can be expressed as:

$$n = \frac{(w+c+x\tan\theta_1)}{p}\left\{\frac{(d_z+d_1)\tan\alpha-x}{d_z+d_1+w+c}\cos\theta_1\right.$$

$$+\frac{x}{d_z+w+c}\cos\theta_1+\sin\theta_1\left[\frac{(d_z+d_1)\tan\alpha-x}{d_z+d_1+w+c}\tan\left(\theta_1-\tan^{-1}\frac{x}{d_z+w+c}\right)\right.$$

$$\left.\left.+\frac{x}{d_z+w+c}\tan\left(\tan^{-1}\frac{(d_z+d_1)\tan\alpha-x}{d_z+d_1+w+c}+\theta_1\right)\right]\right\} \tag{1}$$

If the grating is further rotated in the plane y-z (Fig. 2) by an angle θ_2 and in the plane z-x by an angle θ_3 (Fig. 3) Eq. (1) can be made to represent the most general expression for the fringe order at a point when $p/\cos\theta_3$ is substituted for p and $d-(h-z)\sin\theta_2$ for d_z where h is the total height of the model. Equation (1) can also be written in terms of the total distance from the grating to the model by letting $w_t = w + c + x\tan\theta_1$. A detailed derivation of Eq. (1) is presented in the Appendix.

An explicit solution of Eq. (1) for either w or w_t does not seem easy to obtain. However, it is possible to solve it numerically by redefining it in terms of functions of w or w_t and combining them into the form $F_1(w,n) = F_2(w)$ that can be solved by a trial and error algorithm.

Influence of Rotations and Translation on the Sensitivity: In practical cases it is convenient to position the camera and light source sufficiently far from the model so that d_z is very large compared to $(w + c)$ and to x and consequently $\gamma \simeq \alpha$ and $\phi \simeq 0$. Under these conditions it is possible to explicitly show the influence of the rotations as well as the translation of the grating plane on the sensitivity of the response of the system. The sensitivity can be quantified as the ratio of the fringe order at a point to its actual distance to the grating (Fig. 1).

a) Rotatión about the z axis: Letting $\theta_2 = \theta_3 = 0$ Eq. (1) reduces to

$$n = \frac{1}{p}(w + c + x \tan \theta_1)(\tan\alpha \cos\theta_1 + \sin\theta_1 \tan\alpha \tan\theta_1)$$

therefore the sensitivity

$$s_z = n/w_t = \frac{1}{p} \tan \alpha \sec \theta_1 \tag{2}$$

By examining Eq. (2), it is seen that for a given depth and pitch the sensitivity varies as the secant of the rotation θ_1. From 0 to 20 degrees, the sensitivity increases only 6% but reaches 41% for θ_1 equal to 45°. However, large values of θ_1 bring the grating plane further apart from the model producing the fading of the fringes. The increase in sensitivity has to be compromised with the quality of the fringes.

b) Rotation about x axis: Letting $\theta_1 = \theta_2 = 0$, Eq. (1) becomes

$$n/w_t = \frac{1}{p} \frac{(d_z+d_1)\tan\alpha - x}{d_z+d_1+w+c} = \frac{1}{p}\tan\alpha \tag{3}$$

Since $\tan\alpha = f/(d_z + d_1)$ and $d_z = d-(h-z)\sin\theta_2$ the sensitivity is given by

$$s_x = \frac{1}{p} \frac{f}{d-(h-z)\sin\theta_2} \tag{4}$$

If $d \gg (h-z)$ the change in sensitivity obtained by rotating the grating plane about the x axis is neglible.

c) Rotation about y axis: Letting $\theta_1 = \theta_2 = 0$ in Eq. (1) gives

$$s_y = \frac{1}{p} \cos\theta_3 \tan\alpha \tag{5}$$

In this instance the sensitivity is proportional to the cosine of the angle of rotation. As θ_3 approaches 90° the pitch becomes very large ($p' = p/\cos\theta_3$) and the fringes fade. The sensitivity decreases as θ_3 increases.

d) Rigid translation in the x-y plane: In this case $\theta_1 = \theta_2 = \theta_3 = 0$ and the sensitivity is expressed by

$$s_t = \frac{1}{p}\tan\alpha \tag{6}$$

Translation of the grating plane does not bring about any change in the sensitivity but as in case a) will tend to produce weaker fringes. Translation of the grating plane is of no practical interest.

Simplified Equations: Three cases of practical importance, determined by the number of elements used in the system will be considered. The grating will touch the body and its plane will be perpendicular to the axis of the camera.

a) Point source and camera: In Eq. (1) let $\theta_1 = \theta_2 = \theta_3 = c=0$ (Fig. 4) The distance d_z becomes constant and the fringe order at representative points 1 and 2 is given by

$$n_{1,2} = \frac{w}{p}\left[\frac{(d+d_1)\tan\alpha - x}{d + d_1 + w} + \frac{x}{d + w}\right] \tag{7}$$

It can be noted from Eq. (7) that if the body has a plane of symmetry coinciding with the y - z plane, the fringe pattern will not be symmetric because of the sign of x. In practice the zero fringe order is arbitrarily assigned to the point of contact between the grating and the surfaces. Knowing the fringe order, the grating pitch and the geometry of the optical system, Eq. (7) can be solved for w as follows

$$w = -\frac{1}{2}k \pm \sqrt{\frac{1}{4}k^2 + \frac{npd}{\mathrm{tg}\alpha}} \tag{8}$$

where

$$k = \frac{d[(d+d_1)\ \mathrm{tg}\alpha - x - np] - np(d+d_1) + x(d+d_1)}{(d+d_1)\ \mathrm{tg}\alpha - np}$$

The + or - sign in Eq. (8) is selected to obtain the desired sign for w. Eq. (8) is valid for any z.

A case of practical importance occurs when the source and the camera focusing plane lay on a plane that is parallel to the grating plane. Letting $d_1 = 0$ in Eq. (7) the fringe order is given by

$$n_{1,2} = \frac{w}{p}\left[\frac{d\,\mathrm{tg}\alpha - x}{d+w} + \frac{x}{d+w}\right] = \frac{d\,\mathrm{tg}\alpha}{d+w}\ \frac{w}{p} \tag{9}$$

It can be observed from this expression that the fringe pattern will be symmetric with respect to x if the body is symmetric with respect to z. Substituting now $\tan\alpha$ by f/d, the displacement w is given by

$$w = \frac{npd}{f-np} \tag{10}$$

b) Collimated incident light and camera: If in the previous set-up a collimator is added between the source and the grating Fig. (5) the object receives incident parallel rays of light. This is equivalent to locating the source at an infinite distance from the object. Taking the limit of Eq. (7), as d_1 approaches infinity, the fringe order is given by

$$n_{1,2} = \frac{w}{p}\left[tg\alpha + \frac{x}{d+w}\right] \tag{11}$$

Again the pattern corresponding to a symmetric body will not be symmetric because of the sign of x. Solving for w in Eq. (11)

$$w = -\frac{1}{2}k_1 - \sqrt{\frac{1}{4}k_1^2 + \frac{npd}{tg\alpha}} \tag{12}$$

where

$$k_1 = \frac{d - np + x}{tg\alpha}$$

c) Collimated incident and reflected light: This condition can be obtained by inserting another collimator, this one between the grating and the camera as depicted in Fig. (6). In this instance the fringe order can be arrived at by letting $d_1 + d$ go to infinity in Eq. (7). Then

$$n_{1,2} = \frac{w}{p} - tg\alpha$$

thus (13)

$$w = \frac{np}{tg\alpha}$$

Experimental Considerations

Set up: Of the simplified cases presented above, the last one seems to be the most attractive because of the simplicity of Eq. (13). Unfortunately, as the bodies grow larger, wider collimated beams are required, which in turn implies larger diameter lenses. When of good quality, these lenses are difficult to obtain at a reasonable cost.

The use of a point source of light and a camera appears to have practical advantages although the analysis of the pattern is more complicated (Eq. 8). However, a simplified expression can be obtained when the source and camera are on a plane parallel to the grating (Eq. 10). This three element system (source, grating and camera) is frequently the most advantageous.

Sensitivity: The sensitivity of the method is a direct function of the pitch of the grating and the angle of observation. Very small pitches cannot be used because the very fine gratings diffract the light. The The appropriate pitch and angle of observation have to be selected for the particular range of heights to be measured.

Fringe Definition: When coarse gratings are used, it is advantageous to have a thin light source parallel to the direction of the grating lines to minimize the penumbra. The thinner the source the sharper the fringes. It should be remembered that shadow-moiré fringes are formed at planes located at different distances from the grating plane and should be photographed using small diaphram apertures. When the bodies to be studied have a plane of symmetry, it is advisable to use two light sources symmetrically located with respect to the camera axis to avoid that the fringes that appear on the side near one source of light present different intensity than the fringes at the opposite side.

Fringe definition may also be lost at areas at which the gradients are steep. In these cases it is recommended to move the grating during the exposure[6,11].

Sources: White light sources of about 500 to 1000 watts are usually required to generate proper intensity. Incandescent filament lamps are most commonly used and the filament is then aligned with the grating lines. To sharpen the fringes illumination can be passed through a slit, but this decreases the intensity.

Application

To experimentally verify some of the previously developed equations, a sphere 6.9 in (175 mm) in diameter was analyzed. For convenience, the selected set-up consisted of a point light source and a camera laying on a plane parallel to the grating. The fringe pattern shown in Fig. 7 is symmetric as predicted by Eq. (9) for symmetric surfaces. Surface depths referred to the grating plane (tangent to the sphere) were computed using Eq. (10) and compared to the values obtained from geometry. Fig. 8 shows this comparison. The agreement is very good.

As a more general application of the shadow-moiré technique, the anticlastic surface of a perforated tube, turned inside out was also studied (Fig. 9). The optical arrangement consisted of collimated incident light and camera. In this instance, even though the surface is symmetric with respect to x, the fringe order is not (Fig. 10) in accordance with Eq. (11), where the alternate sign of x for two points symmetrically located about the vertical axis yields different values of n. To further illustrate this effect, the fringe order n has been represented along three different lines on the surface of the tube. On the vertical axis, where $x = 0$, the fringe distribution exhibits complete symmetry. However, for any other off-the-vertical-axis line asymmetry appears, reaching a maximum along the horizontal axis. The actual depths, on the other hand, have to remain symmetric and can be obtained using Eq. (12).

Acknowledgments

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APPENDIX: General Equation for Interpretation of Shadow-Moiré

Referring to Fig. 1, consider the grating plane that has been translated along the y axis an amount c from the highest point on an arbitrary surface S and subsequently rotated through an angle θ_1, about the z axis (positive rotation according to the vectorial product convention). The fringe order corresponding to the interference of the shadow cast at the point P and the grating itself is simply,

$$n = a'/p \tag{A.1}$$

a' being the distance measured on the grating plane between the incident and reflected rays, and p the pitch of the grating. Since only one grating is used, dark areas of the grating interfere with their own shadows and consequently the integer fringe orders correspond to light fringes while the half orders to dark fringes[14] (Fig. 7)

From geometry it is seen that

$$a' = [(a_1+a_2)\cos\theta_1 + e + g)]/p \tag{A.2}$$

where

$$a_1+a_2 = (w+c+x\ \tan\theta_1)(\tan\gamma + \tan\phi) \tag{A.3}$$

$$e = a_2\ \sin\theta_1\tan(\theta_1-\phi) = (w+c+x\ \tan\theta_1)\tan\gamma\ \sin\theta_1\tan(\theta_1-\phi) \tag{A.4}$$

and

$$g = a_1\ \sin\theta_1\tan(\theta_1+\gamma) = (w+c+x\ \tan\theta_1)\tan\phi\sin\theta_1\tan(\theta_1+\gamma) \tag{A.5}$$

γ and ϕ are the angles the incident and the reflected rays make with the normal to the contacting normal position of the grating plane respectively and w the depth of the point P from that position.

Equations (A.3), (A.4) and (A.5) are now substituted into Eq. (A.2). After collecting terms a' is

$$a' = (w+c+x\ \tan\theta_1)\{(\tan\gamma+\tan\phi)\cos\theta_1+\sin\theta_1[\tan\gamma\tan(\theta_1-\phi)+\tan\phi\tan(\gamma+\theta_1)]\} \tag{A.6}$$

$$\tan\gamma = \frac{f-x}{d_z+d_1+w+c} \tag{A.7}$$

and

$$\tan\phi = \frac{x}{d_z + w + c} \tag{A.8}$$

where d_z is the variable distance between the camera and the grating plane for the different x = constant planes as produced by the rotation of the grating about the x axis. According to Fig. 2, d_z can be expressed as

$$d_z = d - (h-z)\sin\theta_2 \tag{A.9}$$

h representing the total height of the model and d the constant distant between the camera and the original position of the grating. Moreover, the distance f between the camera optical axis and the point light source is

$$f = (d_z + d_1)\tan\alpha \tag{A.10}$$

α is referred to as the illumination angle and d_1 is the distance between the camera and the source measured along the axis of the camera.

Using the expressions for d_z and f in (A.7) and (A.8), the tangents of γ and ϕ are then given by

$$\tan\gamma = \frac{[d-(h-z)\sin\theta_2 + d_1]\tan\alpha}{d-(h-z)\sin\theta_2 + d_1 + w + c} \tag{A.11}$$

$$\tan\phi = \frac{x}{d-(h-z)\sin\theta_2 + w + c} \tag{A.12}$$

Now, eliminating the $\tan\gamma$ and $\tan\phi$ from (A.6) using (A.11) and (A.12), and dividing through by the pitch modified by the grating rotation θ_3 (Fig. 3) about y, the fringe order as determined by Eq. (A.1) finally is obtained as

$$n = \frac{(w+c+x\tan\theta_1)}{p\cos\theta_3}\Big\{\Big[\frac{\langle d-(h-z)\sin\theta_2+d_1\rangle\tan\alpha - x}{d-(h-z)\sin\theta_2+d_1+w+c} + \frac{x}{d-(h-z)\sin\theta_2+w+c}\Big]\cos\theta_1$$

$$+ \sin\theta_1\Big[\frac{\langle d-(h-z)\sin\theta_2+d_1\rangle\tan\alpha - x}{d-(h-z)\sin\theta_2+d_1+w+c}\tan\Big(\theta_1 - \tan^{-1}\frac{x}{d-(h-z)\sin\theta_2+w+c}\Big)$$

$$+ \frac{x}{d-(h-z)\sin\theta_2+w+c}\tan\Big(\tan^{-1}\frac{\langle d-(h-z)\sin\theta_2+d_1\rangle\tan\alpha - x}{d-(h-z)\sin\theta_2+d_1+w+c} + \theta_1\Big)\Big]\Big\} \tag{A.13}$$

It can be concluded, therefore, that the most general interpretation of the shadow moiré fringe order at any point on an arbitrary surface can completely be defined in terms of the geometry of the optical system lay-out and the rotations and translation of the grating plane. Translations perpendicular to the camera axis do not affect the fringe value.

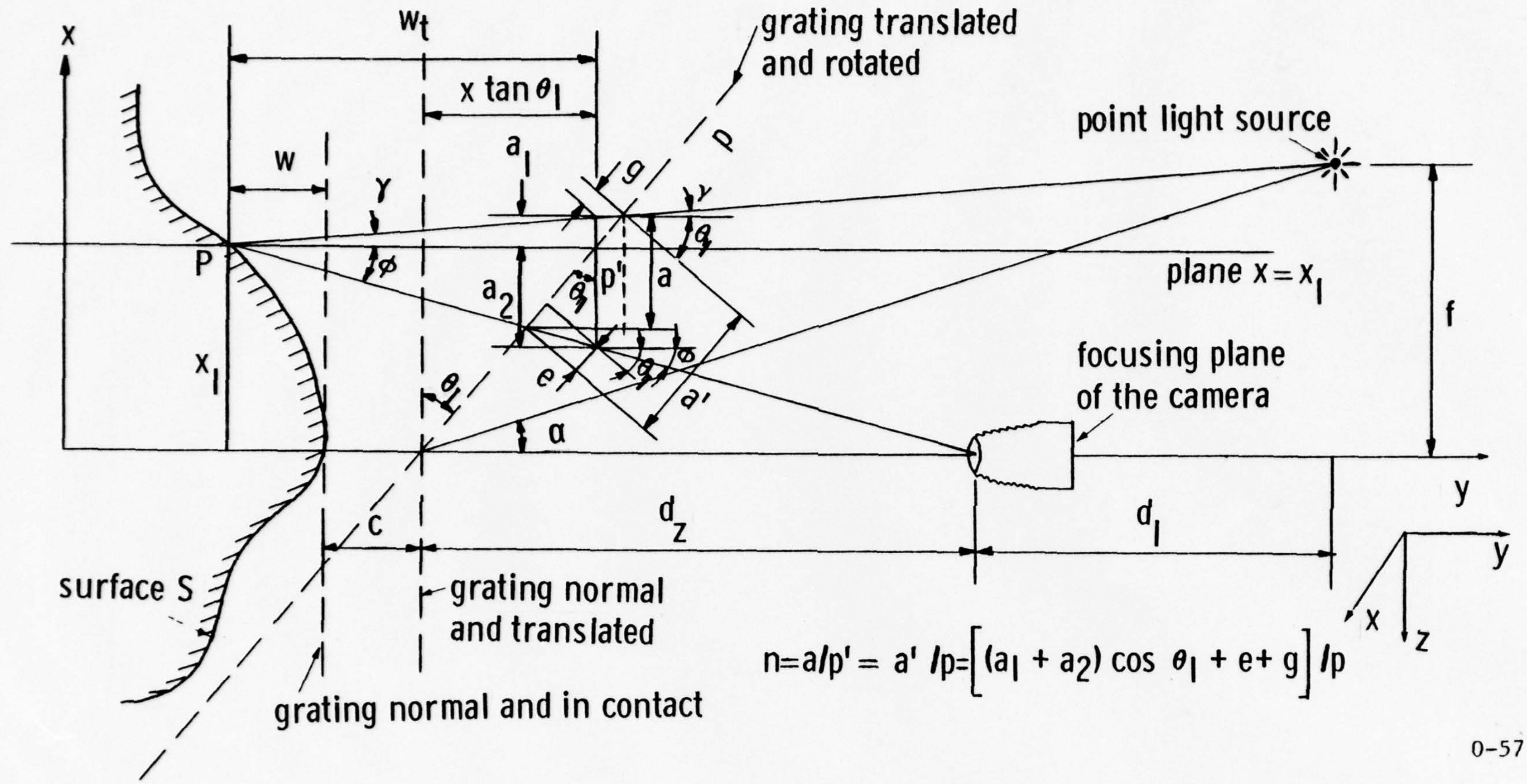

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Fig. 1 Shadow Moire Fringe Interpretation on an Arbitrary Surface Using a Point Source and Camera When the Grating Has Been Rotated and Translated in the x-y Plane

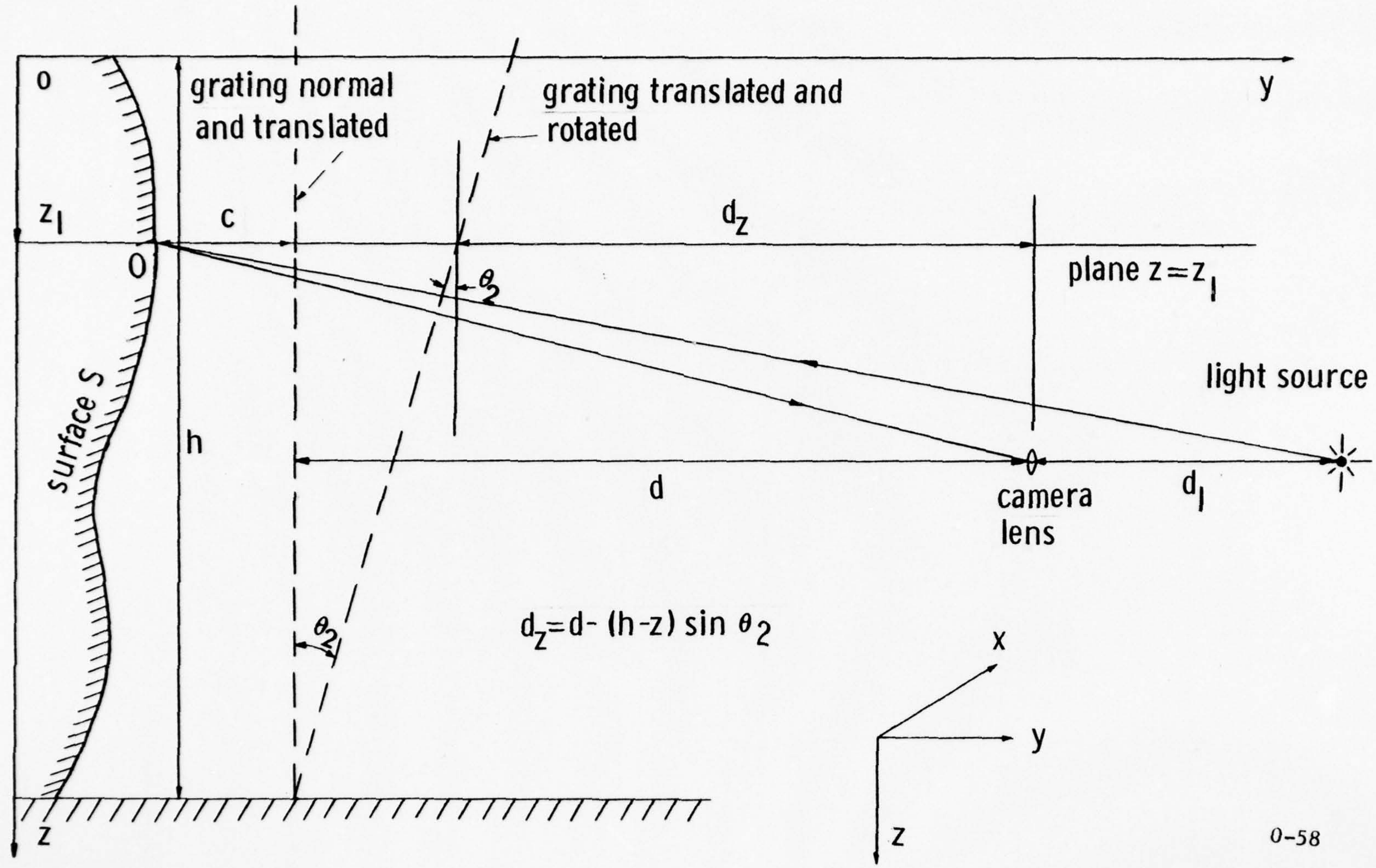

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Fig. 2 Effect of the Rotation of theGrating in the y-z Plane on the Interpretation of the Shadow Moire Fringes Obtained Using a Point Source and Camera, on an Arbitrary Surface

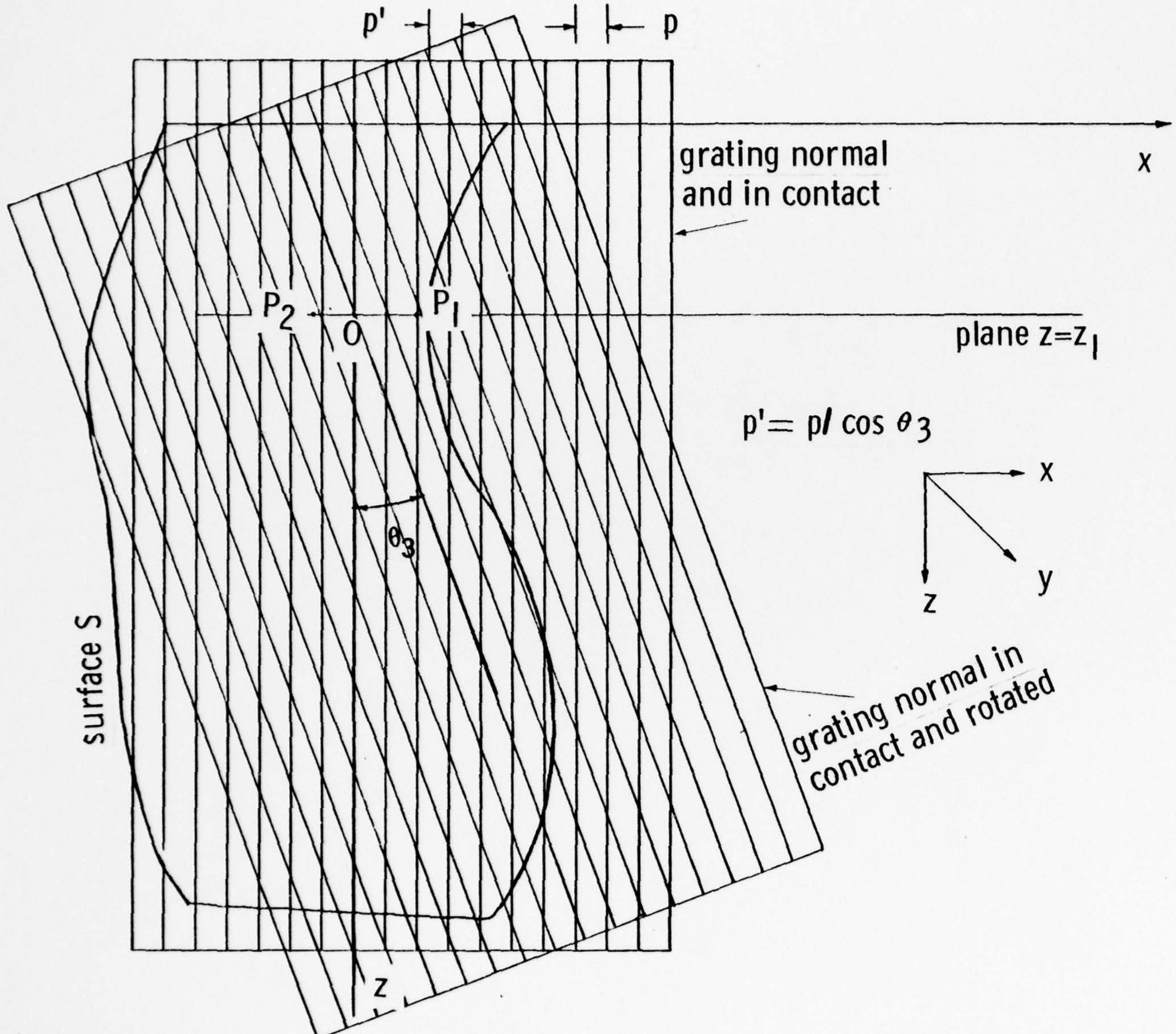

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Fig. 3 Effect of the Rotation of the Grating in the z-x Plane on the Interpretation of the Shadow Moire Fringes Obtained Using a Point Source and Camera, on an Arbitrary Surface

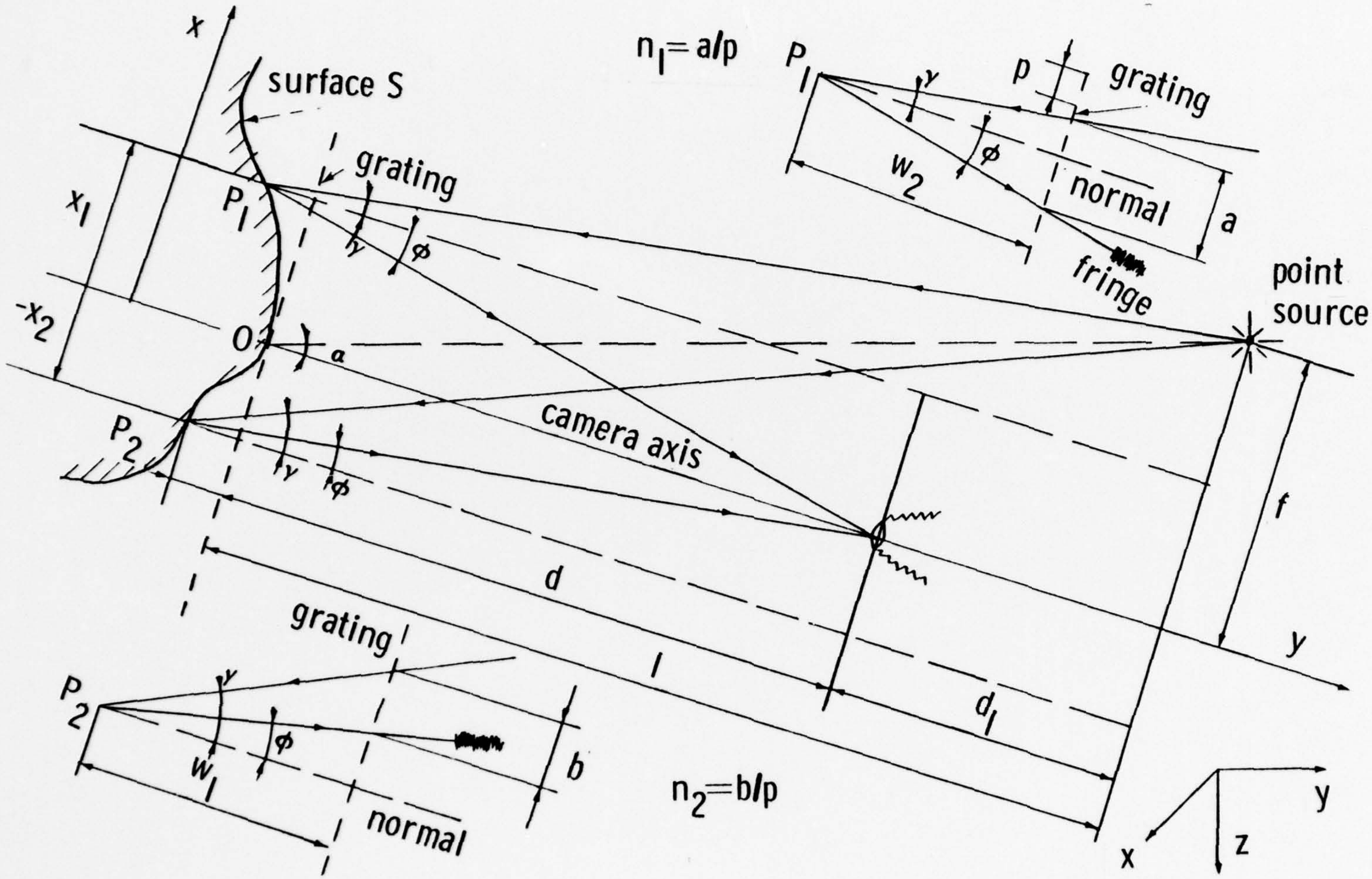

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Fig. 4 Interpretation of the Shadow Moire Fringes Obtained Using A Point Source and Camera, on an Arbitrary Surface

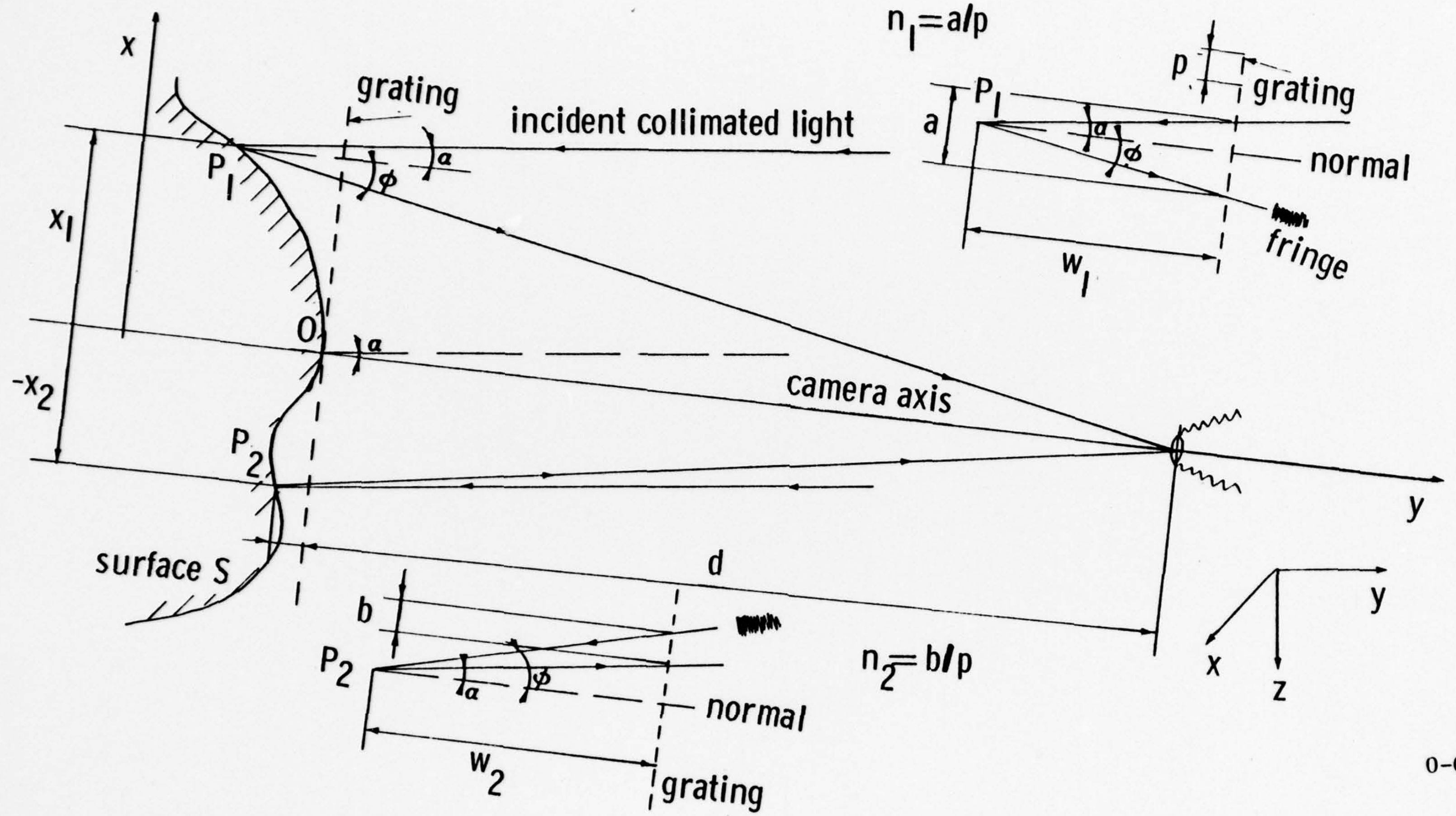

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Fig. 5 Interpretation of the Shadow Moire Fringes Obtained Using Incident Collimated Light and Camera, on an Arbitrary Surface

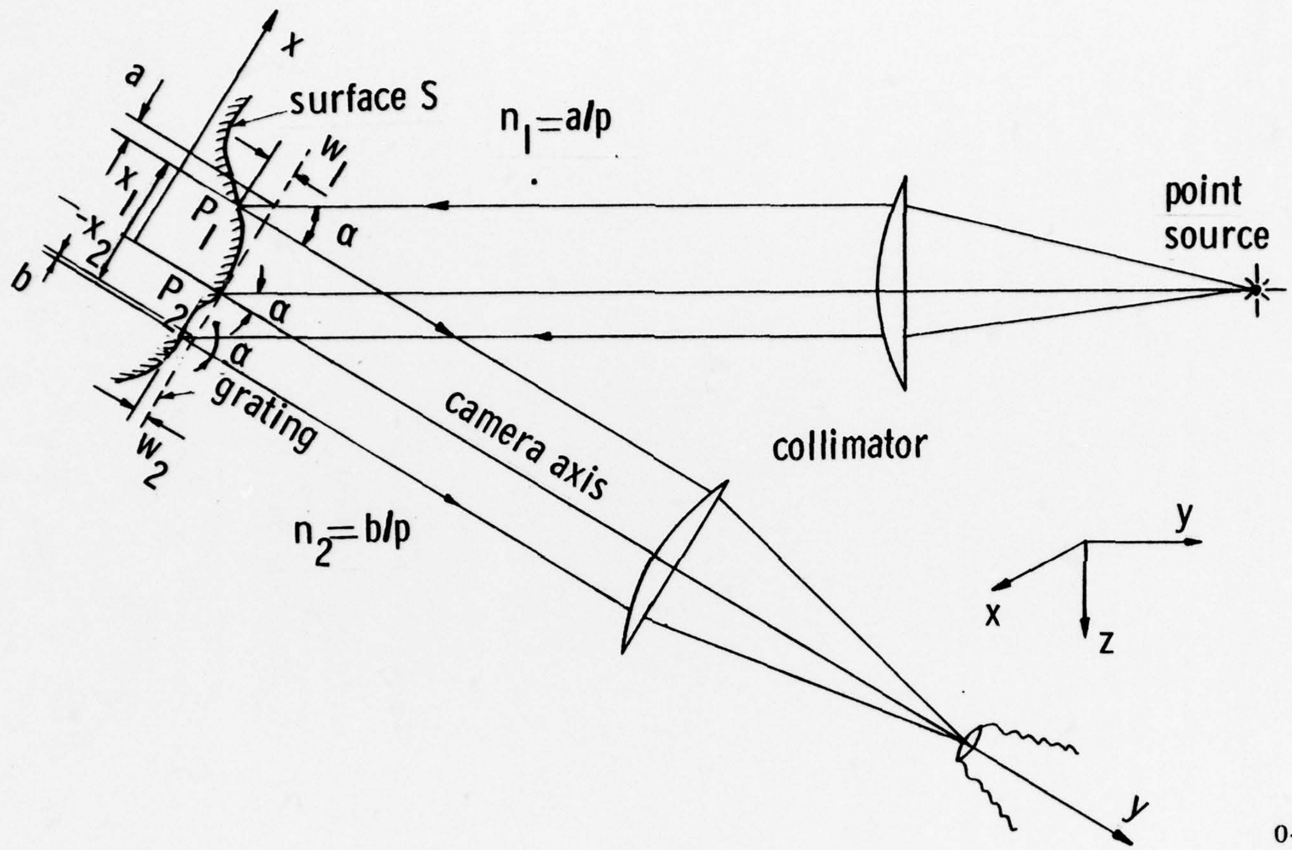

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Fig. 6 Interpretation of the Shadow Moire fringes Obtained Using Collimated Incident and Reflected Light, on an Arbitrary Surface

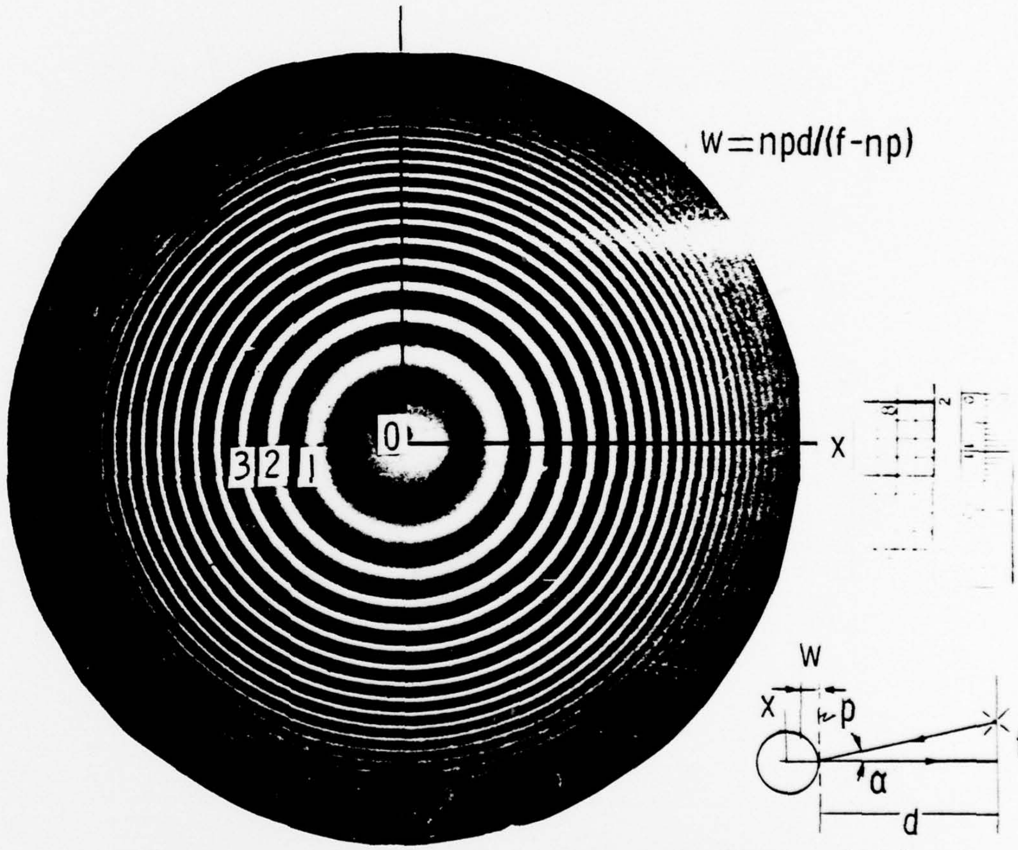

Fig. 7 Isostathmics of a Spherical Surface Obtained Using Shadow-Moire. Camera and Light Source on a Plane Parallel to the Grating

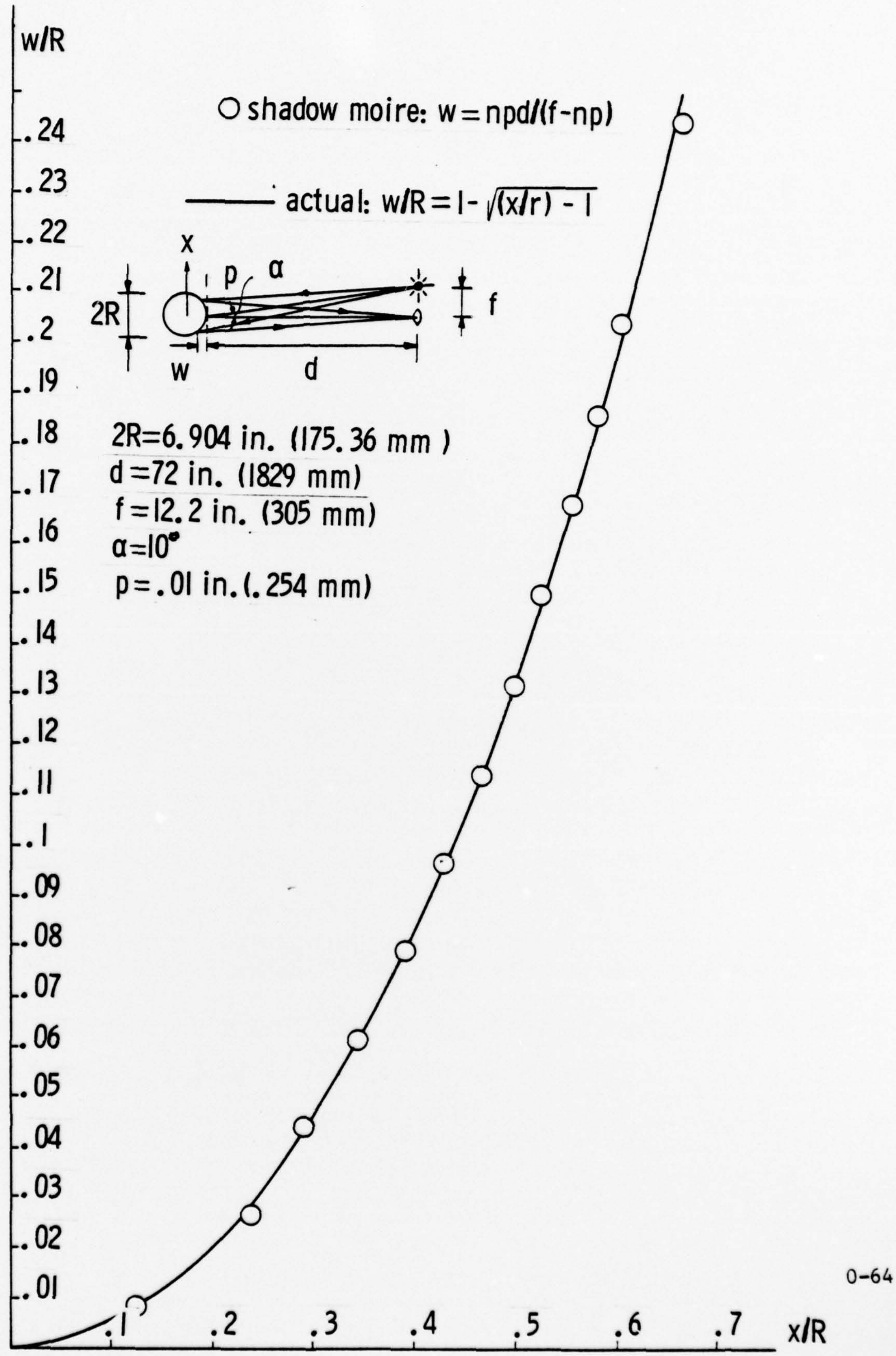

Fig. 8 Depth of a Spherical Surface from the Grating Plane Tangent to the Surface, Obtained Using Shadow Moire Fringes Produced Having the Camera and Light Source on a Plane Parallel to the Grating

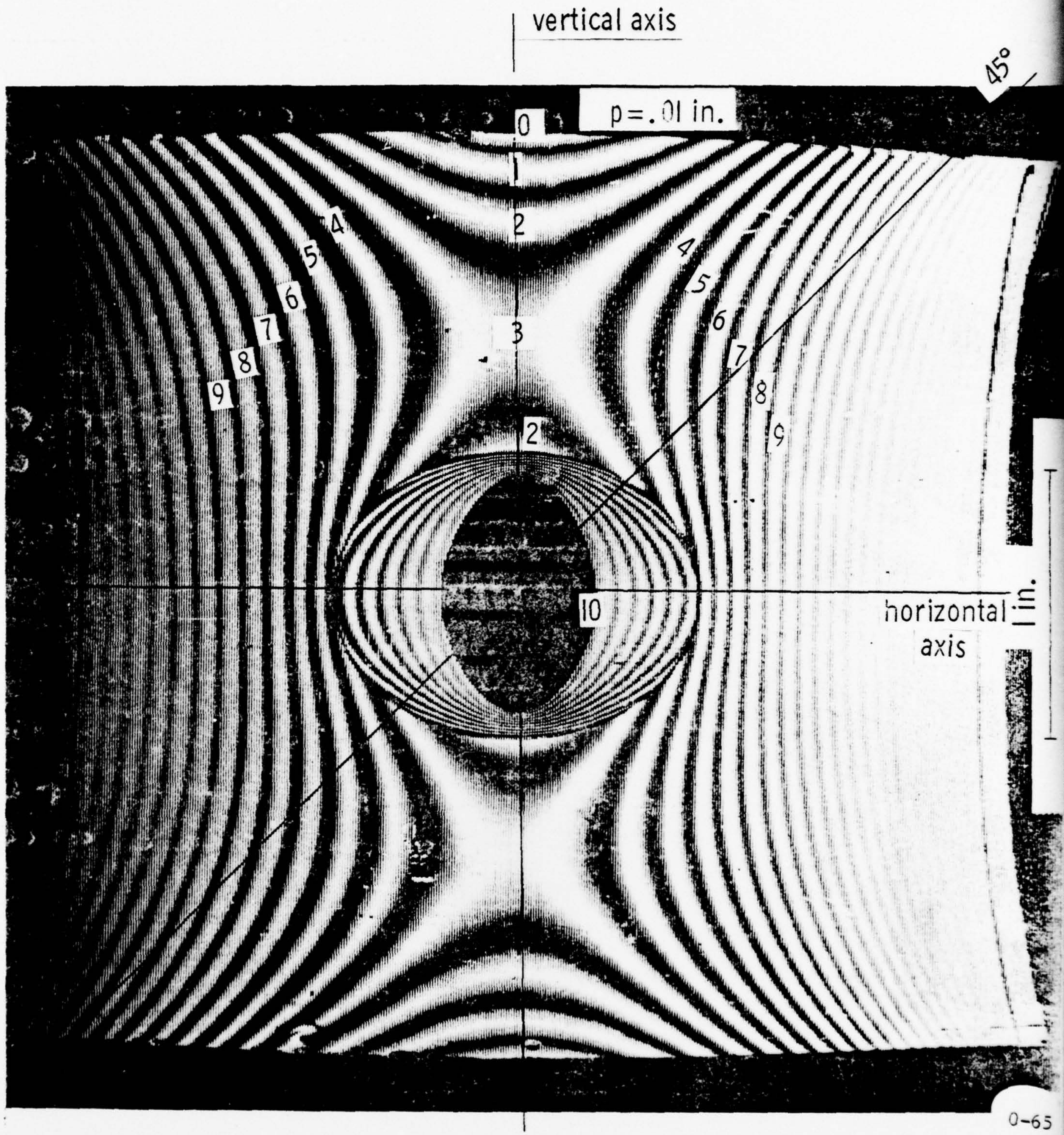

Fig. 9 Loci of Points of Equal Depth on the Outer Surface of a Multi-Perforated Inverted Tube Obtained Using Shadow-Moire. Parallel Incident Light and Camera $n=(w/p)(\tan\alpha + x/(d+w))$

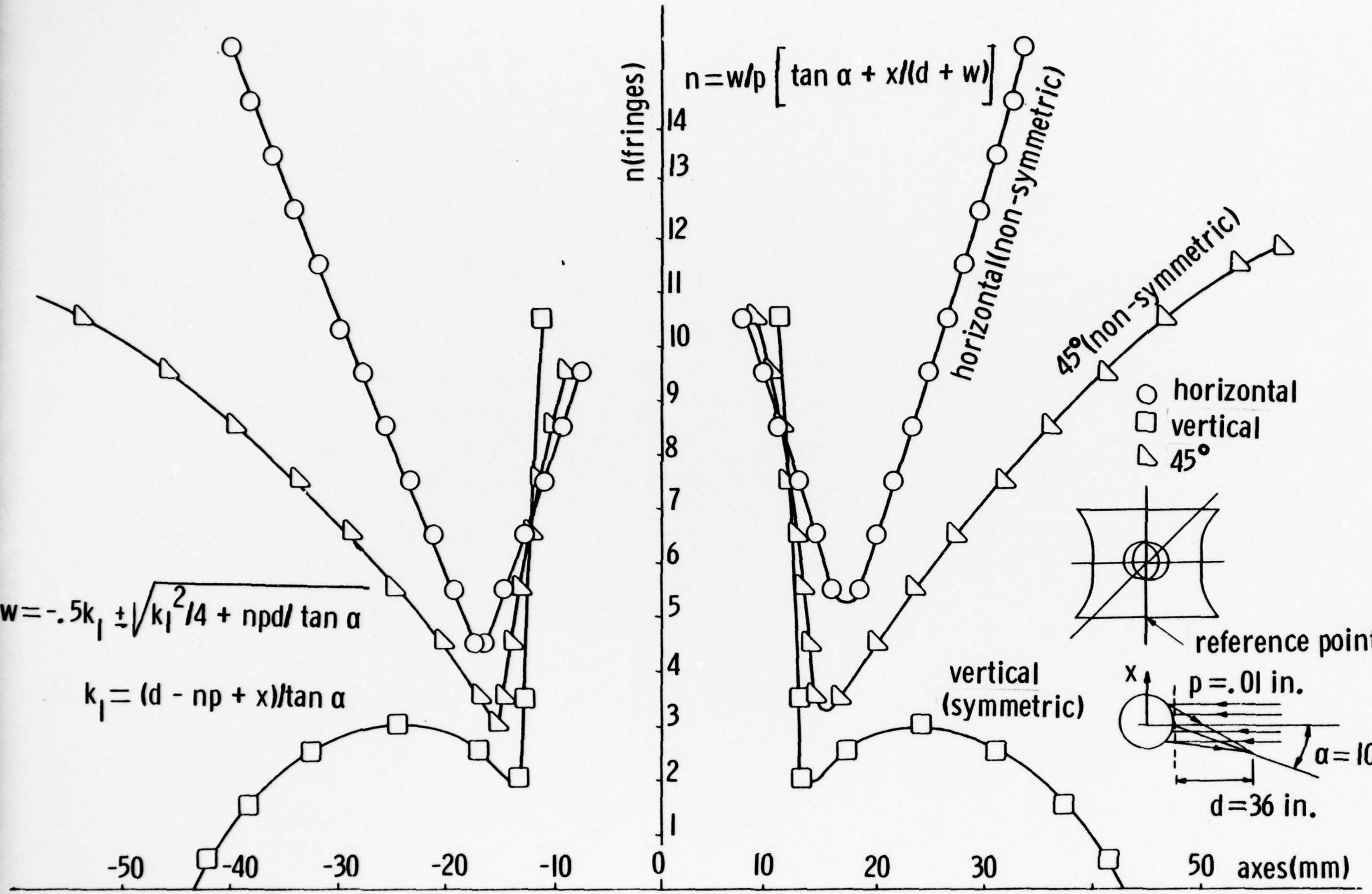

Fig. 10 Shadow Moire Fringe Order on Vertical, Horizontal, and at 45° Lines on the Outer Surface of a Multi-Perforated Inverted Tube

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Shadow-moiré is presented as one of several means of determining loci of constant height in curved surfaces (isotathmics). A general expression to interpret the fringes is derived taking into consideration possible rotations and translation of the grating. A discussion of the influence of such rotations and translation on the sensitivity of the response is also presented. Furthermore, simplified equations are obtained for particular cases of practical interest. An example of application to the

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