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AIRCRAFT HYDRAULIC SYSTEMS DYNAMIC ANALYSIS

VOLUME IV
FREQUENCY RESPONSE (HSFR)

COMPUTER PROGRAM TECHNICAL DESCRIPTION

MCDONNELL AIRCRAFT COMPANY
MCDONNELL DOUGLAS CORPORATION
ST. LOUIS, MISSOURI

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AIR FORCE AERO PROPULSION LABORATORY
AIR FORCE SYSTEMS COMMAND
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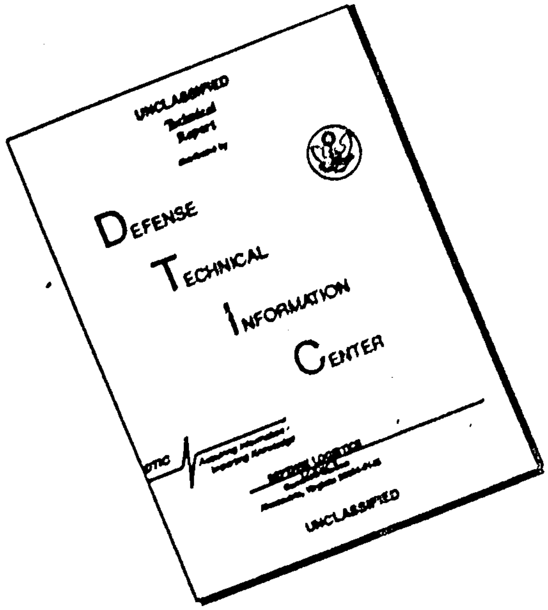

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For a selected system pressure, temperature, flow, and pump speed range, the program calculates the pulsation pressure and energy levels generated by the pump. It predicts the amplitude and location of the resulting

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20. ABSTRACT (Continued)

acoustical standing waves, and how these waves are transmitted and attenuated throughout the hydraulic system. The program may be used for acoustical analysis in the pressure side or both the pressure and return sides of the hydraulic system.

Estimated line lengths and sizes from preliminary design work give a good estimate of hydraulic system natural frequencies and pressure amplitudes. The program outputs plots of the peak flow, pressure, and impedance amplitudes of any selected harmonic of the pulsation noise versus pump speed for selected locations in the system. In addition, the program outputs plots of total acoustic energy density and intensity (power) versus pump speed.

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SUMMARY

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For a selected system pressure, temperature, flow, and pump speed range, the program calculates the pulsation pressure and energy levels generated by the pump. It predicts the amplitude and location of the resulting acoustical standing waves, and how these waves are transmitted and attenuated throughout the hydraulic system. The program may be used for acoustical analysis in the pressure side or both the pressure and return sides of the hydraulic system. Estimated line lengths and sizes from preliminary design work give a good estimate of hydraulic system natural frequencies and pressure amplitudes. The program outputs plots of the peak flow, pressure, and impedance amplitudes of any selected harmonic of the pulsation noise versus pump speed for selected locations in the system. In addition, the program outputs plots of total acoustic energy density and intensity (power) versus pump speed.

1.0 INTRODUCTION

The Hydraulic System Frequency Response (HSFR) program predicts how oscillatory flows and pressures caused by the acoustical energy content of a hydraulic pump output are transmitted through the lines and components of a hydraulic system. Resonance can occur if a frequency of this oscillatory output coincides with a natural frequency of the system, or of part of the system. Patterns of standing waves similar to those observed in organ pipes are generated. A resonant condition can produce large oscillatory pressure and flow amplitudes. Resonant acoustical noise can cause excessive line motion and stresses, resulting in premature failure of system lines or components.

The HSFR program predicts the pump speeds at which major resonances occur, and defines the amplitude and location of the oscillatory pressure and flow standing waves. The description of the system being simulated is easily changed to investigate various practical system modifications for the attenuation and relocation of the major resonant conditions. This capability allows potential problems related to hydraulic acoustic energy to be eliminated during the design stage.

The HSFR program includes subroutines which model lines, a rotating axial piston pump, and other components found in aircraft hydraulic systems. The modular construction of the program allows additional subroutines describing other components to be easily inserted. A special functional subroutine, GRAPH2, is called by the main program for plotting the calculated variables vs. pump speed. The program calculates the required fluid properties at the specified temperature and pressure via a special functional subroutine, FLUID.

A detailed technical description of the HSFR computer program and all subroutines used by the program are included herein. Volume III of this report, under separate cover, contains a user's manual for the HSFR computer program.

The program is written in Fortran IV language for use on the CDC 6600 or IBM 360 computer systems. A punched card deck is used to input the program to the machine while output is by line printer.

2.0 TECHNICAL SUMMARY

The HSFR program, which operates in the frequency domain, examines each element of the system in turn, excluding the pump, and via the appropriate subroutine, calculates a 2 x 2 matrix relating input flow and pressure to output flow and pressure for the fundamental harmonic frequency. These matrices are used to calculate the input impedance at each element in the system, starting with the terminating element in the circuit and computing the input impedance at each element back to the pump. Matrix and input impedance calculations are repeated for each harmonic frequency of the pump speed up to and including the harmonic specified in the input data.

Impedance calculated at the pump provides the linear phase and gain relationship between the harmonic flows from the pump into the load (system) and the corresponding pressures across the load. This load impedance is passed to the pump subroutine.

The pump subroutine models a rotating, axial nine-piston hydraulic pump, and establishes the nonlinear relationship between pump output flow and pressure in the time domain. An iterative technique is used to obtain the balance between the output flow and pressure relationships, utilizing the load impedance from the main program. The balance is obtained in the time domain, although a check is performed in the frequency domain. The complex acoustic flow output of the pump is characterized into harmonic flows by Fourier analysis. The pump inlet flow and pressure may also be calculated by the pump subroutine. The calculation technique is the same as that used in the outlet flow calculation. Pump inlet analysis is performed at the option of the user.

Detailed physical dimensional pump data are input along with other system element data. Precompression, decompression, piston stroke, variable valve opening and closing, fluid bulk modulus, pump internal leakage, and cavitation effects are included. Steady state output pressure or swash angle is calculated as a function of pump speed (rpm) and system leakage flow. Swash plate flexibility and compensator valve dynamics are not included.

A simpler pump representation is also available, which depicts the pump as a constant flow source shunted by a parallel impedance. Use of this alternative to the pump subroutine is dictated by the user in the input data, which must include values for the source flow and shunt impedance, or pressure, all of which are derived empirically or from the detailed pump model.

The main program applies the complex flow and pressure from the pump subroutine to the system, using the element matrices to calculate the input flow and pressure at each system element. The program also computes and plots acoustic energy density and/or intensity (power) for selected system elements.

Subroutines are included for pumps, lines, lumped volumes, valves, accumulators, lumped volume and acoustic filter type resonators, and wide band, helical Quincke tubes. The main program handles branch and terminating elements.

3.0 MAIN PROGRAM

3.1 INTRODUCTION AND FLOW DIAGRAM

General flow diagrams for the HSFR main program are shown in Figure 3-1 and 3-2.

The initial section of the HSFR main program (Figure 3-1) provides specification statements for variable names, types, commonality, and direct data inputs. This section also specifies an external MCAUTO dating routine. The main program then provides read and write statements for all the required input data, calls the subroutine FLUID to calculate and write the necessary fluid parameters, and initializes the first calculation point for pump speed. In Figure 3-2, the main program computes the fundamental harmonic frequency for the first pump speed. The program examines each element of the system in turn, excluding the pump, and via the appropriate subroutine, calculates a matrix relating input and output oscillatory flows and pressures for the fundamental harmonic frequency. Information from these matrices is then used to calculate the input impedance at each element in the system, starting with the terminating element in the circuit and computing the input impedance for each element back to the pump. Matrix and input impedance calculations are then repeated for each harmonic frequency of the pump speed up to and including the harmonic specified in the input data.

Impedance calculated at the pump provides the linear phase and gain relationship between the harmonic flows from the pump into the load (system) and the corresponding pressures across the load. This load impedance is then passed to the pump subroutine.

The pump subroutine contains a detailed description of the operation of the pump, and models the nonlinear relationship between pump oscillatory output (or input) flow and pressure in the time domain. An iterative technique is used in the subroutine to obtain the balance between output (or input) flow and pressure for the specified harmonic frequency.

Balanced pump output (or input) flow and pressure are returned to the main program where the matrices for the system elements are again used to obtain the oscillatory input flow and pressure at each element in the system.

The main program then computes acoustic energy density and/or intensity for selected elements only. Values of flow, pressure, impedance, energy density, and/or intensity are stored for the selected elements.

The entire calculation in Figure 3-2 is then repeated for each pump speed over the specified range.

Finally, the main program plots the specified parameters at the specified elements versus pump speed.

The main program is discussed and listings are presented by sections in subsequent paragraphs. Format statements for the main program appear at the end of Section 3.

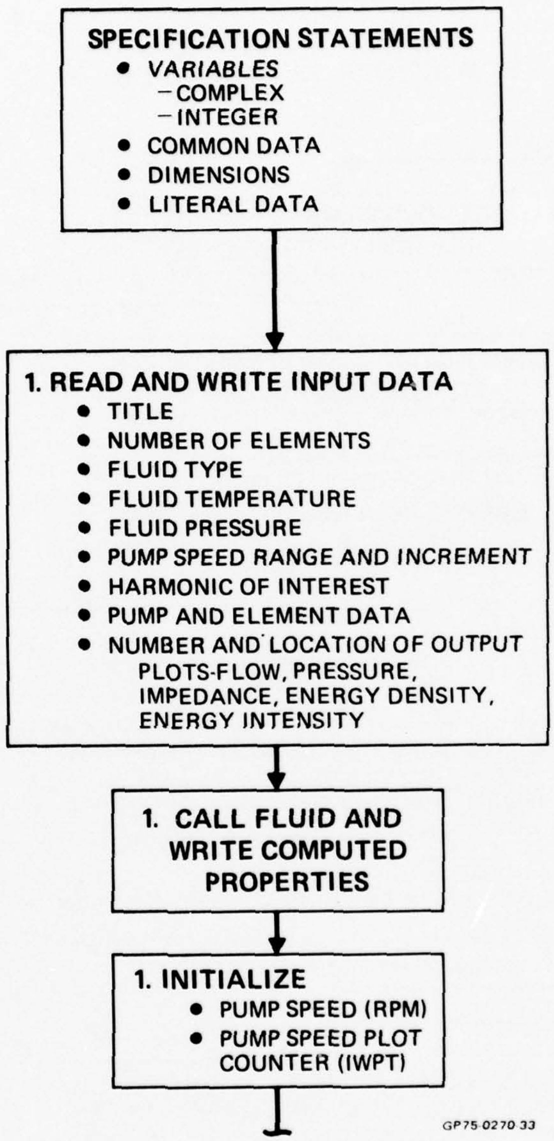
3.1.1 Math Model - See sub-paragraphs in Section 3.

3.1.2 Assumptions - Not applicable.

3.1.3 Computation Method - The HSFR main program, and all the subroutines except PUMP operate in the frequency domain. Pressure, flow, and impedance variables are in complex form and computations are performed in complex form by the computer. The magnitude of the complex vectors are plotted versus pump speed. Impedance values are plotted as a log function (decibels) in order that small amplitudes can be clearly distinguished on the print-plots.

3.1.4 Approximations - See sub-paragraphs in Section 3.

3.1.5 Limitations - Arrays are dimensioned to handle up to 40 circuit element data records. Storage for print-plots is dimensioned to allow



To Calculation Control (FIG. 3-2)

FIGURE 3-1

HSFR MAIN PROGRAM SPECIFICATION AND DATA INPUT FLOW CHART

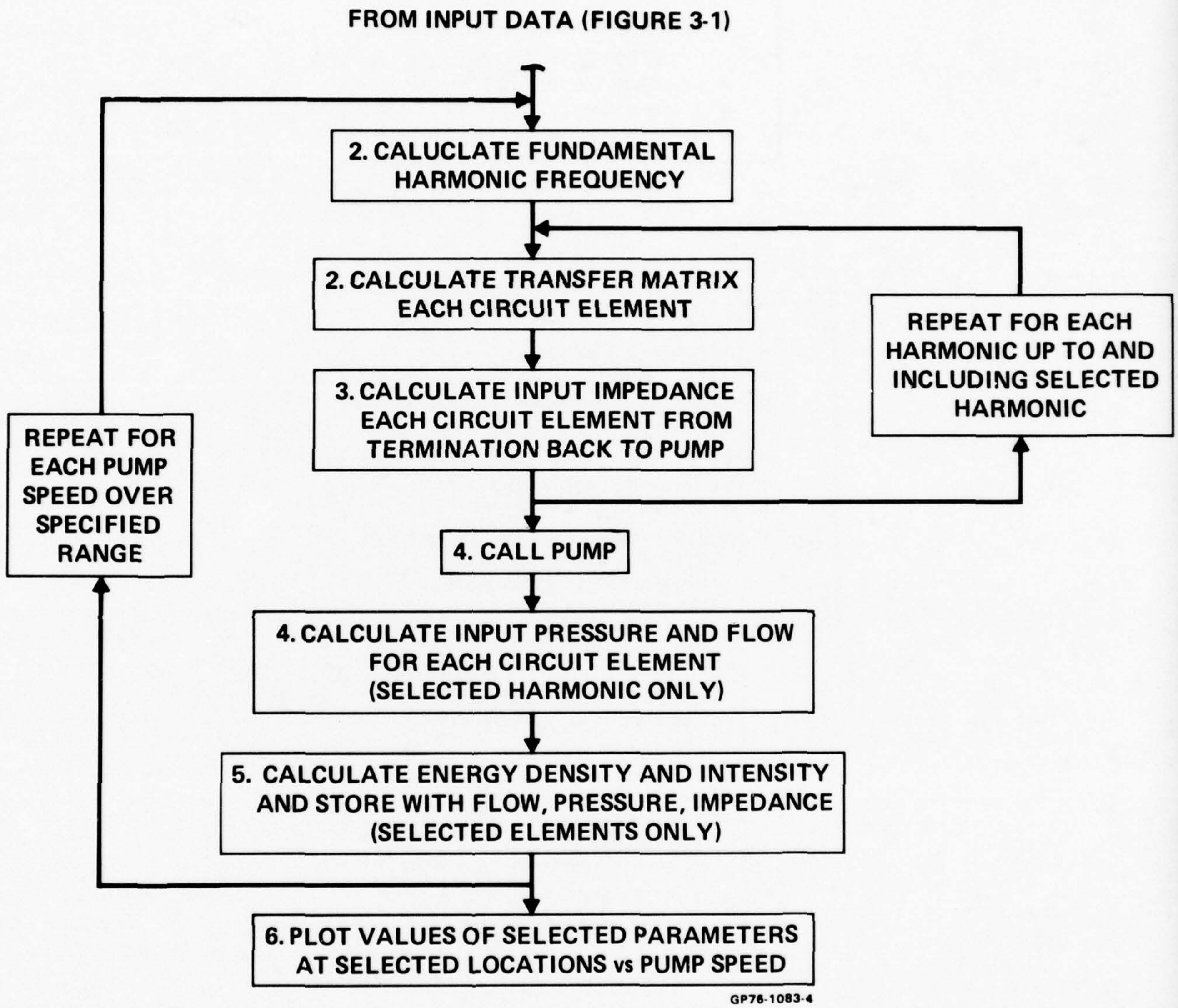


FIGURE 3-2
HSFR MAIN PROGRAM - CALCULATION
CONTROL FLOW CHART

up to 49 output print plots in a single run, i.e., a single system model. Arrays may be redimensioned to save storage cost if dealing with a circuit containing only a few elements. However, redimensioning must be done in the main and all applicable subroutines.

The PARM(-,-) array for storing element input data is dimensioned for eight columns and forty elements, (8,40). The eighth or last column on the first record (card) for an element is reserved for internal program addressing of extra element data records in the PARM array. Therefore, the first element record may contain up to seven physical data inputs. Second and subsequent records on any one element may contain up to eight data inputs.

System input impedance (ZIP) is dimensioned to permit analysis up through the 10th harmonic.

3.1.6 Variable Names - The main program variable names are listed and defined below in alphabetical order. Common variable names used in both the main program and the subroutines are listed separately in paragraph 3.1.6.1.

SYMBOL	DESCRIPTION	UNITS
CHAR( )	Variable name used in title of each print-plot	-
E( )	Acoustic energy intensity at element input	WATTS
EC	Truncated to E for use in print-plot title	-
FLOW	Temporary variable for summing valve flows	IN**3/SEC
I	Integer counter	-
IFLUID	Fluid type in FLUID call statement	-
IIPLT( )	One dimensional array used for storing in sequence the junctions for which flow, pressure, impedance, or energy are to be plotted	-
IPLT	Temporary variable for IIPLT	-
ITYPE	Type of element under examination, extracted from single dimension array NTYPE	-

SYMBOL	DESCRIPTION	UNITS
IWPT	Counter used in incrementing pump speed	-
J	Integer counter	-
K	Integer counter	-
KHARM	Order of harmonic	-
KT	Used in test for improper termination of a branch	-
MM	Identifies existence of extra card data for writing	-
MN	Integer counter	-
NEPLT	Number of energy intensity plots	-
NHARM	Integer form of WHARM	-
NINLT	Pump inlet identification	-
NPLT	Total number of plots	-
NPLTPT	Number of points per plot (= number of pump rpms)	-
NPPLT	Number of pressure plots	-
NQPLT	Number of flow plots	-
NTPLT	Number of energy density plots	-
NTYPE( )	Type of element	-
NZPLT	Number of impedance plots	-
OMEGA( )	Real variable array used to store pump rpm's subsequently used for plotting	-
PANG	Phase angle of complex pressure	RAD
PHASE	Phase angle between complex flow and pressure	RAD
PC	Truncated to P for use as plot title	-
PISTNO	Number on pumping pistons - input data	-
PRESS	Data input steady state pressure	PSI
PM	Not used	-
PO	Pump fundamental complex output pressure at 2500 rpm (Data input for simpler pump representation)	PSI

SYMBOL	DESCRIPTION	UNITS
QANG	Phase angle of complex flow	RAD
QC	Truncated to Q for use plot title	-
QM( )	Not used	-
QO	Pump fundamental output flow at 2500 rpm (Data input for simpler pump representation)	IN**3/SEC
QOVB	Data input valve element steady state flow	IN**3/SEC
STPLT( )	Array used to store in sequence the values for subsequent extraction and plotting	-
T	Acoustic energy density	WATTS/IN**3
TC	Truncated to T for use in plot title	-
TEMP	Data input fluid temperature	°F
TITLE1( )	Data input title for flow plots	-
TITLE2( )	Data input title for pressure plots	-
TITLE3( )	Data input title for impedance plots	-
TITLE4( )	Data input title for energy density plots	-
TITLE5( )	Data input title for intensity plots	-
TITLEN	Temporary variable for plot titles	-
TITLE	Data input for run title	-
TK	Kinetic acoustic energy density	WATTS/IN**3
TP	Potential acoustic energy density	WATTS/IN**3
W	Pump speed for current calculation	RPM
WEND	Data input maximum pump speed for calculation	RPM
WINC	Data input incremental pump speed	RPM
WSTART	Data input first pump speed for calculation	RPM
WHARM	Data input harmonic of interest	-
YPLT( )	Data extracted from STPLT to allow plotting of each curve in turn	-

SYMBOL	DESCRIPTION	UNITS
ZAP( )	Complex impedance of total load on pump inlet for each harmonic	PSI/CIS
ZC	Truncated to 'Z' for use in plot title	-
ZIP( )	Complex impedance of load on pump outlet for each harmonic	PSI/CIS
ZO	Data input equivalent shunt complex impedance for simple pump representation at 2500 rpm	PSI/CIS
ZM	Not used	-
ZP	Total parallel complex impedance of pump equivalent shunt impedance (ZZ) and system input impedance (ZIP)	PSI/CIS
ZZ	ZO corrected for pump speed, viscosity, and density	PSI/CIS

3.1.6.1 Common Variable Names

A	Frequency of harmonic flow	RAD/SEC
BEI	No longer used	
BEIP	No longer used	
BER	No longer used	
BERP	No longer used	
BETA	Frequency dependent friction factor used in LINE and other subroutines	-
BULK	Fluid bulk modulus used for calculation, selected from ABULK tabulation	PSI
G(-,-,-)	Three dimensional array containing transfer matrices produced by subroutines to describe elements	-
IEL	Number of element under examination	-
KTYPE	Used to identify element sub-types	-
NEL	Total number of elements	-
P( )	Pressure at inlet to system element	PSI
PARM(-,-)	Data input for two dimensional array of element physical data	-

SYMBOL	DESCRIPTION	UNITS
PI	The circular measure constant	-
Q( )	Peak input flow into element	CIS
RHØ	Fluid mass density selected from ARHO tabulation	LB-SEC**2/IN**4
VØL	Volume of element	IN**3
VISC	Fluid viscosity selected from AVISC tabulation	IN**2/SEC
XBE	Variable used in calculating BETA	-
Z( )	Complex input impedance of element at harmonic frequency of current calculation	PSI/CIS

3.1.7 Specification Listing - Main Program

```
      PROGRAM HSFR(INPUT,OUTPUT,DATA,TAPE5=DATA,TAPE6=OUTPUT)
C
C * REVISED DECEMBER 1,1975 *
C
C         *VARIABLE TYPES, DIMENSIONS, COMMOMALITY, AND DATA*
C
      COMPLEX BETA,G,P,Q,Q0,P0
      COMPLEX Z,Z0,ZZ,ZP,ZIP,ZAP
      INTEGER TITLE1(10),TITLE2(10),TITLE3(10),TITLE4(10),TITLE5(10),
     +TITLEN(10),TITLE(10)
      COMMON BETA,G,P,Q,Z, XBE,BER,BEI,BERP,BEIP,RHO,BULK,VOL,A,VISC,PAR

     1M,PI, IEL,NEL,KTYPE(40)
      DIMENSION PARM(8,40),G(2,2,40),Z(40),P(40),Q(40)
     +,E(40),T(40)
      DIMENSION NTYPE(40)
      DIMENSION IIPLT(49),STPLT(5000),OMEGA(125),YPLT(125,1)
      DIMENSION QM(40),PM(40),ZM(40)
      DIMENSION CHAR(1)
      DIMENSION ZIP(10),ZAP(10)
      DATA QC/1HQ/,PC/1HP/,ZC/1HZ/,TC/1HT/,EC/1HE/
      DATA TITLE4/8HACOUSTIC,8H ENERGY ,8HDENSITY ,8HIN MILLI,8HWATT-SEC

     +,8H/IN**3 V,8HS PUMP R,8HPM-HARMO,8HNIC NO  ,8H        /
      DATA TITLE5/8HACOUSTIC,8HINTENSIT,8HY(POWER),8H IN WATT,8HS VS PUM

     +,8HP RPM FO,8HR HARMON,8HIC NUMBE,8HR       ,8H        /
      DATA TITLE1/8HPEAK FLO,8HW IN CUB,8HIC INCHE,8HS/SEC VE,8HRSUS PUM

     +,8HP RPM FO,8HR HARMON,8HIC NUMBE,8HR       ,8H        /
      DATA TITLE2/8HPEAK PRE,8HSSURE IN,8H LBS/IN*,8H*2 VERSU,8HS PUMP R

     +,8HPM FOR H,8HARMONIC ,8HNUMBER  ,8H        ,8H        /
      DATA TITLE3/8HIMPEDANC,8HE IN DEC,8HIBELS VE,8HRSUS PUM,8HP RPM FO

     +,8HR HARMON,8HIC NUMBE,8HR       ,8H        ,8H        /
```

3.2 SECTION 1 - READ & WRITE INPUT DATA

In addition to the reading and writing of input data, Section 1 calls the subroutine FLUID for computation and writing of the fluid properties. This section also initializes the first pump speed computation point. All reads and writes for the program are included in this section except the write statements which control the print-plots.

READS

Format #	Description
460	Title of system simulated
470	Number of elements, fluid type, fluid temperature, and fluid pressure
480	First, last, increment pump speed, and harmonic of interest
490, 495	Element type, sub-type, and physical data parameters
560	Plotting requirements for flow, pressure, impedance, acoustic density, and acoustic intensity

WRITES

500	Date and title of system simulated
550,555	Pump speed input data, and harmonic of interest
520	Headings for writing element input data
540,545	System element input data
496	Error message if number of elements exceed storage dimension
620	Error message if circuit in improperly terminated
570,580, 590,593, 595	Plotting requirements

3.2.1 Math Model - Not applicable.

3.2.2 Assumptions - Not applicable.

3.2.3 Computation Method - Not applicable.

3.2.4 Approximations - None.

3.2.5 Limitations - Limitations for reading input data are covered in detail in the user's manual, Volume I of this report.

3.2.6 Variable Names - See paragraph 3.1.6.

3.2.7 Read and Write Listing

```
C
C *SECTION 1- READ AND WRITE INPUT DATA*
C
  100 READ(5,460)(TITLE(I),I=1,10)
      IF(EOF(5))450,101
  101 READ(5,470) NEL,IFLUID,TEMP,PRESS
      READ(5,480)WSTART,WEND,WINC,WHARM,PISTNO
      NHARM=WHARM+.01
      WRITE(6,500)(TITLE(I),I=1,10)
      IF(NHARM.LE.1) GO TO 112
      WRITE(6,550) WSTART,WEND,WINC,NHARM,PISTNO
      GO TO 113
  112 WRITE(6,555) WSTART,WEND,WINC,PISTNO
C
C    COMPUTE AND WRITE FLUID PROPERTIES
C
  113 CALL FLUID(TEMP,PRESS,IFLUID,VISC,BULK,RHO)
      WRITE(6,520)
C
C    READ AND WRITE CIRCUIT ELEMENT INPUT PHYSICAL DATA
C
      K=NEL+1
      DO 103 J=1,NEL
      READ(5,490) NTYPE(J),KTYPE(J),(PARM(I,J),I=1,7)
      IF(NTYPE(J).EQ.6) GO TO 109
      MM=KTYPE(J)/10
      IF(MM.NE.0) GO TO 102
      PARM(8,J)=0
  109 WRITE(6,540) (J),NTYPE(J),KTYPE(J),(PARM(I,J),I=1,7)
      GO TO 103
  102 PARM(8,J)=K
      WRITE(6,540) (J),NTYPE(J),KTYPE(J),(PARM(I,J),I=1,7)
      DO 108 MN=1,MM
      READ(5,495) (PARM(I,K),I=1,8)
      WRITE(6,545) (PARM(I,K),I=1,8)
      K=K+1
      IF(K.GT.40) GO TO 106
  108 CONTINUE
  103 CONTINUE
      GO TO 111
  106 WRITE(6,496)
      STOP 7700
C
C    READ, COMPUTE, AND WRITE OUTPUT PLOT REQUIREMENTS
C
  111 READ(5,560) NQPLT,(IIPLT(I),I=1,NQPLT)
      READ(5,560) NPPLT,(IIPLT(I+NQPLT),I=1,NPPLT)
      READ(5,560) NZPLT,(IIPLT(I+NQPLT+NPPLT),I=1,NZPLT)
      READ(5,560)NTPLT,(IIPLT(I+NQPLT+NPPLT+NZPLT),I=1,NTPLT)
      READ(5,560)NEPLT,(IIPLT(I+NQPLT+NPPLT+NZPLT+NTPLT),I=1,NEPLT)
```

3.2.7 (Cont.)

```
      PI=3.1416
      NPLTPT=(WEND-WSTART)/WINC+1.01
      NPLT=NQPLT+NPPLT+NZPLT+NTPLT+NEPLT
      IWPT=1
      W=WSTART
      DO 105 IEL=1,NEL
      IF(NTYPE(IEL).NE.6) GO TO 105
      KT=KTYPE(IEL)
      KT=KT+IEL
      IF(NTYPE(KT).LT.10) GO TO 104
  105 CONTINUE
      IF(NTYPE(NEL).GT.10) GO TO 115
  104 WRITE(6,620)
      STOP
  115 CONTINUE
      IF(NQPLT.LT.1.0) GO TO 370
      WRITE(6,570)NQPLT,(IIPLT(I),I=1,NQPLT)
  370 IF(NPPLT.LT.1.0) GO TO 380
      WRITE(6,580)NPPLT,(IIPLT(I+NQPLT),I=1,NPPLT)
  380 IF(NZPLT.LT.1.0) GO TO 383
      WRITE(6,590)NZPLT,(IIPLT(I+NQPLT+NPPLT),I=1,NZPLT)
  383 IF(NTPLT.LT.1.0) GO TO 385
      WRITE(6,593)NTPLT,(IIPLT(I+NQPLT+NPPLT+NZPLT),I=1,NTPLT)
  385 IF(NEPLT.LT.1.0) GO TO 390
      WRITE(6,595)NEPLT,(IIPLT(I+NQPLT+NPPLT+NZPLT+NTPLT),I=1,NEPLT)
  390 CONTINUE
```

3.3 SECTIONS 2 and 3 - ELEMENT TRANSFER MATRIX AND IMPEDANCE CALCULATIONS

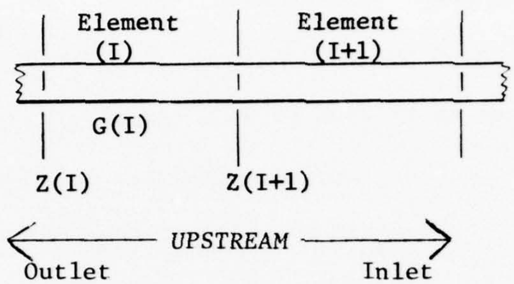
Section 2 first calculates the fundamental harmonic frequency of the pump speed. Each element in the circuit is then examined sequentially starting with the first element, the pump. The transfer matrix for the pump is initialized for the first time through the DO 250 loop. The applicable subroutine for each of the other elements if then called, one at a time, up through the terminating element. Each subroutine calculates a 2 X 2 transfer matrix which relates input and output flow and pressure at the fundamental harmonic of the current calculation pump speed. The transfer matrix is initialized for dummy (NTYPE = 7) branch (NTYPE = 6) and pump elements. If pump inlet analysis is desired, the inlet is automatically assigned its sequential element number (NINLT = IE). After the last terminating element in the circuit has been examined, control passes to Section 3 which calculates the input impedance for each element starting with the last element in the circuit.

Input impedance Z(I) for the next element upstream I =(I-1) is then calculated where Z(I+1) is the input impedance of the last downstream element. This calculation repeats until Z(1) is calculated, which is the system load impedance at the pump outlet for the fundamental harmonic of the current calculation pump speed. Return system load impedance (ZAP) is assigned to the pump inlet, if applicable.

Section 2 and 3 are then repeated for each harmonic of the current pump speed up to and including the specified harmonic.

Progression of impedance calculation is such that the calculated input impedance for the harmonic of interest is stored for each element in the system. Values for the system inlet and outlet load impedances are stored in the ZIP(-) and ZAP (-) arrays for each harmonic, starting with the fundamental, up to and including the harmonic of interest.

3.3.1 <u>Math Model</u> - Math models for transfer matrix calculations are covered in Section 4 of Volume 2, under the subroutine descriptions. The basic math model for computation of element input impedances in Section 2 of the main program is as follows:



Input impedance of the upstream element (I) is computed from the transfer matrix G(I) for element (I), calculated by the subroutines, and the impedance of the downstream element (I+1). The pressure and flow across element (I) are related by the matrix form $\begin{bmatrix} G(I) \end{bmatrix} \cdot \begin{bmatrix} Q(I) \\ P(I) \end{bmatrix} = \begin{bmatrix} Q(I+1) \\ P(I+1) \end{bmatrix}$

or

$$\begin{bmatrix} G(1,1,I) & G(1,2,I) \\ G(2,1,I) & G(2,2,I) \end{bmatrix} \cdot \begin{bmatrix} Q(I) \\ P(I) \end{bmatrix} = \begin{bmatrix} Q(I+1) \\ P(I+1) \end{bmatrix}$$

In equation form

$$G(1,1,I)Q(I) + G(1,2,I)P(I) = Q(I+1) \tag{1}$$

$$G(2,1,I)Q(I) + G(2,2,I)P(I) = P(I+1) \quad . \tag{2}$$

By definition

$$P = QZ \quad . \tag{3}$$

Dividing equation (2) by (1) yields

$$\frac{G(2,1,I)Q(I) + G(2,2,I)P(I)}{G(1,1,I)Q(I) + G(1,2,I)P(I)} = \frac{P(I+1)}{Q(I+1)} \quad . \tag{4}$$

From equation (3)

$$\frac{P(I+1)}{Q(I+1)} = Z(I+1), \ \frac{P(I)}{Q(I)} = Z(I) \quad . \qquad (5)(6)$$

Using (5) and dividing the left side of (4) by Q(I) yields

$$\frac{G(2,1,I) + G(2,2,I)\mathcal{Z}(I)}{G(1,1,I) + G(1,2,I)\mathcal{Z}(I)} = \mathcal{Z}(I+1) \quad .$$

Solving for $\mathcal{Z}(I)$ gives

$$\mathcal{Z}(I) = \frac{G(1,1,I)\mathcal{Z}(I+1) - G(2,1,I)}{G(2,2,I) - G(1,2,I)\mathcal{Z}(I+1)} \ , \qquad (7)$$

which is the basic equation used for impedance calculations.

3.3.1.1 Circuit Termination

If the circuit is closed at the terminating element (I), then Q(I+1) = 0. Equation (1) then becomes

$$G(1,1,I)Q(I) + G(1,2,I)(PI) = 0.$$

Rearranging

$$\frac{P(I)}{Q(I)} = \frac{-G(1,1,I)}{G(1,2,I)} \ , \ \text{or}$$

$$\mathcal{Z}(I) = \frac{-G(1,1,I)}{G(1,2,I)} \quad . \qquad (8)$$

If the circuit is open due to flow at a valve element (I), then P(I+1) = 0. Equation (2) then becomes

$$G(2,1,I)Q(I) + G(2,2,I)P(I) = 0 \quad .$$

Rearranging

$$\frac{P(I)}{Q(I)} = -\frac{G(2,1,I)}{G(2,2,I)} \ , \ \text{or}$$

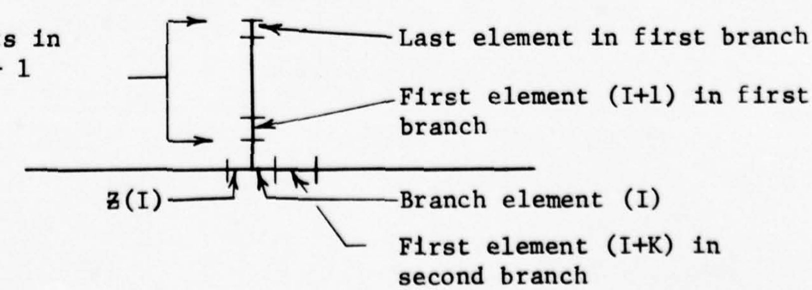
$$\mathcal{Z}(I) = -\frac{G(2,1,I)}{G(2,2,I)} \quad . \qquad (9)$$

3.3.1.2 Branch Elements

Input impedance for a branch element (NTYPE=6) is considered to be the equivalent impedance of a two branch parallel network.

K = no. of elements in first branch + 1

K = KTYPE(I)+1



$$\frac{1}{Z(I)} = \frac{1}{Z(I+1)} + \frac{1}{Z(I+K)}$$

$$Z(I) = \frac{Z(I+1)Z(I+K)}{Z(I+1) + Z(I+K)} \qquad (10)$$

3.3.2 Assumptions

The impedance of all branch terminating elements, except valves, is assumed to be that of a closed circuit per equation (8), i.e. the output flow of the element is set to zero. Terminating valve elements (NTYPE=14) are assumed to be open circuit with input impedance per equation (9), where the output pressure is set to zero to simulate an open ended termination. The open circuit representation allows simulation of servovalve flow.

As can be seen from equation (10) above, it does not matter which branch circuit comes first in the data deck.

3.3.3 Computation Method - Not applicable.

3.3.4 Approximations - None.

3.3.5 Limitations - All branch circuits must be terminated and have the correct number of elements in the branch. An error message is generated from the data read section if a branch circuit is not properly described and terminated. Correctly equating pressures and summing flows at branch junctions is a very important aspect of the main program.

3.3.6 Variable Names - See Paragraph 3.1.6.

3.3.7 Transfer Matrix and Impedance Calculation Listing

```
C
C *SECTION 2- COMPUTE TRANSFER MATRIX FOR EACH CIRCUIT ELEMENT*
C
      IF(NHARM.EQ.0) NHARM=1
      QOVB=0.0
  110 CONTINUE
      KHARM=0
      IF(KTYPE(1).EQ.21)NINLT=121
      IF(KTYPE(1).EQ.22) NINLT=122
  120 CONTINUE
      KHARM=KHARM+1
      A=W*PISTNO*PI*KHARM/30.
      DO 250 IEL=1,NEL
      ITYPE=NTYPE(IEL)
      GO TO (130,140,150,160,170,180,175,190,200,250,130,140,150,160,
     +170,180,175,190),ITYPE
  130 CALL LINE
      GO TO 250
  140 CALL RESNTR
      GO TO 250
  150 CALL VOLUME
      GO TO 250
  160 IF(W.NE.WSTART) GO TO 165
      FLOW=PARM(2,IEL)
      QOVB=QOVB+FLOW
  165 CALL VALVE
      GO TO 250
  170 CALL ACCUM
      GO TO 250
  175 IF(KTYPE(IEL).EQ.1) NINLT=IEL
  180 G(1,1,IEL)=CMPLX(1.,0.)
      G(1,2,IEL)=CMPLX(0.,0.)
      G(2,1,IEL)=CMPLX(0.,0.)
      G(2,2,IEL)=CMPLX(1.,0.)
      GO TO 250
  190 CONTINUE
C 190 CALL WHEQUT (WSTART,W)
      GO TO 250
  200 CONTINUE
      IF(KTYPE(IEL).EQ.0) GO TO 220
      IF(KHARM.EQ.NHARM+1) GO TO 240
      IF(KHARM.NE.1) GO TO 250
      GO TO 180
  220 CONTINUE
      G(1,1,1) =CMPLX(1.,0.)
      G(1,2,1) =CMPLX(0.,0.)
      G(2,1,1) =CMPLX(0.,0.)
      G(2,2,1) =CMPLX(1.,0.)
      A=W*PISTNO*PI*NHARM/30.
  250 CONTINUE
```

3.3.7 <u>(Cont.)</u>

```
C
C *SECTION 3- CALCULATE INPUT IMPEDANCE FOR EACH CIRCUIT ELEMENT
C              FROM TERMINATION BACK TO PUMP*
C
      I=NEL
  260 IF(NTYPE(I).GE.11) GO TO 270
      IF(NTYPE(I).EQ.6) GO TO 290
      Z(I)=(G(1,1,I)*Z(I+1)-G(2,1,I))/(G(2,2,I)-G(1,2,I)*Z(I+1))
      GO TO 300
  270 IF(NTYPE(I).EQ.14) GO TO 280
      Z(I)=-G(1,1,I)/G(1,2,I)
      GO TO 300
  280 Z(I)=-G(2,1,I)/G(2,2,I)
      GO TO 300
  290 K=KTYPE(I)+1
      G(1,2,I)=-1/Z(I+K)
      Z(I)=Z(I+1)*Z(I+K)/(Z(I+1)+Z(I+K))
  300 I=I-1
      IF(NINLT.EQ.121.OR.NINLT.EQ.122)ZAP(KHARM)=CMPLX(0.0,0.0)
      IF(I+1.EQ.NINLT) ZAP(KHARM)=Z(I+1)
      IF(I+1.NE.1) GO TO 260
      ZIP(KHARM)=Z(1)
      IF(KTYPE(I+1).NE.0) GO TO 120
```

3.4 SECTION 4 - ELEMENT INPUT PRESSURE AND FLOW CALCULATION

Section 4 of the main program computes the input flow and pressure for each element in the curcuit beginning with the first element and progressing to the last element. Flow and pressure for the first element, the pump, are calculated in the returned from the PUMP subroutine which is called by Section 4. Pump outlet and inlet flow are balanced with the system based on the previously calculated pressure or return system load impedance at the pump. The simpler empirical pump model is used to calculate initial values of pressure (P(1)) and flow (Q(1)). The transfer matrix for the pump, assigned in Section 2, is such that the input flow and pressure for first system element is the same as the pump outlet or inlet value. Input flow and pressure for the next element is then computed using the transfer matrix previously calculated for the first system element.

A test is made to determine if the next element is a branch or termination. For a branch element, the calculation gives the flow out of both legs of the branch and equalizes the pressures. A termination requires no calculation since values of flow and pressure are not calculated for the downstream or terminated end.

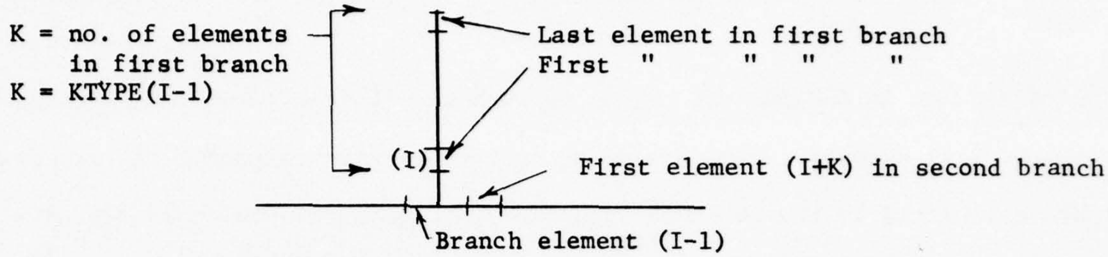
Calculation progresses until the input flow and pressure for the last element is calculated. Values of pressure, flow, and impedance specified by the input plot data are then stored in Section 5.

3.4.1 Math Model - The general models for computing input flow and pressure at each element are the transfer equations (1) and (2). For downstream (upstream for return system) progression, equations (1) and (2) become

$$Q(I) = G(1,1,I-1)Q(I-1) + G(1,2,I-1)P(I-1) \qquad (11)$$

$$P(I) = G(2,1,I-1)Q(I-1) + G(2,2,I-1)P(I-1) \qquad (12)$$

Special considerations are required to handle branch calculations.



Referring to the above sketch, inputs to the first element of each branch leg are calculated by assuming that the pressures are equal in all three legs of the branch.

$$P(I) = P(I-1) = P(I+K) \qquad (13)$$

Flows are then calculated as

$$Q(I) = P(I)/Z(I) \qquad (14)$$

$$Q(I+K) = P(I)/Z(I+K) \qquad (15)$$

General equations (11) and (12) are then used to progress down the first branch until the inputs for the terminating element in that branch are computed. Since the inputs for the first element (I+K) of the second branch have already been calculated using (13) and (15), the progression skips the general calculation for element (I+K).

3.4.2 Assumptions - The assumption of equal pressures in all three legs of a branch element is based on the element being a "point" with no length. Error due to losses in the center of a branch fitting are minimized if each of the three leg elements are assumed to end or begin in the geometric center of the fitting.

3.4.3 Computation Method - 2x2 transfer matrix.

3.4.4 Approximations - None.

3.4.5 Limitations - None.

3.4.6 Variable Names - See para. 3.1.6.

3.4.7 Flow and Pressure Calculations Listing

```
C
C *SECTION 4- CALCULATE INPUT PRESSURE AND FLOW FOR
C              EACH CIRCUIT ELEMENT*
C
      Q0=CMPLX(PARM(1,1),PARM(2,1))
      Z0=CMPLX(PARM(3,1),PARM(4,1))
      Q(1)=Q0*W/2500.
      ZZ=Z0*2500./W
      ZP= ZZ*Z(1)/(ZZ+Z(1))
      P(1)= Q(1)* ZP
      Q(1)= P(1)/Z(1)
      GO TO 310
  240 CONTINUE
      CALL PUMP(WSTART,W,ZIP,NHARM,QOVB,PRESS,WINC,PISTNO,NINLT,ZAP)
  310 CONTINUE
      DO 335 I=2,NEL
      IF(I.EQ.NINLT) GO TO 335
      IF(NTYPE(I-1).EQ.6) GO TO 320
      IF(NTYPE(I-1).GE.11) GO TO 335
      Q(I)=G(1,1,I-1)*Q(I-1)+G(1,2,I-1)*P(I-1)
      P(I)=G(2,1,I-1)*Q(I-1)+G(2,2,I-1)*P(I-1)
      GO TO 335
  320 K=KTYPE(I-1)
      P(I)=P(I-1)
      P(I+K)=P(I)
      Q(I)=P(I)/Z(I)
      Q(I+K)=P(I)/Z(I+K)
  335 CONTINUE
C
```

3.5 SECTION 5 - CALCULATE ENERGY AND STORE PLOT DATA

Section 5 of the main program performs the function of storing in the STPLT(I) array only the data which is to be plotted. The total storage used is equal to the number of plots (NPLT) times the number of plotted points (NPLTPT). The storage dimension is 5000.

Flow and pressure are stored as absolute values (magnitude) of the complex vector. Impedance is stored as 20 LOG_{10} of the absolute value, i.e. decibels. The stored values are used as the vertical axis values for the output plots. The phase angle information is dropped, but can easily be plotted if desired.

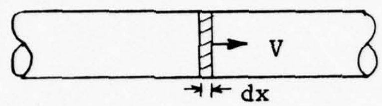
The total acoustic energy density, T(IPLT), is calculated from the absolute values of flow and pressure for those elements specified for plotting. Total acoustic energy density is obtained by summing the calculated kinetic (flow) energy, TK, and potential (pressure) energy, TP. The total energy density is then stored in the STPLT(I) array.

The acoustic intensity or power, E(IPLT), is also calculated from the complex flow and pressure data for those elements specified for plotting. The acoustic power is the product of the pressure and flow vector magnitudes, and the cosine of the phase angle between the pressure and flow vectors. The energy intensity is then stored in the STPLT(I) array.

For each speed calculation point the value of pump speed (rpm) is stored in the OMEGA (IWPT) array, which is subsequently used as the 'X' axis for the output plots.

The pump speed point counter, IWPT, and the pump speed, W, are then incremented, and control is returned to Section 2 for the next calculation.

3.5.1 Math Model - The acoustic energy density is a measure of the total acoustic energy content at the input to a circuit model element. It is composed of kinetic energy (flow) and potential energy (pressure). Consider the following small fluid volume element moving through a hydraulic line.



Acoustic energy density in the fluid element of length dx is given in Paragraph 2.7 of Reference (1) as:

$$t = \text{Kinetic} + \text{Potential}$$

$$t = \frac{RHO\ V^2}{2} + \frac{P^2}{2\ RHO\ C^2} \qquad (16)$$

where: RHO = fluid mass density LB-SEC^2/IN^4

C = velocity of sound in fluid IN/SEC

V = velocity of fluid element IN/SEC

P = peak pressure in fluid element LBS/IN^2.

However, velocity is related to peak flow (Q) and cross sectional area (A) by

$$V = Q/A. \qquad (17)$$

Substituting (17) in (16) yields

$$t = \frac{RHO}{2}(Q/A)^2 + \frac{P^2}{2\ RHO\ C^2}$$

$$t = \frac{RHO\ Q^2}{2\ A^2} + \frac{P^2}{2\ RHO\ C^2} \qquad (18)$$

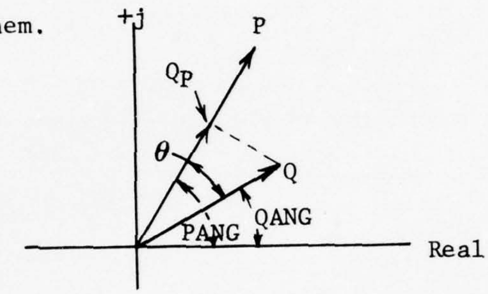
The average or RMS energy density (T) is derived by dividing the peak flow and pressure by $\sqrt{2}$.

$$T = \frac{RHO}{2A^2}\left(\frac{Q}{\sqrt{2}}\right)^2 + \frac{1}{2\ RHO\ C^2}\left(\frac{P}{\sqrt{2}}\right)^2$$

$$T = \frac{RHO\ Q^2}{4A^2} + \frac{P^2}{4\ RHO\ C^2} \quad \frac{IN\text{-}LBS}{IN^3} \qquad (19)$$

To obtain T in MILLIWATTS/IN^3, the results of equation (19) are multiplied by 113.

Acoustic intensity or power is synonomous with the active or in-phase power in an AC electrical circuit. It is obtained by multipling the magnitudes (absolute value) of the pressure and flow vectors by the cosine of the phase angle between them.



The active power in WATTS is

$$E = .113\ PQp = .113\ PQ \cos \theta \qquad (20)$$

where θ = phase angle

$= \text{PANG} - \text{QANG}$

and $\text{PANG} = \text{TAN}^{-1} \left(\frac{\text{IMAG(P)}}{\text{REAL(P)}}\right)$

$\text{QANG} = \text{TAN}^{-1} \left(\frac{\text{IMAG(Q)}}{\text{REAL((Q)}}\right)$.

3.5.2 Assumptions - None

3.5.3 Computation Method - Not applicable.

3.5.4 Approximations - None

3.5.5 Limitations - Since the energy density equation (19) requires element area, the energy density calculation is limited to line elements only.

3.5.6 Variable Names - See para. 3.1.6.

3.5.7 Calculate Energy and Store Plot Data - Listing

```
C
C *SECTION 5- CALCULATE T AND/OR E, STORE THE MAGNITUDE OF EACH
C              Q,P,Z,T, AND/OR E SELECTED FOR PLOTTING*
C
      DO 360 J=1,NPLT
      I=(J-1)*NPLTPT + IWPT
      IPLT=IIPLT(J)
      IF(J.GT.NQPLT)GO TO 340
      IF(IPLT.EQ.39) GO TO 341
      STPLT(I)=CABS(Q(IPLT))
      GO TO 360
  341 STPLT(I)=PHASEM
      GO TO 360
  340 IF(J.GT.NQPLT+NPPLT) GO TO 345
      STPLT(I)=CABS(P(IPLT))
      GO TO 360
  345 IF(J.GT.NQPLT+NPPLT+NZPLT) GO TO 350
      STPLT(I)=20.*ALOG10(CABS(Z(IPLT)))
      GO TO 360
  350 IF(J.GT.NQPLT+NPPLT+NZPLT+NTPLT) GO TO 355
      TK=RHO*(CABS(Q(IPLT)))**2/(PARM(6,IPLT))**2/4.
      TP=(CABS(P(IPLT)))**2/RHO/(PARM(5,IPLT))**2/4.
      T(IPLT)=(TK+TP)*113.
      STPLT(I)=T(IPLT)
      GO TO 360
  355 PANG=ATAN2(AIMAG(P(IPLT)),REAL(P(IPLT)))
      QANG=ATAN2(AIMAG(Q(IPLT)),REAL(Q(IPLT)))
      PHASE=QANG-PANG
      IF(PHASE.GT.180.) PHASE=PHASE-360.
      IF(PHASE.LT.-180.) PHASE=PHASE+360.
      E(IPLT)=CABS(P(IPLT))*CABS(Q(IPLT))*COS(PHASE)*.113
      STPLT(I)=E(IPLT)
  360 CONTINUE
      OMEGA(IWPT)=W
      IWPT=IWPT+1
      WRITE(6,630) Q(5),QANGM,P(5),PANGM,PHASEM,Q(6),P(6),W
  630 FORMAT(10X,7(F10.4),/,10X,5(F10.4))
      W=W+WINC
      IF(W.LE.WEND)GO TO 110
C
```

3.6 SECTION 6 - PLOTTING STORED INFORMATION

Information previously stored in STPLT(I+K) is transferred one block at a time into YPLT(K,1) and then plotted. YPLT(K,1) is used in the call statement to GRAPH2 as the 'Y' arrav, as is the 'X' array OMEGA which contains the values of speed used in the calculations. The call statement to GRAPH2 also contains the number of plot points NPLTPT and the plot character CHAR.

After control is returned from GRAPH2, the applicable plot title is read into the array TITLEN. The main program then writes the plot title and main run title.

The above procedure is repeated for each plot in each plot category as specified in the input data.

The entire program is restarted if a second data deck has been loaded, otherwise the end of file stop 1001 is executed.

The use of the special print plotting routine GRAPH2 is cheaper and provides an easier to read plot format than the print plot routine PLOTR used on the CDC 6600 system.

3.6.1 Math Model - Not applicable.

3.6.2 Assumptions - None

3.6.3 Computation Method - No applicable

3.6.4 Approximations - None

3.6.5 Limitations - None

3.6.6 Variable Names - See paragraph 3.1.6

3.6.7 Plotting of Stored Information - Listing

```
C *SECTION 6- TAKE THE MAGNITUDE OF PRESSURE,FLOW, IMPEDANCE,
C              ENERGY DENSITY, AND/OR ENERGY INTENSITY FOR
C              SELECTED ELEMENTS FROM STORAGE AND PLOT*
C
      DO 440 J=1,NPLT
      I=(J-1)*NPLTPT
      DO 400 K=1,NPLTPT
  400 YPLT(K,1)=STPLT(I+K)
      IF(J.GT.NQPLT) GO TO 410
      CHAR(1)=QC
      CALL GRAPH2(OMEGA,YPLT,NPLTPT,CHAR)
C360  CALL PPLOT (OMEGA,YPLT,NPLTPT,125,1,41H/IMPEDANCE IN DECIBELS VERS

C360 1US PUMP RPM // )
      DO 405 I=1,10
  405 TITLEN(I)=TITLE1(I)
      GO TO 430
  410 IF(J.GT.NQPLT +NPPLT) GO TO 420
      CHAR(1)=PC
      CALL GRAPH2(OMEGA,YPLT,NPLTPT,CHAR)
C360  CALL PPLOT (OMEGA,YPLT,NPLTPT,125,1,39H/PRESSURE IN PSI VERSUS
C360 1  PUMP RPM// )
      DO 415 I=1,10
  415 TITLEN(I)=TITLE2(I)
      GO TO 430
  420 IF(J.GT.NQPLT+NPPLT+NZPLT) GO TO 426
      CHAR(1)=ZC
      CALL GRAPH2(OMEGA,YPLT,NPLTPT,CHAR)
C360 CALL PPLOT (OMEGA,YPLT,NPLTPT,125,1,35H/ FLOW IN CIS RMS VERSUS PU
C360 1MP RPM// )
      DO 425 I=1,10
  425 TITLEN(I)=TITLE3(I)
      GO TO 430
  426 IF(J.GT.NQPLT+NPPLT+NZPLT+NTPLT) GO TO 428
      CHAR(1)=TC
      CALL GRAPH2(OMEGA,YPLT,NPLTPT,CHAR)
      DO 427 I=1,10
  427 TITLEN(I)=TITLE4(I)
      GO TO 430
  428 CHAR(1)=EC
      CALL GRAPH2(OMEGA,YPLT,NPLTPT,CHAR)
      DO 429 I=1,10
  429 TITLEN(I)=TITLE5(I)
  430 CONTINUE
      WRITE(6,610)CHAR(1),IIPLT(J),TITLEN,NHARM
      WRITE(6,600)(TITLE(I),I=1,10)
  440 CONTINUE
      GO TO 100
  450 STOP 1001
  460 FORMAT(10A3)
  470 FORMAT(2I5,2F10.0)
```

3.7 FORMAT STATEMENTS - MAIN PROGRAM

Main program format statements contain all read formats and all write formats except for the writing of fluid properties in FLUID, and plot writing in GRAPH2.

3.7.1 Format Statements, Main Program - Listing

```
480 FORMAT (8F10.0)
490 FORMAT(2I5,7F10.0)
495 FORMAT(8F10.0)
496 FORMAT(10X,70H INPUT DATA EXCEEDS ARRAY DIMENSIONS - PROGRAM TERMI

   1NATED                              )
500 FORMAT (1H1,44X,43HHYDRAULIC SYSTEM FREQUENCY RESPONSE PROGRAM,
   +//,30X,10A8)
520 FORMAT(//,8H ELEMENT,1X,48(1H*),25HSYSTEM ELEMENT INPUT DATA,
   1 48(1H*),/,7H NUMBER,//,10X,6HN     K,6X,48(1H.),
   2 13HPHYSICAL DATA,48(1H.),/,8X,10HTYPE  TYPE,//)
540 FORMAT(1H  ,I3,5X,I3,2X,I3,7F14.3,/)
545 FORMAT(1H ,16X,8F14.5,/)
550 FORMAT(//,20X,27HRESPONSE IS CALCULATED FROM ,F9.2,4H TO  ,F9.2,
   1    24H R.P.M. IN INCREMENTS OF ,F9.2,7H R.P.M. ,//,42X,
   2 41HRESPONSE IS PLOTTED FOR HARMONIC NUMBER ,I2,
   3//,50X,28HNUMBER OF PUMPING ELEMENTS= ,F5.0)
555 FORMAT(//,20X,27HRESPONSE IS CALCULATED FROM ,F9.2,4H TO  ,F9.2,
   1    24H R.P.M. IN INCREMENTS OF ,F9.2,7H R.P.M. ,//,40X,
   2 55HRESPONSE IS PLOTTED FOR THE -FIRST- HARMONIC FREQUENCY   ,
   3//,50X,28HNUMBER OF PUMPING ELEMENTS= ,F5.0)
560 FORMAT(16I5)
570 FORMAT(10X,I3,37H Q PLOTS FOR INPUT TO ELEMENT NUMBERS,20I4,
   +/,50X,20I4)
580 FORMAT(10X,I3,37H P PLOTS FOR INPUT TO ELEMENT NUMBERS,20I4,
   +/,50X,20I4)
590 FORMAT(10X,I3,37H Z PLOTS FOR INPUT TO ELEMENT NUMBERS,20I4,
   +/,50X,20I4)
593 FORMAT(10X,I3,37H T PLOTS FOR INPUT TO ELEMENT NUMBERS,20I4,
   +/,50X,20I4)
595 FORMAT(10X,I3,37H E PLOTS FOR INPUT TO ELEMENT NUMBERS,20I4,
   +/,50X,20I4)
600 FORMAT(/,45X,10A8)
610 FORMAT(/,38X,A1,I2,2X,10A8,I2)
620 FORMAT(  36H     THE PROGRAM HAS BEEN TERMINATED.,//,72HEITHER A BR

   AANCH OR THE END OF THE SYSTEM HAS BEEN I PROPERLY TERMINATED.,//,2

   B8HSUGGEST YOU CHECK YOUR DATA.)
    END
```

4.0 PUMP SUBROUTINE

4.1 INTRODUCTION AND FLOW DIAGRAM

Subroutine PUMP is a general, detailed model of a rotating, axial, nine-piston, pressure-compensated hydraulic pump. The model computes the ability of the pump to deliver flow against an output pressure by modeling the non-linear relationship between pump output flow and pressure in the time domain. The main program calculates the harmonic load impedance of the rest of the circuit, and this provides the linear phase and gain relationship between the harmonic flows into the load and the corresponding pressures across the load, in the frequency domain. The balance is obtained in the time domain, although a check is performed in the frequency domain. The model can also calculate inlet flow based on the return system load impedance which is calculated by the main program.

PUMP considers valving areas, pre-compression, de-compression, steady state swash angle or pressure, fluid bulk modulus, pump internal leakage, circuit termination flow, and piston motion. Steady state swash angle is calculated and balanced as a function of pump internal leakage, circuit over-board leakage, and pump speed. If the swash angle is maximum, the steady state pressure is calculated. Swashplate flexibility and swashplate-compensator dynamics are not included.

Piston pressure at the beginning of pre-compression is assumed constant and equal to the inputed steady state inlet pressure if inlet side analysis is not selected. Otherwise, piston pressure is calculated continuously for the full pump revolution.

Figure 4-1 is a general flow chart of the PUMP subroutine. The specification section includes initialization of variables from input data, and calculates several constants. Specification statements are followed by initialization of pump variables from the main program input data, and calculation of valve port areas for a full 360° revolution. These operations are performed only once, when PUMP is called on the first pump speed. Steady state swash angle or outlet pressure are then calculated, followed by the precompression pressures. Pump outlet flow is calculated and then analyzed by Fourier analysis. The steady state component of the Fourier analysis is then compared to the total steady state circuit flow. If these differ excessively, the swash angle or outlet pressure are corrected and the pump outlet flow is recalculated. When the swash angle or outlet pressure produce a steady state flow essentially equal to the pump and circuit termination leakage flows, the Fourier analysis is completed to calculate harmonic flows up through the harmonic of interest. Harmonic pressure and flow are then balanced dynamically by reconstructing the time dependent output pressure and recomputing flow from section 4. If inlet analysis is not selected, pump outlet flow and pressure for the harmonic of interest are returned to the main program.

If inlet analysis is selected, piston decompression and inlet flow is calculated. If return system analysis is selected, Fourier analysis of inlet flow is performed, followed by dynamic balancing of inlet flow with the return system load. Otherwise, inlet flow is based on the constant steady state inlet pressure and the program goes directly to the hanger torque calculations. Inlet and outlet flow and pressure for the harmonic of interest are then returned to the main program.

The PUMP subroutine is divided into thirteen sections. Each section is discussed and a listing of that section is presented individually in subsequent paragraphs.

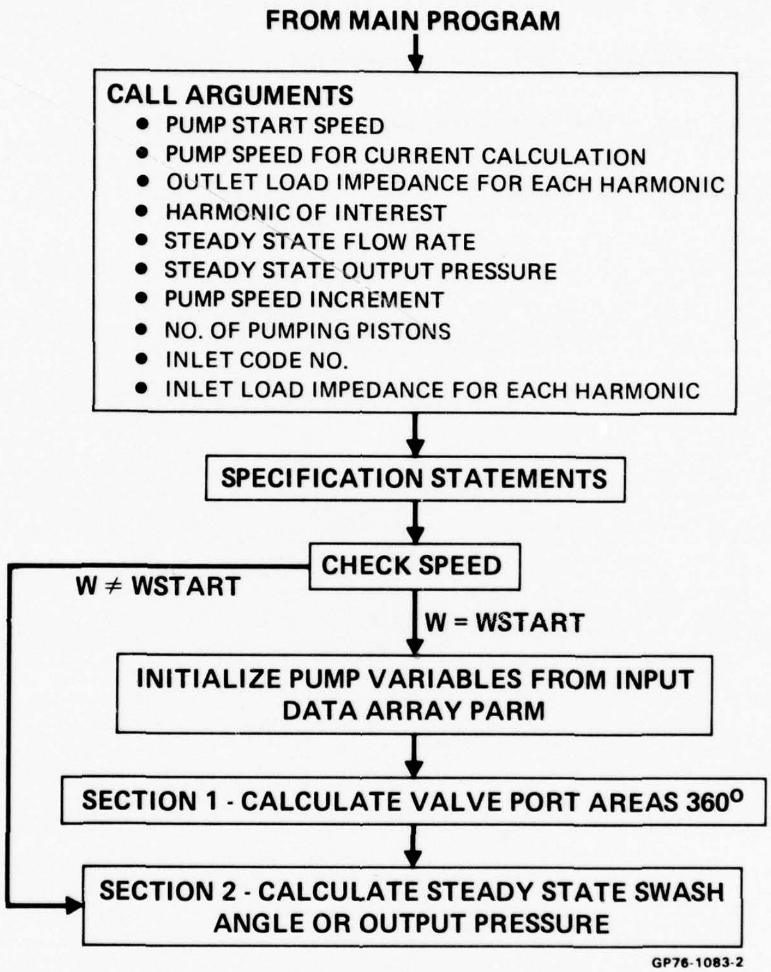
4.1.1 Math Model - See sub-paragraphs in section 4.

4.1.2 Assumptions - See sub-paragraphs in section 4.

4.1.3 Computation Method - See sub-paragraphs of section 4.

4.1.4 Approximations - See sub-paragraphs of section 4.

4.1.5 Limitations - See sub-paragraphs of section 4.



(SEE NEXT PAGE)

FIGURE 4-1
HSFR COMPUTER PROGRAM - PUMP SUBROUTINE FLOW CHART

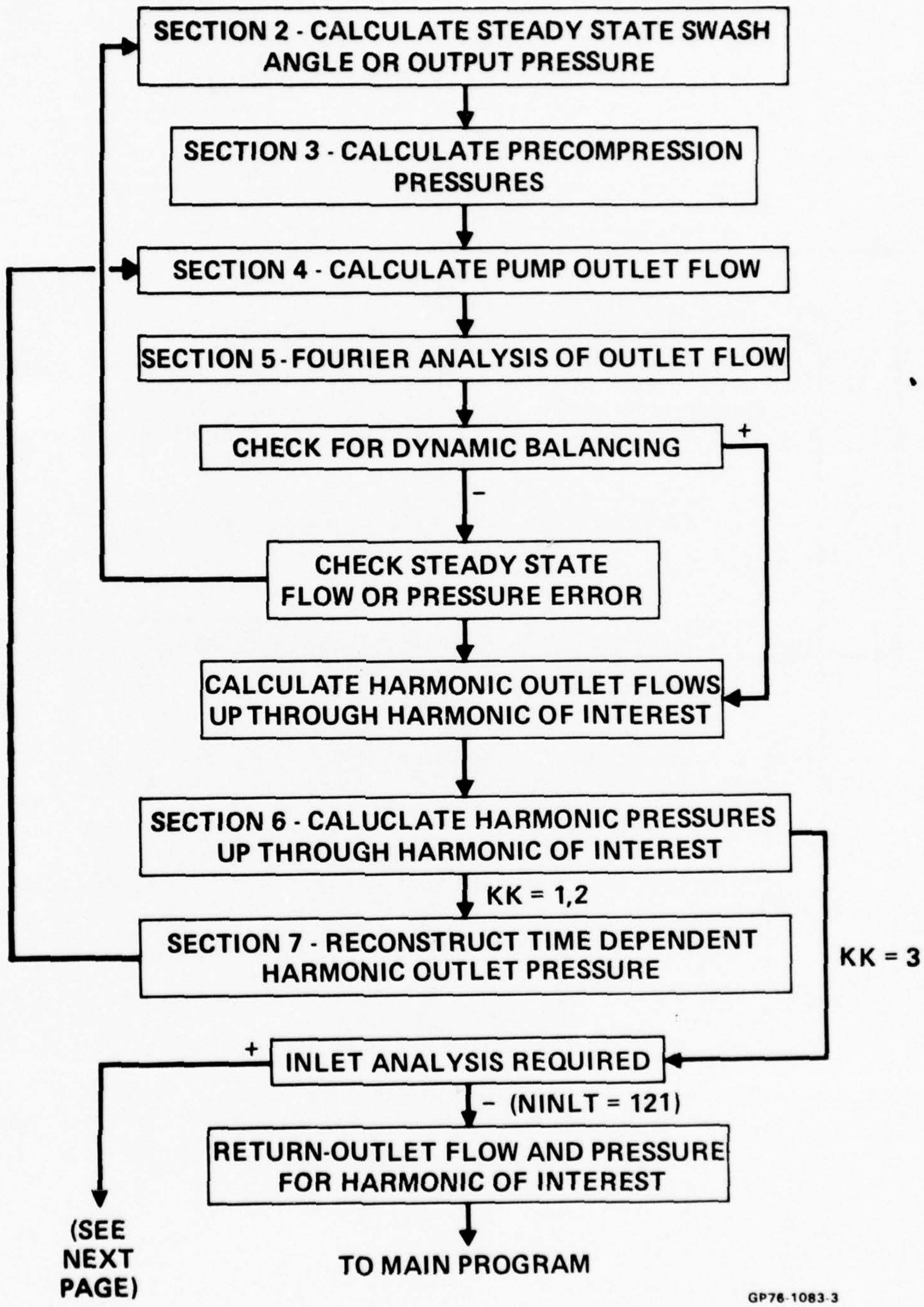


FIGURE 4-1 (Continued)
HSFR COMPUTER PROGRAM - PUMP
SUBROUTINE FLOW CHART

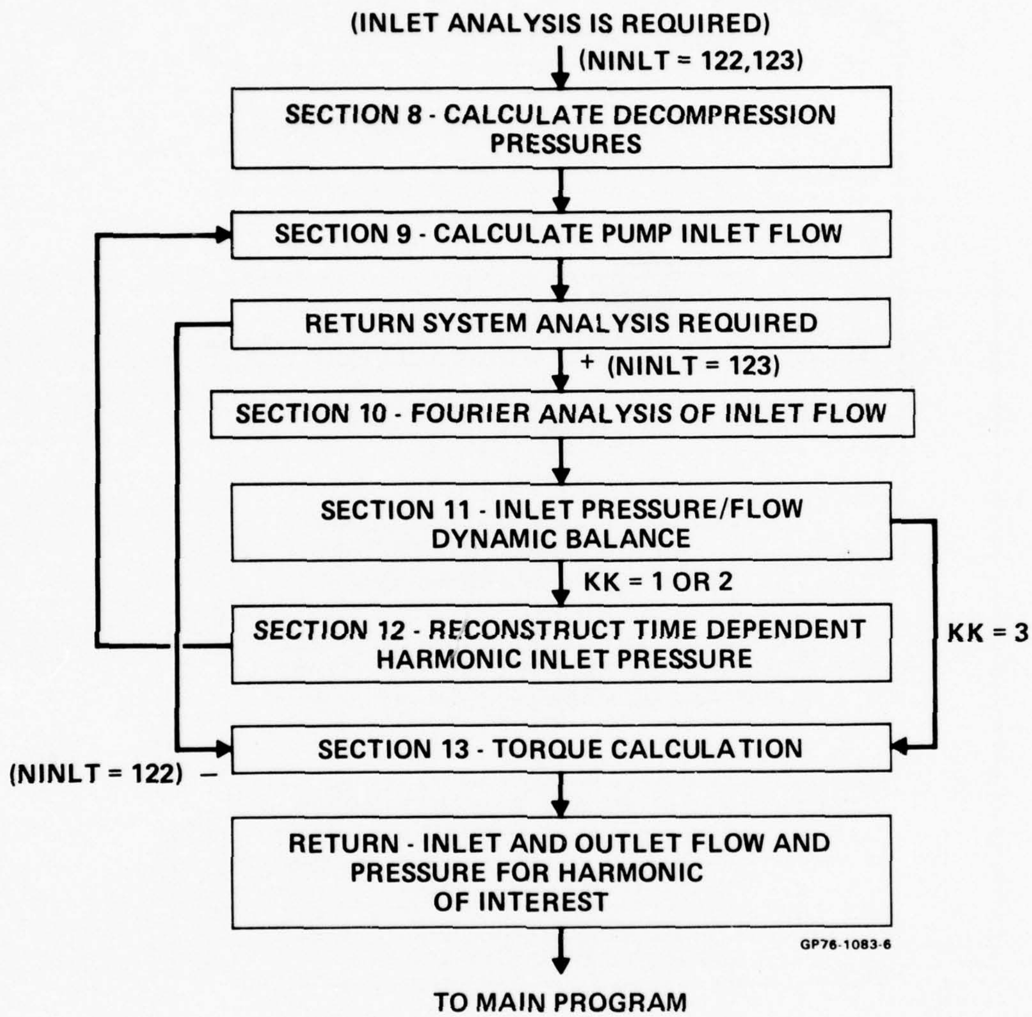


FIGURE 4-1 (Continued)
HSFR COMPUTER PROGRAM - PUMP
SUBROUTINE FLOW CHART

4.1.6 Variable Names - Variable names unique to the PUMP subroutine are listed below. Common variables are discussed in the main program paragraph 3.1.6.1.

SYMBOL	DESCRIPTION	UNITS
ABETA	Angle obtained during calculation of valve area	RAD
ACTLEV	Swashplate actuator lever arm	IN
ACTLEVO	Swashplate actuator lever arm at zero angle	IN
AINC	Incremental shaft rotation angle	DEGREES
ALPHA	Angle obtained during calculation of valve area	RAD
ANG	Angular value of valve opening	RAD
ANGCR	Input data swashplate fixed cross angle	DEGREES
ARACT	Area of swashplate actuator	IN**2
ASWASH	Swashplate variable angle	RAD
ASWAST	Swashplate variable angle for writing in output	DEGREES
ATOFF	Angle of swashplate offset	DEGREES
AVAREA	Temporary variable in calculation of valve area	-
AT	Angular position of pistons	RAD
ATPR	Temporary variable	-
ATSU	Temporary variable	-
BULKP	Bulk modulus during pre-compression	PSI
C	Temporary variable used in Fourier calculation	-
CAVOL	Piston cavitation volume	IN**3
C1	Temporary variable used in Fourier calculation	-
CPRESS	Case pressure - steady state	PSI
CSPRESS	Case to inlet pressure at zero case drain flow	PSI
COEF	Temporary variable used in Fourier calculation	-
DANG	Incremental shaft rotation angle used in pre-compression calculation	RAD

SYMBOL	DESCRIPTION	UNITS
DIACT	Diameter of swashplate actuator - input data	IN
DIAPIS	Input data pumping piston diameter	IN
DISAM	Maximum actuator displacement	IN
DLEAK	Leakage from one cylinder during rotation through incremental angle (DANG)	IN**3
DPRESD	Pressure change in cylinder during decompression	PSI
DPRESP	Pressure change in cylinder during precompression	PSI
DSWASH	Incremental swashangle due to incremental speed change	RAD
DT	Incremental time for rotation through incremental angle (DANG)	SEC
DVOL	Incremental displacement of piston for rotation through (DANG)	IN**3
DX	Incremental piston stroke for rotation through (DANG)	IN
ETA(J)	Percentage error between predicted and resulting J^{th} harmonic pressure	%
FNTZ	Temporary variable used in Fourier analysis	-
FQ1(I,KK)	Complex output flow of the I^{th} harmonic, for the KK^{th} test, from Fourier analysis	CIS
FQ1(KKN,1)	Complex flow for the next harmonic, KKN, equals the last Fourier flow for the next harmonic, calculated from the final KK=3 test balanced flow from the last harmonic-FQ1(KKN,1) = FQ1(KKN,3)	CIS
FQ1I(-,-)	Complex inlet flow	CIS
HPRESS	Pump outlet steady state pressure	PSI
HOFF	Swashplate offset	IN
I	Integer counter	-
IFL	Integer counter for number of steady state balance loop iterations	-
J	Integer counter	-
JK	Not used	-

SYMBOL	DESCRIPTION	UNITS
KK	Integer counter for dynamic balancing test	-
KKN	Order of harmonic (1,2,3,---)	-
LEAK	Constant for piston lap leakage	CIS/PSI
LK1 - LK8	Temporary variables used in flow calculations	-
LPRESS	Input data-steady state inlet pressure	PSI
LPRESD	Piston pressure in de-compression calculation	PSI
LPRESP	Piston pressure in pre-compression calculation	PSI
LVOL	Piston volume in pre-compression calculation	IN**3
M	Integer counter	-
NAPP	Number of active pistons pumping	-
NAPS	Number of active pistons sucking	-
N, NJ, NP	Integer counter	-
NDEG	Integer counter for stepping cylinder rotation in 1/2 degree increments, beginning with NDEG=1 at piston bottom dead center on the swashplate	-
ND1-ND9	Index positions for stepping cylinder rotation through a full revolution, 360°	-
NHARM	Integer form of WHARM	-
NINLT	Inlet identification code number	-
NKM	Index position of first, second, third, or fourth piston during flow calculation	-
NPRSOP	Index position when cylinder slot starts to open to pressure slot	-
NPROP	Index position when cylinder slot is fully open to pressure slot	-
NPRSCL	Index position when cylinder slot starts to close to pressure slot	-
NPRCL	Index position when cylinder slot is fully closed to pressure slot	-
NSUSOP	Index position when cylinder slot starts to open to suction slot	-

SYMBOL	DESCRIPTION	UNITS
NSUOP	Index position when cylinder slot is fully open to suction slot	-
NSUSCL	Index position when cylinder slot starts to close to suction slot	-
NSUCL	Index position when cylinder slot is fully closed to suction slot	-
NSTEPD	Number of steps in de-compression calculation	-
NSTEPP	Number of steps in pre-compression calculation	-
N2-N6	Integer variables for writing valve areas	-
ORF	Orifice coefficient of valve	IN**2/SEC/LB**.5
PACTU	Actuator pressure	PSI
PIA	Piston area	IN**2
PISTNO	Number of pumping pistons	-
POVOL	Input data oil volume between piston at midstroke and port face	IN**3
PIMASS	Piston mass	LBS-SEC**2/IN
PP( ), PPI( )	Internal pressure in pistons 1, 2, 3, or 4 at a given index position	PSI
PISPR	Piston pressure	PSI
PPM(I)	Magnitude of the I^{th} harmonic peak pressure	PSI
PLEAK	Leakage out of cylinder bore	CIS/PSI
PPP(I)	Phase angle of the I^{th} harmonic peak pressure	RAD
PPT( )	Time dependent amplitude of pump output pressure for each rotation index position during output cycle	PSI
PQ1(I,KK)	Complex output test pressure of the I^{th} harmonic, for the KK^{th} test in dynamic balancing	PSI
PQ1I(I,KK)	Complex inlet test pressure of the I^{th} harmonic	PSI
PRESS	Input data for steady state pump output pressure	PSI
PSPG	Actuator pressure due to spring force	PSI
PTP	Not used	

SYMBOL	DESCRIPTION	UNITS
P3,P5,P6,P7	Not Used	
QERR	Error between input and Fourier analysis steady state flow	CIS
QIN	Flow into piston	CIS
QMAX	Maximum flow capability at full swashangle	CIS/RPM
QOUT	Flow out of piston	CIS
QOVB	Total overboard steady state leakage from main program	CIS
QOVBT	Test overboard flow for steady state balancing	CIS
QQ( ), QQI( )	Flow out (in) of piston 1, 2, 3, or 4 at a given index position	CIS
QQFC(I) QIFC(I)	COSINE peak amplitude of pump output (inlet) flow from Fourier analysis for I^{th} harmonic	CIS
QQFS(I) QIFS(I)	SINE amplitude of pump output (inlet) flow from Fourier analysis for I^{th} harmonic	CIS
QQT(N)	Time dependent output flow from pump	CIS
QO	Not used	-
RBORC	Input data cylinder centerline circle radius	IN
RV	Input data cylinder and valve plate slot center line radius	IN
RVX	Width of valve open area	IN
RO,R3	Temporary variables in area calculation	IN
R1	Input data cylinder slot radius	IN
R2	INPUT data valve plate pressure slot radius	IN
R4	Valve plate suction slot radius	IN
S	Temporary variable in Fourier analysis	-
SA	Half amplitude of piston stroke due to controlled angle of swashplate	IN
SSA	Half amplitude of piston stroke due to fixed swash-plate cross angle	IN

SYMBOL	DESCRIPTION	UNITS
SAM	Maximum piston half stroke at maximum swashangle	IN
SLEAK	Leakage into cylinder bore	CIS/PSI
SLOTW	Cylinder slot width	IN
SLTHAG	Cylinder slot width half angle	RAD
SWASH	Input data for maximum swashangle	DEG
THPRS	Input data valve plate pressure slot start angle	DEG
THPRE	Input data valve plate pressure slot end angle	DEG
THSUCS	Input data valve plate suction slot start angle	DEG
THSUCE	Input data valve plate suction slot end angle	DEG
THETA	Angular position of cylinder slot centerline	RAD
THEOLD	Last Angular position of cylinder slot centerline	RAD
TLEAK	Input data pump internal leakage to case	CIS
TLEAKT	Test pump internal leakage for steady state balancing	CIS
TORAAV	Average total torque due to piston inertia	IN-LBS
TORPAV	Average total torque due to piston pressure	IN-LBS
TORQ	Total swashplate torque at each calculation point	IN-LBS
TORQAC	Swashplate torque due to piston inertia at each calculation point	IN-LBS
TORQAS	Summation of TORQAC	IN-LBS
TORQPR	Swashplate torque due to piston pressure at each calculation point	IN-LBS
TORQPS	Summation of TORQPR	IN-LBS
TORQSU	Summation of TORQ	
TORQ1	Temporary variable	-
TORTAV	Total average torque	IN-LBS
TRACAV	Average torque due to cross angle	IN-LBS
TRQACR	Torque due to cross angle at each calculation point	IN-LBS

SYMBOL	DESCRIPTION	UNITS
TRQACS	Summation of TRQACR	IN-LBS
U0,U1,U2, U3	Temporary variables used in Fourier analysis	-
VA	Piston volume at a given index position	IN**3
VAREA( )	Cylinder slot flow area at each index position, 0-360° in 1/2° increments	IN**2
WINC	Input data pump speed increment	RPM
W	Harmonic frequency (same as A in main program)	RAD/SEC
WSTART	Input data first pump speed calculation point	RPM
XA	Piston position at a given index position	IN
XLAST, XOLD	Position of piston at last index position	IN
XNEW	Position of piston at new index position	IN
Y	Current calculation pump speed (same as W in main program)	RPM
ZO	Pump shunt impedance for I^{th} harmonic	PSI/CIS
ZIP( )	Complex impedance of load on pump outlet for each harmonic	PSI/CIS
ZAP( )	Complex impedance of load on pump inlet for each harmonic	PSI/CIS

4.1.7 Specifications and Initialization - Listing

```
      SUBROUTINE PUMP (WSTART,Y,ZIP,NHARM,QOVB,PRESS,WINC,PISTNO,
     +NINLT,ZAP)
C
C * REVISED MARCH 3,1975 *
C
C          *VARIABLE TYPES, DIMENSIONS, COMMONALITY*
C
      REAL LPRESS,LVOL,LEAK,LK1,LK2,LK3,LK4,LK5,LK6,LK7,LK8
      REAL LPRESP,LPRESD
      COMPLEX BETA,G,P,Q,Z,ZV2,ZV
      COMPLEX Q0,Z0,ZIP,P3,ZAP
      COMPLEX FQ1,PQ1,ZEQ,PQ1I,FQ1I
      COMMON BETA,G,P,Q,Z, XBE,BER,BEI,BERP,BEIP,RHO,BULK,VOL,W,VISC,PAR
     1M,PI, IEL,NEL,KTYPE(40)
      DIMENSION G(2,2,40),PARM(8,40),P(40),Q(40),Z(40)
      DIMENSION QQT(81),PPT(81),QQFC(11),QQFS(11),PPM(11) ,PPP(11)
      DIMENSION PP(6),QQ(6),PTP(81)
      DIMENSION ZIP(10),ZAP(10)
      DIMENSION XL(6),VAREA(800)
      DIMENSION FQ1(10,3),PQ1(10,3),ZEQ(10),ETA(10)
      DIMENSION PISPR(800),QQI(81),PPI(81)
      DIMENSION QIFC(11),QIFS(11),FQ1I(10,3),PQ1I(10,3)
      DATA VAREA/800*0.0/,PISPR/800*0.0/
C
C                         *SOLUTION METHOD*
C
C *INITIALIZE VARIABLES FROM INPUT DATA OR MAIN PROGRAM*
C
      IF(Y.NE.WSTART) GO TO 140
      R1=PARM(1,1)
      SLOTW=PARM(2,1)
      RV=PARM(3,1)
      RBORC=PARM(4,1)
      DIAPIS=PARM(5,1)
      POVOL=PARM(6,1)
      R2=PARM(7,1)
      R4=PARM(1,NEL+1)
      SWASH=PARM(2,NEL+1)
      TLEAK=PARM(3,NEL+1)
      ANGCR=PARM(4,NEL+1)
      THPRS=PARM(5,NEL+1)
      THPRE=PARM(6,NEL+1)
      THSUCS=PARM(7,NEL+1)
      THSUCE=PARM(8,NEL+1)
      LPRESS=PARM(1,NEL+2)
      HOFF=PARM(2,NEL+2)
      DISAM=PARM(3,NEL+2)
      ACTLEV0=PARM(4,NEL+2)
      PIMASS=PARM(5,NEL+2)
      CPRESS=PARM(6,NEL+2)
```

4.1.7 Cont'd

```
CSPRES=PARM(7,NEL+2)
DIACT=PARM(8,NEL+2)
ARACT=DIACT**2*PI/4.
QOVBT=QOVB
TLEAKT=TLEAK
ASWASH=SWASH/57.3
NOPIST=PISTNO
SLTHAG=(ASIN(SLOTW/(2.*RV)))*57.3
NAPP=(180.+2.*SLTHAG-THPRS-THPRE)/360.*PISTNO+1.
NAPS=(180.+2.*SLTHAG-THSUCS-THSUCE)/360.*PISTNO+1.
```

4.2 SECTION 1 - VALVE AREA CALCULATION

Figures 4-2, 4-3, and 4-4 illustrate modeling parameters for a typical aircraft rotating piston hydraulic pump, including those required for area calculations.

Section 1 is a generalized computation of the valve flow area for one cylinder at every 1/2 degree position in a full revolution of the cylinder barrel. Area calculation is done once, the first time PUMP is called by the main program for the starting pump speed (W = WSTART). Computed valve areas for each 1/2 degree position are stored in the VAREA array. Valve areas are used in PUMP for outlet and inlet flow, pre-compression, and de-compression calculations.

Section 1 first calculates the angular increment, based on the number of pistons, to produce 720 divisions per revolution, (.5° for a 9 piston pump) and the index positions for the beginning and end of the valve plate pressure and suction slots. Valve areas are then calculated for each portion of the cylinder revolution using the index positions to set up the various DO loop ranges. Area calculations begin with the cylinder centerline at bottom dead center on the swash plate based on the controlled swash angle (NDEG = 1 in Figure 4-2).

4.2.1 Math Model - The basic valve area calculation model is illustrated in Figure 4-5 for the opening of the cylinder slot to the pressure slot. Variables are identified in paragraph 4.1.6. Valve area is initially due to two intersecting circles. Overlapping slot areas are calculated and finally the area is limited by the cylinder slot width itself. As the cylinder slot closes to the pressure slot, areas are obtained by merely reversing the values computed during opening.

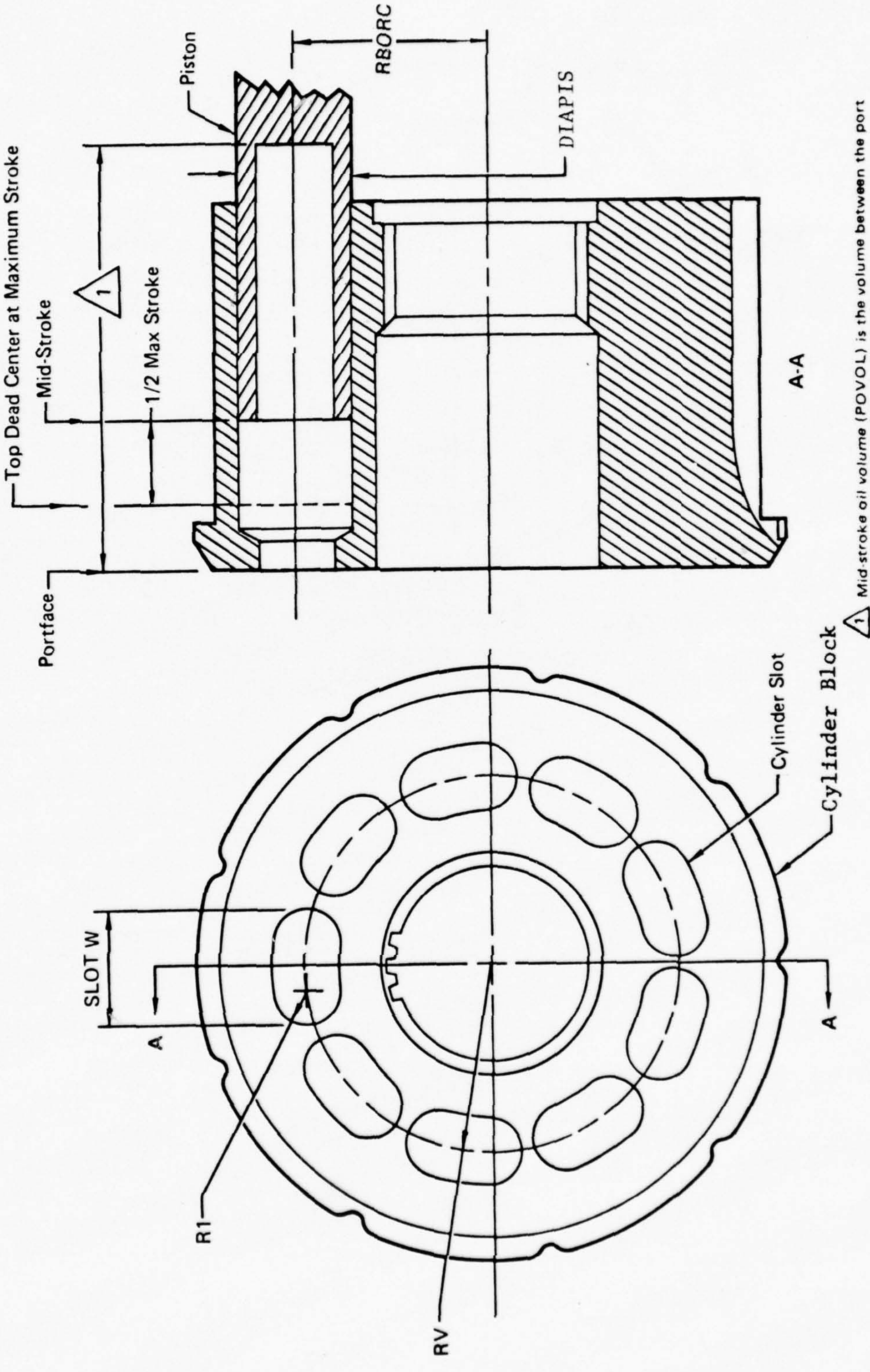


FIGURE 4-2
PUMP CYLINDER BLOCK PARAMETERS

GP75-0108-16

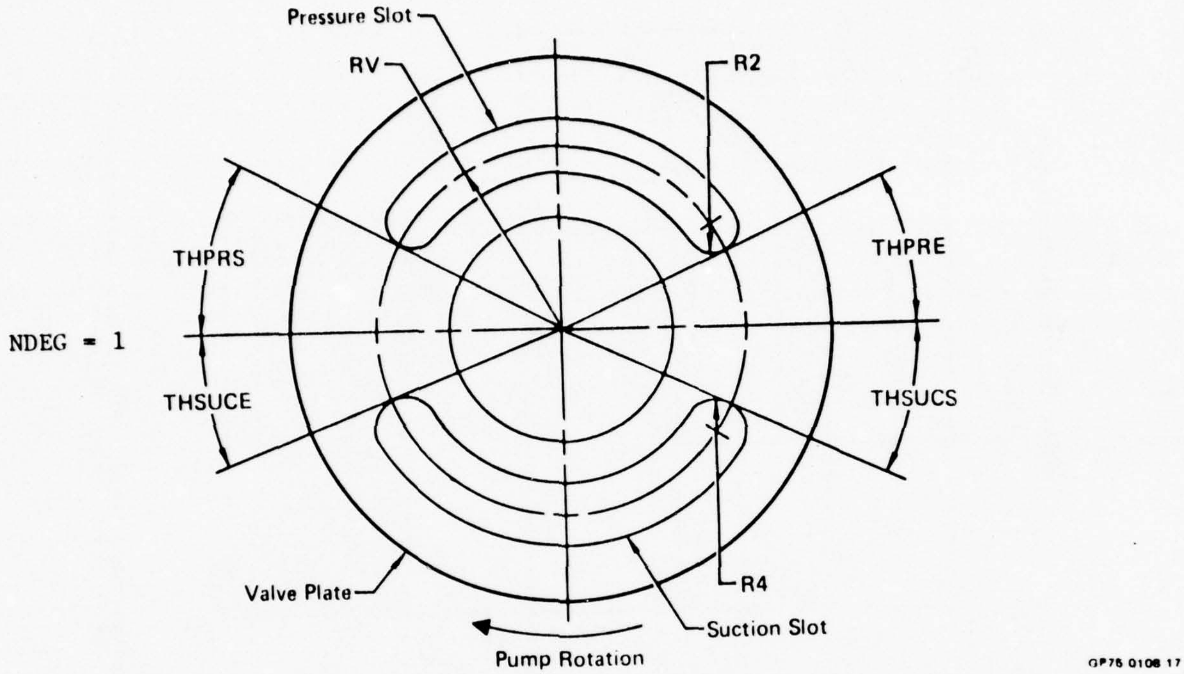


FIGURE 4-3
PUMP VALVE PLATE PARAMETERS

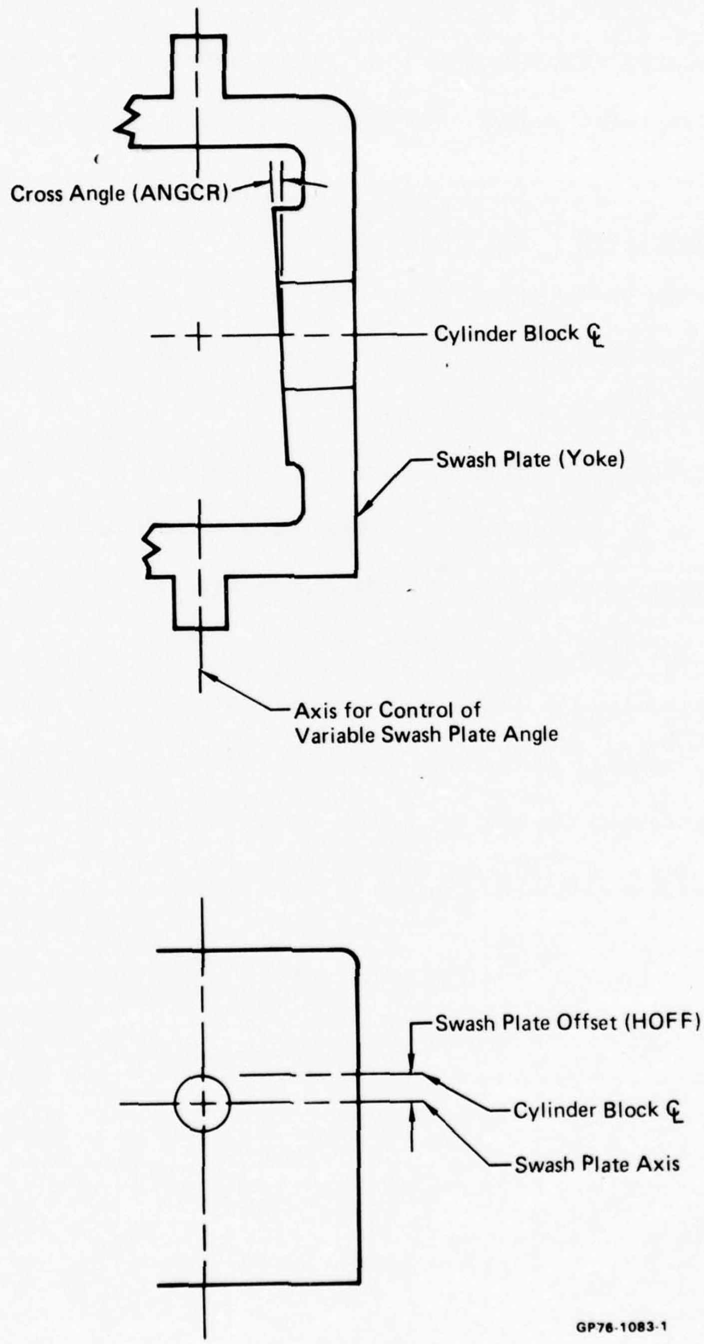


FIGURE 4-4
SWASH PLATE PARAMETERS

Areas for opening and closing the cylinder slot to the suction slot are also calculated with the Figure 4-5 math model, except that the valve plate suction slot radius (R4) is used instead of the pressure slot radius (R2).

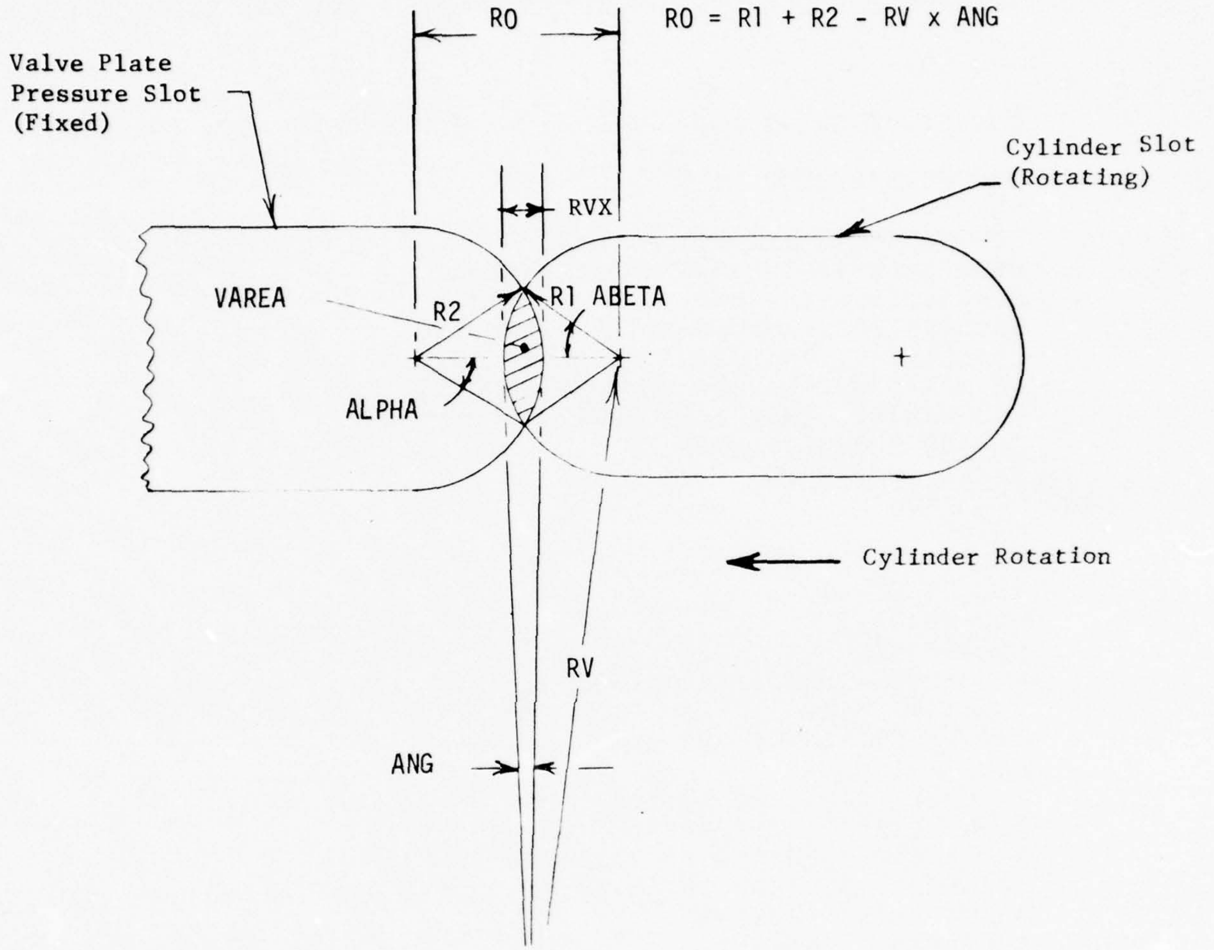
4.2.2 Assumptions - The cylinder slot has semi-circular ends, and is assumed to be rectangular rather than a circular segment. The centerline radii (RV) of all slots are assumed to be equal, i.e. cylinder slot, valve plate pressure and suction slots.

4.2.3 Computation Method - Areas are calculated for each 1/2 degree increment of cylinder block rotation from 0 to 360 degrees.

4.2.4 Approximations - Area formulas in Figure 4-5 are not exact, however, the error is less than .1%.

4.2.5 Limitations - Pumps with non-typical timing (valve plate configurations) may require that valve areas be computed manually and read into the main program, instead of being computed in PUMP.

4.2.6 Variable Names - See paragraph 4.1.6.



$$\cos(ALPHA) = (RO^2 - R1^2 + R2^2)/(2 \times RO \times R2)$$

$$\cos(ABETA) = (RO^2 + R1^2 - R2^2)/(2 \times RO \times R1)$$

$$VAREA = R1^2(ABETA - \sin(ABETA) \times \cos(ABETA)) + R2^2(ALPHA - \sin(ALPHA) \times \cos(ALPHA)) \quad (1)$$

when $RO < 0$

$$VAREA = .5 \times \pi (R1^2 + R2^2) - 2 \times RO \times R3 \quad (2)$$

where: $R3 = R1$ if $R2 > R1$
or $R3 = R2$ if $R1 > R2$

Figure 4-5 Pump Port Valve Area Calculations

4.2.7 Section 1-- Valve Area Calculation - Listing

```
C       SECTION 1 VALVE AREA CALCULATION FOR FULL 360 DEGREES REVOLUTION
C
C
C
C       COMPUTE VALVING INDEX POSITIONS FOR EIGHTY FLOW INCREMENTS
C
        AINC=4.5/PISTNO
        NPRSOP=(THPRS-SLTHAG)/AINC+1.
        NPROP=(THPRS+SLTHAG)/AINC+1.
        NPRSCL=(180.-THPRE-SLTHAG)/AINC+1.
        NPRCL=(180.-THPRE+SLTHAG)/AINC+1.
        NSUSOP=(180.+THSUCS-SLTHAG)/AINC+1.
        NSUOP=(180.+THSUCS+SLTHAG)/AINC+1.
        NSUSCL=(360.-THSUCE-SLTHAG)/AINC+1.
        NSUCL=(360.-THSUCE+SLTHAG)/AINC
        DO 400 NDEG=1,NPRSOP
        VAREA(NDEG)=0.0
  400   CONTINUE
        R3=R1
        IF(R1.GT.R2) R3=R2
        ND1=NPRSOP+1
        ND2=NPROP-1
        DO 500 NDEG=ND1,ND2
        ANG=(NDEG-NPRSOP)*AINC/57.3
        RVX =ANG * RV
        IF(RVX.GT.SLOTW) RVX=SLOTW
        R0=R1+R2 -RVX
        IF(R0.LE.0.01) GO TO 410
        ALPHA=(R0**2-R1**2+R2**2)/(2.0*R0*R2)
        ABETA=(R0**2+R1**2-R2**2)/(2.0*R0*R1)
        IF(ALPHA.GT..9999.OR.ABETA.GT..9999) GO TO 420
        ABETA =ACOS (ABETA)
C360    ABETA=ARCOS(ABETA)
        ALPHA=ACOS (ALPHA)
C360    ALPHA=ARCOS(ALPHA)
        AVAREA=R1*R1*(ABETA-SIN(ABETA)*COS(ABETA))
        VAREA(NDEG)=R2*R2*(ALPHA-SIN(ALPHA)*COS(ALPHA))+AVAREA
        GO TO 500
  410   CONTINUE
        VAREA(NDEG)= 0.5*PI*(R1*R1+R2*R2)-2.0*R0*R3
        GO TO 500
  420   VAREA(NDEG)=0.0
  500   CONTINUE
        DO 550 NDEG=NPROP,NPRSCL
        VAREA(NDEG)=PI*(R3**2)+(SLOTW-2.*R3)*2.*R3
  550   CONTINUE
        ND3=NPRSCL+1
        ND4=NPRCL-1
        I=1
        DO 600 NDEG=ND3,ND4
        VAREA(NDEG)=VAREA(NPROP-I)
        I=I+1
  600   CONTINUE
```

4.2.7 Cont'd

```
      DO 650 NDEG=NPRCL,NSUSOP
      VAREA(NDEG)=0.0
  650 CONTINUE
      IF(R1.GT.R4) R3=R4
      ND5=NSUSOP+1
      ND6=NSUOP-1
      DO 700 NDEG=ND5,ND6
      ANG=(NDEG-NSUSOP)*AINC/57.3
      RVX=ANG*RV
      IF(RVX.GT.SLOTW) RVX=SLOTW
      R0=R1+R2-RVX
      IF(R0.LE.0.01) GO TO 660
      ALPHA=(R0**2-R1**2+R2**2)/(2.0*R0*R2)
      ABETA=(R0**2+R1**2-R2**2)/(2.0*R0*R1)
      IF(ALPHA.GT..9999.OR.ABETA.GT..9999) GO TO 670
      ABETA=ACOS(ABETA)
C360  ABETA=ARCOS(ABETA)
      ALPHA=ACOS(ALPHA)
C360  ALPHA=ARCOS(ALPHA)
      AVAREA= R1*R1*(ABETA-SIN(ABETA)*COS(ABETA))
      VAREA(NDEG)= R2*R2*(ALPHA-SIN(ALPHA)*COS(ALPHA))+AVAREA
      GO TO 700
  660 CONTINUE
      VAREA(NDEG)=0.5*PI*(R1*R1+R2*R2)-2.0*R0*R3
      GO TO 700
  670 VAREA(NDEG)=0.0
  700 CONTINUE
      DO 750 NDEG=NSUOP,NSUSCL
      VAREA(NDEG)=PI*(R3**2)+(SLOTW-2.*R3)*2.*R3
  750 CONTINUE
      ND7=NSUSCL+1
      ND8=NSUCL-1
      I=1
      DO 800 NDEG=ND7,ND8
      VAREA(NDEG)=VAREA(NSUOP-I)
      I=I+1
  800 CONTINUE
      ND9=360./AINC
      DO 850 NDEG=NSUCL,ND9
      VAREA(NDEG)=0.0
  850 CONTINUE
      WRITE(6,840) NPRSOP,NPROP,NPRSCL,NPRCL,NSUSOP,NSUOP,NSUSCL,NSUCL
  840 FORMAT(8(5X,I5))
      NJ=ND9/6
      DO 860 J=1,NJ
      N2=J+NJ
      N3=J+2*NJ
      N4=J+3*NJ
      N5=J+4*NJ
      N6=J+5*NJ
  860 CONTINUE
```

4.3 SECTION 2 STEADY STATE SWASH ANGLE AND OUTPUT PRESSURE CALCULATION

Section 2 calculates the steady state swash angle as a function of pump speed, circuit overboard flow (valve leakage), and pump internal leakage. When the swash angle is maximum, the steady state pump output pressure is calculated.

On the first call of PUMP, certain constants are calculated; cylinder bore area (PIA), maximum half stroke (SAM), maximum flow per rpm (QMAX), orifice flow coefficient (ORF), and the cylinder cavitation volume is set to zero. These calculations are bypassed on all subsequent calls of PUMP. The steady state loop counter (IFL) is initialized to (1) each time PUMP is called for a new pump speed.

Swash angle (ASWASH) is first estimated as a function of the pump speed and the input data for circuit overboard flow and pump internal leakage. If the pump is at full stroke, control passes to the last part of section 2 where a steady state pressure (HPRESS) is calculated. If the pump speed is high enough to meet inputed circuit flow demands, the new swash angle is estimated by adjusting the old angle with an incremental swash angle adjustment calculated from the new and old pump speeds. Various tests are included to insure a stable calculation, and smooth transition from full swash angle at low pump speeds, and from one pump speed to the next.

4.3.1 Math Model

4.3.1.1 Swash Angle - Two methods of calculating swash angle are used. One calculates swash angle by multiplying the maximum swash angle (SWASH) by the ratio of the required total input flow demand (QOVB + TLEAK) to the pump delivery capability at the current speed (QMAX*Y).

$$\text{ASWASH} = \frac{\text{SWASH}}{57.3} \times \frac{(\text{QOVB} + \text{TLEAK})}{(\text{QMAX} \times \text{Y})} \qquad (3)$$

Equation (3) is used for recalculating ASWASH to balance flow, except that adjusted values of flow are used (QOVBT + TLEAKT).

The other method computes an incremental swash angle (DSWASH) based on the new and old pump speeds, where

$$\text{DSWASH} = \text{ASWASH}(2) - \text{ASWASH}(1)$$

From equation (3)

$$\text{DSWASH} = \frac{\text{SWASH(QOVB=TLEAK)}}{57.3\ \text{QMAX}} \times \frac{1}{\text{Y2}} - \frac{1}{\text{Y1}} \quad (4)$$

where Y1 = old pump speed
Y2 = new pump speed = Y (current calculation value)

However,

$$\frac{1}{\text{Y2}} - \frac{1}{\text{Y1}} = \frac{\text{Y1} - \text{Y2}}{\text{Y2Y1}} \quad (5)$$

and the speed increment

$$\text{WINC} = \text{Y2} - \text{Y1}. \quad (6)$$

Combining (5) and (6), and substituting in (4) yields,

$$\frac{1}{\text{Y2}} - \frac{1}{\text{Y1}} = \frac{-\text{WINC}}{\text{Y2} - \text{YxWINC}} \quad (7)$$

$$\text{DSWASH} = \frac{\text{SWASH(QOVB+TLEAK)}}{57.3\ \text{QMAX}} \times \frac{-\text{WINC}}{\text{Y2} - \text{YxWINC}} \quad (8)$$

4.3.1.2 Output Pressure - If the pump is at maximum stroke, steady state pump output pressure (HPRESS) is calculated as a function of the pump maximum delivery capacity, the input flow demand (QOVB + TLEAK), and the input steady state pressure (PRESS).

For an open circuit, the initial estimate of output pressure is

$$\text{HPRESS} = \text{PRESS} \left(\frac{(\text{QMAX x Y})}{(\text{QOVB+TLEAK})}\right)^{1.05} \quad (9)$$

The flow exponent value (1.05) is derived from a typical multi-branch system with integrated actuators having both electrical and mechanical servovalves. Second and subsequent iterations for steady state pressure balancing are calculated by correcting the last pressure, based on the error between the new overboard flow (QOVBT) and the last steady state Fourier analysis flow (QQFC(1)).

$$\text{HPRESS(New)} = \text{HPRESS(old)} \times (1 - (\text{QOVBT}-\text{QQFC}(1))/25.) \quad (10)$$

In a closed circuit, the overboard flow is zero, and

$$\text{HPRESS(New)} = \text{HPRESS(old)} \times \frac{\text{QMAX} \times \text{Y}}{\text{TLEAKT}} \quad (11)$$

Equation (11) is used for both initial and subsequent iterations in closed circuit balancing. The pressure is assumed to vary linearly with flow, based on characterizing the pump internal leakage (TLEAKT) as viscous flow.

4.3.2 Assumptions - See 4.3.1 above.

4.3.3 Computation Method - See 4.3.1 above.

4.3.4 Approximations - See 4.3.1 above.

4.3.5 Limitations - The steady state calculation is not sensitive enough to balance rapidly for very low inputed flow demands at very high pump speeds, e.g. above 10,000 rpm. It may be too sensitive to balance with very high inputed flow demands at very low pump speeds, e.g. 50 gpm at 50 rpm.

4.3.6 Variable Names - See 4.1.6.

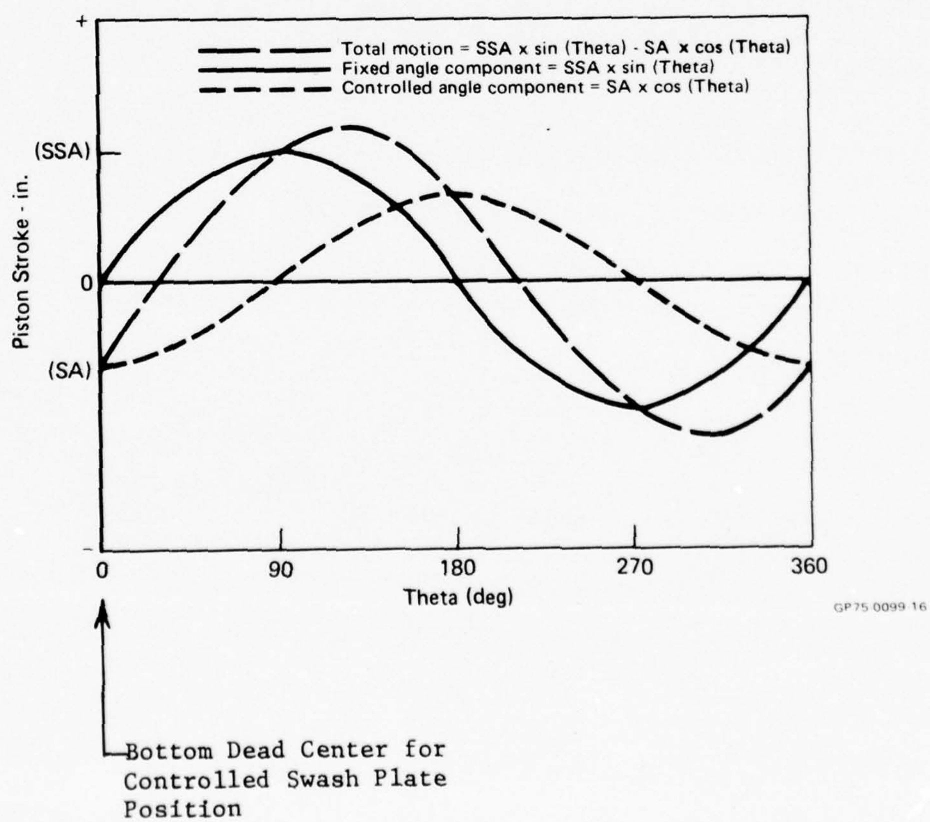
4.3.7 Steady State Swash Angle & Pressure Calculation - Listing

```
C      SECTION 2  SWASH ANGLE AND STEADY STATE OUTPUT PRESSURE CALCULATIO
N
C
       PIA=DIAPIS**2*PI/4.0
       PISPR(NSUCL)=LPRESS
       CAVOL=0.00
       SAM=RBORC*TAN(SWASH/57.3)
       QMAX=0.3*SAM*PIA
       ORF=2.0/RHO
       ORF=.65*SQRT(ORF)
   140 CONTINUE
       IFL=1
       IF(ASWASH.LT.SWASH/57.3) GO TO 151
       ASWASH=SWASH*(QOVB+TLEAK)/(QMAX*Y)/57.3
       IF(ASWASH.GT.SWASH/57.3) ASWASH=SWASH/57.3
       GO TO 158
   151 IF(Y.EQ.WINC) GO TO 156
       DSWASH=WINC/((Y)**2.-Y*WINC)
       DSWASH=SWASH*(QOVB+TLEAK)*DSWASH/QMAX/57.3
       IF(DSWASH.GE.ASWASH) GO TO 156
       ASWASH=ASWASH-DSWASH
       GO TO 156
   155 ASWASH=SWASH*(QOVBT+TLEAKT)/(QMAX*Y)/57.3
   156 IF(ASWASH.LT.SWASH/57.3) GO TO 178
       ASWASH=SWASH/57.3
   158 IF(QOVB.EQ.0.0) GO TO 165
       IF(IFL.NE.1) GO TO 159
       HPRESS=PRESS*((QMAX*Y)/(QOVB+TLEAK))**1.05
       GO TO 175
   159 HPRESS=HPRESS*(1.-(QOVBT-QQFC(1))/25.)
       GO TO 175
   165 HPRESS=PRESS*QMAX*Y/TLEAKT
   175 IF(HPRESS.GE.PRESS.OR.HPRESS.LT.1.0) HPRESS=PRESS
       GO TO 179
   178 HPRESS=PRESS
```

4.4 SECTION 3 - CALCULATION OF PISTON PRE-COMPRESSION PRESSURE

Section 3 calculates the cylinder pressure which exists just before the cylinder slot starts to open to the valve plate pressure slot. This pressure is the result of pre-compression in the cylinder during that portion of cylinder block rotation when the cylinder slot is blocked by the valve plate, between the suction and pressure slots.

4.4.1 Math Model - Piston motion is the sum of two sinusoidal motions as shown in the following diagram.



Total piston motion is the sum of the fixed angle (ANGCR) component and the controlled angle (ASWASH) component. A pressure dependent factor for leakage from each cylinder to case is estimated for cylinder pressures above inputed case pressure as

PLEAK = TLEAK /(PRESS*NAPP). (12)

Leakage from the case to each cylinder for cylinder pressures below case pressure is estimated from the factor

SLEAK = -(TLEAK/NAPS/SQRT(CSPRES)). (13)

Cavitation volume in the cylinder, if any, is calculated and tracked throughout the cylinder block revolution. Piston pressures are stored for each position throughout the 360° calculation. Time dependent oscillatory outlet pressure is initialized to zero PSI.

4.4.2 Assumptions - If inlet analysis is not selected, the piston is assumed to be completely filled on the suction stroke, and the initial cylinder pressure is assumed to be the inputed steady state value. Initial cylinder pressure is assumed to be that of the last speed calculation when inlet analysis is performed. It is started at the steady state value for the first calculation at the first speed. Pressure dependent leakage, and sinusoidal piston motion are assumed. Bulk modulus is recalculated at each step based on the last step cylinder pressure, and the bulk modulus formula used in FLUID.

4.4.3 Computation Method - The calculation is performed in 1/2 degree increments (DANG) with the initial cylinder slot centerline angle (THETA) computed from the suction slot end angle (THSUCE) and the cylinder slot half-angle (SLTHAG). The number of calculation steps (NSTEPP) is calculated based on index positions defining the end of the suction slot and the beginning of the pressure slot in the value plate.

4.4.4 Approximations - See 4.4.2 above.

4.4.5 Limitations - Dynamic effects on the initial inlet pressure are not included.

4.4.6 Variable Names - See 4.1.6.

4.4.7 Section 3 - Calculation of Piston Pre-Compression Pressure - Listing

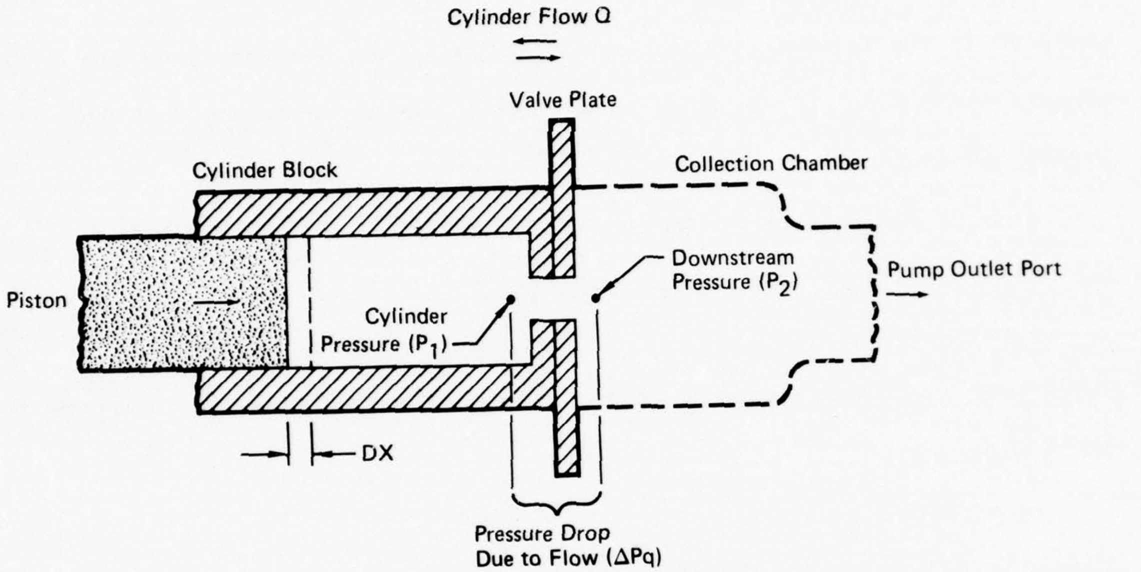
```
C      SECTION 3- PISTON PRECOMPRESSION CALCULATION
C
  179 NSTEPP=ND1+ND9-ND8-2
      DPRESP=0.0
      LPRESP=PISPR(NSUCL)
      THETA=(SLTHAG-THSUCE)/57.3
      PLEAK=TLEAK/(PRESS*NAPP)
      SLEAK=-(TLEAK/NAPS/SQRT(CSPRES))
      DANG=AINC/57.3
      DT=AINC/(Y*6.)
      SA=RBORC*TAN(ASWASH)
      SSA=RBORC*TAN(ANGCR/57.3)
      XLAST =SSA*SIN(THETA)-SA*COS(THETA)
C     WRITE(6,925) CAVOL,NSUCL,LPRESP,XLAST,THETA
      DO 160 I=1,NSTEPP
      IF(LPRESP.LT.CPRESS) GO TO 162
      DLEAK=(LPRESP+DPRESP/2.-CPRESS)*PLEAK*DT
      GO TO 164
  162 DLEAK=SQRT(CPRESS-LPRESP)*SLEAK*DT
  164 BULKP=BULK+12.*(LPRESP-PRESS)
      THETA=THETA +DANG
      XNEW =SSA*SIN(THETA)-SA*COS(THETA)
      DX= XNEW -XLAST
      DVOL=DX*PIA
      LVOL=POVOL-XNEW*PIA
      DPRESP=(DVOL-DLEAK-CAVOL)/LVOL*BULKP
      XLAST=XNEW
      LPRESP=LPRESP+DPRESP
      NP=NSUCL+I
      IF(NP.GE.ND9+1) NP=NP-ND9
      IF(LPRESP.GT.0.01) GO TO 166
      CAVOL=CAVOL-DVOL+DLEAK
      LPRESP=0.01
      DPRESP=0.0
      GO TO 169
  166 CAVOL=0.0
  169 PISPR(NP)=LPRESP
C     IF(Y.EQ.50.) GO TO 161
C     IF(Y.EQ.1000.) GO TO 161
      IF(Y.EQ.3600.) GO TO 161
      GO TO 160
  161 WRITE(6,930) CAVOL,NP,LPRESP,
     +XLAST,DX,LVOL,DLEAK,BULKP,THETA
  160 CONTINUE
      CAVOLD=CAVOL
      XOLD=XLAST
      THEOLD=THETA
      DO 150 N=1,81
  150 PPT(N)=0.0
      KKN = 1
      KK = 1
  170 CONTINUE
```

4.5 SECTION 4 - PUMP OUTLET FLOW CALCULATION

Section 4 calculates the total output flow from the pump for one cycle, i.e., 40° of cylinder block rotation for a nine piston pump. Each of the active pumping pistons (NAPP) is sequentially incremented through 80 steps. The total outlet flow is determined by summing that from each piston. The calculation is started one index step after the pre-compression ends. Pressure in the first cylinder is initially the final pre-compression value. Pressure in the other open cylinders is equal to the sum of the previously calculated steady state output pressure (HPRESS) and time dependent oscillating pressure (PPT).

Cylinder pressure and flow calculated at each step taking into account piston and valve plate leakage, pressure drop across the valve, piston motion, and fluid compressibility. QQT(1) is made equal to QQT(81) to reduce the effects of the calculation start-up discontinuity, caused by assumed initial cylinder pressures.

4.5.1 Math Model - Consider an active pump cylinder with its slot exposed to the valve plate pressure slot, and stroking through an incremented stroke (DX).



Assuming that flow through the slots follows the classic orifice flow equation, then

$$Q = C_d \times VAREA\sqrt{\frac{2\,\Delta P_q}{RHO}} \qquad (14)$$

where C_d = orifice flow coefficient
= .65

Squaring both sides of (32)

$$Q^2 = \frac{2C_d^2 x VAREA^2}{RHO} \times \Delta Pq$$

If $LK4 = \frac{2C_d^2 \times VAREA^2}{RHO}$, then

$$Q^2 = LK4 \cdot \Delta Pq \quad . \qquad (15)$$

Cylinder pressure (P_1) at the end of the incremental stroke DX may be estimated two ways. The first estimate includes the rise due to stroke compression, the loss due to leakage, and the loss due to flow, such that

$$P_1 = \frac{PP + \frac{BULK\ DX\ PIA}{VA}}{1 + \frac{BULK\ LEAK\ DT}{VA}} - \Delta P_f \quad (16)$$

or $$P_1 = LK2 - \Delta P_f \quad (17)$$

where PP is the initial cylinder pressure, ΔP_f is the pressure loss due to an average fluid flow Q over time DT, and LK2 is the net pressure resulting from stroke and leakage.

Another estimate of P_1 may be derived from the downstream pressure (P_2) and the orifice pressure drop (ΔP_q) such that

$$P1 = P_2 + \Delta P_q. \quad (18)$$

Downstream pressure is the sum of the steady state (HPRESS) and the harmonic (PPT) pressures.

$$P_2 = HPRESS + PPT \quad (19)$$

Combining (18) and (19) yields

$$P_1 = HPRESS + PPT + \Delta P_q\ . \quad (20)$$

Equating (17) and (20) gives

$$LK2 - \Delta P_f = HPRESS + PPT + \Delta P_q$$

$$\Delta P_q = LK2 + \Delta P_f - HPRESS - PPT \quad (21)$$

Pressure loss due to fluid flow over time DT is estimated as

$$\Delta P_f = \left(\frac{DT \cdot BULK}{VA \cdot LK1}\right) \cdot Q = LK3 \cdot Q \quad (22)$$

where $LK1 = 1 + \left(\frac{BULK \cdot LEAK \cdot DT}{VA}\right)$, i.e. the dimensionless leakage factor from (34). Using (22) in (21) then gives

$$\Delta P_q = LK2 - LK3 \cdot Q - HPRESS - PPT \quad (23)$$

Substituting (23) in (15) gives

$$Q^2 = (LK2 - LK3 \cdot Q - HPRESS - PPT)LK4$$

$$Q^2 + LK4\ LK3 \cdot Q - LK4\ (LK2 - HPRESS - PPT) = 0. \quad (24)$$

Equation (24) may be expressed as

$$Q^2 + 2 \cdot LK5 \cdot Q - LK6 = 0 \tag{25}$$

where

$$LK5 = \frac{LK4 \cdot LK3}{2} \tag{26}$$

$$LK6 = LK4(LK2 - HPRESS - PPT). \tag{27}$$

The solution to Equation (25) is

$$Q = SIGN(LK6) \cdot (-LK5 + \sqrt{LK5^2 + |LK6|}) \tag{28}$$

Letting $LK7 = LK5^2 + |LK6|$ (29)

and $LK8 = -LK5 + \sqrt{LK7}$ (30)

then the new average total flow from the cylinder is

$$QOUT = SIGN(LK8, LK6). \tag{31}$$

The new internal pressure for the Mth piston at the end of the incremental stroke is then estimated as

$$PISPR(NKM) = LK2 - QOUT \cdot LK3 \tag{32}$$

4.5.2 Assumptions - Assumptions used for calculating pump flow are listed below.

a) flow through the cylinder and valve plate slots is described by the orifice equation, using a low orifice coefficient (.65) applicable to high Reynolds numbers, for all area conditions from full open to closed.

b) the last fluid flow calculated is taken as an average flow over the incremental time period.

c) internal leakage from the cylinder is constant over the incremental time period.

d) flow calculated for each active cylinder is additive with no interaction effects between cylinders.

4.5.3 Computation Method - Flow is calculated for one output cycle of 80 increments regardless of the increment size.

4.5.4 Approximations - See paragraph 4.5.1.

4.5.5 Limitations - Early development of the flow math model disclosed instability in the calculation when the old and new flow were averaged. Using the newly calculated flow value as an average flow over the time period produced a stable calculation.

4.5.6 Variable Names - See paragraph 4.1.6 and 4.5.1.

4.5.7 Section 4 - Pump Output Flow Calculation - Listing

```
C       SECTION 4- PUMP OUTPUT FLOW CALCULATION
C
        CAVOL=CAVOLD
        XLAST=XOLD
        THETA=THEOLD
        DO 180 N=1,81
  180   QQT(N)=0.0
C       WRITE(6,925) CAVOL,NP,LPRESP,XLAST,THETA
        DO 200 M=1,NAPP
        DO 190 N=1,80
        NKM=NPRSOP+N+(M-1)*80
        IF(NKM.GE.NPRCL+1) GO TO 190
        THETA=THETA+DANG
        XNEW=SSA*SIN(THETA)-SA*COS(THETA)
        DX=XNEW-XLAST
        VA=POVOL-XNEW*PIA
        XLAST=XNEW
        IF(CAVOL.GT.0.00) GO TO 187
        BULKP=BULK+12.*(PISPR(NKM-1)-PRESS)
        IF(PISPR(NKM-1).GE.CPRESS) GO TO 186
        LEAK=SLEAK
        LK1=1.+SQRT(BULKP)*LEAK*DT/VA
        GO TO 185
  186   LEAK=PLEAK
        LK1=1.+BULKP*LEAK*DT/VA
  185   LK2=(PISPR(NKM-1)+BULKP/VA*DX*PIA)/LK1
        LK3=BULKP*DT/VA/LK1
        LK4= (ORF*VAREA(NKM))**2
        LK5 =LK3*LK4 *0.5
        LK6 = LK4 *(LK2 -HPRESS-PPT(N))
        LK7 =LK5*LK5 +ABS(LK6)
        LK8 = -LK5 +SQRT(LK7)
        QOUT= SIGN(LK8,LK6)
        PISPR(NKM)=LK2-QOUT*LK3
        IF(PISPR(NKM).GT..01) GO TO 188
  187   CONTINUE
        LEAK=SLEAK
        QOUT=-(0.65*VAREA(NKM)*SQRT(2.*(HPRESS+PPT(N))/RHO))
        PISPR(NKM)=.01
        CAVOL=CAVOL-DX*PIA+SLEAK*DT*CPRESS+QOUT*DT
        IF(CAVOL.LE.0.00) CAVOL=0.00
  188   QQT(N)=QOUT+QQT(N)
C       IF(Y.NE.5000.) GO TO 190
C       WRITE(6,930) CAVOL,NKM,PISPR(NKM),XLAST,DX,VA,
C      +LEAK,BULKP,THETA,QQT(N)
  190   CONTINUE
  200   CONTINUE
        QQT(81)=QQT(1)
```

4.6 SECTION 5 - FOURIER ANALYSIS OF PUMP OUTLET FLOW

Section 5 performs a mathematical harmonic analysis of the time dependent pump total output flow calculated in Section 4. Flow is calculated over the cycle period for each harmonic from the fundamental up to and including the input harmonic of interest.

Steady state flow (QQFC(1)) calculated by the Fourier analysis is compared to the inputed steady state overboard flow (QOVB) for swash angles below the maximum angle. If swash angle is maximum, the Fourier flow is compared to the corrected value of overboard flow (QOVBT). If the error is not acceptable, a new estimate is made for pump leakage (TLEAKT) when the model is closed circuit (QOVB=0). For an open circuit, a new estimate of overboard flow (QOVBT) is computed, including a new pump leakage (TLEAKT) if the swash angle is maximum.

New estimates of overboard flow and/or pump leakage are than used to recalculate the steady state swash angle or pressure. Pump output flow and the Fourier analysis sections are then repeated until steady state balance is obtained. Harmonic flows (FQ1(I,KK)) are then calculated from the balanced steady state conditions.

Steady state balancing is by-passed during subsequent dynamic balancing in Section 6.

4.6.1 Math Model - The Fourier analysis is an IBM subroutine (Reference (2)) which has been modified to reduce the cost of running. SINE and COSINE amplitudes are combined into FQ1(I,KK), which is in complex form with its phase related to the opening angle of the first cylinder. A phase of 0 or 2π is at the moment when the first cylinder opens to the pressure slot. The real part is the sine wave amplitude, and the complex part is the cosine wave amplitude.

4.6.2 Assumptions - None

4.6.3 Computation Method - Fourier analysis

4.6.4 Approximations - None

4.6.5 Limitations - None

4.6.6 Variable Names - See 4.1.6

4.6.7 Section 5 - Fourier Analysis of Pump Output Flow - Listing

```
C       SECTION 5- FOURIER ANALYSIS OF PUMP OUTPUT FLOW
C
        COEF=.02469
        C1=.07753
        S1 =SIN(C1)
        C1 =COS(C1)
        S =0.0
        C =1.0
        FNTZ=QQT(1)
        J =1
    210 U2=0.0
        U1=0.0
        I= 81
C         FORM FOURIER COEFFICIENTS RECURSIVELY
    220 U0=QQT(I)+2.0*C*U1-U2
        U2=U1
        U1=U0
        I=I-1
        IF(I-1) 230,230,220
    230 QQFC(J)=COEF*(FNTZ+C*U1-U2)
        QQFS(J)=COEF*S*U1
        IF(J-NHARM-1) 240,250,250
    240 U3=C1*C-S1*S
        S =C1*S+S1*C
        C =U3
        J = J+1
        GO TO 210
    250 QQFC(1)=QQFC(1)*0.5
        IF(QQFC(1).LE.0.01) QQFC(1)=0.01
C
C         CHECK STEADY STATE FLOW ERROR
C
        IF(KK-2) 252,255,255
    252 IF(ASWASH.GE.SWASH/57.3) GO TO 253
        QERR=ABS(QQFC(1)-QOVB)
        GO TO 256
    253 QERR=ABS(QQFC(1)-QOVBT)
    256 ASWAST=ASWASH*57.3
C       WRITE(6,900) ASWAST,SWASH,HPRESS,PRESS,QQFC(1),
C      +QOVB,QOVBT,TLEAKT,TLEAK,Y
    900 FORMAT(/,9(F10.4,3X),F10.0,/)
        IF(QERR.LE.0.0100) GO TO 255
C
C         CORRECT OVERBOARD (VALVE) FLOW AND/OR PUMP INTERNAL LEAKAGE
C
        IF(QOVB.NE.0.0) GO TO 254
        TLEAKT=TLEAKT-QQFC(1)/3.
        IF(TLEAKT.LE.0.001) TLEAKT=0.001
        GO TO 258
    254 IF(ASWASH.LT.SWASH/57.3) GO TO 257
```

4.6.7 (Cont.)

```
      QOVBT=QOVBT-(QOVBT-QQFC(1))/3.0
      GO TO 259
  257 QOVBT=QOVBT+(QOVB-QQFC(1))/3.0
  259 TLEAKT=TLEAK*HPRESS/PRESS
      IF(QOVBT.LE.0.001) QOVBT=0.001
  258 CONTINUE
      IFL=IFL+1
      IF(IFL.EQ.50) GO TO 975
      GO TO 980
  975 STOP 2222
  980 CONTINUE
      IF(ASWASH.GE.SWASH/57.3.AND.HPRESS.LT.PRESS) GO TO 158
      GO TO 155
C
C        COMPUTE HARMONIC FLOWS FROM FOURIER ANALYSIS
C
  255 DO 260 I=1,NHARM
      FQ1(I,KK) = CMPLX(QQFS(I+1), QQFC(I+1))
  260 CONTINUE
```

4.7 SECTION 6 - OUTLET PRESSURE-FLOW BALANCE CALCULATION

After the calculation of pump output flow and its Fourier analysis in Sections 4 and 5, Section 6 estimates the shunt impedance (ZO). Shunt impedance is then combined with the system load impedance (ZIP) to give the total impedance seen by the pump.

To analyze the pump/system interaction, 1/5 of the calculated dynamic flow FQ1(I,KK) is applied to calculate a time dependent pressure output (PPT(J)) in Section 7. This new output test pressure is then used to recalculate the pump output flow from Section 4. The change in output flow thus produced is then used to recalculate the shunt impedance followed by a new estimate of total impedance, and a new dynamic pressure. The resulting change in dynamic pressure is then used to update the time dependent output pressure. Again, pump output flow is recalculated from Section 4 using the new output pressure. Finally, a third output pressure is calculated using the third estimate of output flow and the load impedance. Earlier simulation has shown that this balancing routine produces a very small error ETA ( ), which is calculated so that it may be written out, if desired.

Dynamic balancing is repeated for each harmonic up to and including the input harmonic of interest, beginning with the fundamental harmonic.

4.7.1 Math Model - In earlier empirical work, it was found that the pump dynamic flow amplitude varied with the system input impedance, or more specifically with the dynamic output pressure. Initial calculations of pump output flow were based on a constant output pressure which is the same as working into a zero impedance load. Earlier simulations showed that the reduction in output flow was somewhat analogous to putting a shunt impedance across the output in parallel with the system input impedance. In reality this shunt impedance does not exist because the output flow is reduced by the very presence of the dynamic pressure. However, using the shunt impedance analogy allows the use of conventional theory and has produced some interesting results.

The idea of predicting the pump/system interaction is complicated by the fact that the system input impedance, pump output flow, pump output pressure, and analogous shunt impedance, are all complex quantities with interacting phase and amplitude relationships. It is difficult to understand the actual mechanism that creates the above noted relationship, but the method seems to work.

4.7.2 Assumptions - See 4.7.1 above.

4.7.3 Computation Method - See 4.7.1 above.

4.7.4 Approximations - See 4.7.1 above.

4.7.5 Limitations - Unknown.

4.7.6 Variable Names - See 4.1.6.

4.7.7 Section 6 - Pressure Flow Balance - Listing


```
C       SECTION 6- OUTLET PRESSURE-FLOW BALANCE CALCULATION
C
  270 CONTINUE
      I = KKN
      IF (KK - 2) 280,290,300
  280 CONTINUE
      W=Y*PISTNO*PI*KKN/30.
      Z0=1.0E5/W*(.2,-1.)
      PQ1(I,KK) = FQ1(I,KK )*Z0*ZIP(I)/(Z0 +ZIP(I)) / 5.0
      P6=CABS(PQ1(I,KK))*5.0
      IF(Y.NE.5000.) GO TO 281
      WRITE(6,1682) P6,QQFC(1)
  281 P3=PQ1(I,KK)
      JK=0
      TO= -45
      MNQ= 10
      PPM(I) = CABS(PQ1(I,KK))
      GO TO 320
  290 CONTINUE
      Z0 = PQ1(I,KK-1) /(FQ1(I,KK-1)-FQ1(I,KK))
      PQ1(I,KK) = FQ1(I,KK-1)*Z0*ZIP(I)/(Z0 +ZIP(I))
      P6=CABS(PQ1(I,KK))
      IF(Y.NE.5000.) GO TO 291
      WRITE(6,1682) P6,QQFC(1)
  291 P3=PQ1(I,KK)-PQ1(I,KK-1)
      PQ1(I,KK+1) = PQ1(I,KK) - PQ1(I,KK-1)
      PPP(I) = ATAN2(AIMAG(PQ1(I,KK+1)),REAL(PQ1(I,KK+1)))
      PPM(I) = CABS(PQ1(I,KK+1))
      GO TO 320
  300 CONTINUE
      J=KKN
      PQ1(J,KK) = ZIP(J) * FQ1(J,KK)
      P6=CABS(PQ1(I,KK))
      IF(Y.NE.5000.) GO TO 301
      WRITE(6,1682) P6,QQFC(1)
  301 CONTINUE
      ETA(J) = CABS(100 * (PQ1(J,KK) - PQ1(J,KK-1)) / PQ1(J,KK))
      KKN = KKN + 1
      IF(KKN.GT.NHARM) GO TO 335
      KK = 1
      FQ1(KKN,1) = FQ1(KKN,3)
      GO TO 270
  320 CONTINUE
```

4.8 SECTION 7 - RECONSTRUCTION OF TIME DEPENDENT PRESSURE

Section 7 computes the time dependent output pressure (PPT) from each estimate of complex dynamic output pressure (P3) from Section 6. The dynamic balance counter (KK) is incremented and control is returned to Section 4 until dynamic balancing is completed. Complex pump output balanced flow (Q(1)) and pressure (P(1)) for the harmonic of interest are then stored for returning to the main program. The pump inlet code number (NINLT) is then tested. If inlet analysis is required, execution continues to Section 9. If not, control is returned to the main program.

4.8.1 Math Model - PPT(J) is the array which stores the 80 values of the time dependent pressure for the pump cycle period. The array values, which are initialized to zero for each new RPM, are accumulative as the calculation progresses for each balance iteration and each harmonic.

The value of PPT(J) is added to with each test pressure and harmonic so that it contains the total of the balanced pressures for each harmonic up through the selected harmonic. The final values in the PPT(J) array then represent the actual time dependent total pressure waveform on the downstream side of the pump valve plate.

The PPT(J) value is made up of the last PPT value, and the magnitudes of last complex dynamic pressure (P3) by

$$\text{PPT(J)} = \text{PPT(J)} + \text{REAL(P3)}\cdot\text{SIN(THETA)} + \text{AIMAG(P3)}*\text{COS(THETA)} \qquad (33)$$

where THETA is the cylinder block rotation angle for each incremental position translated into a 2π cycle period, in multiples of the harmonic number (I).

4.8.2 Assumptions - None.

4.8.3 Computation Method - Not applicable.

4.8.4 <u>Approximations</u> - None

4.8.5 <u>Limitations</u> - None

4.8.6 <u>Variable Names</u> - See 4.1.6.

4.8.7 Section 7 - Reconstruct Time Dependent Pressure - Listing

```
C      SECTION 7- RECONSTRUCTION OF TIME DEPENDENT OUTLET PRESSURE
C
       DO 330 J=1,81
       THETA= (J-1)*I*0.07854
       PPT(J) =PPT(J) + REAL(P3)* SIN(THETA) +AIMAG(P3)* COS(THETA)
   330 CONTINUE
       KK = KK + 1
       GO TO 170
   335 CONTINUE
       ASWAST=ASWASH*57.3
       WRITE(6,900) ASWAST,SWASH,HPRESS,PRESS,QQFC(1),
      +QOVB,QOVBT,TLEAKT,TLEAK,Y
       Q(1)=FQ1(NHARM,3)
       P(1)=PQ1(NHARM,3)
       IF(NINLT.EQ.121) GO TO 1900
```

4.9 SECTION 8 - PISTON DECOMPRESSION CALCULATION

The calculation of piston pressure during de-compression are identical to the pre-compression calculation described in Section 3. Index numbers for the de-compression portion of the block revolution are used. Cylinder cavitation volume is tracked continuously. Piston pressure is limited to .01 PSI if the cylinder cavitates.

4.9.1 Section 8 - Piston Decompression Calculation - Listing

```
C       SECTION 8- PISTON DECOMPRESSION CALCULATION
C
        LPRESD=PISPR(NPRCL)
        NSTEPD=NSUSOP-NPRCL
        DPRESD=0.0
        WRITE(6,925) CAVOL,NPRCL,LPRESD,XLAST,THETA
        DO 1000 I=1,NSTEPD
        IF(LPRESD.LT.CPRESS) GO TO 360
        DLEAK=(LPRESD+DPRESD/2.-CPRESS)*PLEAK*DT
        GO TO 365
    360 DLEAK=SQRT(CPRESS-LPRESD)*SLEAK*DT
    365 BULKP=BULK+12.*(LPRESD-PRESS)
        THETA=THETA +DANG
        XNEW =SSA*SIN(THETA)-SA*COS(THETA)
        DX= XNEW -XLAST
        DVOL = DX * PIA
        LVOL= POVOL-XNEW *PIA
        DPRESD=(DVOL-DLEAK-CAVOL)/LVOL*BULKP
        XLAST=XNEW
        LPRESD=LPRESD+DPRESD
        NP=NPRCL+I
        IF(LPRESD.GT.0.01) GO TO 918
        CAVOL=CAVOL-DVOL+DLEAK
        LPRESD=0.01
        DPRESD=0.0
        GO TO 920
    918 CAVOL=0.0
    920 PISPR(NP)=LPRESD
C       IF(Y.EQ.50.) GO TO 922
C       IF(Y.EQ.1000.) GO TO 922
C       IF(Y.EQ.5000.) GO TO 922
C       GO TO 1000
C   922 WRITE(6,930) CAVOL,NP,LPRESD,
C      +XLAST,DX,LVOL,DLEAK,BULKP,THETA
    930 FORMAT(F10.7,5X,I3,5X,F8.2,2X,4(F10.6,2X),F10.0,5X,2(F10.3,5X))
    925 FORMAT(F10.7,5X,I3,5X,F8.2,2X,F10.6,53X,F10.3)
   1000 CONTINUE
        CAVOLD=CAVOL
        XOLD=XLAST
        THEOLD=THETA
        KKN = 1
        KK = 1
        DO 1050 N=1,81
   1050 PPI(N)=0.0
   1100 CONTINUE
```

4.10 SECTION 9 - PUMP INLET FLOW CALCULATIONS

The calculation of pump inlet flow is identical to that described in Section 4 for outlet flow. The initial calculation is based on the inputed steady state inlet pressure. The pump inlet code number (NINLT) is checked at the end of Section 9. If no return system load exists, control passes to Section 13 where swashplate torques are calculated. This omits dynamic balancing at the inlet, but provides a good estimate of torques without modeling the return system. If return system load exists, control passes on to Section 10.

4.10.1 Section 9 - Pump Inlet Flow Calculation - Listing

```
C      SECTION 9- PUMP INLET FLOW CALCULATION
C
       CAVOL=CAVOLD
       XLAST=XOLD
       THETA=THEOLD
       DO 1150 N=1,81
 1150  QQI(N)=0.0
C      WRITE(6,925) CAVOL,NP,LPRESD,XLAST,THETA
       DO 1500 M=1,NAPS
       DO 1400 N=1,80
       NKM=NSUSOP+N+(M-1)*80
       IF(NKM.GE.NSUCL+1) GO TO 1400
       THETA=THETA+DANG
       XNEW=SSA*SIN(THETA)-SA*COS(THETA)
       DX=XNEW-XLAST
       VA=POVOL-XNEW*PIA
       XLAST=XNEW
       IF(CAVOL.GT.0.00) GO TO 1370
       BULKP=BULK+12.*(PISPR(NKM-1)-PRESS)
       IF(PISPR(NKM-1).GE.CPRESS) GO TO 1360
       LEAK=SLEAK
       LK1=1.+SQRT(BULKP)*LEAK*DT/VA
       GO TO 1365
 1360  LEAK=PLEAK
       LK1=1.+BULKP*LEAK*DT/VA
 1365  LK2=(PISPR(NKM-1)+BULKP/VA*DX*PIA)/LK1
       LK3 = BULKP*DT/VA/LK1
       LK4= (ORF*VAREA(NKM))**2
       LK5 =LK3*LK4 *0.5
       LK6=LK4*(LK2-LPRESS-PPI(N))
       LK7 =LK5*LK5 +ABS(LK6)
       LK8 = -LK5 +SQRT(LK7)
       QIN=SIGN(LK8,LK6)
       PISPR(NKM)=LK2-QIN*LK3
       IF(PISPR(NKM).GT..01) GO TO 1380
 1370  CONTINUE
       LEAK=SLEAK
       QIN=-(0.65*VAREA(NKM)*SQRT(2.*(LPRESS+PPI(N))/RHO))
       PISPR(NKM)=.01
       CAVOL=CAVOL-DX*PIA+SLEAK*DT*CPRESS+QIN*DT
       IF(CAVOL.LE.0.00) CAVOL=0.00
 1380  QQI(N)=QIN+QQI(N)
C      IF(Y.NE.5000.) GO TO 1400
C      WRITE(6,930) CAVOL,NKM,PISPR(NKM),XLAST,DX,VA,
C     +LEAK,BULKP,THETA,QQI(N)
 1400  CONTINUE
 1500  CONTINUE
       QQI(81)=QQI(1)
       IF(NINLT.EQ.122) GO TO 350
```

4.11 SECTION 10 - FOURIER ANALYSIS OF PUMP INLET FLOW

The pump inlet flow is mathematically analyzed as described in Section 5 for the outlet flow, except that steady state balancing is not performed.

4.11.1 Section 10 - Fourier Analysis of Pump Inlet Flow - *Listing*

```
C       SECTION 10- FOURIER ANALYSIS OF PUMP INLET FLOW
C
        COEF=.02469
        C1=.07753
        S1 =SIN(C1)
        C1 =COS(C1)
        S =0.0
        C =1.0
        FNTZ=QQI(1)
        J =1
 1610   U2=0.0
        U1=0.0
        I= 81
C         FORM FOURIER COEFFICIENTS RECURSIVELY
 1620   U0=QQI(I)+2.0*C*U1-U2
        U2=U1
        U1=U0
        I=I-1
        IF(I-1) 1630,1630,1620
 1630   QIFC(J)=COEF*(FNTZ+C*U1-U2)
        QIFS(J)=COEF*S*U1
        IF(J-NHARM-1) 1640,1650,1650
 1640   U3=C1*C-S1*S
        S =C1*S+S1*C
        C =U3
        J = J+1
        GO TO 1610
 1650   QIFC(1)=QIFC(1)*.5
C
C         COMPUTE HARMONIC FLOWS FROM FOURIER ANALYSIS
C
 1655   DO 1660 I=1,NHARM
        FQ1I(I,KK) = CMPLX(QIFS(I+1), QIFC(I+1))
 1660   CONTINUE
```

4.12 SECTION 11 - INLET PRESSURE-FLOW BALANCE CALCULATION

The pump inlet flow is dynamically balanced as described for the outlet flow in Section 6.

4.12.1 <u>Section 11 - Inlet Pressure - Flow Balance Calculation - Listing</u>

```
C      SECTION 11- INLET PRESSURE-FLOW BALANCE CALCULATION
C
 1670 CONTINUE
      I = KKN
      IF (KK - 2) 1680,1690,1700
 1680 CONTINUE
      W=Y*PISTNO*PI*KKN/30.
      Z0=1.0E5/W*(.2,-1.)
      PQ1I(I,KK) = FQ1I(I,KK )*Z0*ZAP(I)/(Z0 +ZAP(I)) / 5.0
      P5=CABS(PQ1I(I,KK))
      P6=LPRESS/P5
      IF(Y.NE.5000.) GO TO 1681
      P7=P5*5.0
      WRITE(6,1682) P7,QIFC(1)
 1682 FORMAT(10X,2(F10.4),//)
 1681 P3=PQ1I(I,KK)
      GO TO 1720
 1690 CONTINUE
      Z0 = PQ1I(I,KK-1) /(FQ1I(I,KK-1)-FQ1I(I,KK))
      PQ1I(I,KK) = FQ1I(I,KK-1)*Z0*ZAP(I)/(Z0 +ZAP(I))
      P5=CABS(PQ1I(I,KK))
      P6=LPRESS/P5
      IF(Y.NE.5000.) GO TO 1691
      WRITE(6,1682) P5,QIFC(1)
 1691 P3=PQ1I(I,KK)-PQ1I(I,KK-1)
      PQ1I(I,KK+1) = PQ1I(I,KK) - PQ1I(I,KK-1)
      PPP(I) = ATAN2(AIMAG(PQ1I(I,KK+1)),REAL(PQ1I(I,KK+1)))
      GO TO 1720
 1700 CONTINUE
      J=KKN
      PQ1I(J,KK) = ZAP(J) * FQ1I(J,KK)
      P5=CABS(PQ1I(J,KK))
      P6=LPRESS/P5
 1713 IF(Y.NE.5000.) GO TO 1715
      WRITE(6,1682) P5,QIFC(1)
 1715 ETA(J)=CABS(100*(PQ1I(J,KK)-PQ1I(J,KK-1))/PQ1I(J,KK))
      KKN = KKN + 1
      IF(KKN.GT.NHARM) GO TO 340
      KK = 1
      FQ1I(KKN,1) = FQ1I(KKN,3)
      GO TO 1670
 1720 CONTINUE
```

4.13 SECTION 12 - RECONSTRUCTION OF TIME DEPENDENT INLET PRESSURE

Pump inlet dynamic pressures are reconstructed as described for the outlet pressure in Section 7, except that the minimum inlet pressure is limited to .01 PSI. Balanced inlet flow and pressure are stored for return to the main program.

4.13.1 <u>Section 12 - Reconstruction of Time Dependent Inlet Pressure - Listing</u>

```
C      SECTION 12- RECONSTRUCTION OF TIME DEPENDENT INLET PRESSURE
C
       DO 1730 J=1,81
       THETA= (J-1)*I*0.07854
       PPI(J) =PPI(J) + REAL(P3)* SIN(THETA) +AIMAG(P3)* COS(THETA)
       IF(PPI(J).LT.-LPRESS) PPI(J)=-LPRESS
 1730  CONTINUE
       KK = KK + 1
       GO TO 1100
  340  CONTINUE
       Q(NINLT)=FQ1I(NHARM,3)
       P(NINLT)=PQ1I(NHARM,3)
  350  CONTINUE
```

4.14 SECTION 13 - TORQUE CALCULATION

This section calculates the average torque acting on the swashplate. These include torques due to spring force, piston mass acceleration forces, and piston pressure forces. Certain pump variables are pre-defined for output plotting, if desired. These include swashangle, pre-compression mismatch, piston pressure at the start of de-compression, cavitation volume, de-compression mismatch, piston pressure at the start of pre-compression, actuator pressure due to spring force, and total actuator pressure.

4.14.1 Math Model - The lever arm (ACTLEV) of the swashplate actuator is estimated based on actual pump geometry as a linear function of swashangle.

$$ACTLEV = ACTLEVO + ASWASH*1.428 \tag{34}$$

The swashangle is inversely proportion to pump speed for constant flow. This must be changed to suit the particular pump being modeled, since it is not derived from input data.

The swashplate torque due to piston acceleration is the sum of torques due to inertial forces in the variable and steady state motion planes:

$$TORQAC = Tv + TRQACR \tag{35}$$

$$\text{where } Tv = Fv * TORQ1 \tag{36}$$

$$Tv = M * a * TORQ1 \tag{37}$$

$$Tv = PIMASS * a * TORQ1. \tag{38}$$

The piston force moment arm at each increment of piston position (shaft rotation angle) is

$$TORQ1 = RBORC*Cos(AT) + HOFF \tag{39}$$

For simple harmonic motion, piston acceleration is

$$a = DY * w^2 * Cos(AT). \tag{40}$$

Piston motion maximum amplitude is

DY = (RBORC + HOFF) (DISAM - DISACT)/ACTLEV (41)

for shaft rotation angles from

$[3\pi/2 - SIN^{-1} (HOFF/RBORC)]$ to $[\pi/2 + SIN^{-1} (HOFF/RBORC)]$

and DY = (RBORC - HOFF) (DISAM - DISACT)/ACTLEV

for shaft rotation angles from

$[\pi/2 + SIN^{-1} (HOFF/RBORC)]$ to $[3\pi/2 - SIN^{-1} (HOFF/RBORC)]$,

where (DISAM - DISACT)/ACTLEV = TAN(ASWASH).

The shaft rotation frequency (W) as a function of pumping frequency is

$w = Y\pi/30.$ or (42)

w = W/PISTNO

Inertial piston torque due to the steady state swashplate angle is

$TRQACR = PIMASS * A_c * TORQ1$

where $A_c = w^2 * SSA * SIN(AT)$

Therefore,

$TRQACR = -w^2 * SSA * SIN(AT) * PIMASS * TORQ1.$ (43)

Substituting (41) and (42) into (40), and then into (38) gives

$T_v = ((Y*\pi/30.)**2*(RBORC + HOFF)*(DISAM - DISACT)$

$*COS(AT)/ACTLEV)*PIMASS*TORQ1$ (44)

Substituting (43) and (44) into (35) yields the total piston acceleration torque on the swashplate at any piston position.

Swashplate torque due to piston pressure is

TORQPR = PISPR(NKM) * PIA * TORQ1.

Total swashplate torque due to piston pressure and acceleration is then

TORQ = TORQAC + TORQPR

The torque at each increment of shaft angle is then summed, divided by the number of calculation points, and multiplied by the number of pistons to obtain total average torque (TORTAV);

TORQSU = TORQSU + TORQ

TORTAV = TORQSU/ND9 * PISTNO (45)

4.14.2 Assumptions - Swashplate actuator lower arm is assumed to vary linearly with swashangle.

4.14.3 Computation Method - Not applicable.

4.14.4 Approximations - None.

4.14.5 Limitations - Unknown.

4.14.6 Variable Names - See 4.1.6.

4.14.7 <u>Section 13 - Torque Calculations - Listing</u>

```
C       SECTION 13 - TORQUE CALCULATION
C
        ACTLEV=ACTLEV0+ASWASH*1.428
        TRQACS=0.0
        TORQSU=0.0
        TORQPS=0.0
        TORQAS=0.0
        ATOFF=ASIN(HOFF/RBORC)
        ATPR=0.5*PI+ATOFF
        ATSU=1.5*PI-ATOFF
        DISACT=DISAM-ACTLEV*TAN(ASWASH)
        DO 1810 M=1,NOPIST
        DO 1820 N=1,80
        NKM=NSUCL+N+(M-1)*80
        IF(NKM.GE.ND9+1) NKM=NKM-ND9
        AT=(NKM-1)*AINC/57.3
        TORQ1=RBORC*COS(AT)+HOFF
        TORQPR=PISPR(NKM)*PIA*TORQ1
        TRQACR=-((Y*PI/30.)**2*SSA*SIN(AT)*PIMASS*TORQ1)
        IF(AT.GE.ATSU.OR.AT.LE.ATPR) GO TO 1830
        TORQAC=((Y*PI/30.)**2*(RBORC-HOFF)*(DISAM-DISACT)*
       +COS(AT)/ACTLEV)*PIMASS*TORQ1+TRQACR
        GO TO 1860
  1830  TORQAC=((W/PISTNO)**2*(RBORC+HOFF)*(DISAM-DISACT)*
       +COS(AT)/ACTLEV)*PIMASS*TORQ1+TRQACR
  1860  TORQ=TORQPR+TORQAC
        TORQSU=TORQSU+TORQ
        TORQPS=TORQPS+TORQPR
        TORQAS=TORQAS+TORQAC
        TRQACS=TRQACS+TRQACR
C       IF(Y.EQ.5000.) GO TO 1840
C       GO TO 1820
C1840   CONTINUE
  1820  CONTINUE
  1810  CONTINUE
        TORTAV=TORQSU/ND9*PISTNO
        TORPAV=TORQPS/ND9*PISTNO
        TORAAV=TORQAS/ND9*PISTNO
        TRACAV=TRQACS/ND9*PISTNO
        WRITE(6,1890) TORPAV,TRACAV,TORAAV,TORTAV,Y
  1890  FORMAT(27X,F12.2,F12.6,2(F12.2),F10.0,//)
  1870  CONTINUE
  1880  Q(37)=CMPLX(ASWAST,0.0)
        Q(38)=CMPLX((PISPR(NPRSOP)-HPRESS),0.0)
        Q(39)=CMPLX(PISPR(NPRCL),0.0)
        Q(40)=CMPLX(CAVOLD*1000.,0.0)
        P(37)=CMPLX((PISPR(NSUSOP)-LPRESS),0.0)
        P(39)=CMPLX(PISPR(NSUCL),0.0)
        PSPG=(150.-ASWASH*400.)/ARACT
        PACTU=TORTAV/ACTLEV/ARACT+PSPG+CPRESS
        P(38)=CMPLX(PSPG,0.0)
        P(40)=CMPLX(PACTU,0.0)
  1900  CONTINUE
        RETURN
        END
```

5.0 LINE SUBROUTINE

5.1 Introduction

Subroutine LINE computes the element values of a 2 x 2 matrix which represents the complex relationship between input and output flow and pressure. Both rigid line (tubing) and hose elements are modeled in LINE.

Specification statements are followed by initialization of variables from main program input data, including computation of the line or hose element internal radius from input outside diameter and wall thickness. A frequency dependent friction damping factor (BETA) is then computed. The velocity of sound is then computed depending on whether the circuit element is a rigid tube (KTYPE = 0) or a hose (KTYPE = 1). Element characteristic impedance is computed, and then used to compute the four matrix element values which are returned to the main program.

5.2 Math Model

The LINE math model is represented by the following equation in matrix form.

$$\begin{bmatrix} Q_{(n+1)} \\ P_{(n+1)} \end{bmatrix} = \begin{bmatrix} \cos\left(\frac{jW\ BETA\ L}{C}\right), & -\frac{\pi R^2}{BETA\ RHO\ C} \times \sin\left(\frac{jW\ BETA\ L}{C}\right) \\ \frac{BETA\ RHO\ C}{\pi R^2} \times \sin\left(\frac{jW\ BETA\ L}{C}\right), & \cos\left(\frac{jW\ BETA\ L}{C}\right) \end{bmatrix} \times \begin{bmatrix} Q_{(n)} \\ P_{(n)} \end{bmatrix} \quad (1)$$

Where: (n)is the line element input

(n + 1)is the line element output

BETA = Frequency Dependent Friction Damping Factor

$$= 4.0\left[\frac{1}{R}\sqrt{\frac{VISC}{2W}}\right]^2 - \left(\frac{1}{R}\sqrt{\frac{VISC}{2W}}\right) + \left[\left(\frac{1}{R}\sqrt{\frac{VISC}{2W}}\right) - 1.0\right] j \quad (2)$$

C = Corrected Velocity of Sound Through Fluid

$$= \sqrt{\frac{BULK}{RHO\left(1 + \frac{K \cdot R \cdot BULK}{EA}\right)}} \qquad \underline{\text{Rigid tube}} \quad (3)$$

$$C = \sqrt{\frac{E}{RHO}} \qquad \underline{\text{Hose}} \qquad (4)$$

Equation (1) is from Reference (3), which is reprinted in Appendix A of this report. Use of the friction factor BETA is suggested by References (3) and (4). In early model development, BETA was evaluated from a tabulation based on previously calculated Thompson functions. However, it was found that the approximation for BETA per equation (2) saved computation time, with an error of less than one percent over the frequency range of interest.

Equation (3) takes into account the elastic modulus of the line material and the method of support by modifying the speed of sound through the fluid as suggested in Reference (5). Equation (4) calculates the velocity of sound in a hose using experimental data centered about the steady state operating pressure for an equivalent bulk modulus of elasticity (E), which includes both hose material and fluid elasticity per Equation (5) below.

$$E = \frac{\text{(Pressure Change)}}{\text{(Volume Change)}} \times \text{(Total Hose Volume)} \qquad (5)$$

In the absence of good experimental data, damping losses due to the hysteresis in the hose wall expansion and contraction are not included.

5.3 Assumptions

Basic assumptions relating to equation (1) and (2) are covered in Reference (3). Frequency dependent friction is based on laminar flow. The dynamic friction model may be inaccurate for steady state turbulent conditions. Reference (9) suggests that steady state turbulence does have an effect on laminar viscous conditions at a tube or hose wall. Fortunately, most of the frequency analysis of an aircraft central hydraulic system is conducted for steady state system flow which is laminar in the large central system lines.

Line mounting factors vary from 1.7 to 1.9 per Reference (5) for lines with an expansion joint between anchors to one anchored at one end only. Equation (3) assumes a mounting factor of 1.8.

5.4 Computation Method - Not applicable.

5.5 Approximations - See 5.2 above.

5.6 Limitations - Unknown.

5.7 Variable Names

Variable names unique to the LINE subroutine and/or math model are listed below. Common variables are described in paragraph 3.1.6.1.

SYMBOL	DESCRIPTION	UNITS
A	Line wall thickness	IN
AREA	Line or hose internal cross-sectional area	IN**2
C1	Temporary variable	--
C2	Temporary variable	--
C	Speed of sound in fluid	IN/SEC
E	Line material modulus of elasticity or hose bulk modulus of elasticity	PSI
G1	Complex cosine term in matrix	--
G2	Complex sin term in matrix	--
j	$\sqrt{-1}$	--
jw	Laplace operator	--
K	Line mounting factor = 1.8	--
R	Line internal radius	IN
XL	Line length	IN
Y1	Trig function argument (real)	--
Y	Trig function argument (complex)	--
Z5	Characteristic impedance	--

5.8 LINE Subroutine - Listing

```
      SUBROUTINE LINE
C
C * REVISED MARCH 3,1975 *
C
C         *VARIABLE TYPES, DIMENSIONS, AND COMMONALITY*
C
      COMMON BETA,G,P,Q,Z, XBE,BER,BEI,BERP,BEIP,RHO,BULK,VOL,W,VISC,PAR
     1M,PI, IEL,NEL,KTYPE(40)
      COMPLEX BETA,G,P,Q,Z,Y1,Y,G1,G2,Z5
      DIMENSION G(2,2,40),PARM(8,40),P(40),Q(40),Z(40)
C
C                          *SOLUTION METHOD*
C
C *INITIALIZE VARIABLES FROM INPUT DATA ARRAY*
C
C     LINE OR HOSE LENGTH
C
      XL=PARM(1,IEL)
C
C     LINE WALL THICKNESS(A=0.0 FOR HOSE)
C
      A=PARM(3,IEL)
C
C     LINE OR HOSE INTERNAL RADIUS
C
      R=PARM(2,IEL)/2.0 - A
C
C     LINE MATERIAL MODULUS OR HOSE BULK MODULUS OF ELASTICITY
C
      E=PARM(4,IEL)
C
C *FREQUENCY DEPENDENT FRICTION DAMPING FACTOR*
C
  200 XBE=SQRT(VISC/(2.0*W))/R
      BETA=4.0*XBE**2-XBE+(0.0,1.0)*(XBE-1.0)
C
C *TEST FOR HOSE ELEMENT*
C
      IF(KTYPE(IEL).EQ.1)GO TO 300
C
C *VELOCITY OF SOUND IN LINE*
C
      AREA=PI*R**2
      PARM(6,IEL)=AREA
      C1=1.8*(R)   *BULK
      C2=BULK/(RHO*(1.+C1/(A*E)))
      C=SQRT (C2)
      PARM(5,IEL)=C
      GO TO 400
C
```

5.8 <u>LINE Subroutine - Listing (Continued)</u>

```
C *VELOCITY OF SOUND IN HOSE*
C
  300 C=SQRT(E/RHO)
      PARM(5,IEL)=C
C
C *CALCULATE FLOW & PRESSURE TRANSFER FUNCTION MATRIX ELEMENTS*
C
  400 Y1=W*BETA*XL/C
      Y=Y1*(0.,1.)
      Z5=BETA*RHO*C/(PI*R*R)
      G1=CCOS(Y)
      G2=CSIN(Y)
      G(1,1,IEL)=G1
      G(1,2,IEL)=-(1./Z5)*G2
      G(2,1,IEL)=Z5*G2
      G(2,2,IEL)=G1
      RETURN
      END
```

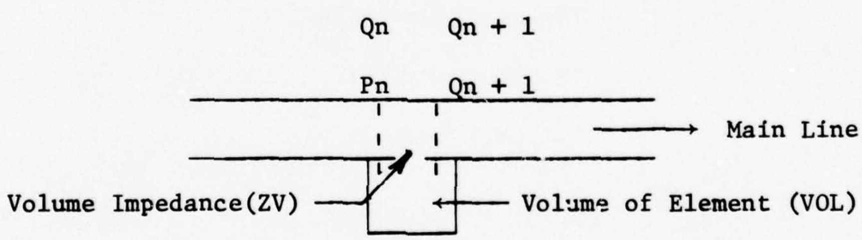
6.0 COMPONENT SUBROUTINES

6.1 VOLUME SUBROUTINE

Subroutine VOLUME computes the element values of a 2 x 2 matrix which represents the complex relationship between input and output flow and pressure across a lumped volume element in the main circuit. Lumped volume elements are treated as pure capacitive elements, with no dynamic inlet and outlet losses.

Specification statements are followed by initialization of the element volume from input data. Volume impedance (ZV) is then computed and used in assigning the matrix values, which are returned to the main program. A test is included to prevent an indefinite value for G (1,2) should the input volume be zero.

6.1.1 Math Model - Lumped volumes are treated as capacitive elements with no dynamic inlet and outlet losses.



Flow and pressure across the volume element are related by:

$$\begin{bmatrix} 1 & -\frac{1}{ZV} \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} Qn \\ Pn \end{bmatrix} = \begin{bmatrix} Qn + 1 \\ Pn + 1 \end{bmatrix}$$

or:

$$Qn - \frac{Pn}{ZV} = Q\ n + 1$$

$$Pn = P\ n + 1 \quad .$$

Impedance for a capacitance is:

$$ZV = \frac{BULK}{j \times W \times VOL} \quad . \qquad (1)$$

6.1.2 Assumptions - The effect on the predicted resonant frequency of ignoring the inlet and outlet losses are minimized by modeling the inlet and outlet lengths as lines up to the volume cavity. The resonant pump speeds predicted have agreed well with those measured on a test stand and the F-15 iron bird, over a wide range of system temperatures.

6.1.3 Computation Method - Not applicable.

6.1.4 Approximations - See 6.1.2 above.

6.1.5 Limitations - None.

6.1.6 Variable Names - Variable names unique to the VOLUME subroutine and/or math model are listed below. Common variables are described in paragraph 3.1.6.1.

SYMBOL	DESCRIPTION	UNITS
j	$\sqrt{-1}$	--
VOL	Volume of element	IN**3
ZV1	Temporary variable	--
ZV2	Temporary variable	--
ZV	Volume impedance	PSI/CIS

6.1.7 VOLUME Subroutine - Listing

```
      SUBROUTINE VOLUME
C
C**** REVISED SEPTEMBER 3,1974 ****
C
C         *VARIABLE TYPES,DIMENSIONS, AND COMMONALITY*
C
      COMPLEX BETA,G,P,Q,Z,ZV2,ZV
      COMMON BETA,G,P,Q,Z, XBE,BER,BEI,BERP,BEIP,RHO,BULK,VOL,W,VISC,PAR

     IM,PI, IEL,NEL,KTYPE(40)
      DIMENSION G(2,2,40),PARM(8,40),P(40),Q(40),Z(40)
C
C                             *SOLUTION METHOD*
C
C *INITIALIZE VOLUME FROM INPUT DATA ARRAY*
C
      VOL=PARM(1,IEL)
C
C *CALCULATE VOLUME IMPEDANCE*
C
      ZV1=W*VOL
      ZV2=CMPLX(0.,ZV1)
      ZV=BULK/ZV2
C
C *ASSIGN MAIN LINE MATRIX ELEMENT VALUES*
C
      G(1,1,IEL)=CMPLX(1.,0.)
      G(2,1,IEL)=CMPLX(0.,0.)
      G(2,2,IEL)=CMPLX(1.,0.)
      IF(VOL.EQ.0.)GO TO 100
      G(1,2,IEL)=-1./ZV
      RETURN
  100 G(1,2,IEL)=(0.,0.)
      RETURN
      END
```

6.2 Resonator Subroutine (RESNTR)

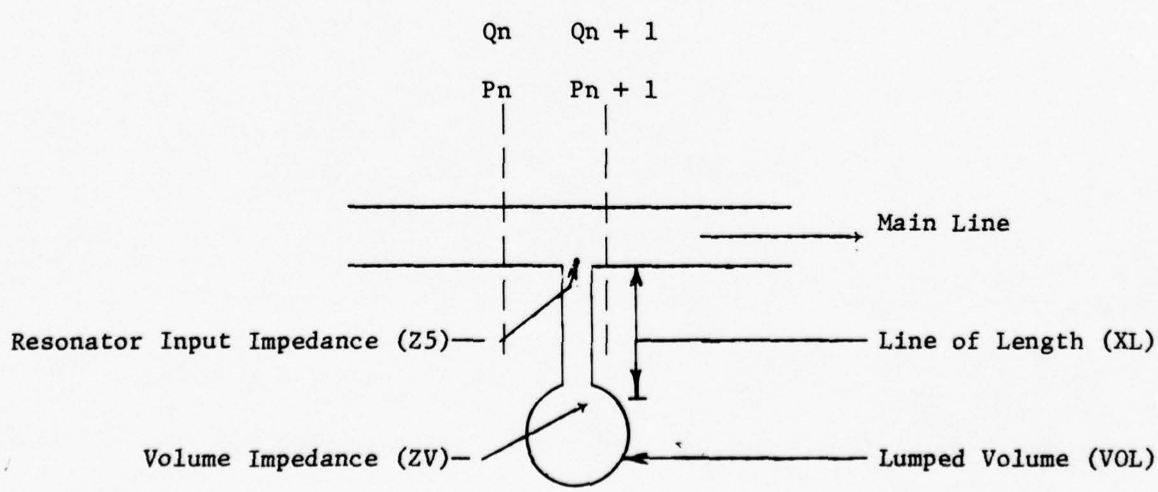
The RESNTR subroutine models two acoustic devices; a short line/lumped volume resonator, and a Pulsco acoustic filter.

RESNTR computes the element values of a 2 x 2 matrix which represents the complex relationship between main line input and output flow across the unit.

Specification statements are followed by initialization of variables for the applicable unit from the main program input data. The appropriate model is selected based on the KTYPE number.

6.2.1 Lumped Volume Resonator (KTYPE 0) - The initial section of RESNTR describes a lumped volume type resonator connected to the main circuit with a length of line. After initialization of variables RESNTR computes the frequency dependent friction factor (BETA), and the velocity of sound in the resonator line. Input impedance (Z5) at the main line junction is then computed, and used to assign matrix values which are then returned to the main program.

6.2.1.1 Math Model - The resonator is modeled by combining two basic elements; a lumped volume, and a line.



The resonator is treated as a line that is terminated by the volume impedance (ZV). Resonator impedance (Z5) is then the input impedance of the line. The subroutine solves for the impedance (Z5) by using the characteristic LINE matrix equation, with the volume (VOL) as the terminating impedance.

Flow and pressure across the line length (XL) are related by:

$$\begin{bmatrix} G11 & G12 \\ G21 & G22 \end{bmatrix} \cdot \begin{bmatrix} Q_5 \\ P_5 \end{bmatrix} = \begin{bmatrix} QV \\ PV \end{bmatrix} .$$

or

$$G11 \times Q5 + G12 \times P5 = QV \tag{1}$$

$$G21 \times Q5 + G22 \times P5 = PV \tag{2}$$

Dividing (2) by (1)

$$\frac{G21 \times Q5 + G22 \times P5}{G11 \times Q5 + G12 \times P5} = \frac{PV}{QV} \tag{3}$$

but

$$\frac{PV}{QV} = ZV \quad . \tag{4}$$

Substituting (4) in (3), and dividing the left side of (3) by Q5 yields

$$\frac{G21 \times \frac{Q5}{Q5} + G22 \times \frac{P5}{Q5}}{G11 \times \frac{Q5}{Q5} + G12 \times \frac{P5}{Q5}} = ZV \quad . \tag{5}$$

Using $Z5 = \frac{P5}{Q5}$ in (5), and then solving for Z5 gives:

$$Z5 = \frac{ZV \times G11 - G21}{G22 + ZV \times G12}$$

or

$$Z5 = (Z1 - Z2)/(Z3 - Z4) \quad . \tag{6}$$

Matrix element values in (6) for the branch line element are computed as described in paragraph 5.1.1 of the LINE subroutine. Volume impedance (ZV) is described in paragraph 6.1.1.

6.2.1.1 *(Continued)*

Matrix values for the main line are determined as

$$\begin{bmatrix} 1 & \frac{1}{Z5} \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} Qn \\ Pn \end{bmatrix} = \begin{bmatrix} Q\ n+1 \\ P\ n+1 \end{bmatrix} \qquad (7)$$

or

$$Qn - \frac{Pn}{Z5} = Q\ n + 1 \qquad (8)$$

$$Pn = P\ n + 1 \qquad (9)$$

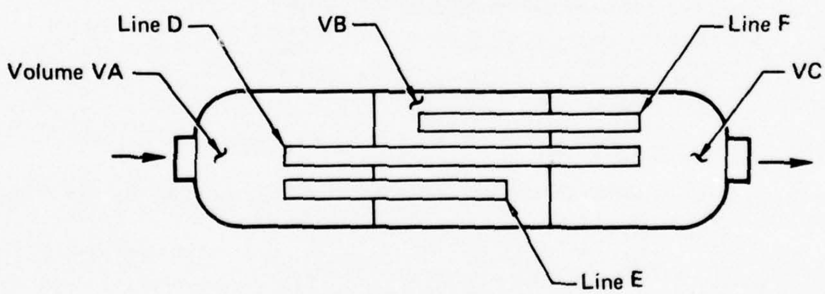
6.2.1.2 Assumptions - Same as 5.1.2 and 6.1.2.

6.2.1.3 Computation Method - Not applicable.

6.2.1.4 Approximations - None.

6.2.1.5 Limitations - Entrance angle effects, if any, are not included.

6.2.2 Pulsco Acoustic Filter (KTYPE = 32) -



NOTE: This device is manufactured and marketed by the

Pulsco Division
American Air Filter Co., Inc.
Louisville, Ky.

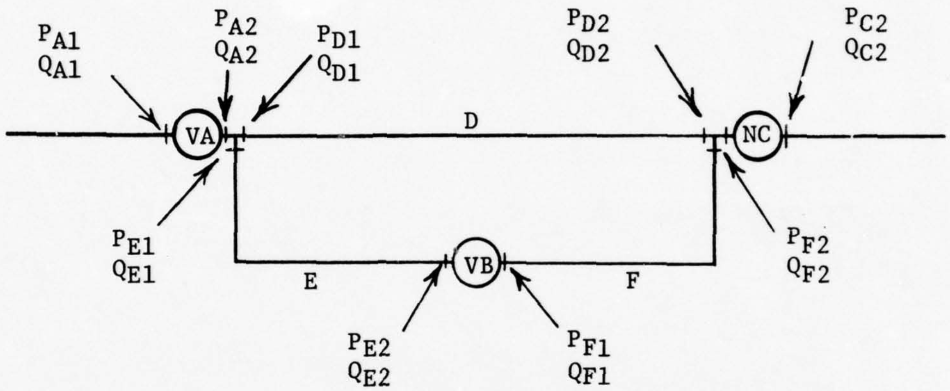
The design disclosed herein is the property of American Air Filter Co., Inc., Pulsco Division.

The Pulsco acoustic filter consists basically of three volumes interconnected as shown by three lines. For KTYPE 32, RESNTR moves to the section for the acoustic filter and initializes the three volume values. Volume impedance values are then calculated. Line variables are initialized and line matrix

6.2.2 (Continued)

values are calculated for all three lines in the DO 120 loop. Main line matrix values are then calculated and assigned to the G(-, -, -) array.

6.2.2.1 Math Model - The Pulsco acoustic filter is characterized as two parallel flow paths between the end volumes



The model is derived by combining the dynamics of three basic elements; volumes, lines, branches. Equations relating the pressure and flow across the various basic elements are as follows based on math models in sections 3.3, 5.0, and 6.1.

Volume (VA)

$$QA1 - PA1/ZVA = QA2$$

but $QA2 = QD1 + QE1$

therefore

$$QA1 - PA/ZVA = QD1 + QE1 \qquad (10)$$

where, for the first branch

$$PA = PA1 = PA2 = PD1 = PE1.$$

Volume (VB)

$$QE2 - PE2/ZVB = QF1$$

and $PB = PE2 = PF1$

therefore

$$QE2 - PB/ZVB = QF1 \qquad (11)$$

6.2.2.1 (Continued)

Volume (VC)

$$QC1 - PC/ZVC = QC2$$

but $QD2 + QF2 = QC1$

therefore

$$QD2 + QF2 - PC/ZVC = QC2 \quad (12)$$

where $PC = PF2 = PC1 = PC2 = PD2$ (13)

Line (D)

$$D11 \cdot QD1 + D12 \cdot PA = QD2 \quad (14)$$

$$D21 \cdot QD1 + D22 \cdot PA = PC \quad (15)$$

Line (E)

$$E11 \cdot QE1 + E12 \cdot PA = QE2 \quad (16)$$

$$E21 \cdot QE1 + E22 \cdot PA = PB \quad (17)$$

Line (F)

$$F11 \cdot QF1 + F12 \cdot PB = QF2 \quad (18)$$

$$F21 \cdot QF1 + F22 \cdot PB = PC \quad (19)$$

First analyzing the branch path E - VB - F, starting from (11)

$$QE2 = QF1 + PB/ZVB.$$

Substituting for QE2 in (16) gives

$$E11 \cdot QE1 + E12 \cdot PA = QF1 + PB/ZVB.$$

From (18)

$$QF1 = (QF2 - F12 \cdot PB)/F11,$$

therefore

$$E11 \cdot QE1 + E12 \cdot PA = (QF2 - F12 \cdot PB)/F11 + PB/ZVB.$$

By using (17) to eliminate PB, it can be shown that

$$A \cdot QE1 + B \cdot PA = QF2 \quad (20)$$

where:

$$A = F11 \cdot E11 + F12 \cdot E21 - (F11 \cdot E21)/ZVB$$

$$B = F11 \cdot E12 + F12 \cdot E22 - (F11 \cdot E22)/ZVB.$$

6.2.2.1 (Continued)

From (18) and (19) it can be shown that

$$PB = (F11 \cdot PC - F21 \cdot QF2)/(F11 \cdot F22 - F21 \cdot F12).$$

Substituting this into (17) for PB, and (20) into (17) for QF2 yields

$$C \cdot QE1 + D \cdot PA = PC \qquad (21)$$

where:

$$C = F22 \cdot E21 - (F21 \cdot F12 \cdot E21)/F11 + (F21 \cdot A)/F11$$

$$D = F22 \cdot E22 - (F21 \cdot F12 \cdot E22)/F11 + (F21 \cdot B)/F11.$$

The first branch may be analyzed by starting with (10), substituting (15) for QD1 and eliminating QE1 using (21) to yield

$$E \cdot QA1 + F \cdot PA = PC \qquad (22)$$

where:

$$E = (D21 \cdot C)/(C + D21)$$

$$F = (D22 \cdot C + D21 \cdot D - (D21 \cdot C)/ZVA)/(C + D21).$$

The second branch may be analyzed by first eliminating QD1 from (14) using (15) to yield

$$QD2 = (D11 \cdot PC)/D21 - (D11 \cdot D22 \cdot PA)/D21 + D12 \cdot PA. \qquad (23)$$

Substituting (23) into (12) to eliminate QD2, (22) to eliminate PC, and (20) to eliminate QF2 yields

$$(D11 \cdot E \cdot QA1)/D21 + (D11 \cdot F \cdot PA)/D21 - (D11 \cdot D22 \cdot PA)/D21 + D12 \cdot PA$$
$$+ A \cdot QE1 + B \cdot PA - PC/ZVC = QC2. \qquad (24)$$

From (21) and (22)

$$QE1 = E \cdot QA1/C + F \cdot PA/C - D \cdot PA/C. \qquad (25)$$

Using (22) and (25) in (23) yields

$$G \cdot QA1 + H \cdot PA = QC2 \qquad (26)$$

where:

$$G = D11 \cdot E/D21 + A \cdot E/C - E/ZVC$$

$$H = D11 \cdot F/D21 - D11 \cdot D22/D21 + D12 + A \cdot F/C - A \cdot D/C + B - F/ZVC$$

6.2.2.1 (Continued)

The dynamic pressure and flow across the filter are then given by (22) and (26), which in the standard 2 x 2 matrix form used for all components is

$$\begin{bmatrix} G & H \\ E & F \end{bmatrix} \cdot \begin{bmatrix} QA1 \\ PA \end{bmatrix} = \begin{bmatrix} QC2 \\ PC \end{bmatrix} \quad (27)$$

The matrix values E, F, G, H are computed in the model as ZE, ZF, ZG, and ZH. The matrix values for the three lines are computed in the model as values in the GL (-, -, -) array for lines 1, 2, and 3. For instance, the "12" matrix value for line D (line 1) is GL(1, 2, 1). Volume impedances are computed as derived, i.e. ZVA, ZVB, and ZVC. Matrix values for the acoustic filter (ZE, ZF, ZG, and ZH) are then assigned to the G(-, -, IEL) array for return to the main program.

6.2.2.2 Assumptions - Same as for line and volume elements.

6.2.2.3 Computation Method - Same as for lines and volumes.

6.2.2.4 Approximations - Unknown.

6.2.2.5 Limitations - Unknown.

6.2.3 Variable Names - Variable names unique to the RESNTR subroutine and/or math model are listed below. Common variables are described in Paragraph 3.1.6.1.

SYMBOL	DESCRIPTION	UNITS
A	Wall thickness of neck line	IN
AR	Internal cross-sectional area of neck line	IN**2
C1	Temporary variable	--
C2	Temporary variable	--
C	Speed of sound in neck fluid	IN/SEC
E	Neck material modulus of elasticity	PSI
EL(-)	Material modulus of elasticity for acoustic filter line elements	
G1	Complex cosine term in line matrix	--
G2	Complex sin term in line matrix	--

6.2.3 (Continued)

SYMBOL	DESCRIPTION	UNITS
GL(-,-,-)	Matrix values for acoustic filter line elements	--
j	$\sqrt{-1}$	
jW	Laplace operator	
K	*Line mounting factor = 1.8*	--
RL(-)	Internal radius of acoustic filter lines	IN
R	Neck internal radius	IN
VA,VB,VC	Volumes in acoustic filter	IN**3
VOL	Volume of resonator cavity	IN**3
WTH(-)	Wall thickness of acoustic filter lines	IN
XL	Length of neck	IN
YL(-)	Length of acoustic filter lines	IN
Y1	Trig function argument (real)	--
Y	Trig function argument (complex)	--
Z1,Z2,Z3,Z4,Z5	Temporary variables	--
ZA,ZB,ZC,ZD	Temporary variables	--
ZE,ZF,ZG,ZH	Matrix values for acoustic filter	--
Z5	Resonator input impedance (last)	PSI/CIS
ZVA1,ZVA2	Temporary variables	--
ZVB1,ZVB2	Temporary variables	
ZVC1,ZVC2	Temporary variables	

6.2.4 RESNTR Subroutine - Listing

```
      SUBROUTINE RESNTR
C
C * REVISED JULY 4, 1976 *
C
C         *VARIABLE TYPES, DIMENSIONS, AND COMMONALITY*
C
      COMMON BETA,G,P,Q,Z, XBE,BER,BEI,BERP,BEIP,RHO,BULK,VOL,W,VISC,PAR

     1M,PI, IEL,NEL,KTYPE(40)
      COMPLEX BETA,G,P,Q,Z,Z1,Z2,Z3,Z4,Z5,G1,G2,Y1,Y,ZVA2,ZVA,ZVB2,ZVB,
     1ZVC2,ZVC,GL,ZA,ZB,ZC,ZD,ZE,ZF,ZG,ZH
      DIMENSION G(2,2,40),PARM(8,40),P(40),Q(40),Z(40),YL(3),WTH(3),RL(3

     1),EL(3),GL(2,2,3)
C
C *TEST FOR RESONATOR TYPE*
C
      IF (KTYPE(IEL).EQ.32) GO TO 110
C
C                       *SOLUTION METHOD - LUMPED VOLUME RESONATOR*
C
C *INITIALIZE VARIABLES FROM INPUT DATA ARRAY*
C
C    LENGTH OF NECK
C
      XL=PARM(1,IEL)
C
C    INTERNAL RADIUS OF NECK
C
      R=PARM(2,IEL)
C
C    WALL THICKNESS OF NECK
C
      A=PARM(3,IEL)
C
C    NECK MATERIAL MODULUS OF ELASTICITY
C
      E=PARM(4,IEL)
C
C    VOLUME OF RESONATOR CAVITY
C
      VOL=PARM(5,IEL)
C
C *FREQUENCY DEPENDENT FRICTION DAMPING FACTOR*
C
      XBE=SQRT(VISC/(2.0*W))/R
      BETA=4.0*XBE**2-XBE+(0.0,1.0)*(XBE-1.0)
C
C *VELOCITY OF SOUND*
C
      C1=1.8*(R+A)*BULK
      C2=BULK/(RHO*(1.+C1/(A*E)))
```

6.2.3 (Continued)

SYMBOL	DESCRIPTION	UNITS
GL(-,-,-)	Matrix values for acoustic filter line elements	--
j	$\sqrt{-1}$	
jW	Laplace operator	
K	Line mounting factor = 1.8	--
RL(-)	Internal radius of acoustic filter lines	IN
R	Neck internal radius	IN
VA,VB,VC	Volumes in acoustic filter	IN**3
VOL	Volume of resonator cavity	IN**3
WTH(-)	Wall thickness of acoustic filter lines	IN
XL	Length of neck	IN
YL(-)	Length of acoustic filter lines	IN
Y1	Trig function argument (real)	--
Y	Trig function argument (complex)	--
Z1,Z2,Z3,Z4,Z5	Temporary variables	--
ZA,ZB,ZC,ZD	Temporary variables	--
ZE,ZF,ZG,ZH	Matrix values for acoustic filter	--
Z5	Resonator input impedance (last)	PSI/CIS
ZVA1,ZVA2	Temporary variables	--
ZVB1,ZVB2	Temporary variables	
ZVC1,ZVC2	Temporary variables	

6.2.4 (Cont.)

```
      C=SQRT(C2)
C
C *VARIOUS LINE MATRIX TERMS*
C
      Y1=W*BETA*XL/C
      Y=Y1*(0.,1.)
      G1=CCOS(Y)
      G2=CSIN(Y)
      AR=PI*R*R
      Z5=BETA*RHO*C
C
C *SOLVE FOR LINE INPUT IMPEDANCE*
C
      Z1=BULK*Z5*AR*G1
      Z2=W*VOL*Z5*Z5*G2*(0.,1.)
      Z3=W*VOL*Z5*AR*G1*(0.,1.)
      Z4=BULK*AR*AR*G2
      Z5=(Z1-Z2)/(Z3+Z4)
C
C *ASSIGN MAIN LINE MATRIX ELEMENT VALUES*
C
      G(1,1,IEL)=1.
      G(2,1,IEL)=0.
      G(2,2,IEL)=1.
      G(1,2,IEL)=-1./Z5
      GO TO 250
  110 CONTINUE
C
C                               *SOLUTION METHOD - ACOUSTIC FILTER*
C
C
C *INITIALIZE VOLUMES FROM INPUT DATA ARRAY*
C
      VA=PARM(1,IEL)
      VB=PARM(2,IEL)
      VC=PARM(3,IEL)
C
C *CALCULATED VOLUME IMPEDANCE VALUES*
C
      ZVA1=W*VA
      ZVA2=CMPLX(0.,ZVA1)
      ZVA=BULK/ZVA2
      ZVB1=W*VB
      ZVB2=CMPLX(0.,ZVB1)
      ZVB=BULK/ZVB2
      ZVC1=W*VC
      ZVC2=CMPLX(0.,ZVC1)
      ZVC=BULK/ZVC2
C
```

6.2.4 RESNTR Subroutine - Listing

```
      SUBROUTINE RESNTR
C
C * REVISED JULY 4, 1976 *
C
C          *VARIABLE TYPES, DIMENSIONS, AND COMMONALITY*
C
      COMMON BETA,G,P,Q,Z, XBE,BER,BEI,BERP,BEIP,RHO,BULK,VOL,W,VISC,PAR

     1M,PI, IEL,NEL,KTYPE(40)
      COMPLEX BETA,G,P,Q,Z,Z1,Z2,Z3,Z4,Z5,G1,G2,Y1,Y,ZVA2,ZVA,ZVB2,ZVB,
     1ZVC2,ZVC,GL,ZA,ZB,ZC,ZD,ZE,ZF,ZG,ZH
      DIMENSION G(2,2,40),PARM(8,40),P(40),Q(40),Z(40),YL(3),WTH(3),RL(3

     1),EL(3),GL(2,2,3)
C
C *TEST FOR RESONATOR TYPE*
C
      IF (KTYPE(IEL).EQ.32) GO TO 110
C
C                        *SOLUTION METHOD - LUMPED VOLUME RESONATOR*
C
C *INITIALIZE VARIABLES FROM INPUT DATA ARRAY*
C
C     LENGTH OF NECK
C
      XL=PARM(1,IEL)
C
C     INTERNAL RADIUS OF NECK
C
      R=PARM(2,IEL)
C
C     WALL THICKNESS OF NECK
C
      A=PARM(3,IEL)
C
C     NECK MATERIAL MODULUS OF ELASTICITY
C
      E=PARM(4,IEL)
C
C     VOLUME OF RESONATOR CAVITY
C
      VOL=PARM(5,IEL)
C
C *FREQUENCY DEPENDENT FRICTION DAMPING FACTOR*
C
      XBE=SQRT(VISC/(2.0*W))/R
      BETA=4.0*XBE**2-XBE+(0.0,1.0)*(XBE-1.0)
C
C *VELOCITY OF SOUND*
C
      C1=1.8*(R+A)*BULK
      C2=BULK/(RHO*(1.+C1/(A*E)))
```

6.2.4 (Cont.)

```
      C=SQRT(C2)
C
C *VARIOUS LINE MATRIX TERMS*
C
      Y1=W*BETA*XL/C
      Y=Y1*(0.,1.)
      G1=CCOS(Y)
      G2=CSIN(Y)
      AR=PI*R*R
      Z5=BETA*RHO*C
C
C *SOLVE FOR LINE INPUT IMPEDANCE*
C
      Z1=BULK*Z5*AR*G1
      Z2=W*VOL*Z5*Z5*G2*(0.,1.)
      Z3=W*VOL*Z5*AR*G1*(0.,1.)
      Z4=BULK*AR*AR*G2
      Z5=(Z1-Z2)/(Z3+Z4)
C
C *ASSIGN MAIN LINE MATRIX ELEMENT VALUES*
C
      G(1,1,IEL)=1.
      G(2,1,IEL)=0.
      G(2,2,IEL)=1.
      G(1,2,IEL)=-1./Z5
      GO TO 250
  110 CONTINUE
C
C                          *SOLUTION METHOD - ACOUSTIC FILTER*
C
C
C *INITIALIZE VOLUMES FROM INPUT DATA ARRAY*
C
      VA=PARM(1,IEL)
      VB=PARM(2,IEL)
      VC=PARM(3,IEL)
C
C *CALCULATED VOLUME IMPEDANCE VALUES*
C
      ZVA1=W*VA
      ZVA2=CMPLX(0.,ZVA1)
      ZVA=BULK/ZVA2
      ZVB1=W*VB
      ZVB2=CMPLX(0.,ZVB1)
      ZVB=BULK/ZVB2
      ZVC1=W*VC
      ZVC2=CMPLX(0.,ZVC1)
      ZVC=BULK/ZVC2
C
```

6.2.4 (Cont.)

```
C *CALCULATE LINE MATRIX VALUES FOR RESONATOR LINES*
C
      KK=PARM(8,IEL)
      DO 120 I=1,3
C
C     INITIALIZE VARIABLES FROM INPUT DATA ARRAY
C
C     -LINE LENGTH-
C
      YL(I)=PARM(1,KK)
C
C     -LINE WALL THICKNESS-
C
      WTH(I)=PARM(3,KK)
C
C     -LINE INTERNAL RADIUS-
C
      RL(I)=PARM(2,KK)/2.0-WTH(I)
C
C     -LINE MATERIAL MODULUS OF ELASTICITY-
C
      EL(I)=PARM(4,KK)
C
C     FREQUENCY DEPENDENT FRICTION DAMPING FACTOR
C
      XBE=SQRT(VISC/(2.0*W))/RL(I)
      BETA=4.0*XBE**2-XBE+(0.0,1.0)*(XBE-1.0)
C
C     VELOCITY OF SOUND IN LINE
C
      C1=1.8*(RL(I)+WTH(I))*BULK
      C2=BULK/(RHO*(1.+C1/(WTH(I)*EL(I))))
      C=SQRT(C2)
C
C     CALCULATE LINE MATRIX TERMS
C
      Y1=W*BETA*YL(I)/C
      Y=Y1*(0.,1.)
      Z5=BETA*RHO*C/(PI*RL(I)*RL(I))
      G1=CCOS(Y)
      G2=CSIN(Y)
      GL(1,1,I)=G1
      GL(1,2,I)=-(1./Z5)*G2
      GL(2,1,I)=Z5*G2
      GL(2,2,I)=G1
      KK=KK+1
  120 CONTINUE
C
```

6.2.4 (Cont.)

```
C *CALCULATE G MATRIX VALUES FOR MAIN PROGRAM*
C
      ZA=GL(1,1,3)*GL(1,1,2)+GL(1,2,3)*GL(2,1,2)-GL(1,1,3)*GL(2,1,2)/ZVB

      ZB=GL(1,1,3)*GL(1,2,2)+GL(1,2,3)*GL(2,2,2)-GL(1,1,3)*GL(2,2,2)/ZVB

      ZC=GL(2,2,3)*GL(2,1,2)-GL(2,1,3)*GL(1,2,3)*GL(2,1,2)/GL(1,1,3)+GL(

     +2,1,3)*ZA/GL(1,1,3)
      ZD=GL(2,2,3)*GL(2,2,2)-GL(2,1,3)*GL(1,2,3)*GL(2,2,2)/GL(1,1,3)+GL(

     +2,1,3)*ZB/GL(1,1,3)
      ZE=GL(2,1,1)*ZC/(ZC+GL(2,1,1))
      ZF=(GL(2,2,1)*ZC+GL(2,1,1)*ZD-GL(2,1,1)*ZC/ZVA)/(ZC+GL(2,1,1))
      ZG=GL(1,1,1)*ZE/GL(2,1,1)+ZA*ZE/ZC-ZE/ZVC
      ZH=GL(1,1,1)*ZF/GL(2,1,1)-GL(1,1,1)*GL(2,2,1)/GL(2,1,1)+GL(1,2,1)+

     +ZA*ZF/ZC-ZA*ZD/ZC+ZB-ZF/ZVC
C
C *ASSIGN MATRIX ELEMENT VALUES*
C
      G(1,1,IEL)=ZG
      G(1,2,IEL)=ZH
      G(2,1,IEL)=ZE
      G(2,2,IEL)=ZF
  250 CONTINUE
      RETURN
      END
```

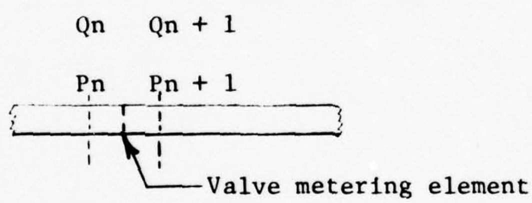
6.3 VALVE SUBROUTINE

Subroutine VALVE defines the element values of a 2x2 matrix which represents the complex relationship between input and output flow and pressure across a valve element in the circuit.

Specification statements are followed by initialization of valve impedance (K) from input data. Matrix values are then assigned using the valve impedance.

6.3.1 Math Model

Subroutine VALVE uses a linearized impedance (Z) of the valve at the steady-state flow condition; no dynamic effects are included.



Flow and pressure across the valve are related by

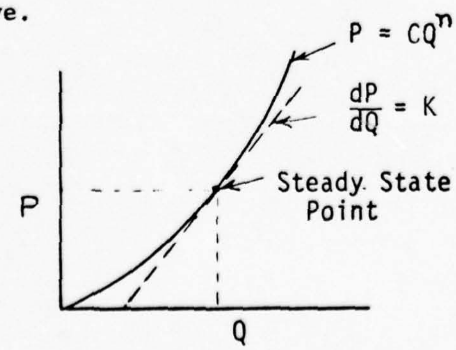
$$\begin{bmatrix} 1 & , & 0 \\ -K & , & 1 \end{bmatrix} \times \begin{bmatrix} Qn \\ Pn \end{bmatrix} = \begin{bmatrix} Qn+1 \\ Pn+1 \end{bmatrix} , \qquad (1)$$

or

$$Qn = Qn+1 \qquad (2)$$

$$-KQn + Pn = Pn+1 \qquad (3)$$

Valve impedance (K) is taken from the pressure-flow characteristic by using the tangential slope for the estimated steady state flow rate through the valve.



For a valve pressure/flow relationship of the form

$$P = CQ^n$$

where: P = pressure drop (psi)
Q = flow rate (cis)
C = constant
n = flow exponent

the linearized valve impedance (K) is determined as

$$K = \frac{dP}{dQ} = nCQ^{n-1} \quad ,$$

but

$$C = \frac{P}{Q^n} \quad ,$$

therefore

$$K = \frac{nP}{Q} \quad . \tag{4}$$

If valve flow can be characterized as an orifice (n = 2), then the impedance is $K = {}^{2P}/_{Q}$. The orifice relationship is typical of electrohydraulic servo valve *steady state control flow.* If valve flow can be characterized as laminar for the steady state condition, then n = 1 and the impedance is $K = {}^{P}/_{Q}$. The laminar relationship is typical for null leakage flow across lapped spool valves, e.g. mechanical servovalves, and second stages of electrohydraulic valves.

Parallel valve elements, for instance those within an electro-mechanical integrated servoactuator, may be combined for modeling as a single valve element by computing an equivalent impedance (K_e) *for all the parallel flow paths.*

$$\frac{1}{K_e} = \frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3} \quad . \; . \; . \tag{5}$$

6.3.2 Assumptions - Valve impedance as described above, is assumed to correctly define the reflection and transmission of the acoustical signals at the valve.

6.3.3 Computation Method - Not applicable.

6.3.4 Approximations - None

6.3.5 Limitations - Empirical pressure-flow data, if available, is used to calculate the impedance at the estimated steady state flow condition. If data is not available on the modeled valve, it may be necessary to use data on a similar valve. Inlet and outlet passages should be modeled as close to the actual valve metering element as possible, so as to avoid errors in length and hence in the resonant frequency predictions.

6.3.6 Variable Names - Variable names unique to the VALVE subroutine and/or math model are listed below. Common variables are described in paragraph 3.1.6.1.

SYMBOL	DESCRIPTION	UNITS
K	Valve linearized impedance	PSI/CIS

6.3.7 VALVE Subroutine - Listing

```
      SUBROUTINE VALVE
C
C**** REVISED SEPTEMBER 3,1974 ****
C
C         *VARIABLE TYPES, DIMENSIONS, AND COMMONALITY*
C
      COMPLEX BETA,G,P,Q,Z
      REAL K
      COMMON BETA,G,P,Q,Z, XBE,BER,BEI,BERP,BEIP,RHO,BULK,VOL,W,VISC,PAR

     1M,PI, IEL,NEL,KTYPE(40)
      DIMENSION G(2,2,40),PARM(8,40),P(40),Q(40),Z(40)
C
C                        *SOLUTION METHOD*
C
C *INITIALIZE VALVE IMPEDANCE FROM INPUT DATA ARRAY*
C
      K=PARM(1,IEL)
C
C *ASSIGN TRANSFER MATRIX ELEMENT VALUES*
C
      G(1,1,IEL)=CMPLX(1.,0.)
      G(1,2,IEL)=CMPLX(0.,0.)
      G(2,1,IEL)=CMPLX(-K,0.)
      G(2,2,IEL)=CMPLX(1.,0.)
      RETURN
      END
```

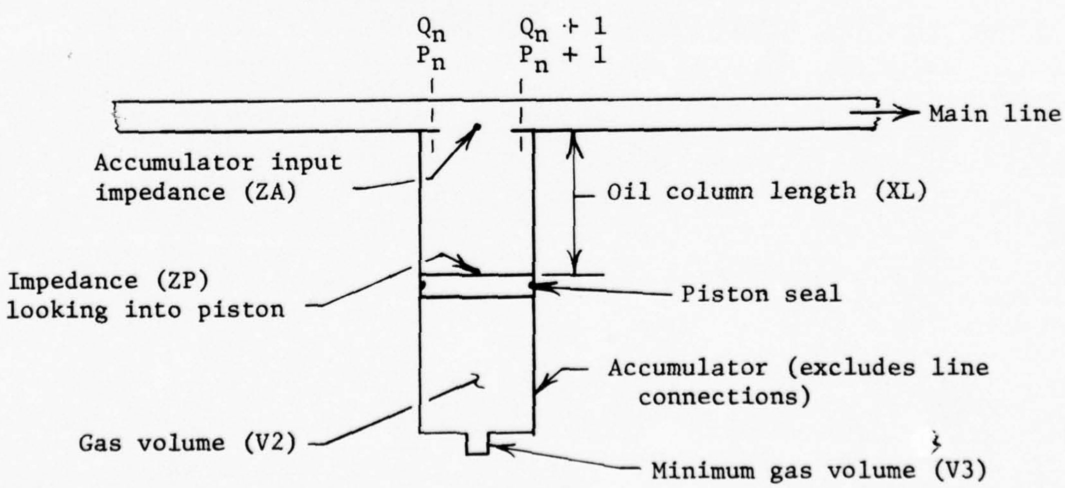
6.4 ACCUMULATOR SUBROUTINE (ACCUM)

Subroutine ACCUM computes the element values for a 2x2 matrix which represents the complex relationship between input and output pressure and flow, for an impedance looking into an attached accumulator. ACCUM models a piston-type accumulator as three series elements; line, piston, and gas volume. Piston dynamics and seal friction are included in the model.

Specification statements are followed by initialization of variables from input data. ACCUM then computes the frequency dependent friction factor, velocity of sound in oil, piston area, gas volume, and piston position. A test for piston bottoming on the gas side is included to limit the computed gas volume. Dynamic input impedance of the accumulator is computed, and used in computing matrix element G(1,2). Matrix element G(1,2) is given different values if the piston is bottomed at either end of the stroke.

6.4.1 Math Model

ACCUM models a piston-type accumulator as a line, a piston with mass and friction, and a gas volume as three series elements.



Representing the oil column as a line element, and using equation (6) derived in paragraph 6.2.1 for a resonator, the accumulator input impedance may be written as

$$ZA = \frac{ZP \times G11 - G21}{G22 + ZP \times G12} \qquad (1)$$

where

$$G11 = \cos(Y) = G22 = G1$$

$$G12 = \frac{\sin(Y)}{ZC} = G2$$

$$G21 = ZC \times \sin(Y)$$

$$Y = j \times W \times BETA \times XL/C$$

$$ZC = BETA \times RHO \times C/AREA \;.$$

The termination impedance (ZP) of the line, looking into the piston, is the summation of piston friction and inductive (mass) effects, and the gas volume capacitive effect such that

$$ZP = \frac{B}{A^2} + j\left(\frac{M \times W}{A^2}\right) + \left(\frac{1.4 \times PW}{j \times W \times V2}\right) . \qquad (2)$$

Piston friction constitutes a steady state effect and may be represented in units of impedance as

$$ZR = \frac{B}{A^2} \quad (PSI/CIS) \;. \qquad (3)$$

Piston mass produces an inductive dynamic effect, and may be represented as the complex quantity

$$\frac{j \times M \times W}{A^2} \quad (PSI/CIS) \;. \qquad (4)$$

The gas volume produces a capacitive dynamic effect represented by the complex quantity

$$\frac{1.4 \times PW}{j \times W \times V2} \quad (PSI/CIS) \;. \qquad (5)$$

where (1.4 x PW) is the adiabatic bulk modulus for gaseous nitrogen (Reference 6).

Equations (1) - (5) form the basic quantities in the ACCUM listing for solving the dynamic impedance of the piston-type accumulator.

Matrix elements for the main line are assigned in accordance with the equation

$$\begin{bmatrix} 1 & , & -\frac{1}{ZA} \\ 0 & , & 1 \end{bmatrix} \times \begin{bmatrix} Qn \\ Pn \end{bmatrix} = \begin{bmatrix} Qn+1 \\ Pn+1 \end{bmatrix} \tag{6}$$

or

$$Qn - \frac{Pn}{ZA} = Qn+1 \tag{7}$$

$$Pn = Pn+1 \tag{8}$$

If the piston is bottomed on the gas side (V2 = V3), the line (XL) is then a terminated element and equation (1) becomes

$$ZA = -\frac{G11}{G12}$$

$$= -\frac{\cos(Y) \times ZC}{\sin(Y)}$$

$$ZA = -\frac{G1 \times ZC}{G2} \tag{9}$$

Equation (9) is used instead of (1) to calculate the G(1,2,IEL) matrix element for this special case.

If the piston is bottomed on the oil side (PR > PW), the accumulator is essentially eliminated from the circuit by making the G(1,2,IEL) element equal to zero.

6.4.2 Assumptions - Gaseous nitrogen is assumed to be the pressurizing agent in the accumulator. An adiabatic expansion exponent of 1.4 is assumed for nitrogen.

6.4.3 Computation Method - Not applicable.

6.4.4 Approximations - The piston friction damping coefficient (B) must be determined experimentally. An estimated value of 0.1 is suggested.

6.4.5 Limitations - ACCUM is currently restricted to modeling a piston-type accumulator. Bladder-type accumulators will be incorporated in the model at a later date.

6.4.6 Variable Names - Variable names unique to the ACCUM subroutine and/or math model are listed below. Common variables are discussed in paragraph 3.1.6.1.

SYMBOL	DESCRIPTION	UNITS
A	Input data accumulator wall thickness	IN
AREA	Piston area	IN**2
B	Input data piston friction damping coefficient	LB*SEC/IN
C	Velocity of sound in oil	IN/SEC
C1	Temporary variable	--
C2	Temporary variable	--
E	Input data Young's modulus of wall material	PSI
G1	Temporary variable	--
G2	Temporary variable	--
KK	Address of second record input data	--
M	Input data piston mass	LB*SEC**2/IN
PR	Input data gas precharge pressure	PSIA
PW	Input data working pressure	PSIA
R	Input data internal radius of accumulator	IN

SYMBOL	DESCRIPTION	UNITS
V1	Input data initial gas volume at precharge	IN**3
V2	Actual gas volume at working pressure	IN**3
V3	Input data minimum gas volume	IN**3
XL	Piston position (length of oil column)	IN
Y	Temporary variable	--
Y1	Temporary variable	--
ZA	Complex impedance of accumulator	PSI/CIS
ZB	Temporary variable	--
ZC	Characteristic impedance of line (oil column)	PSI/CIS
ZR	Friction impedance (complex)	PSI/CIS
ZP	Impedance looking into piston (complex)	PSI/CIS

6.4.7 ACCUM Subroutine - Listing

```
      SUBROUTINE ACCUM
C
C * REVISED MARCH 3,1975 *
C
C              *VARIABLE TYPES, DIMENSIONS, AND COMMONALITY*
C
C     REAL M
      COMMON BETA,G,P,Q,Z, XBE,BER,BEI,BERP,BEIP,RHO,BULK,VOL,W,VISC,PAR
     1M,PI, IEL,NEL,KTYPE(40)
      COMPLEX BETA,G,P,Q,Z,Y1,Y,G1,G2,ZP,ZA,ZB,ZC
      DIMENSION G(2,2,40),PARM(8,40),P(40),Q(40),Z(40)
C
C                         *SOLUTION METHOD*
C
C *INITIALIZE VARIABLES FROM INPUT DATA ARRAY*
C
C     PISTON MASS
C
      M=PARM(1,IEL)
C
C     INTERNAL RADIUS OF ACCUMULATOR
C
      R=PARM(2,IEL)
C
C     ACCUMULATOR WALL THICKNESS
C
      A=PARM(3,IEL)
C
C     WALL MATERIAL MODULUS OF ELASTICITY
C
      E=PARM(4,IEL)
C
C     GAS PRECHARGE VOLUME
C
      V1=PARM(5,IEL)
C
C     MINIMUM GAS VOLUME
C
      V3=PARM(6,IEL)
C
C     GAS PRECHARGE PRESSURE
C
      PR=PARM(7,IEL)
C
C     WORKING PRESSURE
C
      KK=PARM(8,IEL)
      PW=PARM(1,KK)
C
C     PISTON FRICTION DAMPING COEFFICIENT
```

6.4.7 (Cont.)

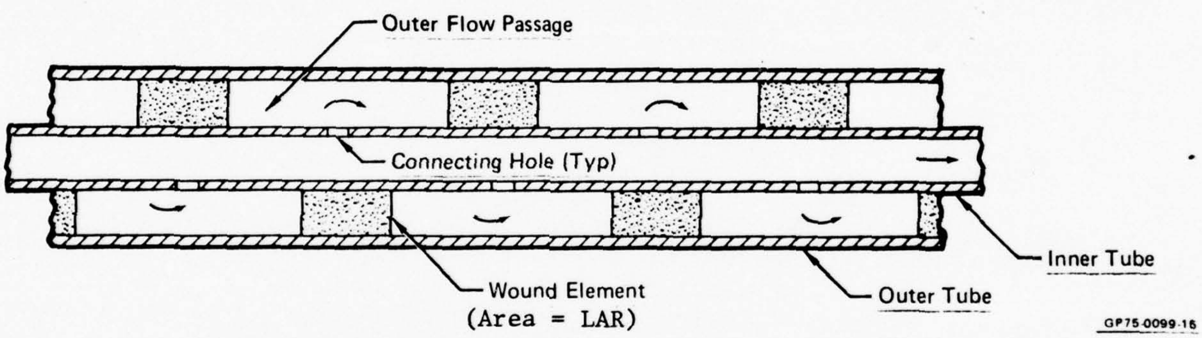
```
C
      B=PARM(2,KK)
C
C *LINE FREQUENCY DEPENDENT FRICTION DAMPING FACTOR*
C
      XBE=SQRT(VISC/(2.0*W))/R
      BETA=4.0*XBE**2-XBE+(0.0,1.0)*(XBE-1.0)
C
C *VELOCITY OF SOUND*
C
      C1=1.8*(R+A)*BULK
      C2=BULK/(RHO*(1.+C1/(A*E)))
      C=SQRT (C2)
C
C *PISTON AREA*
C
      AREA=PI*R*R
C
C *GAS VOLUME AT WORKING PRESSURE*
C
      V2=PR*V1/PW
C
C *TEST FOR BOTTOMING ON GAS SIDE*
C
      IF(V2.LT.V3) V2=V3
C
C *PISTON POSITION (LENGTH OF OIL COLUMN)*
C
      XL=(V1-V2)/AREA
C
C *SOLVE FOR DYNAMIC IMPEDANCE OF ACCUMULATOR*
C
      Y1=W*BETA*XL/C
      Y=Y1*(0.,1.)
      ZC=BETA*RHO*C/AREA
      G1=CCOS(Y)
      G2=CSIN(Y)
      ZR=B/(AREA**2)
      ZB=M*W/(AREA**2)-1.4*PW/(V2*W)
      ZP=ZR+ZB*(0.,1.)
      ZA=(ZP*G1-ZC*G2)/(G1+ZP*G2/ZC)
C
C *ASSIGN MAIN LINE MATRIX ELEMENT VALUES*
C
      G(1,1,IEL)=1.
      G(1,2,IEL)=-1./ZA
      G(2,1,IEL)=0.
      G(2,2,IEL)=1.
C
C *MATRIX ELEMENT 1,2 IF BOTTOMED ON GAS SIDE*
```

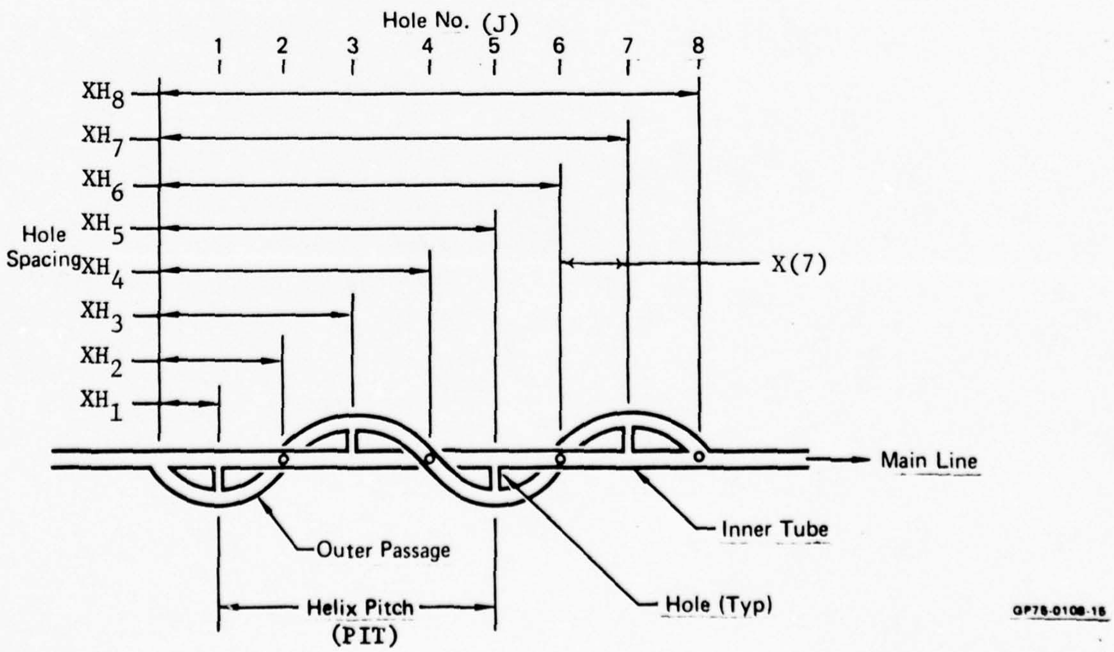
6.4.7 (Cont.)

```
C
      IF(V2.EQ.V3) G(1,2,IEL)=G2/(G1*ZC)
C
C *MATRIX ELEMENT 1,2 IF BOTTOMED ON OIL SIDE*
C
      IF(PR.GT.PW) G(1,2,IEL)=0.0001
      RETURN
      END
```

6.5 QUINCKE TUBE SUBROUTINE (WHEQUT)

WHEQUT models a wide-band, helical Quincke tube, which may be used to dampen acoustic noise in the resonant mode. The modeled Quincke tube configuration is shown below. An outer spiraled passage is formed by winding a solid element around the straight inner tube, which is then enclosed in another straight outer tube.



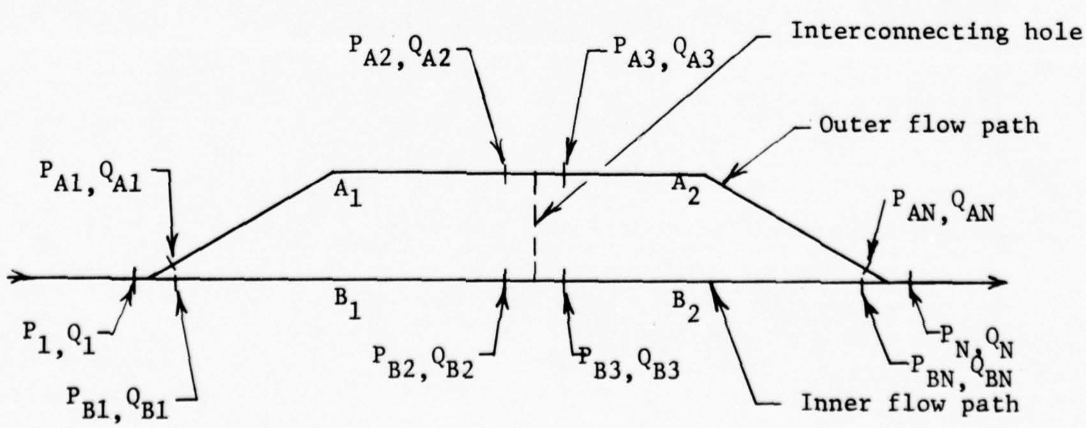


After specification and initialization of data, WHEQUT uses the same basic calculations as the LINE subroutine to find the transfer matrix for each element of the inner and outer flow paths. Matrices used in the network analysis are then initialized. Subsequently, a loop calculation performs a network analysis which computes the initial element values of a 4x4 matrix $(\hat{M})$ representing the pressure and flow across the Quincke tube device. Computations are then performed to "shrink" the 4x4 matrix to the standard 2x2 matrix (G) required to represent the Quincke tube element in the main program impedance calculations. The 2x2 matrix values are then returned to the main program.

6.5.1 Math Models

6.5.1.1 Network Analysis Math Model - A network analysis of flow in the two parallel paths and interconnecting holes provides the usual 2x2 matrix element values representing the main line input and output flow and pressure across the Quincke tube element.

Consider the following simplified Quincke tube with a single interconnecting hole between the inner and outer flow paths.



Equations for flow and pressure across line elements A_1 and B_1 may be written in matrix form by combining the 2x2 matrices for each line into a 4x4 matrix.

$$\begin{bmatrix} MA \end{bmatrix} \cdot \begin{bmatrix} Q_{A1} \\ P_{A1} \\ Q_{B1} \\ P_{B1} \end{bmatrix} = \begin{bmatrix} Q_{A2} \\ P_{A2} \\ Q_{B2} \\ P_{B2} \end{bmatrix}$$

$$\begin{bmatrix} \multicolumn{2}{c}{\text{Line } A_1} & 0 & 0 \\ \multicolumn{2}{c}{\text{2x2 Matrix}} & 0 & 0 \\ 0 & 0 & \multicolumn{2}{c}{\text{Line } B_1} \\ 0 & 0 & \multicolumn{2}{c}{\text{2x2 Matrix}} \end{bmatrix} \cdot \begin{bmatrix} Q_{A1} \\ P_{A1} \\ Q_{B1} \\ P_{B1} \end{bmatrix} = \begin{bmatrix} Q_{A2} \\ P_{A2} \\ Q_{B2} \\ P_{B2} \end{bmatrix} \qquad (1)$$

Equations for flow and pressure across the interconnecting hole may be expressed in a 4x4 matrix as

$$\begin{bmatrix} MC \end{bmatrix} \cdot \begin{bmatrix} Q_{A2} \\ P_{A2} \\ Q_{B2} \\ P_{B2} \end{bmatrix} = \begin{bmatrix} Q_{A3} \\ P_{A3} \\ Q_{B3} \\ P_{B3} \end{bmatrix}$$

$$\begin{bmatrix} 1 & -\frac{1}{ZH} & 0 & \frac{1}{ZH} \\ 0 & 1 & 0 & 0 \\ 0 & \frac{1}{ZH} & 1 & -\frac{1}{ZH} \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} Q_{A2} \\ P_{A2} \\ Q_{B2} \\ P_{B2} \end{bmatrix} = \begin{bmatrix} Q_{A3} \\ P_{A3} \\ Q_{B3} \\ P_{B3} \end{bmatrix} \qquad (2)$$

where ZH = Impedance of interconnecting hole

$$= \frac{4LH*RHO}{DH^2}\left(\sqrt{\frac{8WxVISC}{DH}} + Wj\right) \qquad (3)$$

Combining (1) and (2) gives a general 4x4 matrix equation, which is applicable to a network with (n-1) elements.

$$\begin{bmatrix} MA \end{bmatrix} \cdot \begin{bmatrix} MC \end{bmatrix} \cdot \begin{bmatrix} Q_{A1} \\ P_{A1} \\ Q_{B1} \\ P_{B1} \end{bmatrix} = \begin{bmatrix} Q_{AN} \\ P_{AN} \\ Q_{BN} \\ P_{BN} \end{bmatrix}$$

$$\begin{bmatrix} M \end{bmatrix} \cdot \begin{bmatrix} Q_{A1} \\ P_{A1} \\ Q_{B1} \\ P_{B1} \end{bmatrix} = \begin{bmatrix} Q_{AN} \\ P_{AN} \\ Q_{BN} \\ P_{BN} \end{bmatrix}$$

$$\begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{bmatrix} \cdot \begin{bmatrix} Q_{A1} \\ P_{A1} \\ Q_{B1} \\ P_{B1} \end{bmatrix} = \begin{bmatrix} Q_{AN} \\ P_{AN} \\ Q_{BN} \\ P_{BN} \end{bmatrix} \qquad (4)$$

Row 1 and 3 equations of (4) may be added to yield

$$(M_{11}+M_{31})Q_{A1}+(M_{12}+M_{32})P_{A1}+(M_{13}+M_{33})Q_{B1}+(M_{14}+M_{34})P_{B1}=Q_{AN}+Q_{BN}. \qquad (5)$$

Since $P_{A1} = P_{B1} = P_1$, the second and fourth terms (columns) may be combined to write

$$(M_{11}+M_{13})Q_{A1}+(M_{12}+M_{32}+M_{14}+M_{34})P_1+(M_{13}+M_{33})Q_{B1} = Q_N \qquad (6)$$

where $Q_N = Q_{AN}+ Q_{BN}$.

Combining the second and fourth terms in the other two equations of (4) gives

$$M_{21}Q_{A1}+(M_{22}+M_{24})P_1 +M_{23}Q_{B1} = P_{AN} \qquad (7)$$

$$M_{41}Q_{A1}+(M_{42}+M_{44})P_1 +M_{43}Q_{B1} = P_{BN} \qquad (8)$$

Equations (6), (7), and (8) may be rewritten into a 3x3 matrix form as

$$\begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{41} & M_{42} & M_{43} \end{bmatrix} \cdot \begin{bmatrix} Q_{A1} \\ P_1 \\ Q_{B1} \end{bmatrix} = \begin{bmatrix} Q_N \\ P_{AN} \\ Q_{BN} \end{bmatrix} \tag{9}$$

where the following equalities are valid for computer operations:

$$M_{11} = M_{11} + M_{31}$$

$$M_{12} = M_{12} + M_{32} + M_{14} + M_{34}$$

$$M_{13} = M_{13} + M_{33}$$

$$M_{22} = M_{22} + M_{24}$$

$$M_{42} = M_{42} + M_{44} \quad .$$

It is necessary to express (9) as a 2x2 matrix to achieve conformability with the standard 2x2 matrix used in the main programs for circuit impedance calculations.

The 2x2 matrix must also relate the input and output variables for the Quincke tube element, i.e. $P_1 - Q_1$, $P_N - Q_N$.

To do this, first eliminate the Q_{A1} term in (9) using the expression

$$Q_{A1} = Q_1 - Q_{B1}$$

to give

$$M_{11}Q_1 + (M_{13}-M_{11})Q_{B1} + M_{12}P_1 = Q_N \tag{10}$$

$$M_{21}Q_1 + (M_{23}-M_{21})Q_{B1} + M_{22}P_1 = P_{AN} \tag{11}$$

$$M_{41}Q_1 + (M_{43}-M_{41})Q_{B1} + M_{42}P_1 = P_{BN} \quad . \tag{12}$$

Solving for Q_{B1} in (12) yields

$$Q_{B1} = \frac{P_{BN}-M_{42}P_1 - M_{41}Q_1}{M_{43}-M_{41}} \quad . \tag{13}$$

Substituting (13) in (11) and (10) gives

$$(M_{21}-AM_{41})Q_1+(M_{22}-AM_{42})P_1 = P_{AN}-AP_{BN} \tag{14}$$

$$(M_{11}-BM_{41})Q_1+(M_{12}-BM_{42})P_1 = Q_N-BP_{BN} \tag{15}$$

where

$$A = \frac{M_{23}-M_{21}}{M_{43}-M_{41}}$$

$$B = \frac{M_{13}- M_{11}}{M_{43}- M_{41}} \quad .$$

Writing (14) and (15) in matrix form using $P_N = P_{AN}= P_{BN}$ gives

$$\begin{bmatrix} M_{11}-BM_{41} & M_{12}-BM_{42} \\ M_{21}-AM_{41} & M_{22}-AM_{42} \end{bmatrix} \cdot \begin{bmatrix} Q_1 \\ P_1 \end{bmatrix} = \begin{bmatrix} 1 & -B \\ 0 & 1-A \end{bmatrix} \cdot \begin{bmatrix} Q_N \\ P_N \end{bmatrix} \tag{16}$$

Evaluating the right matrix determinant and dividing it into the left matrix elements gives

$$\begin{bmatrix} \frac{M_{11}-BM_{41}}{1-A} & \frac{M_{12}-BM_{42}}{1-A} \\ \frac{M_{21}-AM_{41}}{1-A} & \frac{M_{22}-AM_{42}}{1-A} \end{bmatrix} \cdot \begin{bmatrix} Q_1 \\ P_1 \end{bmatrix} = \begin{bmatrix} Q_N \\ P_N \end{bmatrix} \quad . \tag{17}$$

Simplifying the form of (17), and evaluating the matrix elements yields

$$\begin{bmatrix} C & D \\ E & F \end{bmatrix} \cdot \begin{bmatrix} Q_1 \\ P_1 \end{bmatrix} = \begin{bmatrix} Q_N \\ P_N \end{bmatrix} \tag{18}$$

where

$$C = \frac{M_{11}M_{43}-M_{13}M_{41}}{M_{43}-M_{41}-M_{23}+M_{21}}$$

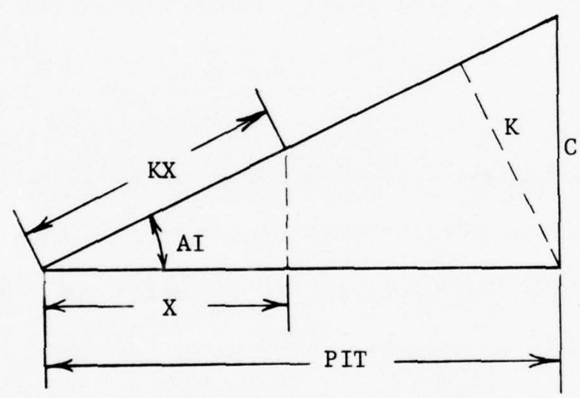
$$D = \frac{M_{12}M_{43}-M_{12}M_{41}-M_{13}M_{42}+M_{11}M_{42}}{M_{43}-M_{41}-M_{23}+M_{21}}$$

$$E = \frac{M_{21}M_{43}-M_{23}M_{41}}{M_{43}-M_{41}-M_{23}+M_{21}}$$

$$F = \frac{M_{22}M_{43}-M_{22}M_{41}-M_{23}M_{42}+M_{21}M_{42}}{M_{43}-M_{41}-M_{23}+M_{21}} .$$

6.5.1.2 Geometric Analysis Math Model

The 2x2 matrix elements for each inner and outer flow path element in equation (1) are derived in the same manner as described in the LINE subroutine using applicable cross-sectional flow areas. The length (X) of the straight line inner elements are related to the equivalent length (KX) and area (AR) of the helical flow path as described below.



X = length of inner tube between successive holes

PIT = helix pitch

C = mean circumference of outer flow path

$C = SI \times \pi$

$$C = \frac{(OID + IOD)}{2} \times \pi$$

AI = helix angle

$$AI = \tan^{-1}\left(\frac{SI \times \pi}{PIT}\right)$$

K = width of outer flow path

K = PIT Sin (AI)

S = depth of outer flow path

S = (OID-IOD)/2

AR = Area of outer flow path

AR = K x S - LAR

KX = Length of outer flow path between successive holes

$$KX = \frac{X}{\text{Cos AI}}$$

6.5.2 Assumptions - Shrinking of the 4x4 matrix to a 2x2 matrix as described in 6.5.1.1 above is assumed to be valid. Frequency dependent friction (BETA) is computed for the inner tube and is assumed to also apply to the outer helical flow path. The velocity of sound in the fluid is not modified by a line support factor or material elasticity.

6.5.3 Computation Method - Matrix algebra.

6.5.4 Approximations - Unknown

6.5.5 Limitations - Unknown

6.5.6 Variable Names - Variable names unique to the WHEQUT subroutine are listed below. Common variables are described in paragraph 3.1.6.1.

SYMBOL	DESCRIPTION	UNITS
AI	Helix angle	RAD
AR	Area of outer flow path	IN**2
C	Velocity of Sound	IN/SEC
CI	Ratio of mean circumference of outer flow to helix pitch	--
DH(-)	Input data for distance from inlet to each connecting hole	IN

SYMBOL	DESCRIPTION	UNITS
GH	Temporary variable	--
G1-G4	Temporary variables	--
HL	Temporary variable	--
I	Integer counter	
IID	Input data for inner tube inside diameter	IN
IOD	Input data for inner tube outside diameter	IN
J	Integer counter	--
K	Width of outer flow path	IN
KK	Address of second record input data	--
KX	Distance on outer flow path	
LAR	Input data wound element cross-sectional area	IN**2
LH	Length of interconnecting holes	IN
M(-)	4x4 array for Quincke tube total matrix	--
MA(-)	4x4 array for respective inner and outer tube transfer matrix	--
MB(-)	Temporary 4x4 array while computing M(-)	--
MC(-)	4x4 array for interconnecting hole transfer matrix	--
NH	Input data for number of interconnecting holes	--
OID	Input data for outer tube inside diameter	IN
OOD	Input data for outer tube outside diameter	IN
PIT	Input data for helix pitch	IN
RH	Temporary variable	--
S	Depth of outer flow path	IN

SYMBOL	DESCRIPTION	UNITS
SI	Mean diameter of outer flow path	IN
X(-)	Array for length of inner tube segments between successive holes	IN
XL	Input data for total length of outer flow path	IN
XH(-)	Input data array for length of each hole from inlet	IN
Y1-Y3	Temporary variables	--
ZH	Impedance of interconnecting hole	--
Z5	Characteristic impedance of inner tube	PSI/CIS
Z6	Characteristic impedance of outer flow path	PSI/CIS

6.5.7 WHEQUT Subroutine - Listing


```
      SUBROUTINE WHEQUT (WSTART,Y)
C * REVISED MARCH 3,1975 *
C
C *VARIABLE TYPES, DIMENSIONS, COMMANALITY*
C
      REAL IID,IOD,LAR,LH,K,KX
      COMMON BETA,G,P,Q,Z, XBE,BER,BEI,BERP,BEIP,RHO,BULK,VOL,W,VISC,PAR
     1M,PI, IEL,NEL,KTYPE(40)
      COMPLEX BETA,G,P,Q,Z,Z5,Y1,Y2,Z6,Y3,G1,G2,G3,G4,M,MA,MB,MC,ZH,GH
     1A,B,D,E,F,O
      DIMENSION G(2,2,40),PARM(8,40),P(40),Q(40),Z(40),M(4,4),MA(4,4),M
     1B(4,4),MC(4,4),XH(18),DH(18),X(18)
C
C *SOLUTION METHOD*
C
C *TEST FOR FIRST RPM*
C
C *INITIALIZE VARIABLES FROM INPUT DATA ARRAY*
C
      IF(Y.NE.WSTART) GO TO 120
      XL=PARM(1,IEL)
      IID=PARM(2,IEL)
      IOD=PARM(3,IEL)
      OID=PARM(4,IEL)
      OOD=PARM(5,IEL)
      LAR=PARM(6,IEL)
      PIT=PARM(7,IEL)
      KK=PARM(8,IEL)
      IF(KK.EQ.0) GO TO 110
      NH=PARM(1,KK)+.01
      LH=PARM(2,KK)
      DO 200 I=1,8
  200 XH(I)=PARM(I,KK+1)
      DO 210 I=9,16
  210 XH(I)=PARM(I,KK+2)
      DO 220 I=1,8
  220 DH(I)=PARM(I,KK+3)
      DO 230 I=9,16
  230 DH(I)=PARM(I,KK+4)
      X(1)=XH(1)
      DO 100 I=2,NH
  100 X(I)=XH(I)-XH(I-1)
      X(I-1)=XL-XH(I)
  110 X(I)=XL
      C=SQRT(BULK/RHO)
  120 XBE=SQRT(VISC/(2.0*W))*2./IID
      BETA=4.0*XBE**2-XBE+(0.0,1.0)*(XBE-1.0)
      Z5=BETA*RHO*C*4.0/(PI*IID*IID)
      Y1=W*BETA/C*(0.,1.)
      SI=(OID+IOD)/2
      S=(OID-IOD)/2
      CI=SI*PI/PIT
      AI=ATAN(CI)
      K=PIT*SIN(AI)
```

6.5.7 (Cont)

```
      AR=K*S-LAR
      Z6=BETA*RHO*C/AR
      DATA MA,MB,MC,M/64*(0.0)/
      DO 130 I=1,4
      MA(I,I)=(1.,0.)
      MB(I,I)=(1.,0.)
      MC(I,I)=(1.,0.)
  130 M(I,I)=(1.,0.)
      J=1
  140 Y2=X(J)*Y1
      KX=X(J)/CCOS(AI)
      Y3=KX*Y2
      G1=CCOS(Y2)
      G2=CSIN(Y2)
      G3=CCOS(Y3)
      G4=CSIN(Y3)
      MA(1,1)=G1
      MA(1,2)=-(1./Z5)*G2
      MA(2,1)=Z5*G2
      MA(2,2)=G1
      MA(3,3)=G3
      MA(4,3)=Z6*G4
      MA(3,4)=-(1./Z6)*G4
      MA(4,4)=G3
      CALL GMPRD(MA,MB,M,4,4,4)
      IF(J.EQ.NH+1) GO TO 150
      IF(NH.LE.0.0) GO TO 150
      RH=8.0*W*VISC
      RH= SQRT(RH)/DH(J)
      HL=LH*4.0*RHO/(DH(J)*DH(J))
      ZH= HL*RH+HL*W*(0.,1.)
      GH=1.0/ZH
      MC(1,2)=-GH
      MC(1,4)=GH
      MC(3,2)=GH
      MC(3,4)=-GH
      CALL GMPRD(MC,M,MB,4,4,4)
      J=J+1
      GO TO 140
  150 M(1,1)=M(1,1)+M(3,1)
      M(1,2)=M(1,2)+M(3,2)+M(1,4)+M(3,4)
      M(1,3)=M(1,3)+M(3,3)
      M(4,2)=M(4,2)+M(4,4)
      M(2,2)=M(2,2)+M(2,4)
      COM=M(4,3)-M(4,1)-M(2,3):M(2,1)
      G(1,1,IEL)=(M(1,1)*M(4,3)-M(1,3)*M(4,1))/COM
      G(1,2,IEL)=(M(1,2)*M(4,3)-M(1,2)*M(4,1)-M(1,3)*M(4,2)
     1+M(1,1)*M(4,2))/COM
      G(2,1,IEL)=(M(2,1)*M(4,3)-M(2,3)*M(4,1))/COM
      G(2,2,IEL)=(M(2,2)*M(4,3)-M(2,2)*M(4,1)-M(2,3)*M(4,2)
     1+M(2,1)*M(4,2))/COM
      RETURN
      END
```

6.6 NEW COMPONENT SUBROUTINES

Additional component model subroutines may be incorporated in the frequency response program by following the general instructions outlined below.

6.6.1 Main Program Call Requirements (Ref. Para. 3.3) - The component for a new subroutine must be assigned NTYPE and KTYPE number identifications to allow the new subroutine to be called by the existing computed go to statement in the "DO 250" loop of the main program. In order for currently programmed circuit termination logic to remain valid, a new subroutine must be assigned to an existing element family, i.e., NTYPE 1,2,3,4,5,8, or 9. NTYPE's 6 (branch) and 7 (dummy) should not be used for new subroutines. It is desirable, but not necessary, that a family contain similar types of elements. KTYPE numbers may be used for calling the applicable subroutine. KTYPE numbers for this purpose should be single digits from 0-9, since the second (left) KTYPE digit is used to indicate the number of extra data cards for an element.

As an example, suppose that a new type of valve element (KTYPE = 1) is to be added to the valve family (NTYPE = 4). The present call statement (#165) in the main program (paragraph 3.3.7) becomes

```
165 IF (KTYPE(IEL).EQ.0) CALL VALVE
    IF (KTYPE(IEL).EQ.1) CALL VALVE1.
```

If the new subroutine requires the exchange of information other than common data with the main program, it must be included in the call statement.

```
165 IF (KTYPE(IEL).EQ.0) CALL VALVE
    IF (KTYPE(IEL).EQ.1) CALL VALVE1(XXX)
```

6.6.2 Structure of New Subroutine - The first line of a new subroutine contains the subroutine name, with the same argument as the main program call statement.

Example

SUBROUTINE VALVE1(XXX)

The subroutine name is followed by specification statements. A "common" statement is included, identical to the other subroutine "commons". The dimension statement for G, PARM, P, Q, and Z is also included. Type statements are as required based on the variable names used in the new subroutine.

Specification statements are followed by initialization of applicable variables from the input data array, PARM(-,-). Computation of the new component impedance follows, and finally assignment and/or computation of the component 2X2 transfer matrix values is required. Control is then returned to the main program.

7.0 OUTPUT SUBROUTINES

7.1 GRAPH2 AND SCALED SUBROUTINES

Subroutine GRAPH2 plots the selected component and line element parameters versus pump speed. Section 6 of the main program identifies the array addresses for a particular parameter, and calls GRAPH2 for the actual plotting. GRAPH2 sets up the plot axis scaling, writes the axis scales, and plots the parameter values. Plot titles are written by Section 6 of the main program.

Subroutine SCALED is called by GRAPH2, and controls the vertical (Y) and horizontal (X) axes scaling of the plots.

7.1.2 Variable Names - GRAPH2

SYMBOL	NAME	UNITS
I	Integer counter	-
ICHART (-)	X and Y axis write characters	-
IPCHAR (1)	Plot character	-
ISP	Integer counter for Y axis	-
ISPACE (-)	Temporary variable for writing X and Y axis scales	-
LINE	Integer counter for plot line number	-
NPLTPT	Number of points per plot	-
OMEGA (-)	Pump speed values	RPM
XAX	Temporary variable for writing X axis scale values	-
XMIN	First (lowest) X axis value	RPM
Y	Temporary variable - Y axis scale value	-
YDELTA	Y-axis scale incremental value	-
YLAST	Last Y-axis scale value	-
YLO	Lowest value in YPLT search range	-
YMAX	Maximum value to be plotted	-
YMIN	Minimum value to be plotted	-

Symbol	Name	Units
YPLT (-,-)	Array of values to be plotted	-
YUP	Highest value in YPLT search range	-

7.1.3 Variable Names - SCALED

AMAX	Maximum value to be plotted	-
AMIN	Minimum value to be plotted	-
IBOT	Variable used to calculate Y axis scale values	-
IEMAX	Variable used to calculate Y axis scale values	-
IEXP	Variable used to calculate Y axis scale values	-
ITOP	Variable used to calculate Y axis scale values	-
J	Integer counter	-
MANT	Variable used to calculate Y axis scale values	-
RANGE	Range of values to be plotted	-
RMAX	Maximum Y axis scale value	-
RMIN	Minimum Y axis scale value	-
SCALE (-)	Scale factors for Y axis	-

7.1.3 Subroutine GRAPH2 - Listing

```
      SUBROUTINE GRAPH2(OMEGA,YPLT,NPLTPT,IPCHAR)
C
C * REVISED MARCH 3,1975 *
C
      DIMENSION OMEGA(125),YPLT(125,1), ISPACE(101),
     +XAX(6),ICHART(4),IPCHAR(1)
      DATA ICHART/1HI,1H-,1H+,1H /
C-----SCALE X
      XMAX=OMEGA(NPLTPT)
      XMIN=OMEGA(1)
      CALL SCALED(XMAX,XMIN)
      XDELTA=(XMAX-XMIN)/100.
C-----FIND Y-PARAMETERS
    2 YMAX=YPLT(1,1)
      YMIN=YMAX
      DO 902 I=2, NPLTPT
      YMAX=AMAX1(YMAX,YPLT(I,1))
  902 YMIN=AMIN1(YMIN,YPLT(I,1))
      IF(YMAX.NE.YMIN)GO TO 9025
      YMAX=YMAX+25.
      YMIN=YMIN-25.
 9025 CALL SCALED(YMAX,YMIN)
      YDELTA=(YMAX-YMIN)/50.
C-----ADVANCE TO TOP OF NEXT PAGE
      WRITE(6,601)
  601 FORMAT (1H1)
C-----LOOP FOR EACH PLOT LINE
      Y=YMAX + YDELTA
    7 DO 907 LINE=1, 51
      YLAST=Y
      Y=Y-YDELTA
      YUP=Y+YDELTA/2.
      YLO=YUP-YDELTA
C-----FIRST + LAST CHARACTER ON LINE = *I*
      ISPACE(1)=ICHART(1)
      ISPACE(101)=ICHART(1)
C-----FIRST + LAST LINES ALL *-*, EXCEPT *+* IN COL. 11,21,31,...81,91
        IF(LINE.NE.1.AND.LINE.NE.51) GO TO 11
    9 DO 909 ISP=2,100
      IF((ISP-1).EQ.(ISP-1)/10*10)GO TO 10
      ISPACE(ISP)=ICHART(2)
      GO TO 909
   10 ISPACE(ISP)=ICHART(3)
  909 CONTINUE
      GO TO 14
C-----INITIALIZE COL.2-100 OF LINES 2-50 TO * *, OR **------*--* IF AX
   11 IF(Y.LE.0..AND.YLAST.GT.0.)GO TO 13
   12 DO 912 ISP=2,100
  912 ISPACE(ISP)=ICHART(4)
      GO TO 14
```

7.1.3 Subroutine GRAPH2 - Listing (Continued)

```
   13 DO 913 ISP=2,100
      ISPACE(ISP)=ICHART(2)
  913 IF((ISP-1).EQ.(ISP-1)/10*10)ISPACE(ISP)=ICHART(3)
C-----SEARCH YPLT FOR VALUES IN RANGE YLO.LT.YPLT.GE.YUP
   14 DO 914 I=1, NPLTPT
      IF(YPLT(I,1).GT.YLO .AND. YPLT(I,1).LE.YUP)GO TO 145
      GO TO 914
C-----FIND COLUMN NEAREST TO I-TH VALUE OF OMEGA
  145 ISP=(OMEGA(I)-XMIN)/XDELTA + 1.5
      IF(ISP) 914,15,16
   15 ISP=1
      GO TO 18
   16 IF(ISP-102)18,17,914
   17 ISP=101
   18 ISPACE(ISP)=IPCHAR(1)
  914 CONTINUE
C-----LINES 1,11,...,41,51 HAVE Y-VALUES---THESE, PLUS LINES 6,16,26,3
C-----+ 46 ALSO HAVE *+* IN COL 1+101 IF NO PLOT CHARACTER PRESENT
      IF((LINE-1).NE.(LINE-1)/5*5)GO TO 19
      IF(ISPACE(1).NE.IPCHAR(1))ISPACE(1)=ICHART(3)
      IF(ISPACE(101).NE.IPCHAR(1))ISPACE(101)=ICHART(3)
      IF((LINE-1).NE.(LINE-1)/10*10)GO TO 19
C-----WRITE PLOT LINE
      WRITE(6,602)Y, ISPACE
  602 FORMAT(1X,17X,F9.2,2X,I 1A1)
      GO TO 9 7
   19 WRITE(6,6 3)ISPACE
  6 3 FORMAT(1X,28X,1 1A1)
  9 7 CONTINUE
C-----CALCULATE + PRINT X-AXIS VALUES
   2  DO 92  I=1, 6
   92 XAX(I)=XMIN + (I-1)*2 .*XDELTA
      WRITE(6,6 4) XAX
  6 4 FORMAT(1X,22X,5(F9.2,11X),F9.2)
      RETURN
      END
```

7.1.4 Subroutine SCALED - Listing

```
      SUBROUTINE SCALED(RMAX,RMIN)
C
C * REVISED MARCH 3,1975 *
C
      DIMENSION SCALE(6)
      DATA SCALE/.5,1.,2.,5.,10.,20./
      RANGE=RMAX-RMIN
      AMAX=RMAX
      AMIN=RMIN
      IEXP=ALOG10(RANGE)
      MANT=RANGE/10.**IEXP
      IF(RANGE.GT.MANT*10.**IEXP)MANT=MANT+1
      GO TO(80,90,100,100,100,110,110,110,110,70), MANT
   70 MANT=1
      IEXP=IEXP+1
   80 J=2
      GO TO 120
   90 J=3
      GO TO 120
  100 J=4
      GO TO 120
  110 J=5
  120 IEMAX=ALOG10(RMAX)
      RMAX=INT(AMAX/10.**IEMAX)*10.**IEMAX
  121 IF(RMAX.GE.AMAX)GO TO 130
      RMAX=RMAX+.05*SCALE(J)*10.**IEXP
      GO TO 121
  130 RMIN=RMAX-SCALE(J)*10.**IEXP
      IF(RMIN.LE.AMIN)GO TO 150
      J=J+1
      IF(J.LT.5.5)GO TO 130
      J=1
      IEXP=IEXP+1
      GO TO 130
  150 IF(RMIN*AMIN.GT.0.)GO TO 170
      RMIN=0.
      RMAX=SCALE(J)*10.**IEXP
      IF(RMAX.LT.SCALE(J-1)*10.**IEXP)RMAX=SCALE(J-1)*10.**IEXP
  170 IF(RMIN.LT.0.)GO TO 180
      IF(RMIN.GT..1*RMAX)GO TO 180
      RMIN=0.
      RMAX=SCALE(J)*10.**IEXP
      IF(RMAX.LT.AMAX)RMAX=SCALE(J+1)*10.**IEXP
      RETURN
  180 ITOP=(RMAX-AMAX)/(.05*SCALE(J)*10.**IEXP)
      IBOT=(AMIN-RMIN)/(.05*SCALE(J)*10.**IEXP)
      IF(ITOP.EQ.IBOT)RETURN
      RMIN=RMIN+IABS((ITOP-IBOT)/2)*.05*SCALE(J)*10.**IEXP
      RMAX=RMIN+SCALE(J)*10.**IEXP
      RETURN
      END
```

8.0 UTILITY SUBROUTINES

8.1 GMPRD SUBROUTINE

Subroutine GMPRD is a modified IBM scientific computational subroutine (Reference (2)) which multiplies two complex general matrices to form a resultant complex general matrix such that

$$[A]\cdot[B]=[R] \tag{1}$$

GMPRD is currently called only by the WHEQUT subroutine, where it is used to multiply complex matrices of 4 x 4 order.

8.1.1 Variable Names - GMPRD variable names are listed below.

SYMBOL	DESCRIPTION	UNITS
A	Name of first input matrix	-
B	Name of second input matrix	-
R	Name of output matrix	-
N	Number of rows in A	-
M	Number of columns in A and rows in B	-
L	Number of columns in B	-

8.1.2 GMPRD Subroutine - Listing

```
SUBROUTINE  GMPRD   TRACE                                    CDC 6600
                    SUBROUTINE GMPRD(A,B,R,N,M,L)
              C
              C**** REVISED SEPTEMBER 3,1974 ****
              C
                    COMPLEX A,B,R
                    DIMENSION A(1),B(1),R(1)
              C
                    IR=0
                    IK=-M
                    DO 100 K=1,L
                    IK=IK+M
                    DO 100 J=1,N
                    IR=IR+1
                    JI=J-N
                    IB=IK
                    R(IR)=0
                    DO 100 I=1,M
                    JI=JI+N
                    IB=IB+1
                100 R(IR)=R(IR)+A(JI)*B(IB)
                    RETURN
                    END
```

8.2 FLUID SUBROUTINE

Subroutine FLUID calculates and writes the fluid density, adiabatic bulk modulus, and kinematic viscosity at the fluid steady state temperature (TEMP) and pressure (PRESS) specified in the frequency program input data. Data are currently included in FLUID for three hydraulic fluids; MIL-H-5606B, MIL-H-83282, and SKYDROL 500B. FLUID is presently dimensioned to accept data on three additional fluids. Fluid data sources and calculation of properties are discussed in Appendix B, along with tabulations of data presently included in FLUID. Data sources are also contained in the FLUID subroutine itself via comment records. Volume 1 of this report describes the option whereby the user may specify and input his own fluid data to the HSFR program, if the user desires to not use FLUID.

The FLUID subroutine argument requires the input data values for the temperature, pressure, and the fluid type identification number. Also, the argument includes the variable names of the three fluid properties, since they are not in "common". Parameters are then dimensioned for nine input data points and six fluids. Data statements are then used to input the name of each fluid, the nine temperature data points for each fluid, and the bulk modulus and viscosity data corresponding to the nine temperature points for each fluid. Only two points are used for density input data since a straight line interpotation is used over the entire temperature range for density calculations.

Subroutine INTERP is then called to estimate the fluid property value at the actual fluid operating temperature. Viscosity is then converted from metric (centistokes) to English units (newts). Density, bulk modulus, and viscosity values are then corrected to the steady state operating pressure.

Finally FLUID writes the computed properties before returning control to the main program.

8.2.1 Math Model - Not applicable.

8.2.3 Computation Method - FLUID calls subroutine INTERP (Ref. paragraph 8.3) which derives the fluid property at operating conditions from input property data.

8.2.4 Approximations - Not applicable.

8.2.5 Limitations - Not applicable.

8.2.6 Variable Names

SYMBOL	NAME	UNITS
A	Temperature ratio	-
ABULK(-,-)	Array for ten adiabatic bulk modulus input data points for six fluids	PSI
ATEMP(-,-)	Array for ten temperature data points for six fluids	°F
AVISC(-,-)	Array for ten viscosity input data points for six fluids	CENTISTOKES
B	Viscosity correction exponent	-
BRHO(-,-)	Array for two density input data points for six fluids	LB*SEC**2/IN
BTEMP(-)	First and next to last temperature input points	°F
BULK	Bulk modulus at operating conditions	PSI
COEFF	Viscosity correction factor	-
IF	Input data fluid type identification number	-
IFLUNM(-,-)	Array for fluid names	-
IK	Number of input temperature points	-
IND	Error indicator	-
J	Integer counter	-
PRESS	Input data fluid operating pressure	PSIG

8.2.6 Variable Names (Cont'd)

SYMBOL	NAME	UNITS
RHO	Density at operating conditions	LBS*SEC**2/IN
TEMP	Input data fluid operating temperature	°F
VISC	Viscosity at operating conditions	IN**2/SEC

8.2.7 FLUID Subroutine Listing

```
      SUBROUTINE FLUID (TEMP,PRESS,IF,VISC,BULK,RHO)
C**** REVISED MARCH 3, 1975 ****
      DIMENSION ATEMP(10,6),AVISC(10,6),ABULK(10,6),BRHO(2,6),
     1BTEMP(2),COEFF(6),IK(6),IFLUNM(3,6)
C
C     SECOND SUBSCRIPT REFERS TO FLUID TYPE (IF PARAMETER)
C
      DATA IFLUNM
     1/8H FOR MIL,8H-H-5606B,8H AT      ,
     28H FOR MIL,8H-H-83282,8H AT      ,
     38H FOR SKY,8HDROL 500,8HB AT     ,
     46*8H        ,
     58H        ,8H        ,8H    AT  /
C
      DATA ATEMP /
     1-65.,-40.,0.,50.,100.,150.,200.,250.,300.,300.,
     2-65.,-40.,0.,50.,100.,150.,200.,250.,300.,300.,
     3-65.,-40.,0.,50.,100.,150.,200.,250.,300.,300.,
     430*10. /
C
      DATA BTEMP /
     1-65.,275. /
C
C       RHO,BULK AND VISC DATA ARE FOR 0.0 PSIG
C
C     RHO DATA SOURCE:
C     1-MDC REPORT A2686 DATED 4/74
C     2-MDC REPORT A2686 DATED 4/74
C     3-MONSANTO DATA SHEET DATED 6/67(DOUGLAS HYD MANUAL)
      DATA BRHO /
     18.57E-5,7.63E-5,
     28.49E-5,7.3E-5,
     310.3E-5,8.9E-5,6*10./
C
C     BULK DATA SOURCE:
C     1-LETTER TO G.AMIES FROM J.W.NOONAN DATED 11/70
C     2-TECHNICAL REPORT AFML-TR-73-81 DATED 4/73
C     3-LETTER TO G.AMIES FROM J.W.NOONAN DATED 11/70
      DATA ABULK /
     113.47E5,3.25E5,2.9E5,2.48E5,2.08E5,1.73E5,1.42E5,1.19E5,.98E5,
     A.98E5,
     213.47E5,3.25E5,2.9E5,2.48E5,2.08E5,1.73E5,1.42E5,1.19E5,.98E5,
     A.98E5,
     334.26E5,4.05E5,3.64E5,3.18E5,2.7E5,2.29E5,1.94E5,1.62E5,1.38E5,
     A1.38E5,30*10. /
C
C     VISC DATA SOURCE:
C     1-MDC REPORT A2686 DATED 4/74
C     2-MDC REPORT A2686 DATED 4/74
C     3-MONSANTO DATA SHEET DATED 6/67(DOUGLAS HYD MANUAL)
      DATA AVISC /
```

8.2.7 FLUID Subroutine Listing - Cont'd

```
      11993.5,482.3,134.4,34.85,14.47,7.46,4.58,3.19,2.39,2.39,
      211446.9,2019.3,269.45,48.87,15.95,7.46,4.24,2.83,2.04,2.04,
      33485.5,598.07,104.18,27.9,11.7,6.5,4.18,2.89,2.15,2.15,
      A30*10./
C
       DATA IK/3*9,3*10/
       DATA COEFF/.335,.33,.42,3*10./
C
       IF(IF.EQ.0) GO TO 100
       CALL INTERP(TEMP,ATEMP(1,IF),ABULK(1,IF),20,IK(IF),
      1BULK,IND)
       CALL INTERP (TEMP,BTEMP,BRHO(1,IF),10,2,
      1RHO,IND)
       CALL INTERP (TEMP,ATEMP(1,IF),AVISC(1,IF),11,
      1 IK(IF),VISC,IND)
       CALL INTERP(TEMP,ATEMP(1,IF),AVISC(1,IF),12,
      1 IK(IF),COEFF(IF),IND)
C
C      VIsC IS CONVERTED FROM CENTISTOKES TO NEWTS
       VISC=VISC*1.555E-3
C      DENSITY, VISC AND BULK ARE ADJUSTED TO PRESSURE 'PRESS'
       RHO=RHO*(1.+PRESS/2.5E5)
       BULK=BULK+12.*PRESS
       A=560./(TEMP+460.)
       B=((COEFF(IF))**A)*PRESS*2.3E-4
       VISC=VISC*EXP(B)
 200   WRITE(6,601) (IFLUNM(J,IF),J=1,3),PRESS,TEMP,VISC,RHO,BULK
 601   FORMAT(//25X,11H FLUID DATA ,3A8,F7.1,9H PSIG AND ,F6.1,
      A7H DEG F ,//,35X,
      1          14HVISCOSITY    -,  E10.3,2X,9HIN**2/SEC,/,35X,
      2          14HDENSITY      -,  E10.3,2X,17H(LB-SEC**2)/IN**4,
      3          /,35X,14HBULK MODULUS -,  E10.3,2X,3HPSI )
       GO TO 222
  100  CONTINUE
       IF=6
       WRITE(6,602) (IFLUNM(J,6),J=1,3),TEMP,VISC,RHO,BULK
 602   FORMAT(//25X,16H FLUID DATA FOR ,3A8,12H     PSIG AND,F6.1,
      A7H DEG F ,//,35X,
      1          14HVISCOSITY    -,E10.3,2X,9HIN**2/SEC,/,35X,
      2          14HDENSITY      -,E10.3,2X,17H(LB-SEC**2)/IN**4,
      3          /,35X,14HBULK MODULUS -,E10.3,2X,3HPSI )
 222   CONTINUE
       RETURN
       END
```

8.3 INTERP Subroutine

The INTERP subroutine provides interpolation for continuous or discontinuous functions of the form Y = f(X). INTERP is a shortened version of a MCAUTO library functional subroutine named DISCOT. (Reference (7)).

INTERP uses two other subroutines, DISER1 and LAGRAN, to derive the dependent variable from tabulated data input by the programmer or already existing in the program subroutine. Subroutine DISER1 gives the data points around the X variable. Lagrange's interpolation formula is used in the subroutine LAGRAN to obtain a Y value. For an X value lying outside the range of the tabulated data, the Y value will be extrapolated. Fluid viscosities are calculated using a modified Walther equation (Reference (8)).

8.3.1 Solution Method. The INTERP subroutine provides the necessary control parameters to DISER1 and LAGRAN to yield a dependent variable. The subroutine arguments are:

Subroutine INTERP (X,TABX, TABY, NC, NY, Y, IND)

where:

X - Argument of function Y = f(X)

TABX - X array of independent variables in ascending order

TABY - Y array of dependent variables in ascending order

NC - Control word

Tens Digit - Degree of interpolation

Units Digit - 1 = Walther equation

0 = LAGRAN interpolation

NY - Number of data points in the Y array

Y - Dependent variable

IND - Error indicator

0 = Normal interpolation

1 = Extrapolation outside range of data points.

8.3.2 Assumptions. Not applicable

8.3.3 Computations. The degree of interpolation is decoded from the control word NC in the INTERP subroutine argument and passed to DISER1. The error indicator IND is set to zero. On finding the data point closest to the X value from DISER1, it is entered into the LAGRAN subroutine argument. If the modified Walther equation is to be used for a viscosity calculation, IDX is set equal to -1.

8.3.4 Approximations. Not applicable

8.3.5 Limitations. The X and Y data points must be entered in an ascending order. When tabulating a discontinuous function the independent variable (X) at the point of discontinuity is repeated, i.e.,

$$X_1, X_2, X_3, X_3, X_4, X_5$$

$$Y_1, Y_2, Y_3, Y_4, Y_5, Y_6$$

Thus for discontinuous functions there must be K + 1 points above and below the discontinuity, where K is the degree of interpolation.

8.3.6 INTERP Variable Names.

SYMBOL	NAME	UNITS
IDX	Degree of interpolation	-
IND	Solution indicator	-
	= 0 Normal interpolation	
	= 1 Extrapolation outside of data range	
NC	Control word	-
NPX	Dummy array	-
NPX1	Location of data point X, Y for interpolation	-
NY	Number of Y data points	-
TABX	X array of data points	-
TABY	Y array of data points	-

SYMBOL	NAME	UNITS
X, XA	Independent variable	-
Y	Dependent variable	-

8.3.7 Subroutine INTERP - Listing

```
SUBROUTINE INTERP(X,TABX,TABY,NC,NY,Y,IND)
DIMENSION TABX(1),TABY(1),NPX(8)
IDX=(NC-(NC/100)*100)/10
IND=0
XA=X
CALL DISER1(XA,TABX,1,NY,IDX,NPX,IND)
NPX1=NPX(1)
IF((NC-IDX*10).EQ.1)IDX=-1
IF((NC-IDX*10).EQ.2)IDX=-2
CALL LAGRAN(XA,TABX(NPX1),TABY(NPX1),IDX+1,Y)
RETURN
END
```

8.4 <u>DISER1 Subroutine</u>

The subroutine DISER1 returns the array location of the lower bound value of the interval in which the independent variable lies. DISER1 is a modification of a MCAUTO library subroutine named DISSER, (Reference (7)). The arguments for the DISER1 subroutine are as follows:

Subroutine DISER1 (XA, TAB, I, NX, ID, NPX, IND)

XA - Independent variable

TAB - X array

I - Tabulated data location

NX - Number of points in the independent array

ID - Degree of interpolation

NPX - Location of lower bound for data point XA, in the TAB array

IND - Indicator

8.4.1 Solution Method. Not applicable

8.4.2 Assumptions. Not applicable

8.4.3 Computations. On entry of the independent variable, XA, and the tabulated data form the TABX array, DISER1 finds the tabulated data values that bound XA, and returns the smaller one to the calling program. If XA lies outside the lower end of the data, DISER1 returns the first data point as the lower bound. Should XA lie outside the upper tabulated value, the second from the last data point location is returned by DISER1.

8.4.4 Approximations. Not applicable

8.4.5 Limitations. Not applicable

8.4.6 DISER1 Variable Names

Symbol	Name	Units
IND	Solution indicator	-
I, ID, IT, J, NLOC, NLOW, NPB, NPT, NPU, NPX, NUPP, NX, NXX	Integer counters	-
TAB	Array of independent variables	-
XA	Independent variable	-

8.4.7 Subroutine DISER1 - Listing

```
      SUBROUTINE DISER1(XA,TAB,I,NX,ID,NPX,IND)
      DIMENSION TAB(1)
      IF(XA-TAB(I))71,73,72
71    IND=IND+1
      NPX=I
      RETURN
73    XA=TAB(I)
      NPX=I
      RETURN
72    J=I+NX-1
      IF(XA-TAB(J))1,77,76
76    IND=IND+1
      NPX=J-ID
      RETURN
77    XA=TAB(J)
      NPX=J-ID
      RETURN
1     NPT=ID+1
      NPB=NPT/2
      NPU=NPT-NPB
      IF(NX-NPT)4,5,10
4     ID=NX-1
      GO TO 1
5     NPX=I
      RETURN
10    NLOW=I+NPB
      NUPP=I+NX-(NPU+1)
      IF(NX-20)18,18,11
11    NXX=NX/2+I
      IF(XA-TAB(NXX))12,17,13
12    NXX=NXX-NX/4
      IF(XA-TAB(NXX))18,17,17
13    NXX=NXX+NX/4
      IF(XA-TAB(NXX))14,17,17
14    NLOW=NXX-NX/4
      GO TO 18
17    NLOW=NXX
18    DO 19 II=NLOW,NUPP
      NLOC=II
      IF(TAB(II)-XA)19,20,20
19    CONTINUE
      NPX=NUPP-NPB+1
      RETURN
20    NPX=NLOC-NPB
      RETURN
      END
```

8.5 LAGRAN Subroutine

The LAGRAN subroutine interpolates or extrapolates a data point from two known tabulated values. In addition, LAGRAN calculates viscosity using a modified Walther equation. The LAGRAN subroutine arguments are

Subroutine LAGRAN (XA, X, Y, N, ANS)

XA - Independent variable

X - X array

Y - Y array

N - Degree of interpolation

ANS - Dependent variable

8.5.1 Math Model. LAGRANGES interpolation equation is used in this subroutine to calculate the dependent variable. The LAGRANGE formula is

$$P(x) = \sum_{i=0}^{m} L_i(x)\, y_i \tag{2}$$

where

$L_i(x)$ is the Lagrange multiplier function.

$$L_i(x) = \frac{(x-x_0)(x-x_1)\cdots(x-x_{i-1})(x-x_{i+1})\cdots(x-x_n)}{(X_i-x_0)(x_i-x_1)\cdots(x_i-x_{i-1})(x_i-x_{i+1})\cdots(x_i-x_n)} \tag{3}$$

The LAGRANGE equation generates a polynomial between two data points. The degree of the polynominal is that specified by the index value n. The dependent variable is returned as ANS in the subroutine argument.

A modified version of the Walther equation taken from Reference (8) is used in the calculation of viscosity. The ASTM charts are based on this equation.

$$\mathrm{LOG}\,[\mathrm{LOG}\,(v+c)] = A\ \mathrm{LOG}\ {}^{\circ}R + B \tag{4}$$

where

c = a constant

°R = Temperature, °RANKINE

v = Viscosity, cSt

A,B = Constants for each fluid

LOG = Log to the base 10

The ASTM chart expresses c as a constant varying from 0.75 at 0.4 cSt to 0.6 at 1.5 cSt and above.

8.5.2 Assumptions. The Lagrangian equation generated by the subroutine only uses the data points around the dependent variable to generate a polynomial for interpolation. The last or first set of two data points is used for extrapolation. The equation used to determine the viscosity uses a constant factor that is applicable to viscosity values of 2 centistokes or more.

8.5.3 Computation. The procedure LANGRAN performs, whether it be interpolation or the viscosity calculation, is always recognized by testing the n argument in the subroutine statement. If n is equal to zero, then the viscosity is calculated using the modified Walther equation. Otherwise n specifies the degree of interpolation to be used by the Langrange formula. Both results are returned to the calling program through the variable named ANS. The LAGRAN interpolation is a direct application of equation (2) to the given data.

Before evaluating the viscosity equation (4) for the viscosity value at XA temperature, the constants A and B must be calculated. They are determined using the data points that surround the dependent variable, or the first or last set of two data points if the dependent variable lies outside the range of the tabulated data. With the constants calculated for this fluid the viscosity can be computed from Equation (4).

8.5.4 Approximations. In the viscosity calculation, 0.6 was used as a constant factor for all ranges of viscosity.

8.5.5 Limitations. Since the LAGRANGE method only uses two data points to interpolate it can become inaccurate for remotely spaced tabulated data points. Any degree of interpolation greater than two can lead to erroneous results.

For the viscosity equation, any computed value of viscosity less than two centistokes cannot be considered accurate, and should be weighed in the final results.

8.5.6 LAGRAN Subroutine Variable Names

SYMBOL	NAME	UNITS
A	Constant for viscosity	-
ANS	Dependent variable	-
B	Constant for viscosity	-
I,J	Integer counters	-
N	Method of solution	-
	N = 0 Viscosity calculation	
	N > 0 Degree of interpolation	
PROD	Lagrange partial product	-
P1	LOG LOG of (Y(1) + C)	cSt
P2	LOG LOG of (Y(2) + C)	cSt
T1	LOG of T(1)	°R
T2	LOG of T(2)	°R
X	X-array	-
XA	Independent variable	-
Y	Y-array	-

8.5.7 Subroutine LAGRAN - Listing

```
      SUBROUTINE LAGRAN(XA,X,Y,N,ANS)
      DIMENSION X(1),Y(1)
      IF(N.EQ.-1) GO TO 20
      IF(N.EQ.0)GO TO 10
      SUM=0.0
      DO 3 I=1,N
      PROD=Y(I)
      DO 2 J=1,N
      A=X(I)-X(J)
      IF(A) 1,2,1
1     B=(XA-X(J))/A
      PROD=PROD*B
2     CONTINUE
3     SUM=SUM+PROD
      ANS=SUM
      RETURN
C     VISCOSITY CALCULATION
   10 CONTINUE
      A1=0.
      IF(Y(1).LE.2.)A1=EXP(-1.47-1.84*Y(1)-.51*Y(1)**2)
      A2=0.
      IF(Y(2).LE.2.)A2=EXP(-1.47-1.84*Y(2)-.51*Y(2)**2)
      P1=ALOG10(ALOG10(Y(1)+.7+A1))
      P2=ALOG10(ALOG10(Y(2)+.7+A2))
      T1=ALOG10(X(1)+460.)
      T2=ALOG10(X(2)+460.)
      B=(P1-P2)/(T2-T1)
      A=P1+B*T1
      Z=10**(10**(A-B*ALOG10(XA+460.)))
      IF(Z.LE.2.7)GO TO 11
      ANS=Z-.7
      RETURN
   11 ANS=(Z-.7)-EXP(-.7487-3.295*(Z-.7)+.6119*(Z-.7)**2
     +-.3193*(Z-.7)**3)
      RETURN
C     FLUIDE CALCULATION
   20 CONTINUE
      P1=ALOG10(ALOG10(Y(1)+.6))
      P2=ALOG10(ALOG10(Y(2)+.6))
      T1=ALOG10(X(1)+460.)
      T2=ALOG10(X(2)+460.)
      B=(P1-P2)/(T2-T1)
      A=P1+B*T1
      V0=10**(10**(A-B*ALOG10(XA+460.)))-.6
      T5=10**((.125989+A)/B)
      T1000=10**((-.477159+A)/B)
      S=ALOG10((T5)/100.+1.)-ALOG10((T1000)/100.+1)
      DELX=65.10979*S
      S=6.65/DELX
      ALPHA=3.23523-11.3886*S+13.1735*S*S-4.8881*S*S*S
      BETA=-5.33425+19.9521*S-23.9448*S*S+10.155*S*S*S
      CHI=3.35452-13.1273*S+17.1712*S*S-7.6551*S*S*S
      ANS=ALPHA+BETA*ALOG10(V0)+CHI*(ALOG10(V0))**2
      IF(ANS.LT.0.)ANS=0
      RETURN
      END
```

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APPENDIX A

DYNAMIC RESPONSE OF FLUID LINES

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Dynamic Response of Fluid Lines

The use of hydraulic transmission lines in automatic control, liquid-propellant rocket, and other systems requires accurate knowledge of their dynamic response. In this paper the effects of fluid viscosity and compressibility are included to derive transfer functions relating the pressure and velocity variables at the two cross sections of a line. The results of theoretical analysis are compared with experimental data obtained from frequency-response tests. The analysis includes the significant effect on the dynamic response caused by the natural frequency of vibration of the line in the longitudinal direction. It is shown that in small-diameter lines the viscous effects cannot be neglected. The theoretical analysis may be used to improve the performance of systems incorporating hydraulic networks.

Introduction

THE purpose of early investigation on unsteady fluid flow in straight pipes was to analyze the surging phenomena or "water hammer" in power plants employing large-diameter conduits and to determine the velocity of sound in the fluid. The classical work was done by Joukowski [1][1] and Allievi [2] around 1900. More recently, unsteady flow in small-diameter pipes plays a major role in liquid-propellant rocket systems, hydraulic and pneumatic control systems, the circulation system of the blood, and elsewhere. Ezekiel and Paynter [3] obtained ordinary differential equations in hyperbolic operators relating pressures and flows at two cross sections of a hydraulic line. Oldenburger and Donelson [4] verified the validity of these equations in tests at the Tennessee Valley Authority on a hydroelectric installation with an 8-mile 18-ft-dia tunnel. Regetz [5] at the National Aeronautics and Space Administration Lewis Research Center did the same for a 1-in-dia line. Oldenburger and Goodson [6] simplified the Paynter equations. In most of the studies the fluid is taken as nonviscous, or at most the viscosity effect is included in the form of pressure loss due to friction. For smaller diameter lines the nonviscous water-hammer theory fails to agree accurately with experiment [7].

Walker, Kirkpatrick, and Rouleau [8] added a viscosity term due to the velocity gradient in the direction of flow, but neglected the gradient across the tube cross section. Their results are applicable to the wave motion in a viscous fluid outside the boundary layer and do not explain the internal flow in a pipe where the viscous effects of the boundary-layer shearing of one fluid layer on another are of a much larger order of magnitude. The case of a viscous, incompressible fluid has been treated by Uchida [9], neglecting the variations of the pressure gradient and velocity along the length of the pipe as in this case the velocity of propagation of disturbances in the fluid is infinite. Brown [10] has treated a compressible fluid but also has neglected the variations of the pressure gradient and velocity along the length of the line [see equations (56) and (57) of Brown]. This theory is valid for an infinite or long line.

If not properly accounted for, the dynamics of small-diameter hydraulic lines may cause combustion instability in rockets or unsatisfactory performance elsewhere. It is the objective of this paper to derive transfer functions of hydraulic networks employing small-diameter pipes, so that their performance may be correctly and easily predicted and the design of the systems incorporating them improved.

The Basic Differential Equations

Assumptions

The following assumptions are made:

(i) The elasticity of the pipe walls may be neglected when compared with the compressibility of the fluid. As shown in Appendix 1, this is a fairly good assumption for small-diameter pipes.

(ii) The temperature variations are small enough so that the fluid viscosity may be considered to be constant.

(iii) The velocity and the change of all dependent variables in the ϕ (circumferential) direction are negligible due to rotational symmetry.

(iv) The flow is laminar. This assumption implies that the Reynolds number is about 2000 or less.

Differential Equations

It is convenient to use cylindrical coordinates whose x-axis is identified with the center line of the pipe as shown in Fig. 1. Let r be the coordinate in the radial direction and t denote time.

[1] Numbers in brackets designate References at end of paper.

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Nomenclature

a = inside radius of tube
$A(s)$ = constant of integration
A_1 = inside cross-sectional area of pipe
A_2 = cross-sectional area of pipe wall
A_3 = area of orifice
b = speed of sound in material of pipe wall
ber, bei = Thomson functions of first kind and order zero
ber′, bei′ = first derivatives of ber and bei
$B(s)$ = constant of integration
$B_1(s)$ = constant of integration
$B_2(s)$ = constant of integration
c = speed of sound in fluid
E = modulus of elasticity of pipe
$f(x, s)$ = defined by equation (18)
$G(s)$ = transfer matrix
h = wall thickness of pipe
j = $\sqrt{-1}$
J_0, J_1 = Bessel functions of first kind
K = bulk modulus of liquid
K_0 = defined by equation (29)
L = distance between sections 1 and 2
m = mass of orifice block
n = defined by equation (39)
p = pressure deviation, $p(x, r, t)$
$\bar{p}$ = cross-sectional average pressure deviation, $\bar{p}(x, t)$
$\bar{P}(x, s)$ = Laplace transform of $\bar{p}(x, t)$
q = defined by equation (39)
Q = volume
r = coordinate in radial direction
s = Laplace variable, transformation with respect to time
t = time

(Continued on next page)

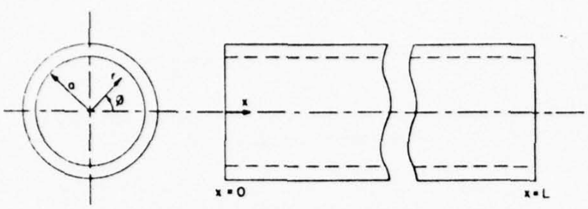

Fig. 1 Pipe coordinates

The deviations of the velocities in the x and r-directions from their steady-state values are denoted by $u(x, r, t)$ and $v(x, r, t)$, respectively, and the deviation of the pressure by $p(x, r, t)$. Let ρ designate the fluid density, μ the absolute viscosity, ν the kinematic viscosity, and K the fluid bulk modulus.

The complete Navier-Stokes' equations in cylindrical coordinates are given by Pai [11]. With the foregoing assumptions, these equations for the deviations are simplified and given in the following text.

Equation of Motion: *x*-Direction

$$\rho\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r}\right] = -\frac{\partial p}{\partial x} + \mu\left[\frac{4}{3}\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{3}\frac{\partial}{\partial x}\left(\frac{\partial v}{\partial r} + \frac{v}{r}\right)\right] \quad (1)$$

Equation of Motion: *r*-Direction

$$\rho\left[\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial r}\right] = -\frac{\partial p}{\partial r} + \mu\left[\frac{4}{3}\frac{\partial^2 v}{\partial r^2} + \frac{4}{3}\frac{1}{r}\frac{\partial v}{\partial r} - \frac{4}{3}\frac{v}{r^2} + \frac{\partial}{\partial x}\left(\frac{1}{3}\frac{\partial u}{\partial r} + \frac{\partial v}{\partial x}\right)\right] \quad (2)$$

Continuity Equation

$$\frac{\partial \rho}{\partial t} + \rho\frac{\partial v}{\partial r} + \rho\frac{v}{r} + \rho\frac{\partial u}{\partial x} + v\frac{\partial \rho}{\partial r} + u\frac{\partial \rho}{\partial x} = 0 \quad (3)$$

Equation of State for a Liquid

$$\frac{\partial \rho}{\rho} = \frac{\partial p}{K} \quad (4)$$

The following further assumptions are made:

(i) Since $u \gg v$, we neglect equation (2). Neglecting this equation implies that the pressure is constant across the cross section of the tube and becomes a function only of x and t.

(ii) As shown in Appendix 2, $\partial u/\partial t \gg u(\partial u/\partial x)$ and $\partial u/\partial t \gg v(\partial u/\partial r)$. Hence in equation (1) the nonlinear convective acceleration terms on the left-hand side may be neglected.

(iii) It is also shown in Appendix 2 that the only important viscous terms on the right-hand side of equation (1) are $\partial^2 u/\partial r^2$ and $(1/r)\,\partial u/\partial r$. The other viscous terms in that equation may be neglected.

(iv) In equation (3), the terms $v(\partial\rho/\partial r)$ and $u(\partial\rho/\partial x)$ may be neglected when compared with the other terms. This assumption may be justified on the ground that the bulk modulus for liquid is of the order of 300×10^3 psi.

With the foregoing assumptions equations (1) to (4) reduce to the following two differential equations:

$$\rho\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \mu\left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right] \quad (5)$$

$$\frac{1}{K}\frac{\partial p}{\partial t} + \frac{\partial v}{\partial r} + \frac{v}{r} + \frac{\partial u}{\partial x} = 0 \quad (6)$$

If in equation (5) μ is assumed to be zero, the equation reduces to the basic water-hammer equation of reference [6].

Solution of Differential Equations

We define $\bar{p}$ as the average pressure and $\bar{u}$ as the average velocity in the x-direction across the tube cross section; i.e.,

$$\pi a^2 \bar{p}(x, t) = \int_0^a 2\pi r p\,dr$$

and

$$\pi a^2 \bar{u}(x, t) = \int_0^a 2\pi r u\,dr$$

where a is the internal radius of the tube.

From assumption (i), it may be noted that $p = \bar{p}$.

Let

$$U(x, r, s) = L[u(x, r, t)]$$

and

$$\bar{P}(x, s) = L[\bar{p}(x, t)]$$

Taking the Laplace transform of both sides of equation (5) with zero initial conditions, i.e., $u(x, r, 0) = 0$ and $\bar{p}(x, 0) = 0$, we get

$$\frac{\partial^2 U}{\partial r^2} + \frac{1}{r}\frac{\partial U}{\partial r} - \frac{s}{\nu}\left(U + \frac{1}{\rho s}\frac{\partial \bar{P}}{\partial x}\right) = 0 \quad (7)$$

Let V be defined by

$$V = U + \frac{1}{\rho s}\frac{\partial \bar{P}}{\partial x} \quad (8)$$

Substituting for V of equation (8) in equation (7), we obtain

$$\frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r} - \frac{s}{\nu}V = 0$$

The foregoing equation has only one solution which is finite at $r = 0$ and this is given by

$$V = f(x, s)\left\{J_0\left[jr\left(\frac{s}{\nu}\right)^{1/2}\right]\right\}$$

Nomenclature

u = deviation of velocity component $u(x, r, t)$ in x-direction

$\bar{u}$ = deviation of cross-sectional average velocity $\bar{u}(x, t)$ in x-direction

$\bar{U}(x, s)$ = Laplace transform of $\bar{u}(x, t)$

v = deviation of velocity component $v(x, r, t)$ in r-direction

V = defined by equation (8)

x = coordinate in axial direction

y = displacement $y(x, t)$ of pipe in x-direction

$Y(x, s)$ = Laplace transform of $y(x, t)$

α = density of material of pipe

β = defined by equation (17)

γ = real part of β

δ = imaginary part of β

ζ = coefficient of damping

μ = fluid absolute viscosity

ν = fluid kinematic viscosity

ρ = fluid density

ϕ = coordinate in circumferential direction

ω = frequency, radians per sec

Subscripts

1 = upstream section 1

2 = downstream section 2

s = steady-state value

where J_0 is the Bessel function of first kind and order zero, and $f(x, s)$ is as yet an unknown function of x and s. Thus

$$U = f(x, s)\left\{J_0\left[jr\left(\frac{s}{\nu}\right)^{1/2}\right]\right\} - \frac{1}{\rho s}\frac{\partial \bar{P}}{\partial x} \tag{9}$$

Equation (9) has to satisfy the boundary condition that $U = 0$ at $r = a$. This will be true if

$$\frac{\partial \bar{P}}{\partial x} = \rho s f(x, s) J_0\left[ja\left(\frac{s}{\nu}\right)^{1/2}\right] \tag{10}$$

Hence

$$U = f(x, s)\left\{J_0\left[jr\left(\frac{s}{\nu}\right)^{1/2}\right] - J_0\left[ja\left(\frac{s}{\nu}\right)^{1/2}\right]\right\} \tag{11}$$

Multiplying both sides of equation (11) by $2\pi r$ and integrating with respect to r from 0 to a, we have

$$\bar{U} = f(x, s)\left\{\frac{2}{ja\left(\frac{s}{\nu}\right)^{1/2}} J_1\left[ja\left(\frac{s}{\nu}\right)^{1/2}\right] - J_0\left[ja\left(\frac{s}{\nu}\right)^{1/2}\right]\right\} \tag{12}$$

Similarly multiplying both sides of equation (6) by $2\pi r$ and integrating with respect to r from 0 to a, we get

$$\pi a^2 \frac{1}{K}\frac{\partial p}{\partial t} + 2\pi \int_0^a r\frac{\partial v}{\partial r}\,dr + 2\pi\int_0^a v\,dr + \pi a^2 \frac{\partial \bar{u}}{\partial x} = 0 \tag{13}$$

Integration by parts yields

$$2\pi\int_0^a v\,dr = 2\pi v r\Big|_0^a - 2\pi\int_0^a r\frac{\partial v}{\partial r}\,dr = 0 - 2\pi\int_0^a r\frac{\partial v}{\partial r}\,dr$$

Equation (13) is therefore reduced to

$$\frac{\partial \bar{p}}{\partial t} = -K\frac{\partial \bar{u}}{\partial x} \tag{14}$$

Taking the Laplace transform of both sides of equation (14) with zero initial condition, i.e., $\bar{p}(x, 0) = 0$, it follows that

$$\bar{P} = -\frac{K}{s}\frac{\partial \bar{U}}{\partial x}$$

whence

$$\frac{\partial \bar{P}}{\partial x} = -\frac{K}{s}\frac{\partial^2 \bar{U}}{\partial x^2} \tag{15}$$

Substituting for $\partial\bar{P}/\partial x$ from equation (10) and for $\bar{U}$ from equation (12) in equation (15), we have

$$\frac{d^2 f(x, s)}{dx^2}\left\{\frac{2}{ja\left(\frac{s}{\nu}\right)^{1/2}}\frac{J_1\left[ja\left(\frac{s}{\nu}\right)^{1/2}\right]}{J_0\left[ja\left(\frac{s}{\nu}\right)^{1/2}\right]} - 1\right\} + \frac{s^2}{c^2}f(x, s) = 0 \tag{16}$$

where c is $(K/\rho)^{1/2}$; i.e., the speed of sound in the fluid.

Let β be defined by

$$\frac{1}{\beta^2} = \left\{\frac{2}{ja\left(\frac{s}{\nu}\right)^{1/2}}\frac{J_1\left[ja\left(\frac{s}{\nu}\right)^{1/2}\right]}{J_0\left[ja\left(\frac{s}{\nu}\right)^{1/2}\right]} - 1\right\} \tag{17}$$

Equation (16) now becomes

$$\frac{d^2 f(x, s)}{dx^2} + \frac{\beta^2 s^2}{c^2}f(x, s) = 0$$

The solution of the foregoing equation is given by

$$f(x, s) = A(s)\cos\frac{s\beta}{c}x + B(s)\sin\frac{s\beta}{c}x \tag{18}$$

where $A(s)$ and $B(s)$ are the constants of integration and may be obtained from the two boundary conditions on x. Hence

$$\bar{U}(x, s) = \left[A(s)\cos\frac{s\beta}{c}x + B(s)\sin\frac{s\beta}{c}x\right]\frac{J_0\left[ja\left(\frac{s}{\nu}\right)^{1/2}\right]}{\beta^2} \tag{19}$$

and

$$\bar{P}(x, s) = \left[A(s)\sin\frac{s\beta}{c}x - B(s)\cos\frac{s\beta}{c}x\right]\frac{\rho c}{\beta}J_0\left[ja\left(\frac{s}{\nu}\right)^{1/2}\right] \tag{20}$$

Let $\bar{P}_1(s)$ and $\bar{U}_1(s)$ be the Laplace transforms of the pressure and velocity, respectively, at the upstream section where $x = 0$. Then $A(s)$ and $B(s)$ may be eliminated from equations (19) and (20) and are given by

$$A(s) = \bar{U}_1\frac{\beta^2}{J_0\left[ja\left(\frac{s}{\nu}\right)^{1/2}\right]}$$

and

$$B(s) = -\bar{P}_1\frac{\beta}{\rho c J_0\left[ja\left(\frac{s}{\nu}\right)^{1/2}\right]}$$

Hence at a section a distance x from the upstream end the Laplace transforms of the velocity and pressure are given by the following two equations:

$$\bar{U}(x, s) = \bar{U}_1\cos\frac{s\beta}{c}x - \bar{P}_1\frac{1}{\beta\rho c}\sin\frac{s\beta}{c}x$$

$$\bar{P}(x, s) = \bar{U}_1\beta\rho c\sin\frac{s\beta}{c}x + \bar{P}_1\cos\frac{s\beta}{c}x$$

At the downstream end where $x = L$, the Laplace transforms of velocity and pressure are denoted by $\bar{U}_2$ and $\bar{P}_2$ and are given by

$$\bar{U}_2 = \bar{U}_1\cos\frac{s\beta L}{c} - \bar{P}_1\frac{1}{\beta\rho c}\sin\frac{s\beta L}{c} \tag{21}$$

$$\bar{P}_2 = \bar{U}_1\beta\rho c\sin\frac{s\beta L}{c} + \bar{P}_1\cos\frac{s\beta L}{c} \tag{22}$$

Let $G(s)$ be the transfer matrix defined by

$$G(s) = \begin{pmatrix}\cos\frac{s\beta L}{c} & -\frac{1}{\beta\rho c}\sin\frac{s\beta L}{c}\\ \beta\rho c\sin\frac{s\beta L}{c} & \cos\frac{s\beta L}{c}\end{pmatrix}$$

Equations (21) and (22) are expressions relating variables of a four-terminal network. In matrix form, they are given by

$$\Big(G(s)\Big)\begin{Bmatrix}\bar{U}_1\\ \bar{P}_1\end{Bmatrix} = \begin{Bmatrix}\bar{U}_2\\ \bar{P}_2\end{Bmatrix} \tag{23}$$

Frequency-Response Equations

From the transfer matrix equation (23) which represents the dynamic characteristics of the line, we get

$$\frac{\bar{U}_1}{\bar{P}_1} = \frac{\frac{\bar{U}_2}{\bar{P}_2}\cos\frac{s\beta L}{c} + \frac{1}{\beta\rho c}\sin\frac{s\beta L}{c}}{\cos\frac{s\beta L}{c} - \frac{\bar{U}_2}{\bar{P}_2}\beta\rho c\sin\frac{s\beta L}{c}} \tag{24}$$

From equation (22), we have

$$\frac{\bar{P}_2}{\bar{P}_1} = \cos\frac{s\beta L}{c} + \frac{\bar{U}_1}{\bar{P}_1}\beta\rho c \sin\frac{s\beta L}{c}$$

Substituting for $\bar{U}_1/\bar{P}_1$ in the foregoing equation from equation (24) and simplifying, we obtain

$$\frac{\bar{P}_2}{\bar{P}_1} = \frac{1}{\cos\dfrac{s\beta L}{c} - \dfrac{\bar{U}_2}{\bar{P}_2}\beta\rho c \sin\dfrac{s\beta L}{c}} \qquad (25)$$

For frequency-response analysis $s = j\omega$ and it follows that

$$\beta = \frac{1}{\left\{\dfrac{2}{a\left(\dfrac{\omega}{\nu}\right)^{1/2} j^{3/2}}\,\dfrac{J_1\left[a\left(\dfrac{\omega}{\nu}\right)^{1/2} j^{3/2}\right]}{J_0\left[a\left(\dfrac{\omega}{\nu}\right)^{1/2} j^{3/2}\right]} - 1\right\}^{1/2}}$$

Hence

$$\beta = \frac{1}{\left[\dfrac{2}{a\left(\dfrac{\omega}{\nu}\right)^{1/2} j^{3/2}}\,\dfrac{\operatorname{ber}_1 a\left(\dfrac{\omega}{\nu}\right)^{1/2} + j\operatorname{bei}_1 a\left(\dfrac{\omega}{\nu}\right)^{1/2}}{\operatorname{ber} a\left(\dfrac{\omega}{\nu}\right)^{1/2} + j\operatorname{bei} a\left(\dfrac{\omega}{\nu}\right)^{1/2}} - 1\right]^{1/2}}$$

where ber and bei are Thomson functions of the first kind.[2] Now,

$$\operatorname{ber}_1 a\left(\frac{\omega}{\nu}\right)^{1/2} = \frac{1}{\sqrt{2}}\left[\operatorname{ber}' a\left(\frac{\omega}{\nu}\right)^{1/2} - \operatorname{bei}' a\left(\frac{\omega}{\nu}\right)^{1/2}\right]$$

$$\operatorname{bei}_1 a\left(\frac{\omega}{\nu}\right)^{1/2} = \frac{1}{\sqrt{2}}\left[\operatorname{ber}' a\left(\frac{\omega}{\nu}\right)^{1/2} + \operatorname{bei}' a\left(\frac{\omega}{\nu}\right)^{1/2}\right]$$

where the prime denotes differentiation. Hence

$$\beta = \frac{1}{\left\langle \dfrac{2}{a\left(\dfrac{\omega}{\nu}\right)^{1/2} j^{3/2}}\,\dfrac{\dfrac{1}{\sqrt{2}}\left\{\left[\operatorname{ber}' a\left(\dfrac{\omega}{\nu}\right)^{1/2} - \operatorname{bei}' a\left(\dfrac{\omega}{\nu}\right)^{1/2}\right] + j\left[\operatorname{ber}' a\left(\dfrac{\omega}{\nu}\right)^{1/2} + \operatorname{bei}' a\left(\dfrac{\omega}{\nu}\right)^{1/2}\right]\right\}}{\operatorname{ber} a\left(\dfrac{\omega}{\nu}\right)^{1/2} + j\operatorname{bei} a\left(\dfrac{\omega}{\nu}\right)^{1/2}} - 1\right\rangle^{1/2}} \qquad (26)$$

The Thomson functions of zero order and their first derivatives are extensively tabulated by Nosova [12]. Using these tables and an electronic digital computer, the values of β were calculated for different frequencies of interest and may be found in reference [13]. It may be noted that β is a complex function of the frequency.

Let $\beta = \gamma + j\delta$ where γ is the real part and δ the imaginary part of β. For very high frequencies γ tends to zero and δ to -1 so that β tends to $-j$. If $\beta = -j$ then the viscous theory yields the same results as the nonviscous water-hammer equations of reference [6]. Therefore the dynamics of the fluid flow are characterized by the parameter $a\,(\omega/\nu)^{1/2}$.

Substituting for β in equations (24) and (25) with $s = j\omega$, we get

$$\frac{\bar{U}_1(j\omega)}{\bar{P}_1(j\omega)} = \frac{\dfrac{\bar{U}_2(j\omega)}{\bar{P}_2(j\omega)}\cos\dfrac{\omega L}{c}(-\delta + j\gamma) + \dfrac{1}{(\gamma + j\delta)\rho c}\sin\dfrac{\omega L}{c}(-\delta + j\gamma)}{\cos\dfrac{\omega L}{c}(-\delta + j\gamma) - \dfrac{\bar{U}_2(j\omega)}{\bar{P}_2(j\omega)}\rho c(\gamma + j\delta)\sin\dfrac{\omega L}{c}(-\delta + j\gamma)} \qquad (27)$$

$$\frac{\bar{P}_2(j\omega)}{\bar{P}_1(j\omega)} = \frac{1}{\cos\dfrac{\omega L}{c}(-\delta + j\gamma) - \dfrac{\bar{U}_2(j\omega)}{\bar{P}_2(j\omega)}\rho c(\gamma + j\delta)\sin\dfrac{\omega L}{c}(-\delta + j\gamma)} \qquad (28)$$

Equations (27) and (28) are the desired frequency-response equations.

Experimental Test Setup

The experimental data used in this analysis are obtained from frequency-response tests on a line consisting of a stainless-steel tube of 0.495-in. ID and 0.065-in. wall thickness. The tests were carried out in the Automatic Control Laboratory of the School of Mechanical Engineering, Purdue University. The theoretical analysis is based on the assumption that the fluid viscosity is constant. In the case of non-Newtonian fluids, the effective viscosity is a function of the shear rate and therefore changes with the steady-state flow and the internal pipe radius. For a line of a given internal radius, the effective viscosity of a non-Newtonian fluid may be easily calculated from the steady-state pressure drop for each steady-state flow condition. If the flow deviation from its steady-state value is small enough or if the fluid is only slightly non-Newtonian, the change of the effective viscosity resulting from the flow deviation may be neglected and it may be considered to be constant for a given steady-state flow condition. The theory developed in this analysis may then be applied to non-Newtonian fluids. Hydraulic oil Mil-0-5606, which is a non-Newtonian fluid, was supplied to the test line through an electrohydraulic servovalve near section 1 and the oil discharged to atmosphere through a sharp-edged orifice near section 2. The pressure deviations $\bar{P}_1$ and $\bar{P}_2$ were measured by pressure transducers and the velocity deviations $\bar{U}_1$ and $\bar{U}_2$ by hot-wire anemometers, and the results were read on a transfer function analyzer. The schematic diagram of the experimental setup is shown in Fig. 2 and the details are given by Roberts [7]. The line and fluid physical constants are given in Table 1.

For the frequency-response runs analyzed here, the range of test frequencies was from 1 to 100 cps. The amplitude of the signal input to the servovalve was kept constant throughout the frequency range. The steady-state pressure at section 1 was 225 psig, the steady-state flow 1.9 gpm (i.e., the steady-state velocity was 3.1 fps), the Reynolds number 650, and the temperature was held nearly constant at 83 deg F. At 1 cps the amplitude of the sinusoidal pressure deviation at section 1 was 50 psi.

The values of the steady-state pressure and flow for the line and outlet orifice used for the runs are shown in Fig. 3. Steady-state pressures at sections 1 and 2 are denoted by P_{s1} and P_{s2}, respectively.

Theoretical and Experimental Comparison Using a Boundary Condition Without Line Vibration

The upstream end of the line was bolted to a steel block supporting the servovalve and the block was securely fastened to the floor. The line was supported at every 6 ft on Styrofoam blocks and sandbags were placed on the top. The downstream end of the line was connected through the orifice block to the outlet tank by a short length of hose. The upstream end could then be considered as fixed and the downstream end was relatively free to move in the axial direction, but the lowest natural frequency of longitudinal vibrations of the line was much above the experimental frequency range.

The experimental values of the magnitude and phase angle of

[2] For order zero, the subscript is omitted in notations for Thomson functions.

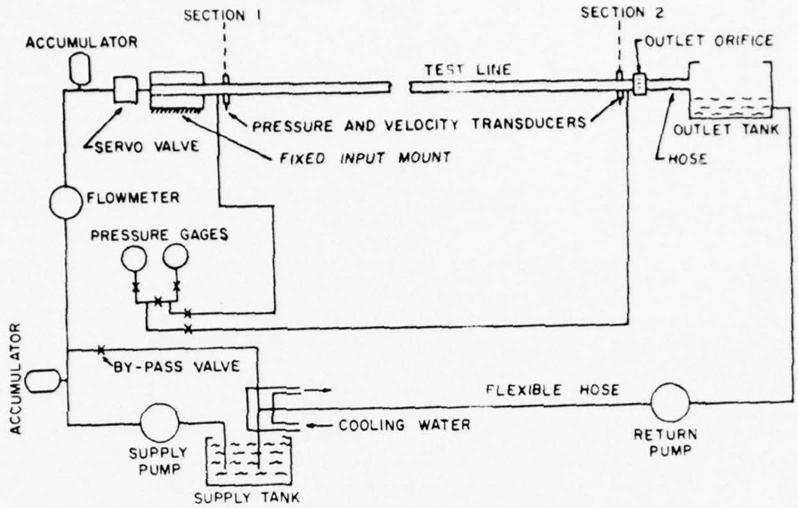

Fig. 2 Schematic diagram of test setup

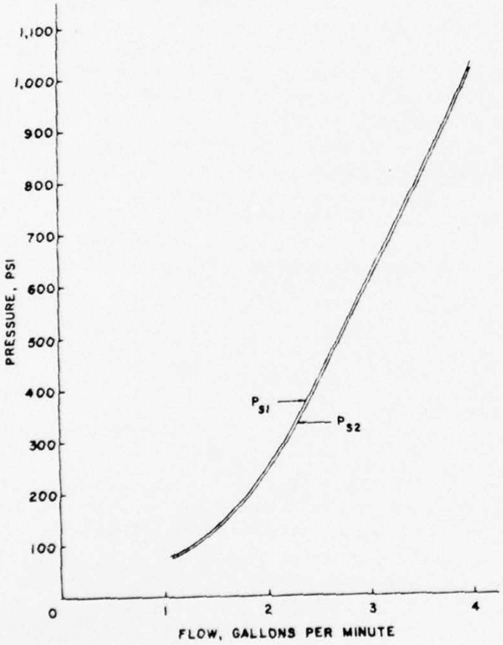

Fig. 3 Steady-state pressure versus flow

$$\frac{\bar{U}_2(j\omega)/\bar{P}_2(j\omega)}{\bar{U}_2(j\,31.41)/\bar{P}_2(j31.41)}$$

are shown in Fig. 4. The ratio $\bar{U}_2(j\omega)/\bar{P}_2(j\omega)$ is normalized by dividing it by $\bar{U}_2(j\,31.41)/(\bar{P}_2(j\,31.41)$ which is the experimental ratio at 5 cps; i.e., $\omega = 31.41$ radians per sec.

It is observed from Fig. 4 that the magnitude is approximately constant and the phase angle is almost negligible for the experimental frequency range. Hence it may be assumed that

$$\frac{\bar{U}_2}{\bar{P}_2} = \frac{1}{K_0} \tag{29}$$

The value of the constant K_0 may be determined from the slope of the steady-state pressure P_{s2}-curve in Fig. 3. At the steady-state pressure of 225 psi at section 1, this slope is given by

$$\frac{dP_{s2}}{dQ} = 266 \text{ lb min/in.}^2 \text{ gal}$$

Table 1

Constant	Value
a—internal radius of tube	0.020625 ft; i.e., 0.248 in.
L—length of line between sections 1 and 2	40.25 ft
c—velocity of sound in fluid (experimental)	4250 fps
ρ—density of fluid	1.663 lb sec²/ft⁴
ν—kinematic viscosity of fluid (effective)	197 × 10⁻⁶ ft²/sec

Hence, $K_0 = 23{,}400$ lb sec/ft³.

Substituting in the right-hand side of equations (27) and (28) from equation (29), it follows that

$$\frac{\bar{U}_1(j\omega)}{\bar{P}_1(j\omega)} = \frac{\frac{1}{K_0}\cos\frac{\omega L}{c}(-\delta + j\gamma) + \frac{1}{(\gamma + j\delta)\rho c}\sin\frac{\omega L}{c}(-\delta + j\gamma)}{\cos\frac{\omega L}{c}(-\delta + j\gamma) - \frac{\rho c}{K_0}(\gamma + j\delta)\sin\frac{\omega L}{c}(-\delta + j\gamma)} \tag{30}$$

and

$$\frac{\bar{P}_2(j\omega)}{\bar{P}_1(j\omega)} = \frac{1}{\cos\frac{\omega L}{c}(-\delta + j\gamma) - \frac{\rho c}{K_0}(\gamma + j\delta)\sin\frac{\omega L}{c}(-\delta + j\gamma)} \tag{31}$$

With the constants given previously, the magnitude and phase of $\bar{U}_1(j\omega)/\bar{P}_1(j\omega)$ and of $\bar{P}_2(j\omega)/\bar{P}_1(j\omega)$ were computed from equations (30) and (31) for different frequencies and are shown in Figs. 5(*a*) to 6(*b*). The ratio $\bar{U}_1(j\omega)/\bar{P}_1(j\omega)$ is normalized by dividing it by $\bar{U}_1(j\,6.283)/\bar{P}_1(j\,6.283)$ which is the ratio at 1 cps; i.e., $\omega =$ 6.283 radians per sec. The experimental ratio has also been corrected for the frequency responses of the transducers themselves which may be found in reference [7]. There is good agreement between the theoretical and experimental results of this investigation. Figs. 6(*a*) and 6(*b*) also show the theoretical curves obtained from the water-hammer equations of reference [6]. It is seen that the attenuation and phase shift caused by viscosity are maximum at the frequencies where the magnitude ratio has its peaks.

Theoretical and Experimental Comparison Using a Boundary Condition With Line Vibration

For a second experiment the upstream and downstream ends of the line were connected exactly in the same manner as stated in the preceding section. The line was supported at every 6 ft by thin steel wires hung from steel angles which were fastened to the

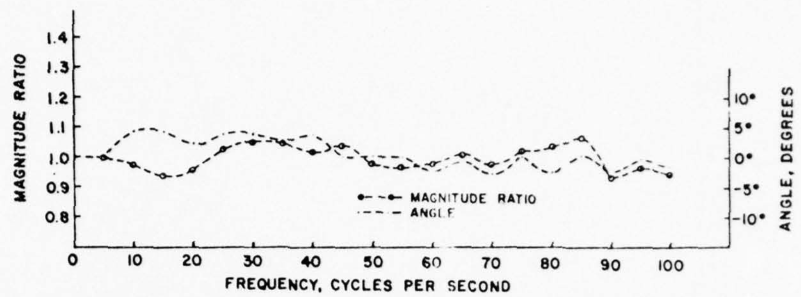

Fig. 4 **Normalized magnitude ratio of velocity to pressure deviations and their relative phase angle at section 2 versus frequency**

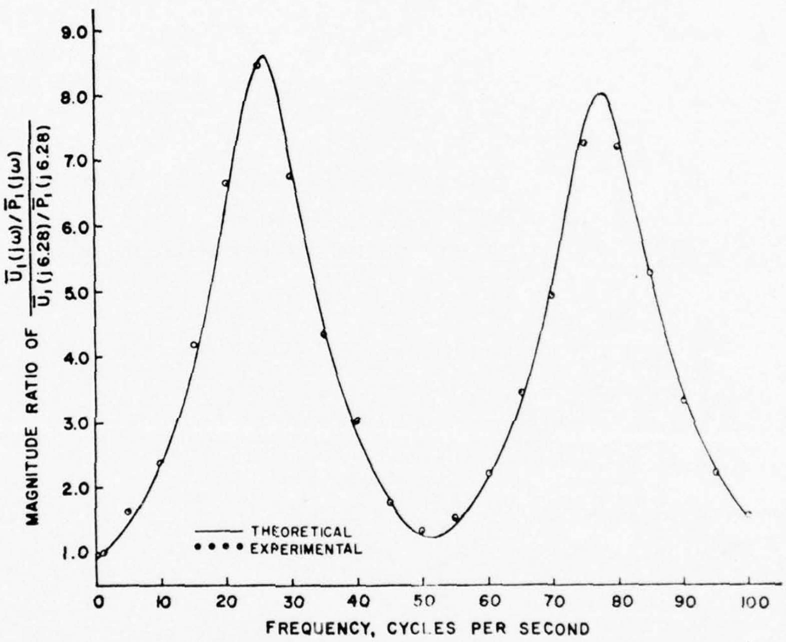

Fig. 5(a) **Normalized magnitude ratio of velocity to pressure deviations at section 1 versus frequency**

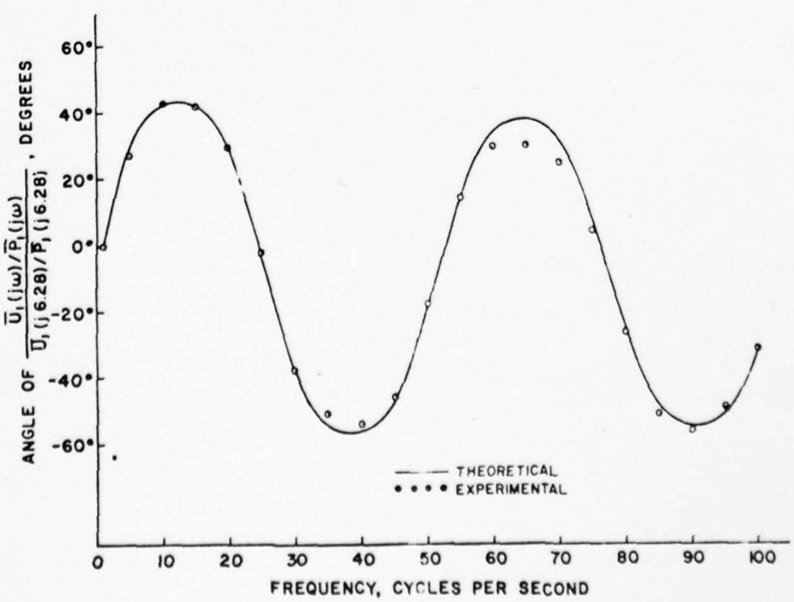

Fig. 5(b) **Phase angle of velocity deviation relative to pressure deviation at section 1 versus frequency**

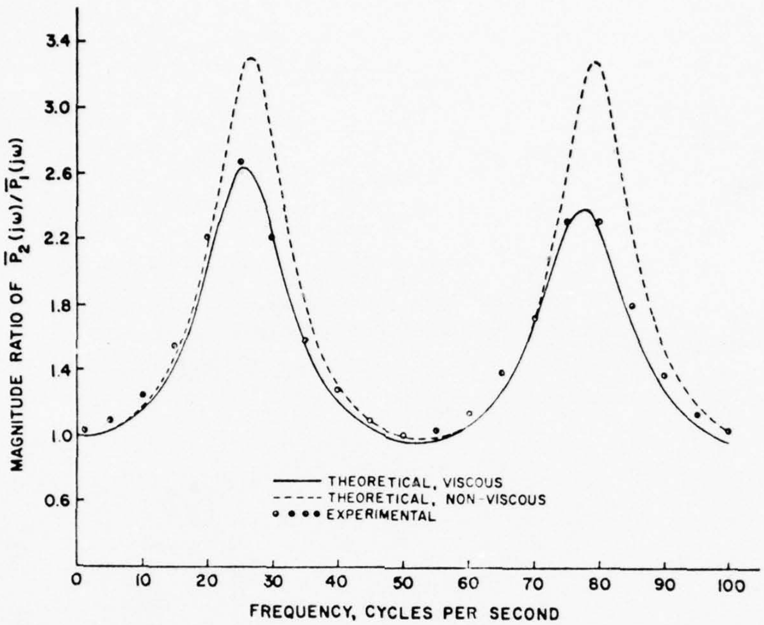

Fig. 6(a) Magnitude ratio of pressure deviation at section 2 to pressure deviation at section 1 versus frequency

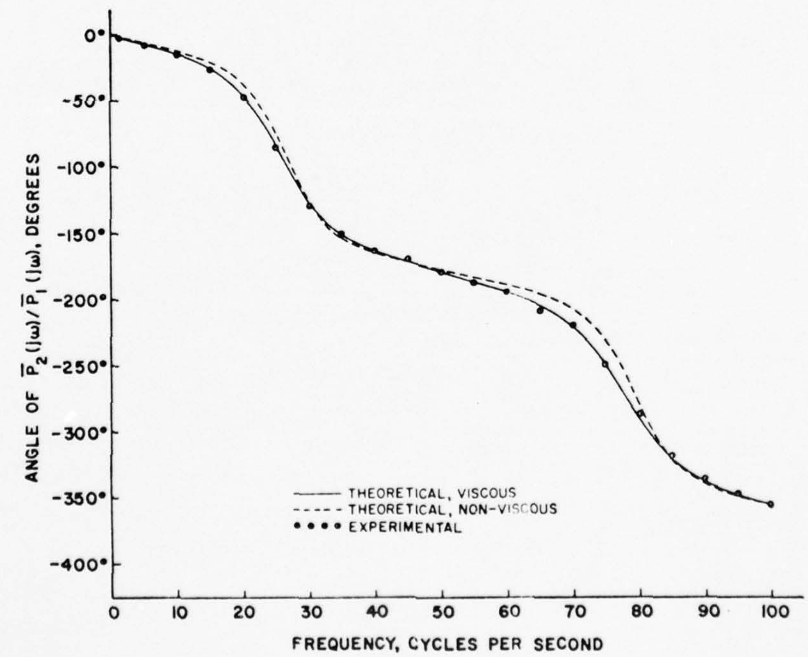

Fig. 6(b) Phase angle of pressure deviation at section 2 relative to pressure deviation at section 1 versus frequency

floor. The lowest natural frequency of longitudinal vibrations of the line was now within the experimental frequency range.

The equation for the undamped longitudinal vibrations of the pipe is given by Jacobsen and Ayre [15] and is

$$\frac{\partial^2 y}{\partial t^2} = b^2 \frac{\partial^2 y}{\partial x^2} \tag{32}$$

with $b^2 = E/\alpha$, where E is the modulus of elasticity of the pipe, α the density of the material, and y is the displacement of the pipe in the x-direction.

Taking the Laplace transform of both sides of equation (32) with zero initial conditions, i.e., $y(x, 0) = 0$ and $[\partial y(x, t)/\partial t]_{t=0} = 0$, we get

$$\frac{d^2 Y}{dx^2} - \frac{s^2}{b^2} Y = 0 \tag{33}$$

where

$$Y(x, s) = L[y(x, t)]$$

The solution of equation (33) is given by

$$Y(x, s) = B_1(s)\cosh\frac{sx}{b} + B_2(s)\sinh\frac{sx}{b} \tag{34}$$

The boundary conditions are as follows:

$$y = 0 \qquad \text{at } x = 0 \tag{35}$$

$$A_2E\frac{\partial y}{\partial x}(L, t) + m\frac{\partial^2 y}{\partial t^2}(L, t) + \zeta\frac{\partial y}{\partial t}(L, t)$$

$$= p_2(A_1 - A_3) \text{ at } x = L \tag{36}$$

where A_1 is the inside cross-sectional area of the pipe, A_2 the cross-sectional area of the pipe wall, and A_3 the area of the orifice.

The first term on the left-hand side of equation (36) represents the force resulting from the stress in the material of the pipe, the second term the inertia force due to the mass of the orifice block, and the third term the damping force caused mostly by the short length of hose. The term on the right-hand side is the net pressure force.

From equation (35), we have $B_1(s) = 0$ and taking the Laplace transform of both sides of equation (36), we get

$$A_2E\frac{dY}{dx}(L, s) + ms^2Y(L, s) + \zeta sY(L, s) = \bar{P}_2(s)[A_1 - A_3] \tag{37}$$

Substituting for $Y(L, s)$ in equation (37) from equation (34) it follows that

$$B_2(s) = \frac{\bar{P}_2(A_1 - A_3)}{A_2E\frac{s}{b}\cosh\frac{sL}{b} + ms^2\sinh\frac{sL}{b} + \zeta s\sinh\frac{sL}{b}}$$

Hence

$$Y(L, s) = \frac{\bar{P}_2(A_1 - A_3)}{s\left[ms + \zeta + \frac{A_2E}{b}\coth\frac{sL}{b}\right]}$$

The transfer function relating the velocity of the end of the pipe to the pressure $\bar{P}_2$ is given by

$$\frac{sY(L, s)}{\bar{P}_2(s)} = \frac{A_1 - A_3}{ms + \zeta + \frac{A_2E}{b}\coth\frac{sL}{b}} \tag{38}$$

For frequency response $s = j\omega$ and we have

$$\frac{j\omega Y(L, s)}{\bar{P}_2(j\omega)} = \frac{A_1 - A_3}{j\omega m + \zeta + \frac{A_2E}{b}\coth\frac{j\omega L}{b}}$$

Let

$$\frac{A_1 - A_3}{(j\omega m + \zeta) + \frac{A_2E}{b}\coth\frac{j\omega L}{b}} = n + jq \tag{39}$$

The values of n and q were computed for different frequencies by using the constants given in Table 2.

The boundary condition now becomes

$$\frac{\bar{U}_2(j\omega)}{\bar{P}_2(j\omega)} = \frac{1}{K_0} + n + jq \tag{40}$$

Substituting in the right-hand side of equation (28) from equation (40), it follows that

$$\frac{\bar{P}_2(j\omega)}{\bar{P}_1(j\omega)} = \frac{1}{\cos\frac{\omega L}{c}(-\delta + j\gamma) - \rho c\left(\frac{1}{K_0} + n + jq\right)(\gamma + j\delta)\sin\frac{\omega l}{c}(-\delta + j\gamma)} \tag{41}$$

Table 2

A_1—inside cross-sectional area of pipe, sq ft	0.001336
A_2—cross-sectional area of pipe wall, sq ft	0.000793
A_3—area of orifice, sq ft	0.0001914
m—mass of orifice block, lb sec²/ft	0.135
ζ—coefficient of damping, lb sec/ft	7.0
E—modulus of elasticity of pipe, psf	$30 \times 144 \times 10^6$
b—velocity of sound in material of pipe, fps	16,910
L—length of line between sections 1 and 2, ft	40.25
ρ—density of material of pipe, lb sec²/ft⁴	15.1

With the constants given previously the magnitude and phase of $\bar{P}_2(j\omega)/\bar{P}_1(j\omega)$ were computed from equation (41) for different frequencies and are shown in Figs. 7(*a*) and 7(*b*), respectively, along with the experimental values. The figures also show the theoretical curves obtained from equation (31) in the absence of line vibrations. It is evident that the natural frequency of vibration of the pipe in the longitudinal direction has a marked effect on the dynamic response. The maximum deviations in magnitude and phase occur near the natural frequency of longitudinal vibrations of the line which in this case was 83 cps.

Summary and Conclusions

Using the basic Navier-Stokes' equations, a transfer matrix is derived relating the dynamic pressure and velocity for small-diameter hydraulic lines. The results of the theoretical analysis are compared with experimental data obtained from frequency response runs on a 1/2-in-ID test line.

It is concluded that the dynamic response of small-diameter lines can be predicted correctly by the viscous theory developed in this paper. In the case of Newtonian fluids, as the frequency and the diameter of the line are increased, the viscosity effect decreases and the water-hammer equations yield fairly good results for fluid of low viscosity. Then the dynamics of the line are characterized by the parameter $a(\omega/\nu)^{1/2}$. When $a(\omega/\nu)^{1/2}$ is small, i.e., when highly viscous liquid oscillates at a small frequency in a small-diameter line, the viscosity effect is not negligible. When $a(\omega/\nu)^{1/2} \to \infty$, i.e., when a liquid of low viscosity oscillates at very high frequency in a large-diameter tube, viscosity and friction may be neglected.

In the case of non-Newtonian fluids such as Mil-0-5606 and some other aircraft hydraulic oils, the effective viscosity decreases with increasing shear rate. The rate of shear necessary to cause appreciable change in viscosity, either transient or permanent, varies with the fluid. Then as the radius a decreases, the effective viscosity ν also decreases for the same rate of flow, and the relative change in the parameter $a/\nu^{1/2}$ determines the effect of viscosity.

If the natural frequency of longitudinal vibrations of the line falls within the frequency range of interest, then the dynamic response is modified.

Acknowledgments

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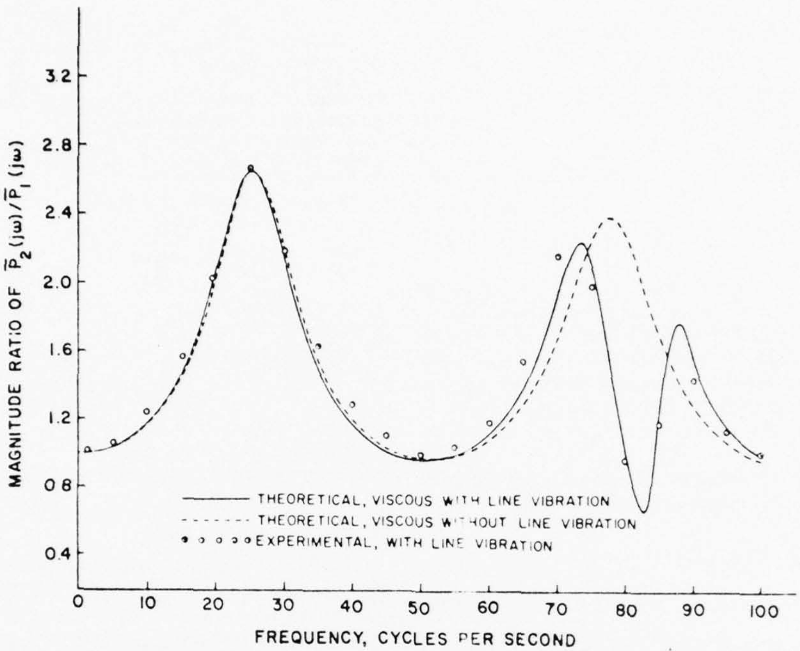

Fig. 7(a) Magnitude ratio of pressure deviation at section 2 to pressure deviation at section 1 versus frequency with line vibrations

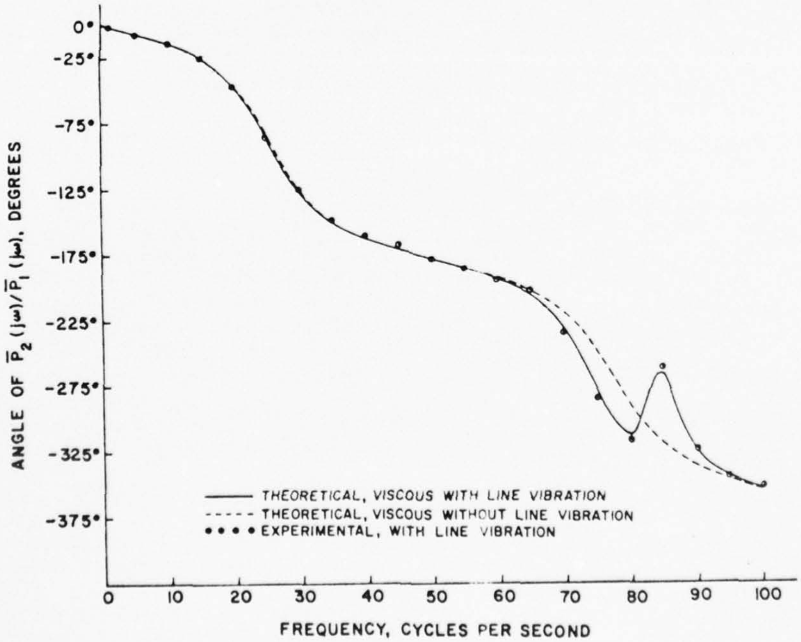

Fig. 7(b) Phase angle of pressure deviation at section 2 relative to pressure deviation at section 1 versus frequency with line vibrations

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APPENDIX 1

Rigidity of Walls of a Small-Diameter Tube

It is customary to include the effect of expansion or contraction of the tube walls in the derivation of the continuity equation of water hammer. However, the assumption that the walls are rigid simplifies the differential equation and the boundary conditions.

Let ΔQ_1 denote the change in volume due to the expansion of the pipe walls and ΔQ_2 the change in volume due to the compression of the liquid element. As shown by Oldenburger [14], ΔQ_1 and ΔQ_2 are given by

$$\Delta Q_1 = \frac{2\pi a^3}{hE - ap}\frac{\partial p}{\partial t}\Delta t \Delta x$$

and

$$\Delta Q_2 = \frac{1}{K}\left[\frac{\partial p}{\partial t}\right]\pi a^2 \Delta t \Delta x$$

Thus

$$\frac{\Delta Q_2}{\Delta Q_1} = \frac{hE - ap}{2Ka} \tag{42}$$

For a stainless-steel tube of $^1/_2$-in. ID and $^1/_{16}$-in. wall thickness carrying a fluid at a pressure of about 3000 psi, typical values of the constants appearing in equation (42) are as follows:

E—modulus of elasticity of tube = 30×10^6 psi
K—bulk modulus of the fluid = 0.3×10^6 psi
a—inside radius of tube = $^1/_4$ in.
h—thickness of the tube walls = $^1/_{16}$ in.
p—pressure = 3000 psi

Substituting the values of these constants in equation (42), we get

$$\frac{\Delta Q_2}{\Delta Q_1} = 12.5$$

If we consider a tube of $^1/_4$-in. ID and $^1/_{16}$-in. wall thickness, then

$$\frac{\Delta Q_2}{\Delta Q_1} = 25$$

Hence, in small-diameter tubes the magnitude of the effect of the compressibility of the liquid is much more than the effect of the elasticity of the wall, which may be assumed to be rigid. However, the theoretical speed of sound can be modified by the proper amount to account for the elasticity of the walls.

APPENDIX 2

The linearization of equation (1) by neglecting the convective acceleration terms is justified by the following argument:

Let L be the characteristic length along the axis of the pipe, U the characteristic velocity, and c the speed of sound in the fluid. Then we have

$$u\frac{\partial u}{\partial x} \underset{=}{0} \frac{U^2}{L}$$

and

$$\frac{\partial u}{\partial t} \underset{=}{0} \frac{U}{L/c}$$

where the symbol $\underset{=}{0}$ designates the order.

A typical value of U is 50 fps while c is about 5000 fps. Hence

$$\frac{u\dfrac{\partial u}{\partial x}}{\dfrac{\partial u}{\partial t}} \underset{=}{0} \frac{\dfrac{U^2}{L}}{\dfrac{U}{L/c}} = \frac{U}{c} = \frac{50}{5000} = \frac{1}{100}$$

Hence $u(\partial u/\partial x)$ and similarly $v(\partial u/\partial r)$ may be neglected when compared with $\partial u/\partial t$.

In equation (1) the viscous terms

$$(1/3)\partial/\partial x[\partial v/\partial r + v/r]$$

may be neglected as they vanish when integrated across the cross section with respect to r from $r = 0$ to $r = a$; i.e.,

$$\frac{1}{3}\frac{\partial}{\partial x}\int_0^a 2\pi r\left(\frac{\partial v}{\partial r} + \frac{v}{r}\right)dr = 0$$

It is now shown that the viscous term $(4/3)\partial^2 u/\partial x^2$ in that equation may also be neglected.

Let u be of order U, x of order L, and r of order ϵL where $\epsilon \ll 1$. Then we have

$$\frac{4}{3}\frac{\partial^2 u}{\partial x^2} \underset{=}{0} \frac{4}{3}\frac{U}{L^2}$$

$$\frac{\partial^2 u}{\partial r^2} \underset{=}{0} \frac{U}{\epsilon^2 L^2}$$

and

$$\frac{1}{r}\frac{\partial u}{\partial r} \underset{=}{0} \frac{U}{\epsilon^2 L^2}$$

Hence, $\partial^2 u/\partial r^2 \gg 4/3(\partial^2 u/\partial x^2)$ and $(1/r)\,\partial u/\partial r \gg (4/3)\partial^2 u/\partial x^2$.

APPENDIX B

HYDRAULIC FLUID DATA

APPENDIX B

HYDRAULIC FLUID DATA

The FLUID subroutine described in paragraph 8.2 of this report uses basic fluid data for viscosity, density, and adiabatic bulk modulus. This appendix discusses fluid data sources and the correction of fluid data from atmospheric to operating pressure. Tabulated data currently input in the FLUID subroutine are presented.

Unfortunately fluid data are not as readily available as one might imagine, particularly for the adiabatic bulk modulus. In literature searches for fluid data, one concludes that much of the required information is not available, or is not traceable to specific measured data. One may also find considerable information on base fluids, but little on the formulated version actually used in hydraulic systems. Required information is either untrustworthy or not available.

In general, data should be obtained for the usable temperature range of the fluid so that there are no limits to the system simulation. For aircraft design, this is typically over the range of -65 to 300°F. Data is currently input in FLUID for nine temperatures; -65, -40, 0, 50, 100, 150, 200, 250, and 300°F.

B1.0 VISCOSITY

Viscosity data is input in the FLUID subroutine in units of centistokes (CM^2/SEC/100) for zero gage pressure. Viscosity is converted to the English unit Newt (IN^2/SEC) in the FLUID subroutine, after viscosity has been interpolated to the operating temperature (TEMP).

Viscosity is then adjusted to the operating pressure (PRESS) using the expression from Reference (B1):

$$VISC_p = VISC_o \; e^B$$

where:

B = $COEFF^{A} \times PRESS \times 2.3 \times 10^{-4}$

$VISC_o$ = Kinematic viscosity (IN^2/SEC) at atmospheric pressure (o psig)

$VISC_p$ = Kinematic viscosity (IN^2/SEC) at pressure (PRESS)

PRESS = Steady state fluid operating pressure (psig)

e = Napierian logarithm base = 2.718

COEFF = Pressure coefficient derived from the Figure B-1 PRL chart from Reference (B1)

$A = \frac{560}{(TEMP+460)}$ = Temperature correction for the pressure coefficient based on a 100°F reference

2.3 = Factor to account for PRL chart being plotted to base 10 logarithm instead of base e.

The pressure coefficient derived from the Figure B-1 PRL chart is based on the atmospheric viscosity (centistokes) at 100°F and the ASTM slope from the Figure B-2 viscosity-temperature chart. E. E. Klaus of Pennsylvania State University confirmed that the Figure B-1 chart was prepared in error using common in lieu of Napierian logarithms, thereby requiring the 2.3 factor noted above.

If the pressure coefficient is not available, it is probably better to estimate a value from a chart on a similar fluid than to omit viscosity pressure correction. Viscosity data and pressure coefficients currently in FLUID are tabulated in Table B-1.

B2.0 BULK MODULUS

FLUID uses the adiabatic (isentropic) bulk modulus (LBS/IN^2) of the fluid. Adiabatic bulk modulus applies under conditions where pressure changes are rapid, and is greater than the isothermal tangent bulk modulus by the ratio of the specific heats (C_p/C_v) of the fluid. Isothermal tangent bulk modulus is the thermodynamically correct value, and represents the true rate of change in fluid elasticity at the steady state pressure of interest.

Unfortunately, adiabatic bulk modulus data are not readily available. Based on a literature search, the best source of adiabatic modulus data is currently the National Research Council of Canada (contact W. J. Noonan). The Canadian source offers published information on direct measurements of adiabatic modulus.

Adiabatic modulus data may be derived from isothermal tangent bulk modulus available from Penn State (E. E. Klaus). However, specific heat ratios available from a paper written by Peeler and Green in 1959, were derived by measuring the adiabatic bulk modulus and comparing it with the isothermal modulus.

In a report dated 24 June 1972 the data for MIL-H-5606 (presumably 'A' type) and MLO-7261 (which is said to conform to MIL-H-83282) is given versus temperature and pressure. This data was presented to the SAE-A-6C Committee in October 1972.

Adiabatic bulk modulus data for 0 psig as shown in Table B-2 are input to the FLUID subroutine, and interpolated to the operating temperature by FLUID. These data are then corrected to the steady operating pressure (PRESS) for all three fluids currently in FLUID using the expression:

$$BULK_p = BULK_o + 12 \times PRESS$$

where: $BULK_p$ = adiabatic modulus at pressure (PRESS)

$BULK_o$ = adiabatic modulus at 0 psig.

The factor (12) is based on bulk modulus versus pressure plots for the three fluids and is considered a good average value for pressures up to 3000 or 4000 psig.

B3.0 FLUID DENSITY

Fluid density data are normally available from sources such as Penn State. These data are normally obtained in the course of measuring the isothermal secant bulk modulus, which is based on the volumetric change from atmospheric to the pressure of interest.

Fluid mass density at atmospheric pressure per Table B-3 is input to the FLUID subroutine for -65°F and +275°F only, and interpolated linearly to the operating temperature. Density is then corrected to the operating pressure using the expression

$$RHO_p = RHO_o \left(1 + \frac{PRESS}{250{,}000}\right) \quad LB \cdot SEC^2/IN$$

where 250,000 (psi) is an average secant bulk modulus at operating conditions for all three fluids.

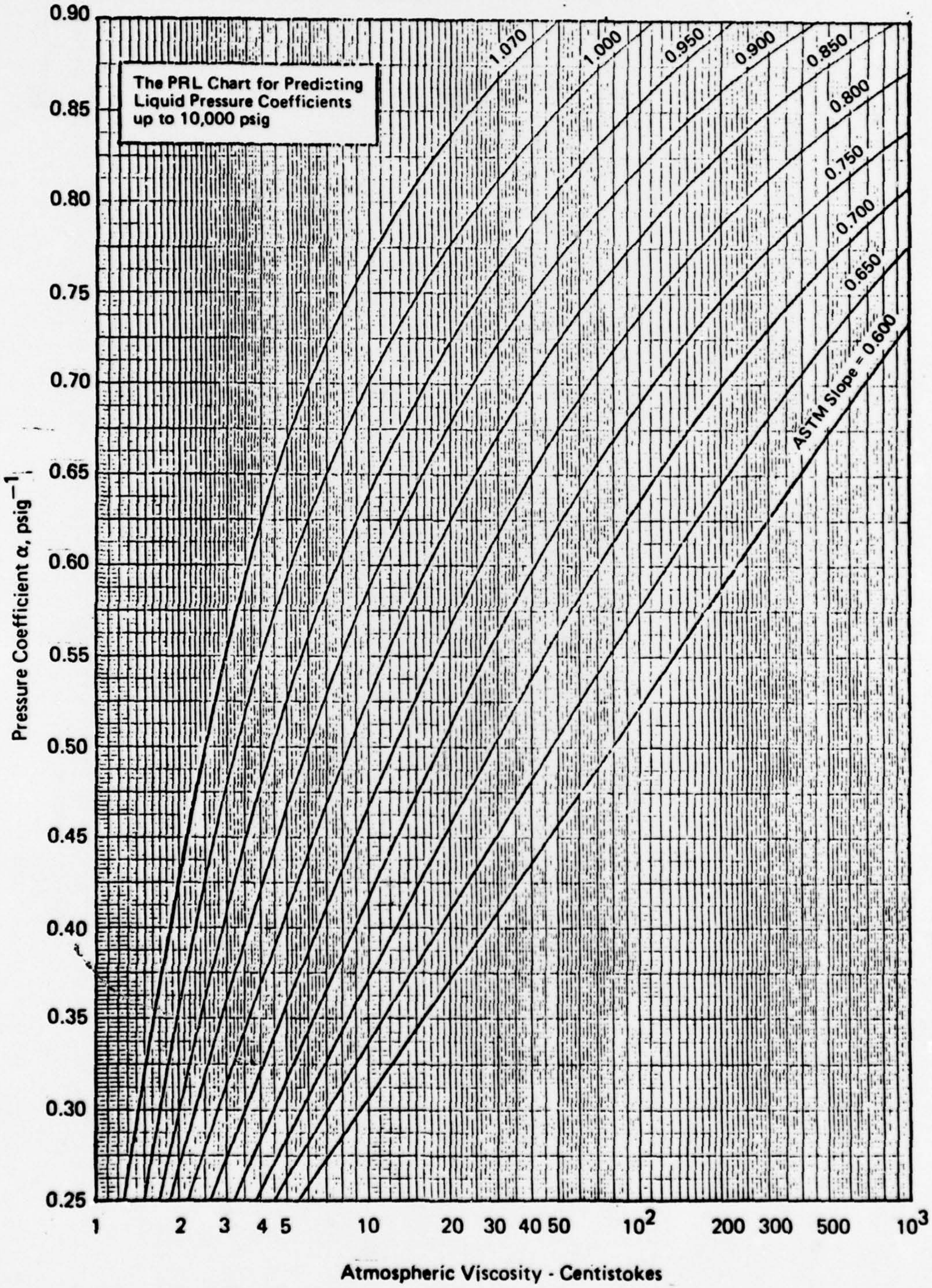

FIGURE B-1
FLUID FACTOR CHART

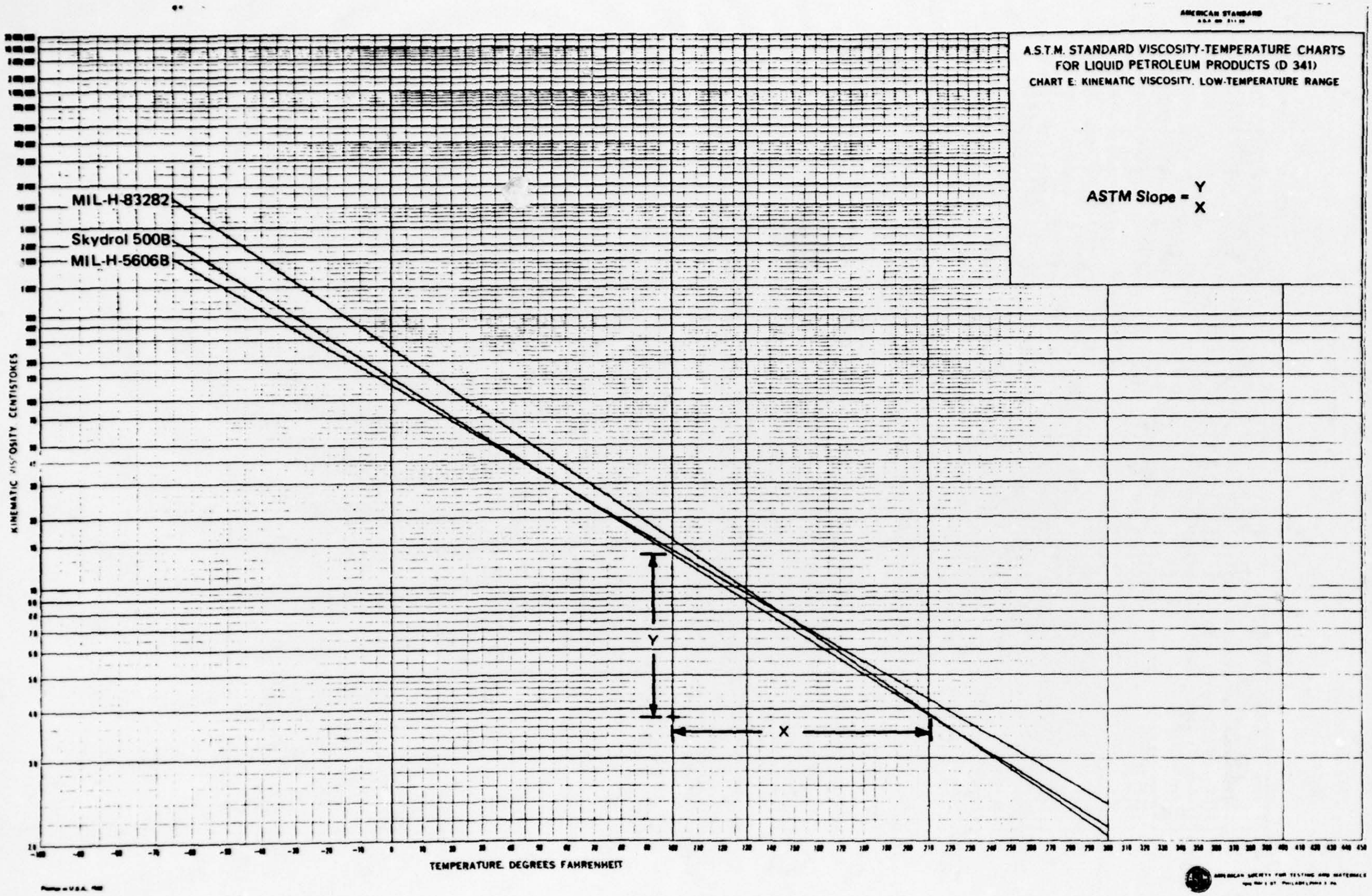

FIGURE B-2
ASTM VISCOSITY-TEMPERATURE CHART

TEMP °F	KINEMATIC VISCOSITY (Centistokes) at 0 psig					
	MIL-H-5606B	MIL-H-83282	SKYDROL 500B			
-65	1993.5	11446.9	3485.5			
-40	482.3	2019.3	598.0			
0	134.4	269.4	104.18			
50	34.85	48.87	27.9			
100	14.47	15.95	11.7			
150	7.46	7.46	6.5			
200	4.58	4.24	4.18			
250	3.19	2.83	2.89			
300	2.39	2.04	2.15			
Data Reference	(B2)	(B2)	(B3)			
COEFF	.335	.330	.42			

TABLE B-1

Viscosity Data in FLUID Subroutine

TEMP °F	ADIABATIC BULK MODULUS(PSI) AT 0 PSIG					
	MIL-H 5606B	MIL-H 83282	SKYDROL 500B			
-65	347,000	347,000	426,000			
-40	325,000	325,000	400,000			
0	290,000	290,000	364,000			
50	240,000	240,000	318,000			
100	208,000	208,000	270,000			
150	173,000	173,000	229,000			
200	142,000	142,000	194,000			
250	119,000	119,000	162,000			
300	98,000	98,000	138,000			
Data Reference	(B4)	(B4) ⚠1	(B4)			

⚠1 HSFR verification tests with MIL-H-5606 and MIL-H-83282 have shown that the Ref. B5 data for MIL-H-83282 is too high. Ref. B4 data for MIL-H-5606 provides better frequency predictions than Ref. B5 in systems using MIL-H-83282.

TABLE B-2

Adiabatic Bulk Modulus Data in FLUID Subroutine

TEMP °F	MASS DENSITY ($LBS\text{-}SEC^2/IN4$) AT 0 PSIG					
	MIL-H 5606B	MIL-H 83282	SKYDROL 500B			
-65	$8.57x10^{-5}$	$8.49x10^{-5}$	$10.3x10^{-5}$			
-40						
0						
50						
100						
150						
200						
250						
275	$7.63x10^{-5}$	$7.30x10^{-5}$	$8.9x10^{-5}$			
Data Reference	(B2)	(B2)	(B3)			

TABLE B-3

Fluid Mass Density Data in FLUID Subroutine

B4.0 REFERENCES

(B1) Air Force Materials Laboratory, "Fluids, Lubricants, Fuels, and Related Materials", Technical Report AFML-TR-67-107, Part 1, March 1967, pages 13-20. (Prepared by the Petroleum Refining Laboratory of Pennsylvania State University.)

(B2) Report MDC A2686, "Evaluation of a Fire-Resistant Hydraulic Fluid", 15 April 1974, Prepared Under contract N62269-73-C-0707 by McDonnell Aircraft Company.

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