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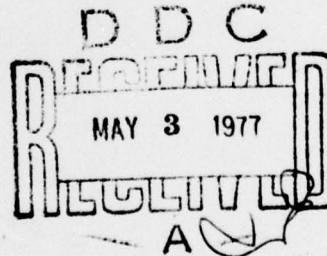
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K. E. Lonngren[†]

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ABSTRACT

If a large negative voltage is applied to an insulated disc inserted in a collision-dominated plasma, the electrons are expelled from the region near the disc leaving a bare cloud of ions. This "transient sheath" is well documented in ion acoustic wave experiments. This paper examines the evolution of the ions in this sheath into a collision-dominated plasma. A new sheath whose dimension is of the order of twice the transient sheath dimension is found.

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[†]On leave from the University of Iowa

EXPANSION OF AN ION SHEATH INTO A COLLISION-DOMINATED PLASMA

K. E. Lonngren[†]

The study of sheaths surrounding biased electrodes inserted in plasmas is a well studied problem. For steady state plasmas, we can easily cite the work of Langmuir as being representative. A sheath which was later discovered by Alexeff and his coworkers was the transient plasma sheath⁽¹⁾. In this sheath, a large negative potential was assumed to be instantaneously applied to an electrode (say a large flat plate) in the plasma. Owing to the mass difference, the electrons are blown into the plasma leaving a bare cloud of ions in the region $0 \leq x \leq x_T$. The plate with voltage $-\phi_0$ applied to it is located at $x = 0$. At the transient sheath dimension x_T , the potential and the electric field are assumed to be zero. Solving Poisson's equation in the transient sheath region with the above boundary conditions, the dimension of the sheath x_T can be computed to be $x_T = [2 e \phi_0 / n e^2 / \epsilon_0]^{1/2}$ in MKS units.

As time progresses, the ions in the transient sheath are "eaten" by the electrode and eventually a steady state Langmuir sheath is set up. In the Langmuir sheath, there exists a balance between the ambipolar ion current to the electrode and the space-charge limited current. For time scales between the creation of the transient sheath and the eventual formation of the Langmuir sheath, the description of the physics involves a numerical solution of the nonlinear partial differential equations of continuity and motion coupled with Poisson's equation and a Boltzmann relation for elections. Widner, et al have done such a calculation and obtained good agreement with experiment.⁽²⁾

In that study, it was found that the ions within the transient sheath decayed in time and one could approximate the ion behavior at $x = 0$ as

$$\rho_+(t) = \frac{1}{1+t} \quad (1)$$

where a suitable normalization has been included. Secondly it was found that the front of the sheath expanded into the plasma, initially at a velocity faster than the ion acoustic velocity C_s .

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At a distance equal to approximately $2 x_T$ from the electrode, it was found that the sheath expansion slowed down to the ion acoustic velocity. Third, large ion currents were initially drawn by the electrode as the ions started to move. A similar result was reported also by others^(3,4). Finally, Oskam and his colleagues noted from experiment that the time that it took for a sheath to reach an equilibrium state (say a Langmuir sheath) decreased if the plasma density increased or if ions of a smaller mass were used^(5,7).

The work that will be described here suggests that the above observations can be understood within the framework of a model which assumes that the ion velocity is proportional to the electric field, i.e. a mobility model. In Section II, we present the model and describe the sheath evolution in Section III. Section IV is the conclusion which relates the predicted behavior with these numerical and laboratory experimental results.

II. The Model.

We shall assume that the plasma can be described by the set of one-dimensional equations

$$\frac{dn}{dt} = -\frac{\partial i}{\partial x} \quad (2)$$

$$\epsilon_0 \frac{\partial E}{\partial x} = \rho_+ - \rho_- \quad (3)$$

$$i = \rho_+ \frac{\partial x}{\partial t} = \mu \rho_+ E \quad (4)$$

$$\nabla \times \vec{B} \sim 0 = \rho_+ \frac{\partial x}{\partial t} + \epsilon_0 \frac{\partial E}{\partial t} \quad (5)$$

which are the equation of continuity, Poisson's equation, definition of current and Maxwell's equation respectively. The set of equations (2-5) has been studied by Many and Rakavy with particular emphasis on the theory of space-charge-limited currents in solids.⁽⁸⁾ Self similar solutions for the set have recently been obtained.⁽⁹⁾ De Oliveira and Leal Ferreira have studied this set with reference to charge neutralization between capacitor plates if charge of one sign filled one half the distance between the plates and charge of the other sign filled the other half.⁽¹⁰⁾ We shall make extensive use of the philosophy of this calculation in the derivation that follows.

We find the characteristics of the system which define the flow lines $x(t)$ for the positive mobile charges

$$\frac{\partial x}{\partial t} = \mu E(x(t), t) . \quad (6)$$

The acceleration along a flow line is given by

$$\begin{aligned} \frac{\partial^2 x}{\partial t^2} &= \mu \left[\frac{\partial E}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial E}{\partial t} \right] \\ &= - \frac{\mu \rho_-}{\epsilon_0} \frac{\partial x}{\partial t} \\ \frac{\partial^2 x}{\partial \tau^2} &= - \frac{\rho_-}{\rho_0} \frac{\partial x}{\partial \tau} \end{aligned} \quad (7)$$

where $\tau = \frac{\mu \rho_0 t}{\epsilon_0}$, ρ_0 is the unperturbed density and we've used (2), (3), (5) and (6). Along the flow line, the variation of positive charge is

$$\begin{aligned} \frac{\partial \rho_+(x(t), t)}{\partial t} &= \frac{\partial \rho_+}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial \rho_+}{\partial t} \\ &= - \frac{\mu \rho_+}{\epsilon_0} (\rho_+ - \rho_-) \\ \frac{\partial (\rho_+ / \rho_0)}{\partial \tau} &= - \frac{\rho_+}{\rho_0} \left(\frac{\rho_+}{\rho_0} - \frac{\rho_-}{\rho_0} \right) \end{aligned} \quad (8)$$

where we've used (2), (4) and (6). Equations (7) and (8) are the starting equations for the derivation of the ion sheath evolution.

III Sheath Evolution

At $\tau = 0$, we shall assume that a large negative potential $-\phi_0$ is applied to the electrode at $x = 0$. Before the ions can move, the electrons are "blown" into the semi-infinite uniform plasma to at least a distance of the transient sheath dimension x_T .⁽¹⁾ The temporal behavior of the ions in the transient sheath region $0 \leq x \leq x_T$ is given by the solution of (8) with $\rho_- = 0$;

$$\frac{\rho_+(\tau)}{\rho_0} = \frac{1}{1 + \tau} \quad (9)$$

As time increases, ions will be "eaten" by the electrode and will expand into the plasma.

As we are more interested in the behavior for $x > x_T$, we shall assume an insulating coating

covers the electrode such that it draws no current. In fact, experiments by Chen and Schott confirm that this technique is a very efficient method to launch ion acoustic waves. (11)

At $x = x_T$, the electric field can be computed from Gauss Law to be

$$E(x_T, \tau) = \rho_+(\tau) \frac{x_T}{\epsilon_0} \quad (10)$$

The velocity of ions for $x > x_T$ can be computed from (7) to be

$$\frac{\partial x}{\partial \tau} = \frac{\partial x}{\partial \tau} \bigg|_{\tau=\tau_0} \epsilon^{-\frac{\rho_-}{\rho_0}(\tau-\tau_0)} \quad (11)$$

The time τ_0 is the time that a particular ion leaves $x = x_T$. The density ratio ρ_-/ρ_0 is the density ratio of excess electrons that are "blown" out of the transient sheath into the neutral plasma. We shall assume that it is uniform for $1 \leq \frac{x}{x_T} \leq (1 + \frac{\rho_0}{\rho_-})$. The value

$$\begin{aligned} \frac{\partial x}{\partial \tau} \bigg|_{\tau=\tau_0} &= \frac{\epsilon_0}{\rho_0} E(x_T, \tau_0) \\ &= \frac{\rho_+(\tau_0)x_T}{\rho_0} \end{aligned} \quad (12)$$

where (10) has been used. We note from (11) that the velocity of the ion cloud slows down as time increases.

The position of the ions in the plasma can be determined by integrating (11) from τ_0 for ions at the sheath edge x_T to x at a time τ . We find

$$\frac{x}{x_T} = 1 + \frac{\rho_0/\rho_-}{1 + \tau_0} \left[1 - \epsilon^{-\frac{\rho_-}{\rho_0}(\tau-\tau_0)} \right] \quad (13)$$

The leading edge of the ion expansion occurs for the ions that leave the transient sheath at $\tau_0 = 0$.

A new sheath criterion is determined as follows. Let us find the time T and position L where the leading edge of the ion sheath expansion slows down to the ion acoustic velocity

$C_s = \tilde{C}_s \epsilon_0 / \mu \rho_0$. From (9), (11) and (12), this time T is defined as

$$\tilde{C}_s = \frac{\partial x}{\partial \tau} = x_T \epsilon^{-\frac{\rho_-}{\rho_0} T} \quad (14)$$

Combining (13) and (14), we obtain

$$\frac{L}{x_T} = 1 + \frac{\rho_0}{\rho_-} \left[1 - \frac{\tilde{C}_s}{x_T} \right] \quad (15)$$

This can be simplified by using the definitions for mobility, the ion acoustic velocity and the transient sheath to yield

$$\frac{L}{x_T} = 1 + \frac{\rho_0}{\rho_-} \left[1 - \sqrt{\frac{kT_e}{2e\phi_0}} \frac{\nu}{\omega_{pi}} \right] \quad (16)$$

where ω_{pi} is the ion plasma frequency with density ρ_0 and ν is the collision frequency. We note that if the excess electrons resided in the region $x_T \leq x \leq 2x_T$ and $\rho_0 = \rho_-$, then $L/x_T \leq 2$.

The ion density for $x \geq x_T$ could in principal be found by eliminating the parameter τ_0 between (6), (11), (13) and Poisson's equation. As this cannot be carried out explicitly, we follow the procedure given by de Oliveira and Leal Ferreira.⁽¹⁰⁾ Differentiate (13) with respect to τ_0

$$\frac{\partial x}{\partial \tau_0} = - \frac{\rho_0 x_T / \rho_-}{(1 + \tau_0)^2} \left[1 + \left\{ \left(\frac{\rho_-}{\rho_0} - 1 \right) + \tau_0 \frac{\rho_-}{\rho_0} \right\} \epsilon^{-\frac{\rho_-}{\rho_0}(\tau - \tau_0)} \right] \quad (17)$$

Along the flow lines, the electric field can be found from (6), (11) and (12) to be

$$E = E(x_T, \tau_0) \epsilon^{-\frac{\rho_-}{\rho_0}(\tau - \tau_0)} \quad (18)$$

Therefore (13) can be rewritten using (10) and (18) as

$$\frac{x}{x_T} = 1 + \frac{\rho_0 / \rho_-}{1 + \tau_0} - \frac{\epsilon_0 E(x_T, \tau_0)}{x_T} \quad (19)$$

Now consider τ_0 to be a function of x and τ and differentiate (19) with respect to x and use (19) and Poisson's equations. One finds the ion density $\rho_+(x > x_T)$ to be

$$\frac{\rho_+(x > x_T)}{\rho_0} = \frac{\rho_- / \rho_0}{1 - \epsilon^{-\frac{\rho_-}{\rho_0}(\tau - \tau_0)} \left[1 - \frac{\rho_-}{\rho_0} (1 + \tau_0) \right]} \quad (20)$$

To obtain $\rho_+(x > x_T)$ as a function of x and τ , we assign values to τ_0 and use equations (13) and (20). Typical results for the value $\rho_- / \rho_0 = 1$ as computed with an HP-25 programmable hand calculator are shown in Figure 1. We note the rapid neutralization of the charge

in the region $x > x_T$.

IV Conclusion

The evolution of an ion sheath into a collision dominated plasma has been computed. The model predicts the temporal ion behavior in the transient sheath and the slow down of the sheath evolution observed by Widner, et al. ⁽²⁾ The unnormalized time that the sheath reaches an equilibrium (say $dx/d\tau = \tilde{C}_s$) is computed from (14) as $\frac{\rho_-}{\rho_0} \frac{\mu \rho_0}{\epsilon} \tilde{t} = \text{constant} = \frac{\rho_- e}{E_0 m_+ v} \tilde{t}$. Therefore this equilibration time $\tilde{t} \propto \frac{m_+}{\rho_-}$ agrees with the Oskam, et al experimental findings. ⁽⁵⁻⁷⁾

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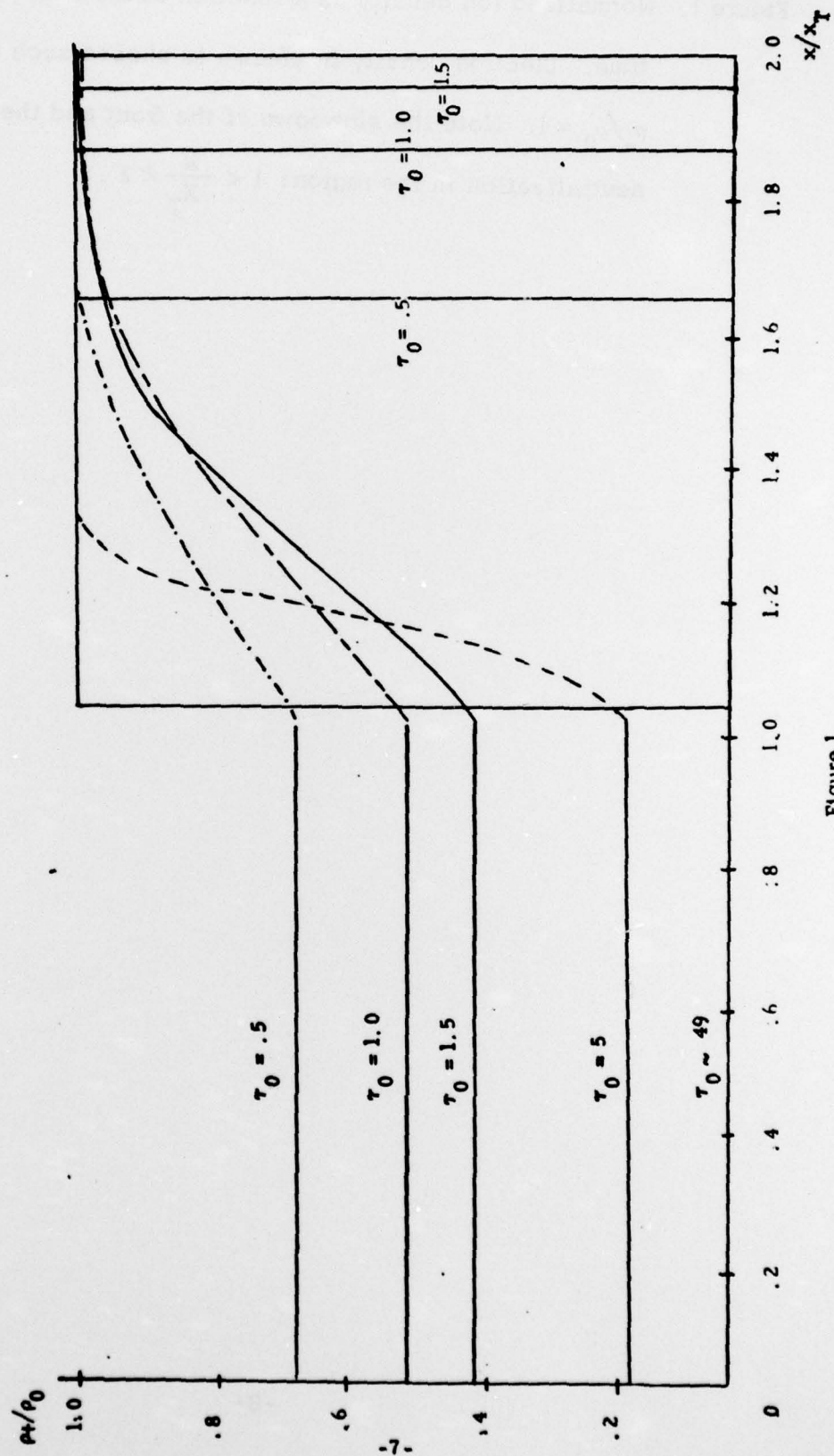


Figure 1

Figure 1. Normalized ion density as a function of distance x/x_T and time. Electron density in plasma chosen such that $\rho_-/\rho_0 = 1$. Note the slowdown of the front and the rapid charge neutralization in the region: $1 < \frac{x}{x_T} < 2$.



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