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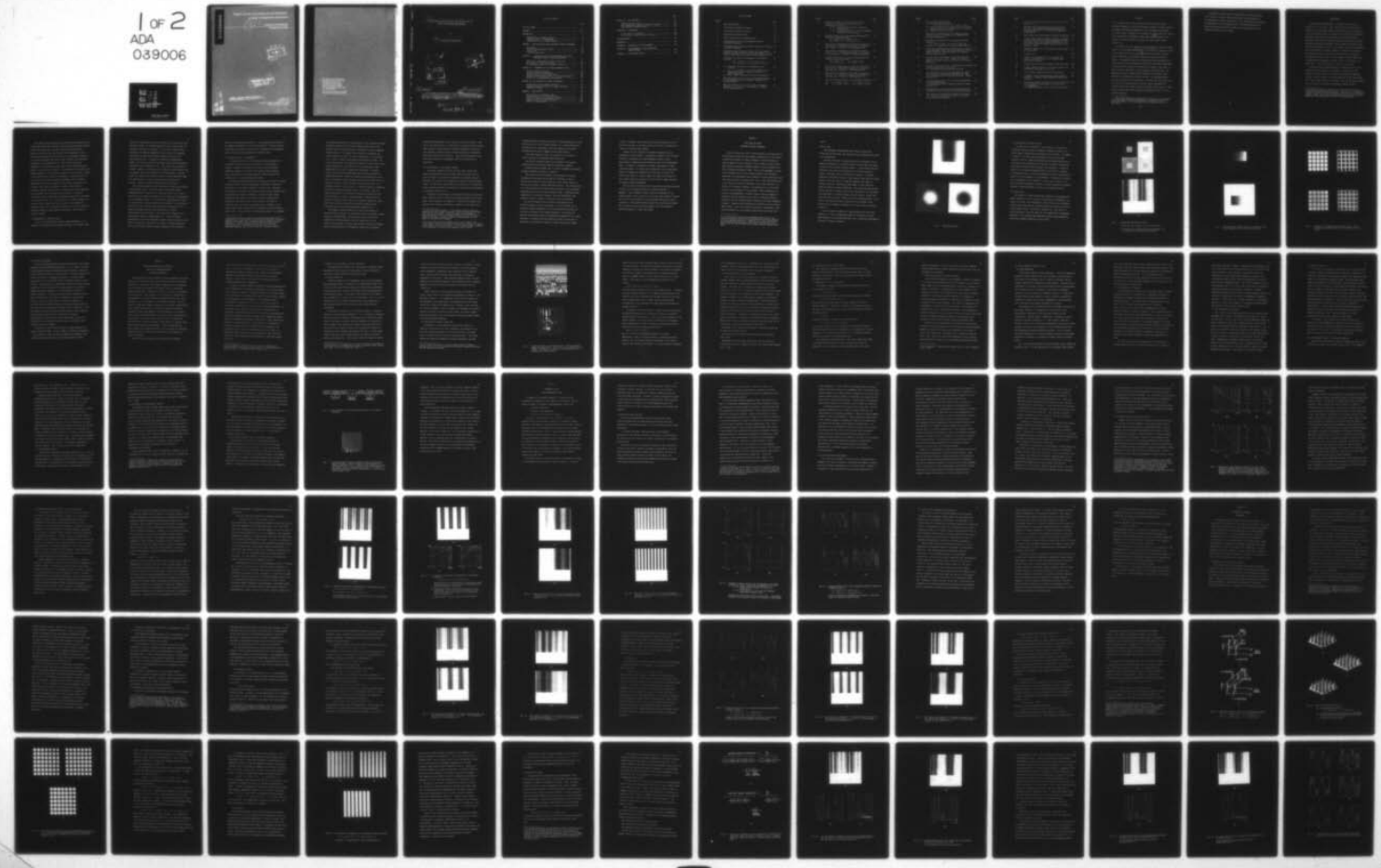

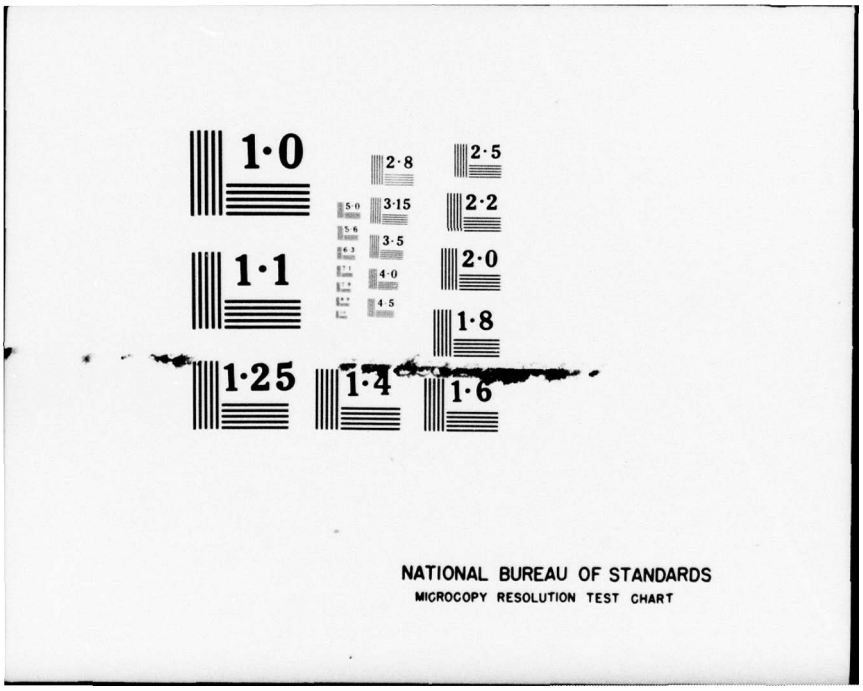
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Digital picture processing and psychophysics: a study of brightness perception

PATRICK COLAS-BAUDELAIRE

UNIVERSITY OF UTAH

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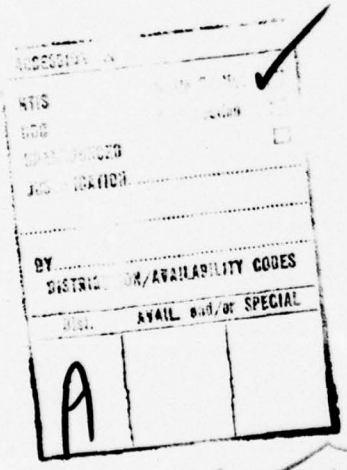
(6) DIGITAL PICTURE PROCESSING AND PSYCHOPHYSICS:
A STUDY OF BRIGHTNESS PERCEPTION,

by

(10) Patrick/Colas-Baudelaire



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ABSTRACT*

A computer driven display system was used to study brightness contrast phenomena, in a project motivated by research in digital picture processing. The modeling approach was that of Stockham and Davidson: the visual system is modeled as the cascade of a linear system (eye optics) and a multiplicative homomorphic system--that is, a logarithmic transformation (retinal receptors), followed by a linear system (neural interaction).

In order to test the linearity of neural interaction, smooth stimulus patterns were utilized, containing only a few sinusoidal components within the low frequency band, and exhibiting classical brightness contrast effects (Mach bands, simultaneous brightness contrast, Hermann grid effect). Data were collected from brightness matching experiments with these smooth patterns. The data were verified in preliminary experiments on similar patterns digitally processed by the inverse of the model, in order to obtain cancellation of the brightness contrast effects. The experimental results showed to be in agreement with Davidson's data, obtained by a fundamentally different method. This new experimental approach indicated that the hypothesis of linearity of neural interaction is justified for smooth patterns. Further studies suggested that intensity edges and contours cause strong departure from linearity. Some steps were also taken toward extending the homomorphic model for color contrast phenomena.

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*This report reproduces a dissertation of the same title submitted to the Department of Electrical Engineering, University of Utah, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

Conclusions are drawn about the implications of these experiments in the fields of computer image processing and visual psychophysics. The advantages of computer techniques in visual experiments are presented; the applications of the homomorphic model of brightness perception to digital picture processing are reviewed, and the implications of the experimental findings are discussed.

INTRODUCTION

The physiology and psychology of perception have fascinated scientists for a long time. Physiologists study the structures and functions of the extremely complex neural network which is responsible for our communicating with the outside world; psychologists analyse the qualitative and quantitative properties of our perceptual system. Sometimes, they attempt to establish theoretical models which formally describe some perceptual phenomena, and are supported by physiological as well as psychophysical facts. More recently, the field of perception has attracted several brands of computer and information scientists; their research interests range widely from artificial intelligence [Feigenbaum and Feldman 1963], to scene analysis [Roberts 1963][1], and to computer-aided psychophysics [Julesz 1971; R.I. Land 1969, 1972; Sekino 1970]. Our motivation here is related to the latter.

One particularly interesting aspect of visual perception is the complexity of the perception of brightness. In particular, some striking *illusions*, known as Mach bands and simultaneous brightness contrast, indicate evidence of some preprocessing of the retinal image at the early stages of the visual pathway. From now on, these phenomena will be referred to altogether as brightness contrast effects.

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[1] Most studies in pattern recognition and scene analysis rely only marginaly on psychophysiological theories. However, it is interesting to observe that the intuitive importance of edge detection in most scene analysis schemes agrees with the experimental notion of edge-oriented mechanism in the visual system, as will be described herein.

This subject gained recognition mainly through the pionneering work of Ernst Mach (1865, 1866 and 1868), and have motivated numerous studies in the last thirty years. Two main forces have initiated these efforts: The first is the role played in these phenomena by neural interaction and inhibition, which prove to be a fundamental ingredient in our sensory systems; the second is that, provided some simplifying assumptions on the properties of the visual system (which seem reasonable), the brightness contrast effect can be modeled using the powerfull tools of Fourier analysis. The possibility of describing such complex phenomena by such a simple formalism is very attractive, subject to the condition that it does not oversimplify or mask the facts. We will show, with the help of computer techniques, that the modeling methods based on Fourier analysis are quite simple, descriptive, and accurate (under certain conditions). Indeed, linear models of brightness contrast have been with us for some time and the importance of this approach can be indicated by the profusion of technical literature, which includes several monographs [Ratliff 1965; von Bekesy 1967; Cornsweet 1970]. But the motivation for a computer scientist to venture on these already well traveled routes exists nonetheless. Besides the fact that it is always interesting to study the information processing aspects of human perception mechanisms, the reasons are basically twofold.

I/ Psychophysics in computer science

First, in the perspective of digital image processing, it is important to understand the fundamental properties of the human visual

system; the human observer is the final stage of any man-machine image processing scheme, and therefore properties of this final element are relevant to the rest of the system. In the case of computer image enhancement and bandwith compression, the existence of a reasonably valid model for human vision should influence the design of the machine-based stages. In particular, Stockham has demonstrated [1968b, 1972] that the notion of processing pictures by multiplicative homomorphic systems [Oppenheim et al. 1968] rather than by additive (linear) systems is in harmony with the physics of image formation and the automatic gain control property of the visual system. The same author also indicated that special attention to these properties might give some insight for a better subjective quality criterion for pictures. Research is being currently pursued in this area.

Still another example would be the problem of proper color balance in photographic prints. As is well known, the human eye shows a tremendous ability to adapt to the illuminating light, be it solar or artificial (this fundamental property will be studied in the perspective of the homomorphic model). On the other hand, photographic film, with its fixed spectral response, records the illuminance of a scene without filtering out the color bias introduced by the illumination. As a result, photographs printed without correction will show an unsatisfactory colored overtone, if the film was not adapted to the illumination (daylight, tungsten lamp, etc.). Since the amateur photographer is usually not prepared to select the film according to the ambiant illumination, a solution is to attempt to balance empirically the colors during the printing process; automation of this procedure

would be an interesting improvement. A good understanding and model of the adaptative properties of the visual system will certainly provide a solid basis for research in digital color compensation of photographic prints, and other related processing of digital images.

II/ Computer science in psychophysics

Our second motivation is that a computer system equipped with a high quality display device[2] is a powerful aid in psychophysics research. Appendix C describes how a digital picture processing equipment, composed of an *off the shelf* general purpose computer, along with sophisticated display apparatus, makes a powerful and original tool in visual psychophysics research.

Von Bekesy introduced the notion of simulating the effect of neural inhibition [1960]; his model approximately implements a convolution process with a crude impulse response, the *neural unit*. Obviously, the computer will provide a much greater versatility and flexibility for testing models by numerical simulation, verifying whether results predicted by the simulation are qualitatively and quantitatively confirmed by experimental facts. When, moreover, the model is invertible, as is the case in the present study, simulation of the inverse model allows the production of compensated pictures [Stockham 1968b, 1972]. In other words, if some invertible model V is considered

---

2 Flying-spot cathode ray tubes with half-tone capabilities are now standard devices; combined with some photographic equipment, and careful calibration procedures, they can produce very high-quality pictures (Appendix C). However, computer driven television monitors will certainly prove more advantageous in the near future, as their technology improves; this will be pointed out on several occasions.

as representing properly the visual system S, then a pattern processed through the inverse system V^{-1} should appear subjectively to the observer like the original pattern is objectively, without brightness contrast effects. This results from processing in cascade by two inverse systems (V^{-1} and $S = V$), which is equivalent to no processing at all. This is indeed a very powerful and attractive concept. As will be shown further, the success and failures of a given model can be clearly demonstrated. The notion of the null experiment is well established in the field of experimental psychology. Von Bekesy [1972a, 1972b] recently studied Mach bands and contour effects by compensation methods. Rather than measuring these phenomena by some sort of brightness matching technique (which generally makes use of an additional stimulus, thereby introducing some perturbation in the phenomenon [Ratliff 1965, pp. 51-52]), a modified stimulus is presented such that the brightness contrast effect is cancelled. Thus, the modification applied to the stimulus indicates the magnitude of the visual phenomenon. However, these experiments, conducted with the perennial color-mixing wheel and similar mechanical apparatus, cannot match the precision and flexibility obtained in computer aided experiments such as the ones presented and described herein.

The computer actually permits one to devise and produce all possible sorts of patterns, for selectively inducing brightness contrast effects of different kinds. The only limit would seem to be the imagination of the experimenter...For the realization of color patterns, the advantage of computer driven display devices over Munsell paper and spinning wheels is overwhelming, despite the calibration

problems when photographic film is used as final output medium[3]. This ability was exploited here, in particular for producing patterns without edges and with a limited number of frequency components, which would show classical brightness contrast effects (Mach bands, simultaneous brightness contrast and grid effect). These patterns permitted a full test of the validity of a homomorphic model in a certain band of frequencies.

III/ Contributions of the present research

It has been suggested for some time [Fry 1948; O'Brien 1958; Davidson and Whiteside 1971] that linear inhibition models were not quite sufficient to elucidate the complexity of brightness contrast phenomena, with regard to the appearance of edges and uniform intensity fields. In particular, these models do not account for the complicated mutual interaction between the perception of contours and the perception of brightness [Ratliff 1971].

In addition, the peripheral visual system[4] has several optical and neural components likely to modify the brightness pattern of the visual image. These components are combined in a fashion which hinders easy probing of the different constituents. However, one can assert that

---

[3] A restriction exists however: if the stimuli used are reflection patterns, one cannot obtain very bright retinal illuminances for which color effects are stronger. Brighter stimuli can be obtained by projecting slides on a high reflectance screen; however, for a quick turn-over procedure, one has to resort to Polaroid films (or much better, a real-time color display!).

[4] This term refers here and thereafter to the first stages of the visual system, namely the eye and the retinal neural network, as opposed to the central (or cortical) stages. It is not meant to apply to vision at the periphery of the retina, as opposed to foveal vision.

brightness contrast is mainly the result of neural processing, and that, from the point of view of Fourier analysis, it is characterized by an attenuation of low frequencies, which are otherwise not seriously pertubated by the optical component of the visual system.

With these premises in mind, we intended in this research, to isolate and probe the inhibitory neural process by:

-limiting the frequency content of the stimuli to the band of frequencies where the retinal network is believed to operate;

-thereby eliminating edges, and, in theory, by-passing other neural processes presumably operating on contours.

Using these tools and techniques just described, an original psychophysical experiment was designed and conducted in order to estimate the frequency transfer properties of the neural network, by testing the brightness contrast phenomenon directly. The results proved to be fully consistent with data obtained previously, from a different experiment based on the same model [Davidson 1968]. Brightness effect compensation on a variety of patterns, using the experimental data, showed the range of validity of this model and its limitations.

The main contribution was the precise investigation of the linearity property of neural interaction. All prior studies on the modulation transfer properties of the neural system have assumed, but not directly verified, the linearity hypothesis. By reducing our ambition to probing the band of low frequencies considered to be of most importance, in relation with the inhibitory process, we were able to demonstrate the reasonable linearity of neural inhibition for smooth

stimuli. Futhermore, more evidence was obtained supporting the view that the invalidity of the model over the full range of frequencies is related to non-linear edge effects.

Chapter III and IV present the computer experiments and the implication of their results. Preliminarily, Chapters I and II prepare the way by introducing the visual phenomena involved, and the physiological, psychological and theoretical basis for the homomorphic model. In Chapter V the question of edges is approached in a more conjectural manner, in relation with hypothetical cortical mechanisms. Chapter VI suggests the extensibility of the model to account for similar color effects. Finally, Chapter VII draws the conclusion of this research from the dual point of view of digital image processing and computer aided psychophysics.

All the pictures illustrating this dissertation have been generated digitally on the equipment and by the procedure described in Appendix C. Some brightness contrast effects are dependent on the viewing distance, since the size of the retinal image changes in inverse ratio to the viewing distance, and the frequency spectrum is scaled accordingly. The standard viewing distance for the pictures presented here, in proper relation with the frequency units (cycle/degree) utilized throughout, is about arms length.

CHAPTER I

MACH BANDS AND OTHER BRIGHTNESS CONTRAST PHENOMENA

Optical illusions have been a favorite subject of investigation by psychologists, because they are dramatic manifestations of the complex unconscious processing of visual information, and help focusing on specific aspects of this processing. Most of these illusions involve quite abstract mechanisms, probably located in cortical stages of the human information processing system. Other visual phenomena, in which we are interested here, can be attributed mainly to the inhibitory properties of the peripheral neural networks, and have correlates in the other sensory systems [von Bekesy 1960, 1967]. They imply modification of the contrast and relative brightness of the visual stimulus, as a function of its spatial variation, by mechanisms built in the neural structure of the peripheral visual system[1]. Before undertaking the study of these mechanisms, we present several classical examples of brightness contrast phenomena, which result in seemingly different visual effects, but are most probably subserved by the same fundamental processes. Although similar phenomena are induced with color patterns [Albers 1963], we will first restrict ourselves to monochromatic black and white stimuli; we are therefore concerned here with foveal cone

---

[1] It is believed that most of this processing takes place in the retina, but similar effects have been demonstrated at other stages of the visual pathway [Jung 1967]. Therefore it is quite possible that this mechanism be physiologically distributed all along the visual system, but this does not bear on the overall modeling approach taken here.

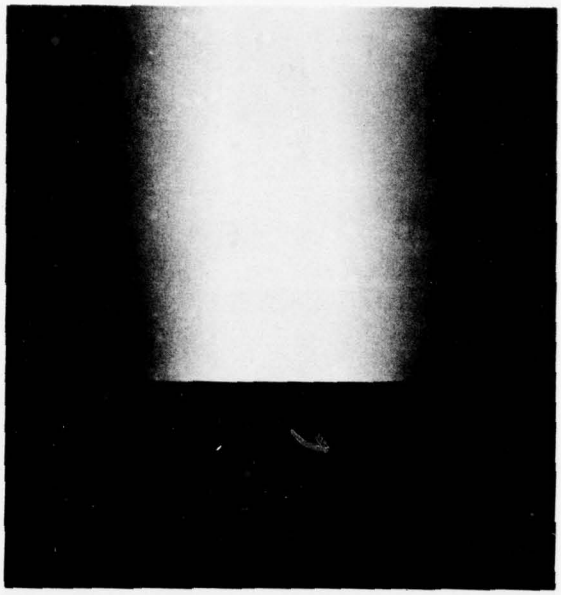
vision.

I/Mach bands

This phenomena, discovered by Mach, has been studied very extensively [Ratliff 1965], and therefore only the important facts need to be reviewed here.

The Mach band effect is characterized by an accentuation of the apparent brightness of an intensity pattern at points where the spatial variation of the intensity changes abruptly: a band brighter than the surround is seen where the inflection of the intensity pattern is convex; a dark band, where the inflection is concave. A classical example is illustrated in Figure 1, where bands of each type are clearly visible at the junctions of a transition ramp with two fields of constant intensity. Quantitative measurements of these subjective bands, have shown noticeable dissymetry: the contrast between the band and the surround is greater for the bright band than for the dark band; the bright band is in general sharper and more distinct, and the dark band wider and more fuzzy. These points will be mentioned again. Up to a point, the Mach band effect increases with the sharpness of the transition, but disappears altogether for a vertical step of intensity (Figure 38).

Since the bands are present for images stabilized on the retina [Riggs et al. 1961], the phenomena cannot be attributed to retinal image movement or after-image mechanisms, and is classically described as the result of neural interaction.



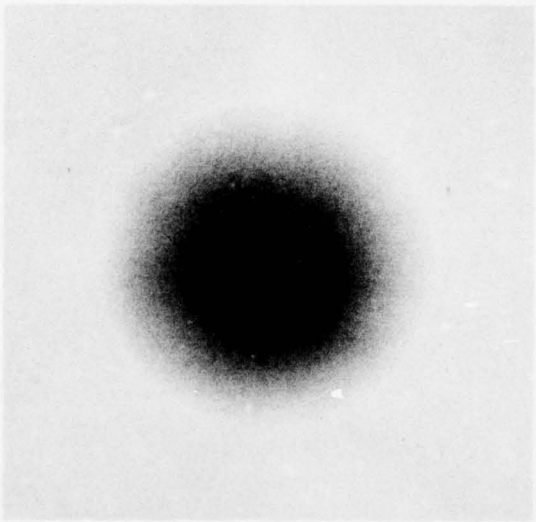


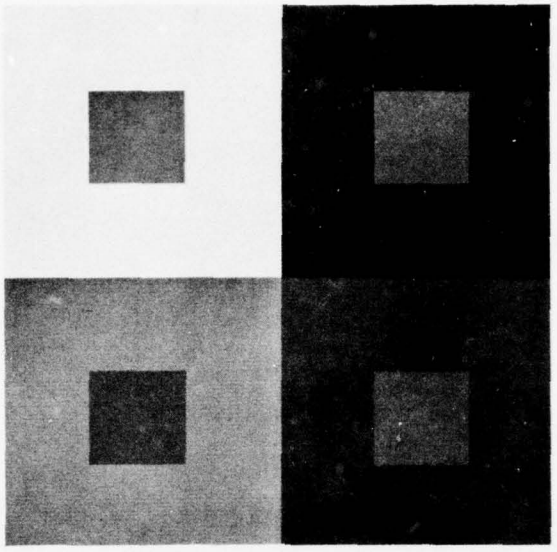
Fig. 1 - Mach Band patterns.

II/ Simultaneous brightness contrast

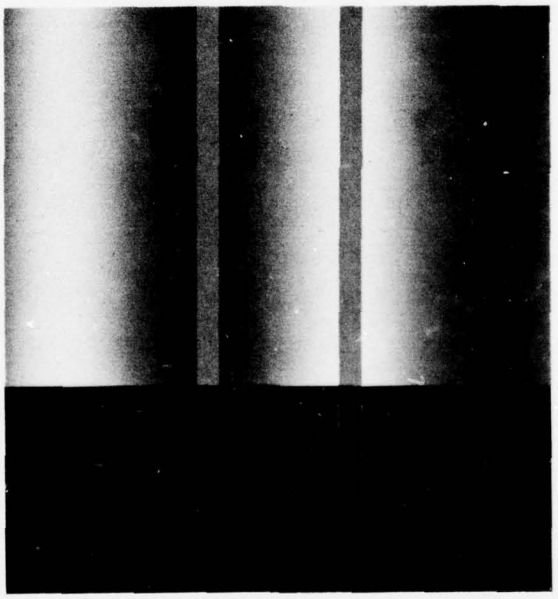
The same cause is attributed to the phenomena of simultaneous brightness contrast, whereby the apparent brightness of parts of the visual image is affected by the intensity of the surround. For example, in Figure 2 a, the four small gray squares have exactly the same intensity, but their apparent brightness is strikingly different and is a function of the brightness of the surround; the darker the surround, the brighter the gray patch will appear, and conversely. As shown on Figure 2 b, the inducing background need not be uniform in intensity, nor extended in space. As will be demonstrated further, the presence of edges is also not necessary to the phenomenon. The spread of influence of the background is quite extended, as can be seen on Figure 3; the gray scale helps estimating the effect of viewing distance on brightness contrast, indicating somehow the extent of the inhibitory action.

III/ Herman grid

The illusion of "fuzzy intersections" apparent on the Hermann grid pattern (Figure 4) is also a result of induction by the background. In Figure 4 (top left), the intersection of the bright bars are surrounded by more bright areas that the straight portions of the same bars, and as a result appear darker. The effect is opposite for a dark grid on a bright background. Here also edges are not necessary for the phenomenon (Chapter IV), and therefore edge mechanisms cannot be held responsible, as had been suggested [reported in Julesz 1970].



(a)



(b)

Fig. 2 - Simultaneous brightness contrast.

(a) The four small squares have same gray level.

(b) The two vertical bands have same gray level; the background is an exponentiated sinusoide.

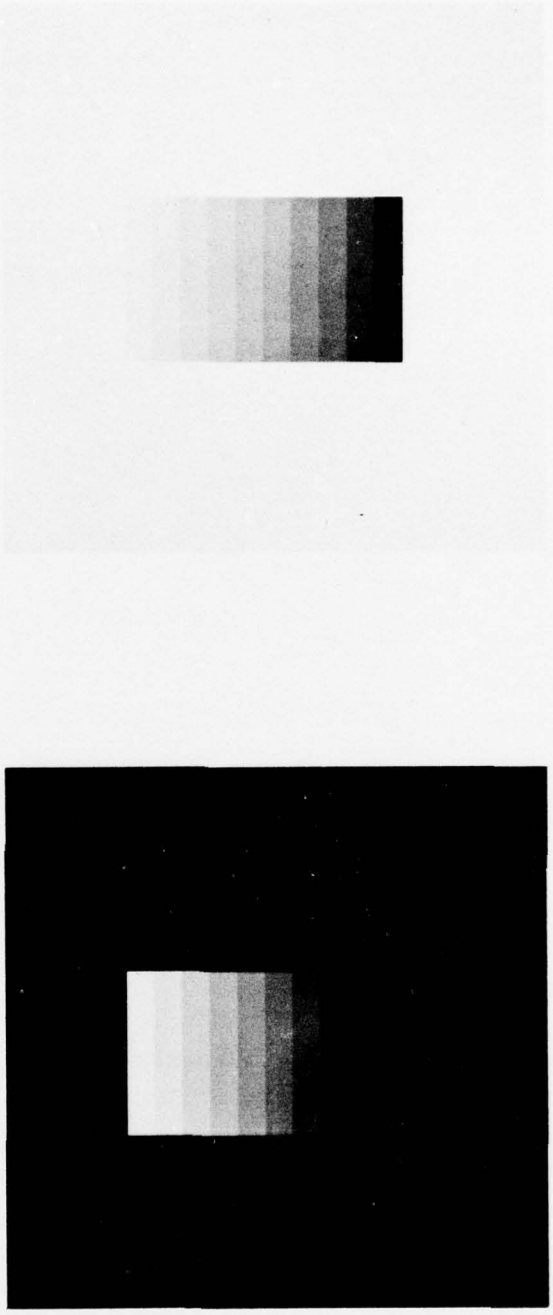


Fig. 3 - Simultaneous brightness contrast: two identical gray scales imbedded in dark and bright surrounds.

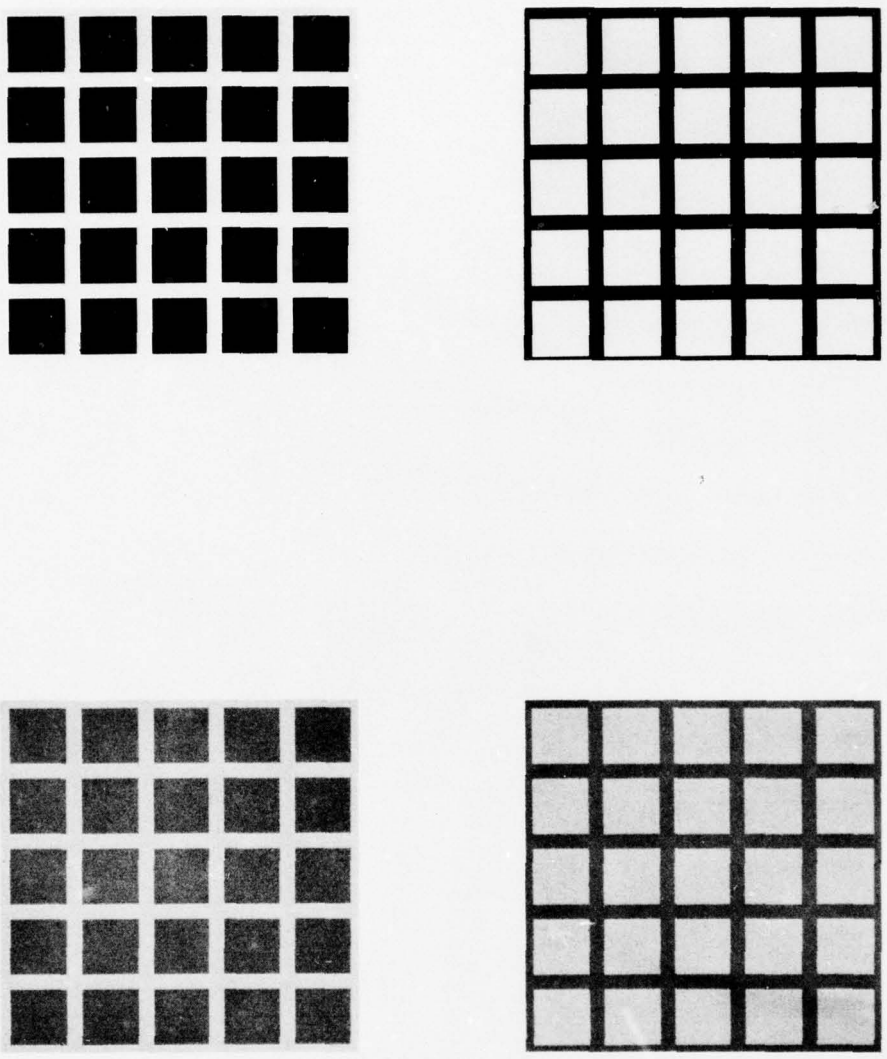


Fig. 4 - Examples of the Hermann grid contrast effect. Notice the darker or brighter spots at the intersections of the grids.

IV/ Inhibitory mechanisms

Let us now indicate briefly how inhibitory mechanisms could explain brightness contrast phenomena qualitatively. Let us consider an array of photoreceptors having the following inhibition property, typical of retinal networks [Poliak 1957; Hartline 1969]: an excited receptor will inhibit the activity of neighboring receptors in a manner roughly proportional to its own activity and as a decreasing function of their distance. This interaction is simultaneous and mutual, and the activity transmitted to the next stage of the neural system is the result of temporal and spatial summation. If we consider now a cluster of adjacent receptors, in an area of uniform retinal illumination, their mutual interaction will not alter the uniformity of their neural activity. But if the same cluster of receptors is exposed to a spatially varying intensity stimulus then contrast between bright and dark areas will be increased by the mutual action of inhibition. It is now apparent how mutual inhibition between adjoining visual receptors would account qualitatively for the sharpening of spatial discontinuities in the intensity pattern (Mach bands), and for the redistribution of brightness of certain areas, as induced by their surround (brightness contrast).

We now turn to a more careful analysis of these processes via a precise convolution model. We will attempt to estimate the parameters of such a model, to study its range and condition of validity, and to relate the inconsistencies with the particular problem of edges.

CHAPTER II

PSYCHO-PHYSIOLOGICAL AND THEORETICAL BASIS FOR A HOMOMORPHIC MODEL OF NEURAL INTERACTION

After presenting diverse brightness contrast phenomena in the prior chapter, we indicated how they can be intuitively linked to inhibitory interaction in the retina. Mach himself suggested that the brightness effect he studied, was probably attributable to neural interaction. However, physiological evidence were lacking at the time for developing this idea further. Mach resorted, for the theoretical formulation of the phenomenon, to diverse algebraic expressions involving the second derivative (i.e. the curvature) of the intensity function. With the advance of physiology, leading in particular to evidence of quasi-linear neural inhibition mechanisms in the retinal receptors of primitive organisms, and with the expansion of Fourier analysis in the fields of optics, strong interest developed in the study of the visual system as a linear system. The research effort prior to 1965 is reviewed and discussed in the excellent monograph on Mach bands by F. Ratliff [1965], which includes the work of Mach himself. Ratliff showed that the earlier models were somewhat equivalent, even if different in formalism, especially since most relied on the hypothesis of the linearity of neural interaction.

Since we intend to propose a non-linear model of brightness

perception, following Davidson [1968] and Stockham [1968b], we will not dwell on the merits and drawbacks of the earlier linear models, appropriately studied in Ratliff's book. However, the *homomorphic*[1] approach still assumes linearity of neural interaction itself. Therefore, we will present in section I the psycho-physiological evidence supporting such an hypothesis.

In short, the earlier studies of brightness contrast phenomena were mainly concerned with the inhibitory interaction in the neural system, but overlooked, or ignored, a fundamental property of the visual system, namely the ability for compressing the range of visual stimuli, revealed by the phenomenon of visual adaptation. The homomorphic model incorporates this property, in the form of a non-linear stage-- a logarithmic transformation-- located at the retinal photoreceptors. As was pointed out by Stockham [1968a, 1968b] the combination of a logarithmic transformation, followed by a linear system attenuating low frequencies implements adequately a dynamic range compression process, typical of the adaptation mechanism in the retina. These ideas will be reviewed in the second section. Justification of the assumption of logarithmic sensitivity of the retina will also be presented. Our contribution will be, then, to demonstrate that, in the frame of a homomorphic model, linearity of neural interaction, under restricted conditions, can be demonstrated by psychophysical experiments (Chapter III and IV).

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[1] For the reader not familiarized with the work of Oppenheim and Stockham [1968] on generalized linear systems, the terminology will be defined further in the present Chapter (Section III).

I/ Basis for a linear model of neural interaction

The present section focus on the justification for modeling neural interaction in the retina as a linear system. Neural interaction is basically of either excitatory of inhibitory nature.

A) Neural inhibition

Inhibition appears to be a fundamental feature of neural systems. Von Bekesy showed that it is present in all human sensory organs [1960, 1967], and that sensory effects analogous to Mach bands are observable in hearing and touch. Lateral neural inhibition has been demonstrated electro-physiologically in the retina of the vertebrate (cat, monkey, frog, goldfish, etc), as well as of the invertebrate. But, it is in the primitive eye of the Limulus (horseshoe crab), where inhibition was discovered by Hartline [1949], that it has been studied the most extensively[2].

The structural simplicity of the Limulus eye makes it a favorite subject of visual physiology experiment. It is a compound eye formed of some 1000 receptors (*ommatidia*), interconnected by a network of neural fibers. The nerves originating from each ommatidium are connected to their neighbors, and collected into bundles forming the optical nerve. The activity of an individual fiber can be recorded and analysed, as individual selected receptors are stimulated by light. By this method, lateral inhibition can be studied spatially and temporally [Ratliff, Hartline and Miller 1963]. These studies show that linearity of neural

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[2] The literature on the physiology of neural interaction is impressive; the reader will find comprehensive coverage in [Hartline 1969], [Ratliff 1965, Chap. I-4], [C.H. Graham 1965, Chap. 5].

interaction is generally well verified, and that the response of retinal receptors, resulting from recurrent interaction with their neighbors, can be expressed by simultaneous linear equations (within threshold limits, and given the restriction that inhibition is not exactly homogeneous in the retina). These equations express the fact that the response of an individual receptor is the simultaneous result (algebraic summation) of positive activity (excitation) and negative activity (inhibition).

Neural inhibition is quite certainly present in the human and primate visual system, as is suggested by psychophysics (Chapter I), and physiology (Figure 5). Its organization is also much more complex in the vertebrate visual system than in the primitive eye of the Limulus [Poliak 1957; Michael 1969; Werblin 1973]. However, if one subscribes to the likely assumption that the perception of brightness is correlated directly to electrical activity in nerve fibers, the study of simple inhibitory mechanisms, as we just described, offers some insight toward an interesting modeling approach.

B) Linear model for neural interaction

In particular, if one extends the physiological evidence of linearity in the Limulus eye, into a convenient (and plausible, as we shall see) linearity hypothesis for neural interaction in the human retina, it results that one can use a convolution model[3]. This method requires the additional assumption of spatial homogeneity, and some

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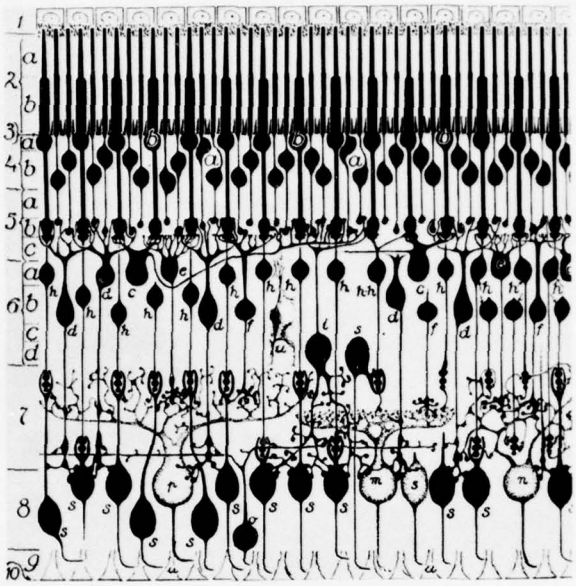
[3] Let us remind the reader that Fourier methods (spatial frequency analysis) and convolution are equivalent approaches, uniquely associated with the property of linearity.



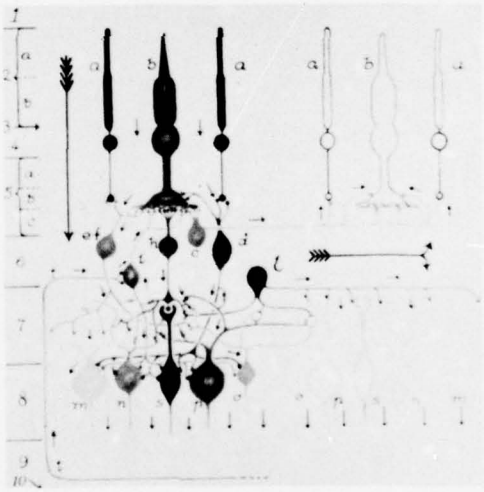


Fig. 5 - Neural structure of the primate retina. Note the complex synaptical connections. [From Polyak, The Vertebrate Visual System. Reproduced by permission of the publisher: The University of Chicago Press].

simplification concerning threshold effects, temporal effects and other limiting conditions. The convolution approach implies that the pattern response of the array of retinal receptors is the point-wise summation of individual responses, each proportional to a standard response, termed the *impulse response* or *point spread function* of the retinal system. This summation can be mathematically expressed by the integral:

$$r(x,y) = \iint e(\xi,\eta)\ \ h(x-\xi,\ y-\eta)\ \ d\xi\ d\eta,$$

relating neural excitation e(x,y) to neural response r(x,y). Following intuitive reasoning (Chapter I), and supported by physiological data [Ratliff 1965], one can describe the important features of the neural impulse response h(x,y) as: a narrow central excitatory peak, surrounded by a wide inhibitory area, whose amplitude decreases with distance (Figure 13).

As results from Fourier analysis, a space invariant linear system can also be characterized by its *frequency response* (or *modulation transfer function*, noted M.T.F.), which is the Fourier transform of the impulse response. As is well known, the advantage of the frequency domain representation is that it reduces the input-output relation defining the system to a simple multiplication:

$$R(u,v) = E(u,v) \cdot H(u,v),$$

where R, E and H are the Fourier transforms of r,e and h, respectively. Then, it is easy to realize, by theoretical Fourier analysis, that the transfer function corresponding to the impulse response just described, will exhibit, as its main feature, attenuation

of low frequencies (Figure 13). Accordingly, one recognizes that the brightness contrast effects presented in the first Chapter (Figure 2), can be related to an attenuation of the low spatial frequencies.

C) The visual system as a linear system

Then, given that an optical system can be described by Fourier analysis [Goodman 1968], a simplifying approach (defective as we shall see) is to idealize the visual system as a cascade of two linear systems: the visual optics, followed by a linear model of neural interaction in the retina. The transfer function of the overall system is then the product of the transfer functions of each component. The frequency response of the visual optics is, typically, a decreasing function of spatial frequencies [Goodman 1968]. The retinal system presents low frequency attenuation (as we have seen, in relation with brightness contrast effects), and also some high frequency attenuation (since the discreteness of the retinal receptor mosaic results in limited resolution). Therefore, the frequency response of the overall system would exhibit a maximum at the intermediate spatial frequencies. These features have been confirmed by several experiments based on the linear model. Experimental methods have consisted in:

-measurement of the subjective brightness of Mach bands [Lowry and De Palma 1961; Brynghdal 1964a];

-measurement of the contrast of sinusoidal gratings [Brynghdal 1964b, 1965];

-measurement of the threshold contrast for sinusoidal gratings [Lowry and De Palma 1962; Campbell and Green 1965; Robson 1966; Watabene et al. 1968].

II/ Non-linearity of the visual system

But, the modeling approach described in the previous section, relies on the linearity of the visual system, whereas there is strong evidence to the contrary. Some of this evidence results directly from the experimental data.

A) Deficiencies in the linear model

For instance, Brynghdal [1965] measured the subjective contrast of an intensity sinusoide

$$L + a \sin(2 \pi f x).$$

According to the linear model, the subjective brightness is in theory:

$$k_o L + k_f a \sin(2 \pi f x),$$

where k_o and k_f are respectively the visual response at DC and at spatial frequency f. Defining the contrast of a sinusoidal grating as the ratio of its amplitude over its mean, then the objective contrast of the test grating is

$$C = a/L,$$

whereas the subjective contrast of the same grating is

$$C' = (k_f a) / (k_o 1) = (k_f/k_o) C.$$

as a result, the ratio C'/C of the subjective and objective contrast should be dependent only on the frequency f, and independent of the mean luminance L, or of the objective contrast a/L. As Brynghdal observed, this was not verified in the experimental data.

Also, Marimont [1963] noted that, if the visual system were linear, then bright and dark bands for a linear intensity ramp should be symmetrical, since inhibition is certainly symmetrical, and a linear

ramp has odd symmetry. But, as we indicated in the first chapter, matching experiments and common observation show Mach bands to be very dissymmetric in brightness.

B) Non-linear response of retinal receptors

Some of these discrepancies could perhaps be related to a property of the retinal photoreceptors which is not included in a total linear model; namely that the electro-physiologic activity of a receptor, not subject to inhibition, is proportional not to the intensity of the stimulus, but rather to its logarithm, over a wide range of stimulus intensity[4]. On these physiological premises, it is tempting to correlate brightness sensitivity, in the absence of inhibition, to the logarithm of the stimulus intensity. This notion is supported by the psychophysical evidence that the visual system is more sensitive to brightness ratio than to brightness difference. This psychophysical property is expressed in the well known Fechner's law, which states that the just noticeable brightness difference is proportional to the logarithm of the stimulus intensity [Hurvitch and Jameson 1966].

Coming back to the physiological significance of the non-linear sensitivity of photo-receptors, one will note that the quasi logarithmic mapping of intensity into neural activity permits to the retina to accept visual inputs over an enormous range of intensities [Hartline 1969]. This ability for visual range compression at the receptors, along with neural interaction, is the fundamental physiological basis

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[4] See for instance: [Tomita 1968], [Ratliff 1965, p. 114], [Cornsweet 1970, p. 249].

for visual adaptation [Werblin 1973].

D) Visual adaptation

There are two aspects of visual adaptation. One type of adaptation happens for brutal change of the light environment (from very dark to very bright, and conversely), and is a rather slow process (several minutes) related to the bleaching of visual pigments. The other type is manifested by the ability of the visual system to adjust instantaneously to the general level of illumination. This aspect of brightness perception is also referred to as brightness constancy. In particular, one knows from everyday experience, that the relative brightness and contrast of a visual scene is fairly independent of the illumination, over a very wide range (several thousand-fold in energy).

This property can be related directly to logarithmic sensitivity of the retinal receptors. The intensity of a visual scene is the multiplicative combination of the reflectance factors and the illumination. Assuming uniform illumination, to simplify, we notice, along with Stockham [Oppenheim et al. 1968], Rushton [1969] and Cornsweet [1970], that a logarithmic transformation will map any uniform change in the level of the illumination, into an additive bias, that some simultaneous feed-back mechanism eliminates. Such an automatic gain control mechanism in the retina, based on the notion of logarithmic sensitivity of receptors, is discussed by Rushton [1969] and Leibovic [1971].

But, as we already mentioned, following Stockham [1968a, 1968b] and Cornsweet [1970], it is the combination of a logarithmic stage followed

by a linear high-pass system (neural interaction), which carries out a more general and more adequate dynamic range compression process. This property appears to be the most significant support for the homomorphic model of brightness perception. It is in agreement with the physiological findings that the brightness sensitivity of the retina is the result of not only logarithmic response of the receptors, but also neural interaction [Werblin 1973].

D) The perception of lightness

The more attractive attribute of the homomorphic model of retinal brightness perception is probably how it implements concurrently a mechanism for brightness contrast and for brightness constancy, or *lightness perception*. We mention here the terminology of E.H. Land [1964, 1971], signifying the perception of the reflectance characteristics of the visual scene, independently of the nearly spatially invariant illumination component. We just indicated how the logarithmic sensitivity of the receptors would, by itself, be the basis of a visual adaptation mechanism when illumination is uniform. We notice now that the attenuation of low spatial frequencies by linear neural interaction (causing brightness contrast effects) will provide an even more flexible mechanism by filtering out, or at least strongly attenuating, non-uniform illumination components, since they are typically slowly varying spatially [Stockham 1968a; Oppenheim et al. 1968].

E.H. Land and McCann [1971] proposed also a retinal model for lightness perception based on pair-wise subtraction of the output of

neighboring logarithmic receptors. Cumulating these differential data along a path connecting two retinal points will yield the ratio of the visual intensity at these two points, and therefore permit the comparison the relative reflectance brightness (lightness) of the corresponding stimulus points, and this independently of uniform or quasi-uniform illumination factors. This retinal mechanism is akin to the homomorphic model, as it relies also on the notion of logarithmic receptors and subtractive interaction (inhibition). However, a fundamental drawback is that it does not include brightness contrast, as the relative brightness of two points is independent of brightness elsewhere. In other words, it is roughly equivalent to a homomorphic system with a crude linear component. The homomorphic model appears therefore more complete and more sophisticated.

E) The dissymetry of Mach bands

But, a critical point seems apparently not to be resolved by the homomorphic model, namely the dissymetry of Mach bands for a linear intensity ramp. One will easily notice that the homomorphic model will predict dissymmetric bands, but in opposite manner to what has been reported from brightness matching experiments [Ratliff 1965, p. 55]. It predicts an undershoot (dark band) larger than the overshoot (bright band), since the log transformation applied to a linear ramp results in a sharper inflection at the juction of the gradient with the dark field. Surprisingly, von Bekesy [1968b] relies on the converse assumption (which seems erroneous) that a logarithmic stage of visual stimulus compression prior to a linear inhibition stage, will produce dissymmetric Mach bands, in conformity with experience, thereby

justifying his approach.

On the other hand, Whiteside and Davidson [1971] experimented with a family of luminance gradients (linear, logarithmic, exponential, in power law), and reported that observers judged the most symmetrical Mach bands to be produced by an exponential ramp. This experiment clearly supports the hypothesis of the logarithmic stage preceding the linear system of neural interaction, since the input to the latter system would then be a symmetric linear ramp, when the external stimulus is an exponential gradient, and would therefore exhibit symmetric Mach bands. Whiteside and Davidson associate the inconsistency reported before, from brightness matching experiments, to the fact that both test stimulus and matching stimulus are affected by the non-linear (logarithmic) transformation. In other words, brightness matching technique would measure the antilog of the subjective brightness rather than the subjective brightness itself. This would explain the inverse dissymmetry obtained by such experimental measurements.

In conclusion, there seems now to be strong evidence, both physiological and psychophysical, supporting not only the principle of a non-linear model for brightness perception, but also the notion of structuring it in the form of a logarithmic stage (retinal receptors) prior to a linear system (neural inhibitory interaction).

III/ The homomorphic model of brightness perception

The notion of logarithmically sensitive receptors was found in some of the earlier models, such as Mach's [Ratliff 1965, p. 273], Taylor's

[Ratliff 1965, p. 118], Marimont's [1962]. Subsequently, several workers mentioned and studied a model including a logarithmic stage followed by a linear system of neural inhibitory interaction.

Brynghdal [1964a] noted that, if the visual system is logarithmically sensitive to stimulus intensity, then, in brightness matching experiments with Mach bands, the linear model of neural interaction should relate, not stimulus intensity (objective measurement) and matching intensity (subjective measurement), as Lowry and De Palma [1961] assumed, but rather the logarithm of these quantities. He used in his experiments a smooth intensity gradient, in order to limit the frequency content of his pattern to the low spatial frequencies. The data appeared to verify his hypothesis, but suffer from the experimental difficulties of brightness matching with weak Mach bands. Von Bekesy [1968b] studied Mach bands in the hypothesis of a stimulus compression stage (logarithmic), followed by a neural network of inhibitory interaction (assumed linear). The same model was proposed and tested by Davidson [1968] and Stockham [1968b], namely the cascade of a linear system (visual optics), a logarithmic transformation (retinal receptors), and a linear system (neural interaction).

A) The homomorphic system

Following the terminology developed by Oppenheim et al. [1968] on generalized linear filtering, we will apply the term homomorphic to such a system composed of a logarithmic stage followed by a linear system. This definition, of mathematical origin, describes the fact that such a system obeys to superposition relatively to multiplication, and is

therefore an example of generalization of linear systems, which obey superposition relatively to addition. The notion of generalized linear systems permits the application of the powerful tool of Fourier analysis to filtering problems involving input signals combined by addition, multiplication and convolution [Gold and Rader 1969, Chap. 8] [Oppenheim et al. 1968].

B) Advantages of the homomorphic approach

The theoretical debate concerns, among other points, the structural relation between the notion of logarithmic sensitivity and the hypothesis of linear interaction, that is to say whether inhibition is additive or multiplicative[5], and, in the former case, whether the visual stimulus compression stage precedes or is concurrent with neural interaction. We have indicated previously how the homomorphic model is in harmony with the notion of visual adaptation to illumination level, and how it implements adequately the appropriate gain control mechanism. As shown by Davidson [1968], Stockham [1968b, 1972] and Cornsweet [1970], and as we will demonstrate in detail herein, the homomorphic model permits a precise investigation of brightness contrast effects. In particular, its fundamental parameters can be estimated by psychophysical experiments.

Also computational ease favors the homomorphic formalism, over the multiplicative (or *shunting*) model of lateral inhibition, although they

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[5] In his comparison of substractive (linear) and multiplicative *(shunting)* inhibition, Furman [1965] overlooks the fact that linear models of inhibition, when coupled with a logarithmic stage, are equivalent to *shunting* models [Sperling 1970], and probably more powerful computation-wise.

have been shown to be equivalent [Sperling 1970]: basically both account for visual adaptation and brightness contrast, and share the same deficiencies for the prediction of brightness near edges and for uniform intensity areas (Chapter V). Therefore, given the fast and well developed techniques for implementation of linear and generalized linear systems on digital computers [Gold and rader 1969], and the experimental compensation technique introduced by Stockham [1968b, 1972] (illustrated in Figure 24), we will study here brightness perception in the theoretical frame of the combined linear-homomorphic system depicted in Figure 6.

Davidson [1968] measured the frequency transfer characteristics of the neural interaction network; his experiment is presented in Chapter III, and the data is compared with results form our own experiment. We present a method which focuses principally on the linearity property of the neural network, and help demonstrate that it is satisfied for patterns without edges.

C) The linear-homomorphic model of brightness perception

Figure 6 depicts the complete linear-homomorphic model of brightness perception, which includes both the optical and neural aspects of the peripheral visual system. In order to facilitate reference to different stages and aspects of this model, let us make some definitions. An intensity (or luminance) stimulus is the external visual input to this system. Following Stockham [1972], we define the logarithm of this objective all-positive quantity as the stimulus density. The output of the total system is the perceived or subjective

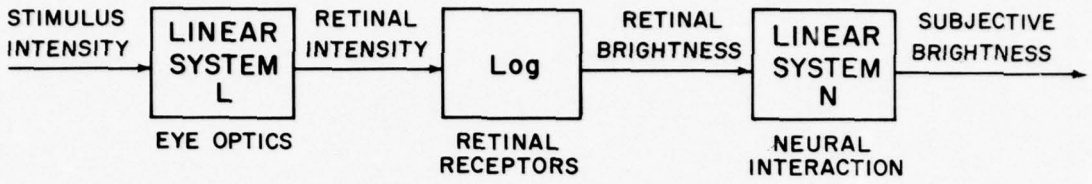

Fig. 6 - Block diagram of the linear-homomorphic model of brightness perception.

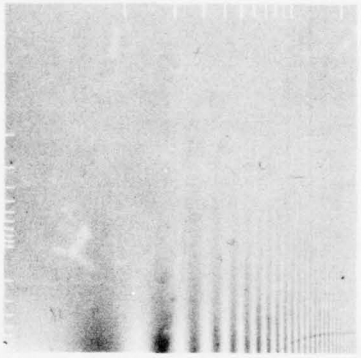

Fig. 7 - Sinusoidal pattern showing threshold contrast in function of spatial frequency. Spatial frequency varies exponentially along the horizontal axis. Contrast varies exponentially along the vertical axis, at constant mean luminance. The pattern outlines the threshold contrast sensitivity on a log-log scale [after Campbell 1968].

brightness. The term retinal intensity (or retinal luminance) applies to the image formed by the optics of the eye on the retina. The term retinal brightness (which we prefer to retinal density) refers to the logarithm of the retinal intensity, that is to say the output of the retinal photo-receptors, and also the input to the linear stage of neural interaction.

A quite simplifying hypothesis about such a model is spatial isotropy. It is probably verified for a non astigmatic eye. It is also convenient to assume that lateral inhibition presents circular symmetry, although it is plausible that complex neural structure have directional preference, and although it is certain that inhibition in the retina is not quite isotropic, as is the case in the Limulus eye. However, the two linear systems of the model will be assumed to have circular symmetry, for the sake of simplicity. As is developed in Appendix B, it is then simpler to study the equivalent one-dimensional system. Therefore, we will experiment here with one-dimensional rectilinear patterns. The transfer functions of the one-dimensional systems will be the radial frequency response of the corresponding two-dimensional systems, and their impulse response the line-spread function of the two-dimensional equivalent.

CHAPTER III

PARAMETERS OF THE LINEAR-HOMOMORPHIC MODEL

In summary of the preceding chapters, it appears that the characteristic properties of the peripheral visual system, which we attempt to model by a combined linear-homomorphic system, are:

a) limited resolution,

b) dynamic range compression,

c) spatial redistribution of brightness.

Although it would seem that a full coverage of the visual homomorphic model should require a study of the modulation transfer functions of the peripheral visual system over the full range of spatial frequencies, this is not necessarily true, insofar as we are mainly interested in properties b) and c). Since brightness contrast phenomena are the result of neural processing, and, as we have already indicated, are characterized by an attenuation of low frequencies, we will focus our attention on the low frequency factors. Therefore we intend to ignore somehow in our experiments the high frequency attenuation factor of the system, and be satisfied with estimates of these characteristics published previously and obtained by contrast threshold methods [Campbell and Green 1965].

Our purpose here is to justify the model block-diagramed in Figure 6, by demonstrating the linearity of neural interaction. Of course,

linearity is meant here in terms of retinal brightness, defined as the logarithm of retinal intensity. At the same time we will provide quantitative data on the modulation of spatial frequencies resulting from this neural interaction. In order to justify our approach, we need to make clear the contributions of the different parts of the peripheral visual system in the context of the linear-homomorphic model, and indicate how we will be able to neglect some portions of the system with impunity.

I/ Optical and neural factors

We have already approximately indicated how the three visual effects mentioned above are associated with the optical and neural components of the peripheral system. Let us review these points in some more detail.

A) Limited resolution, characterized by a high frequency attenuation in the transfer functions of the system, is due partially to the optics of the eye (cornea, lens, pupil) and partially to the retinal neural network.

The optics of the eye is affected by diffraction (as a function of the size of the pupil), chromatic and spherical aberrations; since the light stimulating the retinal receptors has to penetrate its way to the cones and rods through the layer of retinal cells and fibers, the transmitting properties of the retinal membrane should also be included in the optical factors affecting resolution[1].

The retina being a finite mosaic of discrete receptors, its resolving power is affected by the density of receptors, the spatial summation of neural activity and also, the ratio of receptors to nerve fibers emanating from the retina.

B) The non-linear stimulus compression stage, associated in our model to a logarithmic transformation of the retinal intensity stimulus, is located after the optical stage of the system. The notion that this data compression stage is located before a linear inhibition stage was justified in the previous chapter. However, modeling does not imply that the block-diagram structure of the model has to mimic the structure of the neural mechanisms; all that is desired from a successful model is to verify as well as possible the basic psychophysical facts. As we have reported already, a quite different formal approach can be taken: it has been shown that multiplicative interaction mechanisms (labelled *shunting* inhibition), coupled with feedforward and feedback, are quite equivalent in their modeling power to a multiplicative homomorphic system [Sperling 1970]. Therefore, it seems reasonable to take advantage of the power of Fourier analysis by using the homomorphic model approach. Let us note, before leaving the subject, that the claim that *shunting* mechanisms of lateral interaction model more appropriately visual adaptation [Sperling 1970] is superficial. After data compression by a logarithmically sensitive stage, mapping of the stimulus range into the full range of subjective brightness (i.e.

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[1] Optical modulation by the isolated fovea has been recently studied on an excised retina [Ohzu et al. 1972]. It has been found that the rods and cones transfer the optical image in a manner similar to a bundle of optical fibers. The frequency attenuation is of the same order than for the eye optics (lens and cornea).

visual adaptation), is just a matter of appropriate bias and scaling factors in the linear stage of the homomorphic model; the automatic gain control property of the visual system is transparent to our model, which does not include explicit provision for evaluating the proper bias and scale factor, from the input stimulus. In other words, the homomorphic model predicts outputs on an arbitrary scale of subjective brightness.

C) Brightness contrast is the main object of this study. We have previously indicated how it is related to neural interaction, and characterized by low frequency attenuation. However, our model supposes linearity of this neural mechanism. Proper experimental demonstration of the linearity property is necessary in order that the quantitative data obtained on the basis of this model be considered valid. Our experiment, soon to be described, allowed at the same time to evaluate the linearity hypothesis and to collect quantitative measurements. Having related the diverse psycho-physiological aspects of the peripheral visual system with the elements of our model, we can consider the possible experimental methods for measuring its quantitative characteristics. But we now face the problem of estimating experimentally the transfer functions of two linear components of a non-linear system.

II/ Contrast threshold measurements

As we have seen in Chapter II, the non-linear logarithmic stage imbedded in the array of receptors, invalidates any attempt to evaluate the overall transfer characteristics of the visual system. However, it is possible to ignore the effect of the non-linear stage, when the

intensity amplitude (or contrast) of the stimulus is near threshold of perception, since the non-linear transformation which is imbedded in the retinal receptors, presumably introduces negligible distortion in this limit case. Therefore the threshold contrast sensitivity can legitimately be tested for the total visual system, as a function of spatial frequency. The results show generally a maximum contrast sensitivity at about 6 to 10 cycles/degree, and an attenuation at higher and lower frequencies. The reader will get a visual approximation of these features, on a log-log scale, by observing the pattern of Figure 7, after Campbell [1968]; this pattern pictures a sinusoidal grating with frequency increasing exponentially along the horizontal axis, and with amplitude (i.e. contrast) decreasing exponentially along the vertical axis, at a constant luminance level. The triangular contour perceived when observing the pattern, outlines the threshold of contrast perception in function of spatial frequency, and therefore in function of the viewing distance. Not surprisingly, the contrast sensitivity data reflects the combined properties of the modulation transfer functions of the eye optics and the neural network.

However, it is questionable to assert that these data coincide with the multiplicative combination of the two transfer functions. This point of view would imply that a sinusoidal grating is perceived when its contrast is above a given threshold uniform for all frequencies. This is quite debatable, because it is not clear how a neural mechanism, which is linear only as a first approximation, would be able to detect relatively to the same threshold low frequency gratings and high frequency gratings, which produce, for the same amplitude, very

different intensity gradients on the retina. In other words, the additional non-linearity which undoubtedly appears at threshold conditions, is likely to be a function of spatial frequency. In particular, it has been demonstrated [Watebene et al. 1968] that threshold contrast sensitivity varies strongly with mean luminance, a fact which implies non-linearity. On the other hand, as we shall see, our experiment, as well as Davidson's, indicate that linearity is well verified for low frequency patterns. This suggests that spatial frequency attenuation by neural inhibition, and spatial frequency detection, are not related in a simple way.

In addition, several recent studies tend to indicate that spatial frequency detection is mediated by independant frequency channels [Campbell et al. 1970; Sachs et al. 1971; Richards and Spitzberg 1972]. In particular, it has been shown that adaptation to a given spatial frequency raises the contrast threshold at the adaptive frequency, and also that a sinusoidal grating is masked only by noise of similar spatial frequency [Stromeyer and Julesz 1972].

Nonetheless, contrast sensitivity measurements allowed Campbell and Green [1965] to estimate, by an innovative technique, the modulation transfer function of the optics of the eye. We will utilize their data in our model, and, despite the reservations presented above, we will also consider their measure of the contrast sensitivity of the neural system at high frequencies as a good estimate of the blurring characteristics of the retina. There are two main reasons. The first one lies in the difficulty of employing brightness matching techniques

for high frequencies; this will be clear when we present the pattern used in our brightness matching experiment for measuring frequency attenuation by the neural interaction. The other reason is that a first order approximation of the high frequency region will be sufficient, given the more drastic discrepancies in our model which we will encounter later and which can be attributed to non-linear edge effects rather than to defective measurements. We will address these problems in the next two chapters.

III/ Transfer function of the visual optics

Campbell and Green were able to measure the contrast sensitivity, as a function of spatial frequency, of the neural system alone, by creating directly on the retina, by interference methods, a grating stimulus, which was therefore undistorted by the optics of the eye. Their results are plotted in Figure 8c and 8d, for frequencies above approximatelly 6 cycles/degree. Values for lower frequencies were ignored here, because it is for these low frequencies that contrast threshold method are the more questionable[2], and this is actually the band of frequencies which is relevant for studies of brightness contrast phenomena and on which we will focus our attention next. One will notice how the data, above 10 cycles/degree, is well fitted by a linear

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[2] For reasons of retinal anisotropy and non uniformity, these studies on frequency attenuation and sensitivity must be restricted to foveal vision, which implies a limited field of view; as the reader can verify for himself on Figure 7, the detection of contrast for very low frequencies is very uncertain. Campbell and Green also mentioned difficulties in measuring the threshold contrast at very high frequencies (above 40 cycles/degree), as fixation becomes difficult with very narrow fringes.

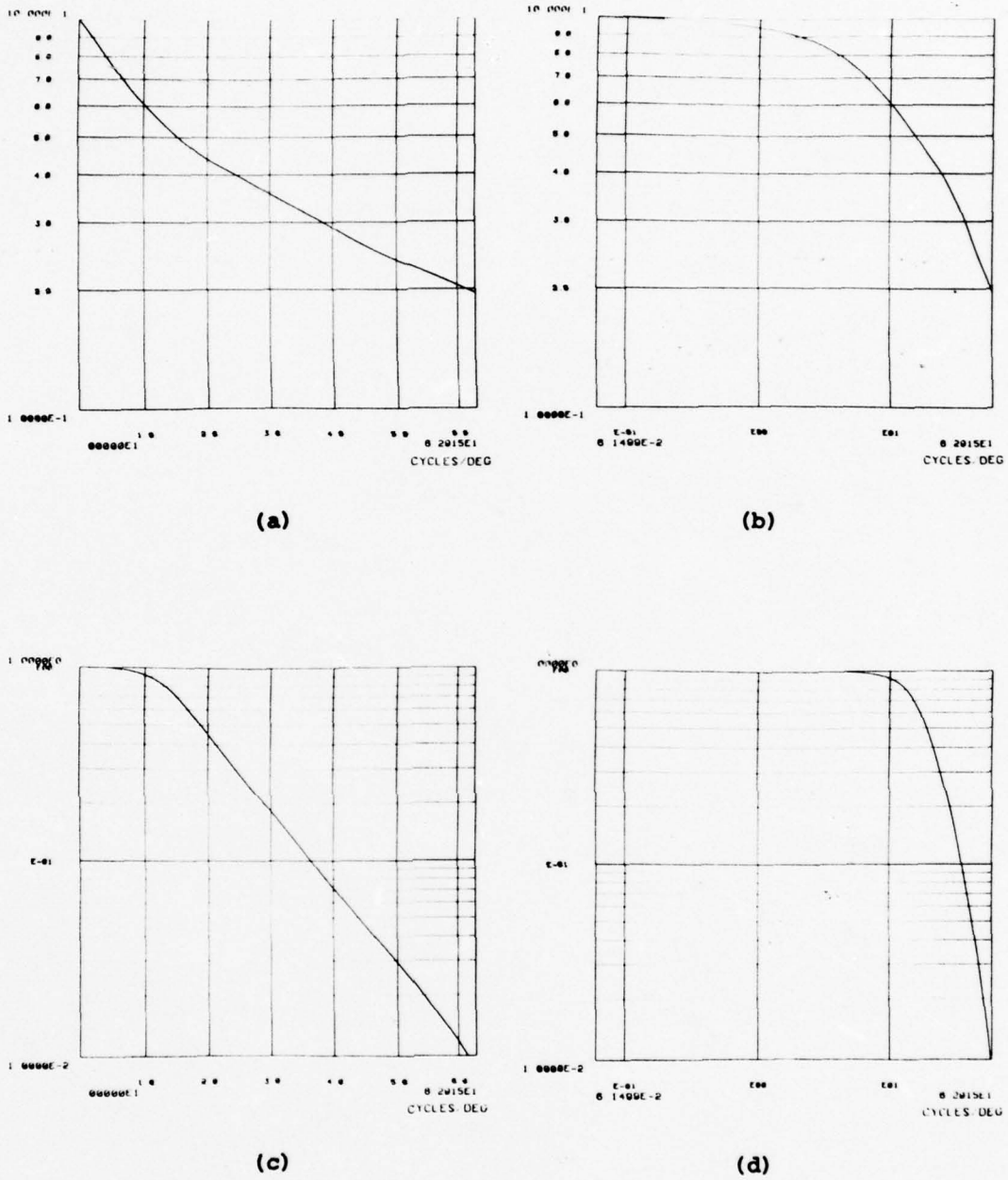


Fig. 8 - Experimental data obtained by Campbell and Green [1965]: modulation transfer function of the visual optics (a & b), contrast sensitivity of the visual neural system (c & d). Modulation and sensitivity are plotted on a logarithmic scale, frequency on a linear (a & c) or logarithmic scale (b & d).

function when plotted on a logarithmic scale of contrast and a linear scale of frequencies.

Campbell and Green also evaluated the resolving power of the total visual system with external sinusoidal gratings of varying frequency and varying contrast. As expected, the contrast sensitivity for external gratings is lower than that for retinal fringes, as a result of attenuation by the visual optics. Ignoring the non-linear stage, since we are at threshold, and assuming that the overall contrast sensitivity of the visual system is the product of the transfer function of the optics by the contrast sensitivity of the neural system itself, the MTF of the eye optics is obtained by the division of the two series of results obtained through these two experiments (Figure 8a and 8b, for a 2 mm pupil). This transfer function is similar to estimates obtained previously by theoretical considerations of diffraction and optical aberrations [Westheimer 1963; Fry 1963], by studies of light scatter on the retina [Gubisch 1967], and optometric methods (reflection of light on the retina); for a review of these researchs, see [Fry 1970]. The transfer function there obtained is typical of a mediocre quality diffraction-limited optical system, but it shows, for a 2 mm pupil, less attenuation than the neural system; in other words, the visual optics appears to resolve better than the retinal receptors.

As results from the considerations above, we can regard these data as a good estimate of the MTF of the eye optics. We will use it in our model, although it will be more often for pointing out that its effect can be neglected, than for actual computation...

IV/ Estimating the transfer function of the neural network

A measurement of the transfer function of the neural network, in the context of the non-linear model of Figure 6, has already been offered by Davidson [1968]. His experiment, however, did not demonstrate convincingly the hypothetical linearity of neural interaction, since he measured the modulation of individual sinusoidal components, and not the modulation of combined frequency components. But the notion of linearity actually involves two distinct properties: additivity and scalability. Davidson's results indicated that the modulation at low frequencies is quite independent of the contrast level (that is of the amplitude of the target sinusoide). This is certainly in agreement with the scalability hypothesis. We will test the other aspect of linearity, perhaps the more important, namely additivity. In order to prevent interference with other factors, we will use patterns at a constant contrast and mean luminance level.

Davidson employed a contrast matching technique: a reference grating of frequency fc is compared to a test grating of frequency f, of variable contrast. The contrast setting such that the two grating appear subjectively to be of comparable contrast, is taken to indicate the relative modulation of the two frequencies f and fo. Davidson experimented over a wide range of frequencies, and at several levels of contrast. The gratings were sinusoides exponentiated for compensation of the logarithm transformation stage, ignoring the negligible effect of the visual optics. These results will be presented further, in parallel with the data from our experiment, which we proceed to describe now.

Since the frequency attenuation carried out by the neural processing is revealed by the "illusion" of brightness contrast, we will attempt a measurement of this modulation in a psychophysical experiment involving a stimulus exhibiting some brightness contrast effect. Futhermore, since our model is based on the hypothesis of linearity of neural interaction, we rely on the analysis of stimulus patterns in terms of sinusoidal components. Therefore, we will search for patterns which are simple in terms of Fourier analysis, and which display brightness contrast effects. The simplest is probably obtained by composition of two sinusoides of frequency f and 3f, in phase, as illustrated in Figure 14a. Since we want to estimate the modulation property of the neural stage alone (linear system N), we desire a stimulus which produces a retinal brightness distribution (refer to Figure 6) of the form

$$\sin(2\pi\ f\ x) + \sin(6\pi\ f\ x).$$

Therefore we want to preprocess this pattern of intensities, in order to compensate for the effect of the visual optics (linear system L) and the logarithmic transformation stage. As is apparent from the data of Figure 8b, the effect of the visual optics, in the band of frequencies that we are going to study (approximately from 0.1 to 4 cycles/degree), can be considered as negligible for only two frequencies f and 3f. For instance, the strongest relative attenuation for the stimuli used in our experiment, would occur on the pattern of highest base frequency (1.23 cycle/degree, Figure 12), and would have a value of about 95%; this is negligible with regards to the intrinsic uncertainties of psychophysical measurements. Therefore we will only compensate for the non-linear

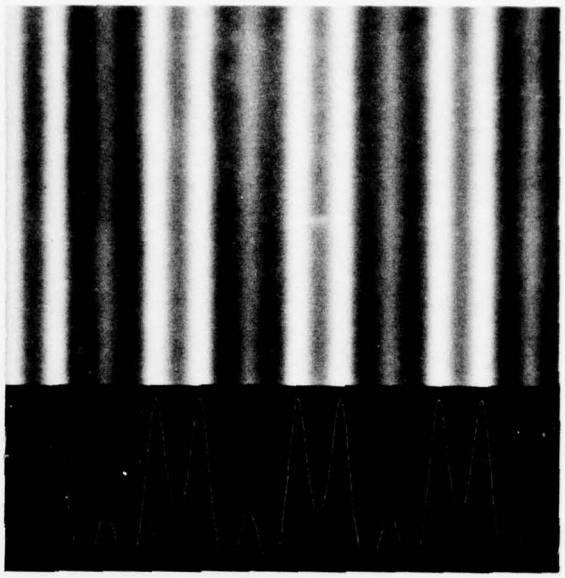
stage at the receptors, by exponentiating the sinusoidal distribution of intensities.

As can be verified on Figure 9a, an intensity distribution

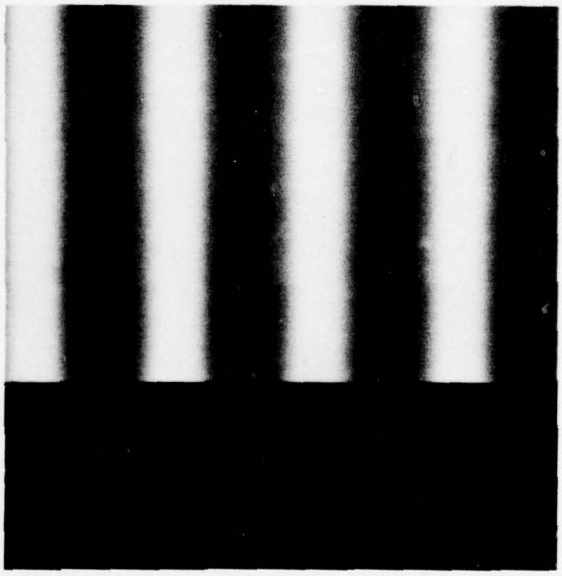
$$\exp(k_1 \sin (2 \pi f x) + k_3 \sin(6 \pi f x)),$$

will produce fairly strong simultaneous brightness contrast when the two sinusoidal components have the same amplitude ($k_1 = k_3$). On the other hand, when the amplitude k_3 of the fast component is only a fraction of that of the slow component ($k_3 = 0.1\ k_1$, in Figure 9b), the background predominates completely. We might consider that as an over-cancellation of the original brightness contrast effect. In a sense, a reversal of the relative apparent brightness of bands exhibiting brightness contrast on the original pattern, occurs between these two extreme cases. In other words, there exists a value of k_3, less than k_1 and greater than $0.1\ k_1$, for which the intermediate bands have the same subjective brightness. For this situation, the ratio k_1/k_3 will indicate the relative amplification of the frequencies f and 3f.

An experiment was set up, with the knowledgeable help of Dr. William Lee, in order to evaluate by this method the relative modulation of frequencies f and 3f, for several values of the base frequency f. This experiment is described in detail in Appendix A. Patterns of five base frequencies were used, from 0.123 to 1.230 cycles/degree; three of them are illustrated here (Figure 9a, 11a and 12a); in order to obtain these exact angular frequencies, the photographic illustrations should be viewed so that the horizontal field of view sustains an angle of approximately 8.1 degree, that is to say from a distance of about 55 cm



(a)



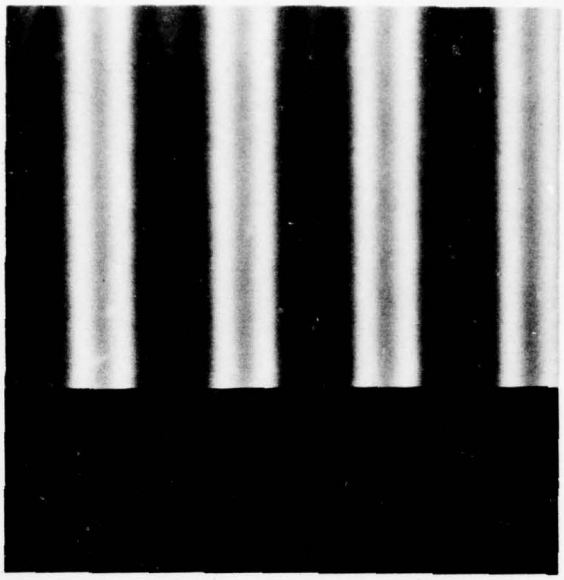
(b)

Fig. 9 - Sinusoidal test pattern corresponding to the density functions:

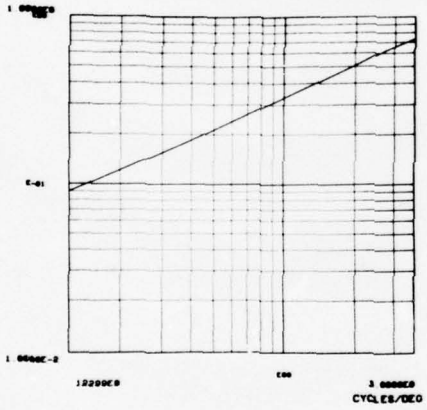
(a) $k_1 \sin(2\pi f x) + k_1 \sin(6\pi f x)$,

(b) $k_1 \sin(2\pi f x) + 0.1\, k_1 \sin(6\pi f x)$.

The background frequency f is approximately 0.50 cycles/degree at arm length viewing distance.



(a)



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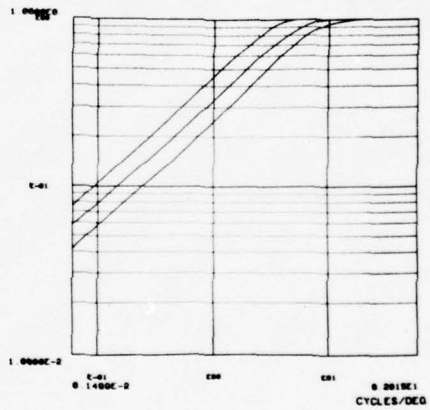


(c)

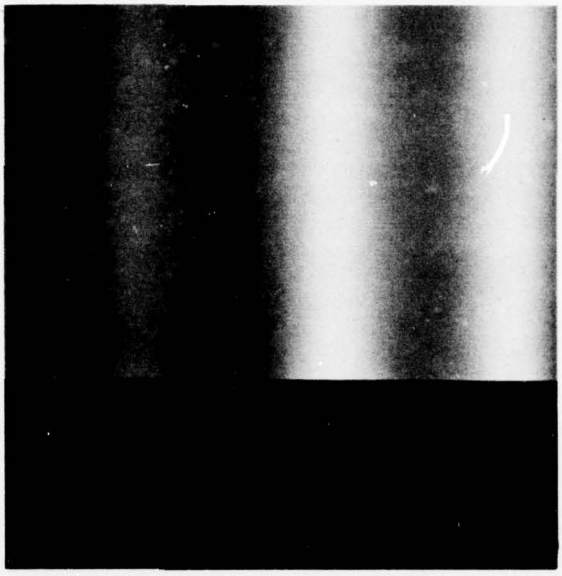
Fig. 10 - (a) Sinusoidal test pattern corresponding to the density function:

$$k_1 \sin(2\pi f x) + 0.5\, k_1 \sin(6\pi f x).$$

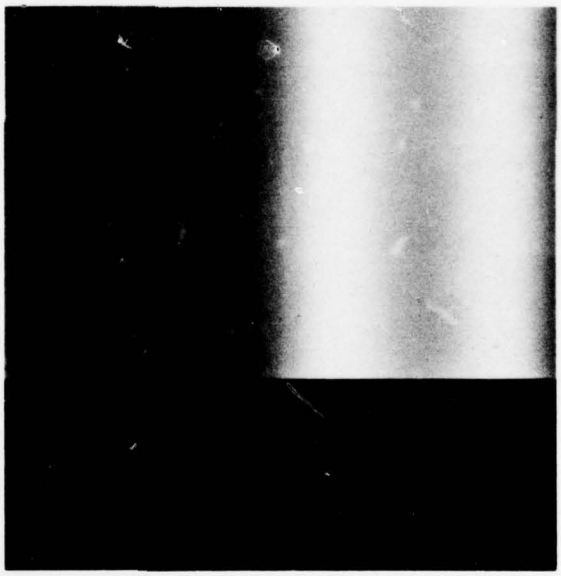
Shows the result of compensating for brightness contrast the pattern of Fig. 9(a), according to the average data from the experiment (Appendix A).

(b) Experimental data for frequency modulation by neural interaction, showing the linear relationship, in the band of frequency tested, between log of modulation and log of frequency.

(c) Three transfer functions having the same property.

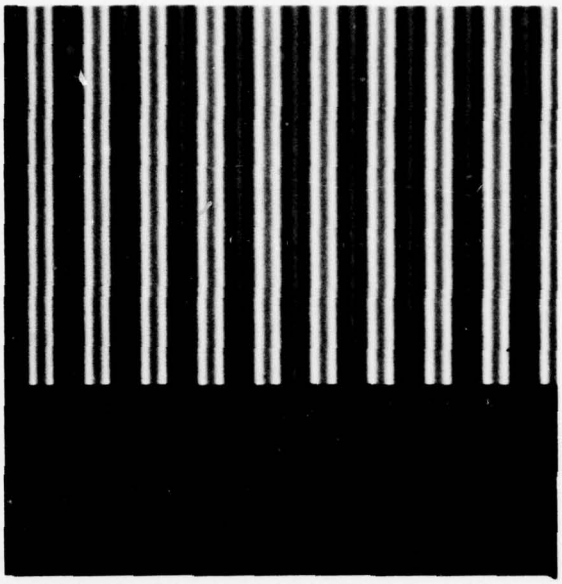


(a)

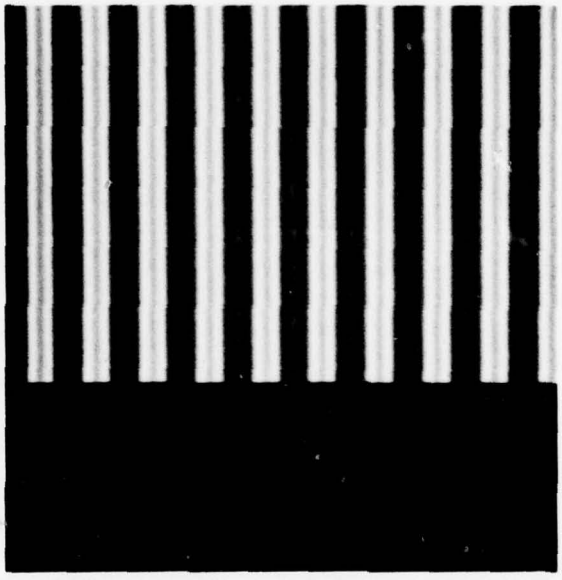


(b)

Fig. 11 - Same test pattern as Fig. 9, for a background frequency of approximately 0.123 cycles/degree: original (a) and compensated (b).



(a)



(b)

Fig. 12 - Same test pattern as Fig. 9 & 10, for a background frequency f of about 1.23 cycles/degree: original (a) and compensated (b).

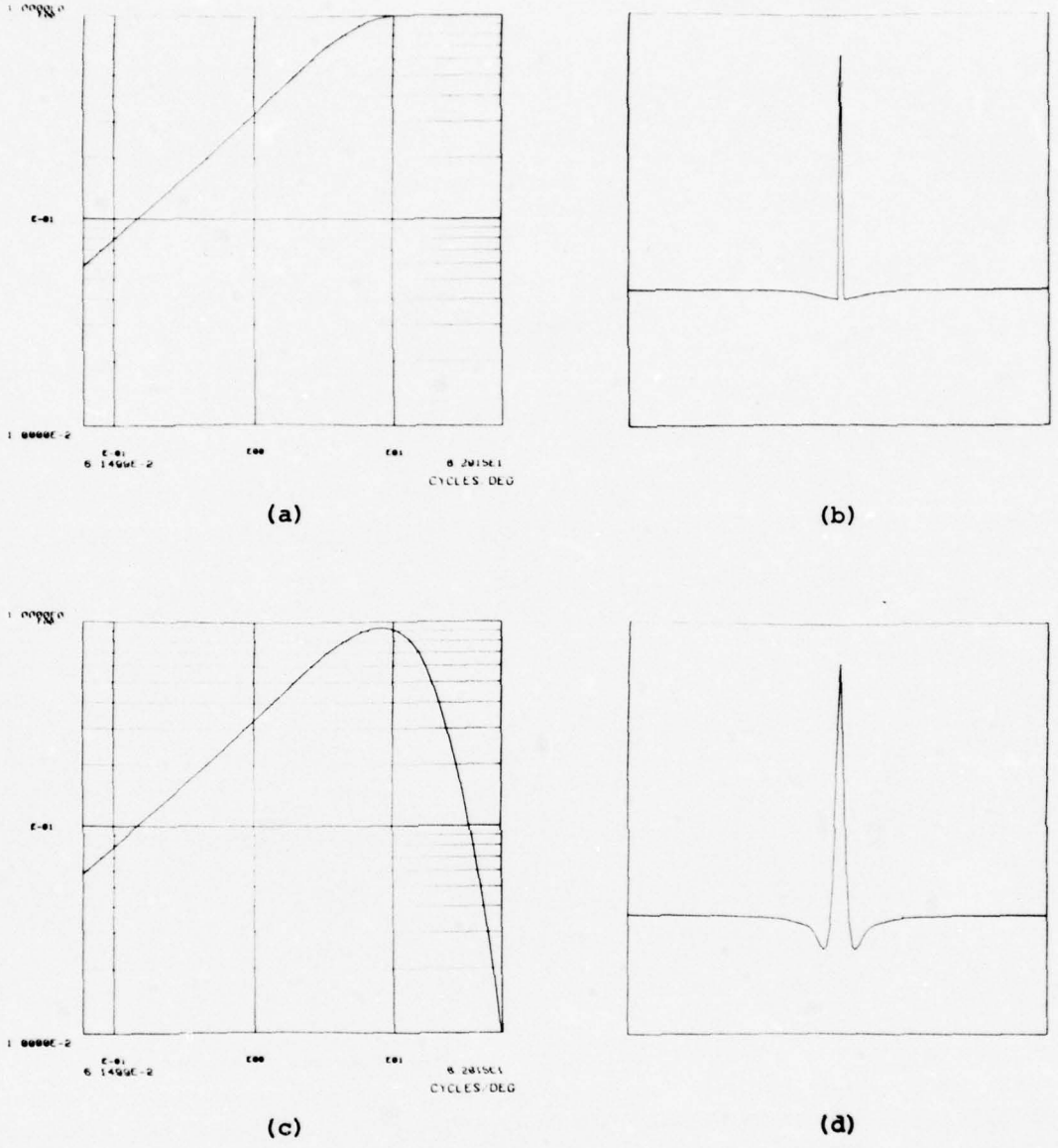


Fig. 13 - Modulation transfer function and corresponding line-spread function for neural interaction (sub-system N on Fig. 6):
(a & b) Ignoring high frequency attenuation by neural blurr;
(c & d) Using data of Fig. 8d, for high frequency attenuation by neural blurr.

Transfer functions are plotted on log-log scale. Line-spread functions are plotted for a span of one degree of visual angle.

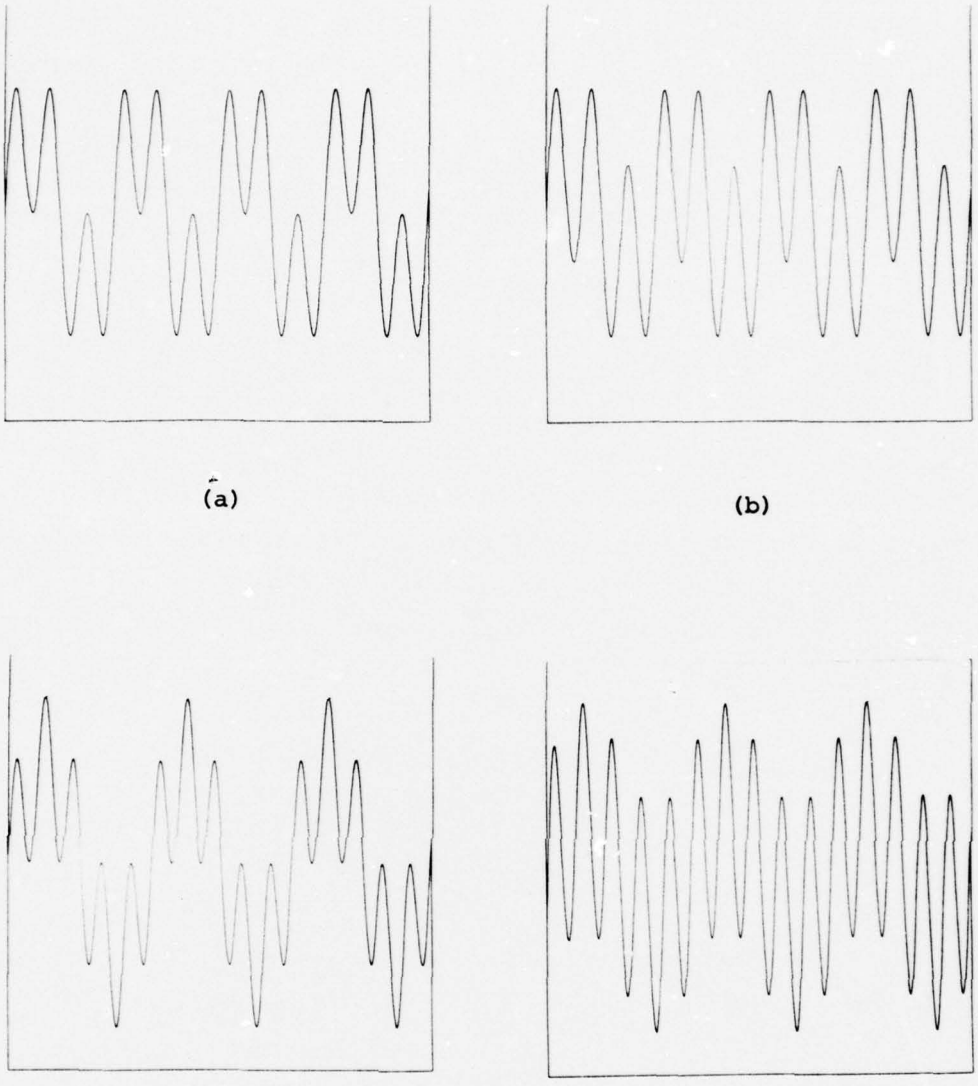


Fig. 14 - Stimulus density (left) and corresponding predicted subjective brightness (right):

(a) $\sin(2\pi f x) + \sin(6\pi f x)$;

(c) $\sin(2\pi f x) + 0.8 \sin(10\pi f x)$.

Scales of density and brightness are arbitrary. Notice the predicted brightness contrast effect.

V/ Results of the psychophysical measurements

As a matter of fact, the viewing distance has no noticeable influence on the brightness contrast effect, for these particular patterns, as is confirmed by the findings of the experiment. The results (Table III) indicate that the relative modulation of frequencies f and 3f is approximately a factor of 2, uniformly over the range of frequencies studied. As a trend, the neural system amplifies twice as much frequencies which are thrice as large...Obviously, these results must be accepted within some margin of uncertainty, as is usual in psychophysical experiments; the data is collected in Appendix A.

These findings imply that the pattern for which $k_3 = k\ 1/2$ is compensated for brightness contrast; this pattern is illustrated in Figure 10a. Some readers might have reservations about the cancellation of the brightness contrast on the photographic illustration. Besides the fact that the experiment was under different viewing conditions, the results clearly indicate, not only a wide variance in the judgement between subjects (Table II), but also a variation of judgement for a given subject and a given stimulus (Table I). The reader will easily experience that the judgement on the relative brightness of the target bands, may vary for a given pattern with the mode of fixation, in particular at low frequencies (Figure 12b). Depending on whether one band, or the other, or the whole field is observed, the relative brightness can be perceived very differently, and this uncertainty increases as the base frequency is lower and the

target bands wider and fuzzier. Von Bekesy [1968] indicated a similar effect with Mach bands of steep luminance gradients. This problem, combined with the necessity of reduced field of view (foveal vision) sets the practical limits of the lower frequencies to about 0.1 cycles/degree (Figure 11). The higher frequencies are also limited for practical reasons, as it becomes increasingly difficult to express a judgement of relative brightness for a high concentration of fringes (Figure 12). In conclusion, the experimental data must be considered only as an average trend. The results could probably be improved in terms of uncertainty, by more carefully analysing the effect of the mode of fixation, and by training the subject in this respect. We will leave to the psychologist the task of experimenting and investigating more precisely in this domain.

On our part, we will be contented by the fact that the data will be satisfactorily confirmed by compensation experiments on other patterns (next chapter), and that they are fully consistent with the results of Davidson [1968], based on the same model and obtained through a different experiment, in different viewing conditions (briefly flashed achromatic photopic fields viewed monocularly). As we have seen, Davidson estimated the modulation of individual sinusoidal components; we have estimated the relative amplification of added sinusoidal components, in quite different illumination and contrast conditions. That the results be similar can certainly be considered as a good indication of the linearity property of the neural processing. We will indicate more evidence in the coming chapter.

If H(f) is the modulation transfer function which we have supposedly estimated in this experiment, then for a band of low frequencies, the following relation holds approximately:

$$H(3f) = 2H(f).$$

Similarly, Davidson's data show that, in the same range, the relation

$$H(3f) = 2.2H(f).$$

is approximately verified. The reader will recognize that these equations express a power law relationship between frequency and modulation, which is characterized by a linear relationship between the logarithm of the modulation H(f) and the logarithm of the frequency f ([Davidson 1968, Figure 10]; our data, Figure 10).

It is interesting to notice, along with Dr. T.G. Stockham, Jr., that the independance (within certain limits) of the modulation of low frequencies from the spatial stretching of the corresponding image, as it results from the power law relation, implies the nice property that the relative brightness of the sizable components of the visual environment will not vary with the viewing distance.

We now proceed to verify whether the experimental data accurately predicts brightness contrast effects, for more complex stimuli; that is to say whether the linearity hypothesis is verified in a more general sense.

CHAPTER IV

THE LINEARITY OF NEURAL INTERACTION

The agreement between Davidson's data and our results is an indication of the linearity of the inhibitory interaction, since in the first case the modulation of single frequency components was measured, in the second case the relative modulation of two added frequency components was tested. It also indicates that such conditions as illumination level, do not alter significantly the fundamental characteristic of the transfer function of the neural system, namely that logarithm of frequency and logarithm of modulation are roughly proportional, at least in the range of low frequencies. It is now desirable to test more generally the model and the data, in order to verify whether the important property of linearity is still valid for less restricted conditions on the stimulus pattern.

I/ Extrapolation of the transfer functions

Since we associate the low frequency attenuation by the neural system with linear inhibitory interaction, and because neural inhibition probably has physically a finite extent, this low frequency attenuation, as estimated in the preceding chapter, should only apply over a limited range. Several studies have indicated that the spread of the inhibitory action is probably no more than 10 to 20 minutes of visual angle [Growney and Weinstein 1972]. Therefore, we may reasonably assume that

the transfer function of the inhibitory system is flat for frequency higher than, say 10 cycles per degree (to be conservative). Such estimates can be obtained, for instance, from theoretical considerations on an idealized gaussian line spread function having some 20 minutes of inhibitory spread [Fry 1970].

We view, here, neural inhibition quite separately from neural summation. Of course, it is only a theoretical convenience to separate the neural system *N* (Figure 6) into the cascade of two sub-systems: a retinal summation sub-system[1], with high frequency attenuation; and a neural inhibitory sub-system, with low frequency attenuation. The total modulation fransfer function of the neural system would be the multiplicative combination of these two MTF's. But it seems clear that, although the two processes are in fact physiologically combined, their modulation effects on spatial frequencies are quite separable, and that our theoretical dichotomy is justified. We discussed already in Chapter IV the reasons for using threshold contrast data for the MTF of the summation sub-system, characterized by blur and loss of resolution. Let us now consider the inhibitory sub-system, and review how much we know about it after our experiment.

Clearly, the knowledge of a power law for the band from 0.1 to 4 cycles per degree, and of uniform attenuation for frequencies higher than 10 cycles/degree, does not permit a unique definition of the

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[1] This term is meant here to include all factors affecting the resolution of the subjective *image*, that is: finite receptor mosaic (since this is equivallent to summing within the window of one receptor), and actual neural summation at different levels of the retinal and neural network.

possible transfer function. However, the situation is not so bad, because the band of frequencies between 0.1 and 4 to 5 cycles/degree really is the important region with regard to brightness contrast effects; extrapolation in other regions should only have a minor effect. For instance, Figure 10c pictures three possible such transfer functions; the intermediate curve, also plotted on Figure 13a, levels off at about 10 cycles/degree, which is about the peak of Davidson's data [1968, Figure 10], but the low frequency attenuation is a little less steep for our data. The corresponding line spread function, represented on Figure 13b for a span of one degree of visual angle, has an inhibitory spread of 15 to 20 minutes of arc.

We will now attempt to test in further detail the linearity hypothesis of neural interaction, by verifying the prediction of the model on more complex patterns exhibiting the diverse brightness contrast effects (Mach bands, simultaneous brightness contrast, Hermann grid effect). As we mentioned in the introduction, given the computing tool at our disposal, a natural method for testing our data is the null experiment. Since the components of the linear-homomorphic model of the visual system are inversible, one should observe cancellation of the brightness "illusion" on pattern pre-processed by the compensation system block diagramed on Figure 24a, after Stockham [1968b, 1972]. However, there are some unknowns in the parameters of our system. This implies that, in order to draw valid conclusions from the forthcoming experiments, we will have to select patterns such that the effect of the necessary extrapolations be minimized. The dubious regions are:

-The very low frequencies (less than 0.1 cycles/degree), for which it is difficult to experiment;

-The intermediate frequencies (from 6 to 10 cycles/degree), where brightness matching experiments become difficult and where neural inhibition and neural blur begin to interfere.

We also mentioned in the previous chapter, that data for high frequencies are based on threshold contrast experiments, and therefore can only be considered as first order approximation of frequency attenuation data. In view of these repeated cautions, our experiments would seem to be based on very precarious foundations. Let us repeat that, in fact, the band of frequencies for which we have been able to collect data *is* the fundamental band as far as brightness contrast effects are concerned.

We proceed to demonstrate this with very simple patterns, displaying the classical brightness contrast effects, and containing only a few frequency components. Therefore, these patterns will be smooth, without edges; as a result, we will avoid a suspected cause of non-linearity in the neural inhibition mechanism[2].

II/ Brightness contrast effects with smooth patterns

We are going to experiment with patterns containing only frequency

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[2] This question has been qualitatively studied, in the context of neural interaction mechanisms [Fry 1948; O'Brien 1958; Ratliff 1971]. Davidson indicated that edge effects were incompatible with the quantitative prediction of the homomorphic (i.e. log-linear model of retinal interaction [Davidson and Whiteside 1971]). We will address this subject further (Chapter V), restricting our ambition here to smooth patterns.

components within the band where our data has been collected. We will also ignore the effect of optical and neural blur, in the same manner and for same reasons as in the experiment described in Chapter IV. Therefore, in this particular case, the model is simplified to a logarithmic transformation followed by a linear inhibition system, for which all the transfer functions of Figure 10c are equivalent.

The following experiments have not been conducted on several subjects, and the comments presented here rely on the subjective judgement of the author as the unique observer. The psychophysiologist would probably desire more careful experimental procedures, but, as we have indicated, our point of view is of a computer scientist, and these subjective judgements will suffice to point out new interesting results.

A) Brightness contrast

In the context of our simplified model (i.e. log transformation followed by high pass linear system of neural interaction, Figure 24b), it is convenient to consider patterns having an intensity distribution of the form

$$I(x) = \exp(k\, D(x)),$$

where the density distribution[3] $D(x)$ is a linear combination of a few sinusoidal terms. By virtue of the simplified model, $D(x)$ is considered to be also the retinal brightness, i.e. the logarithm of the retinal intensity (Figure 6). Then $D(x)$ is the input to the linear system of

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[3] We use here the photographic terminology, which defines the density as the logarithm of the intensity [Stockham 1972]. Our terminology for *brightness* at different stages of the visual model (Figure 6), is defined in Chapter II.

neural interaction, and the corresponding output B(x) is the subjective brightness. Since, throughout this section, D(x) contains only a few frequency components, the computation of B(x) involves only simple mathematical considerations.

Our experiment indicated that for the stimulus pattern with density

$$D(x) = \sin(2 \pi f x) + 2 \sin(6 \pi f x).$$

the brightness contrast was the result of the relative amplification by a factor of about 2 of the frequencies f and 3f. In other words, the subjective brightness is predicted to be approximately

$$B(x) = \sin(2 \pi f x) + 2 \sin(6 \pi f x).$$

This is summarized on Figure 14a and 14b.

If we consider now the stimulus pattern with density

$$D(x) = \sin(2 \pi f x) + 0.8 \sin(10 \pi f x),$$

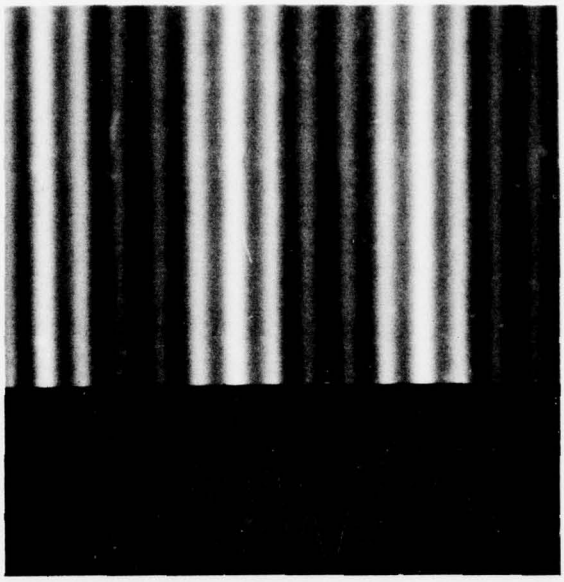
the approximate power law for frequency modulation would predict a relative amplification by a factor of 2.75 of the frequencies f and 5f, i.e. a predicted subjective brightness

$$B(x) = \sin(2 \pi f x) + 2.2 \sin(10 \pi f x),$$

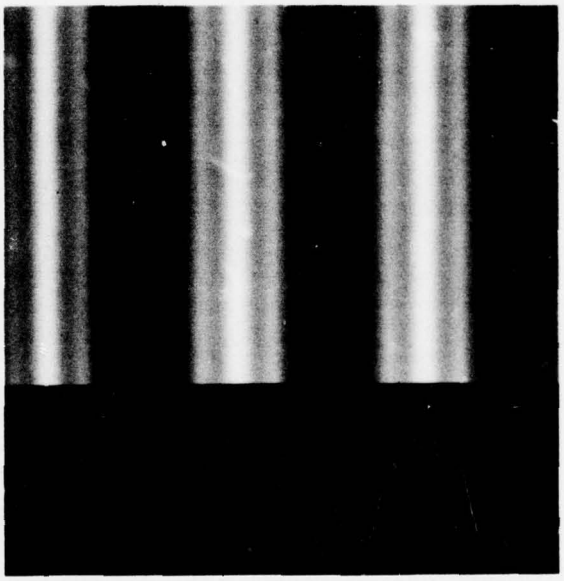
as illustrated in Figure 14c and 14d. The brightness contrast effect is visible on Figure 15a (f = 0.37 cycles/degree) and 16a (f = 0.123 cycles/degree). In order to test this prediction, the compensated patterns corresponding to the density distribution

$$D'(x) = \sin(2 \pi f x) + 0.8/2.75 \sin(10 \pi f x),$$

are presented in Figure 15b and 16b respectively. For the author, as one observer, the cancellation does not seem exactly sufficient, principally in Figure 15b; but for the same observer, it is also the

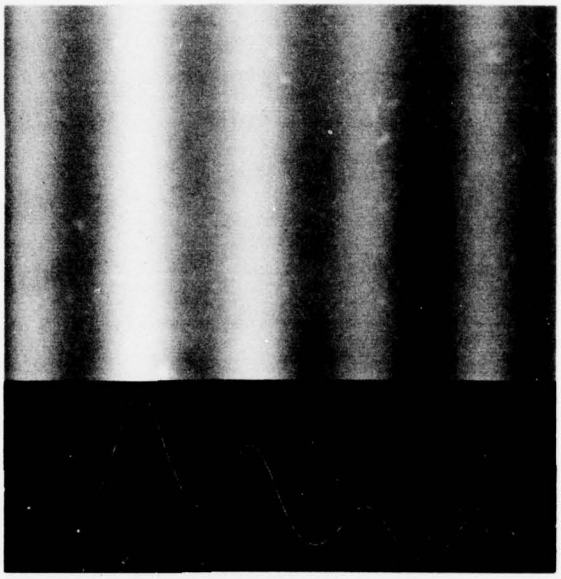


(a)

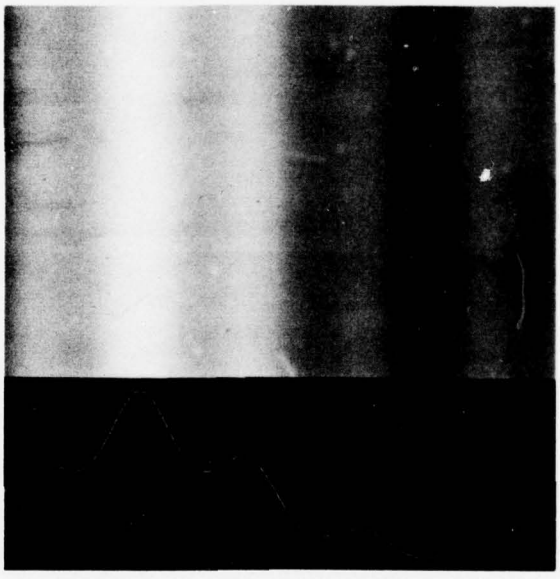


(b)

Fig. 15 - Test pattern corresponding to the density function of Fig. 14c, for a background frequency f of about .37 cycles/degree: original (a) and compensated (b).



(a)



(b)

Fig. 16 - Test pattern corresponding to the density function of Fig. 14c, for a background frequency f of about 0.12 cycles/degree: original (a) and compensated (b).

case for Figure 10a, presumably compensated according to the average of 10 observers. This might probably be a matter of individual differences, and a more carefully study would require that the different stimulus patterns be tested in the same conditions by the same observers. We will leave this for further studies, and we will retain the first order conclusion that, so far, results are roughly in agreement with the linearity hypothesis.

B) Mach bands

Mach bands can be obtained on smooth patterns with a proper change in spatial inflection of the intensity. Such is the case for the density function:

$$D(x) = \sin(2 \pi f x) \cdot (1 - \cos(4 \pi f x)),$$

where an all-positive sinusoide of frequency 2f is modulated by a sinusoide of frequency f (Figure 17a). As linear analysis will show easily, the resulting product actually contains only the frequency terms f and 3f. Accordingly, the different amplification of these two components by the neural system results in Mach bands (Figure 17b), which are readily observable on Figure 18a, although the effect is weaker than for usual Mach bands (Figure 1). This effect is analogous to the "brightness paradox" studied by Bergstrom [1967, 1970], and is exactly a Mach band effect, that is to say resulting from intensity pattern curvature rather than background induction (superficial distinction, however). Inverse modulation blurs out the bands, as can be seen on Figure 18b, thereby cancelling completely the brightness phenomenon.

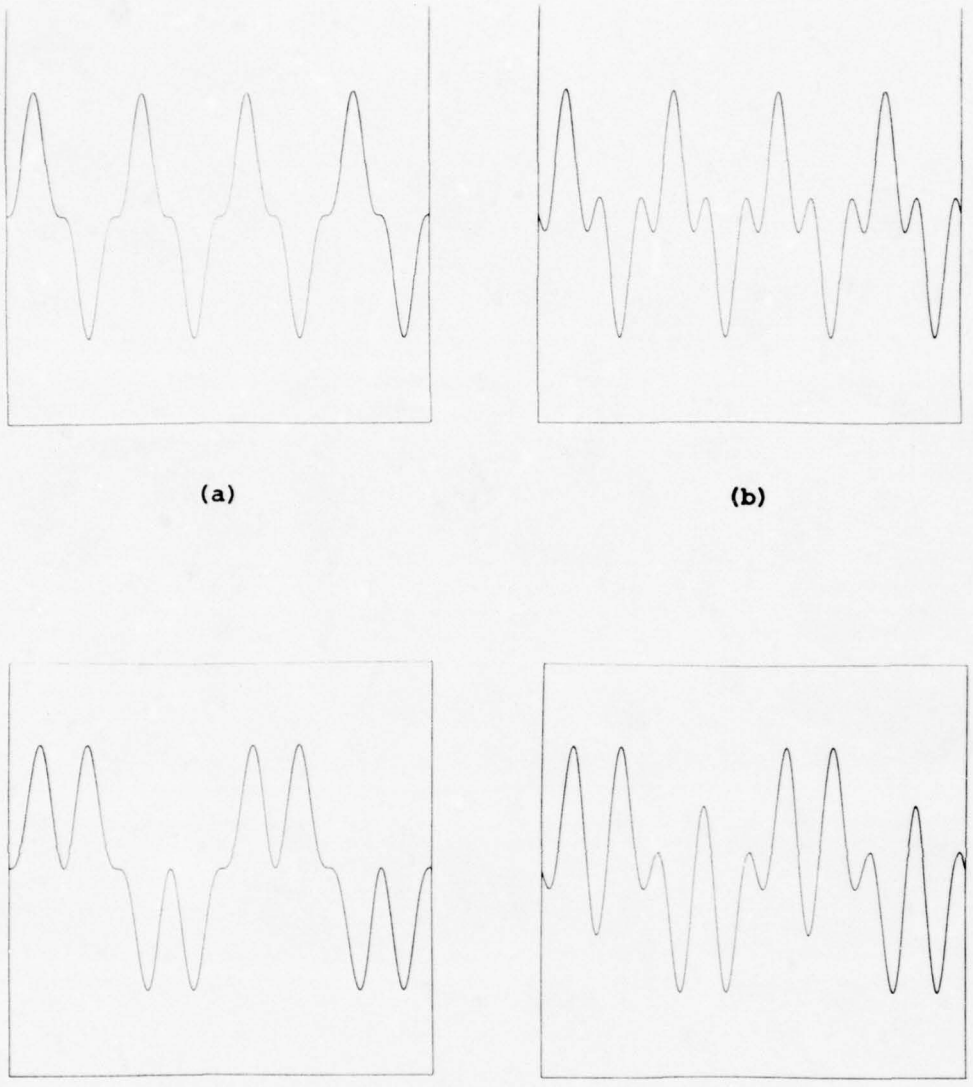
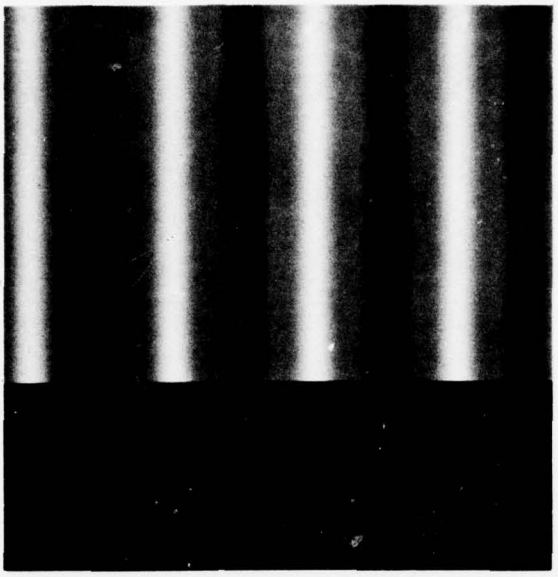


Fig. 17 - Stimulus density (left) and corresponding predicted subjective brightness (right):

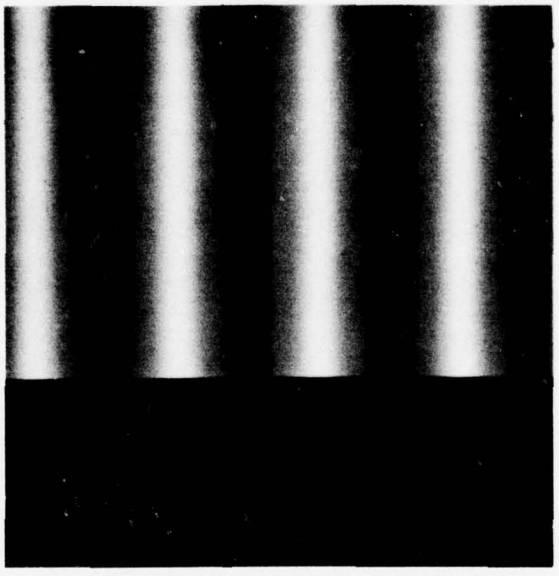
(a) $\sin(2\pi f x) \cdot (1 - \cos(4\pi f x))$;

(c) $\sin(2\pi f x) \cdot (1 - \cos(8\pi f x))$.

Scales of density and brightness are arbitrary. Notice the predicted Mach band and brightness contrast effects.

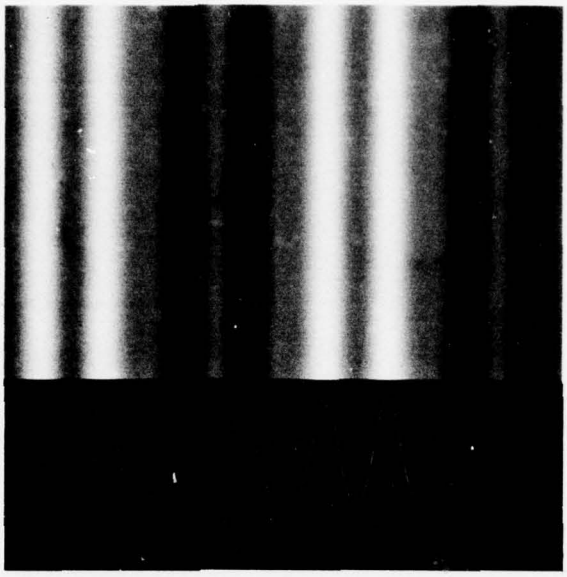


(a)

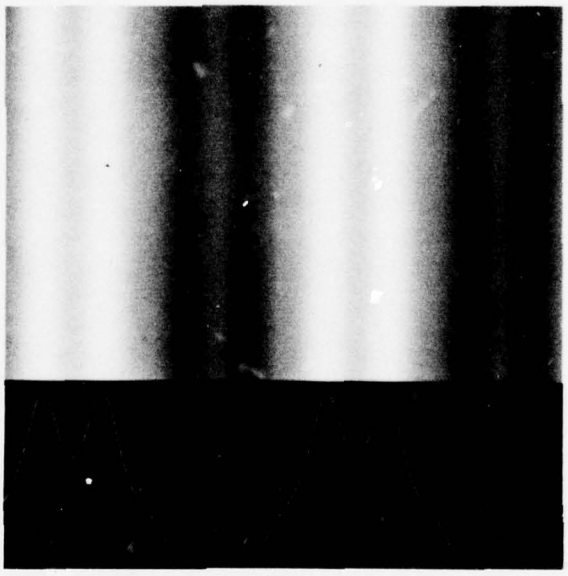


(b)

Fig. 18 - Test pattern corresponding to the density function of Fig. 17a, for a background frequency f of about 0.50 cycles/degree: original (a) and compensated (b).



(a)



(b)

Fig. 19 - Test pattern corresponding to the density function of Fig. 17c, for a background frequency f of about 0.25 cycles/degree: original (a) and compensated (b).

A slightly different density function (Figure 17c),

$$D(x) = \sin(2 \pi f x) \cdot (1 - \cos(3 \pi f x)),$$

containing the frequency terms f, 3f and 5f, exhibits at the same time Mach bands and simultaneous brightness contrast, as observable on Figure 19a, and as predicted on Figure 17d. The effect also appears to be approximately cancelled by inverse modulation (Figure 19b).

One might notice that the Mach band effect is more strongly compensated than simultaneous brightness contrast, but this could probably result from the fact that, for smooth patterns, the Mach band effect is weaker than the brightness contrast effect. So far, the linearity hypothesis appears to be generally well verified, according to the prediction of the model, and as confirmed by cancellation experiments.

C) Hermann grid effect

Let us now turn to the two-dimensional grid effect, noticing that similar phenomena can be obtained from smooth patterns. An interesting mathematical property of the grid patterns displaying this classical "illusion" (Figure 4), is that the intensity function is separable, that is to say is the product of two independant functions of the spatial variables x and y:

$$I(x,y) = I_1(x) \cdot I_2(y).$$

Consider now the following density function:

$$D(x,y) = (1 + \cos(2 \pi f x)) \cdot (1 + \cos(2 \pi f y)),$$

which mathematically describes a surface similar to an infinite corrugated egg crate (Figure 21a). The corresponding stimulus pattern

is illustrated in Figure 22a, for the negated density function -D(x.y). Sign reversal of the density function, as far as the mathematics of our model are concerned, merely results in the physical inversion of black and whites, since the neural system mapping D(x) into B(x) is presumably linear. A bright grid is illustrated in Figure 22a rather than the symmetric dark grid[4], because the gloss of photographic illustrations, makes the effect of brightening at the intersections of the dark grid, less observable than the converse effect for a bright grid.

The darkening at the intersections of the bright grid, observable on Figure 22a, is predicted by the linear model of inhibition (Figure 21b). These results are easy to compute, as a consequence of the separability of the density functions, since the Fourier transform of a separable function is also separable. Figure 20 illustrates this property, and shows the frequency components of the density functions:

$$(1 + a \cos(2 \pi f x)) \cdot (1 + a \cos(2 \pi f y)),$$

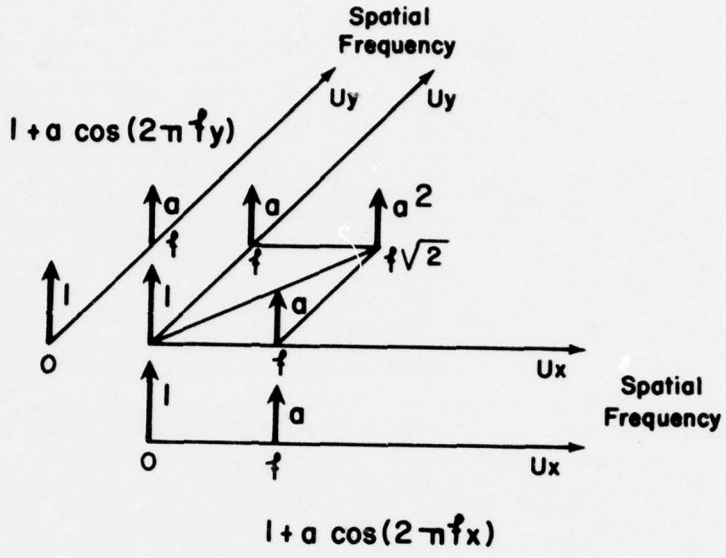
and

$$(1 + a \cos(2 \pi f1 x)) \cdot (1 + b \cos(2 \pi f2 y)).$$

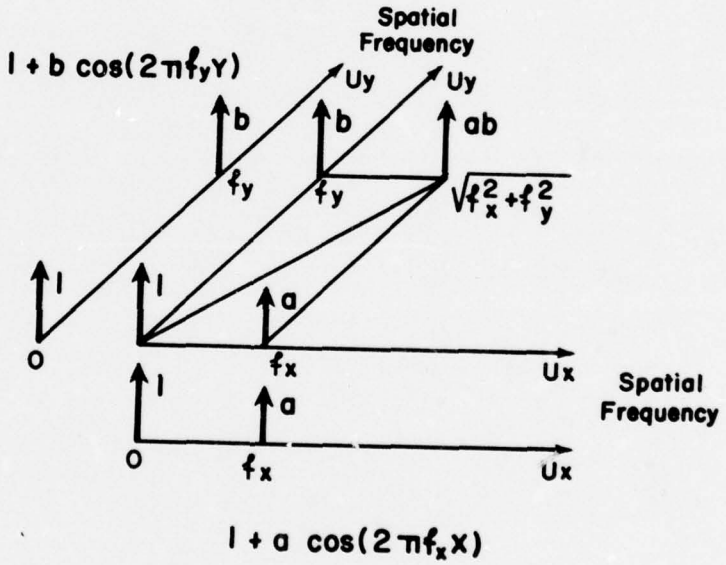
Let us consider, first, the case where the separable functions in x and y have same frequency f. Then, the radial frequency terms for the two-dimensional grid are f and $\sqrt{2}$ f, as shown on Figure 20a. One will

---

[4] The patterns previously presented have totally symmetric density functions, and therefore, there should be, in theory, a complete symmetry between bright and dark regions. However, the reader will observe that it is not exactly the case: bright bands are generally more narrow than "symmetric" dark counterparts. This might be interpreted as an indication that the notion of logarithmic transformation at the level of retinal receptors is approximate, as we mentioned in Chapter II.

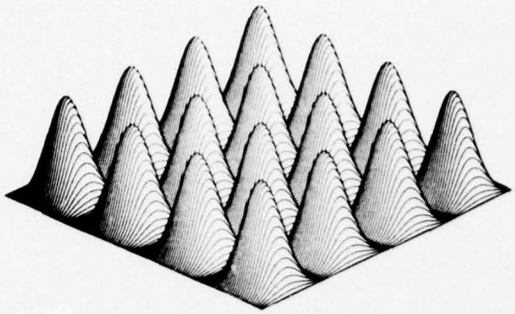


(a)

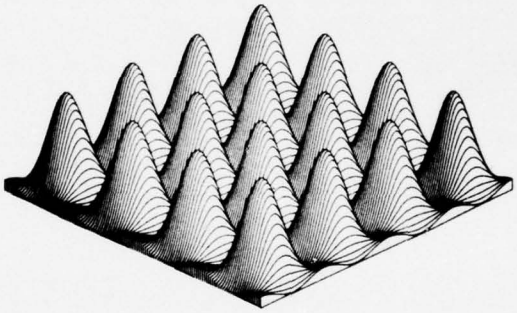


(b)

Fig. 20 - Separable frequency spectra of the separable functions:
(a) $(1 + a \cos(2\pi f x)) \cdot (1 + a \cos(2\pi f y))$;
(b) $(1 + a \cos(2\pi f_x x)) \cdot (1 + b \cos(2\pi f_y y))$.



(a)



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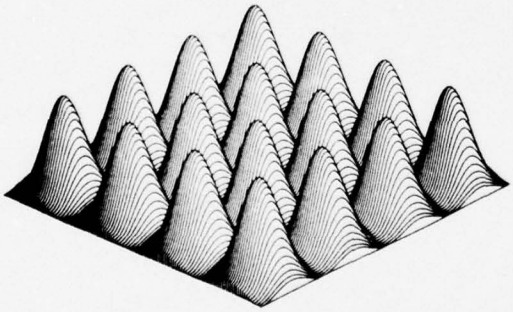


(c)

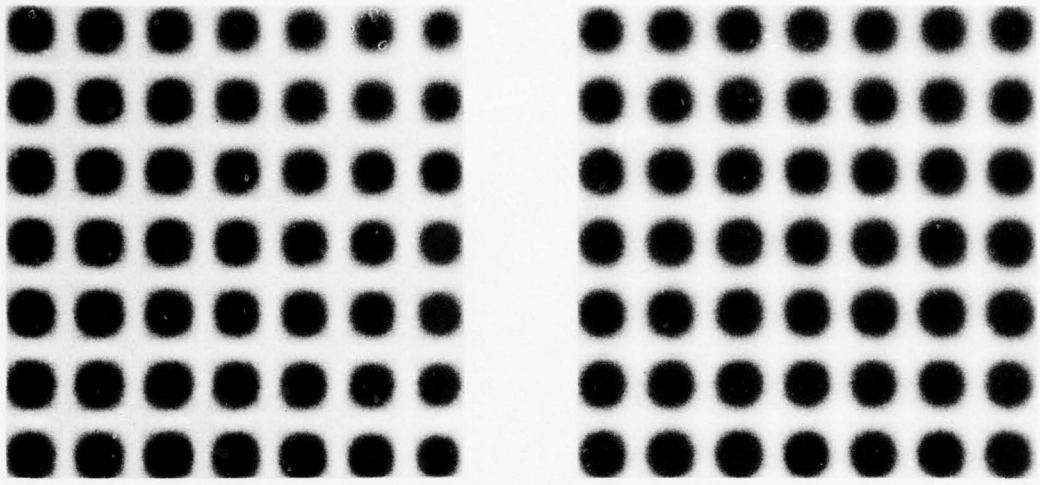
Fig. 21 - Plot on an arbitrary scale of:

(a) Separable stimulus density:

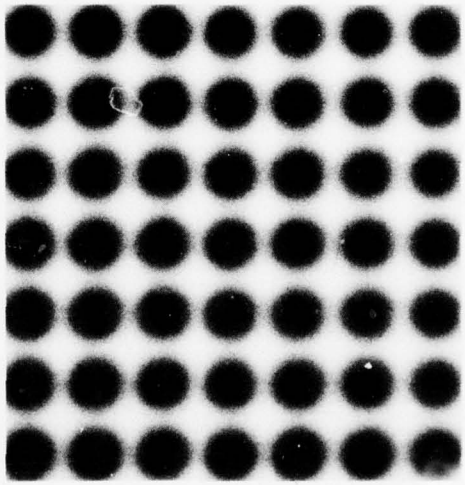
$$(1 + \cos(2\pi\ f\ x)) \cdot (1 + \cos(2\pi\ f\ y))\ ;$$

(b) Predicted subjective brightness (notice brightness contrast effect at the intersections of the grid);

(c) Compensated stimulus density.



(a) (b)



(c)

Fig. 22 - Test pattern corresponding to the separable density function of Fig. 21: original (a), compensated (b) and over-compensated (c). Notice that the grid intersections are darker in (a), and brighter in (c).

note on this diagram that the ration of this two frequency components is equal to a. Therefore the relative modulation of the frequencies f and $\sqrt{2}$ f is directly related to the coefficient a. We will use this property to generate in a simple way a separable smooth grid for any ratio of the frequencies f and $\sqrt{2}$ f.

Our experimental power law, would predict a relative amplification of these two frequencies by a factor of 1.25 (Figure 10b). In other words, to the density function

$$D(x,y) = (1 + \cos(2 \pi f x)) \cdot (1 + \cos(2 \pi f y)),$$

plotted on Figure 21a and displayed on Figure 22a, would correspond the subjective brightness

$$B(x,y) = (1 + 1.25 \cos(2 \pi f x)) \cdot (1 + 1.25 \cos(2 \pi f y)),$$

plotted on Figure 21b. To obtain these expressions, we have ignored DC components in D(x,y) and B(x,y). These effects are qualitatively verified on Figure 22a. In order to confirm quantitatively, we apply the usual compensation procedure. If the relative amplification factor is 1.25, as implied by the experimental data, then the density distribution of Figure 21d:

$$D'(x,y) = (1 + 0.8 \cos(2 \pi f x)) \cdot (1 + 0.8 \cos(2 \pi f y)),$$

should exhibit cancellation of the grid effect. The compensation appears to be quite effective (Figure 22b). An excessive compensation is obtained on Figure 21d, for a = 0.7, as the intersections of the grid now seem much brighter. These findings indicate that for the two dimensional radial frequencies f and $\sqrt{2}$ f, the relative amplification is of the order of 1.25 = 1/0.8, which confirms our prior results.

It is possible to utilize a similar stimulus pattern in order to test our experimental data, which showed the relative amplification of frequencies f and 3f. Figure 20b illustrates the generalization to a separable grid such that the two independant functions of the spatial variables x and y have different frequencies and different amplitudes. Then, it is easy to observe that, in order to test radial frequencies f and 3f, it suffices to generate the pattern with density function:

$$D(x,y) = (1 + \cos(2 \pi f x)) \cdot (1 + \cos(4 \sqrt{2} \pi f y)),$$

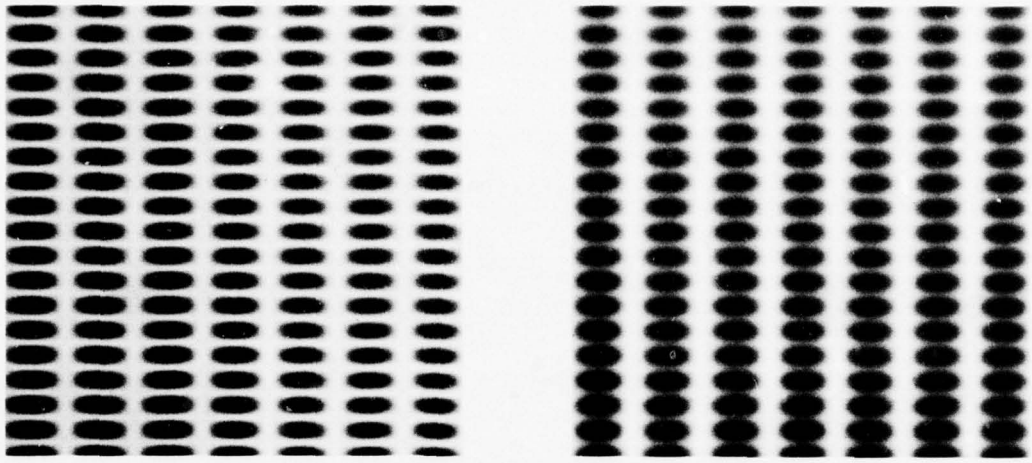
since the frequency terms involved will then be f, $2\sqrt{2}$ f and 3f. Such a pattern is illustrated in Figure 23, and shows the expected grid effect. In order to compensate for the relative amplification of these three frequency components (a factor of 2 for 3f and f; a negligible factor of 1.05 for 3f and $2\sqrt{2}$ f), one displays the density pattern:

$$D'(x,y) = (1 + 0.995 \cos(2 \pi f x)) \cdot (1 + 0.5 \cos(4 \sqrt{2} \pi f y)),$$

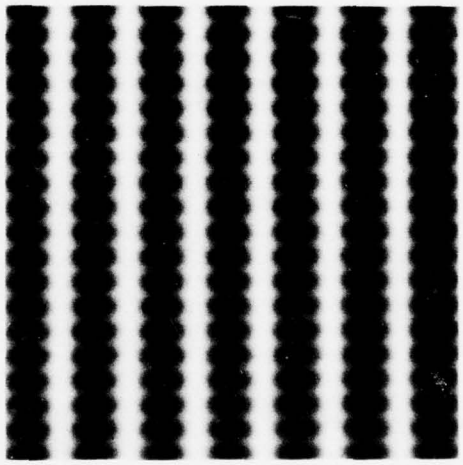
as on Figure 23b. Over-compensation is obtained on Figure 23c. Here again, the predicted cancellation is verified.

D) Conclusions

The various brightness contrast effects obtainable with smooth patterns seem to be in good agreement with the experimental power law, as results from cancellation experiments. We noticed, however, that Mach band effects and grid effects were more strongly cancelled than simultaneous brightness contrast effects. We mentioned that this could be attributed to the fact that the former are weaker effects than the latter. Cancellation experiments, which are not illustrated here, showed better results for the compensation of simultaneous brightness



(a) (b)



(c)

Fig. 23 - Test pattern corresponding to the separable density functions:

$(1 + \cos(2\pi\ f\ x)) \cdot (1 + \cos(2\pi\ 2\sqrt{2}\ f\ y))$.

Original (a), compensated (b) and over-compensated (c).

contrast with a slightly steeper attenuation of low frequencies (i.e. a slope 10% steeper on a log-log scale). Such a value is very close to Davidson's data. Also, we noticed that it is for the patterns of Figure 14c, 15a and 16a, that the brightness compensation is the least successful; this pattern contains frequencies f and 5f. Referring to the plot of Figure 10b, one will realize that, if we assume a power law modulation at low frequencies (as Davidson's data and ours suggest), then experimental error should be less when comparing the modulation of frequencies f and 5f, than when comparing that of frequencies f and 3f. The reason is that errors in estimating the slope of a straight line from pairs of points, diminish when the points are further apart. Then, in brightness matching experiments with the family of patterns containing the frequencies f and n f (where n is odd), one should utilize the largest possible values of n. However, one is limited practically, because patterns for large n, containing $(n-1)/2$ bands per half-period of the background, become confusing for the observer. Also, these remarks are valid only if a power law, i.e. a linear relation on a log-log scale, is sufficiently well verified.

However, we will estimate that our results are consistent enough, to consider that the linearity hypothesis is satisfactorily verified in a limited band of frequencies (approximately, from 0.1 to 4 cycles/degree). Obviously, there is room for more precise and more extensive experimentation, but they were not pursued in the frame of the present study. The concluding chapter indicates the type of equipment which would be desirable for more accurate and more efficient experiments, in the same line of research.

Considering our results conclusive enough for smooth patterns, we will turn now to more complex stimuli. In particular, we are now interested in patterns having more intricate frequency contents, and also edges (since they are fundamental features of the visual environment).

III/ Pattern with edges

We encounter now the problem that we have addressed in some detail before; namely that the only data available for high frequency attenuation is related to threshold of contrast. Thus, in order to minimize errors due to these extrapolated data, we will consider a pattern such that the bulk of its spectral energy is located within the band of frequencies that we have well explored. In fact, any pattern exhibiting strong brightness contrast will have its energy concentrated at low frequencies. But we can be more restrictive, and, for instance, generate a pattern with edges closely related to one of our smooth patterns. As we already noticed, the pattern corresponding to the density function

$$D(x) = \sin(2 \pi f x) \cdot (1 - \cos(8 \pi f x)),$$

illustrated on Figure 17c and 19a, has only three frequency components (f, 3f, 5f); its magnitude Fourier spectrum is plotted on Figure 25b[5].

---

[5] The spectra plotted here and thereafter are digital discrete spectra, since we deal with digital discrete pictures. As a result of the resolution of the pictures processed here, the Fourier spectrum is computed only for 1024 equidistant frequencies, from DC to the Nyquist frequency. Because the plotting program extrapolates linearly between data points, isolated frequency components are actually plotted as triangular spikes.

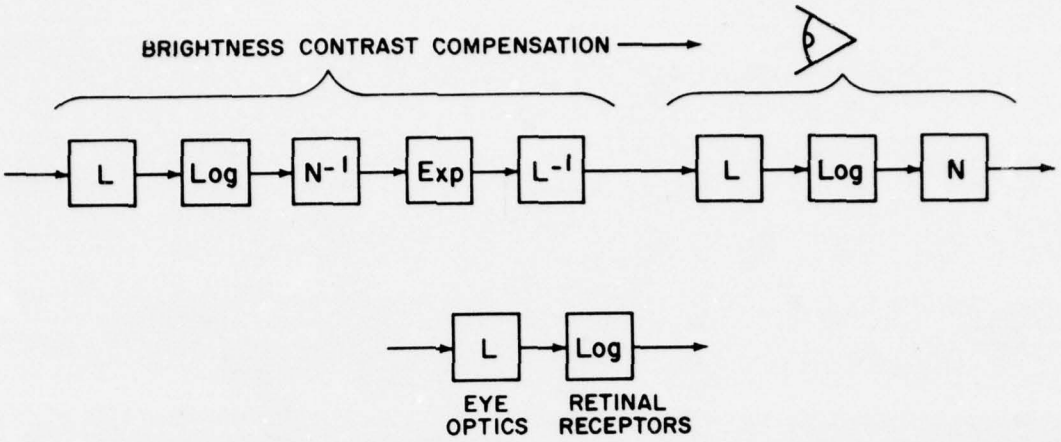
If we modulate the background sinusoide, of frequency f, by an all-positive square wave of pseudo frequency 4f, we obtain the density pattern plotted on Figure 29a, and displayed on Figure 25a. This stimulus exhibits sharp edges and strong simultaneous contrast, which is induced by the sinusoidal background at frequency 0.24 cycles/degree. The reader will verify on Figure 25c that the main components of this patterns are still the frequencies f, 3f and 5f, as in the corresponding smooth pattern.

For such a pattern, we need to consider the full linear-homomorphic model of Figure 6a, extrapolating, as we said, the high frequency terms of the transfer functions. Namely, we utilize the data of Figure 8a [Campbell and Green 1965] for the linear sub-system L (eye optics); we combine the data of Figure 8c [Campbell and Green 1965] and Figure 13a [our data] for the linear sub-system N (neural interaction). The resulting transfer function and line spread function are plotted on Figure 13c and d.

Then, compensating according to the block diagram of Figure 24a, we obtain the pattern displayed on Figure 26a, and the corresponding Fourier spectrum (Figure 26b). Examination of the compensated pattern suggests the following remarks:

- the compensated pattern is strongly blurred;
- some local edge effect has been introduced;
- the brightness contrast cancellation is not quite successful.

We will first take care of the second item, by noticing that the somewhat "artificial" border effect created by the compensation process



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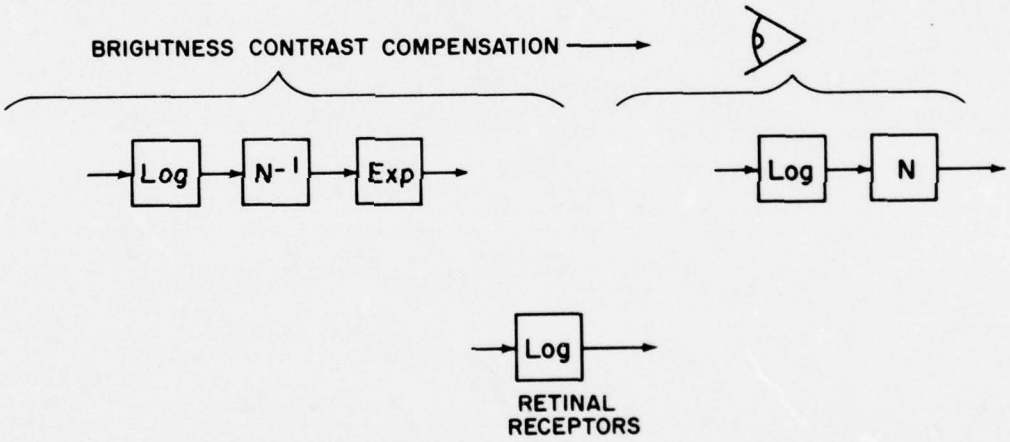
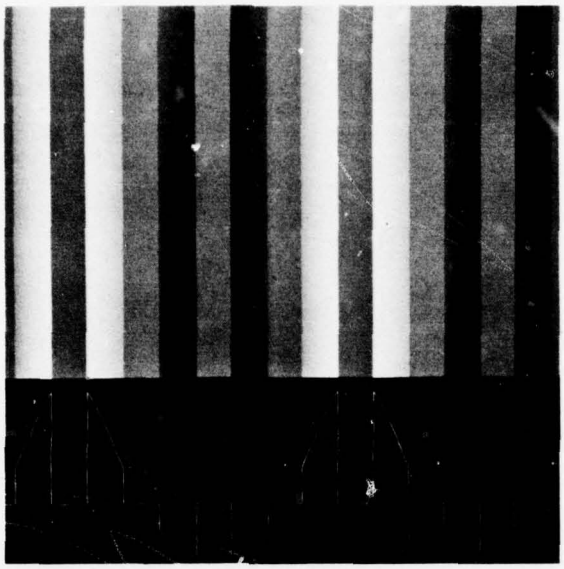
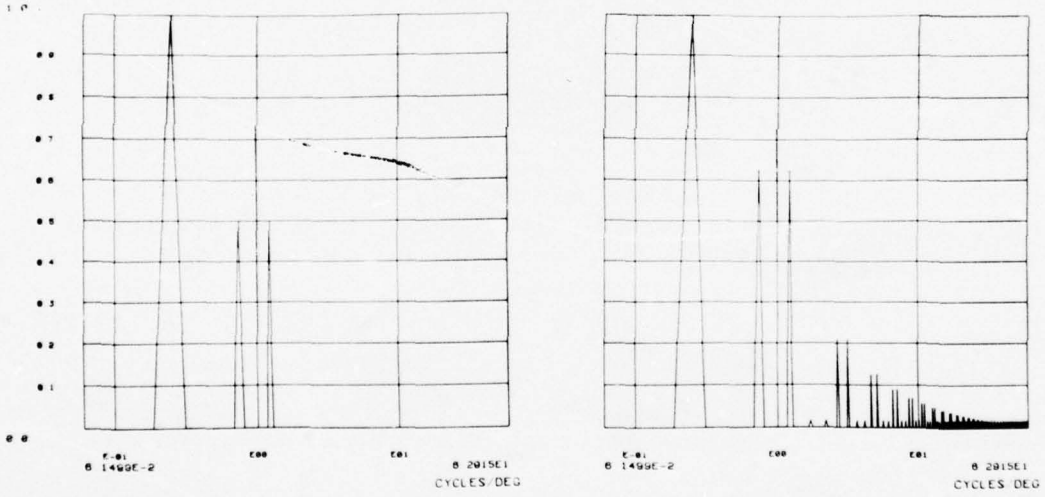


(b)

Fig. 24 - Scheme for brightness contrast compensation, according to the homomorphic model for neural interaction (Fig. 6): complete model (a); simplified model (b), ignoring optical and neural blurr.

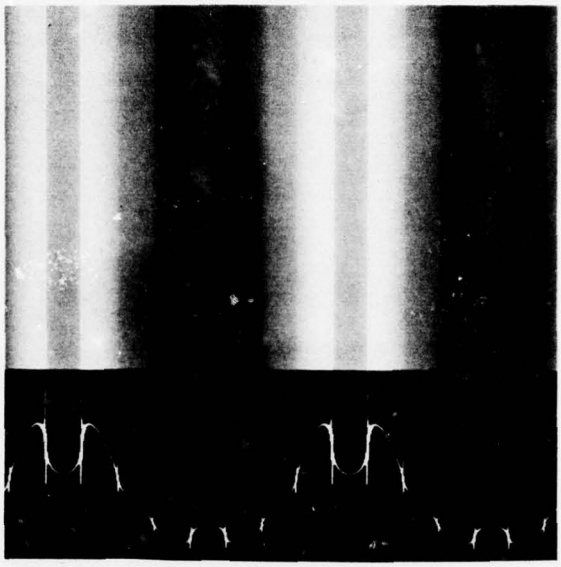


(a)

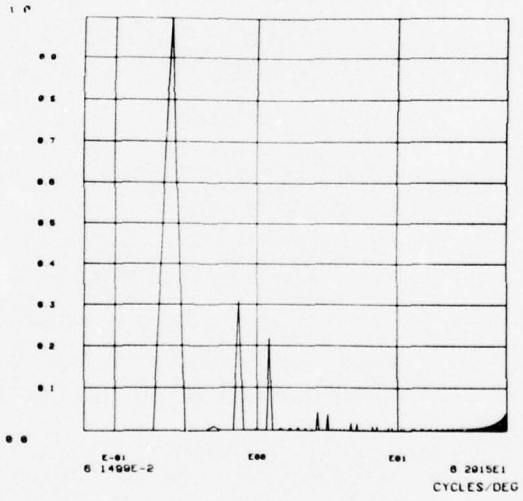


(b) (c)

Fig. 25 - (a) Test pattern with edges, derived from the smooth pattern of Fig. 17c and 19a. Compare its Fourier magnitude spectrum (c) with the spectrum of the smooth pattern (b).



(a)



(b)

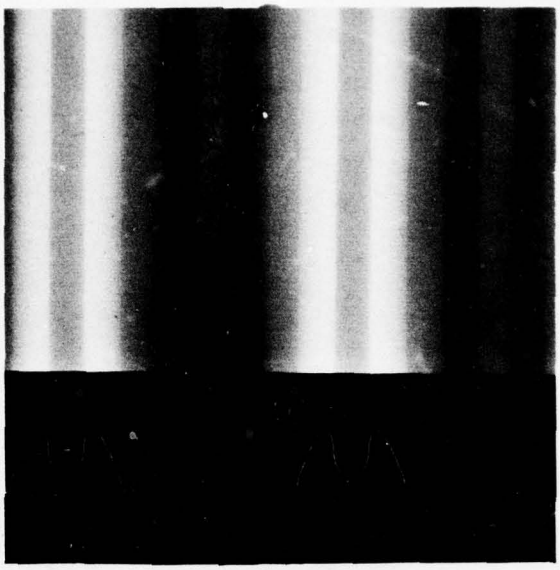
Fig. 26 - (a) Test pattern of Fig. 25a, compensated for brightness contrast by the method of Fig. 24a.

(b) Corresponding Fourier magnitude spectrum.

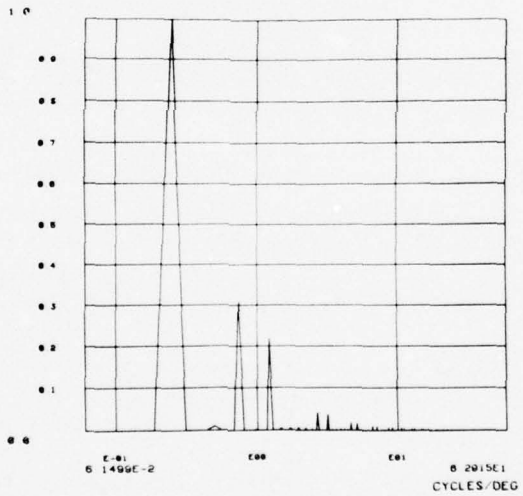
is exclusively the result of high frequency amplification in the inverse sub-systems L^{-1} and N^{-1}. This can be understood from theoretical considerations, since the inverses of the transfer functions of Figure 8 will result in amplification of the high frequencies. We can verify this experimentally; if the same initial pattern is compensated by the simplified model of Figure 24b (ignoring the optical blur-system L-, and ignoring the neural blur-high frequencies of system N-), we obtain the result of Figure 27a and b. The patterns compensated by the complete system (Figure 26a) and by the simplified system (Figure 27a) differ only by the fringing edge effect. But, clearly, these artificial edges do not compensate for the blur, which is a consequence of brightness contrast compensation only. In other words, helped by the particular frequency content of the original pattern, we can separate the result of the inverse system in two separate effects:

- brightness contrast compensation, is mainly the result of modulating the three low frequency terms and causes concurrently a strong blur;
- compensation of optical and neural blur, resulting in the amplification of the high frequency terms, causes a very local and "artificial-looking" edge effect.

Why brightness contrast compensation creates such a strong blur can be understood when realizing how exaggerately sharp is the subjective brightness predicted by the model; Figure 29 (a,b,e,f) illustrates this point. Then, the unsuccessful cancellation of the original pattern can be linked to the fact that the prediction of the model is not in agreement with the visual experience. But the discrepancies between



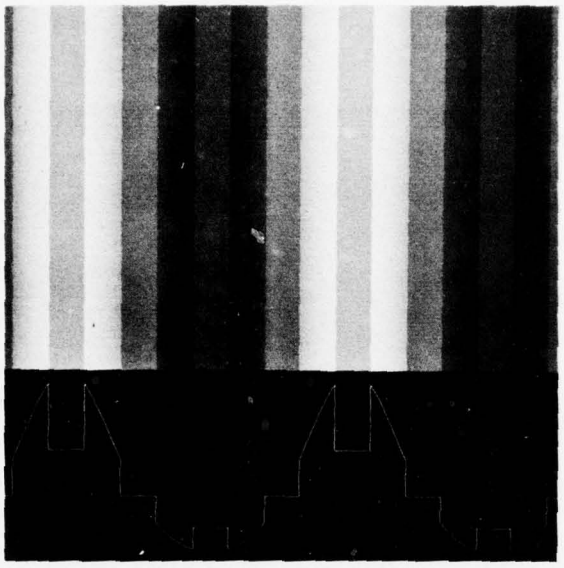
(a)



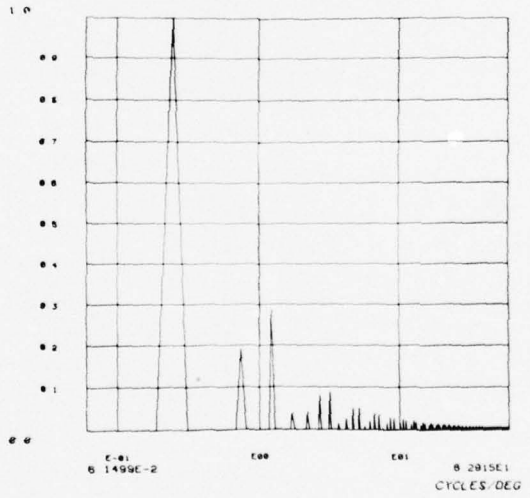
(b)

Fig. 27 - (a) Test pattern of Fig. 25a, compensated for brightness contrast by the simplified method of Fig. 24b.

(b) Corresponding Fourier magnitude spectrum. Compare with Fig. 26.



(a)



(b)

Fig. 28 - (a) Test pattern of Fig. 25a, empirically compensated for brightness contrast.

(b) Corresponding Fourier magnitude spectrum. Compare with Fig. 27.

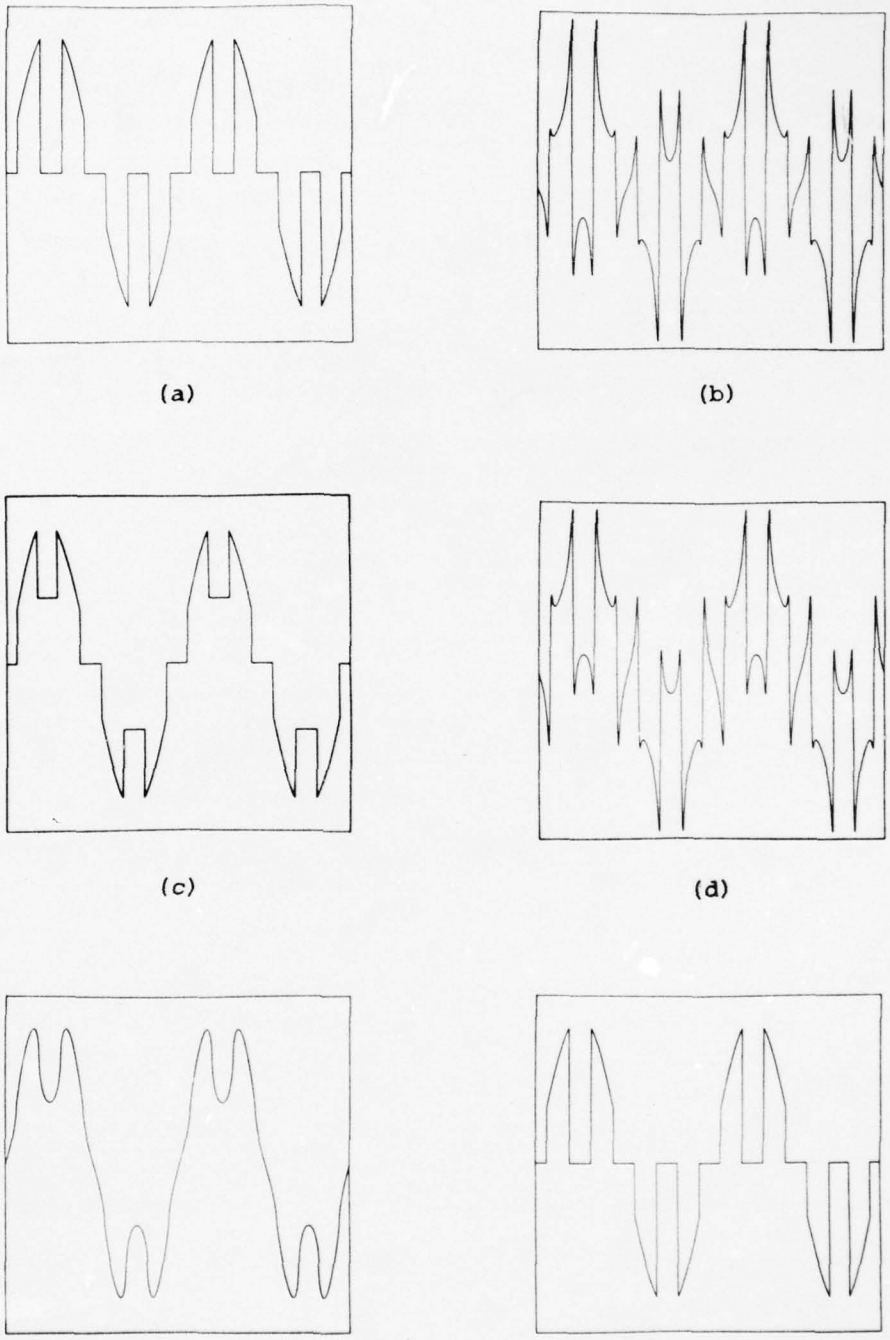


Fig. 29 - Stimulus density (left) and corresponding subjective brightness (right), predicted by the simplified model.

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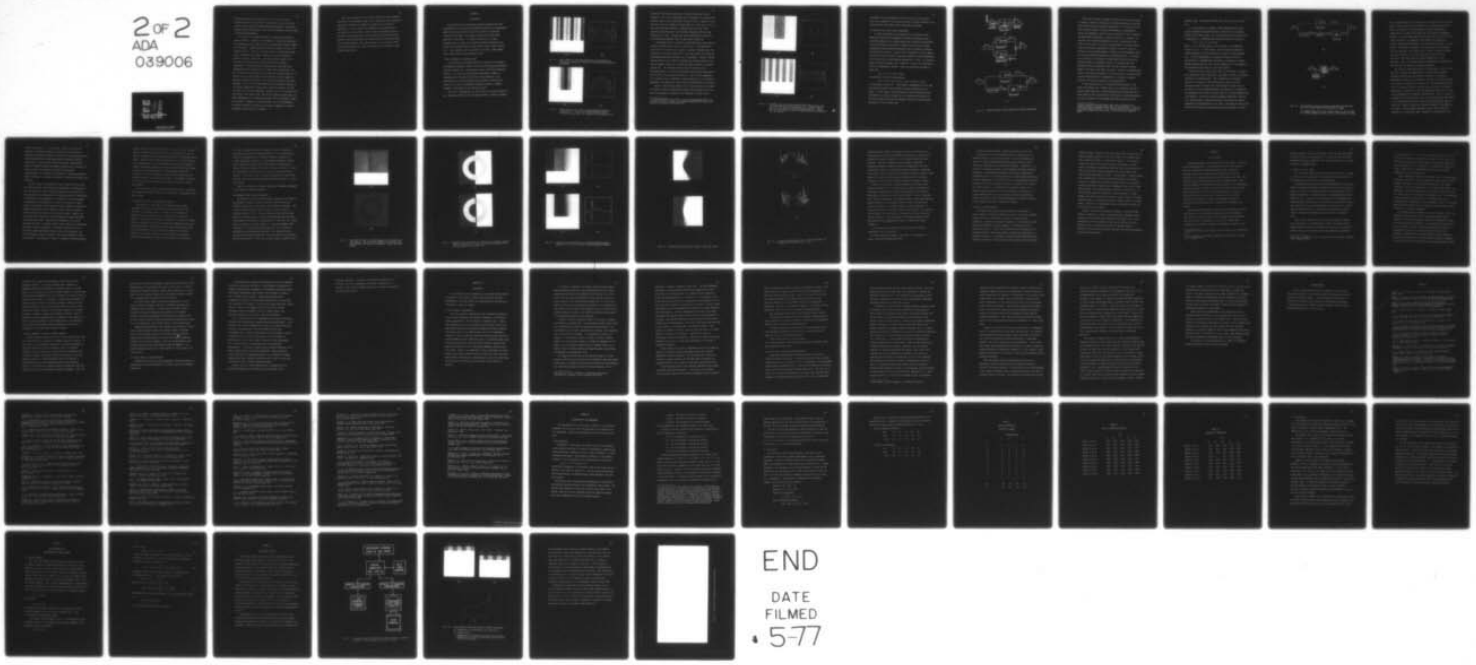

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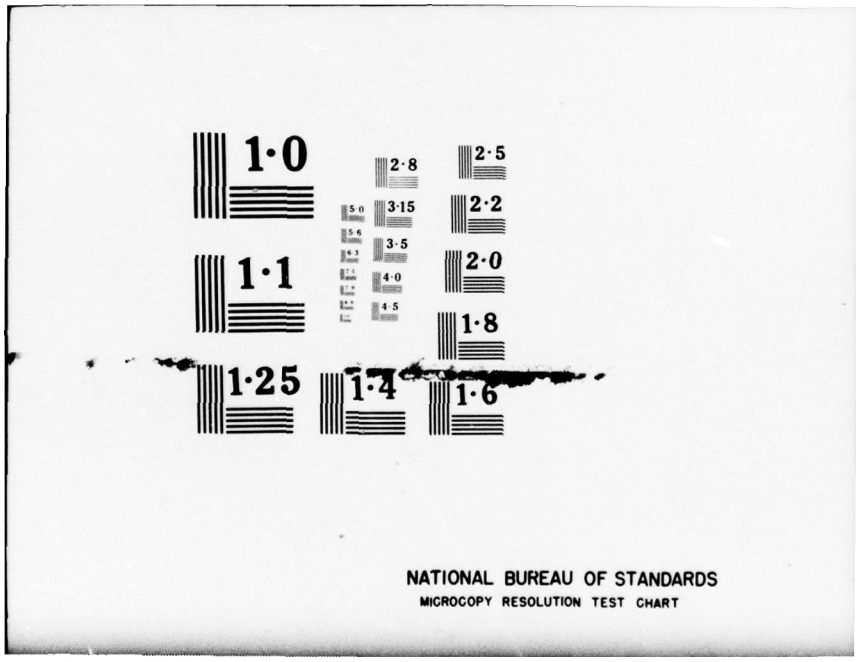
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prediction and reality could be twofold; on one hand, the edge crispening predicted by the model is unrealistic; on the other hand, ignoring for the moment the exaggerated edge sharpening, the question remains whether the magnitude of the over-all brightness contrast effect is predicted approximately.

Of course, it is somewhat difficult to dissociate subjectively these two effects. However, we can attempt to compare the magnitude of the brightness contrast effect for the two unprocessed patterns (smooth and with edges): examine Figures 19a and 25a, Figures 17c and d, and Figures 29a and b. Since it is actually easier to judge subjectively similar brightness than similar contrast, we can also compare the patterns of Figure 27a and 28a. The smooth pattern (Figure 27a) is the result of processing the original picture for brightness compensation. The sharp pattern (Figure 28a) has been empirically compensated from the same picture, to show similar compensation. One will verify that the modulation of the three low frequency terms (f, 3f and 5f) is of the same order--as can be verified on their respective spectrum on Figure 27b and 28b--, and that they differ mostly by their sharpness. One will notice furthermore that the brightness contrast is not exactly compensated in either one of these two patterns, but that the effect is of the same magnitude. Figure 29 (c to f) illustrates the predictions of the model, which, besides the crispening edge effect, are quite comparable and in reasonable agreement with the experience (Figure 27a and 28a). Therefore, it would seem that, at first approximation, for similar modulation of similar low frequency components, a similar brightness contrast effect is obtained.

Then, our conclusion will be that the modulation of low frequency predicted by the homomorphic model and the experimental data (Davidson's and ours) is generally in agreement throughout the cancellation experiments. In other words, the peripheral visual system, and in particular the neural network, seems to behave reasonably linearly in the band of low frequencies, as we verified by using smooth patterns. However, strong discrepancies are revealed when the pattern has edges, which suggest important local non-linearity in the vicinity of sharp intensity discontinuities. The next chapter will be devoted to these complex edges effects.

CHAPTER V

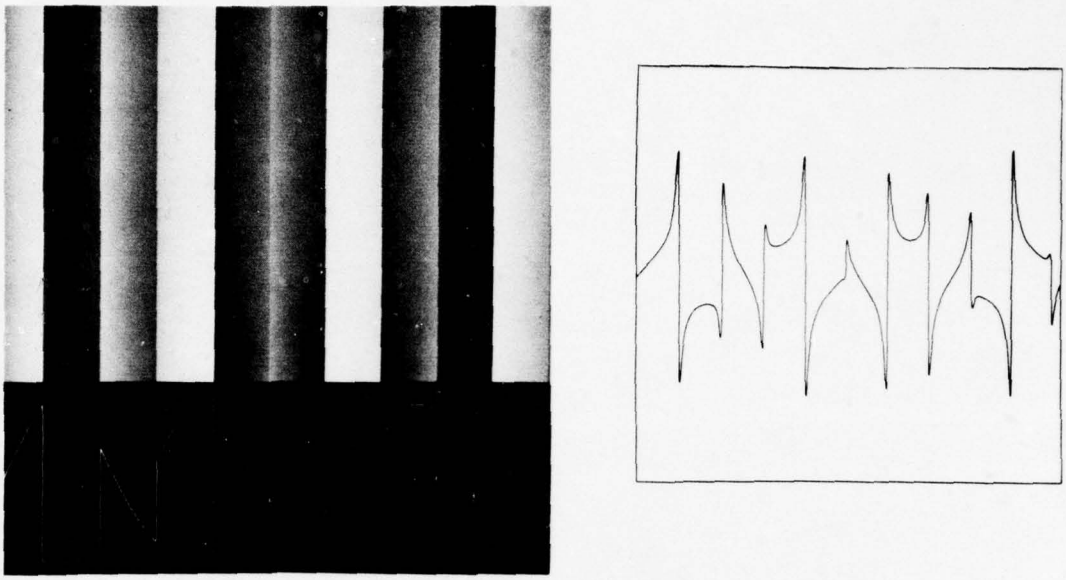
EDGE EFFECTS

The last section of the preceding chapter suggested that some discrepancies in the subjective brightness predicted by the homomorphic model, could be attributed to local non-linearity of inhibitory interaction near edges. The present chapter will address these questions in some detail. Of course, we mean here, by edges, discontinuities or steps of the intensity stimulus projected on the retina, and not some abstract concept such as lines, objects outline, etc... But, obviously, the two notions are related in some complex manner, about which we know certainly little.

I/ The perception of intensity steps

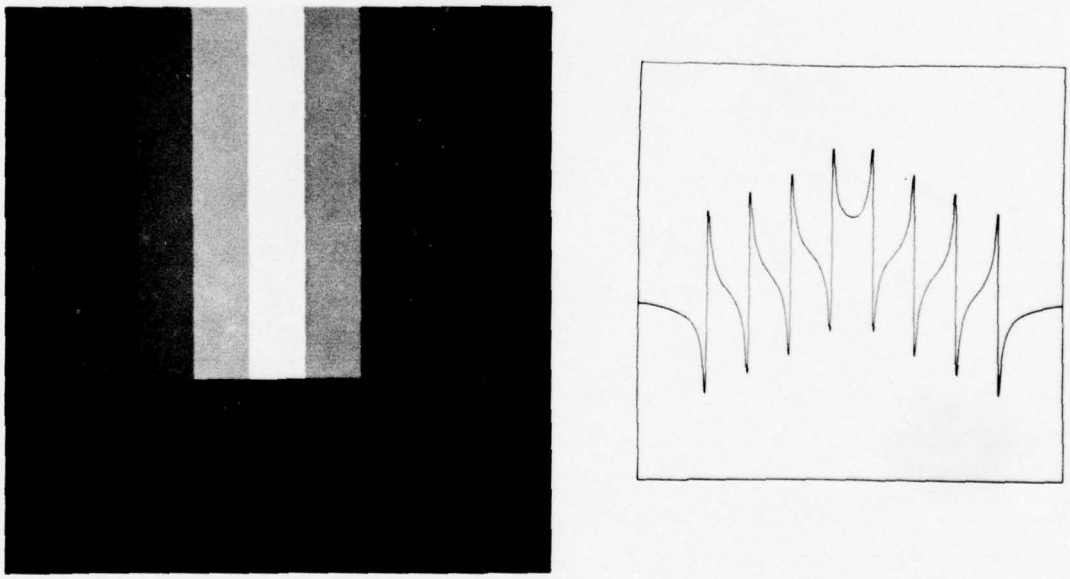
Let us return to the density pattern of Figure 29a, and compare its subjective appearance (Figure 25a) with the brightness predicted by the model (Figure 29b). Focusing particularly on the edge sharpening effects, it is clear that the predicted enhancement of edges is completely out of proportion with their actual appearance. The contradiction is even more apparent for the pattern illustrated on Figure 30, for which the predicted brightness presents a complete reversal of the curvate of the intensity profile.

But, it is a mathematical law, built into the linearity property of the inhibition model, that the crispening effect at points of extreme



(a) (b)

Fig. 30 - Test pattern (a) and predicted subjective brightness (b). Notice the inversion of curvature on the predicted subjective brightness.



(a) (b)

Fig. 31 - Linear density gray scale (a) and predicted subjective brightness (b). Notice the cusped brightness of the intermediate gray steps (a), similar to the prediction (b).

curvature in the intensity pattern, increases drastically with the curvature. As a result, Mach bands will be predicted at a maximum for a vertical step, as can be observed on Figure 37b and d. However, it has been well noted [as reviewed in Ratliff 1965, pp. 55-61] that, if it is true that Mach bands increase in amplitude when the width of the transition ramp diminishes, they altogether disappear when the ramp vanishes into a step. At most, some very narrow bands are perceived, but hardly distinguishable from the edge itself (Figure 38).

A brightness effect of different spatial magnitude is observable on multiple intensity steps, such as on Figure 31a[1]. The brightness of each step is not uniform, but inclined, in general conformity with the prediction of the model (Figure 31b). But, there again, the strong and wide bands predicted by the model are not observable. However, neither the Mach bands, nor the slope in brightness is perceived when only one step is present (Figure 32a), or when the steps alternate in direction (Figure 32c). The difference between these two effects can be experienced more dramatically by alternately covering and uncovering all but one step on the pattern of Figure 31a. That the local appearance of an intensity step be dependent on broad properties of the stimulus is perplexing, and suggests that some higher level mechanisms are involved.

However, before getting deeper into the issues involving the effect of contour on overall brightness perception, and ignoring for the time being the paradoxical differences just mentioned, we would like to

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[1] A standard pattern in the field of technical photography where it is used for calibration purposes; the amateur snapshooter will find it on the boxes of Polaroid films (type 107)...

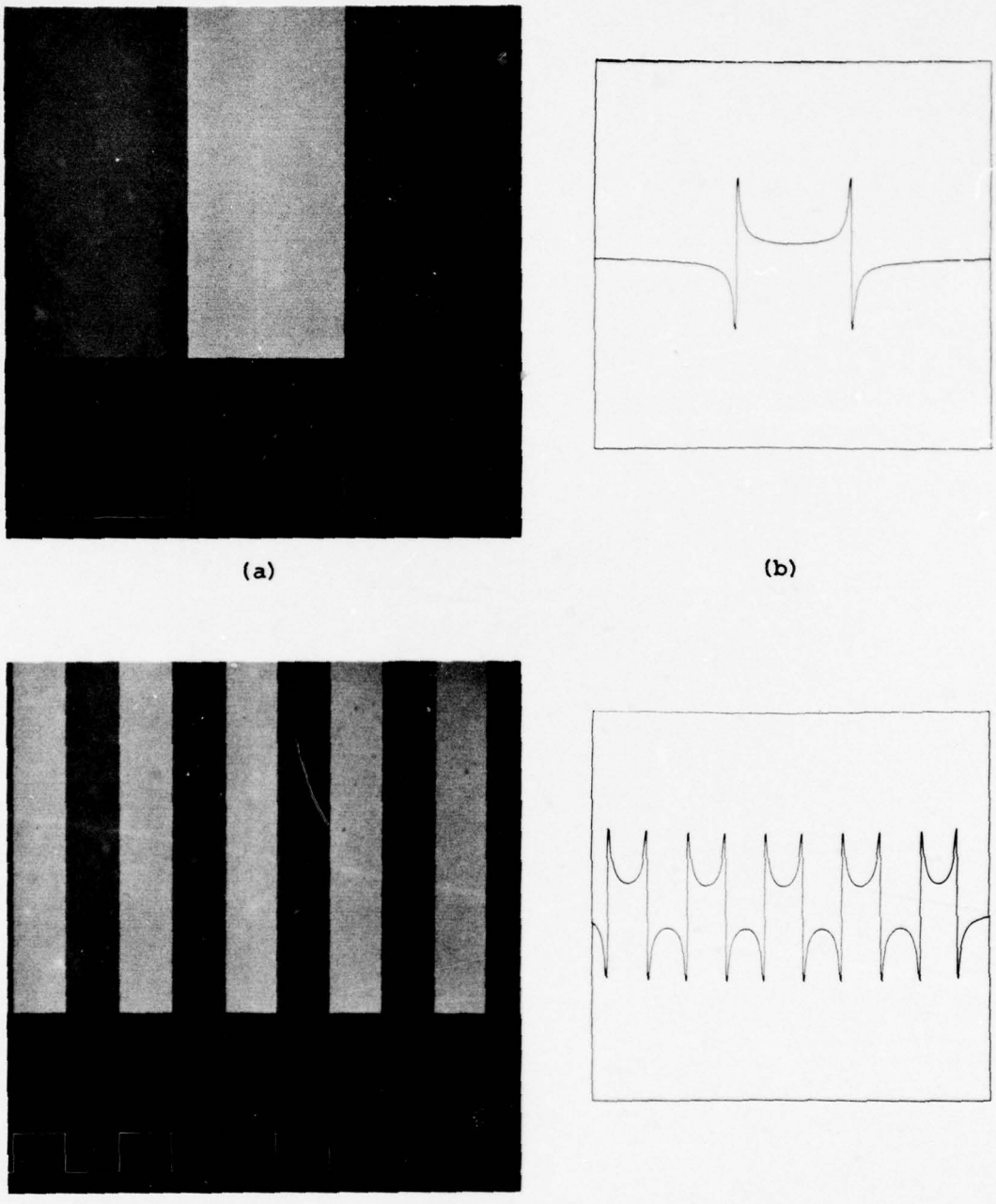


Fig. 32 - Intensity step (a) isolated from the gray scale of Fig. 31a, and predicted subjective brightness (b). Intensity square wave (c), and predicted subjective brightness (d). Notice how the model predicts similar brightness effects for patterns 31a, 32a and 32c.

contemplate first the possibility of some local non-linear process, prior to or imbedded in the convolution model of neural inhibition, and accounting for the different effect observable near edges.

II/ Possible local edge oriented mechanisms

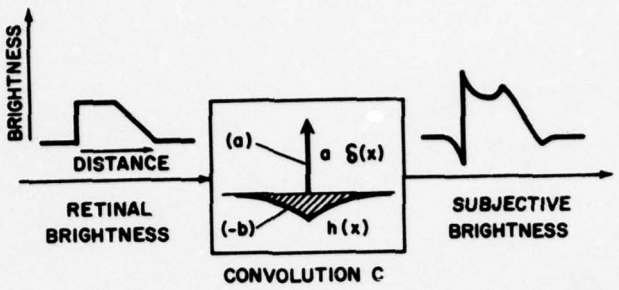
In order to speculate on the possibility of additional neural mechanism controlling the appearance of edges and intensity steps, a convenient approach is to consider Mach bands as additive perturbations to the input density signal and to separate the convolution process representing neural excitation-inhibition into two additive processes, as described in Figure 33. For the sake of simplicity, we will suppose that the excitation process is a perfect transducer, without degradation of the input, with an ideal impulse response $e(x) = a\delta(x)$. We consider a typical inhibition process, with a weight of $-b$ ($0<b<a$, for theoretical as well as experimental reasons). Then, the convolution pictured in Figure 33a:

$$O(x) = I(x) * (a\,\delta(x) + h(x)),$$

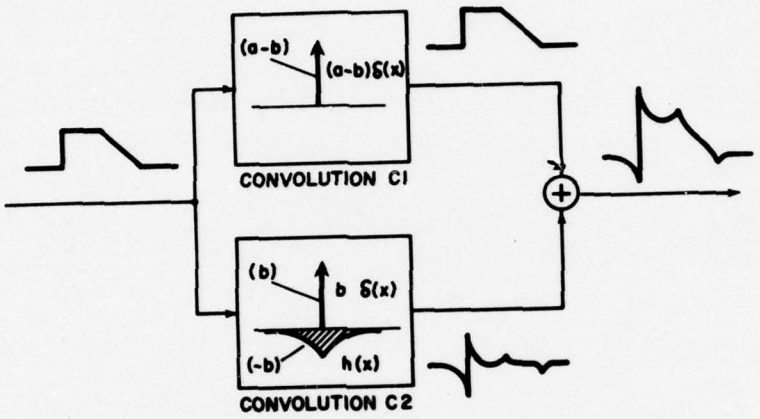
is identical to the sum of convolutions:

$$O(x) = (I(x) * (a-b)\,\delta(x)) + (I(x) * (b\,\delta(x) + h(X))),$$

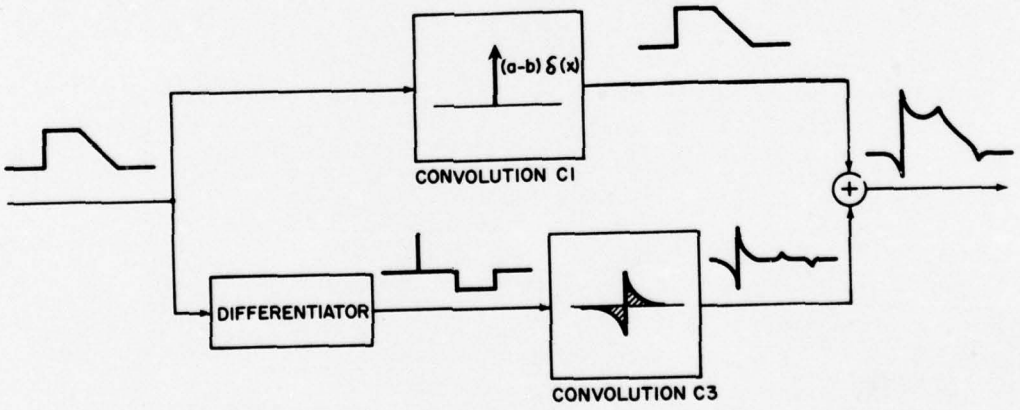
as illustrated in Figure 33b. This additive decomposition is such that the second convolution term involves an impulse response with null weight. In this manner, we have isolated the edge crispening process (sub-system C2, bottom of Figure 33b) as an additive perturbation to the input signal, which would be otherwise arbitrarily scaled through sub-system C1 (top of Figure 33b).



(a)



(b)



(c)

Fig. 33 - Three equivalent linear systems for neural interaction.

With this convenient conceptual dichotomy in mind, we may attempt to imagine a scheme for controlling the relative amplitude of the Mach bands created by the perturbation convolution C2. At this point, the only available evidence on these edge phenomena is psychophysical, whereas the homomorphic model for neural interaction is well supported by physiological experiments, as seen in Chapter II. Therefore, our motivation for considering hypothetical neural mechanisms for edge effects in the retina, is mainly to study whether a convolution model could be theorically generalized or adapted, and not to assert the existence, locus and physiological structure of such mechanisms, since we do not have presently methods for investigating them. As we shall see in Section IV, the problem is complicated by the fact that some central processes are probably involved. We consider first the hypothesis that the peripheral system (retinal inhibitory network) behaves non-linearly when edges are present. However, we probably know enough of the general properties of neural networks to evaluate whether a given mechanism might be "implemented" in a network of neurons; obviously, the last word belongs to the physiologist[2].

A first solution could be a threshold function applied to the output of convolution C2. However this is not satisfying, because it does not seem likely that a fixed threshold would attenuate the Mach bands resulting from a step function sufficiently: These bands would be at least as visible as the brightest Mach bands obtainable with an

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[2] Neural interaction was postulated very early as explanation of brightness contrast phenomena [Mach 1968, in Ratliff 1965, p. 316], but clear physiological evidence of such inhibitory interaction in retinal networks was demonstrated much later, in the Limulus eye [Hartline 1949].

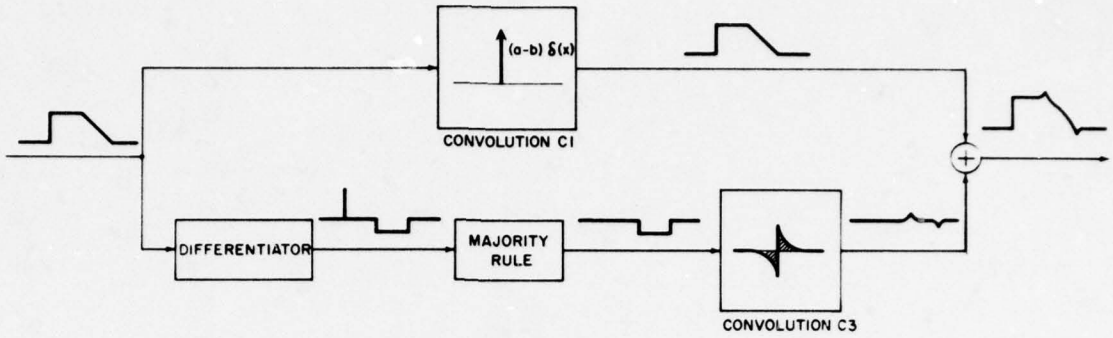
intensity ramp. As has been reported above, this is clearly not the case.

It is also possible to imagine a tuning mechanism which would locally attenuate the crispening effect, according to the intensity gradient of the input signal. Let us first notice that any convolution is equivalent to a differentiation followed by another convolution:

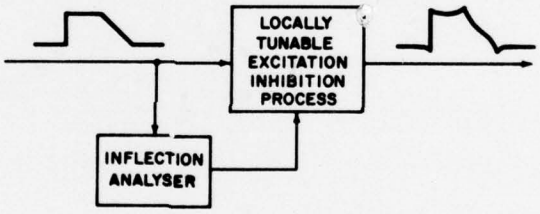
$$f(x) * \delta(x) = f'(x) * s(x),$$

where $f'(x)$ is the derivative of $f(x)$, and $s(x)$ is the undefined integral of $\delta(x)$. In other words, a linear stationary system with impulse response $\delta(x)$, can be analysed in terms of its step response $s(x)$. By this transformation, the system of Figure 33b can be assimilated to a differentiator followed by a convolution C3, whose impulse response (which is also the step response of C2) corresponds to the hypothetical Mach bands of a unit step function.

With this image in mind, we see that it is sufficient to insert a very simple (non-linear) mechanism in the system, in order to eliminate sharpening of intensity steps. Such a mechanism for eliminating impulses at the output of the differentiator, and prior to the convolution stage C3 (Figure 34a), would compare the input value at each point with the neighboring points, and replace, by an average of the surrounding input values, any input which is strongly different from its neighbors. We name this impulse elimination mechanism a *majority rule*, which results in ignoring individual points where the gradient computed by the differentiator is too large. The spread of input of the *majority rule* would partly determine the critical gradient. We have



(a)



(b)

Fig. 34 - (a) Non-linear system for neural interaction, derived from Fig. 33(c), which eliminates sharpening of edges.

(b) Generalization of the non-linear system (a), in the form of a tunable summation system, linear in the absence of edges.

here a satisfying method for sharpening only contours which are not keen enough. This scheme fits in the context of a peripheral system which has as its objective the pre-processing of visual information in preparation for a higher level system for which edges are fundamental features. The notion that edges and contours convey a great part of the visual information is commonplace, and suggests the existence of specific edge-oriented mechanisms in the central visual system. In this context, the effects of peripheral processing would be to accentuate intensity gradients in the visual image, in order to enhance contours, and perhaps compensate for inherent optical and neural loss of resolution. Mach bands would be visible manifestation of this processing. Then, it would make sense if edges which are already well resolved, were not subject to excessive sharpening.

But there are some difficulties with this approach. First, is the fact that, although the implementation of Figure 34a provides for brightness contrast when the intensity variation of the inducing background is gradual (as in Figure 2b), that will not be the case when its variation is stepwise (as in Figure 2a). The second point is that there is no reason to believe that the retinal network, for instance, operates and is structured in the way suggested by Figure 35a; we chose this representation for reasoning convenience only. But as a result of introducing a non-linear process, such as the majority rule, this system is not convertible easily into the more realistic inhibitory model of Figure 33a. However, taking the "black box" approach, it is possible to imagine a locally tunable inhibition process, quasi linear and isotropic in the absence of edges, operating in a similar way to the

system just described. In the vicinity of edges, the spread and amplitude of inhibition would be controlled by an edge detection mechanism (or inflection analyser), resulting in a strong attenuation of the crispening effect, without suppression of brightness contrast (Figure 34b). One will notice that the "black box" just described requires a somewhat flexible and adaptive organisation of neurons, but involves no more than classical excitatory and inhibitory interconnection. For instance, edge detection is readily performed by pairwise connection of neighboring receptors, alternately excitatory and inhibitory.

The main reason for contemplating such a hypothetical non-linear and adaptive system, although precise psychophysical and physiological data are lacking, is that, having shown evidence of the linearity of neural inhibition for smooth stimuli, and having related it conclusively to brightness contrast phenomena, it is natural to try to investigate to comportment of such spatial mechanisms in the vicinity of edges. There has been studies on the spread of lateral inhibition, reviewed in [Growney and Weisstein 1972], showing in general that the spread of lateral inhibition is larger at the vicinity of edges. However, the results are often conflicting, and the experimental designs different, and sometimes too crude, considering the complexity of the neural mechanisms involved. Von Bekesy [1968a, 1972a] preferred to distinguish two types of lateral inhibitions, responsible respectively for the narrow Mach band effect(*Mach inhibition*, with a narrow spread), and for the brightness slope of intensity steps (*Hering inhibition*, with a large spread). His dichotomy is based on experiments showing different

action of factors such as adaptation and flicker on the two brightness effects. Simulating with his *neural unit*, he indicated that the two types of inhibition could be characterized mainly by a different spatial spread. However, this theoretical separation is not really necessary, since we showed that the line spread function obtained by brightness contrast experiments (Chapter III) would also create Mach bands (Figure 37b and d) and brightness slope for intensity steps (Figure 31). Therefore, we favor the notion of a single inhibition mechanism, but spatially adaptive and non-linear near edges. The different influence of the factors mentioned above could certainly be related to strong non-linearity.

The main problem is that some additional neural system, connected with edges, are probably involved in the perception of brightness, as we shall see now.

III/ Effect of contours on brightness perception

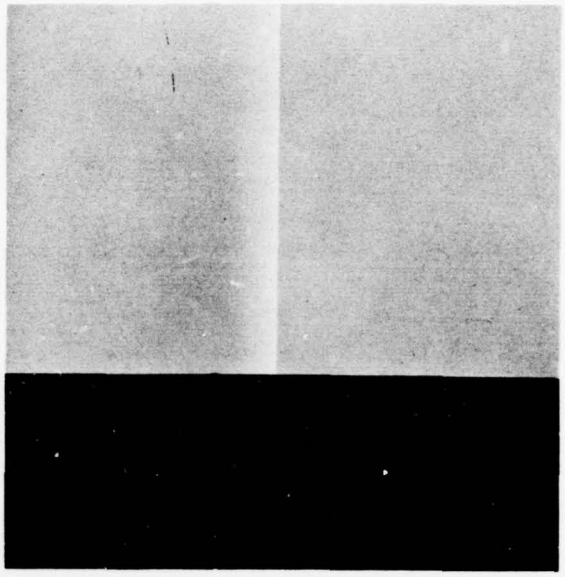
The evidence that edge effects are related to more complex mechanisms that local non-linearity of neural inhibition, is the following: the apparent slope, or fluting, or the brightness of intensity steps is dependent on the characteristics of the whole pattern. In particular, the slope in brightness of a uniform, or nearly uniform, intensity area seems to be subjectively apparent only when the neighboring inducing fields have intensity levels above and below that of the induced field, as is the case for a multiple step function (Figure 31); conversely, this is not observed when the surrounding fields are both brighter or darker than the induced field (Figure 32,

33). But, even when the effect is present, it is not as strong as predicted by a linear model (Figure 31a and b). It is interesting to relate to this phenomenon, the Cornsweet illusion [Cornsweet 1970, p. 272] illustrated on Figure 35, or the effects reported by O'Brien [1958], whereby an intensity cusp at the border of uniform fields induces different brightness for fields of the same intensity. It does not seem possible to explain such an effect by inhibition mechanism (whether it is linear or not), because the phenomenon is uniform over the whole visual field, whereas peripheral neural inhibition is a quite local process.

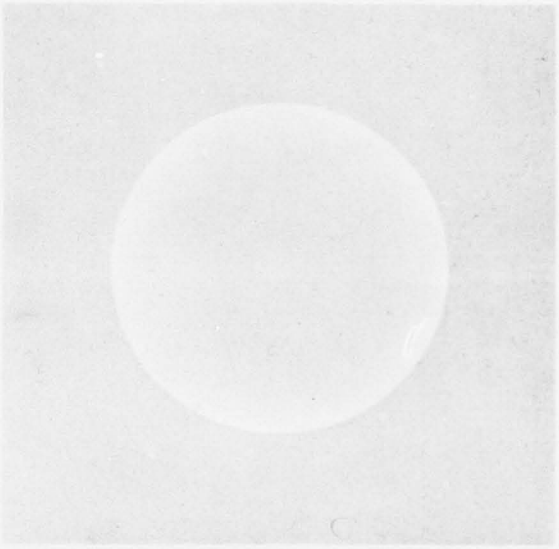
Then, it is tempting to imagine an additional mechanism, subsequent to the non-linear inhibitory process.

IV/ Peripheral and/or central mechanisms

One possibility is an *equalizing* process, applied to the output of the peripheral inhibitory system, which would collect brightness information principally at the edges, and extrapolate in between. Such a theory has been offered earlier [Fry 1948], and the subject is reviewed by Ratliff [1971]. In this context, uniform intensity areas would have brightness slope, only when the edge enhancement resulting from peripheral processing is in opposition, as is the case for a single step in a multiple step wedge (Figure 31). When the peripheral edge crispening is in the same direction (Figure 32, 33), it would be eliminated by the extrapolating mechanism. This empirical rule is actually well verified qualitatively, as can be observed on the various patterns presented here. Then, such a central process, together with an

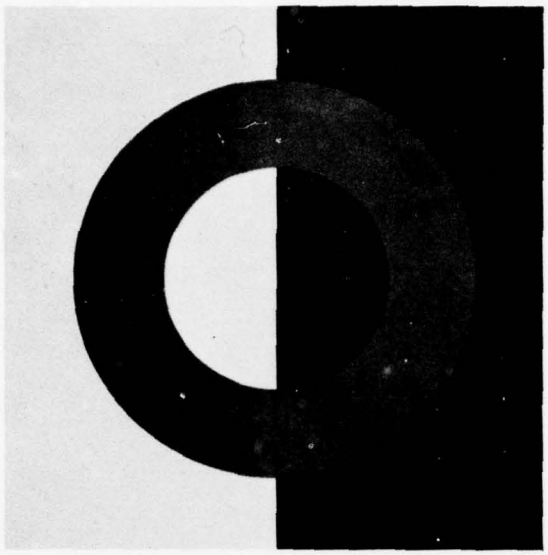


(a)

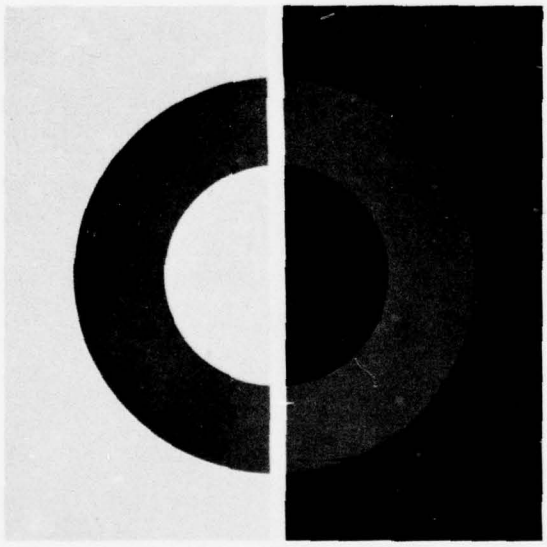


(b)

Fig. 35 - Cornsweet illusion, on a rectilinear (a) and circular (b) test pattern. The two fields composing the picture, have same intensity and different brightness, except near their border.

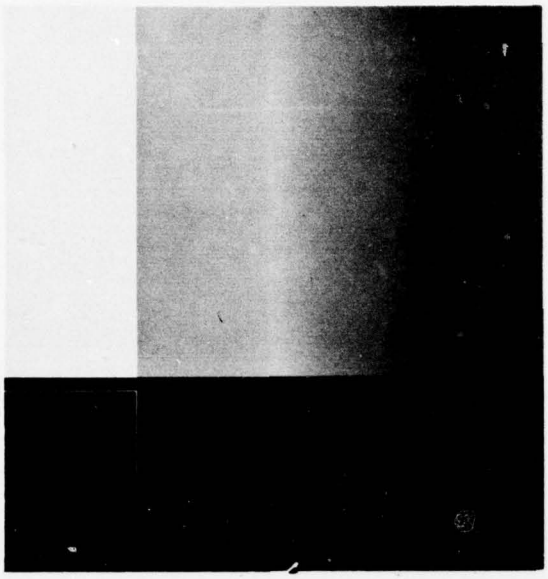


(a)

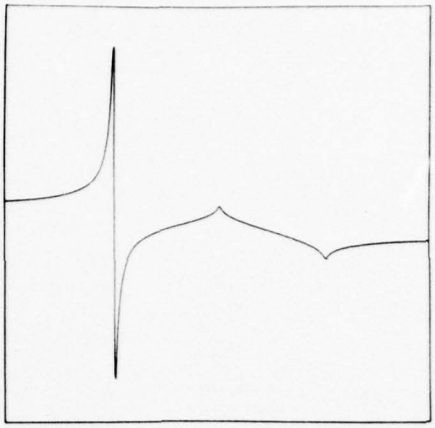


(b)

Fig. 36 - Bennussi ring: notice how the simultaneous brightness contrast *effect is perceived only* when some delimiter separates left and right fields of the pattern (b).



(a)



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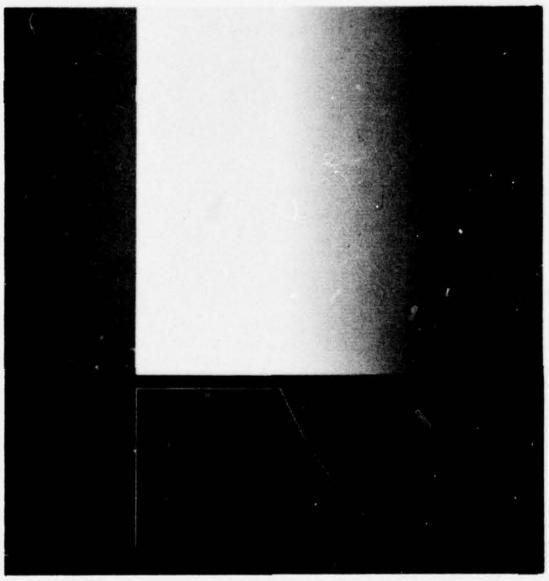
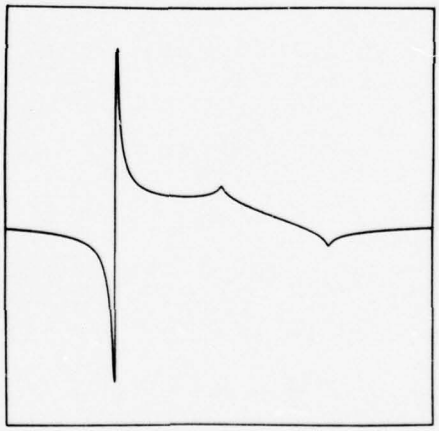


(c)



(d)

Fig. 37 - Compare visual appearance (a & c) and predicted subjective brightness (b & d) of intensity step and intensity gradient.

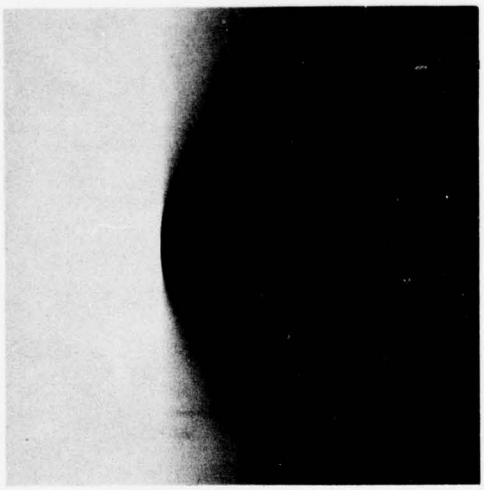
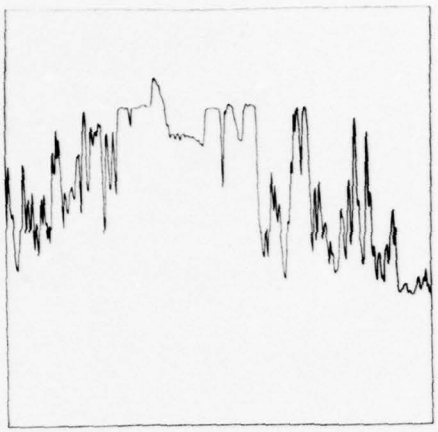
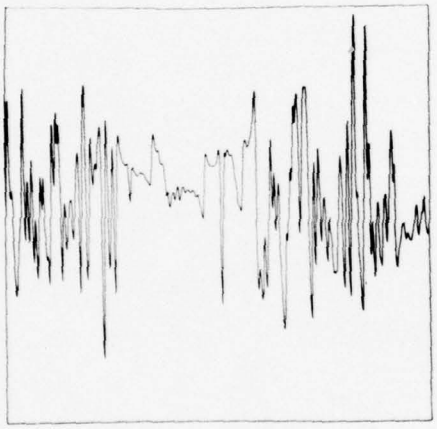


Fig. 38 - Patterns showing transition between a ramp and a step.



(a)



(b)

Fig. 39 - A typical density scan-line of a digital picture (a), processed by the model of Fig. 6 (b).

inhibitory retinal network, non-linear near edges, would explain how three patterns for which the brightness predicted by a linear model of inhibition is similar (Figure 31, 32, 33), appear subjectively quite different. Davidson and Whiteside [1971] propose a brightness integrating mechanism, summing or averaging independently over areas separated by contours, and generating from this operation a brightness more uniform than the output of the peripheral inhibition network. Both schemes just described, would account qualitatively for brightness perception, in the vicinity of edges, including Cornsweet's illusion.

However, at this point, one obvious question is why Mach bands should be visible at all. In other words, why would the systems just described not operate on an uniform field separated on one side by a sharp edge. on the other side by a ramp (Figure 37), like it does when both sides are sharp edges. One possible answer is that sharp edges are easily detectable by the retinal network, and that this particular aspect of the image is a critical element of visual information, to which *both* peripheral and central systems could be tuned. This idea was the motivation for considering the generalized non-linear inhibitory system of Figure 34. But such theory is thoroughly speculative, and physiological and psychophysical evidence is lacking and urgently necessary.

In conclusion, there seems to be some evidence of two basic mechanisms in the visual system

- A neural interaction network, quite linear in the absence of edges, as has been demonstrated herein;

- An edge oriented mechanism, influencing the whole visual field.

Although there are some indications that the former process is basically retinal, and the latter central, it is quite probable that they are in fact intimately mixed and the complex result of neural structures implemented all along the visual pathway. Therefore, it is difficult to estimate how much a given experiment stimulates different mechanisms. The forthcoming challenge in the field of experimental psychophysics will be to develop methods for stimulating and probing the different components of this system in order to gain knowledge about its organisational and physiological structure. The Benussi ring illusion gives some insight of the complexity of the problem (Figure 36); it shows how local brightness contrast is dependent on the presence of a contour separating the ring in two halves on different backgrounds. Berman and Leibowitz [1965] showed experimentally the important effect of the dividing contour for a similar test pattern.

V/ Effect of Stimulus border

In relation with edge and contour phenomena, one question is raised, namely the effect of limitation in space of the visual stimulus, usually finite and surrounded by a bright, or preferably dark, background. Obviously, the spatial truncation of the intensity function introduces extraneous frequency components, as elementary Fourier analysis will tell us. Kelly [1970] showed how this truncation by the surround results in various edge effects influencing the perception of sinusoidal gratings. In particular, threshold contrast measurements are changed by the phase of the truncation of the grating, when the observer

fixates the edge. However, we will not follow Kelly in saying that, in spatial frequency analysis of the visual system, "the standard target size should be infinite," in order to be rigorous. If the subject fixates the center of the stimulus pattern, the problem is simply whether the brightness disturbance generated by the target edges have a spatial extent broad enough to perturbate the central part of the stimulus where measurements are made. In other words, it hardly matters that, mathematically, spatial truncation perturbates the frequency content of an hypothetical infinite intensity pattern, if the neural interaction mechanism responsible for brightness effects, has a finite spatial spread. This applied whether the model is linear or not. In the restricted conditions adopted here (smooth patterns), it seems probable that neural interaction results in a quasi linear and isotrope summation effect, of limited spatial extent (perhaps some 20' of visual angle). Therefore, the experimental targets and methods utilized here are justified in this respect.

However, we also just reviewed some evidence of non-linear edge effects, attributable to some, as yet unspecified, presumably central mechanism, which involves large portions of the field of view. But a characteristic of these effects is that they are uniform in nature. Therefore, they can probably be ignored, for standard viewing conditions, since they do not seem to perturbate the relative brightness inside the target contours.

CHAPTER VI

COLOR CONTRAST

Brightness phenomena are also present for color stimuli. But color being "three-dimensional", in the sense that a given color sensation may vary in *brightness*, *saturation* and *hue*[1], color interaction phenomena result in the subjective modification of not only the brightness of the induced color, but also its saturation and hue. This has been known and used effectively by artists [Albers 1963]. Color contrast phenomena seem qualitatively quite similar to the corresponding achromatic brightness effects, and all of the classical effects (Mach bands--although this still is a controversial matter--, background induction, grid effect) have their correlates for color stimuli. Therefore one is tempted to study how the homomorphic model for peripheral neural processing could be extended to incorporate color effects.

Some empirical formulations, such as von Kries coefficients, have been offered for color induction phenomena, and for the predictive description of subjective colors[2]. But it seems that the homomorphic model would prove a more powerful tool, since it already models reasonably well brightness constancy and brightness contrast, for low

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[1] For the definition of these terms, see for instance [C.H. Graham 1965, p. 60].

[2] See for instance: [Judd 1940], [Richards and Parks 1971; Richards 1972], [Takasaki 1969].

frequency patterns. However, this approach calls for some understanding of the basic color vision mechanisms, or at least for some justifiable hypothesis about these mechanisms. But this issue turns out to be quite controversial [Sheppard 1969].

I/ Theories of color vision

For an in-depth compilation of recent data and theories, the reader is referred to [C.H. Graham 1965][3]; only the most relevant points will be reviewed here, with the risk of over-simplifying.

Trichromacy theory originated from the experimental and theoretical studies of Young and Helmholtz, developed through the work of Hering and others, up to the recent research efforts of De Valois, Hubel, Wiesel, Rushton and McNichol. The theory relies on the psychophysical facts of metamerism (i.e. trivariant color mixture) and on the reasonably well established physiological notion that the retina contains three kinds of cone receptors with different photochemical sensitivity in the visible spectrum; these receptors are characterized by overlapping response curves, with peak at approximately 620, 530 and 470 m.μ [McNichol 1964].

There is also evidence that color vision is probably a complex multi-stage system. The opponent-color theory, initiated by Hering, proposes that the three-color information at the retina is encoded at a posterior stage into two-color on-off signals [Hurvitch and Jameson

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[3] See also: [Teevan 1961] for a collection of the early works, [Lettvin 1967], [Rushton 1969b].

1957, Koedenrink 1972]. E.H. Land has suggested futhermore that the retinal color sensitive systems (probably three of them), operate "in set" over the whole visual field, rather than point-wise [Land 1964; Land and McCann 1971]. We will dwell on this particular theory as it seems to agree with some experimental results on color contrast presented in the following section.

Land's experiments would tend to show that color is not determined point-wise, according to the local response of three different retinal receptors, but rather from the complex combination of the independent responses of the three receptor sets over the whole visual field. This hypothetical retinal-cerebral system, the *retinex*, reduces the role of the receptor sets to the perception of lightness, that is the perception of the reflectance characteristics of the visual scene, in one wavelength band, independently of the illumination factors. The *retinex* mechanism would generate the color sensation from these three retinal "pictures", at different wavelength.

We already indicated in Chapter II how, in achromatic vision, the homomorphic system, modeling neural interaction between neural receptors, accounts for brightness constancy and brightness contrast, and thereby performs the separation of the reflectance factor, that is to say *lightness perception*. This is accomplished by logarithmic transformation which maps the multiplicative reflectance and illumination components into additive components, and by high-pass filtering which eliminates the slowly varying illumination component. We also showed how Land's retinal model for *lightness perception* [Land

and McCann 1971], based on edge detection, could not account for brightness contrast. But we also indicated that, although the homomorphic model describes properly brightness contrast for smooth stimuli, it is defective in the case of edges (Chapter V), and that some contour-oriented mechanisms have to be included in the model in order to generalize its validity. In conclusion, it seems that an hypothetical composite system, following logarithmic receptors, combining an edge detection mechanism, such as Land's retinal model, and a quasi linear mechanism of neural interaction, would account properly for lightness perception, brightness constancy and brightness contrast. Then the hypothesis of three independent sets of receptors is very tempting, in view of the many fundamental visual phenomena which can be modeled by such a system, and considering Land's experiment as well as some experiments on color Mach bands and color contrast.

II/ Three independent inhibitory receptor networks

Earlier observations [Ratliff 1965, pp. 62-63] reported that color Mach bands were visible only when a brightness gradient is present. In other words, no Mach bands were perceived on a pattern representing a transition between two fields of different color, with constant luminance. Similar negative results [Van der Horst and Bouman 1967] lead even to suggest that spatial inhibition was lacking in the color mediating channels; as noted by Ratliff [1965], this would be in contradiction with the well established facts of color induction. However, it was showed more recently [Jacobson and McKinnon 1969] that Mach bands effects can be produced by saturation gradients. Also, the

more extensive and careful experiment conducted by Green and Fast [1969] demonstrated that colored Mach bands were in fact visible on chromatic gradient with constant luminance. Alpern [1964] demonstrated a related result for color contrast, namely that simultaneous brightness contrast is at a maximum when color contrast is at a minimum.

The latter results support the attractive hypothesis of three retinal inhibitory networks, sensitive to different wavelength, but independent from one another. In other words, the kind of neural interaction we have been describing and modeling in the present research, would occur independently within each set of retinal cones. As will be easily observed, three such inhibitory networks would account for all the observable color contrast effects, including Mach bands.

Experimental measurements of the frequency characteristics of these three receptors systems by Mach band increment threshold [Matthews 1967], and by threshold contrast [Van der Horst 1969; Green 1968], showed similar characteristics for the "red" and "green" mechanisms, different for the "blue" mechanism. The "blue" system exhibits lower contrast sensitivity and a wider spread of spatial summation. One possible explanation might be a scarcer density of "blue" cones in the foveal region.

III/ Color effects on smooth patterns

Assuming the existence of three independent inhibitory networks, we experimented with colored patterns such as those used in the achromatic experiments.

Color patterns were created on photographic film by three exposures through blue, green and red filters. It was possible to produce quite striking colored effects, analogous to the achromatic brightness effects (Mach bands, simultaneous color contrast, grid effects), on sharp as well as smooth stimuli patterns. Smooth pattern effects were conpensated successfully by processing independently by the same model used previously (See Chapter III and IV) each primary image (blue, green and red). These results support quite well the notion of three independent inhibitory networks, linear for smooth stimuli, and sensitive respectively in the blue, green and red regions.

But, a critical point in all experiment of this type is whether the different color mechanisms are actually stimulated individually. In our particular case, this is impossible to assert, as too many factors are involved in the generation of the color stimuli on the final photographic print: spectral characteristics of the phosphor, spectral transmitivity of the filters, sensitivity of the film, spectral characteristics of the dyes, chemical interaction during development, etc. For this reason, our experiments must be considered as preliminaries. In addition we neglected any suspected difference in the spatial characteristics of the three mechanisms. Also, generating stimuli patterns by a photographic process, it is difficult to control parameters such that luminance, saturation and hue, and to relate them easily with the blue, green and red digital data.

However, even this crude experiment was in agreement with our theoretical assumptions, and verified the data obtained in the

achromatic experiment. Therefore, the preliminaries appear to be promising, and such experimentation techniques, implemented on a real-time, color, digital display should yield a harvest of interesting and more precise results.

CHAPTER VII

CONCLUSIONS

As we indicated in the introduction, this computer-aided research is relevant to the fields of *digital image processing* and *visual psychophysics*. From the results of our experiments we can draw some conclusions in these two domains.

I/ On the side of psychophysics

This work should have demonstrated what a powerful and versatile aid are digital computing and displaying equipments, for research in visual psychophysics. The term *digital* is important here. Indeed, some fundamental points of our demonstration have made little use of complicated computing and simulating procedures which were available. For instance, most of the simple patterns used in the experiments (Chapter III and IV) could have easily been generated with some ad hoc analog hardware driving a cathode ray tube or a television scope; such tools have actually been built and used for previous studies of the modulation properties of the visual system [Campbell and Green 1965; Green 1968]. However, the advantages of digital techniques are clear, because analog special purpose devices, on one hand, cannot attain the same precision and quality for image production, and on the other hand, do not provide the flexibility for producing a large variety of patterns.

We indicate in Appendix C how display devices and photographic media introduce distortions in the image producing process, and how compensation procedures are fundamental and necessary; it is quite obvious that only digital methods permit such a precise control of picture quality, while allowing broad flexibility. It is also true, however, that there are limitations with respect to the production of color stimuli via the medium of photographic film, as we noted in the preceding chapter, and that a real-time[1] display device would improve the experimental procedures in many respects.

Probably, the more interesting feature of this type of computer aided research is flexibility: a well developed software system allows one to experiment with about any imaginable stimulus pattern. All illustrations in this thesis, excluding block diagrams, were generated with a few interactive programs. In an experiment session on the computer, it is generally a matter of minutes to create a pattern, process it in various ways, display it, change the parameters of the models, etc; if some new software feature is desired, then it is a matter of hours to modify, improve and expand programs significantly, and implement new experimenting ideas.

The computer was used here on one particular aspect of visual perception, but clearly its utility can be extended to many other areas. In particular, the computer has been used previously to study stereopsis with random dot patterns, which would have been laborious, if not

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[1] We mean here a device permitting instantaneous display and observation of a pattern, like on ordinary television.

impossible, to generate otherwise [Julesz 1971]. The most fundamental improvement in computer-aided research would probably be real-time production of the visual stimuli, on a computer-driven TV phosphor, as has been experimented by R. I. Land [1972]. As is apparent in the description and discussion of our experimental design (Appendix A), the very interesting advantage of real-time is the ability to control, while experimenting, the various parameters involved in the visual phenomenon under investigation. However, experiments such as ours, may require computationally complex stimuli and good control of the calibration of the image. The important question is how much trade off must be made between real-time requirements, on one hand, and stimulus complexity and picture qualtiy, on the other hand. The technical answer to this question is probably to utilize a digital video data disk as a refresh memory of a high quality TV monitor. Limited in our case to photographic outputs, and therefore to the study of steady state visual phenomena, we were able to make some contributions to the study of brightness perception.

We have helped investigating a fundamental point in all modeling approach to brightness contrast phenomena, namely the linearity hypothesis of neural interaction. We have showed that linearity is verified for smooth patterns, and indicated that non-linearity is probably located in the vicinity of sharp edges and discontinuities.

We have mentioned earlier that different approaches can be taken in modeling brightness perception. In particular, the shunting inhibition model [Sperling 1970] and the homomorphic model seem able to

describe adequately brightness constancy and brightness contrast. Both methods also fail to predict correctly the subjective appearance of edges and uniform brightness areas. However, it should be clear that the homomorphic method offers computational simplicity, and the important advantage of being testable directly by psychophysical experiments. But also, as discussed in Chapter II, we believe that the homomorphic model rests on a more profound philosophical basis.

Some interesting points should lead to further studies, namely:

- More precise estimation of the low frequency characteristics of neural inhibition, with proper attention to factors such as adaptation, temporal effects, contrast and luminance level;

- Attempt to determine, more precisely, in which manner, and by which mechanisms (peripheral and/or central) do edges have such an important effect on brightness perception;

-Finer investigation of color mechanisms.

In all such endeavors, digital display and computing equipments would most certainly be ideal tools.

II/ On the side of digital picture processing

These experimental results have some bearing on the field of digital processing of pictures. Certainly, they should lead to further research on the line of the work of Dr T.G. Stockham,Jr, on multiplicative image processing [Oppenheim et al. 1968], and the application of vision modeling to picture processing [Stockham 1968b, 1972]. The idea is, in short, that it seems more desirable to process pictures digitally after mapping the available physical intensities into a scale of brightness

having a better relation with the visual perception domain. The first implication is that, in many cases, it is preferable to process the densities of pictures (i.e. the logarithm of the intensities) rather than their intensities, as results from the approximately logarithmic brightness sensitivity of the visual system.

In particular, a precise model of human brightness perception would be welcome in the domain of picture coding[2], as it would provide a better "metric" for judging of the quality of processed pictures, and for estimating the subjective influence of such factors as noise. Earlier concern in human vision, in connection with bandwith compression studies, had mainly to do with contrast sensitivity (Weber's law), spatial resolution and temporal response [Schreiber 1967]. The notion that most of the visual information in pictures is conveyed by edges and contours lead to the *synthetic highs* (or *contour coding*) technique [D.N. Graham 1967]. In this method, the picture is separated in a low-pass component and a contour signal, formed of edges detected by a differentiating operator (gradient, laplacian); the low frequency component is coded by quantization and sampling; edge location and magnitude are coded, transmitted and utilized at the receiver to generate *synthetic highs* to be recombined to the low pass component. The original work was based on additive decomposition of these two terms. Stockham showed that better results could be obtained by multiplicative decomposition, that is to say applying the coding method to picture densities rather than intensities [Oppenheim et al. 1968]. Stockham [1969] also showed how the homomorphic model of brightness

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[2] Also termed: bandwith compression, redundancy reduction.

contrast could be incorporated into Roberts' scheme for picture data compression [Roberts 1962], with an approximated transfer function for neural interaction, and resulted in an improvement of the quality of the coded picture; this property can be interpreted as resulting from a better redestribution of the coding deterioration over the perceptual range of picture brightness. The same author demonstrated [Stockham 1972], that quantization distortion and noise degradation appeared minimal on pictures processed by the homomorphic system modeling brightness perception. These various results plainly justify studies such as ours, for estimating precisely the parameters of a valid visual model.

However, the linearity of the neural interaction is a fundamental asset in the applicability of the ideas just described. For instance, digital bandwith compression methods trade off between, on one hand, pre- and post-transmission processing requirements, transmission bandwith, and, on the other hand, picture degradation. If the visual model helps toward maintaining picture quality, it ought not burden the processing stages by computational complexity. In this respect, linear models are satisfactory, as they rely on the very efficient Fast Fourier Transform techniques.

While the present research provided precise data for the hypothetical visual transfer function and indicated evidence of linearity (for smooth patterns), it also pointed out the large departure of the linearity hypothesis from the subjective reality in the case of luminance edges and contours. For instance, Figure 39a shows a typical

scan line of a digital picture, and Figure 39b illustrates the subjective brightness predicted by the homomorphic model (this is only an approximation, as the line was processed in one dimension). Obviously, the assumption of linearity introduces important distortions, as natural images contain many edges and intensity steps, for which the homomorphic model is in default. Then, when using the visual model, a trade off has to be made between computation speed (minimum for a linear model) and edge distortion (maximum for a linear model). Clearly, the non-linear summation integral which would probably describe somehow the visual system in terms of subjective brightness, if its parameters were readily measurable (which they are not at the present time), would be computationally very expensive. In other words, one might want to look for a computationally efficient first order model of brightness perception.

For instance, Stockham [1972, Fig. 16] used an approximated and empirical transfer function, showing less low frequency attenuation than our experimental data. As a result, brightness contrast is not realized as well, but edge effects are less important. The non-linear model diagramed on Figure 34a, which would be linear in the absence of edges, might provide a basis for an alternate approach; but we already noticed that such a simplified model fails to account properly for brightness contrast phenomena for stepwise variation in density of the inducing background. Still another approach might be an extention of the contour coding technique, in which the density image is first separated in a contour component and a smooth low-pass component, and the low-pass component processed by a linear model of brightness contrast. However,

the low-pass component should not be obtained by linear filtering, which would defeat one's purpose (since the visual model is a high-pass filter!), but rather by some non-linear edges substracting or smoothing method. An additional problem which must be considered, is that the notion of using subjective brightness model in an image coding scheme, involves also the inverse of the model; clearly non-linear systems are potentially troublesome in that respect.

Another area where understanding of the properties of visual perception could be important, is the processing and coding of color pictures, a field still quite at the infancy stage. Clear and concise formalism for describing such subjective notions as color brightness and saturation, and accounting for color contrast and color constancy phenomena, are obviously needed in order to deal competently with such a difficult notion as the quality of a color image. Some steps were taken here toward extending the homomorphic model to color perception.

In conclusion, the full applicability of a model for brightness perception warrants further interesting research.

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APPENDIX A

DESCRIPTION OF THE EXPERIMENT

I am very grateful to Dr William Lee, formerly at the Psychology Department, University of Utah, for helping me in the preparation of this experiment, conducting the experiment itself and collecting the data.

I/ The experiment

Ten subjects of both sexes, with normal vision, were presented a visual stimulus consisting of a vertical, monochromatic, exponentiated sinusoidal pattern, sustaining a field of view of 8 degree, in a dimly illuminated environment. As described in Chapter III, this pattern (Figures 9 to 12) corresponds to the following density distribution:

$$\sin(2 \pi f x) + \alpha \sin(6 \pi f x),$$

For different frequencies f, and several values of the scaling factor α (positif, and inferior or equal to 1). All the stimuli presented had same average density, and therefore same average luminance, and also same contrast.

Two central bands, whose subjective appearance is influenced by simultaneous brightness contrast, were indicated by small markers. The subjects were requested to focus their vision at the center of the pattern, judge the relative brightness of the two bands and verbally report their judgement by one of the following answers:

R (right): The band to the right is brighter;

L (left) : The band to the left is brighter;

E (equal): The two bands are of equal brightness.

It was suggested that the judgement be made at the first glance.

A total of five sets of stimuli were presented, corresponding to the five fundamental frequencies:

f1 = .123 cycles/degree (1 period per picture)

f2 = .246 cycles/degree (2 periods per picture)

f3 = .369 cycles/degree (3 periods per picture)

f4 = .492 cycles/degree (4 periods per picture)

f5 =1.230 cycles/degree (10 periods per picture)

Each set contained ten patterns obtained for ten values of the scaling coefficient α, from 0.1 to 1 by steps of 0.1[1]. For $\alpha = 1$, the pattern produces strong brightness contrast (Fig 9a); for $\alpha = 0.1$, the fast varying component disappears and the target bands are not visible anymore (Fig 9b). Therefore, these two extreme values of the parameter α provide opposite results in the estimation of the relative brightness of the target bands. The purpose of the experiment was to estimate the intermediate value of α for which the subjective brightness of the

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[1] It was noticed that equal steps in the variation of the parameter α do not seem to result in equal step in the variation of the subjective relative brightness, and therefore the averaging method used to obtain the data might be questionable. However, this point was not debated further, considering the relatively important uncertainties intrinsic to the subjective judgement itself. In addition, the technical problems associated with the production of a large number of carefully calibrated projection slides restrained us to the option described here. In this respect, computer driven video monitor of high visual quality, would prove to be of great help for the experimental psychologist.

target bands would be comparable. This threshold value is taken to indicate quantitatively the relative amplification of the frequencies f and 3f by the visual system, according to the model described herein.

Each set of stimuli was presented to every subject four times, the order of the set being reversed from one presentation to the next. Thus the subjects, organized in five groups, were shown up to 5 * 4 * 20 = 400 stimuli. The groups differed by the order in which the five sets (f1, f2 f3 f4 f5) were presented.

II/ The results

The significant data of this experiment is the value of the coefficient α, such that the target bands appear to be of comparable brightness. This value, defined as the *point of subjective equality* (PSE), was estimated, for a given subject and a given set (i.e. a given frequency f), by averaging on the four presentations. The PSE for one presentation was considered to be the mean of the *transition values* T+ and T-, for which the subjective judgments switched from R to E, and E to L, respectively. The procedure is explained in the following example, using the typical data of Table I:

average of T+ = [T+] = .65

average of T- = [T-] = .425

interval of uncertainty

$$IU = [T+] - [T-] = .225$$

point of subjective equality

$$PSE = ([T+] + [T-])/2 = .5375$$

Table II and III contain the results obtained thereby from the experimental data. Averaging of these results over the ten subjects yields the following mean and standard deviation (s. dev.):

Point of subjective equality

set	f1	f2	f3	f4	f5
mean	.47	.51	.48	.49	.57
s. dev.	.13	.10	.10	.15	.08

Interval of uncertainty

set	f1	f2	f3	f4	f5
mean	.14	.26	.23	.29	.30
s. dev.	.04	.17	.13	.17	.18

TABLE I

TYPICAL RESULTS FOR

ONE SET OF STIMULI

	Presentations			
	1	2	3	4
α				
1	R	R	R	R
.9	R	R	R	R
.8	R	R	R	R
.7	R	R	R	E
.6	R	E	E	E
.5	E	E	E	L
.4	E	L	E	L
.3	L	L	L	L
.2	L	L	L	L
.1	L	L	L	L
T+	.55	.65	.65	.75
T-	.35	.45	.35	.55

TABLE II

POINT OF SUBJECTIVE EQUALITY

	Set				
	f1	f2	f3	f4	f5
Subject 1 (gr 1)	.5375	.4563	.4000	.4000	.4813
Subject 2 (gr 1)	.5313	.5625	.5500	.5813	.6750
Subject 3 (gr 2)	.4563	.3313	.5563	.5875	.5375
Subject 4 (gr 2)	.3563	.3875	.2000	.3063	.4688
Subject 5 (gr 3)	.4813	.4938	.4563	.4688	.5188
Subject 6 (gr 3)	.4688	.5188	.4938	.5438	.6875
Subject 7 (gr 4)	.6500	.6063	.5313	.7125	.5750
Subject 8 (gr 4)	.4875	.4013	.5375	.6375	.5438
Subject 9 (gr 5)	.5875	.7063	.5813	.6188	.6938
Subject 10 (gr 5)	.5188	.6000	.4563	.4438	.4813

TABLE III

INTERVAL OF UNCERTAINTY

	Set				
	f1	f2	f3	f4	f5
Subject 1 (gr 1)	.1000	.1625	.1500	.2000	.1625
Subject 2 (gr 1)	.1125	.3500	.2500	.3125	.3500
Subject 3 (gr 2)	.1875	.3125	.3875	.5000	.5000
Subject 4 (gr 2)	.1375	0	.750	.1375	.875
Subject 5 (gr 3)	.1875	.5625	.4375	.3875	.5875
Subject 6 (gr 3)	.1625	.1878	.2625	.1625	.1250
Subject 7 (gr 4)	.1250	.1125	.2125	.1250	.1250
Subject 8 (gr 4)	.2000	.4125	.3750	.2500	.5875
Subject 9 (gr 5)	.1000	.625	.625	.1875	.2625
Subject 10 (gr 5)	.875	.4250	.1375	.6875	.2125

III/ Conclusions

These experimental data were considered to indicate, as an average trend, that the point of subjective equality was about constant over the range of frequencies studied, equal to approximately 0.50. The implications of this result are developed in Chapters III and IV.

The somewhat large values of the intervals of uncertainty reflect the variations in the judgments of each subject for the four presentations of the same stimulus pattern. This variation is apparent on the typical results of Table I. It is believed that this factor could have been improved by some training of the observers, otherwise naive volunteer students. Averaging of the results was accomplished in order to eliminate some of the uncertainty.

However, it is the opinion of the author, from personnal experience, that the technique utilized here should lead nonetheless to more precise measurements than other techniques. In other words, it seems easier for an observer to compare the relative brightness of two target bands, than to estimate the brightness of a Mach band [Lowry and De Palma 1961] [Bryngdahl 1964], or compare the contrast of two gratings [Davidson 1968]. An important advantage of the present technique is that the stimulus is *self-testable*, that is to say no external, adjacent or successive reference stimulus need be presented, as is the case with the other methods.

As we mention in Chapter III, a limitation of the technique is that the range of frequency which can be practically tested is bounded. The restrictions stem from, on the low end, the problems of field of view

and isotropy of the visual system, and on the high end, the confusion of fringes. But, as we also indicated, these limitations are superficial, since we are mainly testing brightness contrast mechanisms and since the band of frequencies which can possibly be tested (roughly from 0.1 to 4 cycles/degree) is the fundamental region, as far as contrast phenomena are concerned.

The main improvement on the experimental design would probably be the use of a real-time digital display. For instance, a high quality TV scope, driven by a digital video data disk, would allow a very flexible experimental procedure. The display program could sense an adjustable potentiometer, so that the observer could control directly the relative brightness of the two target bands, i.e. the parameter α, and adjust the pattern for their best matching. Such a real-time, or nearly real-time, set-up would also allow to use the family of patterns described in Chapter IV, in order to test various contrast effects. The design actually used in the present experiment required the manufacture of numerous calibrated projection slides, and the preparation work involved prevented us from more extensive experimental studies.

APPENDIX B

THE MATHEMATICS OF TWO-DIMENSIONAL LINEAR SYSTEMS

I/ Circular symmetry

Let S be a two-dimensional linear stationary system, such that the impulse response (or point spread function) h(x,y) has circular symmetry, as is the case with a typical diffraction limited optical system [Goodman 1968]. The frequency response (or modulation transfer function, noted MTF) H(u,v), where u and v are the rectangular spatial frequencies associated with the rectangular coordinates x and y, also has circular symmetry. This is easily demonstrated by a transformation into polar coordinates of the Fourier transform equation relating h(x,y) and H(u,v); when expressed in polar coordinates, the impulse response

$$\overline{h}(r) = h(x,y),$$

and the MTF

$$\overline{H}(\rho) = H(u,v),$$

are related by the Hankel transform (for a demonstration of these classical properties, see for instance [Goodman 1968, p. 11]).

II/ Equivalent one-dimensional system

If we restrict now the inputs I(x,y) to a two-dimensional linear stationary system S_2 (not necessarily with circular symmetry), to be dependent on only one variable x:

$$I(x,y) = I_1(x),$$

then the output

$$O(x,y) = I_1(x) * h(x,y) \tag{1}$$

--where the symbol * represents the convolution operator-- is also dependent on the variable x only, as we observe by developing the convolution integral of equation (1):

$$O_1(x) = O(x,y) = \iint_{-\infty}^{+\infty} I_1(x-\xi)\, h(\xi,\eta)\, d\xi\, d\eta. \tag{2}$$

Therefore, in this case, the linear system S_2 is equivalent to a one-dimensional system S_1, with impulse response $h_1(x)$:

$$O_1(x) = I_1(x) * h_1(x).$$

Equation (2) can be written in the form:

$$O_1(x) = \int_{-\infty}^{+\infty} I_1(x-\xi) \left(\int_{-\infty}^{+\infty} h(\xi,\eta)\, d\eta \right) d\xi, \tag{3}$$

which shows that the impulse response $h_1(x)$ of the equivalent system S_1 is:

$$h_1(x) = \int_{-\infty}^{+\infty} h(x,y)\, dy,$$

i.e. the line spread function of system S_2.

APPENDIX C

THE COMPUTER SYSTEM

The digital computer system utilized for experimental picture processing, among other projects, is described in Figure 41. The graphical output devices are its main asset for interactive experiments. The storage scope is used to display graphical information such as frequency responses, line-spread functions, inputs and outputs at various stages of the model, predicted subjective brightness, etc., in one as well as two dimensions (Figure 21).

The test patterns are produced on a high quality cathode ray tube capable of a resolution of at least 1024 by 1024 elements, with some 2048 levels of intensity. However, the display is monitored directly by computer program, and therefore does not have real-time capability. Final output must be obtained on photographic film, for black and white as well as color pictures. Polaroid film is used commonly for preliminary experiments. When more precise control of the experimental viewing conditions are desired (Appendix A), projection transparencies are used.

An important aspect of the picture production routine is the calibration procedure, which guarantees that the final photographic intensities correspond very closely to the digital data intended to be displayed. Important distortions are produced by the CRT phosphor and

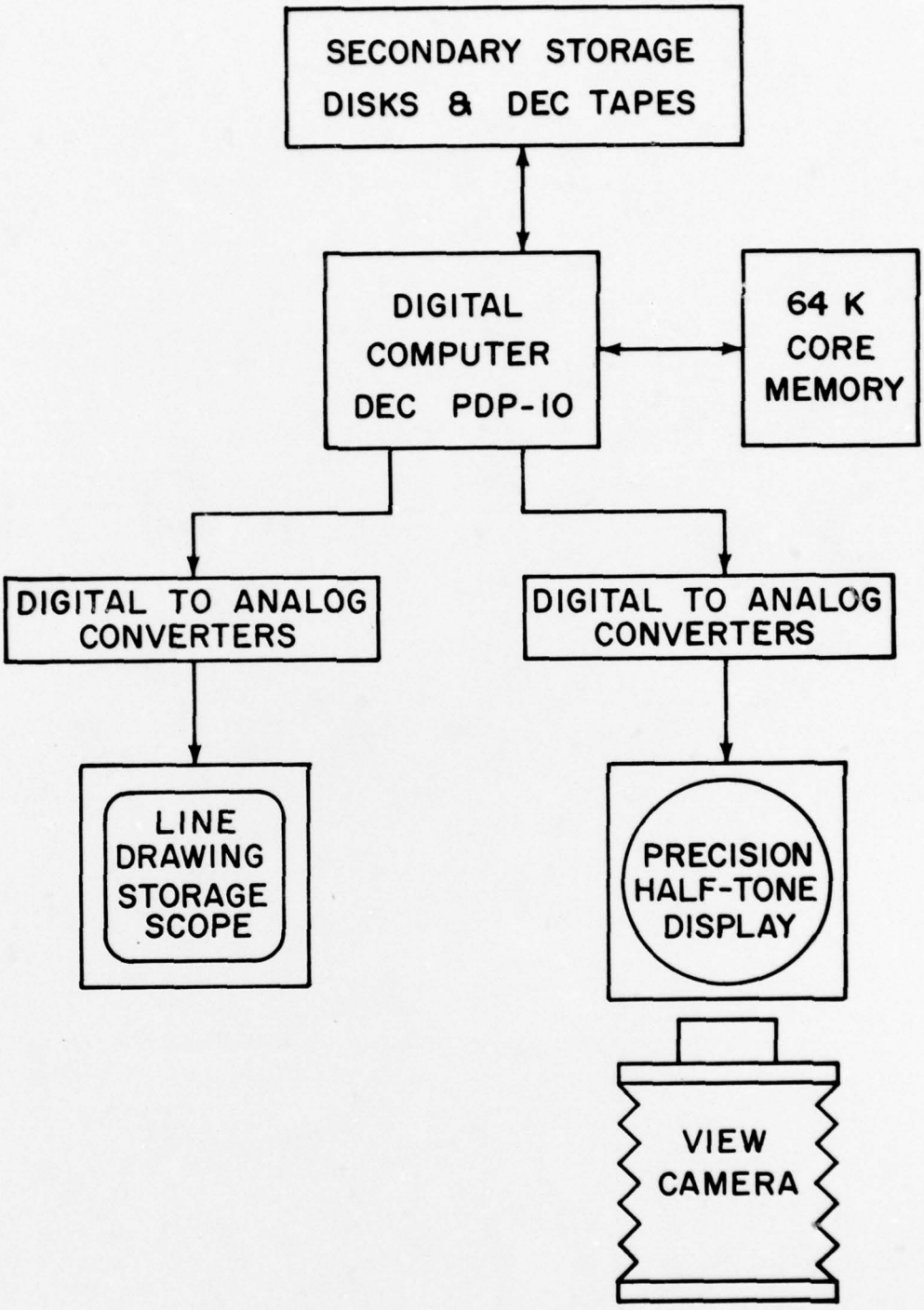
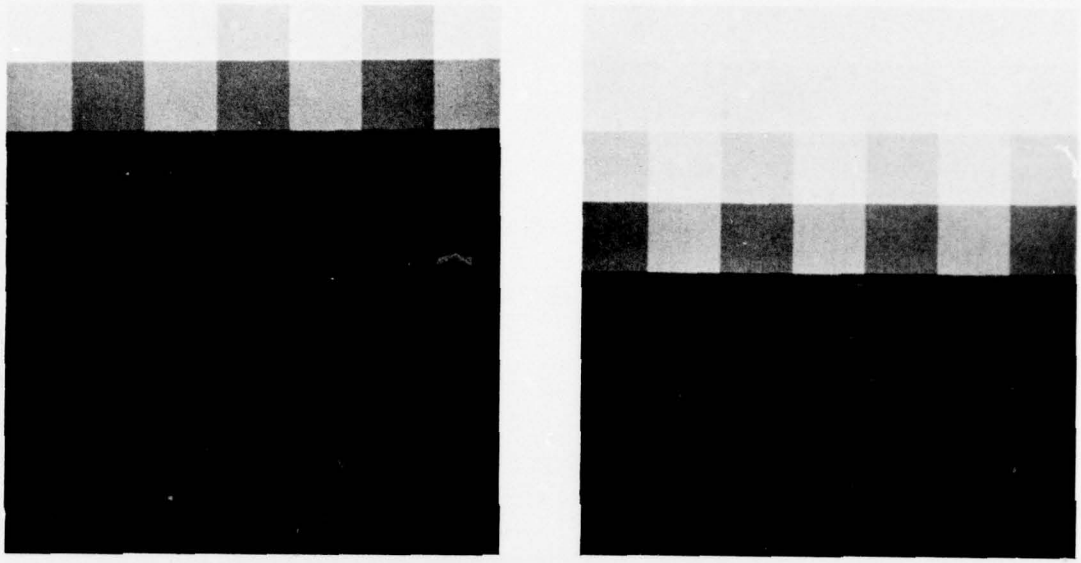
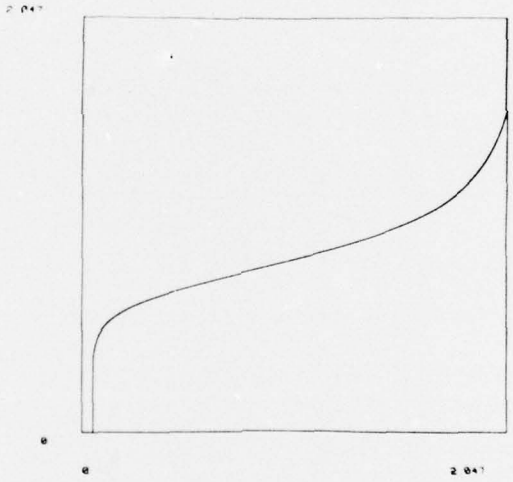


Fig. 41 - A computer system for experimental digital picture processing. Computer Science Department (University of Utah).



(a) (b)



(c)

Fig. 42 - Linear density gray scale used for picture calibration:

(a) compensated for photographic film distortion;

(b) uncompensated.

(c) Compensation curve used for Polaroid film, Type 52; compensation is done by digital table look-up before 11 bit D/A conversion.

the photographic film, which do not respond linearly. The phosphor non-linearity is empirically estimated as a cube power law. The film non-linearity is illustrated in Figure 42b, showing a linear density gray scale compensated for phosphor distortion only. A properly compensated gray scale is shown in Figure 42a: for this pattern a linear relation is verified between the densitometric measurements and the logarithm of the corresponding digital intensity. The compensation is performed digitally by table look-up, using a compensation curve such as that of Figure 42c. The compensation curve is obtained from densitometric measurements of the uncompensated pattern (Figure 42b).

Although this system does not allow real-time display, it has software features allowing interactive and flexible experimentation. It can be conceived of as the equivalent of a desk calculator, operating on waveforms (i.e. arrays) instead of single registers, and performing all conceivably useful unary and binary operations, in addition to storage, input-output, display, and graphical data definition.

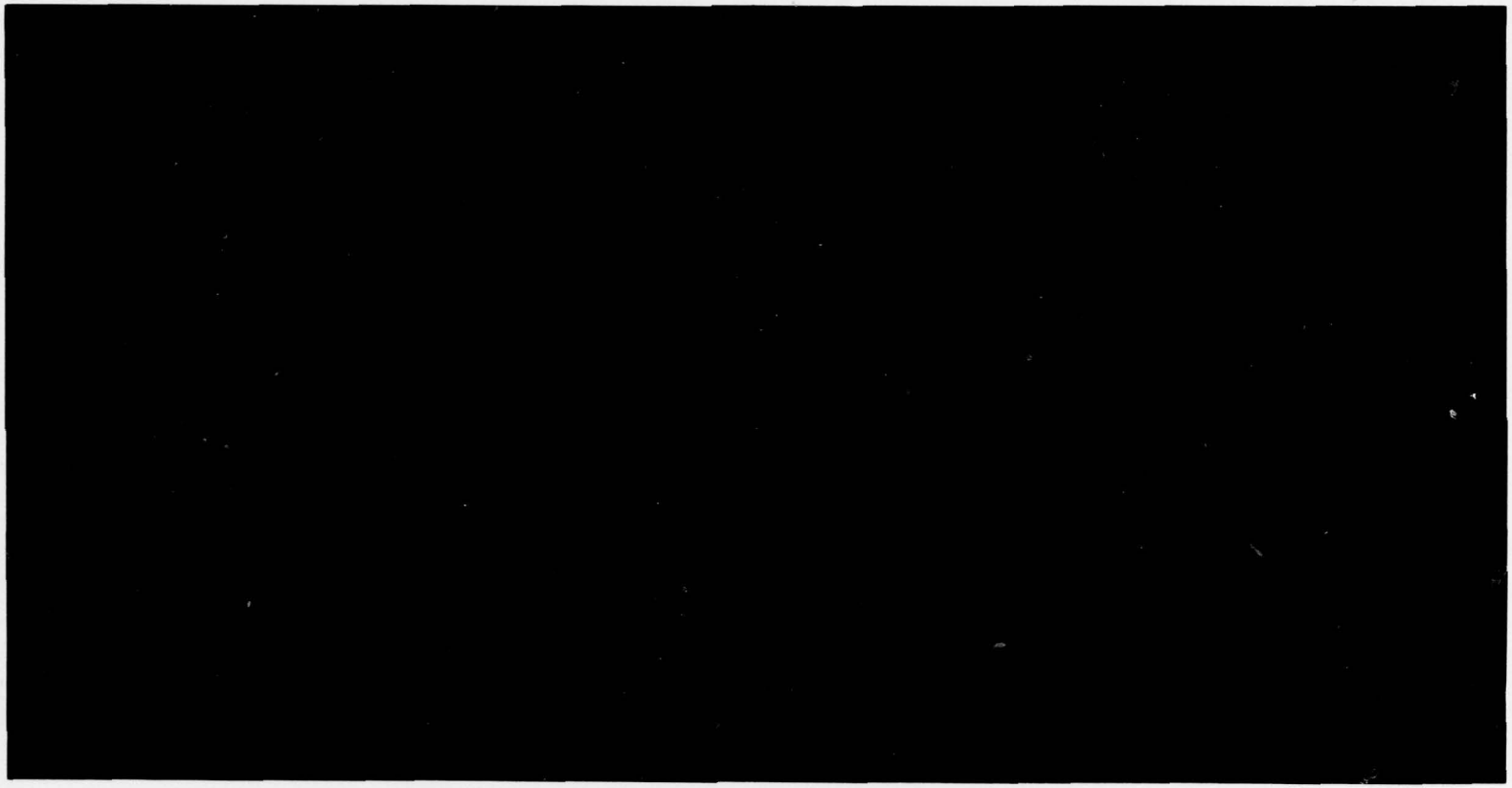


Fig. 40 - Color contrast compensation for smooth patterns.