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PART 1 OF 3

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FOREIGN TECHNOLOGY DIVISION

FLIGHT DYNAMICS

by

A. M. Mkhitaryan

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By: A. M. Mkhitaryan

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FOREIGN TECHNOLOGY DIVISION
WP-AFB, OHIO.

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U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere. When written as ë in Russian, transliterate as yë or ë. The use of diacritical marks is preferred, but such marks may be omitted when expediency dictates.

GREEK ALPHABET

Alpha	A	α	*α*	Nu	N	ν	
Beta	B	β		Xi	Ξ	ξ	
Gamma	Γ	γ		Omicron	O	o	
Delta	Δ	δ		Pi	Π	π	
Epsilon	E	ε	*ϵ*	Rho	P	ρ	*ϱ*
Zeta	Z	ζ		Sigma	Σ	σ	*ς*
Eta	H	η		Tau	T	τ	
Theta	Θ	θ	*ϑ*	Upsilon	T	υ	
Iota	I	ι		Phi	Φ	φ	*ϕ*
Kappa	K	*ϰ*	κ	Chi	X	χ	
Lambda	Λ	λ		Psi	Ψ	ψ	
Mu	M	μ		Omega	Ω	ω	

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English
sin	sin
cos	cos
tg	tan
ctg	cot
sec	sec
cosec	csc
sh	sinh
ch	cosh
th	tanh
cth	coth
sch	sech
csch	csch
arc sin	$\sin^{-1}$
arc cos	$\cos^{-1}$
arc tg	$\tan^{-1}$
arc ctg	$\cot^{-1}$
arc sec	$\sec^{-1}$
arc cosec	$\csc^{-1}$
arc sh	$\sinh^{-1}$
arc ch	$\cosh^{-1}$
arc th	$\tanh^{-1}$
arc cth	$\coth^{-1}$
arc sch	sech^{-1}
arc csch	csch^{-1}
rot	curl
lg	log

GRAPHICS DISCLAIMER

All figures, graphics, tables, equations, etc. merged into this translation were extracted from the best quality copy available.

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FLIGHT DYNAMICS.

A. M. Mkhitaryan.

Pages 1-368.

It is allowed by the Ministry/department of higher and secondary

special education of the USSR as manual for the students of VUZ [99sp4 - Institute of Higher Education], which are trained on specialty "operation of aircraft and engines".

Publishing house "machine-building" Moscow 1971.

Page 2.

Flight dynamics. Edited by doctor of technical sciences, Prof. A. M. Mkhitaryan, M., the "machine-building", of 1971, page 368.

In manual in accordance with the airfoil/profile of training/preparation of the engineers of the civil aviation, are presented the principles the dynamics of the flight of aircraft and helicopter.

Are examined the equations of motion of aircraft, typical of level flight, set of height/altitude and reduction/descent, curvilinear motion, takeoff and landing, distance and duration of

flight. Special attention is devoted to the questions of stability and aircraft handling.

Besides its direct/straight designation/purpose, the book can be recommended to the engineers of the civil aviation and the aircraft industry.

Table 3. illust. 202. References 28 titles.

Page 3.

Foreward.

Manual "flight dynamics" corresponds to the program of the analogous/similar with it course, read the students, who are trained on specialty "operation of aircraft and engines" in the schools of higher education of the Ministry of Civil Aviation of the USSR and in the departments of the civil aviation of VUZ of the Ministry/department of higher and secondary special education of the

USSR.

In manual are examined level flight, the takeoff and the set of height/altitude, reduction/descent and landing; are investigated the distance and duration of flight, the stability and aircraft handling, its behavior at angles of attack beyond stalling and under severe weather conditions. In the book is included the chapter on principles the dynamics of the flight of helicopter.

The systems of the differential equations, which describe the motion of aircraft, are examined in high-speed/velocity and body coordinate systems.

The first of them is utilized for the solution of trajectory problems; the second - during the stability analysis of stability and aircraft handling.

During the solution of trajectory problems, is establish/installed the dependence of aircraft performance on the parameters of the trajectory of its center of mass. The questions,

connected with distance and duration of flight, and also with takeoff and landing, are presented taking into account the special feature/peculiarities of the operation of the aircraft of the civil aviation.

Along with trajectory problems in the present manual, is given sufficient attention to research on stability and to the aircraft handling. The stability analysis of stability of aircraft is based on the linear equations of the disturbed motion. The equations of the disturbed motion are obtained in the single coordinate system, which made it possible to investigate stability and aircraft handling on the basis of the common/general/total stability theory of the motion of mechanical system.

The volume of separate/individual sections of a book and the sequence of their presentation are caused by the specific character of this course and by the many-year experiment of its teaching.

The book is written by the collective of the authors: introduction and chapter XVIII - by Prof. A. M. Mkhitaryan; chapter I - by Prof. R. A. Mezhlumyan and by Prof. V. S. Maximov; chapter II, VI, VII - by Prof. V. S. Maximov; chapter III - by instructor P. S. Laznyuk; chapter IV, V, XVI - instructor V. Ya. by Fridland; chapter VIII, IX, XII, XIV - by Prof. R. A. Mezhlumyan; chapter X, XI, XIII, XV< XVII - instructor I. the L. G. Totiashvili; chapter XIX - by instructor P. S. Laznyuk and instructor E. by I. Sorokin; control questions and problems to chapters are comprised by Prof. V. S. Maximov, by instructor P. S. Laznyuk and instructor V. Ya. by Fridland.

The authors express sincere gratitude to the Honored Scientist and Technologist of the RSFSR, to doctor of technical sciences, to Prof. A. P. Melnikov, doctor of technical sciences, Prof. Ya. to S. Shcherbak and Cand. of tech. sciences, to instructor Yu. A. Val'kov for the valuable observations, made by them with the survey of the manuscript of the book.

The authors also express gratitude of eng. A. G. Baskakovoy for aid in training/preparation of the manuscript for press.

The authors with appreciation accept all observations of the readers about contents of the book which request to direct to to: Moscow, the B-66, 1-1 Basmannyy per., 3, publishing house "machine-building".

Page 5.

Introduction.

V. 1. Object/subject the dynamics of flight.

Flight dynamics is science of the laws of the motion of flight vehicles. In the dynamics of flight, the motion of flight vehicle is considered as totality of two motions: forward motion at the velocity of its center of mass and rotary around the center of mass.

In chapter I, is examined the mechanical analog of flight

vehicle and they are derive/concluded equation of motion of its in high-speed/velocity and body coordinate systems.

In chapter II-VII, is studied the motion of the center of mass of flight vehicle under the action of the main vector R of all external forces. The investigation of the problems, examine/considered in the indicated chapters of course, is based on methods the dynamics of material point with equal in mass to flight vehicle.

Among the wide circle of the tasks of flight dynamics great practical value they have tasks, connected with research on the uniform rectilinear motion of flight vehicle whose solution makes it possible to determine the flight speed, the rate of climb, the distance and the duration of flight and other characteristic parameters of the motion of flight vehicle.

By the steady flight of flight vehicle, is understood this motion, during which the kinematic parameters, which characterize motion, do not depend on time. the steady flight it is possible to present virtually as motion of flight vehicle with negligeably low

accelerations which can arise on the individual sections of the flight trajectory.

Transient flight - the motion of the flight vehicle at which its kinematic parameters change in time.

Investigation of unsteady flight makes it possible to evaluate the maneuverable properties of flight vehicle.

The maneuverability of flight vehicle is called its ability for the determined time interval to change rate, heading and the attitude.

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The behavior of aircraft at angles of attack beyond stalling and under severe weather conditions is described in chapter the XVI and XVIII, but principle the dynamics of the flight of helicopter - in chapter XIX.

V. 2. Principles of the flight of flight vehicles.

for the realization of the flight of any flight vehicle it is necessary to create lift for the overcoming of gravity and thrust for the overcoming of the impedance of air medium. Depending on the method of designing of lift, they distinguish three types of the apparatuses: apparatuses easier than air, that use aerostatic principle of lift formation, apparatuses heavier than air, that use aerodynamic principle of lift formation, and apparatuses by the jet principle of lift formation.

The aerostatic principle of flight is based on Archimedes principle. Examples of the flight vehicles, which use this principle of flight, are the balloons and the airships.

The aerodynamic principle of flight is based on the use of the lift of wings, which appears during the motion of flight vehicle. In this case, in aircraft, the wings are fixed and connected with its

body, but of helicopters they rotate and are propellers.

The jet principle of flight is based on the overcoming of the force of gravitational field by the reaction of masses gases ejected from propulsion nozzle. An example of the flight vehicle of the jet principle of flight is the rocket.

The flight of flight vehicle, which is realized with the aid of the thrust/rod, developed with engine, is called powered flight. Power-off flight is realized with the shut-down engines. For example aircraft during takeoff, the gain of altitude, level flight with those which work with engine completes powered flight, and during powerless glide, it dvigatel[4]misoverwaet passive. Depending on the relationship between the extent of the active and inactive legs of flight, flight vehicles can be divided into two classes: flight vehicles with the large powered flight trajectory of flight (aircraft, helicopters) and flight vehicles with large inactive leg (glider/airframes).

The given classification of flight vehicles according to the principle of their flight is several conditional, since with

development of technology appear the flight vehicles, which use aerostatic and aerodynamic principles of flight. Thus, for instance, airship, equipped with powerful power plant, can accomplish flight at the comparatively high rate (200-300 km/h), at which already it is not possible to disregard its hoisting aerodynamic force.

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Therefore in the contemporary projects of airships, is provided for the very active use of an aerodynamic lift by the imparting to the airship hull of special form. It is cannot also, strictly speaking, to relate contemporary jet aircraft or cruise missile to the flight vehicles, which accomplish flight only on the basis of aerodynamic principle, since the fraction of the direct participation of reaction force in the overcoming of gravity in the various stages of flight is very significant.

The flight vehicles of contemporary civil aviation are related basically to the flight vehicles of the aerodynamic principle of flight.

V. 3. Short outline of advancement of science the dynamics of flight.

The theory of the flight of flight vehicles virtually rises from its from the 80th the years of past century.

In 1881 in Russia, marine officer Aleksandr Fedorovich Mozhayskiy invented flight vehicle heavier than air, named to them "flight projectile". Through four years the apparatus is bygone is constructed, but with takeoff injured. The circuit analysis of the aircraft Mozhaisk shows that its designer completely distinctly visualized the basic condition/positions of the theory of the flight of the vehicle of aerodynamic principle. Its aircraft had the aerodynamic configuration, close to contemporary: wing, fuselage, horizontal and vertical tail assemblies, control surfaces, chassis/landing gear, engine plant.

Hardly compiling this diagram of aircraft can be explained by Aleksandr Fedorovich Mozhayskiy's only intuition. Here great

significance had the experimental work, carried out as it before the construction of aircraft, in particular, experiments on kites and sail law courts, and also distinct knowledge of the theory of ship. it is known also that Mozhaisk the tales are executed the first aerodynamic designs, for example the calculations of the rectilinear steady flight of aircraft.

And only later approximately 20 years, in 1903, the American designers the brothers u. O. Wright constructed the aircraft on which it was possible to complete a series of flights.

In 1892 appeared the work of Nicholas Yegorovich Joukowski "on the soaring/steaming of birds" in which in the essence for the first time of tale were presented the theoretical principles of the motion of aircraft during gliding/planning, were established/installed the dependences between the angle of attack, the flight path angle and the gliding speed.

The same time includes the first works on the theory of the the aerostation of the founder of the cosmonautics of Constantine Eduardovich Tsiolkovskiy, who created the principles of the theory of

the flight of staged rockets and developed the theory of the controlled airships.

In 1894 K. E. Tsiolkovskiy published article "airplane or the bird-like (aviation) flying machine" in which was bygone for the first time was proposed the analytical method of calculation of speed and required power for a level flight.

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Other researchers obtained these results considerably late.

This same time includes the works of the scientists, directed toward compiling the stability theory of flight. It should be noted that the theory of the static stability of flight arose as a result of applying the detailed by this time theory of the static stability of ship whose principles tale were laid in the investigations of Euler's Leonard as early as in 1749.

More complex questions arose in the examination of the stability of motion of aircraft. As principle for compiling the stability theory of aircraft served N. Ye. Joukowski's doctoral dissertation "about strength of motion", which it shielded in 1882. By strength of motion, N. Ye. Joukowski understood the conditions under which is retained the trajectory of the undisturbed motion.

The principles of the contemporary stability theory of the motion of tale are given in 1892 to A. M. Lyapunov in his doctoral dissertation "general problem of stability of motion". The stability theory of A. M. Lyapunov afforded possibility to construct the theory of transient stability of flight vehicles generally and aircraft in particular.

The first two decades of the twentieth century are characteristic by the searches for the aerodynamic configurations of aircraft, providing safe - stable and controlled - flight, which, in turn, required the further development of theory.

In 1909 in the Moscow higher technical school by N. Ye. Joukowski is bygone is for the first time read the course of lectures

on the theoretical principles of aeronautics.

In 1913 N. Ye. Joukowski read the course of lectures for the officer-pilots, whereupon he published in 1913 and 1916 under the identical name of the "dynamics of airplane in elementary presentation" two articles, dedicated the dynamics of the flight of aircraft. In these articles were examined the questions of balance and stability of aircraft, and is also bygone was developed the graphoanalytical method of the airplane performance computation (the so-called method of Joukowski's thrust/rods) who widely is applied up to now. In 1917 is bygone is published its work "aerodynamic design of airplanes". By direct continuer N. Ye. Zhukovskogov stability theory is bygone V. P. Vetchinkin, the for the first time proposed method of the calculation of the forces, applied by pilot to control levers of aircraft during piloting.

S. A. Chaplygin in work "to the common/general/total airfoil theory of monoplane" (1922) generalized the results of investigations in the airfoil theory of aircraft and obtained a series of the new properties of the wing profiles, determining the stability characteristics of flight. This work had exclusively an important effect on the development of investigations in the field of the

theory of longitudinal stability.

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P. N. Nesterova's flight experiments (loop, bank/turn), K. K. Artseulova (corkscrew/spin) in many respects contributed to advancement of science the dynamics of flight.

The large role in the development of the Soviet aviation generally and the theory of flight in particular played the powerful scientific research collectives, organized by the Soviet government: central aerohydrodynamic institute, flight-test institute, etc. Works of V. P. Vetchinkin, B. T. Goroshchenko, V. S. Vedrova, A. N. Zhuravchenko, I. V. Ostoslavskogo, V. S. Pyshnova, B. N. Yur'yeva and other scientists contributed to the further of the development of the Soviet science of the dynamics of flight.

Large services in the expansion/disclosure of the maneuverable and flight properties of aircraft belongs to the gifted pilot-testers of that time - to M. M. Gromov, the V. K. Kokkinak, to V. P. Chkalov

et al.

The renowned designer collectives of O. K. Antonov, S. V. Il'yushin, A. N. Tupolev, A. S. Yakovlev created the constructions, ensuring the high aerodynamic and flight properties of the Soviet aircraft of the civil aviation.

A whole series of the new tasks before flight dynamics placed aviation in connection with the appearance of jet aviation engines. The qualitatively new phenomena, which reveal/detect/exposeed at high velocities of flight of aircraft, required the new solutions of the problems of aerodynamic design, maneuverability, controllability and stability of flight. These problems of tale are successfully solved. Evidence to this - compiling supersonic passenger aircraft Tu-144.

The history of the development of rotating wing aircraft and compiling the theory of their flights also are inseparably connected with Soviet science. Already M. V. Lomonosov in 1754 designed and constructed the flight vehicle, intended for meteorological investigations and which was the prototype of contemporary helicopters. "airfield machine", as named his vehicle brilliant

scientist, had the rotor, which was being given to rotation/revolution by spring, and screw/propeller for the damping of reactionary torque.

tav in 171 years of Academician M. A. Rykachev conducted the first careful experiments in the determination of the aerodynamic properties of rotor. In 1930 in the Soviet Union, were initiated virtually the maiden flights on the helicopter which was bygone was constructed according to the diagram, proposed by B. N. Yur'yev.

From this time begin also serious investigations in the dynamics of the flight of rotating wing aircraft. Large services in compiling this theory belongs to the Soviet scientists to I. P. Bratukhin, to N. I. Kamov, to M. L. Milyu, to A. M. Cheremukhin, to B. N. Yur'yev, to A. S. Yakovlv and by many by others.

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The combination of the special feature/peculiarities of helicopter and aircraft found embodiment in VTOL aircraft in which

are combined the explicit advantages of no-airport basing and the high velocity of level flight. many questions, which relate to the design of such aircraft, are solved in present time.

On the successes of Soviet science, the dynamics of the flight of flight vehicles testify the achievement of the Soviet Union in the mastery/adoption of outer space, which found its expression in the starting/launching of the first in the world artificial Earth satellite on 4 October, 1957, in the first space flight of Yu. A. Gagarin on 12 April, 1961, in a series of the further manned space flights and especially in the automatically controlled flights with soft landing under conditions of lunar gravity and with the return of vehicles on the Earth.

CHAPTER I.

EQUATIONS OF MOTION OF AIRCRAFT

1.1. Basic principles of the mechanics of the driving body. of the coordinate system.

Contemporary aircraft is mechanical system from the interdependent structural cell/elements, which fulfill the different functions also of the having different number degrees of freedom. The mass of aircraft changes in time as a result of the burnout, dropping in flight of loads.

the account of all degrees of freedom and change in the mass during the analysis of the motion of aircraft leads to complex mechanical analog, therefore, compiling an equation of motion, usually they simplify the mechanical analog of aircraft. So, for example, in many instances aircraft represent as one deformed body of constant mass, helicopter - as system of several solid bodies of constant mass, connected between themselves with joints, rocket - as one solid body of variable composition.

Speaking about the motion of body, is implied its displacement

in space and in time relative to any other body, called reference system. The position of the body in question relative to reference system is determined by the appropriate parameters.

In mechanics are distinguished inertial and noninertial reference systems. By inertial reference system is called the coordinate system, connected with the so-called "fixed" stars or with the body of reading, which are moved in space on inertia (at constant velocity) and rectilinearly relative to "fixed" stars. The systems, which do not satisfy these conditions, are called noninertial. In noninertial systems the law of inertia does not occur.

In inertial reference system, the motion is described by two vector equations:

$$\left.\begin{aligned}\frac{d\vec{Q}}{dt}&=\sum_i \vec{F}_i,\\ \frac{d\vec{K}}{dt}&=\sum_i (\vec{r}_i\times\vec{F}_i),\end{aligned}\right\} \qquad (1.1)$$

where the $\vec{Q}=\sum m_i\vec{V}_i$. it is described the main momentum vector of system; $\vec{K}=\sum_i(\vec{r}_i\times m_i\vec{V}_i)$ are the main moment of momentum of system relative to the selected reference point.

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Aircraft is in the general case the deformed body of variable mass. However, changes in its mass are so unessential, that them in

the majority of cases can be disregarded and considered aircraft the body of constant mass.

If we consider aircraft as solid body of constant mass, then system of equations (1.1) it is possible to write in the form

$$\left.\begin{aligned} m\frac{d\vec{V}}{dt} &= \vec{R}, \\ \frac{d\vec{K}}{dt} &= \vec{M}, \end{aligned}\right\} \qquad (1.2)$$

where $\vec{R}$ and $\vec{M}$ - respectively main vector and the main moment of the external forces, which act on aircraft; $\vec{V}$ is a velocity vector of the motion of the center of mass.

The system of equations (1.2), presented in vector form, determines a change in the kinematic parameters of the motion of aircraft as a whole under the action of the assigned external forces. In this case, if we the point of the application/appendix of external

forces combine with the center of mass, then the main vector $\vec{R}$ and the main torque/moment $\vec{M}$ will determine change the parameter of trajectory and angular position of aircraft.

The position of aircraft in space is determined by the integration of the system of differential equations (1.29 under the assigned initial conditions.

For research on the motion of aircraft usually the system of vector equations (1.2) they represent in scalar form, i.e., they examine it in projections on the axis of one coordinate system or the other. The form of scalar differential equations in many respects depends on the coordinate system; therefore the selection of system depends on the character of assigned mission for the target/purpose of lightening its solution.

In the dynamics of flight, are applied the right-handed coordinate systems in which the rotation of axle/axis Ox on $\pi/2$ is glad to its coincidence with axle/axis Oy it is conducted against the course of hour hand, if we look from the end of axle/axis Oz at the origin of coordinates. Passage from one coordinate system to another

it is possible to carry out with three rotations of the system of relatively initial position with its subsequent transfer it began coordinates to coincidence with the origin of the coordinates of another system.

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The reading of angles and senses of the vector of angular rate of rotation in this case is made through the following rule: as positive angle is accepted the angle between the pivoted axle/axis and its projection on the coordinate plane, which contains rotational axis, when rotation he is realized counterclockwise by an observer, who stands facing the origin of coordinates; as the positive direction of the angular velocity vector of rotation/revolution, is accepted the direction, which coincides with the positive direction of axle/axis. During the rotation of system around axle/axis through positive vector angle, of angular rate of rotation is applied in the beginning of coordinates and is directed along the positive direction of the axle/axis around which is realized the rotation.

The coordinate systems are classed according to form, according

to the location of the origin of coordinates and according to the orientation of axle/axes.

By form distinguish the rectangular, spherical, cylindrical, geographical coordinate systems. On the location of the origin of coordinates - geocentric with the origin of coordinates in the center of earth, planet-centered with beginning coordinatesin the center of planet, geotopical with the origin of coordinates on the surface of earth and connected with the origin of coordinates in the center of mass of flight vehicle. The orientation of the axle/axes of the coordinate system in each concrete/specific/actual case can be selected differently depending on assigned mission.

With presentation the dynamics of the flight of aircraft use in essence the rectangular geotopic and body coordinate systems.

Fixed terrestrial rectangular coordinate system

$(x_з' y_з' z_з')$

The origin of coordinates can be arranged in the arbitrary point

of the earth's surface. The axle/axis of Ox'_g lie/rests at the local horizontal plane (direction of the axle/axis of Ox'_g can be selected arbitrarily); the axle/axis of Oy'_g is perpendicular to the local horizontal plane and is directed upward; the axle/axis of Oz'_g is perpendicular to the plane of $x'_gOy'_g$ and is directed to the right from observer, who stands in the beginning of coordinates and viewer along the axle/axis of Ox'_g.

Rectangular coordinate systems, connected with the center of mass of aircraft.

The earth-based coordinate system of $(x_gy_gz_g)$. The direction of the axle/axes of system coincides with the direction of the axle/axes of terrestrial surface system.

The wind coordinate system of $(x_cy_cz_c)$. The axle/axis of Ox_c is directed posigrade flight tangentially toward trajectory; the axle/axis of Oy_c lie/rests at the plane of the symmetry of aircraft; the axle/axis of Oz_c is directed to the side of the right along flight outer plane of wing.

Page 15.

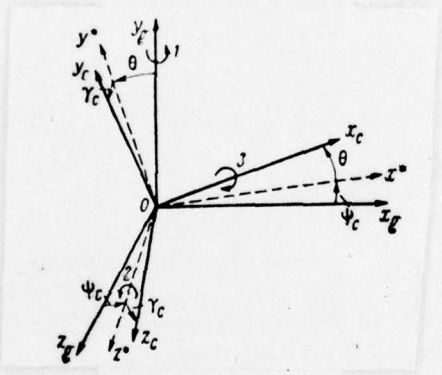
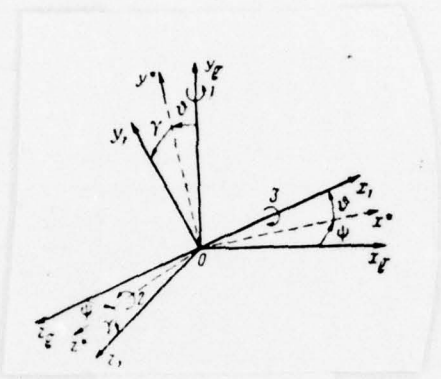


Fig. 1.1. The relative location of terrestrial and wind coordinate systems.

Fig. 1.2. Relative location terrestrial and body coordinate systems.



High-speed/velocity system is oriented relative to earth-based coordinate system by three angles (Fig. 1.1): by the angle of the ψ_c between the projection of velocity vector on the local horizontal plane and the axle/axis of Ox_g; the flight path angle θ - between the axle/axis of γ_c and the local horizontal plane; the angle of the Ox_c between the axle/axis of Oy_c and the local vertical plane, which contains the axle/axis of Ox_c.

In order to pass from earth-based coordinate system to high-speed/velocity, it is necessary to turn terrestrial system relative to the axle/axis of Oy_g to the angle ot ψ_c, then around axle/axis OZ * to angle θ and finally around the axle/axis of Ox_c to the angle of γ_c.

Body coordinate system ($x_1y_1z_1$). Axle/axis Ox_1 is directed forward in parallel to the mean aerodynamic chord of wing and lie/rests at the plane of the symmetry of the aircraft: axle/axis Oy_1 lie/rests at the plane of symmetry and is directed upward; axle/axis Oz_1 the perpendicular to plane x_1Oy_1 and is directed to the side of the right outer plane of wing. Body coordinate system is oriented relative to terrestrial by three angles (Fig. 1.2): by the yaw angle ψ - between the projection of axle/axis Ox_1 on the local horizontal

plane and the axle/axis of Ox_g; the pitch angle ϑ - between axle/axis Ox_1 and the local horizontal plane; the attitude γ - between axle/axis Oy_1 and the local vertical plane, which contains axle/axis Ox_1.

In order to pass from earth-based coordinate system to that which was connected, it is necessary to first turn terrestrial system around the axle/axis of Oy_g to angle of ψ, then around axle/axis Oz * to angle ϑ and finally around axle/axis Ox_1 to angle γ.

High-speed/velocity and body coordinate systems are oriented relative to each other by two angles (Fig. 1.3): by angle of attack α - between the projection of the axle/axis of Ox_c (velocity vector) to the plane of the symmetry of aircraft and axle/axis Ox_1; by angle β - between the axle/axis of Ox_c and the plane of symmetry.

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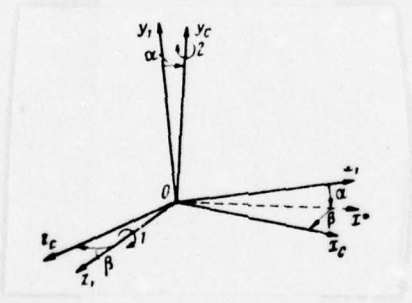

Fig. 1.3. Relative location high-speed/velocity and body coordinate systems.

In order to pass from body coordinate system to high-speed/velocity, it is necessary to turn body-fixed system around axle/axis Oz_1 to angle α, and then around the axle/axis of Oy_c to angle β.

Half-connected coordinate system (x'y'z'). Axle/axis Ox' coincides with the projection of the velocity vector of the center of mass on the plane of the symmetry of aircraft; axle/axis Oy' lie/rests at the plane of the symmetry of aircraft; axle/axis Oz' is perpendicular to plane x'Oy' and is directed to the side of the right outer plane of wing. The half-connected system is oriented by relatively high-speed/velocity angle β.

During positive motion and bank, the half-connected coordinate system coincides with high-speed/velocity.

During the study of the motion of aircraft in the lower layers of the atmosphere the coordinate system can be considered inertial. System of equations (1.2) makes it possible to investigate both the trajectory tasks and the tasks of the orientation of aircraft relative to inertial reference system.

During the study of the trajectory of the motion of the center of mass, usually is accepted the second equation of system (1.2) as that becoming identical and is limited by the solution to only first equation of system (1.2). The indicated condition is provided by the appropriate control displacements of aircraft (by elevators and direction, by ailerons).

During the analysis of the rotation/revolution of the aircraft of its relatively center of mass, it is necessary to examine both equations of system (1.2). The tasks of this circle are one of the stages of the determination of the dynamic properties of aircraft and are tightly connected with the solution to the stability problems and controllability.

1.2. Equations of motion of aircraft in body coordinate system.

By the arbitrary body coordinate system, is understood the system whose axle/axes have fixed attitude relative to aircraft. A

special case of this system is the body-fixed system with beginning in the center of mass. In body-fixed systems torque/moments the inertias of aircraft with the constant of its masses do not depend on time.

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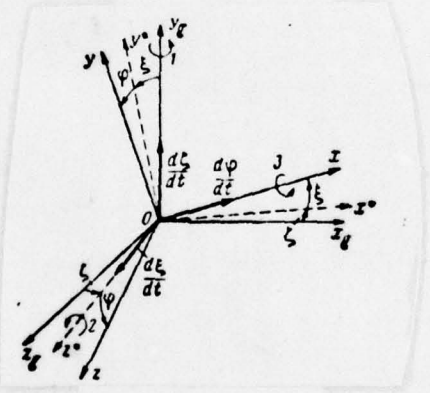

Fig. 1.4. To relationship determination between the kinematic parameters of motion in the connected and earth-based coordinates system.

As it is bygone shown in § 1.1, for research on the motion of aircraft, the system of vector equations (1.2) is examined in projections on the axis of the selected coordinate system, as a result of which instead of equations (1.2) is obtained system six of scalar differential equations of motion, which relate the kinematic parameters of motion with forces and the torque/moments, which act on aircraft.

The selection of the axes as it is shown above, is determined by the character of the tasks in question. So, research on trajectory tasks usually is conducted in wind coordinate system; the tasks of stability and controllability as a rule, are examined in body coordinate system. Both these coordinate systems move in space, since their beginning is combined with the center of mass of aircraft.

Let us examine passage from earth-based coordinate system to arbitrary that whichwas connected. For this purpose is consistent the arbitrary body-fixed system coordinates xyz with the terrestrial $x_g y_g z_g$. Is attained three last three rotations of system xyz relative to $x_g y_g z_g$ (Fig. 1.4): the first - around the axle/axis of Oy_g to angle ζ at the angular velocity $d\zeta/dt$; the second - around axle/axis Oz * to angle ε at the angular velocity $d\varepsilon/dt$ and

the third - around axle/axis Ox to angle φ at the angular velocity $d\varphi/dt$. Then the projections of the $\omega_x, \omega_y, \omega_z$ of angular velocity vector $\vec{\omega}$, which determine the rotation/revolution of system xyz relative to $x_g y_g z_g$, on the coordinate axes of system xyz it is possible to write in the form

$$\left.\begin{aligned}
\omega_x &= \frac{d\varphi}{dt} + \frac{d\zeta}{dt}\sin\xi,\\
\omega_y &= \frac{d\zeta}{dt}\cos\xi\cos\varphi + \frac{d\xi}{dt}\sin\varphi,\\
\omega_z &= \frac{d\xi}{dt}\cos\varphi - \frac{d\zeta}{dt}\cos\xi\sin\varphi.
\end{aligned}\right\} \qquad (1.3)$$

during the rotation/revolution of system xyz relative to $x_g y_g z_g$ unit vectors $\vec{i}$, $\vec{j}$, $\vec{k}$, which determine directions of axle/axes Ox, Oy, Oz, accomplish the rotary motion of the relatively unit vectors of the $\vec{i}_g, \vec{j}_g, \vec{k}_g$ of the determining directions Ox_g, Oy_g, Oz_g.

Since any the assigned in space in module/modulus and direction arbitrary vector $\vec{a}$ does not depend on the selection of the coordinate systems (it is invariant to them), it it is possible to write in the form

$$\vec{a}=a_x\vec{i}+a_y\vec{j}+a_z\vec{k}. \qquad (1.4)$$

differentiating this relationship with respect to time t, we will obtain the following expression for a derivative of a vector $\vec{a}$:

$$\frac{d\vec{a}}{dt}=\frac{da_x}{dt}\vec{i}+\frac{d\vec{a}_y}{dt}j+\frac{da_z}{dt}\vec{k}+a_x\frac{d\vec{i}}{dt}+a_y\frac{d\vec{j}}{dt}+a_z\frac{d\vec{k}}{dt}. \qquad (1.5)$$

unit vectors $\vec{i}$, $\vec{j}$, $\vec{k}$, on module/modulus equal to unity, they rotate at angular velocity $\vec{\omega}$. Therefore their time derivatives are equal to:

$$\left.\begin{aligned} \frac{d\vec{i}}{dt} &= \vec{\omega} \times \vec{i}, \\ \frac{d\vec{j}}{dt} &= \vec{\omega} \times \vec{j}, \\ \frac{d\vec{k}}{dt} &= \vec{\omega} \times \vec{k}. \end{aligned}\right\} \qquad (1.6)$$

substituting (1.6) in (1.5), we will obtain

$$\frac{d\vec{a}}{dt}=\frac{da_x}{dt}\vec{i}+\frac{da_y}{dt}\vec{j}+\frac{da_z}{dt}\vec{k}+a_x(\vec{\omega}\times\vec{i})+a_y(\vec{\omega}\times\vec{j})+a_z(\vec{\omega}\times\vec{k}). \tag{1.7}$$

the first three of terms in equality (1.7) they characterize the rate of change in vector $\vec{a}$, received by observer, who rotates together with the ccordinate system. They define so that called local derivative:

$$\frac{\tilde{d}\vec{a}}{dt}=\frac{da_x}{dt}\vec{i}+\frac{da_y}{dt}\vec{j}+\frac{da_z}{dt}\vec{k}. \tag{1.8}$$

the second three of terms in equality (1.7) it is possible to

write (on the basis of the properties of the product of two vectors) as

$$a_x(\vec{\omega}\times\vec{i})+a_y(\vec{\omega}\times\vec{j})+a_z(\vec{\omega}\times\vec{k})=\vec{\omega}\times\vec{a}. \qquad (1.9)$$

by substituting (1.8) and (1.9) in (1.7), we will obtain

$$\frac{d\vec{a}}{dt}=\frac{\tilde{d}\vec{a}}{dt}+\vec{\omega}\times\vec{a}. \qquad (1.10)$$

Expression (1.10) establish/installs communication/connection between the absolute $\frac{d\vec{a}}{dt}$ and the local $\frac{\tilde{d}\vec{a}}{dt}$ by derivatives.

We will be turned now to system (1.2). Taking into account formulas (1.10) time derivatives of the velocity vectors and moment of momentum can be presented in the form

$$\frac{d\vec{V}}{dt}=\frac{\tilde{d}\vec{V}}{dt}+\vec{\omega}\times\vec{V}, \tag{1.11}$$

$$\frac{d\vec{K}}{dt}=\frac{\tilde{d}\vec{K}}{dt}+\vec{\omega}\times\vec{K}. \tag{1.12}$$

by substituting (1.11) and (1.12) in (1.2), we will obtain equations of motion in the following form:

$$m\left(\frac{d\vec{V}}{dt}+\vec{\omega}\times\vec{V}\right)=\vec{R}, \qquad (1.13)$$

$$\frac{d\vec{K}}{dt}+\vec{\omega}\times\vec{K}=\vec{M}. \qquad (1.14)$$

design/projecting in equations (1.13) and (1.14) vector quanople on the axis of coordinates, we will obtain the following systems of equations of motion in projections on the axis of the arbitrary body coordinate system:

$$\left.\begin{aligned} m\left(\frac{d\tilde{V}_x}{dt}+\omega_y V_z-\omega_z V_y\right)&=\sum X,\\ m\left(\frac{d\tilde{V}_y}{dt}+\omega_z V_x-\omega_x V_z\right)&=\sum Y,\\ m\left(\frac{d\tilde{V}_z}{dt}+\omega_x V_y-\omega_y V_x\right)&=\sum Z; \end{aligned}\right\} \qquad (1.15)$$

$$\left.\begin{aligned} \frac{d\tilde{K}_x}{dt}+\omega_y K_z-\omega_z K_y&=\sum M_x,\\ \frac{d\tilde{K}_y}{dt}+\omega_z K_x-\omega_x K_z&=\sum M_y,\\ \frac{d\tilde{K}_z}{dt}+\omega_x K_y-\omega_y K_x&=\sum M_z. \end{aligned}\right\} \qquad (1.16)$$

here $V_x, V_y, V_z, \omega_x, \omega_y, \omega_z, K_x, K_y, K_z$ - the projection of vectors of $\vec{V}, \vec{\omega}, \vec{K}$ on coordinate axes;

$\Sigma X, \Sigma Y, \Sigma Z, \Sigma M_x, \Sigma M_y, \Sigma M_z$ are projections of the main vector $\vec{R}$ and of the main torque/moment $\vec{M}$ on the same axle/axis.

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Equations (1.15) determine the motion of the center of mass of aircraft, equation (1.16) - the rotation/revolution of aircraft around its center of mass.

The projections of the moment of momentum are determined by the known from mechanics expressions:

$$\left.\begin{aligned} K_x &= \omega_x J_x - \omega_y J_{xy} - \omega_z J_{xz}, \\ K_y &= \omega_y J_y - \omega_z J_{yz} - \omega_x J_{yx}, \\ K_z &= \omega_z J_z - \omega_x J_{zx} - \omega_y J_{zy}, \end{aligned}\right\} \qquad (1.17)$$

where the $J_x, J_y, J_z, J_{xy}, J_{yz}, J_{zx}$ they are determined respectively axial and products of inertia relative to the selected coordinate axes. They are determined by the known from theoretical mechanics relationships:

$$
\left.
\begin{aligned}
J_x &= \sum (y_i^2 + z_i^2)\, m_i, \\
J_y &= \sum (z_i^2 + x_i^2)\, m_i, \\
J_z &= \sum (x_i^2 + y_i^2)\, m_i, \\
J_{xy} &= \sum x_i y_i m_i, \\
J_{yz} &= \sum y_i z_i m_i, \\
J_{zx} &= \sum z_i x_i m_i.
\end{aligned}
\right\} \qquad (1.18)
$$

axial and products of inertia depend on the mass of aircraft, position of its control surfaces and other factors, which in the

general case are the functions of time. In the study of problems, the dynamics of flight frequently on mass change disregard, and controls are considered fixed. In this case the moments of inertia relative to the axle/axes, connected with aircraft, become constants.

By differentiating expressions (1.17) and by substituting derivatives of the projections of the moment of momentum along time into equations (1.16), we will obtain system of equations in projections on the axis xyz, which determines the rotation/revolution of aircraft around the center of mass:

$$\left.\begin{aligned} &J_x\frac{d\omega_x}{dt}+(J_z-J_y)\omega_y\omega_z+J_{xy}\left(\omega_x\omega_z-\frac{d\omega_y}{dt}\right)-\\ &\quad-J_{xz}\left(\omega_x\omega_y+\frac{d\omega_z}{dt}\right)+J_{yz}\left(\omega_z^2-\omega_y^2\right)=\sum M_x,\\ &J_y\frac{d\omega_y}{dt}+(J_x-J_z)\omega_z\omega_x-J_{xy}\left(\omega_y\omega_z+\frac{d\omega_x}{dt}\right)+\\ &\quad+J_{yz}\left(\omega_x\omega_y-\frac{d\omega_z}{dt}\right)+J_{xz}\left(\omega_x^2-\omega_z^2\right)=\sum M_y, \end{aligned}\right\} \quad (1.19)$$

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$$\left.\begin{aligned} J_z\frac{d\omega_z}{dt}+(J_y-J_x)\omega_x\omega_y+J_{xz}\left(\omega_y\omega_z-\frac{d\omega_x}{dt}\right)-\\ -J_{yz}\left(\omega_x\omega_z+\frac{d\omega_y}{dt}\right)+J_{xy}\left(\omega_y^2-\omega_x^2\right)=\sum M_z. \end{aligned}\right\} \quad (1.19)$$

To the system of equations (1.19) in conjunction with system of equations (1.15) and by kinematic relationships (1.3) completely it determines the motion of aircraft.

If aircraft has a plane of material symmetry [1], then, by combining plane xOy with the plane of symmetry, it is possible to considerably simplify system (1.19).

FOOTNOTE [1]One plane of material symmetry they have virtually all contemporary aircraft. ENDFOOTNOTE.

In this case axle/axis Oz will be the principal axis of inertia and the products of inertia of J_{xz}, J_{yz} they will be equal to zero. System (1.19) of signs form

$$\left.\begin{aligned}
J_x\frac{d\omega_x}{dt}+(J_z-J_y)\omega_y\omega_z+J_{xy}\left(\omega_x\omega_z-\frac{d\omega_y}{dt}\right)&=\sum M_x,\\
J_y\frac{d\omega_y}{dt}+(J_x-J_z)\omega_z\omega_x-J_{xy}\left(\omega_y\omega_z+\frac{d\omega_x}{dt}\right)&=\sum M_y,\\
J_z\frac{d\omega_z}{dt}+(J_y-J_x)\omega_x\omega_y+J_{xy}\left(\omega_y^2-\omega_x^2\right)&=\sum M_z.
\end{aligned}\right\} \quad (1.20)$$

If body has an axis of material symmetry, then, by combining axle/axis ox with the axis of symmetry, it is possible to obtain the even simpler form of the recording of equations (1.19). In this case all products of inertia are equal to zero; furthermore, on the strength of the symmetry of $J_y=J_z$. Taking into account these equalities system (1.19) will take form

$$\left.\begin{aligned} J_x\frac{d\omega_x}{dt} &= \sum M_x, \\ J_y\frac{d\omega_y}{dt}+(J_x-J_z)\omega_z\omega_x &= \sum M_y, \\ J_z\frac{d\omega_z}{dt}+(J_y-J_x)\omega_x\omega_y &= \sum M_z. \end{aligned}\right\} \qquad (1.21)$$

equation (1.3), (1.15), (1.19) are made it possible to solve both straight line and borate task the dynamics of the flight of aircraft. Integration of these equations makes it possible: - to determine communication/connection between the forces, which act on aircraft, by the trajectory of its flight and by the kinematic parameters of motion (by velocity and the angular position of aircraft in space); - to determine the stability characteristics and aircraft handling in the different mode/conditions of its flight; -

to determine the required angles of deflection of the controls and value of forces of controls for a flight with respect to the predetermined trajectory.

1.3. Equations of motion of aircraft in the connected $(x_1 y_1 z_1)$ and high-speed/velocity (x_c, y_c, z_c) coordinate systems.

As it is bygone said above, the trajectory problems of the motion of aircraft to conveniently examine in wind coordinate system, but many tasks of stability and controllability simpler are solved in body coordinate system.

For passage to body coordinate system, are necessary in equations (1.3), (1.15) and (1.20) instead of the angles ζ, ξ, ϕ to introduce respectively angles of ψ, ϑ, γ, orienting body coordinate system relative to terrestrial.

By producing the indicated substitution, we will obtain for the aircraft, which has the plane of symmetry $x_1 O y_1$, the following systems of equations of motion and kinematic relationships in body

coordinate system:

$$\left.\begin{aligned} m\left(\frac{dV_{x1}}{dt}+\omega_{y1}V_{z1}-\omega_{z1}V_{y1}\right)&=\sum X_1,\\ m\left(\frac{dV_{y1}}{dt}+\omega_{z1}V_{x1}-\omega_{x1}V_{z1}\right)&=\sum Y_{z1},\\ m\left(\frac{dV_{z1}}{dt}+\omega_{x1}V_{y1}-\omega_{y1}V_{x1}\right)&=\sum Z_1;\end{aligned}\right\} \tag{1.22}$$

$$\left.\begin{aligned} J_{x1}\frac{d\omega_{x1}}{dt}+(J_{z1}-J_{y1})\omega_{y1}\omega_{z1}+J_{x1y1}\left(\omega_{x1}\omega_{z1}-\frac{d\omega_{y1}}{dt}\right)&=\sum M_{x1},\\ J_{y1}\frac{d\omega_{y1}}{dt}+(J_{x1}-J_{z1})\omega_{z1}\omega_{x1}-J_{x1y1}\left(\omega_{y1}\omega_{z1}+\frac{d\omega_{x1}}{dt}\right)&=\sum M_{y1},\\ J_{z1}\frac{d\omega_{z1}}{dt}+(J_{y1}-J_{x1})\omega_{x1}\omega_{y1}+J_{x1y1}\left(\omega_{y1}^2-\omega_{x1}^2\right)&=\sum M_{z1};\end{aligned}\right\} \tag{1.23}$$

$$\left.\begin{aligned} \omega_{x1}&=\frac{d\gamma}{dt}+\frac{d\psi}{dt}\sin\vartheta,\\ \omega_{y1}&=\frac{d\psi}{dt}\cos\vartheta\cos\gamma+\frac{d\vartheta}{dt}\sin\gamma,\\ \omega_{z1}&=\frac{d\vartheta}{dt}\cos\gamma-\frac{d\psi}{dt}\cos\vartheta\sin\gamma.\end{aligned}\right\} \tag{1.24}$$

for passage to wind coordinate system let us realize that the origin of coordinates is located in the center of mass; then in equations (1.3) angles ζ, ε, φ are the angles of ψ_c, θ, γ_c.

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Furthermore, it is necessary to consider that in wind coordinate system the velocity vector of $\vec{V}$ is directed along the axle/axis of Ox_c and, consequently:

$$\left.\begin{aligned} &V_{xc}=V,\quad V_{yc}=0,\quad V_{zc}=0,\\ &\frac{d\tilde{V}_{xc}}{dt}=\frac{dV}{dt},\quad \frac{d\tilde{V}_{yc}}{dt}=0,\quad \frac{d\tilde{V}_{zc}}{dt}=0. \end{aligned}\right\} \qquad (1.25)$$

taking into account these considerations of equation (1.15) and

kinematic relationships (1.3) in wind coordinate system take the form

$$\left.\begin{aligned} m\frac{dV}{dt} &= \sum X_c, \\ mV\omega_{zc} &= \sum Y_c, \\ -mV\omega_{yc} &= \sum Z_c; \end{aligned}\right\} \tag{1.26}$$

$$\left.\begin{aligned} \omega_{xc} &= \frac{d\gamma_c}{dt} + \frac{d\psi_c}{dt}\sin\theta, \\ \omega_{yc} &= \frac{d\psi_c}{dt}\cos\gamma_c\cos\theta + \frac{d\theta}{dt}\sin\gamma_c, \\ \omega_{zc} &= \frac{d\theta}{dt}\cos\gamma_c - \frac{d\psi_c}{dt}\cos\theta\sin\gamma_c, \end{aligned}\right\} \tag{1.27}$$

where the $\Sigma X_c, \Sigma Y_c, \Sigma Z_c$ - the projection of the main vector of external forces on the axis of wind coordinate system.

Contemporary gyroscopes record the projections of the angular velocity vector of the rotation/revolution of aircraft around their center of mass on the axis of body coordinate system. Therefore the projections of angular velocity vector on the axis of wind coordinate system are determined through the measured in flight projections this same vector $\vec{\omega}$ on the axis of body coordinate system.

Let us find communication/connection between the projections of vector $\vec{\omega}$ on the axis high-speed/velocity and body coordinate systems. The projections of $\omega_{x1}, \omega_{y1}, \omega_{z1}$ give on the axis of Ox_c, Oy_c, Oz_c components (Fig. 1.5)

$$\left.\begin{aligned} &\omega_{x1}\cos\alpha\cos\beta+\omega_{z1}\sin\beta-\omega_{y1}\sin\alpha\cos\beta,\\ &\omega_{y1}\cos\alpha+\omega_{x1}\sin\alpha,\\ &-\omega_{x1}\cos\alpha\sin\beta+\omega_{y1}\sin\alpha\sin\beta+\omega_{z1}\cos\beta. \end{aligned}\right\} \qquad (1.28)$$

furthermore, on the same axle/axis are design/projected and vectors $\frac{\vec{d\alpha}}{dt}, \frac{\vec{d\beta}}{dt}$, which determine the rotation of Wind coordinate system around that which was connected at angles α and β and which have form high-speed/velocity and body coordinate systems.

$$-\frac{d\alpha}{dt}\sin\beta, -\frac{d\beta}{dt}, -\frac{d\alpha}{dt}\cos\beta. \qquad (1.29)$$

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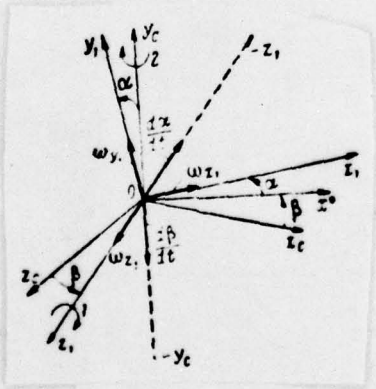

Fig. 1.5. To the determination of communication/connection between the projections of the angular rate of rotation in

By totaling (1.28) and (1.29), we will obtain communication/connection of ω_{xc}, ω_{yc}, ω_{zc} с ω_{x1}, ω_{y1}, ω_{z1} in the form

$$\left.\begin{aligned}
\omega_{xc} &= \omega_{x1}\cos\alpha\cos\beta + \omega_{z1}\sin\beta - \omega_{y1}\sin\alpha\cos\beta - \frac{d\alpha}{dt}\sin\beta,\\
\omega_{yc} &= \omega_{y1}\cos\alpha + \omega_{x1}\sin\alpha - \frac{d\beta}{dt},\\
\omega_{zc} &= -\omega_{x1}\cos\alpha\sin\beta + \omega_{y1}\sin\alpha\sin\beta + \omega_{z1}\cos\beta - \frac{d\alpha}{dt}\cos\beta.
\end{aligned}\right\} \quad (1.30)$$

in cases when flight is realized without slip, relationship (1.30) considerably they are simplified, taking the form

$$\left.\begin{aligned}\omega_{xc} &= \omega_{x1}\cos\alpha - \omega_{y1}\sin\alpha,\\ \omega_{yc} &= \omega_{y1}\cos\alpha + \omega_{x1}\sin\alpha,\\ \omega_{zc} &= \omega_{z1} - \frac{d\alpha}{dt}.\end{aligned}\right\} \quad (1.31)$$

of equation (1.26) together with kinematic relationships (1.30) and (1.31) they determine the motion of the center of mass of aircraft in wind coordinate system.

High-speed/velocity and body coordinate systems most frequently they are applied in the dynamics of flight. However, by this is not eliminated the possibility of applying any another coordinate system, which can turn out to be more convenient during the study of one or the other concrete/specific/actual flight conditions.

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Page 25. Chapter II.

EQUATIONS OF MOTION FOR (Calculating) THE FLIGHT-TRAJECTORY ~~CALCULATION~~ OF AIRCRAFT.

§ 2.1. Forces, which act on aircraft.

In the present chapter are investigated the trajectories of the motion of the center of mass of aircraft in connection with the aircraft of the civil aviation. For the flight-trajectory calculation, as this is bygone shown in chapter I, are utilized the equations, which describe the motion of the center of mass of aircraft under the action of the assigned external forces.

On the aircraft, which moves at relatively small distances from the earth's surface (precisely such we will subsequently examine), act three basic groups of the forces: a) the mass forces, caused by the earth's gravity and the inertia of motion; b) the aerodynamic forces, which appear as a result of the interaction of aircraft with its circumfluent airflow; c) the thrust of the motors, established/installed on aircraft.

The indicated forces determine the main force vector $\vec{R}$, and also, therefore, its projections on the axis of the adopted system of coordinates.

Mass forces. The mass forces, caused by the earth's gravity, include the weight of body G, equal to the product of the mass of

body for free-fall acceleration g.

Free-fall acceleration is different at the different points of terrestial globe, that is caused by the nonuniformity of mass distribution within the volume of earth, and also by its flatness in meridian cuts. Thus, for instance, at equator the value of free-fall acceleration g of the surface of earth amounts on the average to 9.78 m/s^2, and of poles 9.83 m/s^2. With removal/distance from earth, the free-fall acceleration decreases in connection with the weakening of the gravitational field of earth.

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In the problems, connected with the flights of aircraft at small height/altitudes, set/assuming its constant and equal to 9.81 m/s^2.

The mass forces, caused by inertia of motion, include the force of inertia, equal on module/modulus (according to second Newton's law) to the product of the mass of the driving body for the acceleration of his motion.

Aerodynamic forces depend on the form of the body, value and sense of the vector of flight speed relative to body and the characteristics of the environment. In lower layers of the atmosphere (H < 30 km), where the air density is sufficiently great, aerodynamic forces can reach great significance; therefore during flights at these height/altitudes, aerodynamic forces will be those which determine. In upper air (H > 70 km) air density sharply falls, the value of aerodynamic forces decreases by their influence sometimes it is possible to disregard.

Communication/connection between aerodynamic forces and factors, determining them, is establish/installed by experimental aerodynamics by means of formulas

$$\left.\begin{aligned} X &= c_x \frac{\rho V^2}{2} S, \\ Y &= c_y \frac{\rho V^2}{2} S, \\ Z &= c_z \frac{\rho V^2}{2} S, \end{aligned}\right\} \qquad (2.1)$$

where ϱ — the density of the medium in which is realized the flight; S - characteristic is area (usually as characteristic area is accepted wing area); c_x, c_y, c_z — the aerodynamic coefficients, which are determined from the results of the model test of aircraft in wind tunnels or flight tests. There are also theoretical methods of the determination aerodynamic coefficients, but here they are not examined.

The thrust of motors depends on the engine power rating, height/altitude, flight speed and other factors. Direction of thrust

usually is considered coinciding with the direction of the axle/axis of engine, although this and not always so (for example in the engine operation in the mode/conditions of oblique airflow the direction of the thrust of engine can not coincide with its axle/axis). When, on the aircraft, several engines are present, one should consider the gross thrust of all engines and the possible asymmetry of their work.

On the contemporary aircraft of the civil aviation, are applied in essence three types of the engines: pistcn (PD [instrument panel]), turboprop (TVD [turboprop engine]) and turbojet (TRD [turbojet engine]).

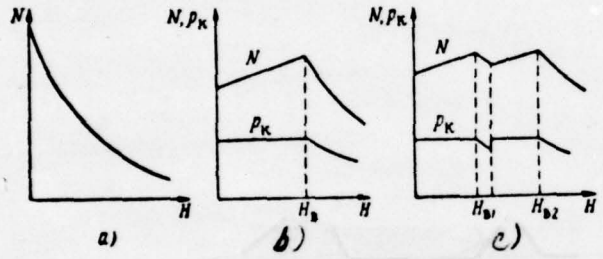


Fig. 2.1. Altitude-speed characteristics of the piston engine: a) low-level PD; b) PD with one-velocity supercharger; c) PD with two-speed supercharger.

The effectiveness of one engine or the other is determined by value created to them thrust/rods (or power); cost-effectiveness/efficiency - by the specific fuel consumption, which are the fuel consumption per unit of thrust/rod (or per unit of power) per unit time.

The engine characteristics are assigned either in the form of the dependence, which links the power of engine with speed and flight altitude (for PD and TVD), or in the form of the dependence between the thrust of engine and flight speed, its height/altitude and the degree of the throttling of engines (for TRD).

Piston engines (PD) structurally can be executed with pressurized system and without it. Pressurized system makes it possible to raise pressure in jugs and thereby to increase its power. Pressurization/supercharging PD is realized usually by the gear-driven centrifugals supercharger (PTsN), which ensure the constancy of the pressurization/supercharging on of the determined height/altitude of H_B, with the called full-throttle height.

Diagrams typical for pistan engines showing the change of power N and change of supercharging pressure p_k according to altitude, without considering flight speed, are shown in Fig. 2.1. When flight speed is considered, engine power will increase thanks to a supercharging increase coming from the ram effect and thanks to an increase in the altitude limits.

Turboprop engines (TVD) unlike PD and TRD realize part of the power in the form of the reactive power which is created by the reaction of gas jets, coming out from engine nozzles, and it is ~10-20o/o of total power TVD. Therefore characteristics TVD usually are assigned for conditional, or the equivalent, power of $N_э$, by that considering shaft horsepower of the engine of $N_в$ and the reactive power of ΔN_R.

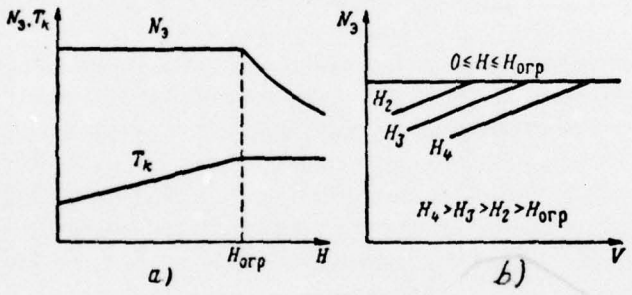
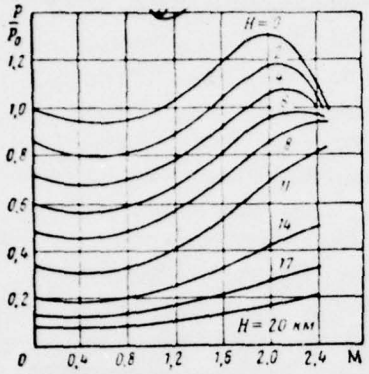


Fig. 2.2. Altitude-speed characteristics of the turboprop engine: a) a change in the power TVD and in the temperature of the gases before the turbine by height: b) a change in the power TVD in speed.

Fig. 2.3. Altitude-speed characteristics of turbojet engine.



The control system TVD in all operating conditions, close to maximum, provides fulfilling the two requirements: the preservation/retention/maintaining of the equivalent horsepower of constant up to the flight altitude of $H_{огр}$ (zone of the limitation of power) and maintaining the temperature of the gases before the turbine of constant in the zone of height/altitudes, large $H_{огр}$. The typical for TVD diagrams of a change in power and temperature of the gases before the turbine of $T_к$ are given in Fig. 2.2a. With an increase in the velocity of flight, the power TVD has a tendency toward increase because of the velocity head which raises ramming and which increases the mass of the passing through the engine air. However, in the zone of the limitation of power (Fig. 2.2b). Higher than the $H_{огр}$ power increases with an increase of velocity.

Turbojet engines (TRD) unlike PD and TVD realize thrust/rod P directly in the form of reaction the scab of the gases, which escape/ensue of engine nozzle. An example of altitude-speed characteristic TRD is given in Fig. 2.3, where P_0 are given the static thrust of engine at height/altitude H = 0.

In the practice of aerodynamic designs, it is necessary to deal with so as those called by the point of tangency and with a power, that ensure the flight of aircraft in air medium. For PD available power can be determined by formula

$$N_p=(N_{c.н}-\Delta N_м)\eta_в+\Delta N_R,$$

where the $N_{c.н}$ - the power, removed from the engine characteristic and corrected for high-speed/velocity pressurization/supercharging; $\Delta N_м$ - the power is lost as a result of an increase in the exhaust backpressure; ΔN_R - the power, created by the reaction of gases on exhaust; $\eta_в$ - the propeller efficiency.

For TVD

$$N_p=N_в\eta_в+\Delta N_R,$$

where the $N_в$ - the power, transferred to screw/propeller; ΔN_R - the power, realized at the nozzle outlet.

Point of tangency TRD is remove/taken directly from the engine characteristics.

The aerodynamic forces and the thrust determine the g-force, which is important aerodynamic characteristic. In the general case by g-force, is understood vector $\vec{n}$, equal to the ratio of a difference in vectors $\vec{R}$ and $\vec{G}$ to the module/modulus of gravitational force G:

$$\vec{n}=\frac{\vec{R}-\vec{G}}{|G|}. \qquad (2.2)$$

The projection of vector $\vec{n}$ on the coordinate axis Ox is called the longitudinal acceleration of (n_x), to axle/axis Oy - by the normal load factor of (n_y), to axle/axis Oz - lateral, or transverse, by the g-force of (n_z). If in relationship (2.2) the quantity of vector R is accepted in accordance with the first equation system (1.2) that for a g-force we will obtain expression

$$\vec{n}=\frac{1}{|g|}\left(\frac{d\vec{V}}{dt}-\vec{g}\right), \qquad (2.3)$$

into right side of which they enter only accelerations. In flight along the ballistic trajectory $\vec{R} = \vec{G}$, i.e., $\vec{n} = 0$, therefore, all bodies within aircraft are found in null gravity state.

§ 2.2. Equations of motion of the center of mass of aircraft.

The motion of the center of mass is conveniently examined in wind coordinate system. In this case the system of equations of the motion of the center of mass in accordance with equations (1.26) [1] will be represented in the form

$$\left.\begin{aligned} m\frac{dV}{dt} &= \sum X, \\ mV\omega_z &= \sum Y, \\ -mV\omega_y &= \sum Z. \end{aligned}\right\} \qquad (2.4)$$

FOOTNOTE [1]. The index of "c" subsequently for the simplicity of recording let us omit. ENDFOOTNOTE.

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1 Внешние силы в проекциях на оси систем, в которых они заданы	2 Оси других систем	3 Оси скоростной системы		
		Ox_c	Oy_c	Oz_c
$-G$	Oy_g	$\sin\theta$	$\cos\theta\cos\gamma_c$	$-\cos\theta\sin\gamma_c$
$P\cos\varphi_{дв}$	Ox_1	$\cos\alpha\cos\beta$	$\sin\alpha$	$-\cos\alpha\sin\beta$
$P\sin\varphi_{дв}$	Oy_1	$-\sin\alpha\cos\beta$	$\cos\alpha$	$\sin\alpha\sin\beta$

Table 2.1.

Key: (1). External forces in projections on the axis of the systems in which they are assigned/prescribed. (2). Axle/axes of other systems. (3). Axle/axes of high-speed/velocity system.

In the right sides of equations (2.4) enter the sums of projections on the coordinate axes of aerodynamic forces, thrust/rod of engines and weight of aircraft. These forces can be assigned/prescribed in the different coordinate systems: aerodynamic forces usually are assigned in high-speed/velocity system, thrust/rod they is assigned in that connected weight - in terrestrial. Therefore during the determination of their projections on the axis of wind coordinate system, it is necessary to consider the direction cosines with the aid of which is realized the passage from the different coordinate systems to high-speed/velocity. These direction cosines are given in Table 2.1.

Since the weight of aircraft G is always directed along the axle/axis of the Oy_g of earth-based coordinate system, then in this table are given only the cosines of the angles between the axle/axis of Oy_g and the axle/axes of Oy_c, Ox_c, Oz_c . The axle/axes of engines can not coincide with axle/axis Ox_1, if they are established/installed at an angle $\pm\varphi_{AB}$ with respect to axle/axis Ox_1, but they always lie in the planes, parallel the planes of symmetry; therefore Table 2.1 shows the only direction cosines, which make it possible to determine the projection of the unit vectors of axle/axes Ox_1 and Oy_1 on the axis of Ox_c, Oy_c, Oz_c .

By substituting in equations (2.4) taking into account the data of Table 2.1 value of the projections of the gravitational forces and thrust/rod on the axis of wind coordinate system and by taking into account according to (1.27) the value of the projections of the angular velocity of ω_z, ω_y and also the projection of aerodynamic force [1], we will obtain the equations of the divzheniya of the center of mass of aircraft in the wind coordinate system of the following form:

$$-X-G\sin\theta+P(\cos\varphi_{AB}\cos\beta\cos\alpha-\sin\alpha\sin\varphi_{AB}\cos\beta)-$$
$$-m\frac{dV}{dt}=0;$$

$$Y-G\cos\theta\sin\gamma_c+P(\cos\varphi_{AB}\sin\alpha+\sin\varphi_{AB}\cos\alpha)-$$
$$-mV\left(\frac{d\theta}{dt}\cos\gamma_c-\frac{d\psi_c}{dt}\cos\theta\sin\gamma_c\right)=0;$$
$$Z+G\cos\theta\sin\gamma_c-P(\cos\varphi_{AB}\cos\alpha\sin\beta-\sin\varphi_{AB}\sin\alpha\sin\beta)+$$
$$+mV\left(\frac{d\theta}{dt}\sin\gamma_c+\frac{d\psi_c}{dt}\cos\theta\cos\gamma_c\right)=0.$$

FOOTNOTE [1]. Since drag is directed to the side, opposite to motion, the projection of aerodynamic force on the axle/axis of Ox_c it is necessary to take with minus sign. ENDFCOTNOTE.

If one considers that

$$\cos\varphi_{\lambda_B}\cos\alpha \mp \sin\varphi_{\lambda_B}\sin\alpha = \cos(\alpha \pm \varphi_{\lambda_B}),$$
$$\cos\varphi_{\lambda_B}\sin\alpha \pm \sin\varphi_{\lambda_B}\cos\alpha = \sin(\alpha \pm \varphi_{\lambda_B}),$$

that the obtained system can be rewritten in the form

$$\left.\begin{aligned}
&P\cos(\alpha \pm \varphi_{\lambda_B})\cos\beta - G\sin\theta - X = m\frac{dV}{dt};\\
&P\sin(\alpha \pm \varphi_{\lambda_B}) - G\cos\theta\cos\gamma_c + Y =\\
&\quad = mV\left(\frac{d\theta}{dt}\cos\gamma_c - \frac{d\psi_c}{dt}\cos\theta\sin\gamma_c\right);\\
&P\cos(\alpha \pm \varphi_{\lambda_B})\sin\beta - G\cos\theta\sin\gamma_c - Z =\\
&\quad = mV\left(\frac{d\theta}{dt}\sin\gamma_c + \frac{d\psi_c}{dt}\cos\theta\cos\gamma_c\right).
\end{aligned}\right\} \qquad (2.5)$$

. System of equations (2.5) corresponds to the general case of the unsteady flight of aircraft along curved path in the arbitrary attitude (with bank and slip) and makes it possible to calculate the kinematic parameters of motion at any point in time. The integration of the system of equations of motion can be executed only in such a case, when the number of equations is equal to the number of unknowns. In an explicit form in equations, systems (2.5) enter 11 variables:

$$P;\ X;\ Y;\ Z;\ G;\ \alpha;\ \beta;\ \theta;\ \gamma_c;\ \psi_c;\ t.$$

. Furthermore, the solution of system (2.5) will depend on the flight altitude H, connected with parameters of V, θ, t with kinematic constraint

$$\frac{dH}{dt} = V \sin\theta.$$

. Consequently, system (2.5) contains 12 variables.

during the solution of system one of the variables - usually time t - accepted as independent variable. The thrust/rod of engine P depends on the engine power rating, and also on height/altitude and flight speed. The components of aerodynamic force are determined by equalities (2.1), in which the aerodynamic

coefficients of c_x, c_y, c_z are the functions of the angles of attack and slip, and also of speed and flight altitude:

$$c_x=f(\alpha, \beta, V, H),\quad c_y=f(\alpha, \beta, V, H),\quad c_z=f(\alpha, \beta, V, H).$$

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Thus, remains seven unknowns during the available four equations - three equations of system (2.5) and equation (2.6).

The missing equations are compensated for by the additional constraints, establish/installed on the basis of research on one or the other concrete/specific/actual problem for which is conducted the calculation. Thus, for instance, in straight flight along inclined trajectory without bank and slip

$$\psi_c=0,\quad \gamma_c=0,\quad \beta=0, \vartheta=const.$$

The number of unknowns is contracted to four system of equations (2.5), (2.6) becomes closed.

Let us determine the projections of vector $\vec{n}$ on the axis of wind coordinate system, by utilizing equations (2.5). Since the left sides

of equations (2.5) are the projection of vector $\vec{R}$ on the axis of wind coordinate system, taking into account formula (2.2) for the projections of vector $\vec{n}$ we will have the following equality:

$$\left.\begin{aligned} n_x &= \frac{P\cos(\alpha \pm \varphi_{дв}) - X}{G}, \\ n_y &= \frac{P\sin(\alpha \pm \varphi_{дв}) + Y}{G}, \\ n_z &= \frac{P\cos(\alpha \pm \varphi_{дв})\sin\beta - Z}{G}. \end{aligned}\right\} \qquad (2.7)$$

By comparing equalities (2.7) and system of equations (2.5), we will obtain the formulas of communication/connection of the projections of g-force with the kinematic parameters of motion in the following form:

$$\left.\begin{aligned} n_x &= \frac{1}{g}\frac{dV}{dt} + \sin\theta, \\ n_y &= \frac{V}{g}\left(\frac{d\theta}{dt}\cos\gamma_c - \frac{d\psi_c}{dt}\cos\theta\sin\gamma_c\right) + \cos\theta\cos\gamma_c, \\ n_z &= \frac{V}{g}\left(\frac{d\theta}{dt}\sin\gamma_c + \frac{d\psi_c}{dt}\cos\theta\cos\gamma_c\right) + \cos\theta\sin\gamma_c. \end{aligned}\right\} \quad (2.8)$$

§ 2.3. Simplification in the equations of motion.

The basic flight conditions of aircraft are characterized by comparatively low angles of attack α, of slip β and by small angle of

the engine installation of $\varphi_{ДБ}$. Therefore approximately it is possible to count

$$P\sin(\alpha\pm\varphi_{Дв})\ll G\cos\theta\cos\gamma,\quad \cos\beta\approx 1{,}0,$$
$$P\cos(\alpha\pm\varphi_{Дв})\sin\beta\ll G\cos\theta\sin\gamma,$$
$$\cos(\alpha\pm\varphi_{Дв})\approx 1{,}0.$$

Taking this into account, system (2.5) somewhat is simplified, taking the form

$$\left.\begin{aligned} &P-G\sin\theta-X=m\frac{dV}{dt},\\ &-G\cos\theta\cos\gamma_c+Y=mV\left(\frac{d\theta}{dt}\cos\gamma_c-\frac{d\psi_c}{dt}\cos\theta\sin\gamma_c\right).\\ &-G\cos\theta\sin\gamma_c-Z=mV\left(\frac{d\theta}{dt}\sin\gamma_c+\frac{d\psi_c}{dt}\cos\gamma_c\cos\theta\right). \end{aligned}\right\} \quad (2.9)$$

In flying practice of the civil aviation, the unsteady flights of aircraft and along curved path compose on time small part. An even less part on time compose flights with bank and slip. Most frequently are encountered the flights of aircraft in vertical plane without bank and slip of $(\gamma_c=0;\ \beta=0)$. At this case the flight trajectory completely lie/rests at the plane of x_cOy_c, to what it corresponds the $\frac{d\psi_c}{dt}=0$, and equations of motion (2.9) take the form

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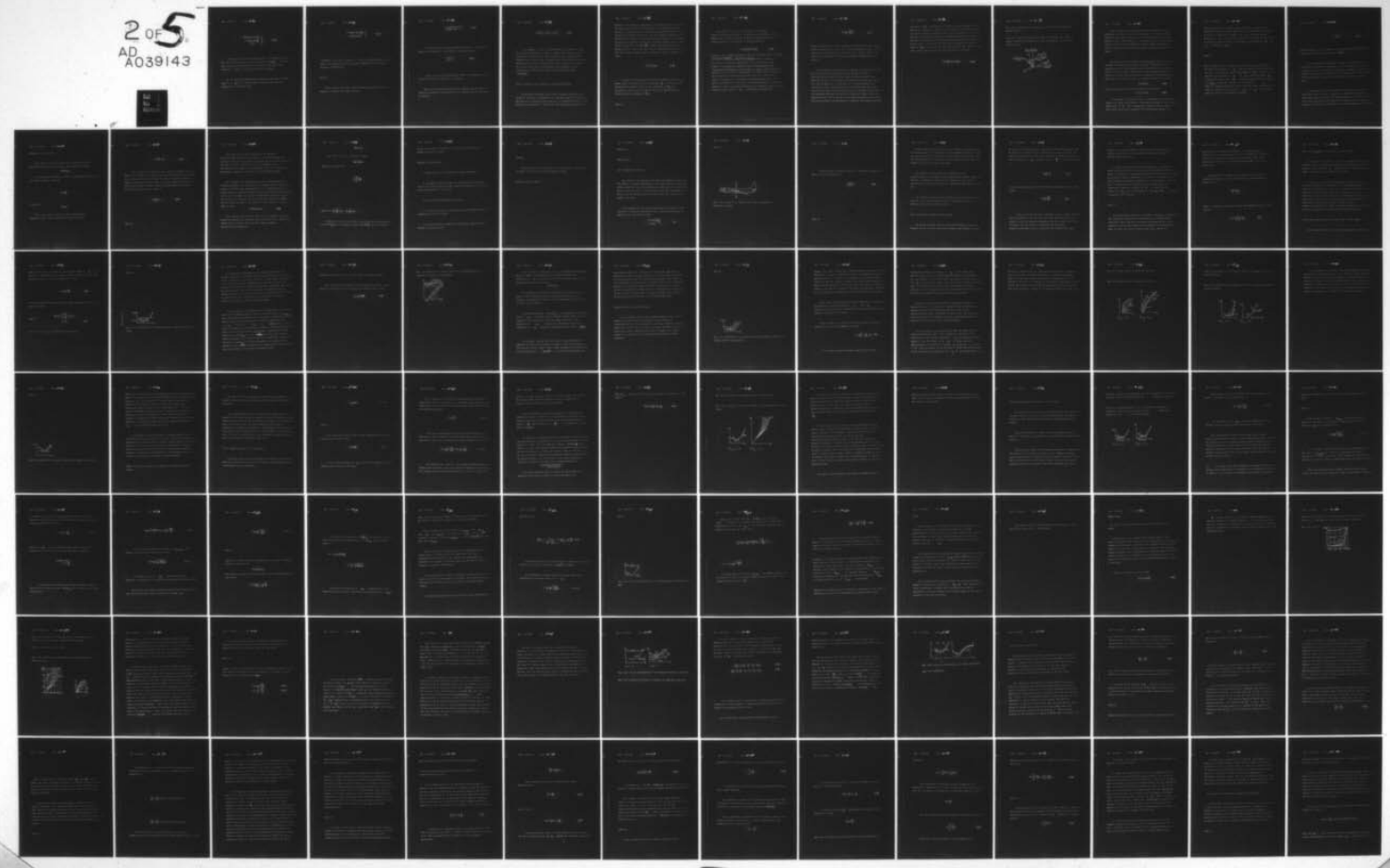

$$\left.\begin{aligned} P - G\sin\theta - X &= m\frac{dV}{dt}, \\ Y - G\cos\theta &= mV\frac{d\theta}{dt}. \\ Z &= 0. \end{aligned}\right\} \qquad (2.10)$$

Equality to zero of the lateral force Z is explained by the fact that in the absence of slip the plane of $x_c O y_c$ coincides with the plane of symmetry, and the forces, which act in the direction of Oz_c in this case they do not appear.

In the case of laterally level flight and slip along straight path $\beta = 0$, $\gamma_c = 0$ θ = const. Equations (2.10) even more are simplified [1], taking the form

$$\left.\begin{aligned} P-G\sin\theta-X&=m\frac{dV}{dt},\\ -G\cos\theta+Y&=0. \end{aligned}\right\} \qquad (2.11)$$

. FOOTNOTE [1]. The third equation Z = 0 we do not record/write, since during the determination of the flight trajectory it no longer is utilized. ENDFOOTNOTE,/

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During steady rectilinear flight without bank and slip of the equation of motion, they take the form

$$\left.\begin{aligned} P - G\sin\theta - X = 0, \\ -G\cos\theta + Y = 0. \end{aligned}\right\} \qquad (2.12)$$

. In flight along horizontal trajectory angle $\theta = 0$; therefore equations of motion are record/written entirely simply:

$$\left.\begin{aligned} P = X, \\ G = Y. \end{aligned}\right\} \qquad (2.13).$$

. Thus, in the level steady flight thrust P is balanced by drag X, and the weight of aircraft G - by lift Y.

Taking into account the made in the beginning of the present paragraph assumptions of projection, the g-forces will be determined by formulas

$$n_x=\frac{P-X}{G},\quad n_y=\frac{Y}{G},\quad n_z=\frac{Z}{G}. \qquad (2.14)$$

From formulas (2.14) it follows that in all cases of flight the normal load factor of n_y is equal to the ratio of lift to the weight of aircraft. For the aircraft of the civil aviation, normal g-force appears under negative lift. This situation can arise only during the sharp input/introduction of aircraft into dive. In the steady level flight in accordance with equalities (2.13) tangential and normal load factors are respectively equal:

$n_x=0,\ n_y=1,0.$

§2.4. Methods of the solution to equations of motion.

Differential equations (2.5), most completely describing the flight of aircraft as displacement of material point, are nonlinear. The methods of obtaining their solutions in analytical form up to now have not been developed - usually for this purpose are utilized the

methods of the numerical integration of equations (2.5) with the aid of computers; therefore in the practice of aerodynamic designs, obtained by extension of the different approximation methods of the solution to equations of motion. Are most thoroughly developed and widely will be utilized the grapho-analytic methods of the solution of systems of equations (2.11) and especially (2.12), (2.13). Their essence entails the following. We convert the first equation of system (2.12) in such a way that thrust P would prove to be on the left side of the equality, and the remaining terms of equation in right:

$$P=X+G\sin\theta. \qquad (2.15)$$

Thrust, the constituting left side of equation (2.15), it is called point of tangency and is designated P_p, a the sum of the projections of forces X and G on the axle/axis of Ox_c, the constituting right side of equation (2.15), is called required thrust/rod and is designated $P_п$.

The point of tangency is determined by the engine characteristics and depends on height/altitude and speed of flight. Taking into account the second equation of system (2.12) for a required thrust it is possible to obtain expression

$$P_{\text{п}} = \frac{G}{K}(\cos\theta + K\sin\theta), \qquad (2.16)$$

linking I will require thrust/rod with the lift-drag ratio K and the flight path angle θ. According to (2.15) in steady rectilinear flight the necessary thrust is equal to that which is available; therefore, if we plot graphs of the change of necessary and available thrust depending upon same characteristic parameter, the solution to equaion (2.15) it will be determined by the point of intersection of these curves. Usually as this parameter is accepted the flight speed of V. The curves of points of tangency are constructed on high-altitude-speed to the engine characteristics; the curves of required thrusts - according to equation (2.16). Communication/connection of required thrust with the required flight speed of V_{η} is determined by expression

$$V_n = \sqrt{\frac{2G\cos\theta}{\varrho S c_{y_n}}}, \qquad (2.17)$$

escape/ensuing of the second equation of system (2.12) taking into account expression for lift (2.1). From expressions (2.16), (2.17) it follows that for graphing of a change in the required thrust from speed it is necessary to have the aerodynamic characteristics of aircraft.

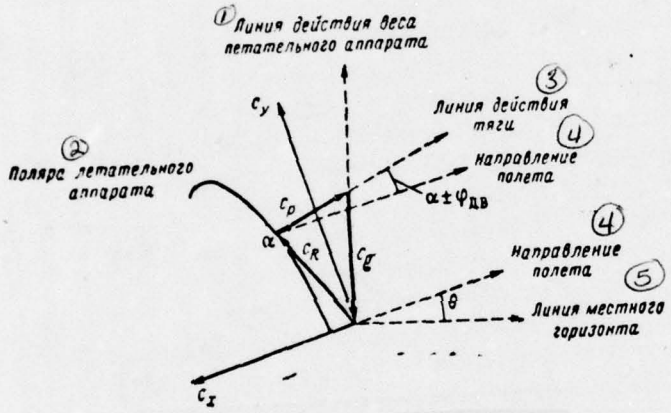
Since required thrust depends on angle θ, and point of tangency and the required flight speed are from flight altitude H, curve/graphs are constructed for constant values θ and H. The described method of the solution to equations of motion was called the name the method of thrust/rods. It is utilized during the calculation of the flight equilibria of aircraft with TRD, since high-altitude-speed characteristics TRD I am assigned in the form of the dependence between the thrust of engine and flight speed, its height/altitude and the degree of the throttling/choking of engines. The altitude-speed characteristics of aircraft with engine-propeller

combination (VMG) as already it is bygone said, are assigned in the form of the dependence, which links the power of engine with speed and flight altitude. Therefore during the calculation of the flight equilibria of aircraft from VMG to conveniently deal with power, but not with thrust/rod. Let us multiply piecemeal equation (2.15) by the flight speed of V. Then on the left side we obtain the available power of N_p, a in right - the required power of $N_п$, which taking into account expression (2.16) can be written in the form

$$N_{\text{п}} = \frac{GV}{K}(\cos\theta + K\sin\theta). \qquad (2.18)$$

Fig. 2.4. Construction of triangle of forces in the rectilinear steady flight.

Key: (1). Line of action of the weight of aircraft. (2). Polar of flight vehicle. (3). Thrust line. (4). Heading. (5). Line of the local horizon.



After constructing from altitude-speed characteristics and expressions (2.17), (2.18) the curve/graphs of the change of the available and required powers and after determining corresponding points of their intersection, it is possible just as in the method of thrust/rods, to find all parameters of the flight equilibria of aircraft from VMG. This method was called the name the method of powers.

The variety of the described grapho-analytic methods is N. Ye. Joukowski's method, which makes it possible to determine the parameters of the steady-state modes of the motion of aircraft also with the aid of graphic constructions. Let us examine system of equations (2.12), describing steady rectilinear flight without bank and slip. In its vector form it is possible to write in the form

$$\vec{P}+\vec{G}+\vec{R}=0, \qquad (2.19)$$

or, by passing over to the dimensionless coefficients:

$$\vec{c}_P+\vec{c}_g+\vec{c}_R=0. \qquad (2.20)$$

On the basis of equation (2.20) is constructed the triangle of forces from which graphically is determined the value of the required coefficient of $\vec{c}_P$, and, consequently, required thrust. Since to each flight conditions corresponds its relationship between the

vectors, which enter equation (2.20), triangle of forces must be constructed for each flight conditions. The construction of triangle of forces is fulfilled as follows (Fig. 2.4). Accepting as the axle/axis of c_x the direction, opposite to heading, is constructed at arbitrary point the polar of aircraft, plot/depositing c_x and c_y on identical scales.

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For a flight with the assigned angle of attack α it is possible to construct the ray/beam of c_R from the origin of coordinates into the point of the polars, which corresponds to preset angle α, and the direction of the beams of c_g and c_p, which are known (c_g is directed always down, c_p along the thrust line at an angle $\alpha \pm \varphi_{AB}$ to heading). The node of the action of and forms the unknown triangle of forces for this flight conditions from which are determined the values of c_p and c_g. The value of required thrust is determined by equality

$$P_{\text{п}} = \frac{c_P}{c_g} G, \qquad (2.21)$$

escape/ensuing of the similarity of the force triangles $\vec{P}$, $\vec{G}$, $\vec{R}$ or of their dimensionless coefficients of $\vec{c}_P, \vec{c}_g, \vec{c}_R$.

In the practice of aerodynamic designs, is utilized and the so-called energy method, developed by V. S. Pyshnov. Method is based on the known from the course of theoretical mechanics position, which confirms that during the motion of solid body in force field the work of external forces is equal to a change in the kinetic energy of body.

By energy method it is not possible to determine the form of trajectory, then it makes it possible to sufficiently simply find some flight characteristics. The application/use of an energy method in concrete/specific/actual examples is examined in the subsequent chapters. Here we will pause only at the certain common/general/total

principles of this method.

Total energy E of the driving body in potential field is expressed by the sum of the kinetic and potential energies:

$$E=E_к+E_п.$$

If the aircraft travels at a speed V at height/altitude H, then its kinetic energy is equal to

$$E_к=\frac{mV^2}{2},$$

a potential

$$E_п=GH.$$

Thus, total energy of the aircraft, which flies at height/altitude H with a velocity of V, will be equal to

$$E=\frac{mV^2}{2}+GH. \qquad (2.22)$$

. After dividing the value of total energy of aircraft E by its weight, let us find the energy of the unit of the weight of aircraft, i.e., specific mechanical energy. The value of specific mechanical energy is measured in linear units and it was called therefore the name energy height/altitude. Energy height/altitude in accordance with formula (2.22) will be equal to

$$H_e=\frac{V^2}{2g}+H. \qquad (2.23)$$

Physically energy height/altitude is the greatest height/altitude to which can be built up the aircraft with the constant values of total energy and weight with an incidence/drop in the speed to zero. Virtually the lift of aircraft to the height/altitude of $H_э$ is unrealizable, since the aircraft with aerodynamic controls at low flight speeds becomes unguided.

Let us examine the application/use of an energy method in a following example. Let us assume that we should find the duration of ascent of aircraft from height/altitude H_1 to height/altitude H_2. Flight speeds at height/altitudes H_1 and H_2 are respectively equal to V_1 and V_2. On cut dl, the aircraft accomplishes the work, equal to the product of a difference in the thrust and resistance on dl, and this work must be equal to the change of total energy of aircraft. Thus, we have

$$dE=(P-X)\,dl. \qquad (2.24)$$

After dividing the left and right sides of equation (2.24) by weight of aircraft and by taking into account the first equality of system (2.14), we will obtain for the change of energy height/altitude expression

$$dH_э = n_x\,dl.$$

Since $dl = V dt$, it is possible to write

$$dH_э = V n_x\,dt,$$

whence it follows that

$$t = \int_{H_{э1}}^{H_{э2}} \frac{dH_э}{V n_x},$$

where the $H_{э1} = \frac{V_1^2}{2g} + H_1, \quad H_{э2} = \frac{V_2^2}{2g} + H_2.$

Knowing the law of the change of speed with the height/altitude of $V = f(H_э)$, it is possible to find $n_x = f(H_э)$ and from formula

(2.25) to determine the duration of ascent of aircraft from one height/altitude to another.

PROBLEMS FOR REPETITION.

1. Which forces do act on aircraft during its flight?

2. Why during flights in upper air the effect of aerodynamic forces can be disregarded? How are related the values of air density at height/altitudes 15 and 70 km?

3. When can arise negative excess load.

4. Of what does consist a difference in the available powers of turboprop and piston engines?

5. When is applied the method of thrust/rods, while when the method of powers and why?

PROBLEM.

Aircraft flies at the altitude of H = 10 km with a velocity of 720 km/h. To find its specific mechanical energy.

Answer/response: 12,038 m.

Chapter III.

LEVEL FLIGHT.

§ 3.1. Equations of motion.

The flight of the aircraft with which the height/altitude does not change, is called horizontal. In the general case the horizontal flight trajectory will be curvilinear, in this case the aircraft can fly with bank and slip. In the particular case the trajectory can be rectilinear and aircraft does not have a bank and a slip. Let us examine this case.

With straight path the flight path angle θ is equal to zero, lift force is directed vertically (Fig. 3.1) and the equation of motion (2.11) they take the form

$$\left.\begin{aligned} P-X&=m\frac{dV}{dt},\\ Y-G&=0. \end{aligned}\right\} \qquad (3.1)$$

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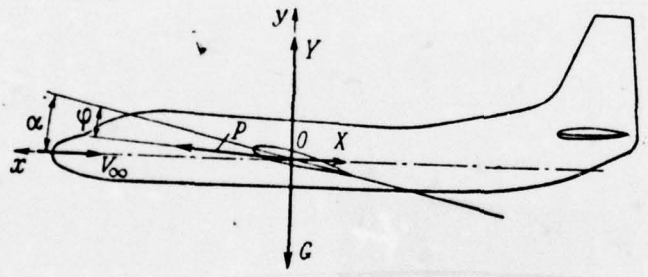

Fig. 3.1. Diagram of the forces, which act on aircraft in horizontally flight.

During steady rectilinear flight (V = const) the equations of motion (3.1) are simplified:

$$\left.\begin{aligned} P &= X, \\ Y &= G. \end{aligned}\right\} \qquad (3.2)$$

Under actual conditions there is no motions of aircraft, which is characterized by the constancy of all parameters, since in flight of aircraft change its weight and the parameters of surrounding air, and consequently, they change the engine thrust, lift and drag. Air currents also affect flight conditions.

The degree of error during the calculation of the characteristics of horizontal flight (minimum and maximum speeds and of, etc) on simplified equations (3.2) depends on the value of thrust-weight ratio $\overline{P}$ whose hearth is implied thrust-to-weight ratio P/G.

For subsonic aircraft this error does not exceed 2-5o/o, with an increase in the thrust-weight ratio, it increases and for a supersonic aircraft can achieve 10o/o.

§ 3.2. Necessary horizontal flight speed.

The speed, necessary for the steady level flight of aircraft relative to air at those which were assigned gross weight and angle

of attack, is called required. Subsequently under flight speed, will be implied the required speed (sometimes this speed is called air). After expressing lift in the second equation of system (3.2) by the lift coefficient of c_y, velocity head of $\frac{\varrho V^2}{2}$ and the wing area S:

$$c_y S \frac{\varrho V^2}{2} = G, \tag{3.3}$$

we will obtain expression for determining the velocity of the level flight:

$$V = \sqrt{\frac{2G}{S \varrho c_y}}. \tag{3.4}$$

. Hence it follows that with constants to the specific wing load G/S and densities ρ with an increase in the angle of attack up to critical value the velocity decreases. With an increase in altitude of flight, the air density ρ decreases; therefore with constant/invariable angle of attack and the specific wing load,

velocity of V must increase. In order that with the increase in altitude the flight speed would remain constant, necessary to increase angle of attack.

In flying practice with known gross weight, the horizontal flight condition usually is assigned by flight altitude and by the value of equivalent or instrument airspeed. Under equivalent airspeed is implied the speed, necessary for a flight at the level of sea under standard atmospheric conditions (ρ = 1.225 kg/m^3, p = 760 mm Hg, T = 288°K) with that velocity head, as under real flight conditions. Equivalent airspeed of V_i is connected with instrument $V_{пр}$ by the relationship of $V_i = V_{пр} + \Delta V$ where ΔV - correction for readings.

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Correction ΔV considers the instrument correction, determined by the laboratory calibration of instrument, and the aerodynamic correction, which considers a possible difference of the static pressure in the sensor station from pressure in the environment, since aircraft and sensor itself distort flow. During the

determination of equivalent airspeed, is considered also the correction in the compressibility of air. Indicated and indicator speeds can be considered equal between themselves only during approximate solutions, since the difference between them can be significant.

The equality of velocity heads in flight of the Earth at equivalent airspeed of V_i and in flight at rated altitude at airspeed of V

$$\frac{\varrho_0 V_i^2}{2} = \frac{\varrho_H V^2}{2},$$

makes it possible to establish/install the dependence between these speeds:

$$V = V_i \sqrt{\frac{\varrho_0}{\varrho_H}} = \frac{V_i}{\sqrt{\Delta}}. \qquad (3.5)$$

where the $\Delta = \rho_H / \rho_0$, the relative density of air,.

In flight of the Earth (H = 0, Δ = 1) indicator and airspeeds coincide. With an increase in altitude, the difference between the airspeed V and indicator V_i increases. So, at height/altitude H = 10,000 m airspeed exceeds indicator 1.72 times.

In flight at different height/altitudes at one and the same equivalent airspeed, velocity head at all height/altitudes is identical. Consequently, in this case of aircraft with fixed weight according to relationship (3.3) lift coefficient, and also, therefore, angle of attack they must be constant/invariable. If lift coefficient decreases, then velocity head and, consequently, also equivalent airspeed must increase. Because of such a communication/connection between equivalent airspeed and coefficient of lift is simplified the monitoring of flight conditions.

§ 3.3. The required thrust of engines for a level flight.

The required thrust in the steady level flight is the force,

which balances drag. The value of the required thrust of $P_п$ it is possible to determine from the first equation of system (3.2), after expressing drag by the drag coefficient of c_x:

$$P_п = c_x S \frac{\varrho V^2}{2}. \qquad (3.6)$$

or by the term-by-term division of the first equation of system (3.2) into the second:

whence

$$\frac{P_п}{G} = \frac{X}{Y} = \frac{c_x S \frac{\varrho V^2}{2}}{c_y S \frac{\varrho V^2}{2}} = \frac{c_x}{c_y} = \frac{1}{K},$$

$$P_п = \frac{G}{K}, \qquad (3.7)$$

where the $K = c_y/c_x$ - aerodynamic fineness ratio.

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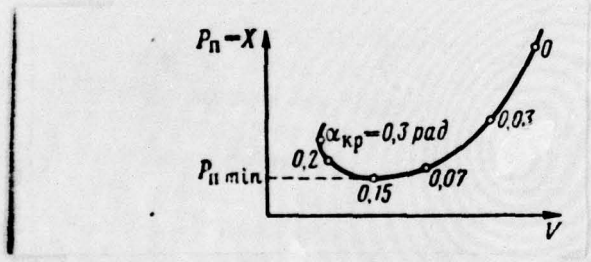


Fig. 3.2. Dependence of the required thrust of subsonic aircraft on speed.

If gross weight approximately is considered constant, then required thrust/rod will depend only on the lift-drag ratio K, which is the function of the angle of attack α, of Mach number of flight (M = V/a) and of the flight configuration of aircraft (landing-gear position, the flaps, the presence of external suspensions and, etc). The characteristic form of the dependence of required thrust on flight speed for the subsonic aircraft in which is absent wave drag, is given in Fig. 3.2. The curve of the dependence of required thrust/rod on speed is called curved Joukowski.

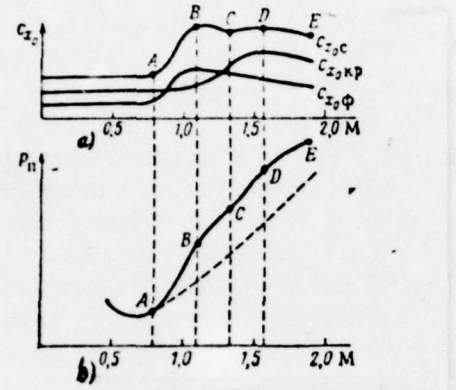
From the course of aerodynamics, it is known that the lift-drag ratio has the maximum value at greatest angle of attack of $\alpha_{наив}$. This determines the character of the dependence of required thrust on flight speed. At the minimum flight speed and, therefore, with $\alpha = \alpha_{кр}$ (in Fig. 32 $\alpha = \alpha_{кр} = 0.3$ rad) quality is low. With an increase in the velocity, the angle of attack decreases, the quality of increases required thrust falls, reaching the minimum value with of $K = K_{max}$ ($\alpha = 0.15$ rad, see Fig. 3.2). During a further increase in the velocity and the corresponding decrease in the angle of attack, the quality decreases, but required thrust begins to increase. Let us examine the compressibility effect of air and other factors on the value of required thrust.

Compressibility effect of air on the value of required thrust.

After replacing in formula (3.6) the velocity of flight on the product of Mach number to the velocity of sound, we will obtain

$$P_{\text{н}} = c_x S \frac{\rho a^2 M^2}{2}. \qquad (3.8)$$

Fig. 3.3. Dependence of profile drag and the required thrust of supersonic aircraft on Mach number.



In this expression the value of c_x can depend on Mach number. From the course of aerodynamics, it is known that the drag coefficient of c_x is equal to the sum of the coefficients of the profile and inductive reactances:

$$c_x = c_{x_0} + c_{xi}.$$

The coefficient of inductive reactance is proportional to the square of lift coefficient, the proportionality factor in the transonic and supersonic zones of flight increasing with an increase in Mach number of flight.

At flight speeds from $M < M_{кр}$, the coefficient of profile drag of c_{x0} in practice barely depends on Mach number. In the transonic field of flight, i.e., with $M_{кр} < M < 1.2$, the coefficient of c_{x0} grow/rises. At supersonic speeds the coefficient of c_{x0} decreases proportional to the value of $\frac{1}{\sqrt{M^2-1}}$.

Of supersonic aircraft with the wing of large sweepback the maximums of profile drag (because of wave) of wing and fuselage, do not coincide in Mach numbers (Fig. 3.3a); therefore the graph/diagram of the dependence of $c_{x0_c} = f(M)$ it can have two maximum. The

first maximum (point B) corresponds to the maximum wave drag of fuselage, the second maximum (point D) - to Mach number, with which the front/leading stern of wing becomes supersonic. This change in the coefficient of profile drag of aircraft leads to the appropriate change in the required thrust: in ranges AB and CD, the required thrust grow/rises more intense about to comparison with ranges BC and DE (Fig. 3.3b). By dotted line is shown the curve of the required thrust which can be obtained, if we disregard wave drag.

Altitude effect on required thrust.

As it is already known, with constants velocity head and the weight of aircraft and in the absence of the increase of compressibility the lift coefficient, and also, therefore, angle of attack they do not depend on flight altitude; therefore lift-drag ratio under these conditions also will be constant and required thrust at the fixed angle of attack will not depend on flight altitude.

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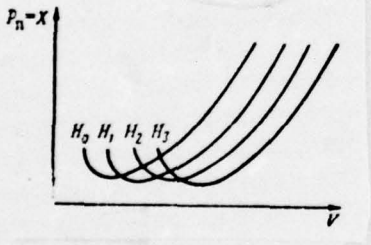

Fig. 3.4. Dependence of the required thrust of subsonic aircraft on flight altitude ($H_0<H_1<H_2<H_3$).

However, the speed, required for a level flight, at constant angle of attack with a change in altitude of flight is changed according to equation (3.5); as a result, in proportion to the increase in the flight altitude the curves of required thrusts, without changing its form, they are displaced to the right (Fig. 3.4). For the plotting of curves of the required thrusts of transonic and supersonic aircraft, it is necessary to preliminarily plot the so-called flight polars.

Flight polar (polar horizontal flight condition) is called the graph/diagram of the dependence of c_y on c_x, constructed for fixed flight altitudes and weight of aircraft at the different values of Mach number.

After replacing in equation (3.3) the velocity of flight by multiplying by the number Ma, we will obtain

$$c_y = \frac{2G}{\varrho V^2 S} = \frac{2G}{\varrho a^2 S} \cdot \frac{1}{M^2}. \quad (3.9)$$

At the base altitude of flight, equation (3.9) gives

communication/connection between of c_y and M. Curve, that connects those which were plotted/applied with the curve/graphs of the $c_x = f(c_y)$ of the points which correspond to rated values of M and c_y, and it will be flight polar for the base altitude of flight (Fig. 3.5). Usually flight polars are constructed for several altitudes. At the subcritical values of Mach number the flight polars of different height/altitudes are fused into one curve.

Figure 3.5 shows that in flight of transonic aircraft at height/altitude H = 10 km with an increase of the velocity, for example, from the mode/conditions, noted by point 1, the mode/conditions, which corresponds to point 6, the lift-drag ratio first increases to the maximum, and then it decreases. This it is possible to note and for other height/altitudes.

The flight polars of supersonic aircraft have their special feature/peculiarities (Fig. 3.6), caused by a decrease in the coefficient of the airfoil impedance of c_{x0} at supersonic flight speeds. At the low values of the c_y of polar, they are shift/sheared to the left (for example, see polars for M = 2.5 and M = 1.2). With an increase in the velocity of flight from the subsonic to the transonic, the polars with of $c_y = 0$ are shift/sheared to

the right, flight polars are deflected to the right (for example the flight polar ABC), and lift-drag ratio on section BC intensively falls. At supersonic speeds flight polar is deflected to the left (CD), and the rate of a decrease in the quality decelerates. Quality change upon transition from subsonic speeds to the field supersonic with respect affects a change in the required thrust (see Fig. 3.3).

Fig. 3.5. Flight polars of transonic aircraft.

Fig. 3.6. Flight polars of supersonic aircraft.

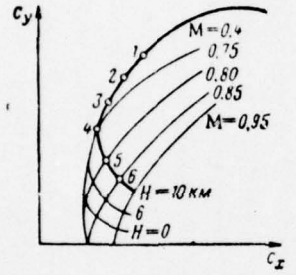

Fig. 3.5.

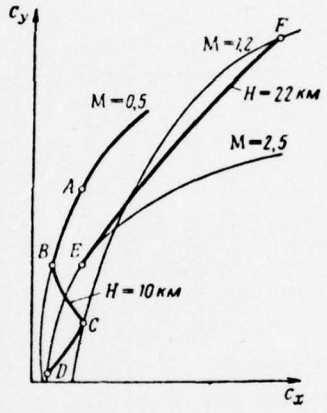

Fig. 3.6.

Fig. 3.7. Dependence of the required thrust of transonic aircraft on flight altitude.

Fig. 3.8. Dependence of the required thrust of supersonic aircraft on flight altitude.

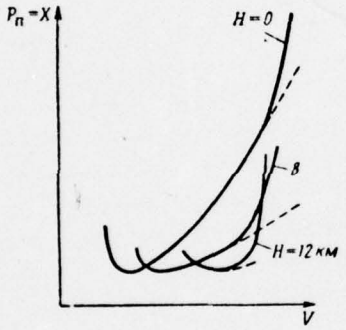

Fig. 3.7.

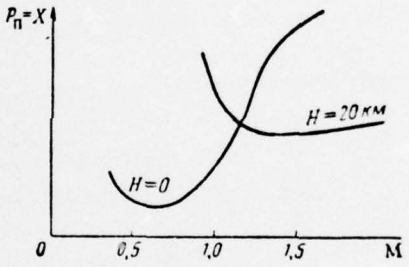

Fig. 3.8.

The peculiarity of the form of the flight polars of transonic and supersonic aircraft shows up in the nature of curved required thrusts at near- and supersonic speeds. Of transonic aircraft in flight at low speeds at the height/altitudes of order $H \leqslant 11$ km the character of curved required thrusts (Fig. 3.7) the same as of aircraft at the subcritical speeds: at high speeds the curves as a result of a decrease in the lift-drag ratio deviate upward.

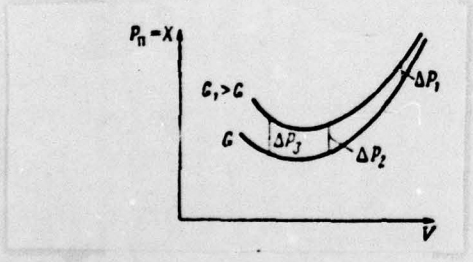

Fig. 3.9. Dependence of required thrust on the weight of aircraft.

The points of the deviation of required thrusts from the direction in which is not considered the compressibility effect, correspond to the appearance of a shock stall, but flight polars in the angles of attack, which correspond to these speeds, they differ from subcritical polar (in Fig. 3.5 for height/altitude H = 10 km the point of deviation is point 4). With an increase in altitude, the beginning of shock stall appears at lower speed, since with an increase in altitude the sound propagation velocity and air density decrease, and for maintaining level flight necessary to increase c_y, a that means and angle of attack.

Of supersonic aircraft the curve of required thrust for low altitudes has similar character (Fig. 3.8), but for high altitudes it considerably it differs from the curved required thrust of transonic aircraft. At low altitudes the required thrust will be minimum at subsonic flight speed, while on large it will be with supersonic, whereupon the minimum required thrust at high altitudes more the minimum required thrust of aircraft at low altitudes.

Effect of the gross weight of aircraft on the value of required thrust.

The effect of the gross weight of aircraft on the value of required thrust especially strongly manifests itself in flight the low speeds.

From relationship (3.9) it is evident that during just one increase in the gross weight an increase in the lift coefficient will increase with a decrease in the velocity of flight. If one considers that at low flight speed the lift coefficient large, and the coefficient of inductive resistance is proportional to the square of the value of lift coefficient, then it will become it is clear that with a the drag of aircraft, and also, therefore, required thrust, it will be more than at high speeds (Fig. 3.9).

§ 3.4. Required power for a level flight.

The power, spent on the overcoming of the drag of the aircraft, which flies with by given speed and weight, is called required power. Is determined it by relationship

$$N_{\text{п}} = P_{\text{п}} V. \qquad (3.10)$$

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After substituting into formula (3.10) expression for required thrust (3.6), we will obtain

$$N_{\text{п}} = \frac{c_x S \varrho V^3}{2}. \qquad (3.11)$$

At the constant values of c_x, S and ρ, the required power is proportional to the cube of speed.

For the analysis of the flight characteristics of aircraft to conveniently use the expression for a required power, obtained by the substitution of the value of the required thrust from equality (3.7) of relationship (3.10):

$$N_{\text{п}} = \frac{GV}{K}. \qquad (3.12)$$

If we into relationship (3.12) instead of the velocity of horizontal flight substitute its value from formula (3.4), then we will obtain expression for determining required power in the form

$$N_{\text{п}} = \frac{G}{K}\sqrt{\frac{2G}{\varrho c_y S}} = 1{,}41\,\frac{c_x}{c_y^{3/2}}\,\frac{G^{3/2}}{1\,\varrho S}. \qquad (3.13)$$

The character of a change of the required power depending on flight speed is shown in Fig. 3.10. With an increase in the velocity, the required power first decreases (from $\alpha = 0.3$ to $\alpha = 0.18$), and

then it increases (from $\alpha = 0.18$ to $\alpha = 0.03$). However, the minimum of required power does not coincide in angle of attack and in velocity with the minimum of required thrust.

Unlike the required thrust whose minimum is determined most advantageous angle of attack and by the maximum of lift-drag ratio, required power will have the minimum value with minimum value of fraction $\frac{c_x}{c_y^{3/2}}$. The relation of $\frac{c_x}{c_y^{3/2}}$ is designated by power factor of power.

The effect of operational factors on the value of required power can be examined in an example of the aircraft with propeller engines, flying, as a rule, with the small Mach numbers $M (M < M_{кр})$, when there is no wave drag. Required powers for different height/altitudes can be determined, if is known the required power of the Earth ($H = 0$). For determining communication/connection between required powers at height/altitude H and of the Earth let us write relationship (3.10) for a flight of the Earth and on height/altitude H:

$$N_{n0} = P_{n0} V_0 \text{ и } N_{nH} = P_{nH} V.$$
[и = and]

Since with constants angle of attack and gross weight the required thrust does not depend on flight altitude, i.e.,

$P_{n0} = P_{nH}$, that, after dividing one equation into another, we will obtain

$$N_{nH} = N_{n\infty} \frac{V}{V_0} = N_{n\infty} \frac{1}{V\Delta}. \qquad (3.14)$$

Fig. 3.10. Dependence of required power on flight speed.

Fig. 3.11. Dependence of required power on height/altitude and flight speed.

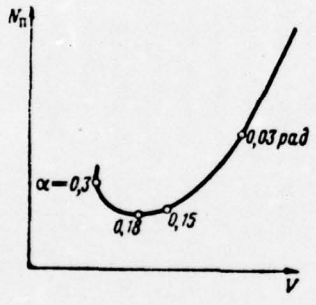

Fig. 3.10.

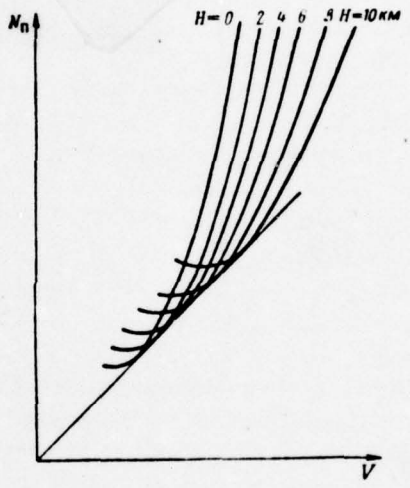

Fig. 3.11.

Thus, during a change in altitude of flight, but with constants angle of attack and gross weight required power and required speed (3.5) vary indirectly square root from the value of the relative density of air Δ. Therefore for obtaining required power on height/altitude H, is sufficient both coordinates of each point of curved required power of the Earth to multiply by the value of $\frac{1}{\sqrt{\Delta}}$.

For this reason for the points, arrange/located on different curved required powers, but correspond one and the same to angle of attack, lie/rest on the ray/beam, coming out from the origin of coordinates. Specifically, all the curves have one common/general/total tangent (Fig. 3.11). Points of contact of tangency correspond to flight at most advantageous angle of attack. Actually, at these points the ratio of required power to flight speed it will be minimum. Since this relation in accordance with formula (3.10) is equal to the required thrust of flight, at point of contact of tangency the required thrust will be minimum, which is possible only in flight with most advantageous angle of attack, i.e., with the maximum quality.

The effect of gross weight on the value of required power is

exhibited more strongly than the effect of required thrust, since with a gain in weight increases not only required thrust, but also the required flight speed.

§3.5. Characteristic velocities of level flight.

The flight fineness ratios are characterized by many parameters, in number of which enter the so-called characteristic velocities: minimum, is most advantageous, economic, cruising, maximum level flight.

Minimum theoretical is called the smallest flight speed, at which the lift still can balance the gross weight of aircraft at base altitude.

Graphically the value of the minimum pitch speed is expressed by the abscissa of the point of contact of the tangency of straight line, parallel axis of ordinates, with the curve of required thrusts for an aircraft with the jet engine (Fig. 3.12) or of the curve of required powers for an aircraft with screw propeller (Fig. 3.13).

Fig. 3.12. Toward the determination of the characteristic velocities of level flight for aircraft with TRD [turbojet engine].

Fig. 3.13. To determination of the characteristic velocities of horizontal flight for an aircraft with TVD [turboprop engine] and PD [instrument panel].

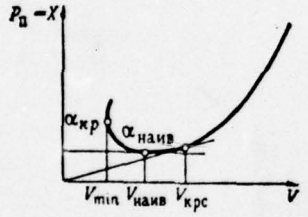

Fig. 3.12.

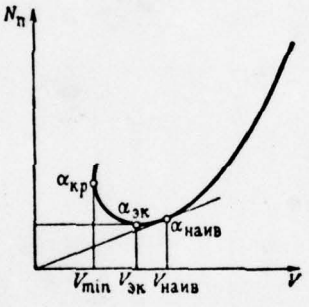

Fig. 3.13.

Analytically the value of minimum speed at the engines of any types is determined from formula (3.4)

$$V_{min}=\sqrt{\frac{2G}{c_{y\,max}S\varrho}}. \qquad (3.15)$$

. The minimum speed of V_{min} usually is called theoretical minimum, since in flying practice this speed is not used.

most advantageous is called the velocity of the steady rectilinear horizontal flight with most advantageous angle of attack, but that means with the minimum required thrust. Graphically the flight conditions at optimum speed for an aircraft with turbojet engine is determined by the point of contact of the tangency of straight line, parallel axis of abscissas, and the curve of required thrusts (see Fig. 3.12).

For an aircraft with screw propeller, the flight conditions at optimum speed graphically is determined by the abscissa of the point of contact of the tangency of straight line, carried out from the

origin of coordinates, and the curve of required powers (see Fig. 3.13).

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Analytically the value of $V_{наив}$ for aircraft with the engines of any types in the absence of wave drag is calculated similarly V_{min} from equality (3.4):

$$V_{наив}=\sqrt{\frac{2G}{c_{y\,наив}Sq}}. \qquad (3.16)$$

. If the flight polars of this aircraft are constructed, then the values of $c_{y\,наив}$ can be defined graphically by polars as ordinates of the point of contact of the tangency of straight line, carried out from the origin of coordinates, with the curve of polars.

During the determination of optimum speed from relationship (3.16) the latter can be converted so that it would make it possible

to estimate the effect of structural/design factors. So, if we disregard compressibility and drag coefficient to express known from course aerodynamics by formula

$$c_x = c_{x0} + \frac{c_y^2}{\pi\lambda_{эф}}, \qquad (3.17)$$

where the $\lambda_{эф}$ are the effective aspect ratio of wing, then respectively lift-drag ratio will be expressed by equality

$$K = \frac{c_y}{c_x} = \frac{c_y}{c_{x0} + \frac{c_y^2}{\pi\lambda_{эф}}}.$$

. In flight with most advantageous angle of attack, lift-drag ratio will be maximum, derived $\partial K/\partial c_y$ will be equal to zero and, consequently:

$$c_{y\ наив} = \sqrt{c_{x_0}\pi\lambda_{эф}} \quad и \quad K_{max} = \frac{1}{2}\sqrt{\frac{\pi\lambda_{эф}}{c_{x0}}}. \qquad (3.18)$$

After the substitution of this value of $c_{y\ наив}$ into formula (3.16) the latter obtains the form:

$$V_{наив} = \sqrt{\frac{2G}{Sq\sqrt{c_{x0}\pi\lambda_{эф}}}}. \qquad (3.19)$$

The economic speed of $V_{эк}$ aircraft with screw propellers corresponds to the minimum required power (see Fig. 3.13).

Analytically the value of economic speed can be calculated as in the preceding/previous cases, according to formula (3.4):

$$V_{эк}=\sqrt{\frac{2G}{c_{y\,эк}S_Q}}. \qquad (3.20)$$

Taking into account that the required power in flight at economic speed is minimum, i.e.,

$$N_{min}=(P_{в}V)_{min},$$

and utilizing equalities (3.4) and (3.7), after transformation we will obtain

$$N_{min}=\left(\frac{c_x}{c_y^{3/2}}\right)_{min} G\sqrt{\frac{2G}{Sq}}.$$

• The value of the relation of $\left(\frac{c_x}{c_y^{3/2}}\right)_{min}$ we find so, as this is made above during the determination of K_{max} for an optimum speed.

Then $c_{y\,эк} = \sqrt{3c_{x_0}\pi\lambda_{эф}}$,

$$V_{эк} = \sqrt{\frac{2G}{S\varrho\sqrt{3c_{x_0}\pi\lambda_{эф}}}}.$$

• During the determination of $V_{ЭК}$ analytical the method gives less precise results, than during the determination of $V_{НАИВ}$,

since economic speed corresponds to high angles of attack with which the polar no longer will coincide with quadratic parabola.

From the comparison of the values of $c_{y\,нав}$ and $c_{y\,эк}$, and also $V_{эк}$ and $V_{нав}$, we note that the $c_{y\,эк}$ of the $\sqrt{3}$ of times is greater than the $c_{y\,нав}$, a of the $V_{эк}$ в $\sqrt{1{,}73}$ of times less than $V_{нав}$.

Cruising speed of aircraft with TRD is characterized by the minimum ratio of thrust/rod to the speed and in the first approximation corresponds to the minimum fuel consumption per kilometer. The concept of cruising speed, as this will be shown in chapter V, somewhat conditionally.

Graphically cruising speed is determined by the abscissa of the point of contact of the tangency of straight line, carried out from the origin of coordinates (see Fig. 3.12), from curved required thrust.

By utilizing equalities (3.7) and (3.4), after transformations

we will obtain

$$\left(\frac{P_n}{V}\right)_{min}=\left(\frac{G}{K\sqrt{\frac{2G}{c_y S\varrho}}}\right)_{min}=\left(\frac{c_x}{c_y^{1/2}}\right)_{min} S\sqrt{\frac{G\varrho}{2S}}. \quad (3.21)$$

Consequently, flight at cruising speed is realized at the angle of attack by which the relation of $c_x/c_y^{1/2}$ is minimal.

For determining cruising speed into formula (3.4) let us substitute the cruising value of the c_y :

$$V_{крс}=\sqrt{\frac{2G}{c_{y\,крс} S\varrho}}. \quad (3.22)$$

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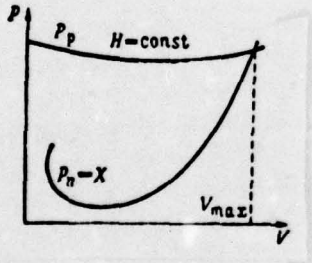

Fig. 3.14. To the determination of the maximum speed of aircraft with TRD.

Taking into account that with $c_y = c_{y\,крс}$ the relation of $c_x/c_y^{1/2}$ is minimal, and assuming that wave drag is absent, let us differentiate in terms of c_y of the indicated relation and, by equating to its zero, we will obtain

$$\frac{\partial}{\partial c_y}\left(\frac{c_x}{c_y^{1,2}}\right)=\frac{\partial}{\partial c_y}\left(\frac{c_{x_0}+c_y^2/\pi\lambda_{эф}}{c_y^{1,2}}\right)=\frac{3\dfrac{c_y^2}{\pi\lambda_{эф}}-c_{x_0}}{2c_y^{3/2}}=0.$$

Hence $c_{y\,крс}=\sqrt{\dfrac{c_{x_0}\pi\lambda_{эф}}{3}}$.

By substituting the value of $c_{y\,крс}$ in equality (3.22), let us compute the value of cruising speed, and also the value of the minimum relation of the $c_x/c_y^{1,2}$:

$$\frac{c_{x\,\text{крс}}}{c_{y\,\text{крс}}^{1\,2}} = \frac{4}{3}\sqrt[4]{\frac{3}{\pi}}\,\frac{c_{x0}^{3/4}}{\lambda_{\text{эф}}^{1/4}}. \qquad (3.23)$$

Characteristic speed for all aircraft is also the maximum speed of level flight. For a turbojet aircraft in flight at maximum speed, the drag is equal to the point of tangency of engines in their work on the nominal rating [1].

FOOTNOTE [1]. In flying practice are distinguished the following engine power ratings of the aircraft: - takeoff, that corresponds to the permissible engine speed and to its full thrust of $P_{взл}$; - nominal, with somewhat by less number of revolutions and by the less thrust/rod of the $P_{ном}$ of component about 0.9 $P_{взл}$; - cruising, determined by a further gear down and thrust/rod; $P_{крс}$ is located from 0.75 to 0.5 $P_{взл}$. ENDFOOTNOTE.

Therefore the maximum speed of aircraft is determined by the point of intersection of the curved available and required thrusts (Fig.

3.14).

For an increase in the maximum flight speed insufficient one increase in the thrust: necessary that with the different modernizations ot aircraft point of tangency would increase faster than required. In this sense advantage belongs to the turbojet engines which make it possible to obtain high thrust at the almost constant value of c_x.

The maximum speeds of contemporary turbojet aircraft are located either in the subsonic range of $V_{max} = 600-1000$ km/h or in the range, which considerably exceed the speed of scund (M > 1.5); in flight by transonic speed very grow/rises the wave drag, and the aircraft, designed for flight at this speed, it turns out to be uneconomical.

For an aircraft with screw propellers in flight with maximum speed, is satisfied the condition of $N_p = N_n$ and, therefore, to flight conditions at maximum speed corresponds the point of intersection of curves required and available powers in the engine operation on the nominal rating.

The maximum speeds of contemporary turbojet transport aircraft are 650-750 km/h, piston - 250-500 km/h.

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3.6. Effect of operating conditions on characteristic flight speeds.

Altitude effect on characteristic flight speeds is most convenient examined with the aid of the curved required and points of tangency and the powers, constructed for different flight altitudes (Figs. 3.15 and 3.16). In the absence of wave drag for determining minimum, economic, most advantageous and cruising speeds on different flight altitudes it is possible also to use formula (3.5), if are known the corresponding speeds for any height/altitude, for example, of the Earth.

Then for an optimum speed we obtain

$$V_{\text{наив}} = V_{\text{0 наив}} \frac{1}{\sqrt{\Delta}}. \qquad (3.24)$$

In a similar manner it is possible to obtain expressions for minimum, economic and cruising speeds. In 3.2 it is bygone shown that, if independent of height/altitude angle of attack is constant, then most advantageous equivalent airspeed will be identical for all height/altitudes.

Fig. 3.15. The available and required thrusts of the aircraft of TU-104 (G = 700,000 N, the engine power rating is maximum).

Key: (1). km/h; (2) N.

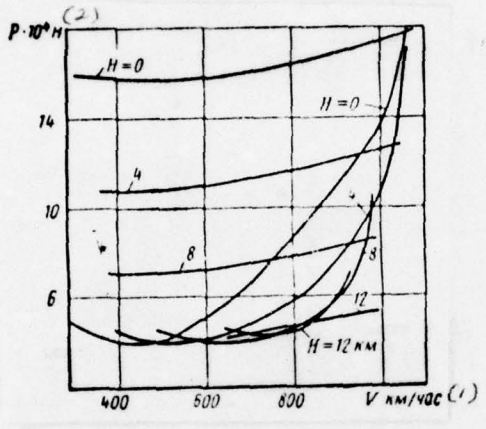

Fig. 3.16. Available and required powers of aircraft an-24 (G = 160.000 n, the engine power rating of-0.85 ratings).

Key: (1). kW. (2). km. (3). km/h.

Fig. 3.17. Change in the characteristic flight speeds with height/altitude.

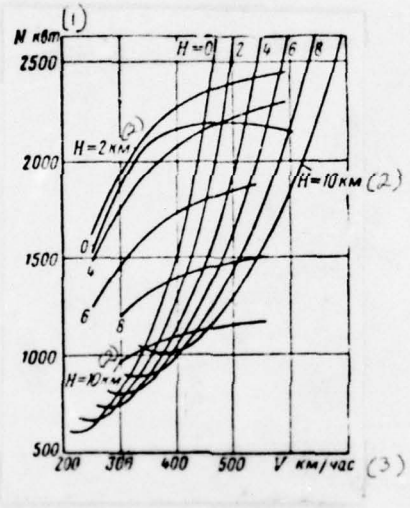

Fig. 3.16.

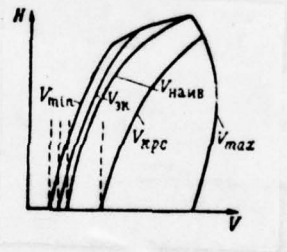

Fig. 3.17.

Consequently, in order to fly at any height/altitude at optimum speed, to pilot sufficient to hold one and the same indicated airspeed (with correction for its readings), and this considerably facilitates aircraft handling. The same it is possible to say about minimum, economic and cruising speeds. Corresponding to them equivalent airspeeds are shown by dotted line in Fig. 3.17.

Of course, this simple rule acts within certain limits outside which probable deviations. First of all, if at high velocities of flight appears the shock stall, which affects the values of c_x and c_y, then the angle of attack, which corresponds to the definite characteristic speed, will change with height/altitude. The law of an increase in the minimum speed can change at the high altitudes of flight due to a decrease in the points of tangency and powers. At high altitude the curves of required thrusts and powers can intersect with those which are had at two points (see Figs. 3.15 and 3.16), whereupon the point of intersection, which corresponds to lower speed, it can prove to be lower than the point, which corresponds to flight conditions from $\alpha_{кр}$. That means the minimum flight speed is restricted no longer not only to aerodynamic flight conditions, but also to the thrust (with a power) of engines. For this reason in the curve of $V_{min}=f(H)$ appears the fracture (see Fig. 3.17).

The value of maximum speed at different height/altitudes is determined from the points of intersection of the available and required thrusts or powers (see Figs. 3.15 and 3.16).

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Since in flight at maximum speed to the available and required power are equal, from formulas (3.6) and (3.11) it is possible to obtain expressions for determining the V_{max}:

$$V_{max} = \sqrt{\frac{2P_p}{c_x S \varrho}}; \qquad (3.25a)$$

$$V_{max} = \sqrt[3]{\frac{2N_p}{c_x S \varrho}}. \qquad (3.25b)$$

For an aircraft with TRD [TPA - turbojet engine] according to formula (3.25a) the maximum speed depends on change by height of the relation of P_p/c_xQ. At height/altitudes to 11 km in the absence of compressibility effect, the c_x is little affected by height; the relation of P_p/Q grow/rises with height/altitude, and with respect grow/rises the V_{max}. At high altitudes the relation of P_p/Q barely depends on height/altitude, but grow/rises the c_x and V_{max} falls. If with an increase in altitude of flight appears wave drag, then the c_x grow/rises and V_{max} with increase in H decreases.

For aircraft with screw propellers according to formula (3.26b) the V_{max} depends on change by height of the relation of $N_p/c_x\varrho$. On aircraft with TVD [ТВД - turboprop engine], the available engine power of which up to altitudes 4-5 km, just as the c_x of aircraft, it is retained approximately constant, the relation of N_p/ϱ grow/rises with height/altitude and with respect grow/rises V_{max}. At high altitudes the available power begins to decrease and V_{max} falls.

The range of values of velocities at which is feasible level flight at the fixed weight of aircraft and flight altitude, is called the speed range of horizontal flight. For each aircraft there is the height/altitude at which the speed range, decreasing in proportion to the increase in the height/altitude, subtends into point (see Fig. 3.17). At this height/altitude of $V_{min}=V_{наив}=V_{max}$. This height/altitude is called the absolute ceiling of aircraft. In other words, the absolute ceiling of aircraft is called the greatest height/altitude at which is still feasible the steady level flight. At this height/altitude the curve of point of tangency (or power) does not intersect the curve of required thrust (or power), but only it concerns it (Fig. 3.18).

The value of maximum speed for a supersonic aircraft is determined as for subsonic aircraft with TRD, by the character of the curved available and required thrusts at different height/altitudes. If the curves of points of tangency with an increase in altitude are displaced down (Fig. 3.19), little changing its configuration, then the curves of required thrusts with an increase in altitude are transformed very considerably. This leads to the fact that in certain altitude range the maximum speed begins to sharply increase, reaching the greatest value at height/altitude on the order of 11 km.

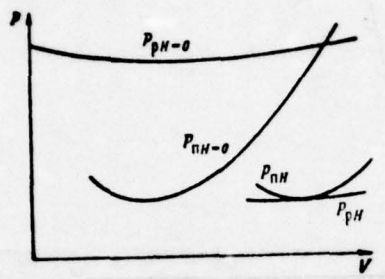

Fig. 3.18.

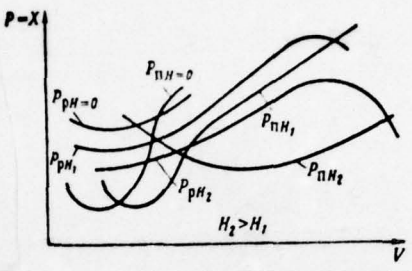

Fig. 3.19.

Fig. 3.18. To the determination of the absolute ceiling of aircraft.

Fig. 3.19. Required and points of tangency of supersonic aircraft.

In order to show, as affect operational factors the value of maximum speed, let us take the logarithm, and then we will differentiate the expressions for determining maximum speed (3.25a) and (3.25b), having preliminarily replaced in formula (3.25b) the available power of aircraft with product propeller efficiency η by the power of $N_{дв}$ by propeller shaft:

$$\frac{dV_{max}}{V_{max}} = \frac{1}{2}\left(\frac{dP}{P} - \frac{d\varrho}{\varrho} - \frac{dS}{S} - \frac{dc_x}{c_x}\right); \tag{3.26}$$

$$\frac{dV_{max}}{V_{max}} = \frac{1}{3}\left(\frac{dN_{дв}}{N_{дв}} + \frac{d\eta}{\eta} - \frac{d\varrho}{\varrho} - \frac{dS}{S} - \frac{dc_x}{c_x}\right). \tag{3.27}$$

. From formula (3.269 it iollows that a change in thrust/rod and coefficient of the impedance of aircraft with TRD to 2o/o leads to a change in the maximum speed to 1o/o.

For aircraft with screw propellers [see formula (3.27)] a

relative change in the maximum speed composes the only third of a relative change in the power of engine, propeller efficiency and drag coefficient.

The obtained numerical ratios are valid only in such a case, when compressibility effect on the character of the flow about the aircraft is not exhibited. Otherwise the effect of the values of c_x and P_p will be especially substantially for aircraft whose curves of the required and points of tangency intersect at small angle, i.e., when $\frac{\partial P_p}{\partial M}$ differs little from $\frac{\partial P_n}{\partial M}$. In turn,, on the parameters of P_p, $N_{дв}$, η, c_x have a considerable effect such operational factors as change in the large temperature range of surrounding air (it affects the P_p, $N_{дв}$), the contamination or the icing of the surface of aircraft (it affects the η, c_x), etc.

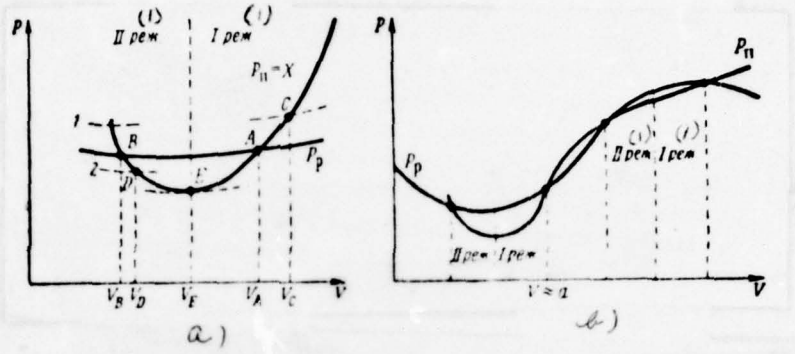

Fig. 3.20. Toward the determination of flight conditions.

Key: (1). conditions.

3.7. Two Flight conditions.

For the preservation/retention/maintaining of the constancy of flight speed, is necessary the equality the available and required thrusts or powers). For a flight at the speed less than maximum, is required the thrust/rod (or power) it is less than nominal. A decrease in the thrust/rod (or power) is achieved by the throttling/choking of engine (by decrease in the fuel feed).

The equality the available and required thrusts (or powers) still not always provides the stable equilibrium of these values. Much is determined by the form of the curved available and required thrusts. If, for example, aircraft with TRD flies with the speed of V_A (Fig. 3.20a) and required thrust is equal that which is had, then during a random decrease in the velocity of flight the resistance of aircraft becomes lower than the engine thrust, appears margin of thrust, under action of which the flight speed will increase until it achieves the value of V_A. With a random increase in the velocity, the margin of thrust will be negative, and

as a result of negative acceleration the velocity decreases to the initial value. Consequently, in point A velocity is supported automatically, the equilibrium of forces of resistance and thrust stable. Let us note that at point A occurs the inequality

$$\frac{\partial P_{\text{п}}}{\partial V} > \frac{\partial P_{\text{р}}}{\partial V}, \qquad (3.28)$$

i.e. at this point the rate of an increase in the required thrust higher than the rate of an increase in the point of tangency.

In flight at the velocity of V_B (pcint B in Fig. 3.20a) equilibrium of forces will not be stable, since, for example, during a random decrease in the velocity of flight required thrust will exceed that which is had.

Therefore velocity not only will not return to its previous value,

but it will be even more from it deflected. At point B, occurs the inequality

$$\frac{\partial P_n}{\partial V} < \frac{\partial P_p}{\partial V}. \qquad (3.29)$$

. If in the process of random changes in the velocity pilot does not change with the deflection of elevator angle of attack, then an increase in the velocity leads to the climb, but a decrease in the velocity - to reduction/descent.

For the flight conditions, determined by points A and B, is characteristic one additional vital difference. For an increase in the velocity of flight for example from V_A to V_C it suffices to increase the engine thrust so that the curve of point of tangency is past through point C. The appearing margin of thrust will lead to a gradual increase of the velocity from V_A to V_C, and, new velocity will be held stable. For a decrease in the velocity in comparison with of V_A, it is necessary to decrease the engine thrust.

For an increase in the velocity of flight from V_B to V_D, it is necessary to first create positive acceleration by a thrust augmentation (is curve 1 in Fig. 3.20a). When in the process of acceleration/dispersal the velocity reaches the value of V_D, required thrust proves to be more than had; therefore for the preservation/retention/maintaining of new velocity necessary to decrease the engine thrust in order to balance required and point of tangency (is curve 2). That means for a change in the velocity of V_B, is required a double change in the engine power rating, and for the preservation/retention/maintaining of the new velocity of V_D, in view of unstable flight conditions, necessary continuously to change the engine power rating.

In the curve of required thrusts it is possible to select the point to the left of which flight conditions will be similar to the flight conditions in point B, and to the right - to the flight conditions in point A. At this point (point E) the rate of an increase in the required and points of tangency is identical, i.e.,

$$\frac{\partial P_n}{\partial V} = \frac{\partial P_p}{\partial V}. \qquad (3.30)$$

point E divides entire speed range from V_{min} to V_{max} by two parts. The flight conditions for which is satisfied condition (3.28), are called the first flight conditions; the flight conditions for which is satisfied condition (3.29), they are called by the second flight conditions.

Flight in the second mode/conditions is undesirable, since subsonic aircraft in this mode/conditions flies at low speed, and that means with a comparatively high angle of attack, which causes poor aircraft handling. Furthermore, the approach/approximation to critical angle of attack increases the danger of stalling on wing and of inlet into corkscrew/spin and makes engine control unusual for a pilot.

By analyzing in the same sequence of the airplane operating conditions with screw propellers, it is possible to obtain the inequalities:

$$\frac{\partial N_u}{\partial V} > \frac{\partial N_p}{\partial V} \text{ (first mode/conditions),}$$

$$\frac{\partial N_u}{\partial V} < \frac{\partial N_p}{\partial V} \text{ (second mode/conditions).}$$

Since the flight of subsonic aircraft in the second mode/conditions is undesirable, the virtually minimum speed of level

flight is the speed on the boundary of two mode/conditions. In flying practice for subsonic passenger aircraft, lower velocity limit usually is establish/installed to the right the boundary of two mode/conditions in order to have certain reserve on speed and not to allow/assume output/yield to the second mode/conditions. The speeds, included between the maximum and virtually minimum, compose the operating range of the velocities of level flight.

The boundary of two mode/conditions of subsonic aircraft with TRD for all height/altitudes is velocity, the very close to most advantageous. This is explained by the fact that the point of tangency TRD weakly depends on flight speed (see Fig. 3.20a); therefore condition (3.30) is satisfied at that point where required thrust is minimum. The boundary of two mode/conditions of aircraft with screw propellers for all height/altitudes is the speed, close to economic. Since and most advantageous, and economic flight speed they increase with height/altitude, the range of the service speed of level flight in proportion to the increase in the height/altitude first insignificantly changes, increasing or decreasing, but higher than the full-throttle height of engine rapidly is contracted, becoming equal to zero at the height/altitude of absolute ceiling. With the landing gear lowering and flaps in takeoff and landing mode/conditions, the speed, which divides the first and second

mode/conditions, decreases; therefore flight is realized virtually in the first mode/conditions.

Of the supersonic aircraft possible two the first and two stalled of flight conditions, one in subsonic and supersonic zones (see Fig. 3.20b). The second mode/conditions in supersonic zone, although it answers condition (3.29), is not faulty, characteristic to the second mode/conditions in subsonic zone, with the exception of the need for a double change in the engine power rating during a change in the flight speed. The second supersonic mode/conditions corresponds to high flight velocities; therefore aircraft is here stable and "hears" the controls well. With high altitudes the flight of supersonic aircraft is made, as a rule, with the second mode/conditions.

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It should be noted that in connection with the wide critical angles of attack of triangular short wing aspect ratios and for the target/purpose of an improvement in the takeoff and landing characteristics in the operation of supersonic passenger aircraft

will be applied also the second subsonic mode/conditions.

3.8. Acceleration/dispersal and braking of aircraft in straight-and-level flight.

By the target/purpose of the calculation of the flight of aircraft with acceleration/dispersal or braking is the determination of time interval, necessary for the assigned change in the velocity, and the determination of the way which in this case passes aircraft. For such calculations it is convenient to use system of equations (3.1). After dividing all terms of the first equation by weight of aircraft and after multiplying them by velocity of V, we will obtain

$$\frac{P-X}{G}V=\frac{V}{g}\frac{dV}{dt}. \qquad (3.31)$$

By returning to expression 2.25), let us note that the right side of equation (3.31) with H = const is equal to the rate of change in the energy height/altitude in time, i.e., let us have a relationship

$$\frac{dH_э}{dt}=\frac{P-X}{G}V.$$

After designating the rate of change in the energy height/altitude

$$V_в^*=\frac{dH_э}{dt}, \tag{3.32}$$

we will obtain

$$V_в^*=\frac{P-X}{G}V=\frac{\Delta PV}{G}. \tag{3.33}$$

From relationship (3.33) it follows that the rate of change in the energy height/altitude of $V_в^*$ depends on the flight conditions

and engine power rating. For an aircraft with screw propeller

$$V_y^* = \frac{N_p - N_n}{G} = \frac{\Delta N}{G}. \qquad (3.34)$$

Differences $\Delta P = P - X$ and $\Delta N = N_p - N_n$ in formulas (3.33) and (3.34) are called margin of thrust and margin of power respectively.

The velocity of a change in the energy height/altitude is positive upon acceleration/dispersal, since in this case the required thrust (or power) it exceeds that which is had. Mentally acceleration/dispersal can be presented as lift of aircraft from the energy height/altitude of $H_{э1}$, which corresponds to flight speed V_1, to the energy height/altitude of $H_{э2}$, which corresponds to flight speed V_2.

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After dividing variables in equation (3.22) and after

integrating the obtained expression, let us find booster duration t:

$$t = \int_{H_{э1}}^{H_{э2}} \frac{dH_э}{V_y^*}. \qquad (3.35)$$

Integral value is located by usually approximately numerical or by graphic methods.

The velocity of a change in the energy height/altitude of V_y^* is negative during braking, i.e., energy height/altitude decreases as a result of a decrease in the kinetic energy of $(H_{э1} < H_{э2})$.

Simpler dependence is obtained for such cases of braking, when point of tangency or power can be accepted equal to zero, and equality (3.33) assumes the form

$$V_y^* = -\frac{X}{G} V.$$

In these cases according to the second equation of system (3.1) G = Y and, therefore,

$$V_1^* = -\frac{X}{Y}V = -\frac{V}{K}. \qquad (3.36)$$

If we this value of V_1^* substitute into integral (3.35) taking into account

$$dH_0 = \frac{VdV}{g},$$

that time of braking with zero thrust/rod will be determined by

integral

$$t=-\frac{1}{g}\int_{V_1}^{V_2}KdV=\frac{1}{g}\int_{V_2}^{V_1}KdV.$$

For determining the speeding-up path or braking, it is necessary to substitute into formula (3.32) the known expression dt = dℓ/V and to integrate obtained after substitution equation (3.32):

$$V_y^*=\frac{VdH_2}{dl}.$$

then the speeding-up path or braking is expressed by integral

$$l=\int_{H_{21}}^{H_{22}}\frac{VdH_2}{V_y^*}. \qquad (3.37)$$

Taking into account dependence (3.33) expression for

determining the speeding-up path or braking can be written also in this form:

$$l=G\int_{H_{s1}}^{H_{s2}}\frac{dH_s}{P-X}=\frac{G}{g}\int_{V_1}^{V_2}\frac{VdV}{P-X}. \qquad (3.38)$$

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During braking with the thrust/rod, equal to zero, the length of the acceleration phase or braking can be determined by substitution into integral (3.37) of the velocity of V_i according to formula (3.36):

$$l=\int_{H_{s2}}^{H_{s1}}KdH_s. \qquad (3.39$$

Integrals (3.35), (3.37), (3.38) and (3.39) are determined by numerical or graphic methods.

We examine the obtained dependences for determining time and path length of acceleration/dispersal or braking. These values in essence are determined by the value of V_u^*. The greater the V_u^* upon acceleration/dispersal, the lesser the time and the speeding-up path. Of aircraft with screw propeller, the value of V_u^* there will be the greatest with the maximum margin of power: $\Delta N = N_p - N_n$ [see formula (3.34)], which usually occurs in flight at economic speed. Of aircraft with TRD, the value of V_u^* will be greatest at speed, several that which exceed most advantageous. Of supersonic aircraft the maximum value of product ΔPV and the respectively maximum rate of change in the energy height/altitude are observed at supersonic speeds (for example, see Fig. 3.19).

If flight speed approaches maximum, then margin of thrust (power) vanishes, therefore, to zero it decreases and the speed of V_u^*. Then the value of integral (3.35) unlimitedly grow/rises, i.e., in level flight aircraft never it reaches maximum speed.

For more rapid deceleration the speed of $V_{\parallel}^{\cdot}$, negative on sign, must have the highest possible absolute value. In order to achieve this, it is necessary either to decrease the engine thrust or to increase the resistance of aircraft. The latter is reached with the aid of special air brakes. According to expression for determining stopping distance with zero thrust/rod (3.39), the lesser the average lift-drag ratio K of aircraft in stopping distance, i.e., the greater its resistance, the shorter the stopping distance.

3.9. Flight of aircraft with unsymmetric thrust/rod.

Under normal conditions of flight the vector of the gross thrust of the engines of aircraft, arrange/located it is symmetrical relative to plane xOy, it lie/rests at the plane of symmetry. Therefore flight can be executed without bank and slip. The flight during which the vector of gross thrust lie/rests outside the plane of symmetry (flight with unsymmetric thrust/rod), it is special.

The most probable reason for the emergence of unsymmetric thrust/rod is failure one or of several engines.

During the emergence of unsymmetric thrust/rod, appears the moment of engine strength relative to axle/axis Oy, in consequence of which flight is made with bank or slip or with the fact and other simultaneously, also, for the balance of aircraft (for greater detail, see chapter XVII, § 2) it is required the deflection of ailerons and rudder. The deflection of the rudder, ailerons and slip increase the drag coefficient of aircraft (Fig. 3.21), due to what increase required thrust and power.

Taking into account supplementary resistances the coefficient of impedance is equal to:

$$c_x = c_{x0} + \frac{c_y^2}{\pi\lambda_{э\phi}} + \Delta c_{x\text{в}} + \Delta c_{x\text{н}} + \Delta c_{x\text{з}} + \Delta c_{x\beta} + \Delta c_{x\text{к}},$$

where the $\Delta c_{x\text{в}}$ - the coefficient of the supplementary resistance of the screw/propeller of failed engine; $\Delta c_{x\text{н}}$ - an increase in the

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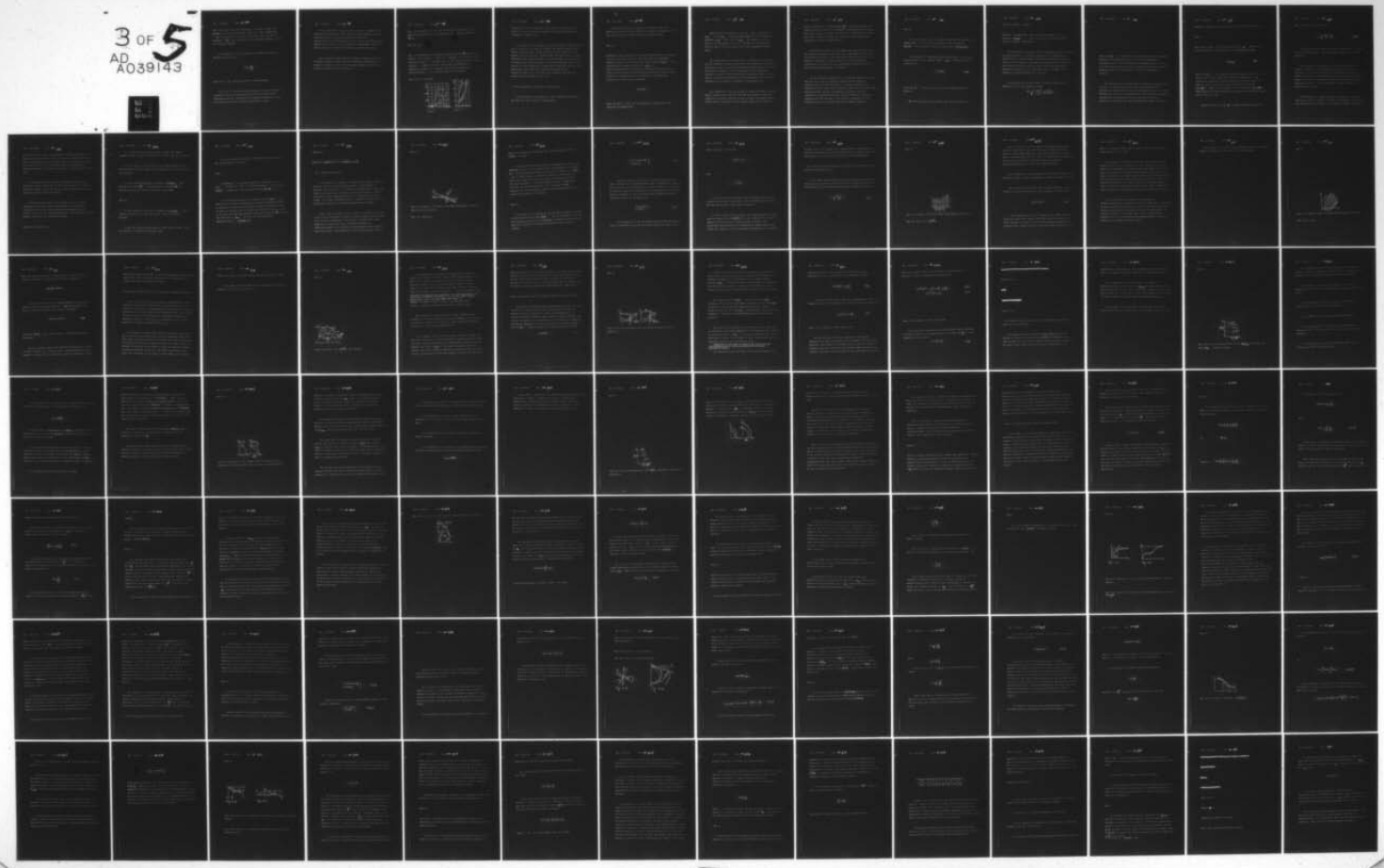

drag coefficient during the deflection of rudder; $\Delta c_{x\vartheta}$ - an increase in the drag coefficient during the aileron deflection; $\Delta c_{x\beta}$ - an increase in the drag coefficient feast the slip of aircraft; Δc_{xn} - the coefficient of the internal impedance of the gas circuit of engine.

The coefficient of the supplementary impedance of ggggqg is determined from formula

$$\Delta c_{x\,n} = \frac{X_n}{S\frac{\varrho V^2}{2}},$$

where the X_n - resisting force of screw/propeller.

The value of negative thrust/rod depends as on the position of propeller blades (screw/propeller is feathered, blade/vane on intermediate detent, screw/propeller removed from detent), so also from speed, the flight altitude and parameters of air.

Flight with failed engine is one of the most complex. Besides unpleasant psychological effect with the engine failure, the crew experience/tests high physical stress, since for the preservation/retention/maintaining of the lateral and longtitudinal equilibrium of aircraft it is required to considerably increase the forces, applied to controls (to pedals and steering control).

Are especially complex that kind flights on aircraft with the turboprop engines in which with the windmilling screw/propeller the negative propeller thrust of the shut-down engine can exceed the positive propeller thrust of engine on.

Fig. 3.21. Dependence of the drag coefficient of aircraft on the aileron angles, rudder and slip angle (in an example of aircraft An-24).

Key: (1). rad

Fig. 3.22. Available and required powers of aircraft An-24 with the failure of one engine; H = 4000 m; G = 180,000 n: 1 - required power in normal flight; 2 - available power in the engine operation of both engines; 3 - required power in flight with one propeller in feathering position; 4 - required power with one windmilling screw/propeller; 5 - the available power of one engine.

Key: (1). kW. (2). km/h.

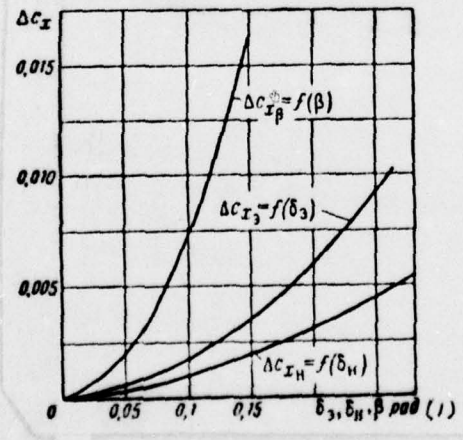

Fig. 3.21.

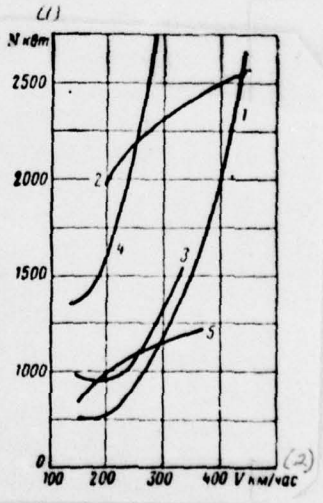

Fig. 3.22.

Furthermore, in these cases due to the varied conditions of the airflow of left and right half wings is created the significant difference in their lifts.

An increase in the impedance of aircraft with the engine failure and a decrease in the point of tangency (or power) sharply change flight conditions (Fig. 3.22): decrease absolute ceiling and the maximum speed of aircraft, narrows itself the speed range of flight. Therefore with the engine failure at high altitude, it is necessary to decrease the flight altitude, and for some aircraft types, level flight with the windmilling screw/propeller generally turns out to be impossible, since points of tangency (or power) for all height/altitudes prove to be less than required. This case for aircraft an-24 is shown in Fig. 3.22.

3.10. Operational limitations of flight speed.

In the operation of aircraft, act the different limitations of the lower and upper limits of flight speed.

Constraints on angle of attack are associated with flow separation from the lifting surfaces of aircraft on leaving of it to wide near-critical and angles of attack beyond stalling.

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Unsymmetric flow separation from lifting surfaces leads to asymmetry of lift on the outer planes of wing and stalling of aircraft; therefore flight in the field of the c_{y_3} close to $c_{y\,кр}$ is not allow/assumed. Usually the value of $c_{y\,доп}$ for concrete/specific/actual aircraft is determined from flight test data. On some aircraft the approach/approximation to critical angle of attack is accompanied by the appearance of vibration and agitation. In these cases is accepted

$$c_{y\,доп} = c_{y\,тр},$$

where the $c_{y\,тр}$ - the lift coefficient, by which appears the agitation of construction.

Tentatively it is possible to accept $c_{y\,доп}$ = (0.75-0.80) $c_{y\,max}$. Since $c_{y\,max}$ depends on Mach number, on Mach number will depend and $c_{y\,доп}$. The value of $c_{y\,доп}$ determines at to base altitude and the weight of aircraft the value of the permissible velocity which, of course, will be more than theoretical minimum speed.

The limitations of the upper limit of velocity are introduced for the different reasons: for providing for strength and rigidity of construction, for the preservation/retention/maintaining of stability and aircraft handling, for the elimination of the vibration of aircraft components (high-speed/velocity agitation, flutter), in the case of the incidence/impingement into vertical air gusts, for the softening of the gradient of pressure in shcck wave, etc.

The limitation of the upper limit of velocity according to Mach number follows either from the condition of strength or from the condition of aircraft handling. At low altitudes the maximum value of flight speed is determined by the allowed values of the external

loads, proportional to velocity head $q\frac{V^2}{2}$. At high altitudes the allowed value of velocity is determined by the critical Mach number, with which center of pressure and the focus of aircraft (see Chapter XII) they are moved back/ago, as a result of which become worse longitudinal control characteristics.

Limitations according to Mach number have very important value for transonic aircraft, since during the development of shock stall are possible the cases of the deficiency of the deviation of control surfaces, inadequacy of the necessary for a control physical possibilities of pilot, and also inadmissibly dangerous changes in the stability characteristic of aircraft.

For the supersonic aircraft of the indicated limitation, there does not exist, since satisfactory changes in the stability characteristics and controllability at subsonic, transonic and supersonic flight speeds can be reached by means of the corresponding design measures. However, for supersonic aircraft there are limitations on spontaneous rolling and the reversal of ailerons, connected with the emergence of the elastic deformations of the constructions of the aircraft, and also on kinetic heating.

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For each aircraft type, is establish/installed its maximum mach number. Thus, for instance, for aircraft of the type TU-104 $M_{пр}=0,90$, for aircraft of the type "Il-18" and An-~~19~~ 10 $M_{пр}=0,63$.

The dependence of maximally possible flight speed at the given height/altitude on the maximum number of $M_{пр}$ is expressed by the relationship:

$$V_{пр}=a_H M_{пр}, \tag{3.40}$$

where the a_H - the speed of sound at the height/altitude in question.

From this relationship it follows that with an increase in

altitude the $V_{пр}$ falls[1].

FOOTNOTE. [1] Maximum speed falls to height/altitude H = 11 km, whereupon $V_{пр} = \text{const}$, since with $11 \leq H \leq 24$ km the speed of sound is constant. ENDFOOTNOTE.

The limitation of the upper limit of velocity on kinetic heating is characteristic for supersonic aircraft. It is connected with a deterioration in the mechanical properties of materials during the heating of the surface of aircraft in connection with an increase in the velocity of flight, and also with the need for maintaining temperature conditions for flight decks, with the danger of the simmering of fuel/propellant for tanks etc.

The extreme value of mach number on kinetic heating can be determined by the approximation formula

$$M_{пр} = \sqrt{\frac{T_{пр} - T}{r\frac{k-1}{2} - T}} = \sqrt{\frac{T_{пр} - T}{0{,}2r - T}},$$

where the T_{mp} - the maximum for the surface of aircraft absolute temperature of sheathing/skin; T is absolute temperature of surrounding air; k - the coefficient of adiabatic curve (k = 1.4); r is a temperature recovery factor in boundary layer. For a laminar boundary layer it is accepted as r = 0.85, for turbulent r = 0.88-0.9.

The limitation of the upper limit of velocity on velocity head is caused by the considerations of the structural strength of aircraft and usually acts at the low altitudes, which do not exceed 5000-6000 m. If at these height/altitudes velocity head, depending on air density, is exaggerated, then the stresses in parts and node/units of aircraft will exceed permissible, which can lead to

undesirable structural distortions and even to their destruction.

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The usually extreme value of velocity head of $q_{пр}$, taken during the calculation of constructions, is determined from formula

$$q_{пр} = kq_{max}, \tag{3.41}$$

where the q_{max} - the maximum permissible value of velocity head in level flight, which depends on the class of aircraft; k - the coefficient, depending on flight conditions; in level flight it is equal to unity. For aircraft of the type "Il-18" and An-10 $q_{max} \sim 12500$ N/m²; for an aircraft of the type TU-104 $q_{max} \sim 19000$ N/m²; value k the mode/conditions of planning for aircraft Il-18 and An-10 is equal to 1.4.

Maximum velocity head of $q_{пр}$ and maximum flight speed of

V_{np} are connected by dependence

$$V_{np}=\sqrt{\frac{2q_{np}}{\varrho}}=\frac{V_{inp}}{\sqrt{\Delta}}. \qquad (3.42)$$

Maximum equivalent airspeed, as it is shown above, is constant for all height/altitudes, which facilitates monitoring of flight conditions.

The limitation of acceleration on g-force is caused by the effect of g-forces on the psychophysical state of the crew and passengers, and also on the structural strength of aircraft. The effect of g-force on the psychophysical state of man depends on the amount of g-force, on direction and duration of its action. Greatest permissible g-force - in spin alignment - breast; smallest - in direction head - basin.

The limitation of flight altitude on sonic boom is connected with the fact that in flight of aircraft at supersonic speed appears the shock wave, which produces an intermittent pressure increase on

the surface of the Earth. The intensity of sonic boom depends on the flight altitude, velocity, volume of aircraft, angle of attack etc. With the removal/distance of aircraft from the surface of the Earth, the intensity of sonic boom decreases. The maximum permissible overpressure above the populated areas must not exceed value $\Delta p \approx 100$ N/m^2.

The indicated value of pressure differential in shock wave determines the height/altitude at which the aircraft can accomplish supersonic flights. Thus, for instance, for an aircraft TU-144 this height/altitude is equal to 10-14 km.

At high altitudes there is no limitations on the pressure differential, but there can be limitations on the velocities, connected with the fact that upon the acceleration/dispersal of aircraft to $M \approx 2.2$ (at operating altitude $H \approx 18$ km) appear the high thermal stresses in the structural elements.

Questions for repetition.

1. In which case the minimum required thrust does remain constant during a change in altitude and in which case it can change?

2. Why in Fig. 3.4 right sides of the curved required thrusts for different height/altitudes do not intersect, but in Fig. 3.7 they do intersect?

3. Which character does have dependence of $H=f(V_{max})$ for aircraft with TRD [ТРД - turbojet engine] and TVD [ТВД - turboprop engine] and as it is expressed graphically?

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4. Which character does have a dependence of $V_{max}=f(H)$ at the different values of the specific load (p = G/S) of transonic aircraft?

5. When the practical speed range of level flight is more - with the released or retracted the landing gear?

6. How is reflected the surface contamination of aircraft in most advantageous flight speed?

Tasks.

1. Determine is the maximum quality and the value of the $c_{y\,max}$ of aircraft TU-104 - at height/altitude H = 4 km, counting $c_{x0}=0{,}018$ (see Fig. 3.15). Answer/response: $K_{max}=17{,}5$.

2. The drag coefficient of the aircraft with of $c_y=0$ as a result of the surface contamination of aircraft increased by 25o/o. To determine the maximum speed loaded aircraft at altitude 6 km, if is known the maximum speed of "pure/clean aircraft" (see Fig. 3.16). It is known also that the inductive reactance of aircraft Ан-24 under the conditions of V_{max} is 15o/o of total resistance.
Answer/response: $V_{max}=446$ km/h.

Chapter IV.

CLIMB AND ~~REDUCTION~~/DESCENT ~~IN THE~~ AIRCRAFT

4.1. Equations of motion.

In this chapter is examined the motion of the center of mass of aircraft along the inclined straight path, arrange/located in vertical plane (Fig. 4.1) in the absence of yawing motion (laterally level flight and slip). The flight of aircraft along this inclined trajectory is more common/general/total in comparison with level flight. The adopted assumptions make it possible to obtain simple and sufficiently precise for practical target/purposes results.

Angle θ path inclination to the horizon is considered positive, if the vertical component of speed is directed upward and contributes to an increase in altitude in the process of flight. This flight conditions is called the climb. The mode/conditions of a reduction/descent in the aircraft is characterized by the negative flight path angle. Reduction/descent with the zero engine thrust is

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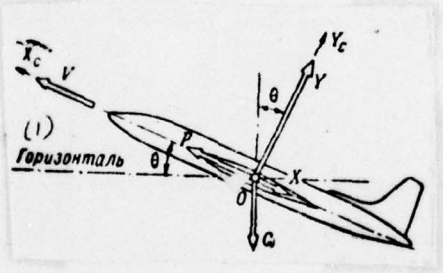

Fig. 4.1. Diagram of the forces, which act on aircraft in flight along inclined trajectory.

Key: (1). Horizontal.

called gliding/planning, and along very steep trajectory ($|\theta| > 0.5$ ~~is glad~~ rad) - by dive.

In flight along inclined trajectory on aircraft, act the forces, indicated in Fig. 4.1. The projection of gravitational force on the Ox_c of wind coordinate system in this case is equal, i.e., $G\sin\theta$. In the climb regime, if trajectory speed does not change, the engine thrust must be higher than during level straight flight. In the mode/conditions of a reduction/descent in the aircraft, the projection of gravitational force on the axle/axis of Ox_c coincides with the direction of flight speed and contributes to the acceleration/dispersal of aircraft.

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For research on the flight of aircraft along inclined trajectory in the absence of bank (with $\gamma_c=0$) and of slip (with $\beta = 0$) are most conveniently applied the equations of motion of aircraft (2.11) which for that which is examine/considered the case change as follows:

$$\left.\begin{aligned} P - X - G\sin\theta &= m\frac{dV}{dt}, \\ Y - G\cos\theta &= 0. \end{aligned}\right\} \tag{4.1}$$

Subsonic passenger and transport aircraft usually make the gain of altitude and reduction/descent with a insignificant change in the trajectory speed. Therefore in a number of cases it is possible in the first equation of system (4.1) to disregard force of inertia; then the equations of the rectilinear steady flight for inclined trajectory accept the following form:

$$\left.\begin{aligned} P - X &= G\sin\theta, \\ Y &= G\cos\theta. \end{aligned}\right\} \tag{4.2}$$

The dependence of lift coefficient cn the flight path angle θ can be traced with the aid of the second equation of system (4.2),

after rewriting it as follows:

$$c_y \frac{\rho V^2}{2} S = G \cos \theta.$$

Then

$$c_y = \frac{2G \cos \theta}{\rho V^2 S}.$$

If speed of V along trajectory does not change, then with an increase in the flight path angle θ lift coefficient will decrease, but that means decrease will angle of attack α.

It must be noted that the angle θ for subsonic aircraft usually does not exceed 0.1-0.15 ~~in glad~~ rad. For such angles of cos θ, very it differs little from unity; therefore angle of attack, lift coefficient and especially lift comparatively barely depend on the flight path angle and in the approximate computations it is possible

to place cos $\theta \approx 1$. However, this simplification is not applicable during the determination of the required thrust or required power in the process of the set of height/altitude or reduction/descent.

4.2. Required speeds, thrust/rods and powers for the steady flight along inclined trajectory.

If are known flight altitude, the flight path angle and angle of attack, then the required flight speed along inclined trajectory can be determined, after solving equations (4.3) relative to speed of V:

$$V = \sqrt{\frac{2G\cos\theta}{\rho S c_y}}. \qquad (4\ 4)$$

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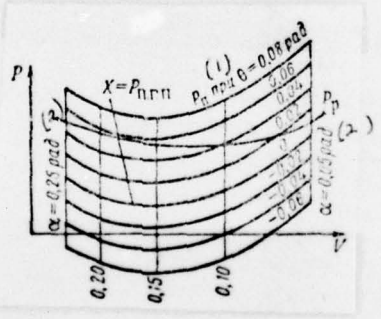

Fig. 4.2. Required thrusts in flight along inclined trajectory.

Key: (1). with. (2). ~~is glad.~~ rad.

From the comparison of the obtained expression with formula (3.4) it follows that during motion along inclined trajectory the required speed, other conditions being equal, (among other things with of identical c_y) it will depend on angle θ, after grow/rising to the value of necessary horizontal flight speed with the approach/approximation of trajectory toward horizontal.

For determining the characteristics of the lift of aircraft with turbojet engines, usually are applied the curves of required thrusts.

The value of required thrust for the steady flight in inclined trajectory is determined from the first equation of system (4.2):

$$P_n = X + G \sin \theta. \tag{4.5}$$

From relationship (4.5) it follows that in flight with the climb the engine thrust must be more than drag to the value of the force component of weight G sine θ. During reduction/descent the required thrust becomes less than the drag, since in this case $\theta < 0$.

With $\theta = 0$ required thrust will be equal to the required thrust of level flight, i.e., to drag.

Force component of weight G sine θ does not depend on flight speed; therefore on the curve/graph of the available and required thrusts, the dependences of required thrust on flight speed with θ = const are depicted as equidistant curves (Fig. 4.2). These curves make it possible to determine required thrust at the different values of the flight speed and flight path angle. If we on the curves of required thrusts plot the curve of the point of tangency of P_p, then the points of intersection of these curves will determine the climb regimes at the maximum trajectory speed at different angles θ.

At the wide flight path angles in the process of reduction/descent, the projection of gravitational force on direction of motion can exceed drag (see Fig. 4.2). In the cases the required thrust becomes negative and aircraft flies with acceleration even during a decrease in the thrust down to zero. In such flight conditions for the preservation/retention/maintaining of constant velocity, are applied aerodynamic brake.

The analysis of the flight of aircraft with piston or turboprop engines is conducted on the curves of required powers.

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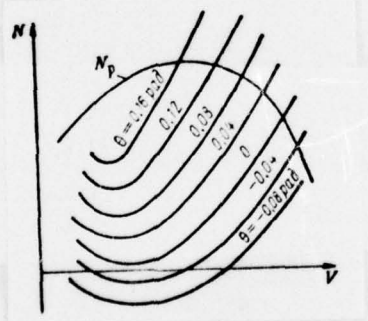

Fig. 4.3. Required powers in flight along inclined trajectory.

Key: (1). is glad.

After multiplying both parts of equation (4.5) to the velocity of flight for inclined trajectory, we will obtain expression

$$P_n V = XV + GV \sin\theta.$$

The product of drag X to the flight speed of V corresponds to required power in the level flight of $N_{n.r.n.}$ (See Chapter III). Thus, the obtained equation can be written in the form

$$N_n = N_{n.r.n} + GV \sin\theta, \qquad (4.6)$$

where the $N_n = P_n V$ are required power in flight along inclined trajectory.

By using equation (4.6), it is possible to construct the curves of required powers for a flight from the inclined trajectory which, however, no longer will be equidistant, since the difference between

the required power during lift or with reduction/descent and required power during level flight will increase with an increase in the flight speed along trajectory (Fig. 4.3).

§ 4.3. Polar of speeds for the steady flight on sloping trajectory.

As it is shown into § 4.2, the point of intersection of curved required thrusts from the curved point of tangency (see Fig. 4.2) they make it possible to establish/install the dependence between the velocities of the steady flight from trajectory and the climb angles. Of aircraft with screw propeller this conformity to conveniently determine according to the points of intersection of the curved required and available powers (see Fig. 4.3).

The dependence between flight speed and the flight path angle can be depicted graphically, also, in polar coordinate system (Fig. 4.4), where the radius-vector is the flight speed. Curve, that welds of the velocity vectors, is called the polar of speeds. In the case, depicted on Fig. 4.4, in all points of polar the vertical component of speed has positive value, i.e., all flight conditions on this polar they correspond to the climb, if engine works in the nominal

rating. This polar is called another the polar of rates of climb.

To the polar of rates, usually will be deposited the angles of attack, as this is made in Fig. 4.4.

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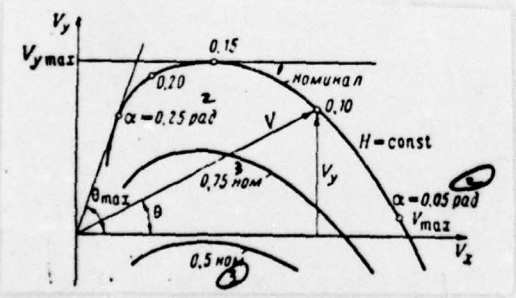

Fig. 4.4. Polar of rates.

Key: (1). Rating. (2). ~~is glad~~ rad. (3). nominal.

If are known points of tangency or powers for the different engine power ratings, that for a each of these mode/conditions it is possible to construct corresponding to it the polar of rates, whereupon the more the engine power rating it differs from the nominal, the lower will be arranged the polar of rates. Beginning from the determined operating mode (in Fig. 4.4 this mode/conditions corresponds approximately 0.5 ratings), the vertical component of speed will be negative everywhere, i.e., at any flight speed a descent will occur. For the flight when point of tangency or power are equal to zero, i.e., aircraft makes gliding/planning, we will obtain the polar of gliding speeds

From chapter III known that with a change in altitude the available and required thrusts also change, therefore it is possible to construct a series of the polars of flight speeds from inclined trajectory for different height/altitudes.

Let us examine in more detail the polar of rates of climb (see Fig. 4.4). The angle of tangent inclination, carried out to the polar of rates of climb from the origin of coordinates, corresponds to the maximum climb angle of $\theta_{max.}$ and point of contact of tangency divides polar by two parts. By comparing in Fig. 4.4 angles of attack and the corresponding to them climb angles, it may be concluded that

under conditions of lift, which correspond to the right side of polar diagram, the rate with an increase in the angle of attack decreases, and the climb angle increases; under conditions of lift, which correspond to the left side of the curve/graph, with an increase in the angle of attack, decrease the climb angle, and flight speed.

§ 4.4. Flight path angle. The vertical velocity during the climb.

The vertical component of the flight speed of aircraft is equal to the rate of change in the flight altitude and is called the vertical velocity. During lift or with reduction/descent, the vertical velocity straddle Oy wind coordinate system, composing with it goal θ; however, in the literature on the dynamics of flight, is accepted to designate the vertical velocity by the symbol of V_y, and not V_{yg}, replacing index g by word "vertical"; therefore

$$V_y = V \sin \theta. \qquad (4.7)$$

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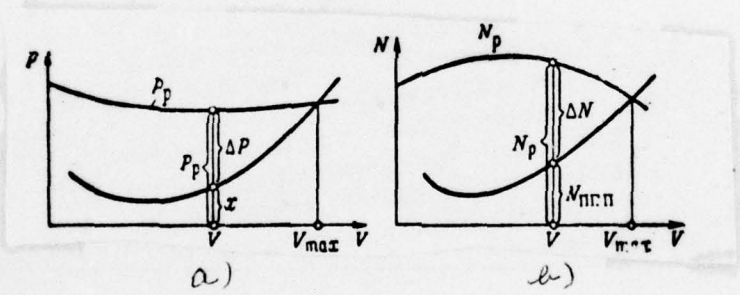

Fig. 4.5. To the determination of the margin of thrust (a) and of power (b).

Changes in the vertical velocity correspond to changes in the flight speed in trajectory. From the different climb regimes it is possible to select one, corresponding to the maximum vertical velocity $V_{y\,max.}$ In Fig. 4.4 this mode/conditions is determined by the ordinate of the point of contact of the tangency of straight line, parallel axis of abscissas, with the polar of rates of climb.

The climb regimes from θ_{max} and at the rate of $V_{y\,max}$ correspond to the varied conditions of flight. If flight from θ_{max} is applied in flying practice comparatively rarely, then the mode/conditions of lift from $V_{y\,max}$ is encountered considerably more frequently, since under these conditions aircraft rapidly gains altitude.

The point of intersection of the polar of rates of climb with the axle/axis of abscissas corresponds to horizontal flight condition at the maximum speed of $V_{max.}$. The vertical velocity under these conditions of flight stops to equal to zero, because whole point of tangency is spent on the overcoming of drag.

Quantities of the angle of slope of the trajectory and vertical speed can be calculated analytically by using equations of motion.

For turbojet aircraft the value of the vertical velocity is

determined from the first equation of system (4.1) by the substitution in it of value sine θ from relationship (4.7):

$$V_y^* = \frac{P - X}{G} V - \frac{V}{g} \frac{dV}{dt}. \qquad (4.8)$$

During the steady flight along the trajectory when $dV/dt = 0$, expression (4.8) for determining the vertical velocity will take form

$$V_y^* = \frac{P - X}{G} V = \frac{\Delta P V}{G}, \qquad (4.9)$$

where ΔP is a margin of thrust (Fig. 4.5a).

For aircraft with the screw propellers of formula for determining the vertical velocity it is possible to obtain in more convenient form. Taking into account the fact that the product of required thrust and velocity is the required power in flight along inclined trajectory, and the product of drag and velocity is required

power during level flight, to expressions (4.8) and (4.9) it is possible to give the respectively following form:

$$V_y^* = \frac{N_p - N_{u.r.n}}{G} = \frac{1}{g}\frac{dV}{dt}V = \frac{\Delta N}{G} = \frac{1}{g}\frac{dV}{dt}V. \tag{4.10}$$

$$V_y^* = \frac{N_p - N_{u.r.n}}{G} = \frac{\Delta N}{G}, \tag{4.11}$$

where ΔN is a margin of power (Fig. 4.5b).

The flight path angle during the steady flight can be determined by formula (4.7), if we substitute into it the value of V_y^* from formula (4.9) or (4.11):

$$\sin\theta = \frac{\Delta P}{G} = \frac{\Delta N}{GV}; \tag{4.12}$$

[redacted]

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[redacted]

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4.5. Maximum vertical velocity and most advantageous trajectory speed during steady climb.

The maximum vertical velocity of $V_{v\,max}$ characterizes the ability of aircraft to heave to base altitude for minimally short time interval. The value of the maximum vertical velocity depends on the engine power rating, but for the sake of simplicity in the

calculations, all formulas of this paragraph are derive/concluded under the assumption that the engines work in the nominal rating.

If are known the polars of velocities for different height/altitudes, then the value of $V_{y\ max}$ for each height/altitude can be found from highest point on the polar of velocities (see Fig. 4.4). However, as a result of the large labor expense of construction for polar, usually is applied simpler auxiliary construction.

From formulas (4.9) and (4.11) it follows that the vertical velocity will have the maximum value at the maximum values ΔPV or ΔN.

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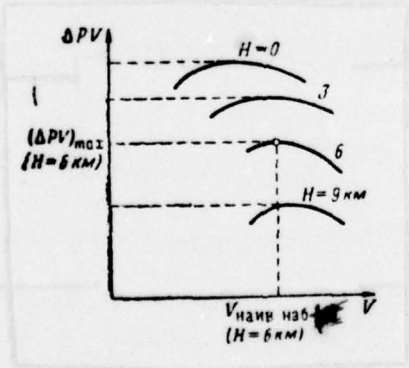

Fig. 4.6. К For the determination of the $(\Delta PV)_{max}$ of aircraft with TRD [ТРД - turbojet engine].

For a turbojet aircraft the determination of the greatest vertical velocity at different height/altitudes is conducted by the following order of auxiliary constructions and calculations:

- the plotting of curves of the required and of points of tangency at the different height/altitudes of level flight (see Fig. 3.15);

- the measurement of margins of thrust ΔP at different velocities and at the taken flight altitudes;

- the calculation of the corresponding products ΔPV;

- the construction of a series of the graph/diagrams of dependence $\Delta PV = f(V)$ (Fig. 4.6) for each of the taken flight altitudes;

- the determination of the maximum values (ΔPV) for the different height/altitudes of flight;

- the calculation of the values of the maximum vertical velocity for these height/altitudes on somewhat modified formula (4.9):

$$V_{y\,max}^{\bullet} = \frac{(\Delta PV)_{max}}{G}.$$

the flight speed, corresponding to $(\Delta PV)_{max}$, is called the most advantageous rate of climb of $(V_{наив.\,наб.})$, , the vertical velocity in this case it will be maximum.

Of subsonic aircraft with TRD, the curve of points of tangency is almost parallel to the axle/axis of abscissas (see Fig. 3.15); therefore the most advantageous rate of climb of $V_{наив.\,наб.}$ will be somewhat more than the optimum speed of the level flight of $V_{наив.}$ with which it is reached the maximum aerodynamic quality of aircraft.

If the maximum vertical velocities for different

height/altitudes are determined, it is possible to construct the graph/diagram of the dependence of $H=f(V_{y\max})$ (Fig. 4.7a, b). Thrust/rod TRD usually decreases with height/altitude, and in this case the plotted function of $H=f(V_{y\max})$ is close to straight line. For a high-altitude turbojet aircraft the curve/graph of $H=f(V_{y\max})$ has a crank point at height/altitude H = 11 km, since beginning with this height/altitude ceases a temperature drop and the engine thrust decreases more intensely.

The point of intersection of the curve of $H=f(V_{y\max})$ with the axle/axis of ordinates determines static theoretical, or the theoretical, ceiling of $H_т$.

Absolute ceiling really existing cannot be achieve/reached by subsonic aircraft. In any case for this, will be required unlimitedly the wide interval of time, since with approach/approximation to absolute ceiling the vertical velocity decreases to zero.

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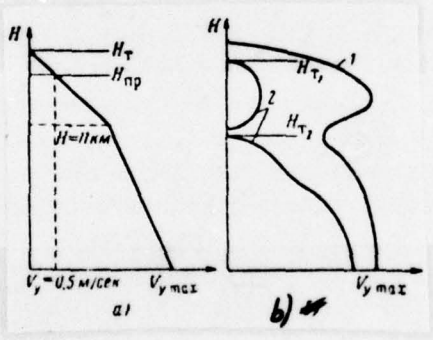

Fig. 4.7. Dependence of the maximum speed of subsonic (a) and supersonic (b) aircraft with turbojet engines on height/altitude.

Therefore the height of different aircraft is compared with respect to their service ceiling of ($H_{пр}$ equal to the height/altitude at which the vertical velocity reaches the determined, in advance fixture. In the Soviet Union as this criterion, is accepted the velocity, equal to 0.5 m/s (see Fig. 4.7).

Of supersonic aircraft in certain altitude range, the margin of thrust ΔP can have considerable maximums with noticeable "failures" between them, what is the reason for a sufficiently complex change in the $V_{y\ max}$ with height/altitude (is curve 1 in Fig. 4.7b).

This means that the aircraft can have two value of absolute ceiling (See Fig. 4.7b - curve 2): subsonic (H_{T2}) and supersonic (H_{T1}). During the lift of this aircraft at the height/altitudes less than the subsonic ceiling, it is necessary first because of margin of thrust to drive away in level flight prior to supersonic speed, and then to already continue ascent.

For aircraft with screw propellers, the determination of the greatest vertical velocity at different height/altitudes somewhat is simplified in comparison with jet and is made in the following order:

- the plotting of curves of the required and of available powers at the different height/altitudes of level flight (see Fig. 3.16);

- the construction of a series of the graph/diagrams of dependence $\Delta N = f(V)$ for each of the taken flight altitudes (Fig. 4.8);

- the determination of the maximum values ΔN for different flight altitudes;

- the calculation of the values of the maximum vertical velocity for these height/altitudes on somewhat modified formula (4.11):

$$V_{y\,max} = \frac{(\Delta N)_{max}}{G}.$$

poston-engined aircraft has the maximum vertical velocity on the full-throttle height of engine (Fig. 4.9a). Of aircraft with high-altitude turboprop engine, the maximum vertical velocity is almost constant to the full-throttle height of engine, and then gradually it begins to decrease with height/altitude to zero.

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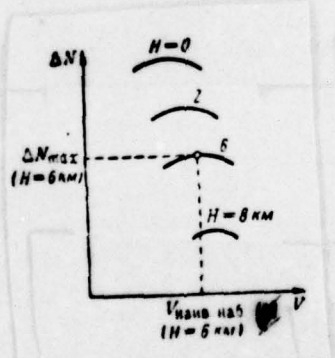
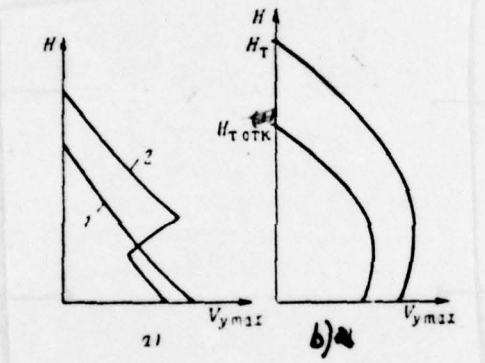


Fig. 4.8. To the determination of the $(\Delta N)_{max}$ of aircraft with screw propellers.

Fig. 4.9. The character of a change in the vertical velocity with height/altitude for an aircraft with the piston (a) and turboprop (b) engines: 1 - low-level PD [ПД - instrument panel]; 2 - PD with preliminary pressurization/supercharging ($H_{T\ OTK}$ are the absolute ceiling of multiengine turboprop aircraft with one failed engine).



The nature of a change in the maximum vertical velocity with height/altitude for a four-engine turboprop aircraft is given in Fig. 4.9b.

The lift of aircraft under the conditions at the maximum vertical velocity is applied in the civil aviation comparatively rarely, since with passenger transportation, besides speed and cost-effectiveness/efficiency of flight, it is necessary to provide the comfort of the passengers and the simplicity of the aircraft control. In order that the passengers painless withstand pressure change in the process of the climb, it is necessary comparatively slowly to change pressure in passenger compartment.

But to the norms of airworthiness for the passenger aircraft of the USSR pressure change in cabin/compartment for one second must not exceed 250 N/m^2 (1.8 mm Hg), that in flight of the Earth corresponds to the vertical velocity approximately 1.6 m/s, at altitude 3-3.5 km - velocity 2 m/s. That means aircraft with nonhermetic cabin/compartment must gain altitude comparatively slowly despite the fact that the power-weight ratio of this aircraft makes it possible to gain altitude considerably faster.

The passenger aircraft, intended for flights at height/altitude more than 3500 m, is equipped by the pressurized cabin pressure in which for any flight altitude must not be less than 75,500 N/m² (565 mm Hg), which corresponds to height/altitude 2400 m on standard atmosphere.

After the takeoff of aircraft, atmospheric pressure in pressurized cabin usually is supported to the height/altitude at which the pressure outside will be below pressure in cabin/compartment on the determined value, different for different types and the constructions of the aircraft.

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With the pressure differential, for example into 50,000 N/m², constant pressure in cabin/compartment is supported from take-off to height/altitude approximately 5 km for the middle latitudes. At a further increase in altitude, is supported a pressure difference 50,000 N/m² up to the base altitude of flight. Since the rate of an

incidence/drop in the atmospheric pressure decreases with height/altitude, the vertical velocity at height/altitude more than 5000 m will comprise approximately 5-6 m/s at the regulated rate of change of the pressure in cabin/compartment. Thus far in the process of the set of cabin altitude is supported "terrestrial" pressure, the vertical velocity to the conditions of comfort is not restricted. During a reduction/descent in the aircraft, act the same limitations.

4.6. the vertical velocity during the unsteady climb.

To the value of the vertical velocity of aircraft at the climb, considerable effect exerts a change in the flight speed. If, for example, during the climb flight speed increases, then the energy of engines is expend/consumed not only on an overcoming of drag and an increase in the potential energy, but also on an increase in the kinetic energy. That means in flight with acceleration/dispersal the vertical velocity, but respectively also the flight path angle they will be less than during the steady flight. Similar flight conditions is applied in the climb by turbojet and turboprop aircraft after takeoff.

If the climb occurs with braking, then the vertical velocity grow/rises, since part of the kinetic energy will pass over to potential, increasing flight altitude.

Taking into account that the ratio of path acceleration dV/dt to free-fall acceleration g numerically equal to g-force with respect to the trajectory of n_x a expression P-X/G according to formula (4.9) equal to the vertical velocity of the V_y^* of the steady flight relationship (4.8) it is possible to write in the form

$$V_y = V_y^* - V n_x. \qquad (4.13)$$

formula (4.13) it makes it possible to quantitatively evaluate the dependence of the vertical velocity on the g-force of n_x and change in the flight speed in trajectory. If, for example, aircraft flies at a rate of 250 m/s, and tangential g-force is equal $n_x = 0.1$ i.e. for one second speed it decreases approximately on 1 m/s, then the vertical velocity increases on 25 m/s. As we see, an increase in the vertical velocity the so large that to them it cannot be disregarded.

To evaluate the effect of a change in the kinetic energy to the value of the vertical velocity, let us make a series of the auxiliary transformations:

$$Vn_x = \frac{V}{g}\frac{dV}{dt} = \frac{V}{g}\frac{dV}{dH}\frac{dH}{dt},$$

but $$\frac{dH}{dt} = V_y,$$

therefore $$Vn_x = \frac{V}{g}\frac{dV}{dH}V_y = V_y\frac{d}{dH}\left(\frac{V^2}{2g}\right).$$

now expression (4.13) assumes the form

$$V_y = V_y^* - V_y \frac{d}{dH}\left(\frac{V^2}{2g}\right).$$

hence

$$V_y = \frac{V_y^*}{1 + \frac{d}{dH}\left(\frac{V^2}{2g}\right)}. \qquad (4.14)$$

Second term in denominator it determines rate of change with the height/altitude of the specific kinetic energy of aircraft, i.e., the kinetic energy, divided by weight of aircraft.

From relationship (4.14) as and from (4.13), it follows that during the climb with acceleration the vertical velocity of V_Y becomes lower than the vertical velocity of V_Y^* with that which was

establish/installing altitude gain along straight path.

Along with a change in the altitude of flight, let us examine a change in the energy height/altitude of $H_э$, for which let us differentiate expression (2.23) by height H:

$$\frac{dH_э}{dH}=1+\frac{d}{dH}\left(\frac{V^2}{2g}\right). \qquad (4.15)$$

taking into account that the denominator in expression (4.14) is the expression of the derivative of $\frac{dH_э}{dH}$, it is possible to determine the dependence of the vertical velocity on a change in the energy height/altitude:

$$V_y=\frac{V_y^\circ}{\frac{dH_э}{dH}}. \qquad (4.16)$$

from equalities (4.15) and (4.16) it follows that with an increase in the velocity with the height/altitude of $\frac{dH_э}{dH}>1$ and

$V_y < V_y^*$.

If one considers that for a subsonic aircraft the rate of climb is little affected, then, obviously, a change of their energy height/altitude in essence depends on a change in the altitude, i.e., $V_y \approx V_y^*$, and then $dH_э \approx dH$.

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For supersonic aircraft second-segment climb occurs with acceleration; therefore for it a difference between between the V_y and V_y^* can be sufficiently considerable. Furthermore, one should consider that of some types of supersonic aircraft in the determined altitude range is possible a sharp decrease in the value of V_y^* up to zero (see Fig. 4.7). At these height/altitudes passage from subsonic flight speeds toward supersonic can be fulfilled in level flight with the low positive values of V_y^* or in flight with reduction/descent, if $V_y^* < 0$.

Since the specific kinetic energy composes the considerable part

of the energy height/altitude of supersonic aircraft, great practical interest is of the greatest specific energy of aircraft and the corresponding to it maximum energy height/altitude, called energy ceiling.

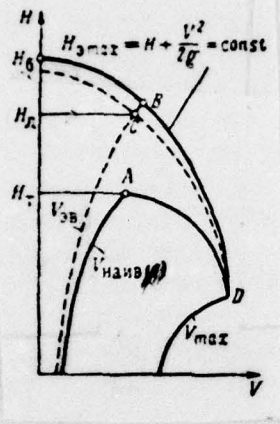
The energy ceiling of $H_{э\,max}$ is reached at the height arrange/located usually lower than absolute ceiling, at flight speed, on close to maximum for all height/altitudes. Therefore if we calculate the energy height/altitude of $H_{э\,пот}$ during the steady flight of supersonic aircraft on absolute ceiling, then always $H_{э\,max} > H_{э\,пот.}$ After acceleration/dispersal to maximum speed at the height which corresponds to energy ceiling, as a result of the conversion of kinetic energy into potential during the unsteady retarding flight it is possible to build up aircraft on the height/altitude, which exceeds static absolute ceiling.

If we realize that entire reserve of kinetic energy is used for the lift, then in this case with a decrease in the velocity of flight down to zero aircraft must is bygone achieve the ballistic ceiling of $H_б$, during which it it would possess only potential energy. However, ballistic ceiling is virtually unattainable for aircraft for the following reasons.

First, for the preservation of aircraft handling its speed must be not lower than the minimum safety speed of $V_{эв}$. The value of safety speed is defined by the value of evolutive velocity head with which the aircraft as vehicle with aerodynamic controllers retains another controllability, i.e., "slushchaetsya" controls. Evolutive velocity head, it is natural, it has constant values for all height/altitudes; therefore safety speed increases with the height/altitude, remaining in this case less minimum ($V_{эв} < V_{min}$), how it is provided aircraft handling under conditions of takeoff and landing.

In the second place, during lift to ballistic ceiling in certain altitude range drag exceeds the engine thrust. And finally upon transition of aircraft from straight flight at the height which corresponds to energy ceiling, to the climb regime it is necessary to increase angle of attack; therefore on certain phase of flight, part of the energy is expend/consumed on the overcoming of the grown inductive reactance.

Fig. 4.10. To the determination of the service ceiling of aircraft.



And although subsequently inductive reactance can be decreased down to zero, all the same yenergeticheskaya aircraft altitude in the process of lift will be less than in the beginning of maneuver, i.e., in flight at the height which corresponds to the energy ceiling of ku (is curve DC in Fig. 4.10).

The greatest height/altitude which can achieve the aircraft during the unsteady controlled flight, is called the service ceiling of $H_{\text{сл}}$. At the height/altitude of service ceiling, the flight speed is equal to safety speed. If we designate the total loss of specific energy by the symbol of $H_{э.п}$, that energy balance during lift from energy ceiling to dynamic will be expressed by relationship

$$H_{э\,max} = H_{\text{д}} + \frac{V_{эп}^2}{2g} + H_{э.п},$$

a the height/altitude of service ceiling - by formula

$$H_A = H_{э\,max} - \frac{V_{зп}^2}{2g} - H_{э.п}.$$

diagram for determining service ceiling is shown in Fig. 4.10. The velocity band of the steady flight is limited by the to the left most advantageous flight speed of $V_{наив}$, to the right - by the maximum speed of V_{max}. Here is plotted/applied the dependence of the safety speed on flight. Point A, at which the $V_{наив} = V_{max}$, determines static absolute ceiling.

The curve BD on curve/graph corresponds to a change in the flight speed with height/altitude in the constant specific energy, equal $H_{э\,max}$. The equation of the curve BD is expressed by formula

$$H_{э\,max} = H + \frac{V^2}{2g}. \quad (4.17)$$

the point of contact of tangency D of the curves BD and AD determines the altitude, which corresponds to the maximum energy height/altitude, i.e., to the energy ceiling of aircraft. In the absence of energy losses, the ballistic ceiling is determined by the point of intersection of the curve of ggggqg with the axle/axis of ordinates.

By disregarding losses, service ceiling can be determined by point B of the intersection of the curve BD with the curve $V_{зв}=f(\dot{H})$. Virtually service ceiling will be determined by point C which is arranged below point B to the rate of energy loss.

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Lower than the point C is arranged the range of unsteady flights. With a decrease in the height/altitude, the speed range of the unsteady level flight is expanded, they increase the horizontal phases of flight with braking.

4.7. Barograms and the trajectories of climb and reduction/descent.

In flying practice are encountered the diverse variants of flight over the inclined trajectory whose accomplishing is connected with the solution of extreme problems. In the number of the most important tasks of this type it is possible to name the climb for the minimum time interval, the set of base altitude with the minimum fuel consumption, steep climb, the gain of altitude or reduction/descent with output/yield to the given point of space for a minimum time and a series of others. The majority of these tasks is related to the range variation.

As an example are set forth below the methods of the determination of the time of climb and trajectory of flight during the assigned climb regime.

Analytically transit time from one flight conditions, characterized by the energy height/altitude of $H_{э1}$, toward another with the energy height/altitude of $H_{э2}$ during unsteady flight it is possible to determine from expression (3.35):

$$t=\int_{H_{э1}}^{H_{э2}}\frac{dH_э}{V_y^*}.$$

this integral is calculated approximately by numerical or graphic methods.

During the steady flight along the trajectory of $dH_э=dH$ and the given formula for determining of t accepts the following form:

$$t=\int_{H_1}^{H_2}\frac{dH}{V_y^*}.$$

thus, during unsteady flight the time of a change in the flight conditions is determined not by the geometric, but energy height/altitude, and in both cases the calculation is conducted not on the true vertical velocity of V_y, a on the velocity of V_y^*, which virtually it can be reached only during the rectilinear steady

climb.

As a rule, during the calculation of integrals the time of climb is calculated from $H_1=H_{01}=0$ to service ceiling.

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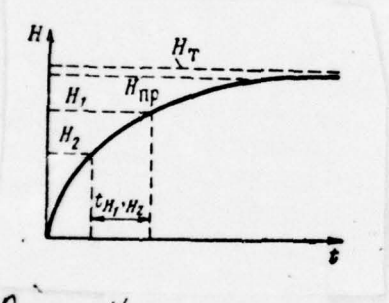

Fig. 4.11

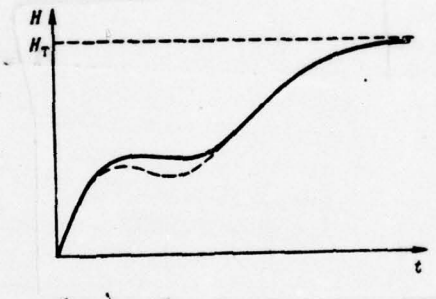

Fig. 4.12

Fig. 4.11. Barogram of the set of the height/altitude of subsonic aircraft.

Fig. 4.12. Barogram of the set of the height/altitude of supersonic aircraft.

The graphic dependence of the climb on time is determined from so the one who is called the barogram of the climb. This name was added historically it is explained by the fact that barogram it can be traced by barograph, i.e., by barometer-chart recorder. Since the height/altitude and air pressure are interconnected, barograph can be thoroughly calibrated not on pressure, but by height, and the recording of instrument will depict the dependence of height/altitude on time, i.e., to barogram.

The barogram of the set of the height/altitude of subsonic aircraft is shown in Fig. 4.11, supersonic - in Fig. 4.12. The horizontal line, carried out at the height/altitude of absolute ceiling, is asymptote for both barogram. Barogram in Fig. 4.12 it has the horizontal area/site, which corresponds to the acceleration/dispersal of aircraft in level flight prior to supersonic speeds, since in this altitude range of level flight the vertical velocity of steady climb for a supersonic aircraft decreases (Fig. 4.7b) or even becomes equal to zero. Contemporary supersonic aircraft, as a rule, have large thrust-weight ratio; therefore for the majority of them of horizontal area/site on barogram (see Fig. 4.12) it can and not be.

If it is required to determine duration of ascent beginning with the torgue/moment of takeoff, then the time, counted off on barogram, will be less approximately on 1-1.5 min. This is explained by the fact that the unstick speed lower than the speed of most advantageous lift; therefore in the beginning of lift aircraft moves accelerated, but its vertical velocity in the beginning of trajectory is lower than maximum.

The length of steady climb can be determined approximately, if is known the rate of climb:

$$L_{наб} = \int_0^t V \cos\theta dt \approx V_{ср} t. \qquad (4.18)$$

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Rate of V subsonic aircraft with height/altitude is little affected; therefore it it is possible to average and to remove from

under integral sign. Furthermore, at comparatively low climb angles it is possible to place $\theta = 1$. More accurately the length of steady climb can be found, after constructing trajectory of climb.

To give the single procedure for the trajectory calculation for all possible cases of unsteady flight is difficult, since the flight trajectory can be both rectilinear and curvilinear. Furthermore, the unsteady flight is usually caused by special requirements, for example by the shortest flight time to the given point in space. During the solution of the last/latter problem, it can seem that the climb from $V_{y\ max}$ will not be optimum, and aircraft faster will achieve the given point at the smaller climb angles and at larger trajectory speed.

A reduction/descent in the aircraft is possible without a change in the engine power rating; however, to avoid the excess of maximum flight speed engines usually are throttled, and then margins of thrust in these mode/conditions, and also, therefore, the vertical velocity they become negative as this evidently from Fig. 4.4.

The similar phenomenon can occur with failure of one of the

engines of multiengine aircraft under the conditions of cruising flight. The character of a change in the $V_{y\max}$, of aircraft with failure of one of the engines is shown in Fig. 4.9b. In this case the aircraft will descend until it achieves the absolute ceiling, determined taking into account the remaining in work engines ($H_{T\,OTK}$ in Fig. 4.9b). Flight at this height/altitude is virtually impossible, since during the least decrease in the flight altitude, for example with the incidence/impingement in the descending gust of air, aircraft no longer will return to absolute ceiling. To confidently keep level of flight pilot will be able only at the height/altitudes less than the new absolute ceiling and which do not exceed the service ceiling, determined taking into account the engine failure.

The calculation the barograms of reduction/descent is conducted accurately in the same manner as for during the climb, only with the changed integration limits (limits must be interchanged the position), since in these calculations the V_y^* will be negative. The distance of reduction/descent is calculated from formula (4.18).

4.8. Gliding/planning aircraft. Flight of glider/airframe.

Gliding/planning - flight conditions with zero or close to zero by thrust/rod. Unpowered flight on specially for this the intended flight vehicles - glider/airframes possesses large attractiveness because of the complex of the subjective perceptions, experience/tested by glider-pilot in flight, and also to the high requirements which are presented to the skill of glider-pilot during the solution of the problems of the maximum use in flight of the technical flight properties of the glider/airframe: the achievement of the maximum range of flight, height/altitude, load capacity, etc.

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Our Soviet glider-pilots on the glider/airframes of the constructions of O. K. Antonov attained the outstanding successes in the sport mastery/adoption of airspace, to them belong many world records in this manly form of sport.

However, interest in the gliding condition of contemporary aircraft is explained in any way not by sport considerations. Good

gliding characteristics of aircraft are a guarantee of flight safety, since for the qualified pilot they make it possible to complete the confident touchdown in the failure of engines.

Set/assuming for the sake of simplicity in the calculations, that the gliding/planning is made along straight path without bank and slips, from equations (4.1) we will obtain for gliiing/planning in vertical plane

$$\left.\begin{aligned} -X-G\sin\theta=m\frac{dV}{dt}, \\ Y-G\cos\theta=0. \end{aligned}\right\} \qquad (4.19)$$

during the steady process of gliding/planning equation (4.19) they are simplified:

$$\left.\begin{aligned} X=-G\sin\theta, \\ Y=G\cos\theta. \end{aligned}\right\} \qquad (4.20)$$

equations (4.19) and (4.20) can be obtained directly from the relationship between the forces, which act on aircraft during gliding/planning (Fig. 4.13).

All the gliding characteristics are slope angle, the vertical velocity and, etc - it is possible to determine of the obtained previously dependences for a flight by inclined trajectory, equating to zero the engine thrust, and also from equations of motion during gliding/planning, especially because these equations are maximally simple.

For determining slope angle during steady glide, it is necessary

to piecemeal divide the first equation of system (4.20) into the second, then

$$\operatorname{tg}\theta = -\frac{X}{Y} = -\frac{c_x}{c_y} = -\frac{1}{K}.$$

gliding angle and the corresponding to it flight speed along trajectory it is possible to determine by the points of intersection of curved required thrusts or powers with the axle/axis of abscissas (see Figs. 4.2 and 4.3). If gliding angle is known, then speed can be determined by formula (4.4).

Fig. 4.13. Diagram of the forces, which act on aircraft during gliding/planning.

Fig. 4.14. Polars of gliding speeds.

Key: (1). Dive. (2). Gliding/planning.

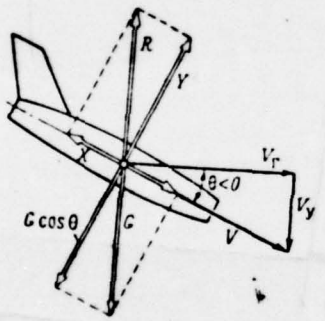

Fig. 4.13

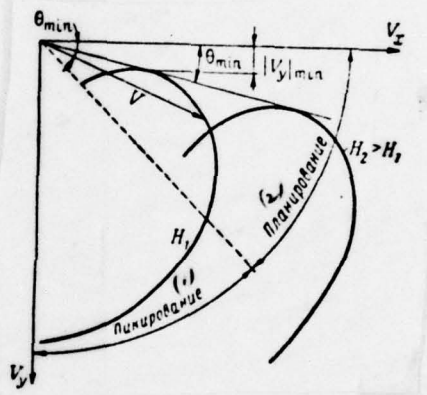

Fig. 4.14

Since each flight speed along trajectory corresponds the defined gliding angle, by plot/depositing velocity vector from the beginning of the coordinate system as from pole, at the appropriate gliding angle, it is possible to obtain the polar diagram (polar) of gliding speeds (Fig. 4.14).

Gliding angle has the minimum absolute magnitude with the maximum lift-drag ratio, i.e.,

$$|\theta|_{min} = \operatorname{arctg} \frac{1}{K_{max}}.$$

vertical and the horizontal components of flight speeds it is possible to find from relationships

$$V_{rop} = V\cos\theta; \quad V_y^* = -V\sin\theta = -\frac{V\cos\theta}{K} = -\frac{V_{rop}}{K}. \qquad (4.22)$$

in flying practice gliding angles usually do not exceed

0.07-0.10 is glad, also, by these cases of $V_{rop} \approx V$.

On the polar of gliding speeds, the point of contact of the tangency of straight line, carried out from the origin of coordinates, with the polar of gliding speeds determines the mode/conditions flat glide of (θ_{min}). The highest point of polar corresponds $|V_y|_{min}$ i.e. to the slowest reduction/descent. For the subsonic passenger aircraft of the speed flat glide of $(|\theta|_{min})$ and slowest reduction/descent in the $(|V|_{v\,min})$ they differ little from each other.

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At the wile gliding angles of $(|\theta| > 0{,}5$ rad, i.e. during dive, the gliding speed is determined from formula (4.4), several that which was altered taking into account relationship $c_y = c_R \cos\theta$:

$$V=\sqrt{\frac{2G}{\varrho S c_R}},$$

where

$$c_R=\sqrt{c_x^2+c_y^2}.$$

during the nose dive of $c_y=0$ and flight speed is expressed by formula

$$V=\sqrt{\frac{2G}{\varrho S c_{x0}}}.$$

flight speed during gliding/planning it increases with a decrease in the air density; therefore different to height/altitudes will correspond the different polar diagrams of speed (see Fig. 4.14).

The distance of the rectilinear steady glide it is possible to determine from relationship

$$l = H \operatorname{ctg} \theta = KH. \qquad (4.2)$$

during the unsteady gliding/planning, i.e., during gliding/planning with acceleration/dispersal or with braking, it is necessary to consider a change in the kinetic energy. If during the steady flight the supplementary potential energy of position is expend/consumed on the completion of work on the overcoming of forces of friction, then during gliding/planning with acceleration/dispersal it is spent on an increase in the kinetic energy. That means during gliding/planning with acceleration/dispersal, the flight path angle will be abrupt/steeper than during steady glide, and during gliding/planning with braking flatter. With respect changes gliding distance.

The length of trajectory during gliding/planning is determined by conversions and integration of differential equation

$$dl = ds\cos\theta = V\cos\theta dt,$$

where ds - the trajectory element, flown for the interval of time dt, but dl - the horizontal projection of this cell/element.

By substituting in previously obtained relationship

$$V_y^* = \frac{dH_3}{dt}$$

the value of V_y^* according to formula (4.22), let us find

$$dt = -\frac{KdH_3}{V\cos\theta}.$$

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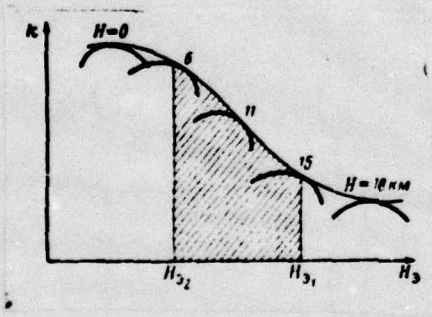

Fig. 4.15. the plotted functions of $k=f(H, H_0)$

By substituting this value dt in (4.24), let us determine dl in the form

$$dl = -KdH_э.$$

or

$$l = -\int_{H_{э1}}^{H_{э2}} KdH_э = \int_{H_{э2}}^{H_{э1}} KdH_э. \qquad (4.25)$$

in the approximate computations the value of lift-drag ratio it is possible to average by height and to remove it from under integral sign. Then we obtain

$$l \approx K_{ср}(H_{э1} - H_{э2}) = K_{ср}\left(H_1 - H_2 + \frac{V_1^2 - V_2^2}{2g}\right). \qquad (4.26)$$

with $V_1 = V_2$ formula (4.26) assumes the form previously obtained formula (4.23).

Gliding distance will be greatest, if gliding condition for each height/altitude is be maintain/withstood with the maximum quality. The maximum quality of subsonic aircraft virtually is equal at all height/altitudes. During calculations of the transonic aircraft of K_{max} for different height/altitudes, it is necessary to average.

The more precise values of the maximum gliding distance are obtained by grapho-analytic method, if are known the curves of the required thrusts of level flight for a series of height/altitudes.

By having curves of required thrusts and after assigning a series of values of velocity, it is possible to determine for each height/altitude and the flight speed of the value of lift-drag ratio and energy height/altitude:

$$K=\frac{G}{P_{\text{п}}};\quad H_{\text{э}}=H+\frac{V^2}{2g}.$$

after which to construct the graph/diagrams of the dependence of $K=f(H, H_{\text{э}})$. Their approximate form is shown in Fig. 4.15. The envelope is the graphic representation of the integrand of integral (4.25) for the optimum gliding conditions at all height/altitudes. The maximum range of gliding/planning is determined by the area, limited by the appropriate energy height/altitudes, the curve/graph of integrand and by the axle/axis of abscissas.

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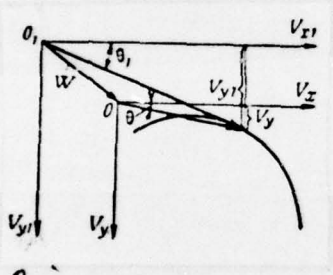

Fig. 4.16

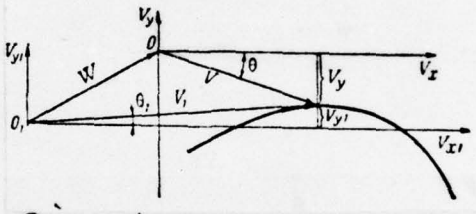

Fig. 4.17

Fig. 4.16. Effect of the incidental down wind on the polar of gliding speeds.

Fig. 4.17. Effect of the incidental upward wind on the polar of gliding speeds.

With the incidence/impingement of aircraft into the air current, characterized speed W, its speed earth referenced V_1, called ground speed, will be equal to the vector sum of wind velocity and gliding speed, i.e.,

$$\vec{V}_1=\vec{V}+\vec{W}.$$

for convenience in the evaluation of the effect of wind velocity on different gliding conditions the diagram of gliding/planning is constructed in the new coordinate system. For this, one should the origin of coordinates (see Fig. 4.14) transpose in the direction, to the reciprocal vector of $\vec{W}$ (Fig. 4.16). In the obtained thus system of coordinates are determined the trajectory speed V_1, the vertical component of the speed of V_{y1} and the flight path angle of θ_1. The character of glide path depends on wind direction: trajectory will be flatter with the incidental and upward wind and abrupt/steeper with wind contrary and descending.

If the vertical component of wind proves to be more than the vertical velocity of flight vehicle, then its reduction/descent will

occur slower than the lift of air, i.e., flight altitude will increase. This is the completely real case, since, for example, of good glider/airframes the smallest rate of descent can be less than 1 m/s, but under conditions of the Soviet Union, fairly often meet the stable updraft vertical component 1-3 m/s and more. Diagram on Fig. 4.17 is constructed for the case when in certain flight envelope the value of the vertical velocity earth referenced becomes positive, i.e., is possible an increase in altitude of flight.

Skillfully by utilizing up drafts, it is possible to gather high altitude and to fly distance several hundreds of kilometers.

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4.9. Effect of operational and structural/design factors on the characteristics of the set of height/altitude, reduction/descent and gliding/planning.

The effect of structural/design and operational factors on the flight of aircraft in vertical plane can be traced, somewhat by

converting the already obtained previously relationships.

So, after fulfilling term-by-term division in formula (4.9), we will obtain

$$V_y^* = V\left(\frac{P}{G} - \frac{X}{G}\right).$$

taking into account that the flight path angle of transport aircraft is usually very low, it is possible in the second term of the right side of the formula to accept $G \approx Y$. Then for determining the vertical velocity we obtain expression

$$V_y^* \approx V\left(\frac{P}{G} - \frac{X}{Y}\right) = V\left(p - \frac{1}{K}\right),$$

where p = P/G - the thrust-weight ratio of aircraft.

The last/latter equality makes it possible to draw the conclusion that an increase in thrust-weight ratio and aerodynamic fineness ratio leads to an increase in the rate of climb.

On the average for contemporary passenger aircraft it is possible to consider that at medium altitudes a change in the thrust (or power) of engine to 1o/o it leads to a change in the vertical velocity with that sign to 1.5-2o/o. The more is loaded aircraft, the considerable the effect of thrust/rod (or power) on the vertical velocity.

An increase in the gross weight with the assigned thrust/rod negatively shows up in the value of the vertical velocity at the climb, mainly, as a result of a decrease in the thrust-weight ratio of aircraft. In the first approximation, it is possible to consider that an increase in the gross weight by 1o/o produces a decrease in the vertical velocity by 1.5-2o/o. A decrease in the vertical velocity in turn, leads to a decrease in the theoretical and service ceilings of aircraft. A change in the weight to 1o/o produces change in the ceiling on 60-70 m, if ceiling exceeds 11 km. On the lower altitudes the gravity effect is more considerably. For example with ceiling in limits 8-10 km each percentage of a change of the weight

produces change in the ceiling approximately on 100 m.

From meteorological conditions most strongly affect the vertical velocity the pressure and the temperature of air. For turbojet engines, for example, a change in the thrust/rod with constant/invariable number of revolutions can be approximately expressed by dependence

$$P = P_0 \Delta \frac{T_0}{T_H},$$

where P_0 is a thrust/rod under standard atmospheric conditions of the Earth; Δ - the relative density of air; T_0 and T_H, - respectively the temperature of the Earth and under conditions of flight.

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Relative density and temperature decrease with height/altitude, whereupon to altitude 11 km an incidence/drop in the density passes a

temperature drop; therefore thrust/rod TRD with height/altitude decreases. At height/altitudes more than 11 km a decrease in the thrust/rod becomes more intense, since at these height/altitudes temperature is constant, and density continues to decrease. For TVD [ТВД - turboprop engine] the character of power change in temperature dependence approximately similar.

For the standard atmosphere of SA-64 (GOST [ГОСТ - All-union State Standard] 4401-64) relation

$$\frac{P}{P_0}=\Delta\frac{T_0}{T_H}$$

for different height/altitudes has the following values:

H км	0	2	4	6	8	10
$\Delta T_0/T_H$	1,000	0,860	0,735	0,684	0,583	0,434

pressure and temperature are subjected to the space-time changes which one should consider. For example the point of tangency of engines in flight at the level of sea in cold frost weather will be significantly higher than in hot weather somewhere in the area of high-mountain airfield. Therefore in the first case the climb is made through more steep trajectory, than in the second.

To gliding characteristics as this is evident from formulas (4.4) and (4.18), in essence affect lift-drag ratio and the weight of of aircraft. The weight of aircraft affects velocity along

trajectory, and lift-drag ratio affec the gliding angle. The greatest lift-drag ratio of contemporary passenger aircraft is 16-17. The temperature and air pressure affect gliding speed, since on them depends the air density, and in practice they do not affect angle and gliding distance.

PROBLEMS FOR REPETITION.

1. Write is the condition of the lift of aircraft for straight path and the corresponding to it equation of motion.

2. What such is the most advantageous rate of climb?

3. How to explain the possibility of two ceilings of supersonic aircraft (see Fig. 4.7b)? Page 93.

4. In which flight conditions during lift, the vertical velocity

will be more - in establish/installed or that which was being unsteady? How is expressed the relationship between them at constant power?

5. What such is the energy ceiling of aircraft?

6. Draw the polars of gliding/planning aircraft without those which were released and with lowered with mechanization/high-lift device and chassis/landing gear with H = const and explain how they they are distinguished.

TASK.

To determine the maximum vertical velocity of the $V_{y\,max}$ of aircraft Ah-24 of the Earth under the following conditions: the equivalent horsepower of one engine N = 1880 kW; the spread/scope of wing l = 29.2 m; wing area 72.46 m^2; the drag coefficient with of $c_y = 0$ is equal to 0.022; propeller efficiency at the flight speeds of $\eta_в = 0.7$ in question the effective aspect ratio of wing less than geometric by 30o/o.

Answer/response: $V_{y\,max} = 7{,}4$ m/s.

[redacted]

[redacted]

[redacted]

[redacted]

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Chapter V.

DISTANCE AND DURATION OF FLIGHT.

§ 5.1. Basic determinations and concepts.

The way l, flown by aircraft, it is possible to divide into three parts: way during the set of the height/altitude of $l_{наб}$, during a reduction/descent in the $l_{сн}$ and during the level flight of $l_{гор}$. Then

$$l=l_{наб}+l_{гор}+l_{сн}.$$

. The way of aircraft during the climb and during reduction/descent is calculated from formula (4.18). The length of the horizontal section of the way of $l_{гор}$ depends on the reserve of fuel/propellant and expenditure/consumption by its engines.

the distance, measured according to the earth's surface, which can fly aircraft without the replenishment of supplies of fuel/propellant, is called flying range. The time during which the aircraft is found in air, is called duration of flight.

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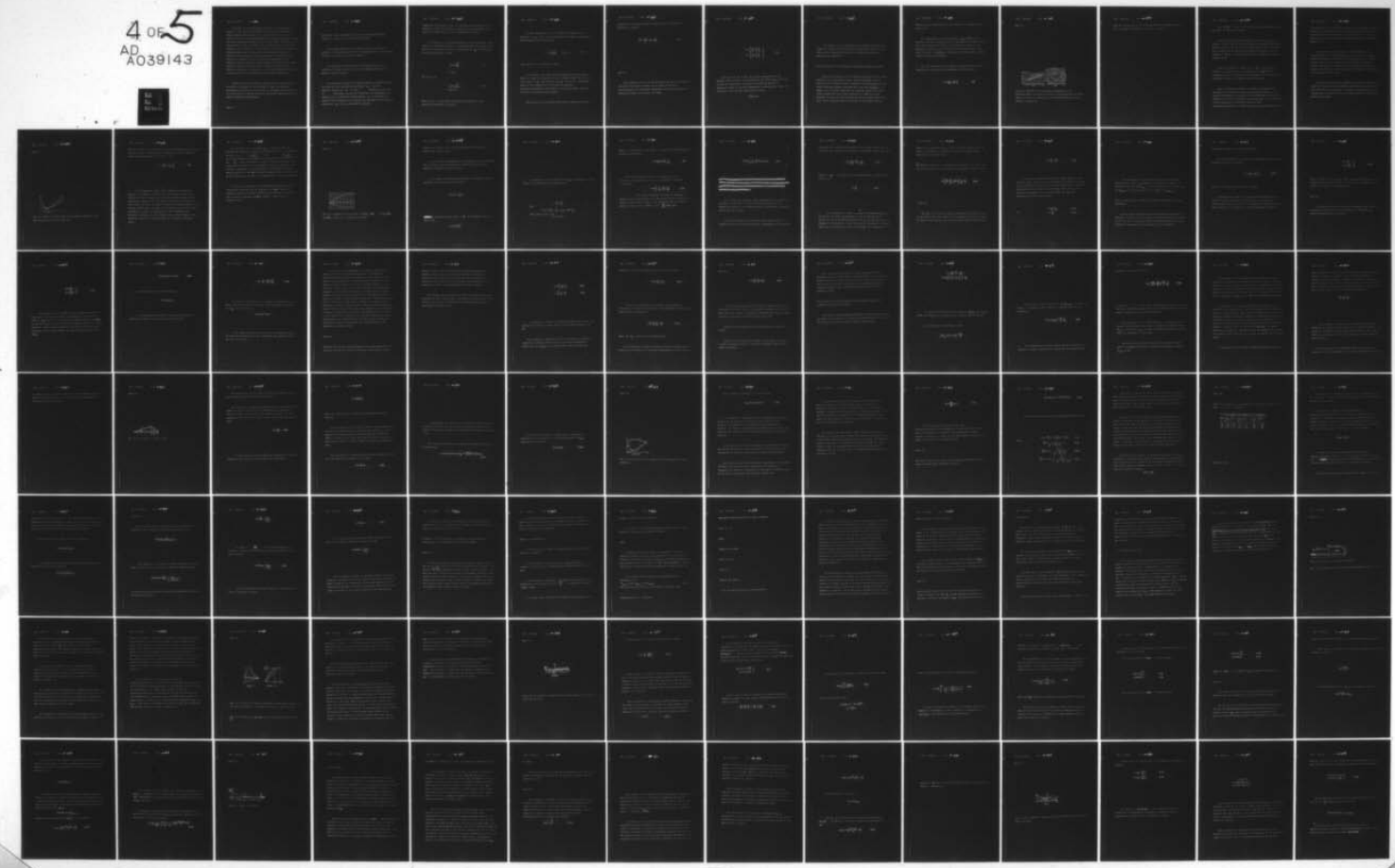

The amount of fuel/propellant, expended on the horizontal section of way, can be determined, if we from the total amount of fuel/propellant deduct part of it, expended during the gain of altitude and with reduction/descent, with starting/launching, testing and engine warm-up, taxiing to start, in the engine operation in the expectation of start, in flight on the landing circle, and also the unproduced fuel remainder in fuel tanks. By the useful expenditure of fuel/propellant, had reserve, is its expenditure on the gain of altitude, level flight and reduction/descent. The obtained flying range in the absence of wind and with the consumption of the available reserve toward the torque/moment of landing with the load, caused by technical specifications, is called technical range. Technical range can be reached as in flight when trajectory lie/rests at one plane, so also in flight along the closed trajectory.

Virtually not entire available fuel reserve is expend/consumed in flight on distance. In the calculation always is considered aeronautical fuel reserve. This reserve is necessary for the case of the complication of weather condition, short-term disorientation and others unforeseen circumstances.

The flying range, calculated taking into account aeronautical reserve, is called the service range of flight.

The greatest distance up to which can the can be removed aircraft from takeoff point with the subsequent return to the initial airfield, is called the radius of action.

The important characteristics of transport and passenger aircraft are maximum range of flight, the maximum endurance, the maximum radius of action.

The necessity of engine for fuel/propellant is characterized by the specific fuel consumption, designated c_p - for TRD [turbojet engine] and c_e - for TVD [turboprop engine] and PD [instrument panel]. The differences in designations are explained by the different dimensionality: c_p is explained the specific fuel consumption in kilograms for one hour for one newton of thrust/rod, c_e they is explained the specific

expenditure/consumption of fuel in kilograms for one hour for one kiloWatt of power. The lesser the specific fuel consumption, other conditions being equal, the more economical the engine.

The specific fuel consumption depends on the engine power rating (degree of throttling/choking), of height/altitude and flight speed. The hourly consumption of the fuel/propellant of $q_ч$ is determined by the relationships: for TRD

$$q_ч = c_p P \frac{(1)\ кг}{час}; \qquad (5.1)$$

(1) kg/h

for TVD and PD

$$q_ч = c_e N \frac{(1)\ кг}{час}, \qquad (5.2)$$

(1) kg/h

where P and N - respectively thrust/rod TRD power TVD or PD, established/installed on aircraft.

The fuel consumption on 1 km way (the fuel consumption per kilometer of $q_к$) finds by the division of the hourly consumption of fuel/propellant for flight speed:

$$q_к = \frac{q_ч}{3{,}6V} \quad kg/km \qquad (5.3)$$

(here speed of V is expressed in m/s).

It is obvious, the length and the duration of the horizontal phase of flight they depend on the fuel reserve aboard the aircraft, on the hour of $q_ч$ and kilometer the $q_к$ of the fuel consumption. Hour and fuel consumption per kilometer as specific expenditure/consumption, they depend on speed and flight altitude, and also on the engine power rating.

Flight time and the distance which will fly aircraft with the

consumption of fuel/propellant in amount dm, it is possible to determine by formulas

$$dl = -\frac{dm}{q_к}; \quad dt = -\frac{dm}{q_ч}. \tag{5.4}$$

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Here negative sign is introduced because the mass of aircraft in the process of flight decreases by the value of the spent fuel/propellant. By integrating expression (5.4), it is possible to determine distance and duration of flight:

$$\left.\begin{aligned} l &= -\int_{m_0}^{m_1} \frac{dm}{q_к} = \int_{m_1}^{m_0} \frac{dm}{q_к}, \\ t &= -\int_{m_0}^{m_1} \frac{dm}{q_ч} = \int_{m_1}^{m_0} \frac{dm}{q_ч}. \end{aligned}\right\} \tag{5.5}$$

here m_0 and m_1 are a mass of aircraft respectively in the beginning and at the end of the section of the level flight for which is made the calculation. A change in the mass of aircraft is numerically equal to the fuel consumption on this phase of flight (if in flight is not made the jettisoning of load):

$$m_1 = m_0 - m_1.$$

. The methods of the determination of distance and duration of flight for aircraft with TRD and for aircraft with TVD and PD have their special feature/peculiarities; therefore both cases are examined here separately.

§ 5.2. Distance and the duration of flight of aircraft with TRD.

distance and duration of level flight they depend on the flight conditions and engine power rating. From relationship (5.1) it follows that the minimum hourly consumption of fuel/propellant, and also, therefore, maximum endurance will occur with $(c_p P_n)_{min}$. In chapter III, is bygone shown that the required thrust for a level flight changes over wide limits depending on speed and flight altitude. The specific fuel consumption changes comparatively little (Fig. 5.1a), increasing with an increase in the flight speed and

decreasing with height/altitude and increase in the engine speed (Fig. 5.1b).

The graph/diagram of the dependence of $q_ч = f(V)$ for an aircraft with TRD is similar with the curve/graph of required thrusts for a level flight (see Figs. 3.2 and 3.3) and is only a little displaced to the side of an increase in the velocity (Fig. 5.2), since the value of $(c_p P_п)_{min}$ corresponds to flight speed, somewhat larger than most advantageous.

The fuel consumption per kilometer for aircraft with TRD is determined in accordance with dependences (5.1) and (5.3):

$$q_к = \frac{q_ч}{3{,}6V} = \frac{c_p}{3{,}6}\frac{P_п}{V}. \qquad (5.6)$$

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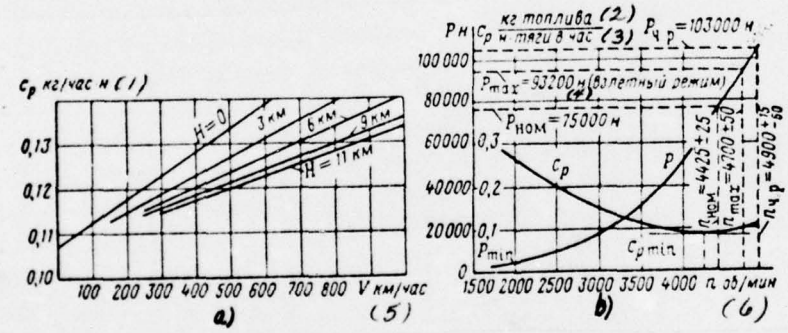

Fig. 5.1. Dependence of the specific consumption of the fuel/propellant of aircraft from TRD on height/altitude and flight speed (a) and of the velocity of the rotation/revolution of the turbine of engine (b).

Key: (1). kg/h•N (2). kg of fuel/propellant (3). N•thrust in hour (4). n (takeoff conditions). (5) km/h ; (6) r/min.

Here $P = P_п$, since in the level steady flight required and the points of tangency are equal.

If we in the first approximation, to assume that the mass of aircraft and the specific fuel consumption are constant/invariable, then in flight at one and the same height/altitude fuel consumption per kilometer will be minimum with minimum the sublimity of the relation of $P_п / V$. This mode/conditions corresponds to flight at cruising speed.

Taking into account a change of the c_p in the process of flight, the minimum value of the expression of $c_p P_п / V$, and, consequently, also the minimum fuel consumption per kilometer will correspond to speed, not how much larger cruising.

With an increase in altitude of flight, the minimum fuel consumption per kilometer sufficiently rapidly decreases. To this largely contributes the increase in the engine speed, caused by an increase in the flight speed with height/altitude. A rate of change in the consumption per kilometer decreases with approach/approximation to service ceiling, and the smallest value of

fuel consumption per kilometer corresponds to the flight of aircraft at the height somewhat the lower altitude of absolute ceiling (on 400-600 m). From this viewpoint a distance flight it is better to realize at the height/altitude of service ceiling or at the height close to it.

It can seem that for a flight at high altitude too much fuel/propellant is expend/consumed in the process of the climb. However, experiment shows that if the horizontal phase of flight composes even the insigniticant part of the entire distance of flight, then flight to nevertheless more favorably accomplish at larger height/altitude, since in this case the amount of expendable fuel/propellant will be less.

In flight of transonic and supersonic aircraft significant role plays wave drag; therefore frequently the flight with the smallest fuel consumption per kilometer corresponds to height/altitude, is considerable (on 3,000-5,000 m) the lower altitude of the absolute ceiling of aircraft.

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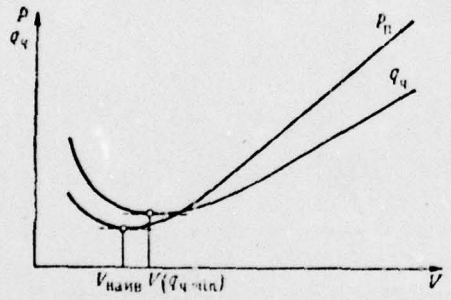

Fig. 5.2. Effect of flight speed on the hourly consumption of the fuel/propellant of aircraft with TRD.

For the analysis of the effect of the flight conditions of supersonic aircraft in fuel consumption per kilometer, we convert expression (5.6) taking into account (3.7) and V = aM:

$$q_{\kappa} = \frac{c_p P}{3{,}6V} = \frac{G}{3{,}6a} \frac{c_p}{KM}. \qquad (5.7)$$

In the supersonic range of Mach numbers, the aerodynamic quality K is gradually decreased with an increase of Mach number, whereupon a rate of change in the lift-drag ratio, beginning with certain Mach number (Fig. 5.3), passes the rate of an increase in Mach number. Therefore the value of product KM reaches maximum at the definite flight speed. The rate of an increase in the specific fuel consumption is small, in summation, the fuel consumption per kilometer, increasing in the transonic range of Mach numbers, then it decreases, reaching the minimum under the conditions of maximum range.

All the given Fig. 5.3 values (c_p, K and, etc) gives for a hypothetical supersonic aircraft, their value with M = 1 are accepted as unity, i.e., $\bar{c}_p = \frac{c_p}{(c_p)_{M=1}}$ and $\bar{K} = \frac{K}{(K)_{M=1}}$. Since the consumption per kilometer is proportional to the factor of the $\frac{\bar{c}_p}{\bar{K}M}$, for this aircraft it will be minimum when $M=2$. From curve/graphs in Fig. 5.3 it is possible to draw the conclusion that the fuel consumption per kilometer with of supersonic velocities weakly depends on flight Mach number; therefore there is a broad band of the flight speeds at which the distance will be close to maximum.

The hourly consumption of fuel/propellant in supersonic zone continuously increases with an increase in the Mach number. The greatest duration of flight of transonic and supersonic aircraft is most frequently possible with Mach numbers = 0.85-1, i.e., in transonic range.

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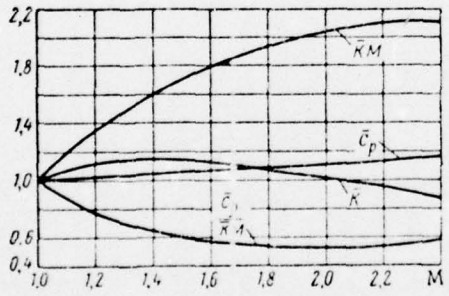

Fig. 5.3. Dependence of the values of $\bar{K}, \bar{c}_p, \bar{K}M$ and $\bar{c}_p/\bar{K}M$ from Mach number for a supersonic aircraft.

Usually this occurs at the speed, several which exceeds most advantageous flight speed.

If hour and fuel consumptions per kilometer for an aircraft with TRD are known, then the distance and duration of flight can be determined by formulas (5.4) and (5.5).

In the steady straight-and-level flight the required thrust in accordance with formula (3.6) is equal to

$$P_{\text{п}} = X = c_x \frac{\varrho V^2}{2} S.$$

By substituting this value of $P_{\text{п}}$ in expression (5.6), we will obtain

$$q_{\text{к}} = \frac{c_{\text{р}}}{3{,}6} \frac{c_x \varrho V S}{2}.$$

By knowing the lift coefficient of c_y, the velocity of level flight can be determined by formula (3.4):

$$V = \sqrt{\frac{2G}{\varrho S c_y}}.$$

Then

$$q_к = \frac{c_p}{3{,}6}\frac{c_x \varrho S}{2}\sqrt{\frac{2G}{\varrho S c_y}} = \frac{c_p}{3{,}6\sqrt{2}}\sqrt{\varrho S G}\,\frac{c_x}{\sqrt{c_y}}.$$

Taking into account that

$$\varrho = \varrho_0 \Delta = 1{,}225\Delta,$$

where ρ_0 - air density of the Earth, $\Delta = \rho/\rho_0$ - the relative density of air, we will obtain

$$q_к = 0{,}68 c_p \sqrt{Sm\Delta}\,\frac{c_x}{\sqrt{c_y}}. \qquad (5.8)$$

After substituting the value of consumption (5.8 per kilometer) into expression (5.5), it is possible to determine flying range:

$$l = 1{,}47 \int_{m_1}^{m_0} \frac{1}{\sqrt{S\Delta}}\,\frac{\sqrt{c_y}}{c_p c_x}\,\frac{dm}{\sqrt{m}}. \qquad (5.9)$$

The obtained integral is solved by numerical or graphic methods. Approximately the calculation can be fulfilled for certain average value of c_p on the assumption that the angle of attack in flight is not changed, i.e., $\frac{\sqrt{c_y}}{c_p c_x} = \text{const}$. Then

$$l=2{,}94\frac{1}{c_F\sqrt{S\Delta}}\frac{\sqrt{c_y}}{c_x}(\sqrt{m_0}-\sqrt{m_1}). \qquad (5.10)$$

In the system of ones MKS, number coefficient in formula (5.10) is equal to 28.8, since in this case mass is replaced by the appropriate weight and the dimensionality of the specific consumption of fuel will be another.

If the fuel reserve does not exceed 35-40 o/o the mass of aircraft, then as it shows practitioner, approximately it is possible

to conduct the calculation according to the average flight mass. In this case from formula (5.9) we obtain sufficiently simple dependence

$$l = \frac{1{,}47}{c_p \sqrt{S\Delta}} \frac{\sqrt{c_y}}{c_x} \frac{m_т}{\sqrt{m_{ср}}}, \qquad (5.11)$$

where the $m_т$ - the spent mass of fuel/propellant, or taking into account (5.7)

$$l = \frac{m_т}{q_к}. \qquad (5.12)$$

If the velocity and flight altitude are assigned/prescribed, the problem is solved unambiguously on any of formulas (5.9) - (5.12). But if is assigned/prescribed only flight altitude and it is required to determine the flight speed, which corresponds to maximum range, then, by utilizing any of the formulas, it is possible to

fulfill the calculation for the range of flight velocities and by means of comparison to determine the flight conditions, which corresponds to maximum range.

The duration of flight is determined from formula (5.5), which with the aid of formulas (5.1) and (3.7) it is possible to give to forms

$$t=\int_{m_1}^{m_0}\frac{dm}{q_{\text{т}}}=\int_{m_1}^{m_0}\frac{1}{c_p}\frac{dm}{P}=\int_{m_1}^{m_0}\frac{K}{gc_p}\frac{dm}{m}. \qquad (5.13)$$

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Integrals in (5.13) are solved by numerical or graphic methods. If one assumes that the specific fuel consumption of fuel and angle of attack do not change, then from expression (5.13) we will obtain

$$t = \frac{K}{gc_p} \ln \frac{m_0}{m_1} . \qquad (5.14)$$

If the fuel consumption during level flight does not exceed 35-40 o/o gross weight, then the calculation of duration of flight can be conducted according to the average flight mass. In this case by averaging of the value of the lift-drag ratio K and of the specific consumption of the fuel/propellant of c_p, from relationship (5.13) we will obtain

$$t = \frac{K}{gc_p} \frac{m_T}{m_{cp}} . \qquad (5.15)$$

or

$$t = \frac{m_T}{q_4} . \qquad (5.16)$$

. From equality (5.15) it follows that if we in the first approximation, accept $c_p = const$, then the greatest duration of flight will be with $K = K_{max}$, i.e., in flight at the optimum speed of $V_{нав}$ on the optimum angle of the attack of $\alpha_{нав}$.

§ 5.3. Distance and the duration of flight of aircraft with screw propellers.

Special feature/peculiarity of the calculation of distance and duration of flight for an aircraft with the screw propellers consists of the need for considering, besides the characteristics of engine, propeller characteristic and especially of its efficiency,

considerably depending on flight conditions.

The fuel consumption per kilometer is determined with the aid of relationships (5.2) and (5.3):

$$q_к = \frac{q_ч}{3{,}6V} = \frac{c_e}{3{,}6}\frac{N}{V}, \qquad (5.17)$$

where N it is determined the power of engine.

Virtually entire power of piston engine is transferred to its shaft, therefore, knowing the efficiency of the propeller of from formulas (5.2) and (5.17), it is possible to determine hour and fuel consumption per kilometer of the piston-engined aircraft:

$$\left.\begin{aligned} q_ч &= \frac{c_e N_п}{\eta_п}; \\ q_к &= \frac{c_e N_п}{3{,}6\eta_п V}, \end{aligned}\right\} \qquad (5.18)$$

where the $N_п$ are the required power, graphically determined by the usually curve of required powers (see Fig. 3.10) and equal in the steady level flight of available power.

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By utilizing dependence (3.12), we will obtain expressions for determining the hour and fuel consumptions per kilometer for poston-engined aircraft in the form

$$\left.\begin{aligned} q_{\text{ч}} &= \frac{c_e G V}{\eta_{\text{пр}} K}, \\ q_{\text{к}} &= \frac{c_e G}{3{,}6\eta_{\text{пр}} K}. \end{aligned}\right\} \qquad (5.19)$$

From chapter II, it is known that of turboprop aircraft the large part of the power of engine is transferred to the propeller shaft of N_B, but less - is realized as reactive power of the ΔN_R of gas jet, coming out from nozzle after turbine. Therefore the total power TVD, called usually equivalent, is equal to the sum of shaft horsepower of the screw/propeller of N_B and reactive power of the ΔN_R:

$$N_e = N_e + PV = N_e + \Delta N_R. \qquad (5.20)$$

Available power finds from relationship

$$N_p = N_e \eta_e + \Delta N_R.$$

Let us introduce the concept of the given efficiency of propeller TVD taking into account the reactive power:

$$\eta_{пр} = \frac{N_p}{N_в} = \frac{N_в \eta_{пр} + \Delta N_R}{N_в + \Delta N_R}. \qquad (5.21)$$

. Virtually in calculation it is possible to consider that the given efficiency of ggggggg is greater than the propeller efficiency of $\eta_{пр}$ by 2-2.5 o/o i.e.

$$\eta_{пр} = \eta_в + (2 \div 2{,}5)\%.$$

. Thus, formulas (5.19) are used also for a turboprop engine, if we during the calculation of fuel consumption per kilometer utilize the given efficiency.

If in the first approximation, the specific expenditure of fuel/propellant and propeller efficiency is set/assumed by constant/invariable, then from formulas (5.19) it follows that the minimum hourly consumption of fuel/propellant corresponds to the minimum required power, i.e., as this is bygone shown in g. III, flight at economic speed. In the same conditions the minimum fuel consumption per kilometer corresponds to flight with the maximum quality, i.e., to flight at optimum speed. Virtually this occurs during the flights of poston-engined aircraft. Of turboprop aircraft with an increase of flight speed, increases the reactive power. Furthermore, during a decrease in the degree of throttling/choking together with an increase of total power increases by the efficiency of engine and decreases the specific expenditure of fuel. According to the character of a change in the power the efficiency of engine and specific fuel consumption, the turboprop engine occupies the intermediate positicn among piston and turbojet engines, more approaching, however, piston.

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Therefore the minimum hourly consumption of fuel/propellant for an aircraft with TVD will occur at the flight speed, which a little

exceeds economic speed; speed with the minimum expenditure per kilometer will be more than most advantageous. The smallest expenditure per kilometer of aircraft with TVD decreases with height/altitude. Therefore for achievement of maximum range, it is necessary to fly at the height/altitude of service ceiling.

The distance and the duration of flight of aircraft with screw propellers we will obtain, after substituting the values of the hour and fuel consumption per kilometer from formulas (5.19) into the integrands of formulas (5.5):

$$l=3{,}6\int_{m_1}^{m_0}\frac{K\eta}{gc_e}\frac{dm}{m}; \qquad (5.22)$$

$$t=\int_{m_1}^{m_0}\frac{K\eta}{gc_eV}\frac{dm}{m}. \qquad (5.23)$$

. Of symbol η, is lowered the index, since here can be used both propeller efficiency of piston engine and the given efficiency for TVD.

The integrals in expressions (5.22) and (5.23) are determined by numerical or graphic methods. If we in the first approximation, assume that the c_e/η is a constant value, then in flight with

constant angle of attack after integration we will obtain

$$l=3{,}6\frac{K\eta}{gc_e}\ln\frac{m_0}{m_1}. \qquad (5.24)$$

But if we in addition to this conduct the calculation according to the average flight mass, then expression for determining flying range is simplified:

$$l=3{,}6\frac{K\eta}{gc_e}\frac{m_т}{m_{ср}}=\frac{m_т}{q_к}, \qquad (5.25)$$

where the $m_т$ - the mass of fuel/propellant.

For determining duration of flight, we will use formula (5.23), which with the averaging of integrand approximately can be written in

the form

$$t=\frac{K\eta}{gVc_e}\frac{m_T}{m_{cp}}=\frac{m_T}{q_u}. \qquad (5.26)$$

The accuracy/precision of the results, obtained with the aid of formulas (5.25) and (5.26) is completely satisfactory, if during level flight the amount of expendable fuel/propellant does not exceed the third of the mass of entire aircraft. Page 104.

Otherwise the calculation can be made through the phases of flight.

Reduction in the weight of aircraft in the process of flight as a result of burnout leads to a slow, but continuous change in the flight conditions.

If we are not add/interfered in engine control, then it is necessary either to lessen angle of attack in order to hold aircraft at base altitude (in this case decreases the inductive reactance and the velocity of aircraft grow/rises), or to maintain/withstand by constant indicated speed, gradually increasing flight altitude (cruising climb).

§ 5.4. Effect of structural/design and operational factors on distance and duration of flight.

The effect of structural/design factors on distance and duration of flight can be examined, by utilizing expressions (5.11) and (5.25) for turbojet and propeller-driven aircraft respectively:

$$l=\frac{1,47}{c_p\sqrt{S\Delta}}\frac{\sqrt{c_y}}{c_x}\frac{m_т}{\sqrt{m_{ср}}};$$

$$l=3,6\frac{K\eta}{gc_e}\frac{m_т}{m_{ср}}=3,6\frac{\eta}{gc_e}\frac{c_y}{c_x}\frac{m_т}{m_{ср}}.$$

In these expressions enter the values of $\sqrt{c_y}/c_x$ and c_y/c_x, which for obtaining the maximum range of flight must be maximal.

In accordance with relationship (3.18):

$$\left(\frac{c_y}{c_x}\right)_{max}=K_{max}=\frac{1}{2}\sqrt{\frac{\pi\lambda_{эф}}{c_{x0}}}.$$

. Taking into account the values of $(c_y/c_x)_{max}$, we find the value of the maximum flying distance of aircraft with the screw propeller:

$$l_{max} = 1{,}8 \frac{\eta}{g c_e} \sqrt{\frac{\pi \lambda_{э\phi}}{c_{x0}}} \frac{m_т}{m_{ср}}. \qquad (5.27)$$

. It is analogous, the using formula (3.23) and (5.11), it is possible to obtain expression for determining the maximum flying

distance of aircraft with TRD:

$$l_{max} = \frac{0,836}{c_p \sqrt{\Delta}} \frac{\lambda_{э\Phi}^{1/4}}{c_{x_0}^{3/4}} \sqrt{\frac{m_{cp}}{S}} \frac{m_т}{m_{cp}}. \qquad (5.28)$$

. Page 105. In formulas (5.27) and (5.28) are taken into account the basic structural/design factors, which affect flying range, which makes it possible to make some conclusions.

For an increase in the flying range, it is necessary to have engines with least possible specific consumption of fuel/propellant (c_e for PD and TVD, c_p for TRD). Aircraft, intended for a flight under the conditions of goal flight.

The wing of this aircraft must have large effective aspect ratio. Of contemporary subsonic aircraft the elongation composes $\lambda_{э\Phi} = 6-10$.

The coefficient of the minimum profile drag must be possible less which is reached by the successful layout of aircraft and by an improvement in the aerodynamic forms. From the comparison of formulas (5.27) and (5.28) it follows that the effect of wing aspect ratio on maximum flying distance of aircraft with TRD is less, but the effect of the coefficient of c_{x0} is more than of aircraft with propellers.

An increase in the gross weight leads to a decrease in the distance, since flight in this case is accomplished at the increased angles of attack and, this means, with the large drag coefficient of aircraft. However, during the comparison of different aircraft types, one should consider that with an increase in the dimensions of aircraft the available fuel reserve increases faster than the gross weight of aircraft, i.e., the relation of m_T/m_{cp} increases, reaching at contemporary aircraft 0.4-0.5. Consequently, the greater the aircraft, fact easier for it to ensure an increase in the flying range.

A contraction of area of wing noticeably manifests itself only

during flights at the speed, close to maximum. A contraction of area of wing, leading to an increase in the specific wing load of m_{cp}/S, makes it possible to decrease the weight of aircraft with the same value of the reserve. During a change in the temperature of air, the hourly consumption of the fuel/propellant of turbojet engine vary directly square root of the absolute temperature:

$$\frac{q_{w1}}{q_{w2}} = \sqrt{\frac{T_1}{T_2}}.$$

Thus, duration of flight with an increase of temperature decreases. As concerns flying range, then it remains unchanged, if in flight is retained the assigned mach number, since in this case the flight speed increases in the same proportion, as fuel consumption per kilometer.

For aircraft with TVD temperature effect on distance and duration of flight approximately the same as for aircraft with TRD.

For piston-engined aircraft a decrease in the temperature produces certain reduction/descent and in the hour of the expenditures of fuel/propellant per kilometer.

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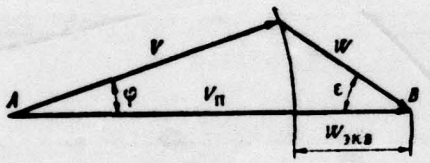

Fig. 5.4. Wind effect on ground speed.

Noticeable effect on the flying range and the radius of action can show wind, especially at low flight speeds.

The velocity of aircraft earth referenced, the so-called ground speed, is equal to vector sum of velocity by air V (velocity of aircraft relative to air) and to wind velocity W (Fig. 5.4). Fuel consumption per kilometer is equal to hour, divided into the ground speed:

$$q_к = \frac{q_ч}{3{,}6V_п}. \quad (5.29)$$

If flight speed and wind velocity are parallel, then the fuel consumption per kilometer is determined from relationship

$$q_к = \frac{q_ч}{3{,}6(V \pm W)},$$

where the positive sign corresponds to tailwind, and "minus" - contrary.

In the general case the velocity vector of wind composes with the vector of ground speed the so-called wind angle ε. In order to maintain the assigned heading, pilot, taking into account wind effect, it must pilot in such a way that the vector of airspeed of V would compose with the assigned heading the angle φ, called drift angle.

The drift angle is determined from velocity triangle (see Fig. 5.4) according to the theorem of the sines:

$$\sin\varphi = \frac{W}{V}\sin\varepsilon. \qquad (5.30)$$

. Consequently, the greater the ratio/relation of wind velocity to airspeed and than more by sine ε, the greater the drift angle. Other conditions being equal, the greatest drift angle will be when $\varepsilon = \pi/2$.

The value of ground speed from velocity triangle is determined by relationship

$$V_n = V\cos\varphi + W\cos\varepsilon = V\left[\sqrt{1-\left(\frac{W}{V}\right)^2\sin^2\varepsilon} + \frac{W}{V}\cos\varepsilon\right]. \tag{5.31}$$

wind effect on the value of ground speed is estimated at calculations at the velocity of the equivalent wind of $W_{экв}$, determined from equality

$$V_п = V + W_{экв}. \qquad (5.32)$$

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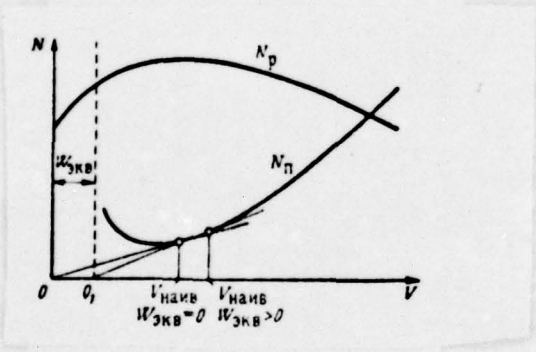

Fig. 5.5. Wind effect on the optimum speed of aircraft with screw propeller.

Hence, utilizing relationship (5.31), we obtain

$$W_{\text{экв}} = V\cos\varphi + W\cos\varepsilon - V. \qquad (5.33)$$

If the value of equivalent wind of definition, then equality (5.32) makes it possible to calculate ground speed, and then according to formula (5.29) to determine fuel consumption per kilometer. The value of equivalent wind velocity can be positive and negative. With respect to this equivalent wind will be incidental or contrary.

In the first case the fuel consumption per kilometer decreases, and distance increases, in the second case - on the contrary, fuel consumption per kilometer increases, and flying range decreases.

Wind affects also the velocity, which corresponds to the minimum consumption per kilometer. This velocity can be obtained, by displacing the origin of coordinates for the curve of required powers by the value of equivalent wind velocity W (Fig. 5.5).

By disregarding the effect of airspeed on the value of the hourly consumption of fuel/propellant, let us arrive at the conclusion that with contrary equivalent wind it is necessary to increase airspeed, and with incidental - to decrease in order to fly with the minimum fuel consumption per kilometer. However, in all cases head wind produces an increase, and incidental - a decrease in the fuel consumption per kilometer.

The value and the direction of wind velocity along course are subjected to the space-time changes, for approximate account of which is determined equivalent wind from route. In the first approximation, route l divide into n of individual sections by length the l_i, in limits of which of equivalent wind velocity can be considered constant. Toga for an entire route of equivalent wind velocity will be defined as sum:

$$W_{экв} = \sum_{i=1}^{n} W_{экв\,i} \frac{l_i}{l}. \qquad (5.34)$$

. Is here registration/accounting the only three-dimensional/space nonuniformity. In order to ascertain that even this approximate calculation gives sufficiently reliable results, we examine the effect of the flight speed, velocity and wind angle on the velocity of equivalent wind.

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For this, from formula (5.33) let us eliminate the value of drift angle ϕ, after using relationship (5.30):

$$W_{экв} = W\cos\varepsilon - V + \sqrt{V^2 - W^2\sin^2\varepsilon}. \qquad (5.35)$$

Then for a velocity increment of equivalent wind we obtain

$$\Delta W_{экв} = \frac{\partial W_{экв}}{\partial V}\Delta V + \frac{\partial W_{экв}}{\partial W}\Delta W + \frac{\partial W_{экв}}{\partial \varepsilon}\Delta\varepsilon, \qquad (5.36)$$

where

$$\frac{\partial W_{экв}}{\partial V} = \frac{1}{\sqrt{1 - \left(\frac{W}{V}\sin\varepsilon\right)^2}} - 1; \qquad (5.37)$$

$$\frac{\partial W_{экв}}{\partial W} = \cos\varepsilon - \frac{\frac{W}{V}\sin^2\varepsilon}{\sqrt{1 - \left(\frac{W}{V}\sin\varepsilon\right)^2}}; \qquad (5.38)$$

$$\frac{\partial W_{экв}}{\partial \varepsilon} = -W\sin\varepsilon\left(1 + \frac{\frac{W}{V}\cos\varepsilon}{\sqrt{1 - \left(\frac{W}{V}\sin\varepsilon\right)^2}}\right). \qquad (5.39)$$

First term in expression (5.36) makes it possible to evaluate the effect of a change in the flight speed to the value of equivalent wind. Calculations show that the change in velocity of flight in sufficiently wide limits barely shows up in the value of equivalent wind even when $\varepsilon = \pm \pi/2$ (Tabl. 5.1).

Data Table 5.1 make it possible to draw the conclusion that the value of equivalent wind, calculated for a flight speed 800 km/h, with sufficient accuracy/precision is valid for the broad band of flight speeds. Even with the wind, which reaches 200 km/h, error during the determination of equivalent wind does not exceed 2 o/o, if the deviation of speed from value 800 km/h does not reach 200 km/h. Consequently, there is no need to make calculations for each flight speed - sufficient to be restricted to calculations for two-three flight speeds, which cover entire range of flight velocities.

The effect of value change of velocity of wind for a velocity increment of equivalent wind depends substantially on wind angle ε as this evidently from the structure of the partial derivative (5.38). The greatest partial derivative will be when $\varepsilon = 0$ or $\varepsilon = \pi$. In this case will occur the equality

$$\Delta W_{экв} = \pm \Delta W.$$

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Table 5.1. Dependence of equivalent wind velocity on flight speed when $\varepsilon = \pi/2$ (V = 800 km/h).

ΔV км/час (1)	$W = 50$ км/час (1)		$W = 100$ км/час (1)		$W = 200$ км/час (1)	
	$\Delta W_{экв}$ км/час (1)	$\frac{\Delta W_{экв}}{V}$ %	$\Delta W_{экв}$ км/час (1)	$\frac{\Delta W_{экв}}{V}$ %	$\Delta W_{экв}$ км/час (1)	$\frac{\Delta W_{экв}}{V}$ %
100	0,15	0,02	0,62	0,07	2,50	0,28
100	0,26	0,04	1,04	0,15	4,32	0,62
200	0,25	0,02	1,01	0,10	4,13	0,41
200	0,70	0,12	2,82	0,48	12,10	2,02

Key: (1). km/h.

It means error in determination the value of equivalent wind at V = const and = const it does not exceed error by the determination of the velocity of wind.

The effect of a change in the wind angle to the accuracy/precision with which is determined of equivalent wind velocity, it is possible to examine, utilizing expression for the partial derivative (5.39) and by velocity triangle in Fig. 5.4. The greatest effect of the change of wind angle manifests itself when $\varepsilon = \pi/2$. In this case with V = const and W = const, we obtain

$$\Delta W_{экв} = -W\Delta\varepsilon.$$

Increase of velocity of equivalent wind is obtained comparatively small. Since W = 0.1 V and increase in the wind angle $\Delta\varepsilon$ = 0.2 rad (11.5°), error in the determination of the velocity of equivalent wind will be 2 o/o from the value of airspeed.

Wind always manifests itself negatively the value of the radius

of action. Let us show this for the case when airspeed and the hourly consumption of fuel/propellant in flight from the place of flight and during return flight are retained unchanged. Let us assume also that in always flight speed and in wind direction are constant.

For the first half of flight, the speed is equal to

$$V_n = V\cos\varphi + W\cos\varepsilon.$$

In accordance with expression (5.29) the fuel consumption per kilometer in this case is equal to

$$q_{k1} = \frac{q_ч}{3{,}6(V\cos\varphi + W\cos\varepsilon)}.$$

For the second half of flight (return route) expression for determining fuel consumption per kilometer will take form

$$q_{к2}=\frac{q_ч}{3{,}6\,[V\cos\varphi+W\cos(\pi-\varepsilon)]}.$$

. Each kilometer of the radius of action aircraft flies twice - back and forth. On this is spent the amcunt of fuel/propellant

$$q_к=q_{к1}+q_{к2}=\frac{2q_ч}{3{,}6V}\,\frac{\cos\varphi}{\cos^2\varphi-\left(\frac{W}{V}\right)^2\cos^2\varepsilon},$$

or after exception/elimination $\cos^2\varepsilon$ from denominator with the aid of relationship (5.30)

$$q_к = \frac{2q_ч}{3,6V} \frac{\cos\varphi}{1-\left(\frac{W}{V}\right)^2}.$$

. The value of $\frac{2q_ч}{3,6V}$ is the fuel consumption per kilometer in flight under conditions of dead calm round trip the ways; therefore

$$q_к = (q_к)_{W=0} \frac{\cos\varphi}{1-\left(\frac{W}{V}\right)^2}. \qquad (5.40)$$

. Since for contemporary aircraft always W < V and cos $\phi \approx 1$, will be correctly inequality

$$q_к > (q_к)_{W=0}. \qquad (5.41)$$

In the particular case when wind blasts accurately in the direction of flight, formula (5.40) will take form

$$q_к = (q_к)_{W=0} \frac{1}{1-\left(\frac{W}{V}\right)^2}.$$

On the duration of flight of aircraft, besides wind, exert themselves the effect and the jet streams, which are encountered on upper boundary of the troposphere. Their speed frequently reaches 100-150 km/h; therefore even for the aircraft, which fly at a high speed, the effect of jet streams turns out to be considerable.

So, from the analysis of 689 flights on route Moscow-Omsk it followed that the hit probability of aircraft into jet streams at a rate of 100 km/h and more is 80 o/o [1].

FOOTNOTE [1]. A. M. Baranov, N. I. Mazurin et al., aeronautical meteorology, l., Gidrometeoizdat, 1966. ENDFOOTNOTE.

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In 1952 the aircraft flew so far from Tokyo in Honolulu (6275 km) for 11 hours instead of the usual 18 hours. During September 1957 aircraft ~~Ty-104,~~ Tu-104, accomplishing voyage Moscow - New-York (about 9,000 km), flew this distance for 13 hours of 30 minutes, and return route from New-York into Moscow flew for 11 hours of 7 minutes. Even if to assume that the speed of jet streams is bygone is constant in value and direction during entire flight time into New-York and back, then of equivalent wind velocity exceeded 80 km/h.

For the target/purposes of the useful use of jet streams in connection with their effect on the voyaging velocity of aircraft in many countries of world, are carried out the intense investigations of this meteorological phenomenon.

PROBLEMS FOR REPETITION.

1. At which angle of attack the flying range of propeller-driven aircraft is maximum?

2. As it changes by height (up to the height/altitude of ceiling) the minimum fuel consumption per kilometer of aircraft from TRD?.

3. You will depict graphically and explain the dependence of the coefficient of the distance of the $\frac{KM}{c_p}$ of supersonic aircraft on Mach number.

4. At which angles of attack, the distance and the duration of

flight of aircraft will be maximum?

5. As affects the ambient temperature, other conditions being equal, the distance and duration of flight?

TASK.

To determine the flying range of aircraft Tu- 104 on the horizontal section of way at height/altitude H = 8 km, flying at cruising speed [mode/conditions $(P/V)_{min}$]. Fuel load, expended on the horizontal phase of flight, is equal $G_T = 140,000$ N. A change in the specific expenditure of fuel/propellant is shown in Fig. 5.1b.

the indicated nominal rating is characterized by the relationship of

$P_{HOM} = 0.8\ P_{BЗЛ}\ (0.8\ P_{max})$. During the sclution of problem, it is necessary to use Fig. 3.15.

Answer/response: L = 2,000 km.

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Chapter VI

TAKEOFF AND LANDING.

6.1. The basic concepts and determinations.

The takeoff and landing are the most critical phases of flight, which require of pilot maximum attention and accuracy/precision. In data statisticians, the greatest percentage of emergencies it falls on the wave-off conditions and landing; therefore to takeoff and landing characteristics of aircraft and their piloting under conditions of takeoff and landing, are presented the special requirements, which cause reliability and flight safety. These requirements are formulated in the norms of airworthiness of the flight vehicles, accepted by the international organization of the civil aviation (ICAO). Along with these norms (" by the international standards") in the separate/individual states, which arrange/locate their own civil aviation, there are their norms of airworthiness (" national standards").

The takeoff and the landing are respectively the initial and final phases of flight of aircraft. In the process of takeoff, the aircraft blows away from the Earth and gains altitude of the assigned echelon, in landing process, it descends from the height/altitude of the assigned echelon and will land to the takeoff and landing strip (runways) of airfield. And in that, and in another case the flight speed changes in the process of flight; therefore the motion in these

mode/conditions is being unsteady.

The aerodynamic characteristics, which determine flight speed, depend on the configuration of aircraft, i.e., the combination of the positions of the wing high-lift device, chassis/landing gear and external suspensions. By the norms of airworthiness are regulated the characteristic flight speeds, which correspond to the assigned configuration of aircraft. let us give the determination of some characteristic flight speeds, which we will operate subsequently.

The practical minimum speed of the steady flight of $V_{\min пр}$ is the speed, reached during a slow decrease in the velocity down to that torque/moment when the steering control of the elevator control is completely is selected by pilot "on itself".

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The practical minimum speed of the steady flight corresponds to values, somewhat less than $\alpha_{кр}$ in the assigned configuration of aircraft; therefore the value of $c_{y\,max}$ in flight at this speed is

not reached.

The speed of the disruption/separation of $V_{ср}$ is the speed, reached during a slow decrease in the velocity (is not more than 1.5 km/h during 1 s) in straight flight, with which appear the oscillations with large amplitude relative to the longitudinal or transverse axis, accompanied by the partial loss of controllability.

The minimum safety speed of the takeoff of $V_{вз.min}$ is the minimum flight speed at which under conditions of failure of one of the engines still is provided the aircraft handling in straight flight with bank not more than 0.087 is glad.

The safe speed of the takeoff of $V_{без}$ is the smallest flight speed at which is allow/assumed the lift of aircraft under conditions of the continuation of takeoff with failed engine to the height/altitude, necessary for the realization of its subsequent landing.

Besides the indicated speeds, great significance to evaluate the

landing data of aircraft has the mentioned previously minimum pitch speed. According to the conditions of flight safety, at this speed are not allow/assumed, since on the approach to of $c_{y\,max}$ is feasible the output/yield of aircraft to angles of attack beyond stalling and its "dumping"; however it is the important characteristic of aircraft, which determines the maximum possibilities of a decrease in the velocity.

6.2. Takeoff of aircraft.

The takeoff of contemporary aircraft can be broken into three stages. During the first stage (Fig. 6.1) is realized take-off run on runway. The basic problem of takeoff/run-up - to acquire at is possible short length this speed which would provide lift-off of the aircraft from the RW. This speed is called of the unstick speed of $V_{отр.}$ According to the norms of the airworthiness of $V_{отр}$ it must exceed not less than by 10o/o minimum pitch speed of V_{min} in takeoff configuration under the conditions of idling or with the zero thrust/rod of power plants and it must be somewhat more than the minimum safety speed of $V_{эк.\,min.}$ The distance at which the speed changes from zero to $V_{отр.}$ is called takeoff run length.

In the second stage is realized the acceleration/dispersal of aircraft before the safe speed of the takeoff of $V_{без}$ with the simultaneous set of the minimum flight altitude. The safe speed of takeoff $V_{без}$ should exceed minimum theoretical speed $V_{мин}$ to 20o/o for twin-engine aircraft and to 15o/o with the larger number of engines and the minimum safety speed of $V_{зв.мин}$ to 10-15o/o. The value of the minimum height/altitude according to the norms of airworthiness is 10 m. The distance at which proceed a change in the speed from $V_{отр}$ to $V_{без}$ and the gain of minimum altitude, is called the length of acceleration/dispersal.

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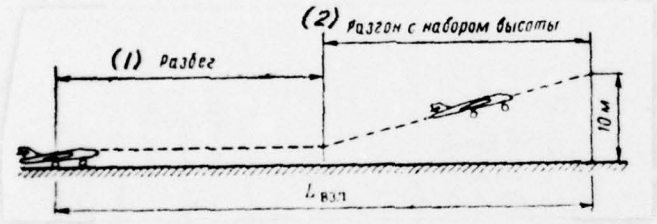

Fig. 6.1. Stages of the takeoff of aircraft.

Key: (1). Takeoff/run-up. (2). Acceleration/dispersal with the climb.

The distance which passes aircraft from the beginning of takeoff/run-up to the end of the acceleration/dispersal, is called the takeoff distance of $L_{взл.}$ The takecff run length and acceleration/dispersal, the takeoff distance, the unstick speeds and the sate speed, the maximum takeoff weight etc. determine the takeoff data of aircraft.

In the third stage which is called the initial climb, the velocity of the aircraft of the is increased, close to most advantageous, and flight altitude reaches approximately 400 m. Respectively change other parameters of the angle of attack, the flight path angle etc.

The profile of the initial climb is determined by the class of the airfield from which is realized the takeoff, since near the Earth of the condition of the flow about the aircraft and, consequently, also the formation of aerodynamic forces largely they differ from the same during flights far from the Earth.

If we compare the distribution of pressure during wing in flight near the Earth and far from it during just one angle of attack, then

it will seem that in flight near the Earth the evacuation/rarefaction on suction side of wing and the pressure on lower grow/rise (Fig. 6.2), that it leads to an increase in the lift. The effect of the Earth on the character of flow around of the wing can be explained by braking flow in space between the wing and the earth/ground and respectively a decrease in the capacity of this space, that leads to an increase in the local velocities above the wing and a decrease their hearth with it. That means at small angles of attack, the eifect of the Earth is expressed in an increase in the coefficient of hoisting.

On the other hand, an increase in the peak of evacuation/rarefaction on suction side of wing leads to an increase in the downstream pressure gradient in its rear part. From the course of aerodynamics, it is known, that the more the peak of evacuation/rarefaction on suction side of wing, the earlier ensues the boundary-layer separation. Thus, in flight near the Earth one should expect a decrease in the critical angle of attack of $\alpha_{кр}$ and $c_{y\,max}$. The tYpical curve/graph of a change in the c_y of wing near and far from the Earth is given in Fig. 6.3.

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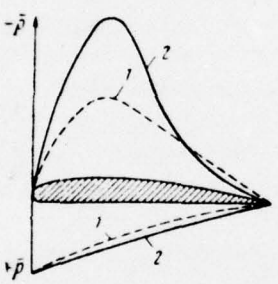

Fig. 6.2.

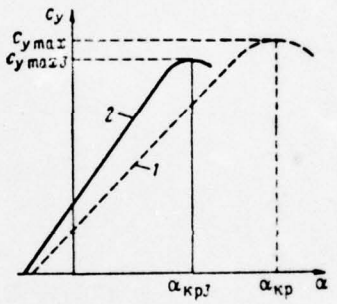

Fig. 6.3.

Fig. 6.2. Pressure distribution according to wing surface during just one angle of attack: 1 - far from the Earth; 2 - near the Earth.

Fig. 6.3. Dependence of $c_y = f(\alpha)$ far from Earth (1) and near Earth (2).

The increase of lift coefficient near the Earth depends on the wing area and distance of aircraft of the Earth. This distance is determined from the excess above the earth/ground of the center of mass of the aircraft either of the trailing edge of the omitted flap or flap.

To each aircraft type corresponds its take-off distance. This length is the most important parameter of takecff data, since it determines dimensions runways and, consequently, also the size/dimensions of airfield.

The takeoff/run-up of contemporary aircraft begins "from brakes", i.e., for a contraction in length of takeoff/run-up, pilot tempers brake only after engines are derived on takeoff conditions. Depending on the type of chassis/landing gear, the takeoff/run-up is realized differently. Of nosewheel airplanes, the takeoff/run-up to velocities ~ (0.6-0.7) $V_{отр}$ is conducted on three wheels; then pilot, selecting knob/stick on itself, it breaks away forepart/nose wheel and continues takeoff/run-up on the main wheels. The breakaway of forepart/nose wheel is allow/assumed upon reaching of the velocity, which ensures the aircraft control, which moves along runway, by means of aerodynamic controllers. Of aircraft with tail

wheel pilot it is direct after the beginning of takeoff/run-up, giving up knob/stick on itself, it noses down of aircraft and breaks away tail wheel, so that the large part of the takeoff/run-up is accomplished on the main wheels.

In the process of takeoff/run-up the velocity of aircraft and together with it lift they grow/rise. At the end of the takeoff/run-up when the velocity of aircraft achieves the value of $V_{отр.}$ and lift will be equaled with the weight of aircraft and somewhat will exceed it, pilot smoothly increases angle of attack to $\alpha_{отр.}$ and aircraft it blows away from the Earth.

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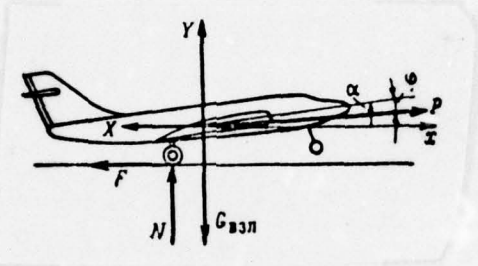

Fig. 6.4. The simplified diagram of the forces, which act on aircraft with takeoff/run-up.

Unstick speed it is possible to determine by formula

$$V_{отр}=\sqrt{\frac{2G_{взл}}{\varrho S c_{y\,отр}}}. \qquad (6.1)$$

unstick speed it is found in direct dependence on load on wing G/S. Of contemporary subsonic passenger liners the load on wing is 4900-5880 N/m², i.e., 4-5 times more than for the aircraft thirtieth - fortieth years. this increase in the load led to the corresponding elongation of the takeoff distance which for contemporary passenger aircraft is 2.0-3.5 km.

With takeoff/run-up on aircraft, besides aerodynamic forces and its weight, act the forces of friction and normal pressure (Fig. 6.4). Force of friction F, as is known, can be expressed by the coefficient of friction f and the force of the normal pressure N:

$$F=fN. \qquad (6.2)$$

if one considers that the trajectory of motion with takeoff/run-up is parallel to runway, then, by design/projecting the acting forces on the axis Ox and Oy and by set/assuming $\cos(\alpha \pm \varphi_{дв}) \approx 1$ in view of the smallness of the angle of $(\alpha \pm \varphi_{дв})$, we will obtain the system of equations, which describes the motion of aircraft with takeoff/run-up in the form

$$\left.\begin{aligned} P_{взл} - X - F &= \frac{mdV}{dt}, \\ Y + N - G_{взл} &= 0. \end{aligned}\right\} \qquad (6.3)$$

system (6.3) it makes it possible to calculate takeoff run length. We convert for this purpose velocity derivative in terms of time as follows:

$$\frac{dV}{dt} = \frac{dV}{dl}\frac{dl}{dt} = V\frac{dV}{dl} = \frac{1}{2}\frac{dV^2}{dl}. \qquad (6.4)$$

then from the first equation of system (6.3) we will obtain

$$l_{\text{разб}}=\frac{1}{2g}\int_0^{V^2_{\text{отр}}}\frac{G_{\text{пад}}dV^2}{P_{\text{раб}}-X-F}, \qquad (6.5)$$

or, if one considers that

$$F=f(G_{\text{пад}}-Y), \quad X=c_x\frac{\rho V^2}{2}S,$$

$$Y=c_y\frac{\rho V^2}{2}S,$$

that the takeoff run length will be expressed by formula

$$l_{раз6} = \frac{1}{2g} \int_0^{V^2_{отр}} \frac{dV^2}{\frac{P_{взл}}{G_{взл}} - f + \frac{\varrho V^2}{2G_{взл}} S (f c_y - c_x)}. \qquad (6.6)$$

in order to calculate according to this formula takeoff run length, it is necessary to know a change of the parameters of $P_{взл}$, f, c_x, c_y in the process of takeoff/run-up [1].

FOOTNOTE [1]. Usually the parameters of $P_{дв}$, f, c_x, c_y are assigned graphically depending on flight speed. ENDFOOTNOTE.

The integration of a similar formula not only analytical, but also by graphic methods differs in terms of large complexity and labor expense, in consequence of which in engineering practice, usually they use the approximation formulas, for example:

$$l_{разб}=\frac{1}{2g}\frac{V^2_{отр}}{\frac{P_{ср}}{G_{взл}}-\frac{1}{3}\frac{1}{K_{взл}}-\frac{2}{3}f}, \qquad (6.7)$$

where the $K_{взл}$ they use aerodynamic fineness ratio with takeoff.

Coefficients 1/3 and 2/3 consider a change of the aerodynamic drag and force of friction in the process of takeoff/run-up. The coefficient of friction f depending on runway conditions varies within the limits of 0.03-0.10.

Unstick speed and average thrust on takeoff approximately can be determined by the formulas:

for aircraft with TRD [ТРД - turbojet engine]

$$V^2_{отр} \approx 1{,}82 \frac{\frac{G_{взл}}{S}}{c_{y\ взл}}; \qquad (6.8)$$

$$P_{ср} \approx 0{,}93 P_{взл}; \qquad (6.9)$$

for aircraft with TVD [ТВД - turboprop engine]

$$V^2_{отр} \approx 1{,}36 \frac{\frac{G_{взл}}{S}}{c_{y\ взл}}; \qquad (6.10)$$

$$P_{ср} \approx 9{,}33 N_{взл}, \qquad (6.11)$$

where the $c_{y\ взл}$ - the value of c_y in takeoff configuration.

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Here thrust of engine and gravitational force is expressed in newtons, power in kilowatts, the speed in meters per second.

The second stage of takeoff as has already been indicated, was intended for the acceleration/dispersal of aircraft to the safe flight speed of $V_{без}$ and simultaneous gain of minimum (10 m) altitude. There are several methods of calculation of the distance of

this stage. Simplest of them is the energy method outlined below.

Total energy of aircraft at the moment of separation of its from runway is equal to

$$E_1 = \frac{m_{взл} V_{отр}^2}{2}.$$

at height/altitude m total energy of aircraft is equal to:

$$E_2 = \frac{m_{взл} V_{без}^2}{2} + 10 G_{взл}.$$

a change in the total energy of aircraft occurs because of the work of the thrust/rod of engine and drag, that act in direction of motion; their difference corresponds to the reserve thrust ΔP whose work is equal to

$$A_{взл}=\Delta P l_{разг.наб}.$$

if we after ΔP accept certain average its values, equal, for example, the half-sum of margins of thrust in the beginning and end of the acceleration/dispersal, then a change in the total energy of aircraft will be equal to the product of the average reserve thrust and the length of the $l_{разг.наб}$:

$$\Delta E_{взл}=E_2-E_1=\Delta P_{ср} l_{разг.наб},$$

whence we obtain formula for $l_{разг.наб}$ in the form

$$l_{разг.наб}=\frac{G_{взл}}{\Delta P_{ср}}\left(\frac{V^2_{без}-V^2_{отр}}{2g}+10\right). \qquad (6.12)$$

from formula (6.12) it follows that during the calculation of $l_{\text{разг. наб}}$ according to the average value of the margin of thrust of $\Delta P_{\text{ср}}$ indifferently, according to which trajectory the aircraft gains minimum altitude.

By totaling takeoff run length according to formula (6.7) and the length of acceleration/dispersal according to formula (6.12), we will obtain the take-off distance:

$$L_{\text{з.т}} = \frac{1}{2g} \frac{V_{\text{отр}}^2}{\frac{P_{\text{ср}}}{G_{\text{взл}}} - \frac{1}{3}\frac{1}{K_{\text{взл}}} - \frac{2}{3} f} + \frac{G_{\text{взл}}}{\Delta P_{\text{ср}}} \left(\frac{V_{\text{без}}^2 - V_{\text{отр}}^2}{2g} + 10 \right). \tag{6.13}$$

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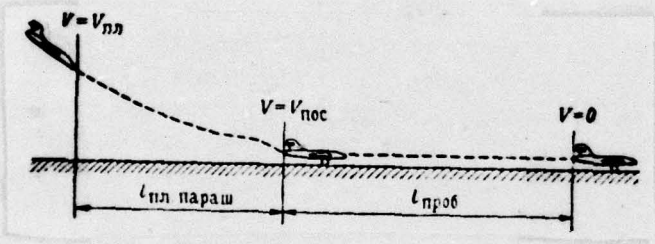

Fig. 6.5. Stages of landing.

6.3. Landing.

In landing process, occurs braking aircraft to the smallest possible (according to the conditions of safety) velocity of level flight in landing configuration with a simultaneous reduction/descent in the aircraft from the minimum height/altitude of landing to height/altitude 0.5-0.8 m above the level runways. According to the norms of airworthiness, the minimum height/altitude of landing is equal to 15 m above the level runways. The distance which it passes aircraft from the point in the trajectory, arrange/located above the runway at height/altitude 15 m, to dead lock, it composes the landing distance of $L_{пос}$.

Landing of early construction with PD [ПД - instrument panel] was made several stages. During the pre-landing glide the aircraft descended rectilinearly to height/altitude 5-8 m, then it was equalized and passed over to level flight to height/altitude 2-3 m. After this in the process of the so-called speed hold it was gradually decreased to the landing, aircraft it parachuted, it landed

on runway and landing run to the earth/ground it completed landing.

On contemporary aircraft all stages of landing, which precede touchdown, are made by single smooth maneuver. The velocity of aircraft at the end of gliding/planning $V_{пл}$ at height/altitude 15 m usually of exceeds minimum speed is more than to 30o/o. To land at this speed it is cannot, it must be decreased down to the $V_{пос}$, which, according to the conditions of flight safety, must be more than minimum speed approximately to 10o/o. Therefore from point at height/altitude 15 m above the runway to touchdown point, speed smoothly decreases from $V_{пл}$ to $V_{пос}$.

Thus, for contemporary aircraft landing process can be divided into two stages (Fig. 6.5). During the first stage occurs gliding/planning aircraft with subsequent alignment/levelling and simultaneous descent to altitude 0.5-0.8 m; in this case the speed decreases from $V_{пл}$ to $V_{пос}$. Upon achieving speed, equal $V_{пос}$, the pilot ceases the selection of lever, the value of lift as a result of the continuous decrease in the velocity becomes less than the weight of aircraft, aircraft parachutes on runway, concerning by its landing gear wheels. After this begins the second stage - range/path on runway. On range/path the velocity of aircraft decreases from $V_{пос}$

to zero.

The landing distance of contemporary aircraft for the latter two decade substantially increased and it comprises for heavy passenger aircraft 1.5-2 km.

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For providing a care/drift to the second circle (in the case of incorrect calculation of landing) during the pre-landing glide the pilot throttles engines, translate/transferring them into the mode/conditions of low flight gas. The thrust/rod which in this case develop the engines, decreases the flight path angle of gliding/planning in accordance with formula

$$\operatorname{tg}\theta = \frac{1-\frac{P}{X}}{K}. \qquad (6.14)$$

this decrease in the flight path angle (especially significant for aircraft with TRD) it is possible to compensate for by the appropriate deviation of wing high-lift device and by decrease in the aerodynamic fineness ratio, for example, because of the use of air brakes. Usually gliding/planning is made with the flaps, deflected to angle δ - 0.52-0.70 ~~is glad.~~ rad.

An increase in the angle of attack for the target/purpose of the preservation/retention/maintaining of lift at the decreasing flight speed is admissible to angles, to 0.04-0.06 is glad less than the critical, determined in landing configuration aircraft. The excess of the indicated limits of angle of attack can lead to unsymmetric flow separation from wing and the stalling of aircraft. In this case, one

should remember that for the aircraft, have tail wheels, angle of attack at the end of the maintaining must not exceed the tail clearance angle of $\alpha_{cr.}$ but for a nosewheel airplane, must be provided for distance 0.2-0.3 m between the lower edge of the tail aircraft component and runway.

With parachuting concludes flight through the air. After touching by wheels runways, aircraft begins range/path on runway. Aircraft with tail wheel realize a landing on the main and tail wheels simultaneously, nosewheel airplanes they are set on the main wheels with somewhat a elevated forepart/nose wheel.

The total length of the sections of gliding/planning, alignment/levelling, maintaining and parachuting can be determined, by utilizing an energy method. A change in the total energy on this section will be equally

$$\Delta E_{\text{пос}} = G_{\text{пос}}\left(\frac{V_{\text{на}}^2 - V_{\text{пос}}^2}{2g} + 15\right).$$

the work of drag is equal to:

$$A_{\text{пос}} = X l_{\text{на.наряш}}.$$

equating both expressions and set/assuming approximately $G_{\text{пос}} \approx Y$, we obtain the following formula for determining the $l_{\text{на.}}$

$$l_{\text{на.наряш}} = K_{\text{ср}}\left(\frac{V_{\text{на}}^2 - V_{\text{пос}}^2}{2g} + 15\right), \qquad (6.15)$$

where the K_{ep} are the average lift-drag ratio on the section of gliding - parachuting.

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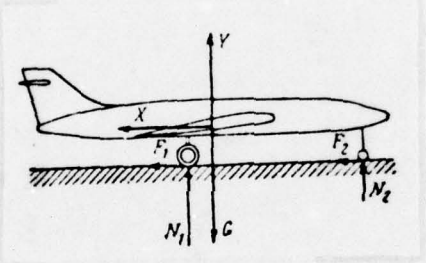

Fig. 6.6. The simplified diagram of the forces, which act on aircraft with range/path.

Gliding speed and landing speed it is possible to determine by formulas

$$V_{пл}=\sqrt{\frac{2G_{пос}}{\varrho S c_{y\,пл}}}; \qquad (6.16)$$

$$V_{пос}=\sqrt{\frac{2G_{пос}}{\varrho S c_{y\,пос}}}. \qquad (6.17)$$

the values of $K_{ср}$, $c_{y\,пл}$, $c_{y\,пос}$ one should take from the aerodynamic characteristics of aircraft, obtained in landing configuration. Tentatively it is possible to accept

$$K_{cp}=6\div7{,}5;$$
$$c_{y\,n\text{л}}=(0{,}5\div0{,}7)\,c_{y\,max.noc};$$
$$c_{y\,noc}=(0{,}7\div0{,}8)\,c_{y\,max.noc}.$$

after touching by wheels runways, aircraft begins the range/path with which concludes the landing. Pilot's basic problem to the period of range/path - as fast as possible to decrease the velocity of aircraft from V_{noc} to zero. In this case, must not arise the loads on the cell/elements of aircraft, which exceed the permissible in conditions strengths.

With range/path on aircraft, act the same forces, as with takeoff/run-up, with the exception of thrust which can be considered equal to zero (Fig. 6.6). By design/projecting all the acting on

aircraft forces on direction of motion and perpendicular to it, we will obtain the following equations of motion with the range/path:

$$\left.\begin{aligned} Y+N_1+N_2-G_{\text{пос}}=0, \\ X+F_1+F_2=-m\frac{dV}{dt}. \end{aligned}\right\} \qquad (6.18)$$

let us introduce the concept of the given coefficient of the friction of $f_{\text{пр}}$, which satisfies conditions

$$f_{\text{пр}}N=f_1N_1+f_2N_2, \quad N=N_1+N_2.$$

the value of the given coefficient of the friction of $f_{\text{пр}}$ it depends on runway conditions and the effectiveness of antiskid drive. Tentatively it is possible to accept $f_{\text{пр}}=0{,}15\div0{,}30$.

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Now equation (6.18) it is possible to write in the form

$$\left.\begin{aligned} Y+N-G_{\text{пос}}&=0,\\ X+f_{\text{пр}}N&=-m_{\text{пос}}\frac{dV}{dt}. \end{aligned}\right\} \qquad (6.19)$$

system (6.19) makes it possible to determine landing run by runway. By substituting in the second equation of system (6.19) value N from the first equation, we will obtain

$$X+f_{\text{пр}}(G_{\text{пос}}-Y)=-m_{\text{пос}}\frac{dV}{dt},$$

whence, taking into account that according to (6.4) the derivative $dV/dt = 1/2\ dV^2/d$, we will obtain the follcwing expression for

determining landing run:

$$l_{\text{проб}} = \frac{G_{\text{пос}}}{2g} \int_0^{V_{\text{пос}}^2} \frac{dV^2}{X + f_{\text{пр}}(G_{\text{пос}} - Y)}. \qquad (6.20)$$

integral (6.20) is calculated by numerical or graphic methods, since a change of parameters X, Y, V in analytic functions usually cannot be expressed. Formula (6.20) can be considerably simplified, if we calculate $l_{\text{проб}}$ from some average values of the forces of friction and impedance. so, by set/assuming in formula (6.20) of $Y \approx \frac{1}{3} G_{\text{пос}}$, the average resisting force [1]

$$X_{\text{ср}} \approx \frac{1}{3} \frac{G_{\text{пос}}}{K_{\text{ст}}},$$

where the $K_{\text{ст}}$ - the aerodynamic fineness ratio with of $\alpha = \alpha_{\text{ст}}$, we will obtain

$$F_{ср} \approx \frac{2}{3} f_{пр} G_{пос}$$

and, consequently:

$$l_{проб} \approx \frac{1}{2g} \frac{V_{пос}^2}{\frac{1}{3}\frac{1}{K_{сг}} + \frac{2}{3} f_{пр}}. \qquad (6.21)$$

FOOTNOTE 1. Formula for determining $X_{ср}$ is located from the condition that the landing is realized with angle of attack, equal to the tail clearance angle of $(\alpha_{пос} - \alpha_{сг})$, and that into the torque/moment of landing drag X can be accepted to the equal required thrust of $P_п = G_{пос}/K_{сг}$. ENDFOOTNOTE.

The overall length of landing distance taking into account formulas (6.15) and (6.21)

$$L_{пос}=K_{ср}\left(\frac{V_{пл}^2-V_{пос}^2}{2g}+15\right)+\frac{1}{2g}\frac{V_{пос}^2}{\frac{1}{3}\frac{1}{K_{сц}}+\frac{2}{3}f_{пр}}. \quad (6.22)$$

landing distance must be shorter than the length runways 1.5 times and in the failure of any out of the systems, which affect the required length of landing distance, it must not exceed the available length of the landing strip, which includes by runway and clear zones (KPB).

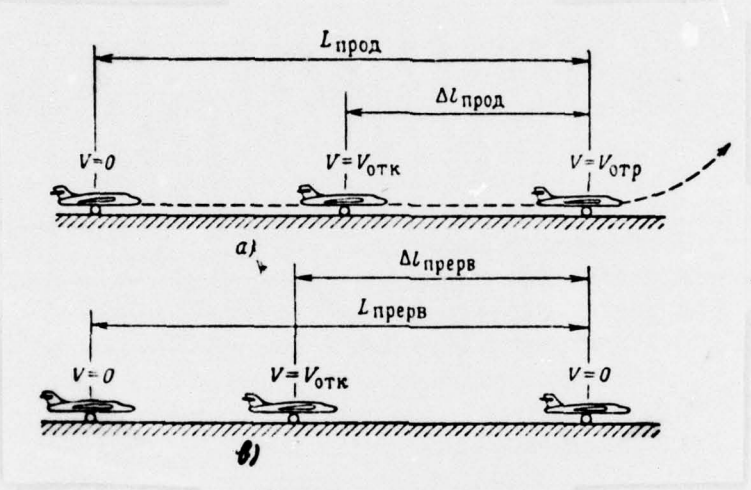
6.4. Special case of takeoff.

The special case of takeoff is the takeoff of multiengine aircraft with failed engine. Contemporary passenger aircraft have a sufficiently large thrust-weight ratio in order to carry out a safe takeoff even with failed engine. Takeoff with failed engine more complex in piloting in comparison with normal takeoff; therefore to its fulfillment are presented the increased requirements.

Depending on that, into which torque/moment occurred the engine failure, takeoff it can be continued or discontinued. In the first case will occur the continued takeoff, in the second - interrupted flight (Fig. 6.7). The selection of one version or the other depends on the speed which it has an aircraft at the torque/moment of the engine failure. If the engine failure occurred at the speeds, close to unstick speed, expedient takeoff to continue, since in this case the distance, necessary for the completion of takeoff, will be less than the distance, necessary for the dead lock of aircraft.

The question concerning the velocity, which determines decision-making to continuation or cessation of takeoff, is connected

Fig. 6.7. Continued (a) and interrupted (b) the takeoffs of aircraft.



with the concept of the critical speed of the takeoff of $V_{кр}$

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By the critical speed of takeoff, is understood greatest the reached by aircraft with takeoff speed by which in the failure of engine are equally possible both the safe continuation of takeoff and the safe cessation of take-off within the limits of the available length of landing strip. If we the distance, passable by aircraft on the continued takeoff, designate $\Delta l_{прод}$ (Fig. 6.7a), and on that which was interrupted - $\Delta l_{прерв}$ (Fig. 6.7b), then critical speed corresponds to equality

$$\Delta l_{прод} = \Delta l_{прерв}.$$

taking into account the concept of critical speed the solution to the question concerning the selection of the version of takeoff can be formulated then: if speed at the torque/moment of the engine failure more critical ($V_{отк} > V_{кр}$), then takeoff must be continued; if

speed at the torque/moment of failure is lower than critical ($V_{отк}<V_{кр}$), then takeoff must be interrupted.

The value of the critical speed of $V_{кр}$ can be determined as follows.

Is expressed $\Delta l_{прерв}$ and the $\Delta l_{прод}$ by the average accelerations of $j_{прерв}$ and $j_{прод}$, which develops the aircraft on these sections:

$$\Delta l_{прерв} = \frac{V_{отк}^2}{2j_{ср.прерв}}; \qquad (6.23)$$

$$\Delta l_{прод} = \frac{V_{отр}^2 - V_{отк}^2}{2j_{ср.прод}}. \qquad (6.24)$$

comparing formulas (6.23) and (6.24) with formulas (6.21) and

(6.7), let us find expressions for determining average accelerations in the failure of one engine:

$$j_{\text{ср.прерв}} = g\left(\frac{1}{3}\,\frac{1}{K_{\text{ст}}} + \frac{2}{3} f_{\text{пр}}\right); \qquad (6.25)$$

$$j_{\text{ср.прод}} = g\left[\left(\frac{P_{\text{взл}}}{G_{\text{взл}}}\right)_{\text{ср}} - \frac{1}{3}\,\frac{1}{K_{\text{взл}}} - \frac{2}{3} f\right], \qquad (6.26)$$

where the $\left(\frac{P_{\text{взл}}}{G_{\text{взл}}}\right)_{\text{ср}}$ - the average/mean thrust-weight ratio of aircraft taking into account the failure of one engine.

On the basis of conditions

$$V_{\text{отк}} = V_{\text{кр}};\quad \Delta l_{\text{прод}} = \Delta l_{\text{прерв}},$$

we will obtain the following expression for determining the critical speed:

$$V_{\text{кр}} = \frac{V_{\text{от}}}{\sqrt{1 + \frac{j_{\text{ср.прод}}}{j_{\text{ср.прерв}}}}}. \qquad (6.27)$$

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Formula (6.27) does not consider time, necessary to pilot for making a decision about the cessation of takeoff; therefore the actual value of $V_{пр}$ must be somewhat less than the value, determined according to this formula.

Interrupted and continued the takeoffs are characterized by the distances of interrupted $L_{взл.\ прерв}$ and continued the $L_{взл.\ прод}$ of takeoffs. Accelerate-stop distance includes distance from the beginning of takeoff/run-up to the dead lock of aircraft after the cessation of takeoff; the distance of the continued takeoff is a

distance from the beginning of takeoff/run-up to the set of the minimum height/altitude 10.0 m above level of runway with the simultaneous acceleration/dispersal of aircraft to the speed, equal to the safe speed of the takeoff of $V_{без.}$ In accordance with the norms of the airworthiness of the distance of interrupted and continued takeoffs they must satisfy the following conditions:

a) each of these distances must be not the more available length of runway;

b) the trajectory of the continued takeoff it must pass with excess not less than 4 m above the external end the runways;

c) during the cessation of the takeoff of the means for braking (brake of landing gear wheels, thrust reversal, interceptor/spoilers, etc.) can be applied only in the range of the velocities, which ensure their safe and reliable application/use and not earlier than 3 s after the torque/moment of the engine failure;

d) the effect of the action of the supplementary means for

braking (besides the wheel brakes) can be considered only in the case of their reliable and stable operation, checked in operation;

e) during the continuation of the takeoff of the means for a thrust augmentation of engines (extreme mode/conditions), if are absent the automatic means for their connection/inclusion, they can be applied not earlier than 3 s after the torque/moment of the engine failure.

6.5. Effect of structural/design factors on takeoff and landing characteristics of aircraft.

The number of structural/design factors, which affect takeoff and landing characteristics, includes the load on wing G/S, thrust-weight ratio P/G, wing high-lift device and the braking system of landing gear wheels. The effect of the indicated factors directly follows from formulas (6.13) and (6.22), the determining length takeoff and landing distances. After replacing in these formulas of $V_{без}$, $V_{отр}$, $V_{пл}$, $V_{пос}$ with their expressions from (6.1), (6.16) and (6.17), let us write formulas (6.13) and (6.22) in the following form:

$$L_{взл}=\frac{1}{g\varrho}\frac{G_{взл}}{S}\left[\frac{1}{\frac{P_{взл}}{G_{взл}}-\frac{1}{3}\frac{1}{K_{взл}}-\frac{2}{3}f}\frac{1}{c_{y\,отр}}+\right.$$
$$\left.+\frac{1}{P/G}\left(\frac{1}{c_{y\,нал}}-\frac{1}{c_{y\,отр}}+\frac{10\varrho}{G/S}\right)\right]; \qquad (6.28)$$

$$L_{пос}=\frac{1}{g\varrho}\frac{G_{пос}}{S}\left[K_{ср}\left(\frac{1}{c_{y\,пл}}-\frac{1}{c_{y\,пос}}+\frac{15\varrho}{G/S}\right)+\right.$$
$$\left.+\frac{1}{c_{y\,пос}}\frac{1}{\frac{1}{3}\frac{1}{K_{ст}}+\frac{2}{3}f_{пр}}\right]. \qquad (6.29)$$

Page 126. Load on wing. From expressions (6.28) and (6.29) it follows that takeoff and landing distance they increase in the first

approximation, proportional to load on wing G/S. Load on the wing of passenger aircraft for the latter 20 years increased 4-5 times, which led to a deterioration in their takeoff and landing characteristics.

The thrust-weight ratio of aircraft has a value only for its takeoff data. From formula (6.28) it follows that with an increase of thrust-weight ratio the takeoff data of aircraft are improved. The thrust-weight ratio of contemporary aircraft for the latter 20 years increased approximately 1.5-2.0 times and achieved value 0.3-0.35. However, this did not lead to a considerable improvement in the takeoff data of aircraft due to a more intense in comparison with thrust-weight ratio increase in the load on wing.

The wing high-lift device is the powerful means, which affects take-off and landing characteristics.

On all contemporary passenger aircraft is utilized one way or another the wing high-lift device under conditions of takeoff - landing. Application/use of mechanization/high-lift device makes it possible to raise $c_{y\,отр}$ and $c_{y\,пос}$ and thereby to decrease the takeoff run length and range/path. As show observations, landing run

with the use of mechanization/high-lift device is decreased by 25-35o/o, and of takeoff/run-up by 10-15o/o. The difference in the effect of mechanization/high-lift device is explained by the fact that the flap deflection of wide angles leads to a decrease in the lift-drag ratio as a result of drag divergence and an increase in the $L_{взл.}$ and therefore on takeoff one should not deflect flaps at angles $\delta > 0.26\text{-}0.35$ ~~is glad~~ rad, whereas on landing flaps can be deflected at angles $\delta = 0.85\text{-}1.1$ ~~is glad~~ rad.

Braking wheels on range/path determines the value of the given coefficient of the friction of $f_{пр.}$ With an increase of the intensity of braking the coefficient of $f_{пр}$ increases and with respect decreases the landing run. Experiment shows that the application/use of a braking makes it possible to reduce the distance of range/path 1.5-2.0 times.

At aircraft with tail wheel braking can be begun directly after landing on runway. In this case, one should bear in mind that extremely intense braking it can lead to nose-over, if resultant force N and F passes behind the center of gravity of aircraft (point T in Fig. 6.8). The possibility of nose-over places limitation on the intensity of braking.

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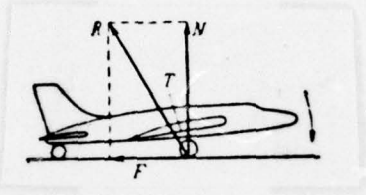

Fig. 6.8. Diagram of forces with nose-over.

Of nosewheel airplanes, the initial phase of range/path is realized on the main wheels with the elevated forepart/nose wheel. In this stage of range/path, the use of braking is undesirable to avoid the sharp lowering of aircraft to forepart/nose wheel. Usually braking an aircraft of such type begins after forepart/nose wheel will touch by runway. The possibility of nose-over of nosewheel airplanes is absent, and the intensity of braking is restricted to the heating of the wheel brakes and to an increase in the wear of pneumatic tires.

Operating experience shows that an increase in the drag coefficient is admissible within the limits of $f_{пр}=0,25\div0,30$; during stronger braking the wheel begins to slip over surface runways.

6.6. Effect of operational factors on takeoff and landing characteristics of aircraft.

To the number of operational factors, which affect takeoff and landing characteristics, they are related: the state of the atmosphere (pressure, temperature, wind and of, etc), location

runways, its gradient, state, etc.

The state of the atmosphere determines the ground speeds of aircraft and the parameters of engines and, thus, it affects its takeoff and landing characteristics.

So, a change in temperature and pressure of medium, on one hand, affects point of tangency with takeoff, with another - the unstick speed, set of height/altitude, gliding/planning and landing.

Air density, temperature and pressure are connected between themselves by equation of state

$$\rho = \frac{p}{RT}, \qquad (6.30)$$

from which, for example, it follows that with an increase in the air pressure at constant temperature occurs an increase in the density, which leads to an increase in the point of tangency of engines. As a

result, takeoff and landing distance in accordance with formulas (6.28) and (6.29) they will be reduced. On the other hand, an increase of temperature it leads to a decrease in the air density and, consequently, also the engine thrust, which, in turn, produces an increase in the takeoff and landing distances.

In the absence of wind, the ground speed of $V_п$ will be equal to the flight speed of V. With wind velocity W, the directed to axle/axis runway at an angle ε, the projection of wind velocity on axle/axis runways is equal $W\cos\varepsilon$, therefore ground speed will comprise

$$V_п = V \pm W\cos\varepsilon. \qquad (6.31)$$

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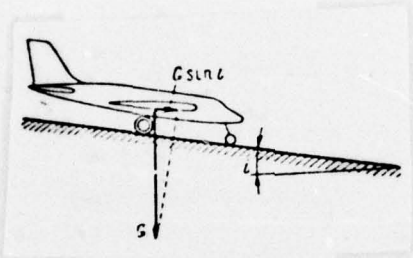

Fig. 6.9. Character of the effect of gradient runways on the value of force of periphery.

Positive sign is taken with tailwind, sign "minus" it is taken with contrary. Consequently, on takeoff with head wind of $V_{отр}$ and $V_{без}$ will be reached at less ground speeds and the take-off distance it decreases, but on downwind take-off, the take-off distance will increase. In exactly the same manner the length of landing distance decreases with head wind and will increase with incidental. Wind effect on the length of takeoff or landing distances decreases with an increase in the $V_{пос}$ and $V_{отр}$, since the more the value of these velocities, the lesser portion/fraction in the value of the ground speed of $V_{п}$ in expression (6.31) it comprises the term of $\pm W\cos\varepsilon$. But sometimes wind can exert a substantial influence on the length of takeoff and landing distances. For example at landing speed 170-200 km/h and the speed of head wind 5 m/s a contraction in length of range/path in comparison with the landing run of the same aircraft with dead calm will be 25-28o/o.

The place of arrangement runways by height above sea level and its gradients also affect the value of the distance of takeoff or landing. With an increase in the absolute mark runways, decreases the air density, and consequently, they increase takeoff and landing distance. Experiment shows that up to height/altitudes 4000 m an increase in the length of takeoff and landing distances approximately to 10o/o.

During takeoff and landing on the runway, which slopes, on aircraft comes into action the secondary force, determined by the projection of the weight of aircraft on direction of motion G sine i (Fig. 6.9). During the motion of aircraft under gradient, this force leads to a decrease in the takeoff/run-up and an increase in the range/path; during the motion of aircraft for lift, on the contrary, it leads to an increase in the takeoff/run-up and a decrease in the range/path.

Runway conditions affects the values of coefficients f and of f_{np} A decrease in coefficients of f and f_{np} leads to an increase in the landing run and a decrease in the takeoff run length and, on the contrary, an increase in coefficients f and in the f_{np} leads to an increase in the takeoff run length and a decrease in the landing run.

A change in coefficients of f and f_{np} can be caused by different reasons (rain, icing runways, etc.). For example icing runways leads to a reduction/descent in the coefficient of f_{np} 1.5-2 times and an increase in the landing run to 30-50o/o; an increase of coefficient f from 0.03 to 0.06 increases takeoff run length to 15-20o/o.

6.7. Ways of an improvement in the takeoff and landing characteristics of aircraft.

The problem of an improvement in the takeoff and landing characteristics of contemporary aircraft is at present sufficiently acute. From its successful resolution in many respects, depends the concurrent ability of air transport in the future, since the deterioration in the takeoff and landing characteristics led to an increase in the postwar years of size/dimensions and cost of airfields and to the contraction of the grid/network of the airfields on which can operate contemporary aircraft.

Above it is bygone said that the basic factors, which affect the length of takeoff and landing distances, are values

$$G/S,\ c_{y\,max},\ \frac{P_{взл}}{G_{взл}},\ K_{взл},\ K_{пос},\ f.$$

these factors actually and determine the possible ways of an improvement in the takeoff and landing characteristics. Let us examine them in more detail.

The decrease in the specific wing load is effective means of an improvement in the takeoff and landing characteristics. However, this way is least acceptable, since an increase in the maximum speed and distance requires increase, but not decrease G/S. Furthermore, decrease G/S, possible either because of decrease in G or because of increase in S, leads to a decrease in the useful load ratio, which makes the cost-effectiveness/efficiency of the operation of aircraft worse.

An increase in the $c_{y\,max}$ is the sufficiently effective means for an improvement in the takeoff and landing characteristics. Is connected it with an increase in the effectiveness of the high-lift wing. At all contemporary aircraft has one wing high-lift device or the other. For the latter 15-20 years the effectiveness of mechanization/high-lift device is bygone is considerably raised because of the application/use of slot and multislot extension flaps, slats, deflected leading edges etc. This made it possible to raise the value of $c_{y\,max}$ to 2.5-2.8. A further increase in the $c_{y\,max}$ is restricted to the flow separation, which appears above flaps during their deviation of wide angles. For dealing with this phenomenon at present increasingly wider begin to be utilized the different methods of boundary layer control (CBL).

The physical essence CBL entails this effect on separated flow, which leads to the liquidation of its breakaway and the restoration/reduction of the evenness of the flow about deflected to the wide angle of flaps. There are several methods CBL. Most investigated of them are suction and blowing of boundary layer.

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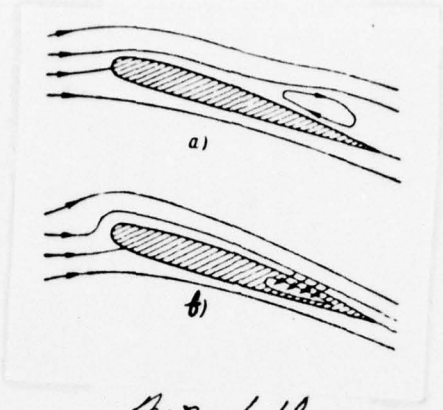

Fig. 6.10.

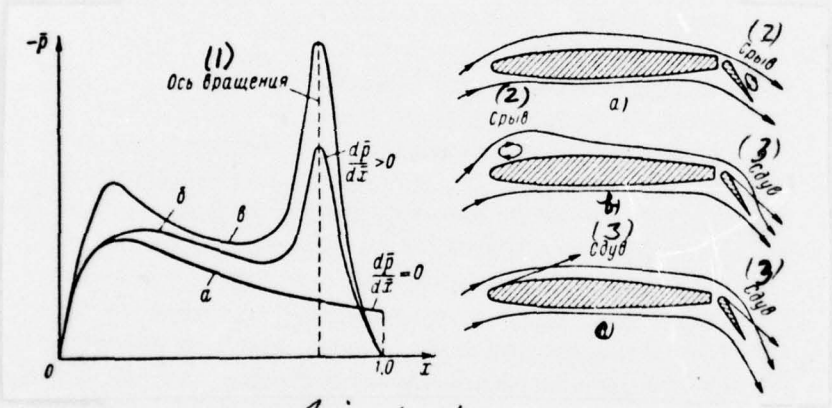

Fig. 6.11.

Fig. 6.10. Flow around of the wing at high angle of attack without CBL (a) and with boundary-layer bleed (b).

Fig. 6.11. The character of the effect of the work of the system of

blowing boundary layer on pressure distribution according to suction side of wing: a) there is no blowing; b) blowing from the spout of flap; c) blowing from the spout of wing and spout of flap.

Key: (1). Rotational axis. (2). Disruption/separation. (3). Blowing.

The suction of boundary layer from breakaway zone (Fig. 6.10) inside wing or flap with the aid of special pump, contributes to the attraction of separated flow and to the restoration/reduction of nonseparated flow. The investigations showed that when the large breakaway zones are present,, occurring above the deflected flaps, suction it has small effectiveness, and therefore it is applied predominantly for the laminarization of boundary layer in the rear end of the wing under cruising conditions of flight.

Blowing boundary layer is more effective. With the use of blowing the nonseparated flow above flaps it is possible to obtain up to angles δ - 1.4-1.55 is glad. The physical flow pattern of airfoil/profile under conditions CBL by blowing is shown in Fig. 6.11. During the flap deflection of the wide angle above the flap, appears the separation of flow (Fig. 6.11a). In breakaway zone, the pressure changes very little, pressure gradient dp/dx is close to zero.

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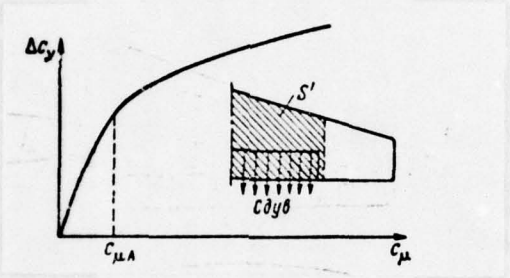

Fig. 6.12. Character of the effect of the size of the coefficient of the intensity of blowing on the increase of lift coefficient.

The injection of jet to the zone of disruption/separation leads to the restoration/reduction of the nonseparated flow, which is characterized by gradient $dp/dx > 0$. In this case, the curvature of flow lines on the upper surface of airfoil/profile increases and with respect increases evacuation/rarefaction.

With intense injection the curvature of flow lines can increase so, that will occur the separation of flow of the leading edge of airfoil/profile (Fig. 6.11b). For the liquidation of breakaway of the leading edge of airfoil/profile, is applied supplementary blowing from leading edge (Fig. 6.11c).

The effectiveness of blowing depends on the value of the momentum/impulse/pulse of the blown out jet, which is characterized by the coefficient of the intensity of blowing

$$c_\mu = \frac{mV_c}{\frac{\rho V^2}{2} S'},$$

where m - the mass flow per second of the blown out air; V_c - the

speed of blowing; $\rho V^2/2$ - velocity head of the incident flow; S' - the wing area, operated CBL (See Fig. 6.12).

The increase in the lift coefficient, obtained as a result of applying blowing, is proportional to the coefficient of the intensity of c_μ, however, the character of its increase is various with different c_μ (Fig. 6.12). With low c_μ occurs a sufficiently energetic increase in the Δc_y, and then after certain value of $c_\mu = c_{\mu A}$ the intensity of an increase in the Δc_y falls. Is explained this phenomenon by the fact that at first in proportion to the increase in the c_μ is eliminated the zone of disruption/separation above the flap and is reduced nonseparated flow. Up to the torque/moment of $c_\mu = c_{\mu A}$ the nonseparated flow is reduced completely. With $c_\mu > c_{\mu A}$ an increase in the Δc_y occurs only as a result of strengthening circulation around wing, and therefore the intensity of the increase of Δc_y decreases.

In practice the coefficients of c_μ are realized only to the values of $c_\mu = c_{\mu A}$. The obtained in this case increases in the Δc_y are ~50-80o/o of the values of c_y without CBL. Thus, the application/use of blowing makes it possible to increase the effectiveness of mechanization/high-lift device 1.5-2.0 times.

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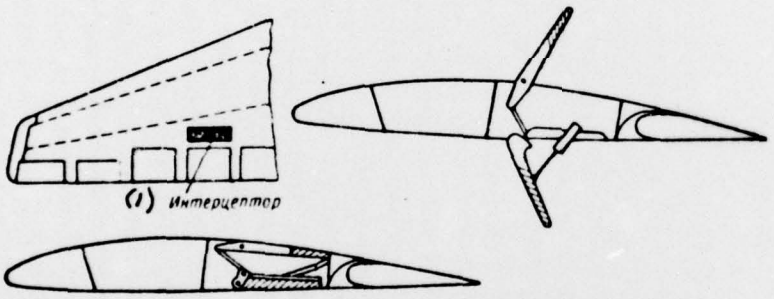

Fig. 6.13. Interceptor/spoiler.

Key: (1). Interceptor/spoiler.

An increase in the take-off thrust of $\frac{P_{взл}}{G_{взл}}$ is checked in operation by the means for an improvement in the takeoff data. It connected with the perfection/improvement of power plants is reached either by means of the boosting of engine operating modes on takeoff or because of the application/use of boosters. In the mode/conditions of afterburning, the take-off thrust is increased to 20-30o/o. Are more effective the starting velocities, which are the supplementary engines of liquid-jet/reactive or powder types, establish/installed in aircraft and detached from it after the completion of takeoff.

A decrease in the lift-drag ratio on landing makes it possible substantially to reduce landing run. In order to maximally decrease the lift-drag ratio, it is necessary to increase c_x and to decrease the c_y. For this purpose are utilized the interceptor/spoilers, air brake, brake parachutes, reversal of the thrust, gripping device (arresting gears), etc.

one of the most effective means for a decrease in the lift-drag ratio is the installation during the wings of interceptor/spoiler-planes, capable of being advanced into flow (Fig. 6.13). During the advancement of interceptor/spoilers, sharply falls lift (as a result of flow separation after interceptor/spoiler) and

increases drag. Interceptor/spoilers make it possible to decrease the lift-drag ratio 8-10 times. Lift convergence leads to an increase in the normal reaction of the wheels of landing gear and as consequence - to an increase in the force of friction of wheels against the earth/ground.

Air brake as interceptor/spoilers, represent by themselves the being advanced in flow areas, establish/installed, as a rule, in fuselage. During the setting up of brakes, the effect of lift convergence is not realized, and lift-drag ratio decreases only because of drag divergence. the same effect creates the brake parachutes, used on very many aircraft, and the arresting gears (Fig. 6.14), used mainly for the aircraft, which realize a landing on aircraft carriers.

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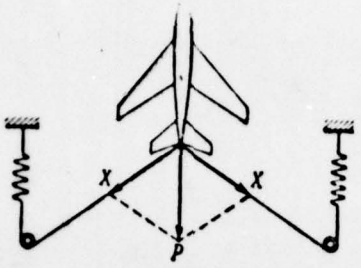

Fig. 6.14.

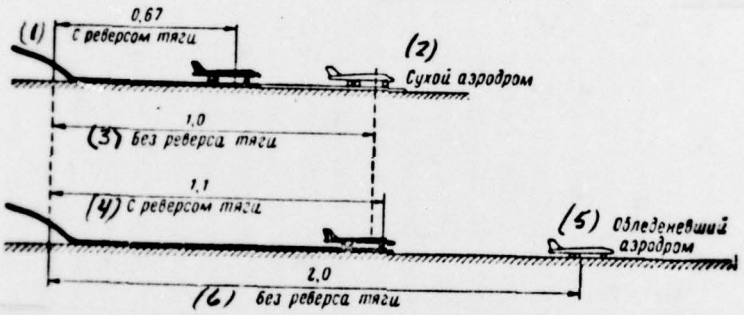

Fig. 6.15.

Fig. 6.14. Diagram of the work of arresting gear.

Fig. 6.15. Character of the effect of thrust reversal on landing run.

Key: (1). With the reverse of thrust/rod. (2). Dry airfield. (3). Without thrust reversal. (4). with reverse of thrust; (5) Iced up airport. (6). Without thrust reversal.

A considerable contraction in length of range/path can be obtained when using a reverse thrust. Of aircraft with TVD [turboprop engine], it is created because of the translation/conversion of propeller blades during range/path into low pitch, realized by removal/taking propeller blades from intermediate stop after aircraft will touch by runway by forepart/nose wheel. Are applied also the so-called reversible-pitchs propeller, which, besides usual mode/conditions, can work in the mode/conditions of reversal of the thrust when propeller blades are establish/installed by the system of reversal in such a way that flow would attack on them at negative angles of attack. The created in this case thrust/rod will in sign coincide with the direction of reverse thrust, and in value it can be regulated by the system of reversal.

Thrust reversal on TRD [- turbojet engine] is obtained because of the rotation of the coming out from nozzle jet. Application/use of a thrust reversal makes it possible to reduce landing run to 25-30o/o. Especially effective turns out to be the application/use of a thrust reversal during landing on the iced over runway (Fig. 6.15).

The further development of the problem of an improvement in the

take-off and landing characteristics of aircraft is the creation of the apparatuses of shortened and vertical takeoff and landing.

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PROBLEMS FOR REPETITION.

1. Why the motion of aircraft during takeoff and landing is being unsteady?

2. How does affect the proximity of the Earth the unstick speed of $V_{отр}$ and landing $V_{пос}$?

3. At which criterion is estimated the need for the cessation of takeoff or its continuation?

4. Which factors do affect the length of takeoff and landing distances? Which of them are most effective?

2. The landing run of aircraft under conditions of horizontal runway is 1000 m. To determine, on how many percent decrease landing run of the same aircraft, if it sits down on the runway, which has the positive gradient i = 0.087 ~~in glad.~~ rad. Value of the given coefficient of friction and quality at ground angle of attack to accept equal $f_{тр}=0,15$; $K_{сг}=12$.

The solution:

PROBLEMS.

1. utilizing a system of equations (6.3) and of expression for the average values of the resisting forces and lift on takeoff/run-up

$$X_{ср} = \frac{1}{3}\frac{G_{взл}}{K_{взл}}; \quad Y_{ср} \approx \frac{1}{3}G_{взл},$$

to show that the complete time of take-off run on runway can be determined by formula

$$t_{разб} = \frac{1}{g}\frac{V_{отр}}{\frac{P_{ср}}{G_{взл}} - \frac{1}{3}\frac{1}{K_{взл}}\frac{2}{3}f}.$$

with range/path for the runway, which slopes i, the system of equations (6.19) of signs the form:

$$\left.\begin{aligned} Y + N - G\cos i = 0, \\ X \pm G\sin i + f_{\text{п}} N = -m\frac{dV}{dt}, \end{aligned}\right\} \qquad (1)$$

(positive sign is related to gradient i > 0, minus sign it is related to gradient i < 0). By utilizing for $X_{\text{ср}}$ the $F_{\text{ср}}$ of the expressions, given into in, it is possible to obtain for the landing run of aircraft from inclined runway formula

$$l_{\text{проб } i\neq 0} = \frac{3}{2g}\,\frac{V_{\text{пос}}^2}{\dfrac{1}{K_{\text{сг}}} \pm 3\sin i + f_{\text{пр}}(3\cos i - 1)}. \qquad (2)$$

from the comparison of formulas (2) and (6.20) it follows that

$$\frac{l_{\text{проб } i\neq 0}}{l_{\text{проб } i=0}} = \frac{1+2f_{\text{пр}}K_{\text{ст}}}{1+K_{\text{ст}}} \frac{1}{[f_{\text{пр}}(3\cos i - 1) \pm 3\sin i]}. \qquad (3)$$

after substituting into formula (3) initial data problems, we will obtain that

$$\frac{l_{\text{проб } i\neq 0}}{l_{\text{проб } i=0}} = 0{,}596.$$

thus, landing length of aircraft on the runway, which slopes i = 0.087 is glad, it decreases by 40o/o.

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