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OPTIMUM DESIGN OF AN IMPEFFECT CORRUGATED TOWER.(U)
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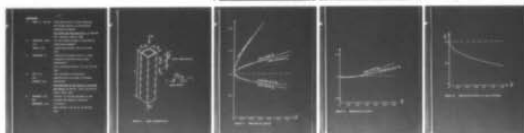
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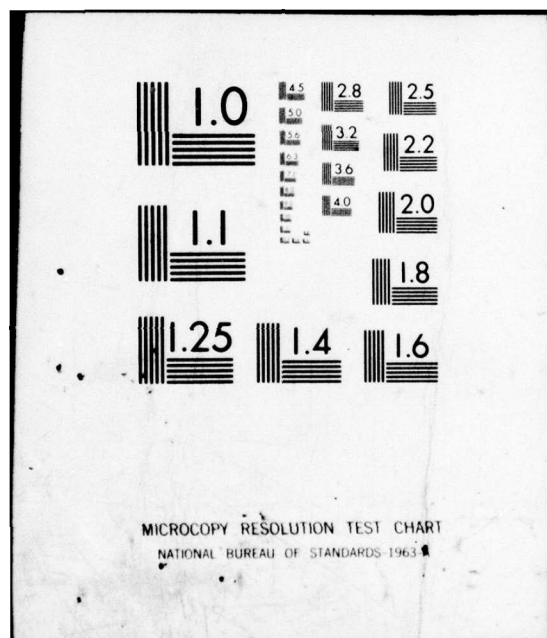
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The optimum design of a thin-walled corrugated tower is considered. The walls are corrugated, and have imperfections. Explicit expressions for the optimum (minimum weight) design are derived, and used to compare weight sensitivity to imperfection magnitude with strength sensitivity.			

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D.M. Richards
Aircraft Design Division
College of Aeronautics
Cranfield Institute of Technology
Cranfield, Bedford MK43 0AL
England

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Aircraft Design Division,
College of Aeronautics
Cranfield Institute of Technology
Cranfield, Bedford MK43 0AL
England

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OPTIMUM DESIGN OF AN IMPERFECT CORRUGATED TOWER

SUMMARY

The optimum design of a square tower to carry axial compressive load is considered, including the effect of a sinusoidal imperfection in the corrugated wall panels. Comparison is made between the imperfection sensitivity of structure weight for given load, and the more drastic effect of imperfections on the strength of the ideal optimum structure for given structure weight.

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D.M. Richards,
Aircraft Design Division,
College of Aeronautics
Cranfield Institute of
Technology,
Cranfield,
Bedford.

INTRODUCTION

The effects of small imperfections on structures which are vulnerable to various forms of instability have been studied extensively recently. Most authors have been concerned to illustrate the way in which the nominal strength of a given structure may be eroded by imperfections. The rate at which strength is reduced as imperfection magnitude increases is characterised by the often ill-defined term 'imperfection sensitivity'.

From the point of view of design, for which the loads are given and the structural dimensions are to be found, a number of authors have discussed these phenomena by illustrating how the strength of structures which are designed without regard for imperfections, may be found to be seriously weakened.

This paper utilises a comparatively simple example to show how, by acknowledging a priori the existence of imperfections in the final structure, optimum designs may be derived to achieve a given strength so that that factor which is most important from the design point of view, i.e. the sensitivity of structure weight to imperfection magnitude, may be quantified.

DESIGN EXAMPLE

The example chosen is a square tower of width B and total height L , which is required to support a uniform axial compressive force P applied evenly over the tower cross-section (see Figure 1).

The tower walls are made from trapezoidally corrugated panels, which are stabilised at regular intervals l by transversely rigid diaphragms of given equivalent thickness t_0 , which provide simple support at their intersections with the wall panels.

The structure is everywhere perfectly manufactured except in one respect; the panels each have an imperfection of sinusoidal form, of wavelength equal to the diaphragm spacing l . The imperfection magnitude is characterised by the quantity $\frac{\delta_0}{k}$, where δ_0 is the initial imperfection amplitude midway between diaphragms, and k is the corrugation local radius of gyration.

The purpose of the analysis is to establish optimum values for corrugation dimensions, tower width and diaphragm spacing, so that total tower weight is minimised.

Linearly elastic material behaviour is assumed throughout.

ANALYSIS

The principal effect of the specified imperfection will be to reduce the axial stiffness of the panels and consequently the overall Euler buckling strength of the tower.

Thus the panels must be designed to work at a load somewhat lower than their own buckling load.

Ideal optimum dimensions for perfectly manufactured trapezoidally corrugated panels are given below (refer to figure 1 for dimensions)

$$t = 0.5881 \left[\frac{wl}{E} \right]^{1/2}$$

$$b = 1.0477 \left[\frac{wl^3}{E} \right]^{1/4}$$

$$a = 0.87 b \quad \dots(1)$$

$$t^x = 1.3423 t$$

$$\theta = 59^\circ$$

where w is design end load per unit width when local buckling coincides with flexural buckling, and t^x is panel equivalent thickness.

Let the nominal strength of the panels be a factor r greater than the given design load, so that

$$w = \frac{rP}{4B}$$

which gives

$$t = 0.2941 \left[\frac{rPl}{EB} \right]^{1/2}$$

$$t^x = 0.3947 \left[\frac{rPl}{EB} \right]^{1/2}$$

$$b = 0.7408 \left[\frac{rPl^3}{EB} \right]^{1/4}$$

$$f = 0.6334 \left[\frac{PE}{rIB} \right]^{1/2}$$

$$\dots(2)$$

where f = surface working stress

The total equivalent cross-section area, to which the weight of the tower is proportional, may now be written down

$$A = 4Bt^{\pi} + \frac{t_o B^2}{l}$$

$$= 1.5788 \left[\frac{rPlB}{E} \right]^{1/2} + \frac{t_o B^2}{l}$$

which is minimised when

$$l = 1.1708 \left[\frac{t_o^2 EB^3}{rP} \right]^{1/3} \quad \dots(3)$$

so that structural material is divided between panels and diaphragms in the ratio 2:1

The list of design variables may now be reformed to give

$$t = 0.3182 \left[\frac{rPt_o}{E} \right]^{1/3}$$

$$b = 0.8338 \left[\frac{t_o B}{E} \right]^{1/2}$$

$$t^{\pi} = 0.4271 \left[\frac{rPt_o}{E} \right]^{1/3}$$

$$f = 0.5854 \left[\frac{P^2 E}{t_o r B^3} \right]^{1/3}$$

$$A = 2.5626 \left[\frac{rPt_o}{E} \right]^{1/3} \cdot B \quad \dots(4)$$

Note that corrugation dimension b is independent of load intensity and Youngs modulus.

The overall stability of the tower must now be considered.

If the top of the tower is free and the base fixed, provided loading is conservative the buckling load is given by

$$P = \frac{\pi^2 E^x I}{4L^2} \quad \dots(5)$$

where I = second moment of area = $\frac{2}{3} t^x B^3$

and E^x = effective axial modulus of imperfect panels given by

$$\frac{E^x}{E} = \frac{\left[1 - \frac{p}{p_E}\right]^3}{\frac{1}{2} \left(\frac{\delta_o}{k}\right)^2 + \left[1 - \frac{p}{p_E}\right]^3}$$

p_E = panel Euler buckling load

p = panel applied load, so that $\frac{p_E}{p} = r$.

$\left(\frac{\delta_o}{k}\right)$ is defined above)

Equation (5) may now be solved for B , the tower width which ensures that the tower has adequate overall buckling strength, giving

$$B = 1.1248 \left\{ \frac{r^{2/3} \left[\frac{1}{2} \left(\frac{\delta_o}{k}\right)^2 + \left(1 - \frac{1}{r}\right)^3 \right]^{1/3}}{\left(1 - \frac{1}{r}\right)^3} \right\} \left[\frac{L^6 p^2}{E^2 t_o} \right]^{1/9} \quad \dots(7)$$

It may be noted from (4) that equivalent cross section area is proportional to B , so that from (7), tower weight will be minimised with respect to panel reserve factor r when the quantity

$$R = \frac{r^{2/3} \left[\frac{1}{2} \left(\frac{\delta_o}{k}\right)^2 + \left(1 - \frac{1}{r}\right)^3 \right]}{\left(1 - \frac{1}{r}\right)^3}$$

is a minimum.

In order to proceed further, it is convenient to write

$$z = 1 - \frac{1}{r} \quad \dots(8)$$

so that

$$R = \frac{\frac{1}{2} \left(\frac{\delta_o}{k} \right)^2 \frac{1}{z^3} + 1}{(1 - z)^{2/3}}$$

which is minimised when

$$\left(\frac{\delta_o}{k} \right)^2 = \frac{4z^4}{9 - 11z} \quad \dots(9)$$

Note that for

$$0 < \frac{\delta_o}{k} < \infty$$

$$0 < z < 9/11$$

and

$$1 < r < 11/2$$

The complete design may now be specified as follows.

$$\begin{aligned} A &= 2.8824 \, \mathfrak{E}_A \left[\frac{L^6 t_o^2 P^5}{E^5} \right]^{\frac{1}{9}} \\ B &= 1.1248 \, \mathfrak{E}_B \left[\frac{L^6 P^2}{E^2 t_o} \right]^{\frac{1}{9}} \\ f &= 0.5204 \, \mathfrak{E}_f \left[\frac{E^5 P^4}{L^6 t_o^2} \right]^{\frac{1}{9}} \\ l &= 1.3169 \, \mathfrak{E}_l \left[\frac{L^6 t_o^5 E}{P} \right]^{\frac{1}{9}} \\ b &= 0.8843 \, \mathfrak{E}_b \left[\frac{L^3 t_o^4 P}{E} \right]^{\frac{1}{9}} \\ t &= 0.3182 \, \mathfrak{E}_t \left[\frac{P t_o}{E} \right]^{\frac{1}{3}} \\ t^x &= 0.4271 \, \mathfrak{E}_t \left[\frac{P t_o}{E} \right]^{\frac{1}{3}} \end{aligned} \quad \dots(10)$$

The $\mathfrak{E}$ functions contain completely the effects of imperfections and are given below

$$\begin{aligned}
 \bar{\delta}_A &= \left[\frac{9(1-z)}{9-11z} \right]^{1/3} \\
 \bar{\delta}_B &= \left[\frac{9(1-z)}{9-11z} \right]^{4/3} \\
 \bar{\delta}_r &= \left[\frac{9-11z}{9(1-z)} \right]^{1/3} \\
 \bar{\delta}_l &= \left[\frac{9(1-z)}{9-11z} \right]^{7/3} \\
 \bar{\delta}_t &= \left[\frac{1}{1-z} \right]^{1/3} \\
 \bar{\delta}_b &= \left[\frac{3(1-z)}{(9-11z)^{1/2}} \right]^{2/3}
 \end{aligned}
 \tag{11}$$

Note that when $\frac{\delta_0}{k} = 0$, $r = 1$, $z = 0$ and all $\bar{\delta} = 1$.

These equations are shown plotted in figures 2 and 3 as functions of imperfection magnitude.

The effect of imperfections on the strength of the tower for given tower weight may be deduced from the first of equations (10), which gives

$$\frac{P}{P_0} = \left[\frac{1}{\bar{\delta}_A} \right]^{9/5} = \left[\frac{(9-11z)}{3(1-z)^{1/3}} \right]^{3/5}
 \tag{12}$$

where P_0 = strength of perfect structure

P = strength of imperfect structure of same weight.

This is plotted in figure 4, which shows how strength for given structure weight appears to be more 'sensitive' to imperfection magnitude, than structure weight for given strength.

CONCLUSIONS

An analysis has been presented which gives optimum dimensions for a square tower with corrugated walls loaded in axial compression. The wall panels are imperfect, and it is shown that structure weight for a given load is thereby somewhat increased. When structure weight is fixed at a value corresponding to the perfect structure, the effect of imperfections on tower strength is found to be very significant.

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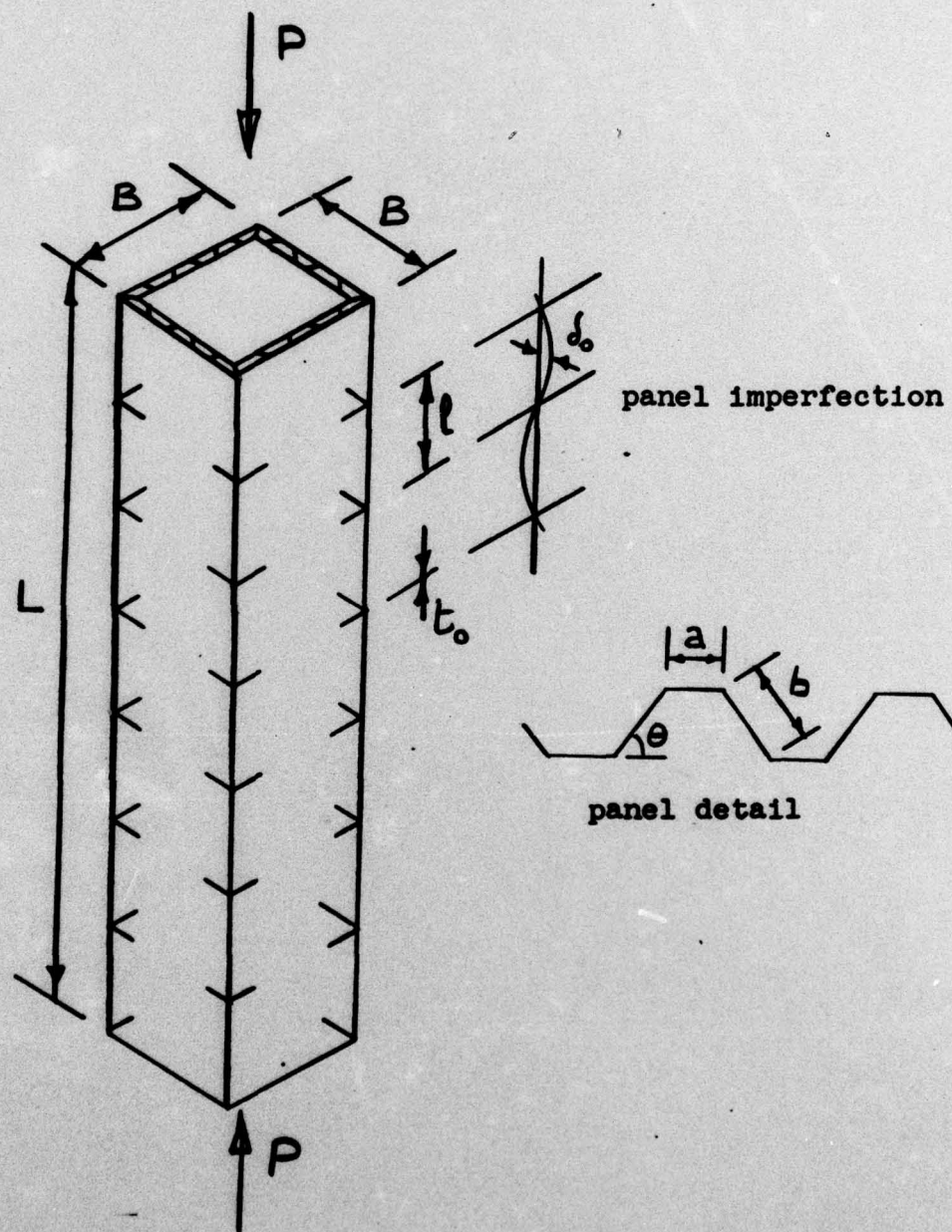


Figure 1. Tower configuration

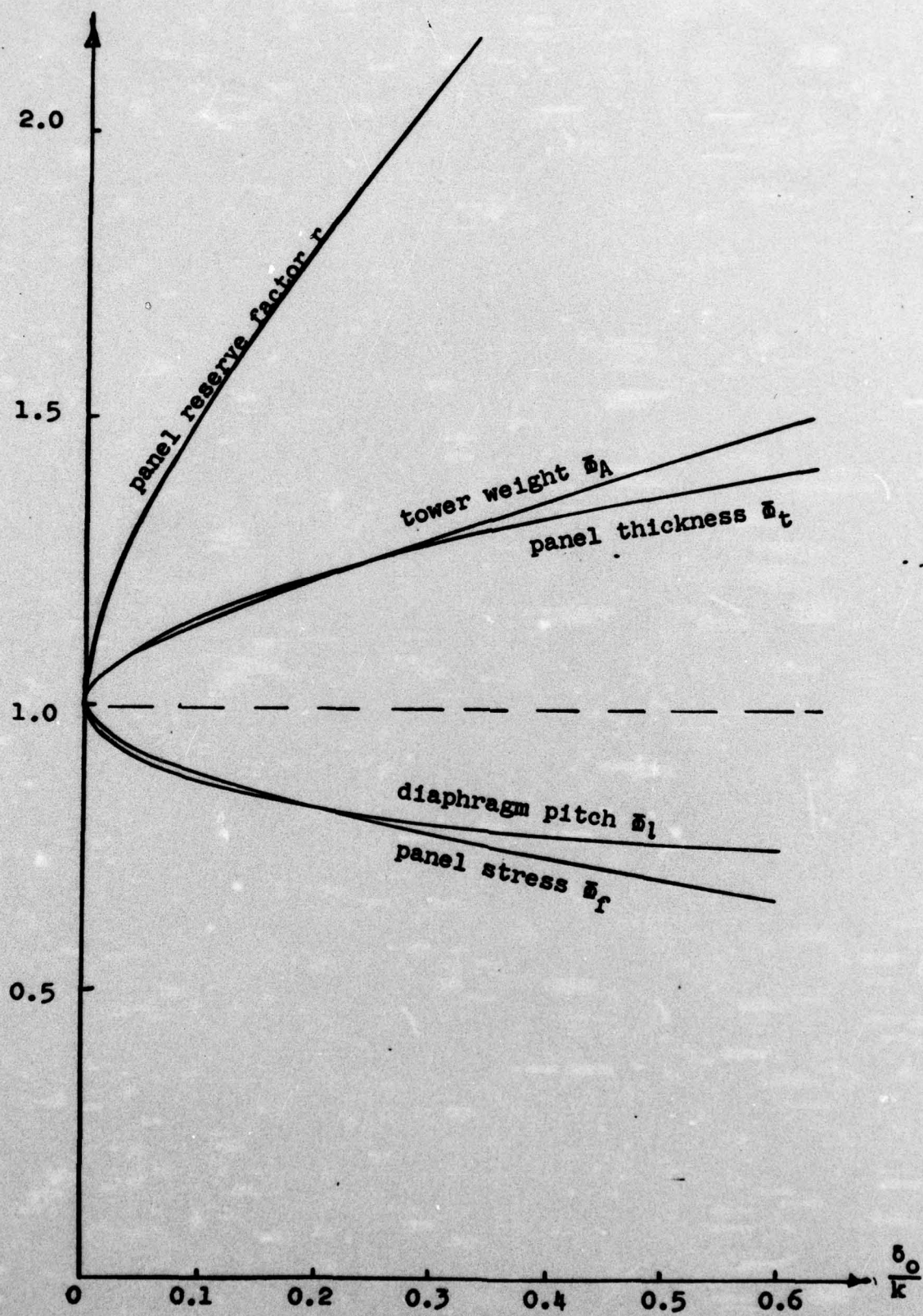


Figure 2. Imperfection factors

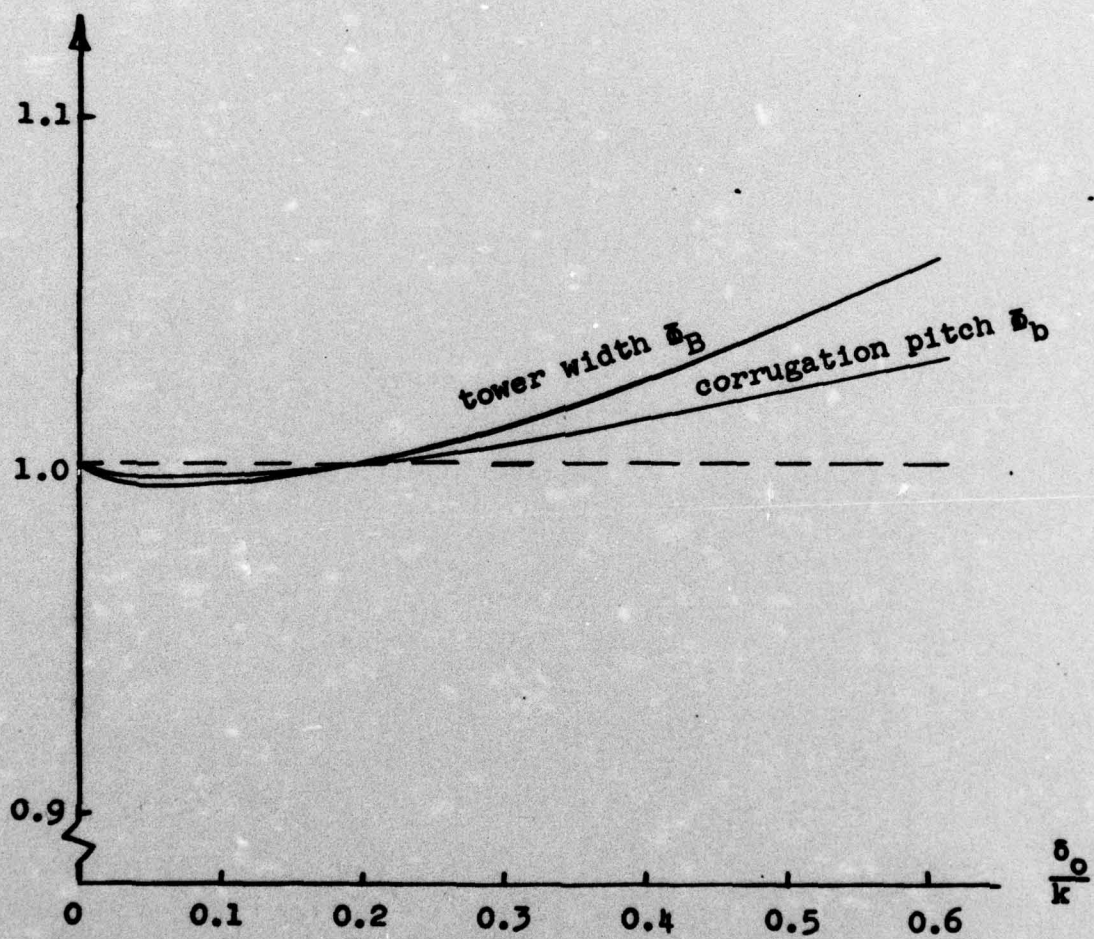


Figure 3. Imperfection factors

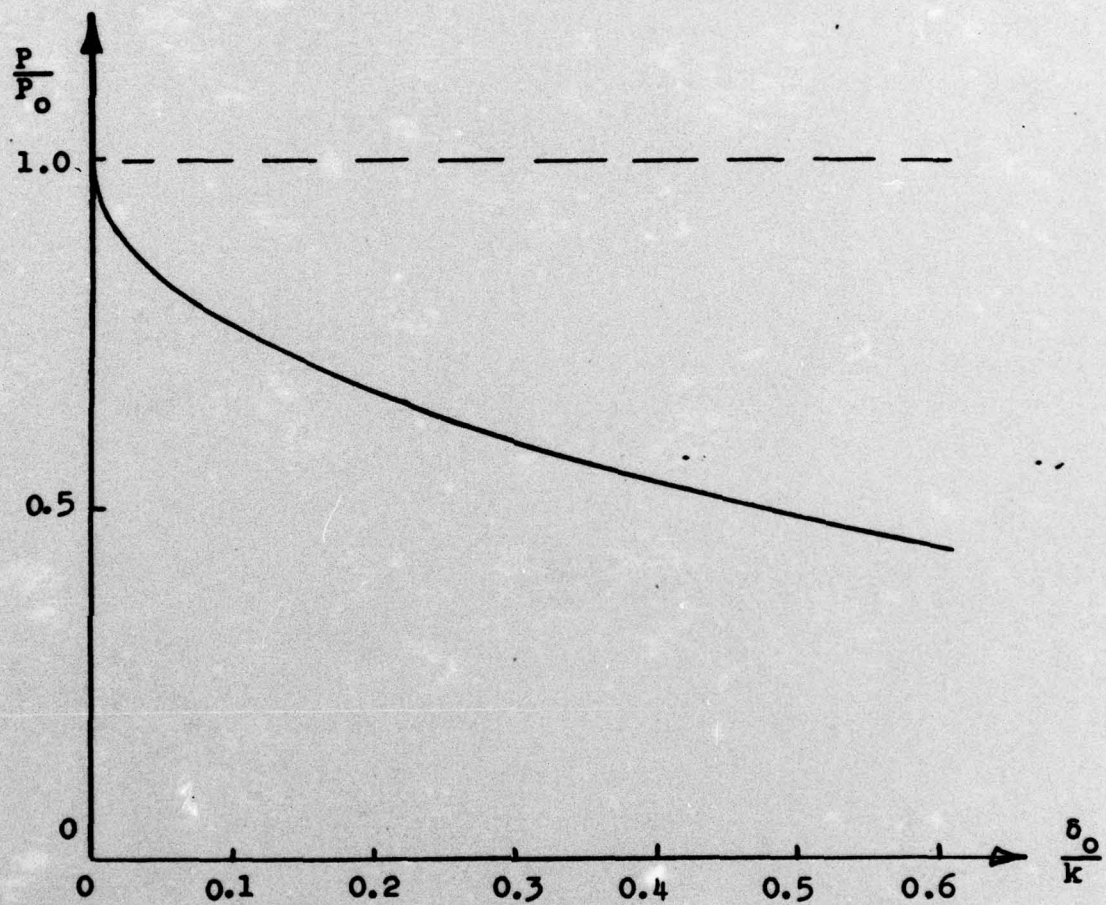


Figure 4. Imperfection effect on tower strength.