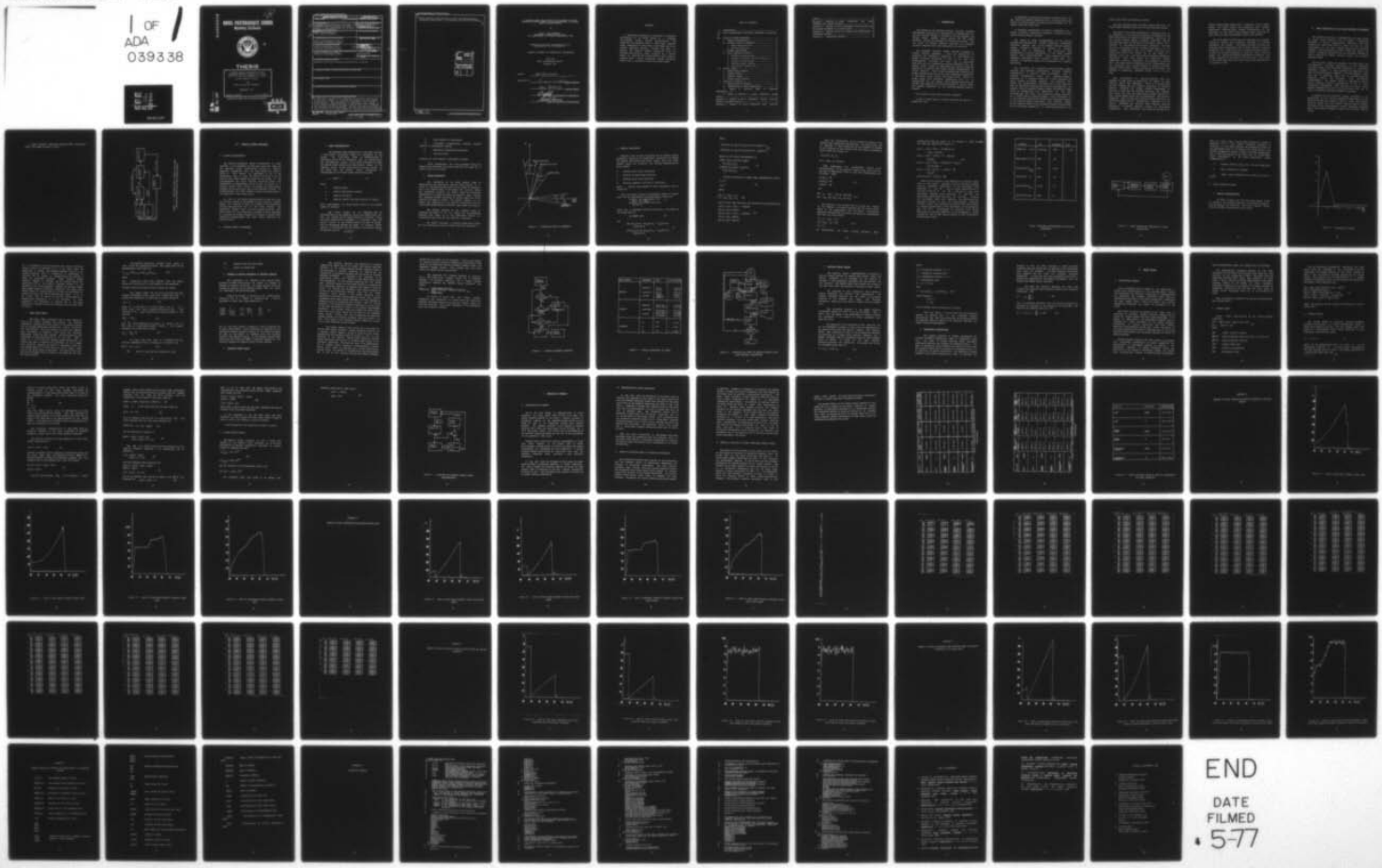
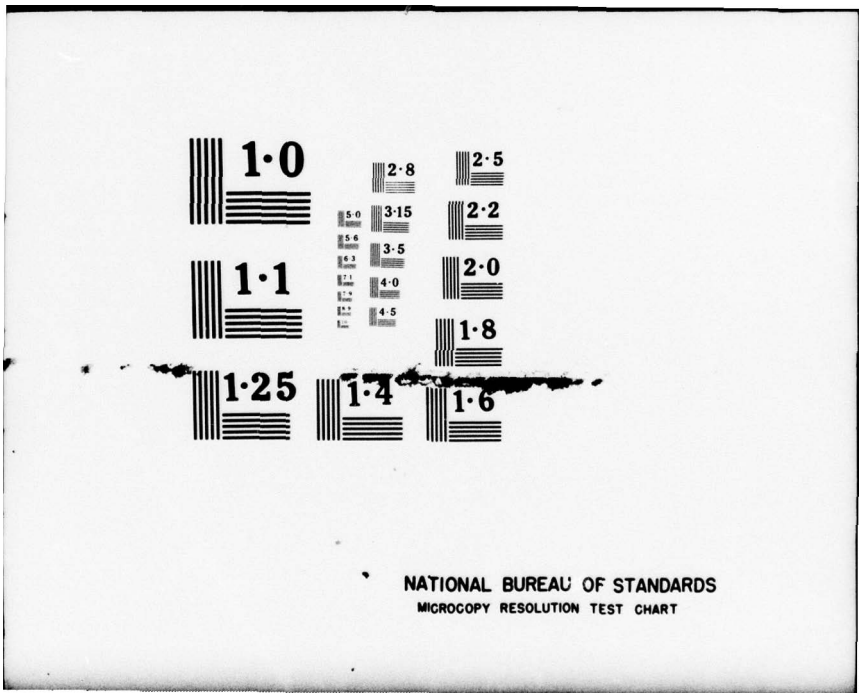
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Monterey, California



THESIS

A KALMAN FILTER APPLICATION TO THE ADVANCED TACTICAL INERTIAL GUIDANCE SYSTEM OF THE AIR-LAUNCHED LOW VOLUME RAMJET CRUISE MISSILE

by

John Archie Van Devender

December 1976

Thesis Advisor: H. A. Titus

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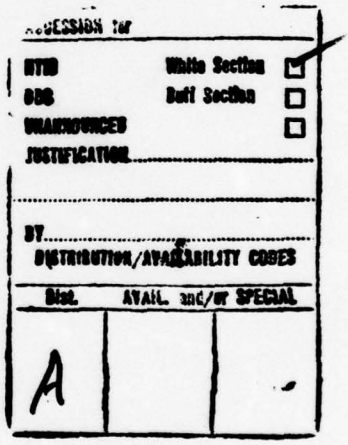
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A KALMAN FILTER APPLICATION TO THE ADVANCED TACTICAL INERTIAL GUIDANCE SYSTEM OF THE AIR-LAUNCHED LOW VOLUME RAMJET CRUISE MISSILE

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ABSTRACT

A Montecarlo simulation is conducted to ascertain performance of the ATIGS system in a proposed air-launched cruise missile configuration. The simulation is conducted within a local-level inertial frame consisting of down-range, cross-range and up as primary reference vectors. Efforts are made to measure the relative effects associated with the intended pure position reset provided by a micrad sensor as compared with those effects which could be expected from a linear suboptimal Kalman filtering scheme used in conjunction with the MICRAD sensor

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I. INTRODUCTION

The purpose of the Advanced Tactical Inertial Guidance System(ATIGS) program is to demonstrate the feasibility of a low cost inertial system to be used in the Air Launched Low Volume Ramjet (ALVRJ) cruise missile for mid course guidance. Within the framework of this stated purpose lies the intent to furnish moderate accuracy in a strapdown inertial navigator with high reliability of operation.

The strapdown inertial system requires a computer to provide inertial reference , hence the possibility of extending the computer's capability by installation of Kalman filtering algorithms is seen as an area for investigation. Previous work (ref. 1,2) in this field indicates that the computational burden associated with the Kalman filter limits its usefulness when position updating systems in the missile give highly accurate measurements of actual position. Most of the aforementioned computational burden resulted from the on-line gain generation required by a non-linear model within the Kalman filter. Hence if a linear model with sufficient performance were to be incorporated and the Kalman gains generated off-line and stored, then possibly the velocity estimation errors which are largely unaffected by the position updates could be reduced.

The purpose of this study was then threefold

1) Test a linear model of missile dynamics for use as a simulation tool.

2) Determine the inertial navigator accuracy within the six degree of freedom simulation when a pure position reset device is installed which provides position updates at two points along the flight path.

3) Determine improvements in missile performance if a Kalman filtering scheme were installed to estimate missile states between position updates.

The means by which accomplishment of the desired purposes was obtained were various Montecarlo simulations utilizing existing data on the proposed inertial guidance system. Extensive work,both in testing of physical equipment and in simulation, had previously been accomplished by various departments of the Naval Weapons Center, China Lake,California. Hence accurate data as to component performance were available. These data were utilized to construct models of the components for computer simulation.

The simulation of a strapdown inertial guidance system requires the nonlinear computations relating observed accelerations to inertial frame coordinates. This is normally accomplished within the guidance-navigator algorithm by a suitably chosen set of state variables and their related non-linear dynamics. The essential idea behind the linearization technique used in this report is that the non-linear calculations relating accelerations and angular rates to velocity changes within the inertial frame could be accomplished upon observation of the said accelerations and rates and utilized as forcing functions for a linear model of system dynamics. This corresponds to a free inertial system with observations physically aligned in the inertial frame of reference. Thus velocity changes in the inertial frame of the strapdown guidance system would be the non-linear combination of accelerations,angles and

angle rates which are treated as inputs

The model dynamics then are simple linear equations for which Kalman filtering gains can be calculated and stored.

The proof of the above linearization technique would be in comparisen of the existing empirical performance of the ATIGS system with the observed corresponding simulation of the system without Kalman filtering installed. Ref.(3) provides ample data of the drift of the ATIGS system as a function of time under actual flight conditions. The data are for a pod mounted version of ATIGS installed on an A-7 aircraft. Information from this report indicated that ground test drift of the system was on the order of 1 nautical mile (nm) per hour under controlled temperature conditions. Under free flight test conditions without temperature control, performance was degraded to 4 nm per hour. The temperature instability was not incorporated into the simulation due to current effort to provide corrective measures within ATIGS. Hence verification of the model was assumed if simulation indicated drifts of 1 to 2 nm per hour.

Once verification of inertial-physical model was assumed, the next phase of observation of effect of pure position update was commenced. Ref.2 in an unclassified portion, contends that the optimal weighting of filtered position estimates and highly accurate measurement of position is such that the filtered estimates are ignored. This being the case, the computational burden imposed by a time varying Kalman filter may be unwarranted. The implicit assumption here is that the velocity errors incurred in an unfiltered system are not significantly decreased by the filter. Therefore the position reset feature would be sufficient to provide required accuracy at mid-course termination. This conclusion was tested by simulation of

missile flight under conditions of increased noise levels within the ATIGS system and comparison with "normal" performance obtained,which allowed visualization of the magnitude of end point error incurred under conditions of pure position reset and different noise level sensors.

The final phase of study was the filtering of the sensor outputs to provide more accurate estimates of velocity throughout the flight. No attempt was made to filter the position update measurements due to their reported accuracy (sigma=50 ft). Thus any gains in performance would have to be from the filtered estimates of position after position update and continuous filtered estimates of velocity. The primary indicator of accuracy in line with ref.2 was taken to be cross range error and cross range error covariance as a function of time.

II. BASIC DESCRIPTION OF THE ATIGS EQUIPMENT UTILIZATION

The approach selected for implementation of ATIGS within an actual cruise missile, was to employ a high-speed processor to handle transformation updating, earth rate torquing and other minor tasks in order to save on time requirements on the more complex navigation and guidance computer. This peripheral processor would then supply the central processing unit(CPU) with the necessary information that it required to compute position within the inertial guidance frame.

Thus the basic ATIGS unit involves ring laser gyros and accelerometers providing information to the peripheral processor wherein after suitable transformation, inertially referenced changes in state variables are supplied to the main navigation-guidance computer. The main computer (CPU) then has an auxiliary input from an external position measuring device, the Microwave Area Correlation fixer(MICRAD). The MICRAD system is designed to provide highly accurate position measurements at two or three preselected checkpoints along the route of flight. These position updates would then be utilized within the CPU to reset the inertial guidance estimate of position.

At present the only filtering system installed is an application to the initial alignment scheme wherein a two stage initialization process is used to align the missile inertial frame with the parent aircraft inertial frame. Filtering is not used presently during midcourse guidance due to the position checkpoint feature and the short time of flight.

A block diagram indicating proposed ATIGS utilization within the ALVRJ is shown in fig.1.

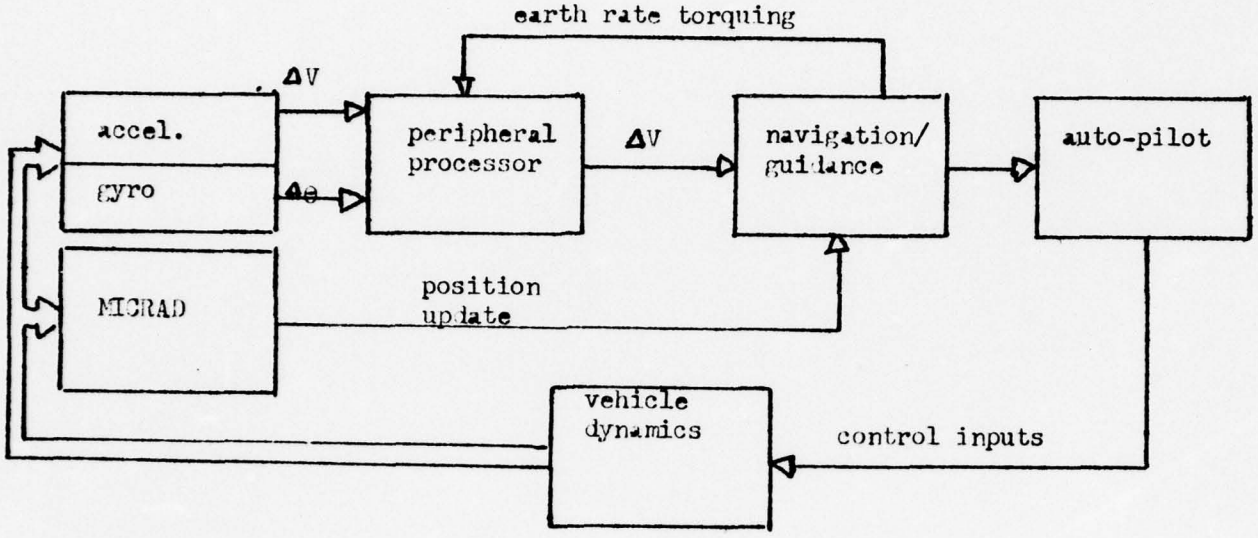


Figure 1 - FUNCTIONAL BLOCK DIAGRAM OF THE ATIGS INSTALLATION WITHIN THE ALVRJ CRUISE MISSILE

III. INERTIAL SYSTEM SIMULATION

A. SYSTEM CONSIDERATIONS

The inertial navigation system incorporated in ATIGS consists of the Honeywell GG-1300 Ring Laser gyro (RLG) and the Sunstrand Q-flex accelerometer acting as sensors. Empirical performance data for these sensors are found in table (1). Both sensors are considered to be of the integrating type in that the output of the RLG is in the form of total angle change per pulse and the output of the accelerometers are total velocity change per pulse. The measurement is accomplished within the RLG by means of a counter system which totals the number of fringe pattern passages during each pulse period and similarly within the accelerometers, the total velocity change is proportional to the magnitude of the output pulse.

In view of the above characteristics it was felt that the inertial system as diagrammed in fig.4 could be modeled simply and linearly by using the outputs of the sensors as forcing functions vice part of the state vectors. The similarity between fig.1 and fig.4 should be noted. This would result in a net reduction in number of state variables by allowing the missile dynamics to consist of second order equations of displacement and first order equations for angular motion.

B. INERTIAL FRAME OF REFERENCE

1. ALVRJ Implementation

The proposed ATIGS application to the ALVRJ utilises a local-level co-ordinate frame for navigation to the target. The local-level frame is characterized by North,East and up as the respective axis of calculations. The non-spherical nature of the earth introduces an angle calculation which relates the local gravity vector to the position vector of the origin from the earth's center. In the ATIGS unit, the gravity vector calculation is accomplished by an inverse square gravitation model

$$G = -(KM/R^3)\ R \qquad (1)$$

where

G	gravity vector
K	earth's gravitation constant
M	mass of the earth
R	position vector from earth center to vehicle

which approximates the local gravity vector to the desired degree of accuracy.

The vector output of an orthogonal set of accelerometers is the geometric sum of all forces which act upon the vehicle and of course gravity is included. Since the above calculation is dependent on position, then the gravity vector is not constant during the time of flight. Thus to distinguish between the effect of external forces applied to the missile and the change in the gravity vector an equation such as

$$F_a = C_a^i \cdot \ddot{R}_i - G \qquad (2)$$

F_a force exerted on instruments

C_a^i coordinate transformation relating inertial axis(i) to accelerometer axis(a)

R_i inertially referenced acceleration

G gravity vector

resolves the time varying accelerometer outputs.

ATIGS accomplishes the above procedure within the missile and calculates the proper direction and range for a direct steer to the target.

2. System Simulation

The simulation of the ATIGS mission began by approximating the local-level inertial frame defined in 1. above as a down-range,cross-range and up frame of reference. Due to the limited range of the missile the gravity vector was considered constant and known, hence the simulation simplified to a simple cartesian co-ordinate space wherein the navigator assumes knowledge of initial position, target position and range to target. The correct heading to the target was assumed to be the positive x-direction with the right-hand system defining positive cross-range accordingly.

The initial position of the inertial frame of reference was taken to be the origin and instantaneous headings were taken to be the difference between the longitudenal body axis and the positive x direction.

The ALVRJ maintains a constant wings level flight and this restriction was also placed upon the simulation.

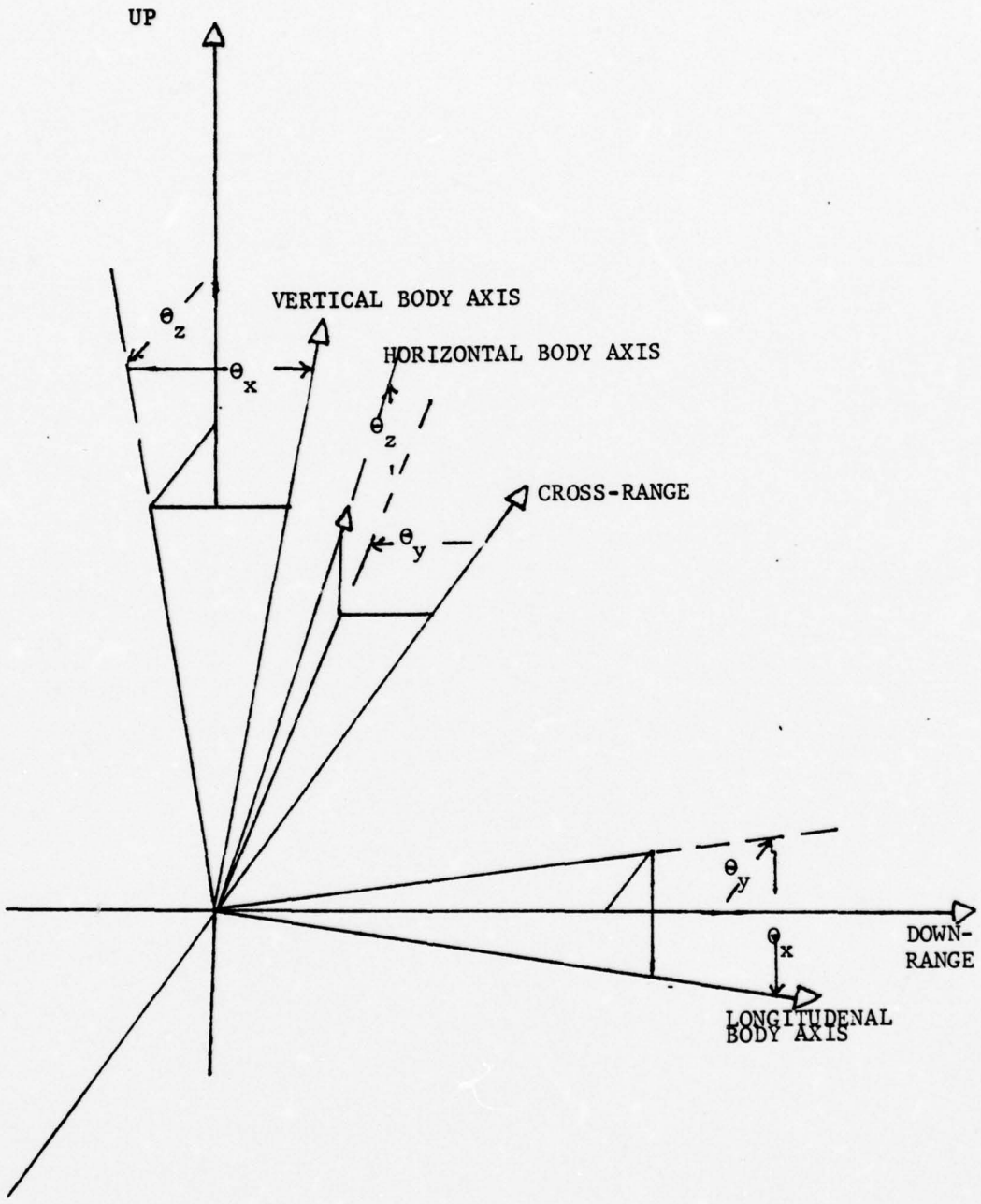


Figure 2 - SIMULATION FRAME OF REFERENCE

C. GENERAL DEVELOPMENT

Defining a set of state variables for an inertial system undergoing arbitrary two dimensional translation and making the careful restriction that small angles and very small angular rates are involved, one obtains (neglecting all noise inputs)

x Distance from origin down-range

$\dot{X}$ Velocity in down-range direction

Y Distance from track centerline

$\dot{Y}$ Velocity component vertical to centerline

Theta Angular displacement of body longitudinal axis to centerline

Due to the small angles and negligible effect of angular rates one can approximate the accelerometer outputs as

$$\begin{aligned} Z_1 &= \text{accel. in longitudenal body axis} = A_{1b} = \Delta V_{1b} \\ Z_2 &= \text{accel. in lateral body axis} = A_{2b} = \Delta V_{2b} \end{aligned} \tag{3}$$

where the B subscript indicates body axis. The output of the single gyro is

$$Z_3 = \dot{\theta}_1 * \Delta T = \Delta \theta_1 \tag{4}$$

Now

$$\begin{aligned} \Delta \dot{X} &= \Delta V_{1b} \cos \theta_1 - \Delta V_{2b} \sin \theta_1 + V_{1b} \Delta \cos \theta_1 \\ &\quad - V_{2b} \Delta \sin \theta_1 \\ \Delta \dot{Y} &= \Delta V_{1b} \sin \theta_1 + \Delta V_{2b} \cos \theta_1 + V_{1b} \Delta \sin \theta_1 \\ &\quad + V_{2b} \Delta \cos \theta_1 \end{aligned} \tag{5}$$

Where

$$\Delta\dot{X}=Z_1\cos\theta_1-Z_2\sin\theta_1+V_{1b}\Delta\cos\theta_1-V_{2b}\Delta\sin\theta_1$$
$$\Delta\dot{Y}=Z_1\sin\theta_1+Z_2\cos\theta_1+V_{1b}\Delta\sin\theta_1+V_{2b}\Delta\cos\theta_1 \quad (6)$$

Which can be further approximated by

$$\begin{aligned}\Delta\dot{X}&\cong Z_1-Z_2\theta_1-V_{1b}\theta_1\Delta\theta_1-V_{2b}\Delta\theta_1\\&\cong Z_1-Z_2\theta_1-V_{2b}Z_3\\\Delta\dot{Y}&\cong Z_1\theta_1+Z_2+V_{1b}\Delta\theta_1-V_{2b}\theta_1\Delta\theta_1\\&\cong Z_1\theta_1+Z_2+V_{1b}Z_3\end{aligned} \quad (7)$$

Further utilization of small angle approximation yields

$$V_{1b}=\dot{X}$$
$$V_{2b}=\dot{Y} \quad (8)$$

Hence

$$\begin{aligned}\Delta\dot{X}&=Z_1-Z_2\theta_1-\dot{Y}Z_3\\\Delta\dot{Y}&=Z_1\theta_1+Z_2+\dot{X}Z_3\end{aligned} \quad (8a)$$

and for unit time intervals the discrete state equations are

$$\begin{aligned}X(k+1)&=X(k)+\dot{X}(k)+.5*\Delta\dot{X}(k)\\\dot{X}(k+1)&=\dot{X}(k)+\Delta\dot{X}(k)\\Y(k+1)&=Y(k)+\dot{Y}(k)+.5*\Delta\dot{Y}(k)\\\dot{Y}(k+1)&=\dot{Y}(k)+\Delta\dot{Y}(k)\\\theta_1(k+1)&=\theta_1(k)+\Delta\theta_1(k)\end{aligned} \quad (9)$$

Thus the observations can be treated as inputs to the system after appropriate substitution. The above model can be expanded to three dimensions and six degrees of freedom by the addition of one cartesian and two angular coordinates which would then consist of

$$Z(k), \dot{Z}(k), \theta_2, \theta_3$$

for a total of 9 states.

This development was accomplished without noise considerations. In the physical system noise would exist in the form of measurement noise in both the accelerometers and the gyros. Hence

$$\begin{aligned} Z_1 &= \Delta V_{1b} + \gamma_1^* \\ Z_2 &= \Delta V_{2b} + \gamma_2^* \\ Z_3 &= \Delta \theta_1 + \varphi_1 \end{aligned} \tag{10}$$

and

$$\begin{aligned} \Delta\dot{X} &= Z_1 - Z_2\theta_1 - \dot{Y}Z_3 - \gamma_1^* - \gamma_2^*\theta_1 - \dot{Y}\varphi_1 \\ \Delta\dot{Y} &= Z_1\theta_1 + Z_2 + \dot{X}Z_3 - \gamma_1^*\theta_1 + \gamma_2^* + \dot{X}\varphi_1 \end{aligned} \tag{11}$$

The purpcse of this approach was to utilize the outputs of the sensors as forcing functions for the linear model. Hence in the preprocessor the non-linear calculations involving observations and states can easily be accomplished such that one then obtains

$$\begin{aligned} U_1' &= Z_1 - Z_2\theta_1 - \dot{Y}Z_3 \\ U_2' &= Z_1\theta_1 + Z_2 + \dot{X}Z_3 \\ U_3' &= Z_3 \end{aligned} \tag{12}$$

as hypothetical and known forcing functions. Thus

substitution into the model of the system of $U_1'(\kappa)$ for $\Delta\dot{x}(k)$ and $U_2'(\kappa)$ for $\Delta\dot{y}(\kappa)$ would result in

$$
\begin{aligned}
X(k+1) &= X(k) + \dot{X}(k) + .5*(\Delta\dot{X}(k) + \gamma_1' \\
&\quad + \gamma_2'\theta_1 + \dot{Y}(k)\varphi_1') \\
\dot{X}(k+1) &= \dot{X}(k) + \Delta\dot{X}(k) + \gamma_1' + \gamma_2'\theta_1(k) \\
&\quad + \dot{Y}(k)\varphi_1' \qquad (13) \\
Y(k+1) &= Y(k) + \dot{Y}(k) + .5*(\Delta\dot{Y}(k) + \gamma_1'\theta_1(k) \\
&\quad + \gamma_2' + \dot{X}(k)\varphi_1') \\
\dot{Y}(k+1) &= \dot{Y}(k) + \Delta\dot{Y}(k) + \gamma_1'\theta_1(k) + \gamma_2' \\
&\quad + \dot{X}(k)\varphi \\
\theta_1(k+1) &= \theta_1(k) + \Delta\theta_1(k) + \varphi_1
\end{aligned}
$$

Hence the net result is the addition of a process noise term to the model. Analysis of this noise term proceeds with the systematic elimination of the non-linear term involving γ' and Θ. This is easily justified due to the small value of γ' and Θ. Thus one is left with down-range process noise involving γ_1' and $\dot{y}\varphi_1'$ and cross-range noise terms involving γ_2' and $\dot{x}\varphi_1'$. Clearly the non-linear terms will dominate. The conclusion that logically follows is that the linearization technique utilized above will obviously be accurate for small angles in a manner proportional to the magnitudes of the quantities $\dot{y}\varphi_1'$ and $\dot{x}\varphi_1'$. Furthermore it indicates that Kalman filtering will be most effective in the estimation of angle information and of much smaller benefit in the filtering of accelerometer noise.

PARAMETER	UNIT	PERFORMANCE	UNITS
RANDOM WALK (^{o}/hr)	GG-1300-RLG	.0075	1 1hr.
BIAS STABILITY (^{o}/hr.)	SAME	.009	1
BIAS SENSITIVITY (^{o}/hr. $_{o}$f)	SAME	.00037	
SCALE FACTOR (%)	SAME	.016	
BIAS UNCERTAINTY (ug)	Q-FLEX	79.0	1
SCALE FACTOR (ug/g)	SAME	63	1

TABLE 1-PUBLISHED UNCERTAINTIES OF THE ATIGS COMPONENTS

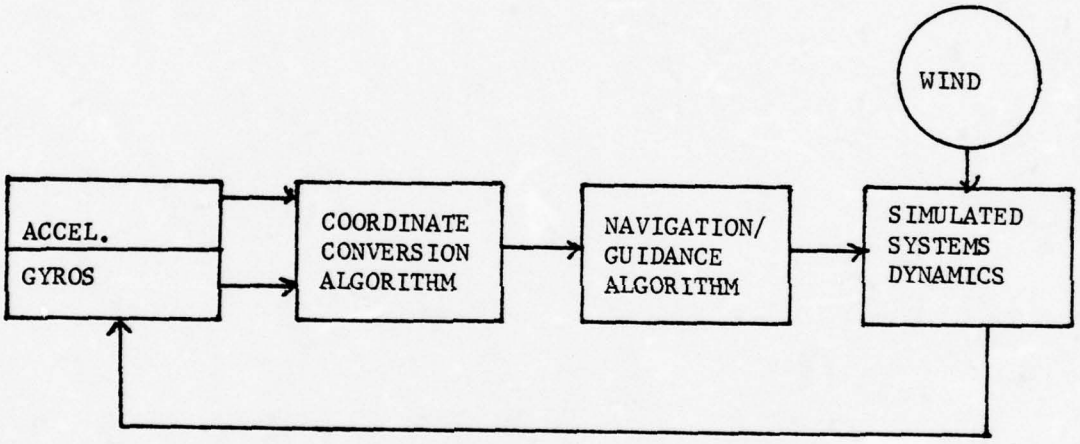


Figure 4 - BLOCK DIAGRAM FOR SIMULATION OF ATIGS INSTALLATION

Thus the total state vector would consist of 9 states as opposed to the 15 state vector considered essential in reference 2. The above technique was inspired by Kortum in his development (ref. 6) on Kalman filter applications. The inertial computation scheme is based on several assumptions, all of which are results of the short time of flight-short range requirements of the ALVRJ application. These assumptions are

1. Constant gravity vector over a 40 nm. flight path

2. Earth torquing not required

3. Small angle assumptions for largest portions of flight

D. BASIC SIMULATION DESIGN

1. General Considerations

In order to verify that the nine state system would be adequate for modeling purposes, a simulation program to test performance was conceived. The actual missile flight profile includes accelerations after launch

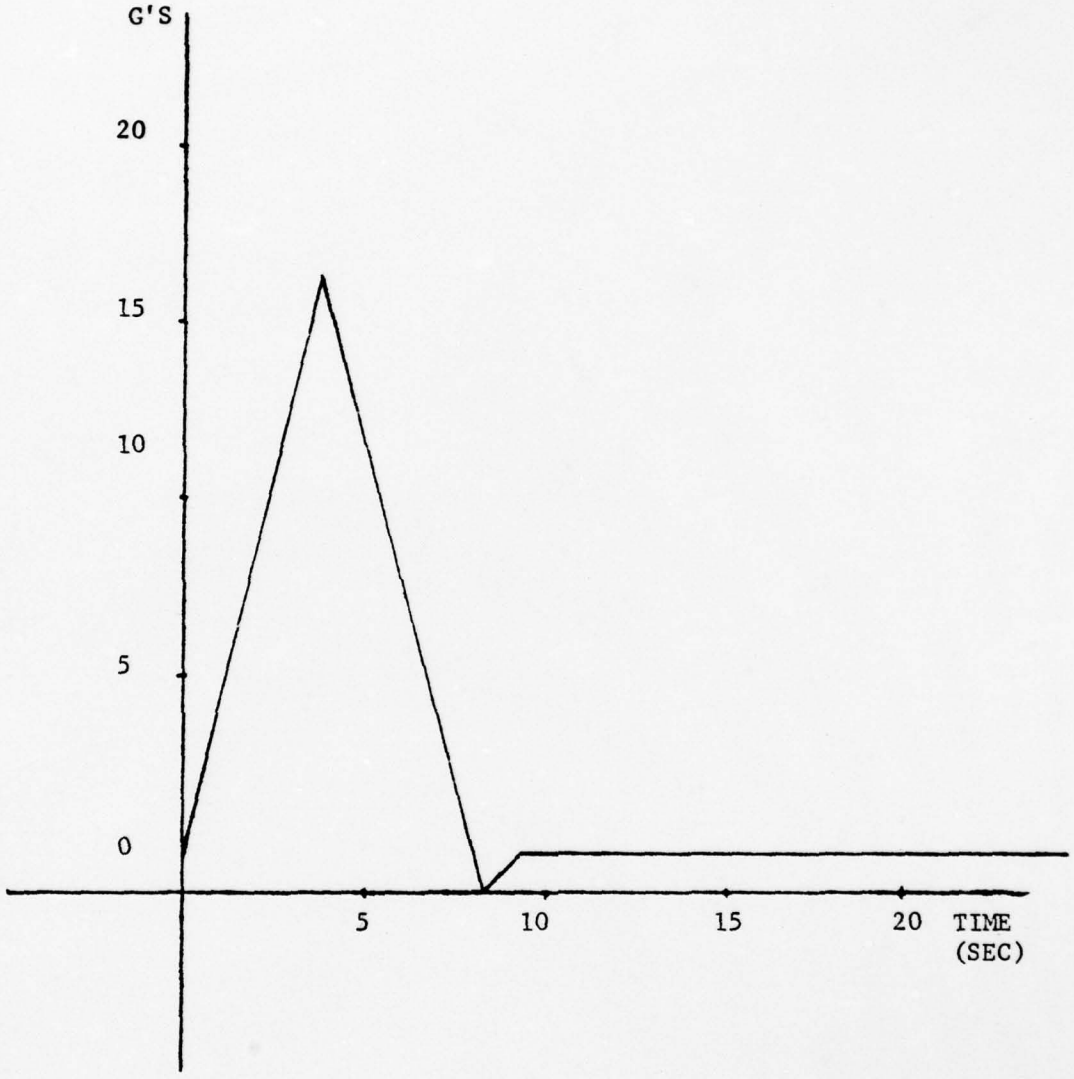


Figure 5 - ACCELERATION PROFILE

up to a maximum of 16 g's(fig.5)from the initial conditions given in fig.6. In addition, it is to be noted that for simplicity a homing type guidance command is given to the missile dynamics. This is recognised as ineffecient in practice but is simple in implementation and provides a basis for comparison purposes. Since missile flight controls respond to airspeed and not inertial speed, the velocities used in the missile dynamics portion of the algorithm are true airspeeds. However the inertial system must always compute in inertial velocities and hence must either adjust its calculations to include this difference or accept any error that this difference may entail. It is to be noted that no means of velocity measurement(i.e. doppler,mach gauge,etc.) is to be provided. In this simulation the difference is ignored due to the large magnitude of inertial velocities obtained and the short time of flight.

2. Noise Input Design

The randcm number generators used in this simulation were of two types; Gaussian and uniform. The Gaussian generators provided the noise inputs to the various sensors and the uniform generators provided the initial conditions and wind effects. The wind effects were such that constant bias directions of positive cross range and negative down-range conditions were imposed. The mean value of wind components in each direction was 30 ft per sec. with a range of + /-8 ft per sec. maximum change per second. No attempt was made to ascertain the relevance of the chosen wind model,its purpose was purely to introduce a bias into the system equations in order that the scale factor noise term of the gyros could be exercised. The scale factor term was finally dropped from the model of the gyros but the wind bias was retained.

The Gaussian generators provided noise inputs to each of the six installed sensors. The chosen model of the accelerometer noise term was

$$\gamma_{1,2,3} = EOG_{1,2,3} + WG_{1,2,3} * A_{1,2,3} \qquad (14)$$

where

EOG random bias term held constant over the entire flight but varied prior to each sample in the montecarlo

WG-scale factor term which varies through the flight

The chosen model for the gyro noise term was more complex consisting of bias terms and a random walk term. A random walk generatior is described in general terms as

$$\dot{E}_r = u_r \qquad (15)$$

where E_r is the error at a given instant and U_r is a white noise term with a standard deviation of σ_{U_r}. The variance of E_r grows linearly with time according to the relation

$$\sigma^2_{E_r} = t\,\sigma^2_{u_r} \qquad (16)$$

with σ_{E_r} given empirically in table 1 as random walk in °/hr with an uncertainty of $1\sigma_{1hr} = .0075$. For a sample generator to be used every second

$$\sigma_{u_r} = \sigma_{E_r} / 60 \qquad (17)$$

An error from each gyro is introduced into the inertial computation which is treated as a change in θ.

$$\varphi(t) = EO + G(t) \qquad (18)$$

PHI error of each gyro per interval of time

EO constant bias term per flight

G result of random walk

3. Changes in Heading Resulting in Velocity Changes

The computation of velocity in the inertial frame consists of terms which relate the change in heading to changes in inertial velocities. For small angular rates the changes in velocity in the inertial frames show as inputs i.e. AWXX,AWXY, AWYY,AWYZ,AWZZ,AWZX.

AWXY is the change in velocity in the X(down-range) direction due to a change in direction Θ_4 . This is expressed in small angle approximations as

$$\begin{vmatrix} AWX \\ AWY \\ AWZ \end{vmatrix} = \begin{vmatrix} 1 & -\Delta\theta_z & \Delta\theta_y \\ \Delta\theta_z & 1 & \Delta\theta_x \\ -\Delta\theta_y & -\Delta\theta_x & 1 \end{vmatrix} \begin{vmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{vmatrix} \qquad (19)$$

AWX is the total change in velocity in the x direction due to small angle change. This calculation is performed in the appropriate missile dynamics portion of the simulation where the $\Delta\Theta$'s are the results of commands from the guidance system of the inertial system. In the inertial system these quantities are treated as $\Delta\Theta$M's or measured changes and all velocity changes are computed based on gyro outputs.

4. Guidance System Design

The specific algorithm for generation of guidance commands was simple due to the homing type control employed. Inherent in the cross-range,down-range reference frame is the knowledge of distance remaining or "time-to-go" for termination of midcourse guidance and initiation of terminal guidance procedures. A parameter that continues to be significant within this project is the small angle, small rate assumption. In the guidance algorithm the one second time intervals chosen for use would require an inordinately long sequencing operation if commands were given in terms of rates. To clarify this statement, rate systems require an initiation and termination command, which for one second intervals would require a two second execution time. Therefore the guidance system employed within this project determines total angle change necessary and then commands an automatic pilot to accomplish this change. Thus the forcing function to the inertial navigator equations is not an input to the rate variables but rather to the angular displacement variables. No process noise was assumed for small angles hence the actual angle change was set equal to the commanded angle change. Notice that this is basically an open-loop process wherein the inertial navigator does not predict the next state based on the commanded heading change but rather on the noisy observed heading change.

The logical question then arises as to the effect of system drift during guidance. Normal procedure would be for a heading change command system in which an error signal generated by the navigator would be driven to null by the rotation of the vehicle. System drift during the heading change operation would result in process noise inputs to the navigator equations. The above was felt to be undesireable due to Kalman filter operation characteristics wherein steady state gains are non-zero for a linear system under process noise. The method of circumventing this discrepancy was to use as the forcing function the observed heading

change for the update of the navigator. Thus in the absence of process noise an accurate indicator of measurement noise would be the difference between observed heading change and commanded heading change. This concept was to be used during the Kalman filter application.

The algorithm for command guidance is given in fig.5. The "time-to-go" concept allows for a continual estimate of distance remaining, and a simple heading calculation computes the heading change necessary for homing.

$$\theta(k)_{required} = \frac{\text{cross-range position}}{\text{final position - present position}} = \frac{Y(k)}{240000-X(k)} \quad (20)$$

It should be noted that the flight profile simulation was terminated at a point where the inertially computed down-range position was greated than/or equal to the final position. This would correspond to the switchover point for terminal guidance.

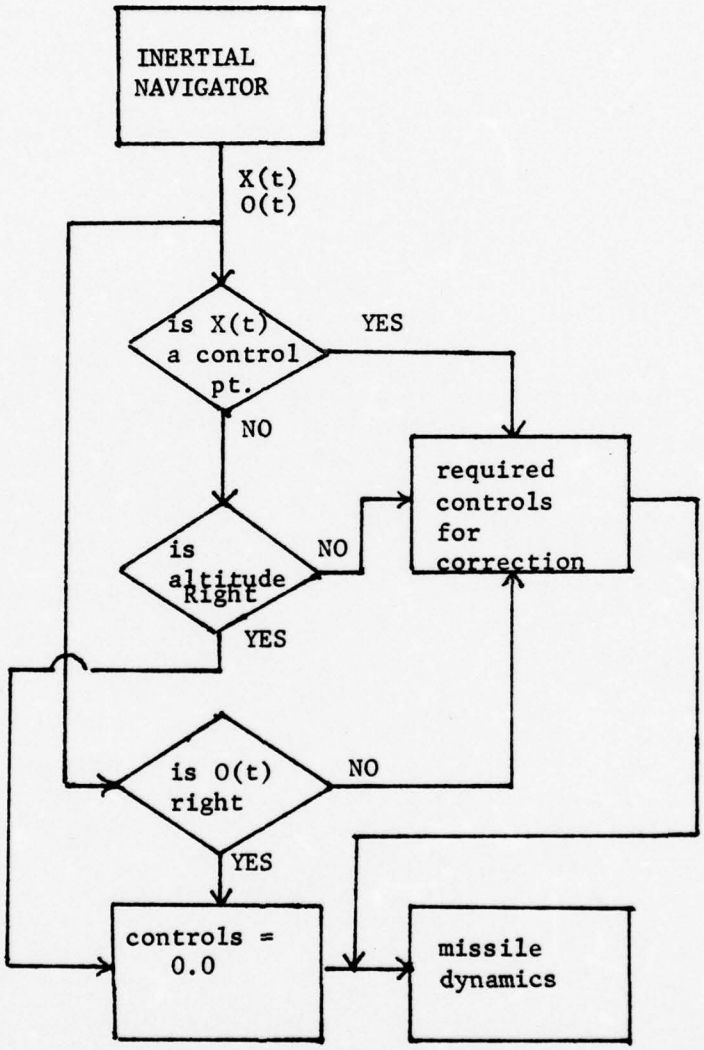


Figure 6 - GUIDANCE COMMANDS ALGORITHM

STATE VARIABLE	COORDINATE	MEAN	STD. DEVIATION
POSITION	downrange	0.0 ft (0.0 m)	1 = 387.0 ft (118 m)
	crossrange	0.0 ft (0.0 m)	1 = 387.0 ft (118 m)
	altitude	35000 ft (10668 m)	1 = 0.0 ft (0.0 m)
VELOCITY	downrange	670 ft/sec (204.2 m/sec)	1 =6.0 ft/s (1.83 m/
	crossrange	670 ft/sec (204.2 m/sec)	1 =6.0 ft/s (1.83 m/s
	vertical	0.0 ft /sec (0.0 m/sec)	1 =0.0 ft/s (0.0 m/s
ALIGNMENT	O_1	0.0°	1 = 2 min
	O_2	0.0°	1 = 2 min
	O_3	0.0°	1 = 2 min

Figure 7 - INITIAL CONDITIONS AT LAUNCH

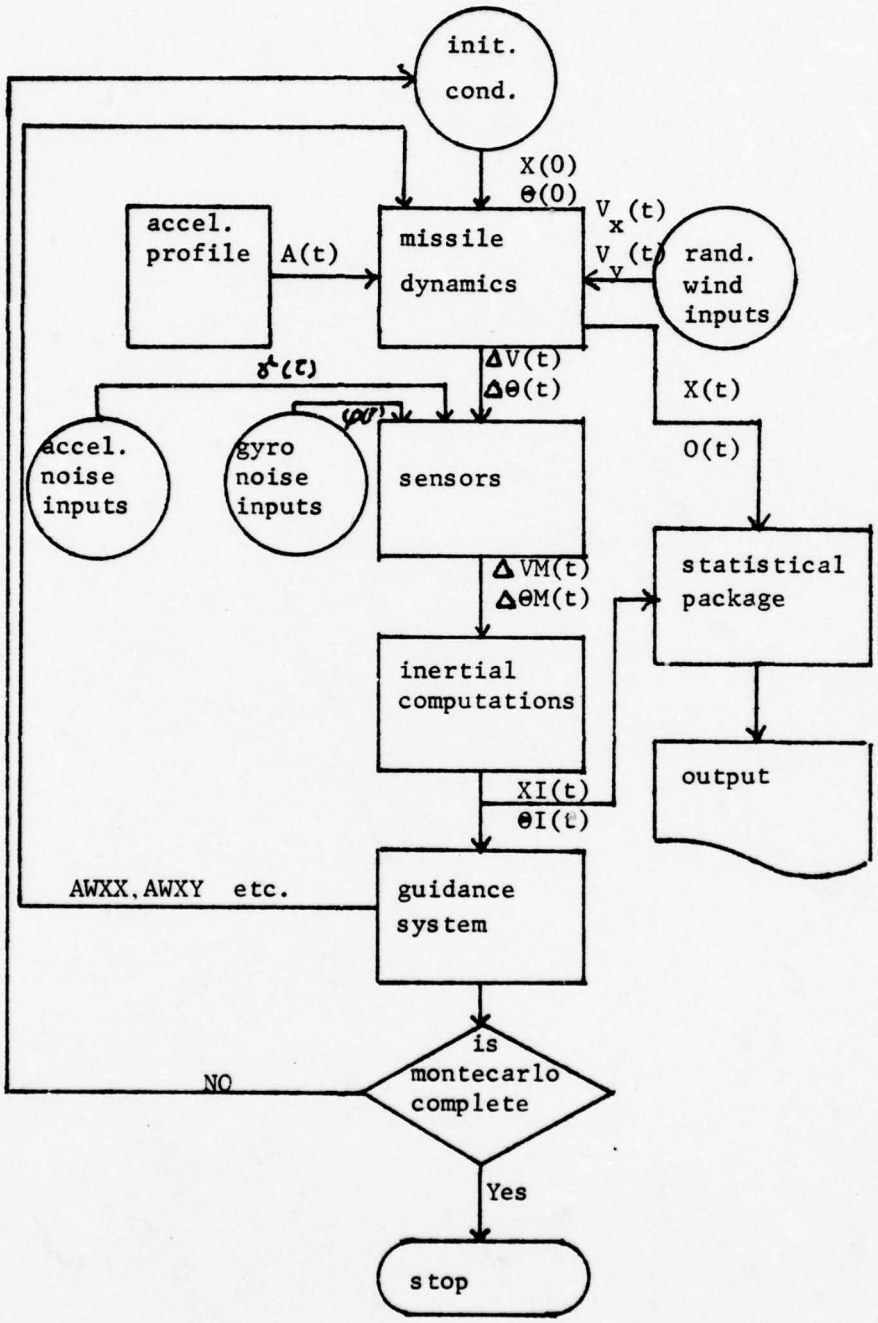


Figure 8 - ALGORITHM FOR MODEL OF MISSILE DYNAMICS WITH PURE INERTIAL COMPUTATIONS

5. Position Update System

The position update system(MICRAD) is designed to fix the missile position at various check points along the flight. Currently it is intended for the missile to navigate to each checkpoint inertially and upon arrival fix its position. The missile would then compute the course to the next check point and proceed to navigate to that point.

For purposes of this simulation and in order to reduce complexity and computer time requirements, the missile estimated position was set equal to its actual position at two discrete time steps. The inertial navigation system was thus reset at time=15 sec and time=80 sec.

The projected accuracy of the MICRAD position measurement system is $\sigma = 50.0$ ft at low altitudes. Thus a noise term was added to the measurement of position within the simulation in an effort to retain agreement with empirical data.

The magnitude of the deviation of the position fix is the fundamental argument in ref.2 for the elimination of the Kalman filter from the inertial system. The rational behind this assertion is that if two estimates of position are available (i.e. Kalman filter position estimate and a MICRAD estimate) then the weighting placed on each estimate would be heavily in favor of the more accurate estimate,logically the MICRAD fix. The optimal mix of the two estimates would then be

$$X = M X_n + (I-M) X_m \qquad (21)$$

where

X_n = navigation estimate = X + p

p = navigation estimate error

X_m = measurement estimate = X + r

r = measurement error

M = r / (r + p)

thus

$$X = (r/(r+p))X_n + (p/(r+p))X_m \quad (22)$$

since logically

$$r << p$$

$$X = X_m$$

and the Kalman filter estimates are ignored.

From the above, it can be seen that filtering for positon is required only in the intervals between observations and that the optimal mix of filtered position and observed position reduces to the observed position for highly accurate measurements.

6. Statistical Formulations

The primary measures of system performance for inertial navigation systems are mean of estimation error (i.e. the mean of actual position minus inertially calculated position) and variance of estimation error. The mean of estimation error reflects the result of bias within the inertial system and the variance of estimation error is an indication of system reliability. The simulation program adopted in this study evaluated the mean of position and

velocity as well as their variance at each one second interval along the flight path in addition to the estimation error mean and variance. The final position states were also computed and the mean and variance presented seperately. It was felt that this information could give a qualitative comparison of the missile performance with and without the Kalman filter installed.

The mean and variance equations for each time interval was computed using standard summation and averaging techniques. Thus

$$\bar{X} = 1/n \sum_{i=1}^{N} x_i \qquad (23)$$

with the inherent assumption that the relative frequency of occurence is analagous to the probability of occurence. The variance was computed similarly with

$$S^2 = (1/(n-1)) \sum_{i=1}^{N} (x_i - \bar{X})^2 \qquad (24)$$

IV. KALMAN FILTER

A. INTRODUCTORY REMARKS

Ref. 12 discusses various aspects of the philosophy of Kalman filter applications in a very concise manner. Within this discussion the practical limitations of implementation are specified; the foremost limitation being that of a prerequisite knowledge of the exact statistical description for each random signal within the system. This a priori information determines the degree of optimality of the filter.

The filter is said to be optimum in the sense that it generates an unbiased, minimum variance estimate of the states of a linear system from some noisy measurement of a subset of those states. The requirements imposed upon the designer are : exact knowledge of the system dynamics, covariances of initial conditions, and noise inputs. Departure from optimality arises when either estimates of the above quantities are used or approximations to the state equations with lesser state variables comprise the model of system dynamics.

Previous studies indicate that best results are obtained with pessimistic estimations of design parameters and linearization of non-linear systems if possible. The pessimistic estimate of design parameters reduce the sensitivity of the design to deviations within the system while linearization results in off-line gain calculations

which significantly reduce the computational requirements.

The linearization technique analysis of this study indicates that the model of the system error terms are most critical in the estimation of theta. The cross-range and down-range position error being most dependent on these terms. Since digital filtering computational requirements increase roughly as the square of state variables, effeciency dictates that the number of filtered variables be minimized. Therefore it was felt that the best usage of Kalman filter techniques would be in the estimation of theta.

This evaluation is supported by various references(6,8) most notably ref.6.

B. GENERAL THEORY

Given a plant characterized by the linear discrete equations:

$$\underset{\sim}{X}(k+1) = \underset{\sim}{\Phi}(k+1,k)\underset{\sim}{X}(k) + \underset{\sim}{\Delta}(k+1,k)\underset{\sim}{U}(k) + \underset{\sim}{W}(k)$$
$$\underset{\sim}{Z}(k) = \underset{\sim}{C}(k)\underset{\sim}{X}(k) + \underset{\sim}{V}(k) \qquad (25)$$

where

$\underset{\sim}{X}(K)$ Column matrix of states

$\underset{\sim}{\Phi}(k+1,k)$ state transition matrix for time k to time k+1

$\underset{\sim}{\Delta}(k+1,k)$ Forcing transfer function

$\underset{\sim}{W}(K)$ Prccess noise term

$\underset{\sim}{Z}(K)$ Matrix of observations

$\underset{\sim}{V}(K)$ Measurement noise

With the appropriate assumptions concerning zero mean noise terms and knowledge of the covariance of initial conditions, the optimal estimate of the state vectors at time k can be arrived at through suitable use of a Kalman filter. This estimate will be characterized by a minimum variance of estimation error as its criteria for optimality. The Kalman filter equations are:

$$
\begin{aligned}
\underset{\sim}{G}(k) &= \underset{\sim}{P}(k/k-1)\underset{\sim}{C}(k)^{t}\left[\underset{\sim}{C}(k)\underset{\sim}{P}(k/k-1)\underset{\sim}{C}(k) + \underset{\sim}{R}(k)\right]^{-1} \\
\underset{\sim}{P}(k/k) &= \left[\underset{\sim}{I}-\underset{\sim}{G}(k)\underset{\sim}{C}(k)\right]\ \underset{\sim}{P}(k/k-1) \\
\underset{\sim}{P}(k+1/k) &= \underset{\sim}{\Phi}(k+1,k)\underset{\sim}{P}(k/k)\,\underset{\sim}{\Phi}^{t}(k+1,k) + \underset{\sim}{Q}(k) \qquad (26) \\
\underset{\sim}{X}(k/k) &= \underset{\sim}{X}(k/k-1) + \underset{\sim}{G}(k)\left[\underset{\sim}{Z}(k) - \underset{\sim}{C}(k)\underset{\sim}{X}(k/k-1)\right] \\
\underset{\sim}{X}(k+1/k) &= \underset{\sim}{\Phi}(k+1,k)\underset{\sim}{X}(k/k) + \underset{\sim}{\Delta}(k+1,k)\underset{\sim}{U}(k)
\end{aligned}
$$

where the notation (k+1/k) implies the estimate at time k+1 given time k.

C. SPECIFIC THEORY

The current method of filtering inertial navigation systems is by building the filter specifically around a given error model of the gyro. The most commonly used error model is a Gauss-Markov drift model described by both correlated noise and bias terms.

$$\dot{D} = -(1/\gamma)D + r \qquad (27)$$

where D is the instantaneous value of error, γ is the correlation time and r is a bias term. Statistically the time function has been found to be roughly equivalent to a random walk model such that

$$\dot{D} = w + r \qquad (28)$$

where W is a white noise term. Since the above drift is colored noise it has been handled previously by incorporating it in the state matrix such that it is part of the estimation process. The state equations would then become

$$\begin{aligned} \dot{\theta}_1 &= \theta_2 \\ \dot{\theta}_2 &= 0 \\ \dot{D} &= w + r \\ Z &= \theta_2 + \dot{D} \end{aligned} \qquad (29)$$

Thus the Kalman filter would be designed for a 3 state vector vice the needed two states. Also the presence of process noise requires a non zero steady state value for the gains of all equations in the update portion of the Kalman filter. This means that improper choice of parameters will bias the estimatior for all time

The overriding consideration of this study being the desire to maintain as few state variables as possible prompted a closer look at the above method of analysis.

The inertial navigator had been modeled as a first order system earlier(sect. I)

$$\theta I(k+1) = \theta I(k) + \theta M(k) \qquad (30)$$

however the Kalman filter requires an observation matrix and the above model uses the observation matrix $\Delta\theta M(k)$ as a forcing function therefore for filtering purposes the rate variables had to be redefined and a new state added.

$$\begin{aligned} \theta I_1(k+1) &= \theta I_1(k) + \theta I_2(k) + \Delta\theta M \\ \theta I_2(k+1) &= \theta I_2(k) \end{aligned} \qquad (31)$$

The new state variable θI_2 is to represent a small

angular rate which should be zero under normal conditions. Notice that the forcing function is solely upon the angular position and not upon the rate variable. Now the observation is the output of the gyro which is

$$\Delta\theta M(k) = \Delta\theta(k) + \theta I_2(k) * \Delta t + \dot{\varphi}(k) * \Delta t \qquad (32)$$

where $\dot{\varphi}$ is the noise term for the gyro given by

$$\dot{\varphi}(k) = EO + G(k) \qquad (33)$$

EO is a constant bias and g(k) is a white noise term. The noise equation then for unit time intervals are

$$\Delta\varphi(k)/\Delta t = EO + G(k) = \Delta\varphi(k) \qquad (34)$$

and the observation equation is

$$\begin{aligned}\Delta\theta M(k) &= \Delta\theta(k) + \theta I_2(k) + \Delta\varphi \\ &= \theta(k) + \theta I_2(k) + EO + G(k)\end{aligned} \qquad (35)$$

Now $\Delta\theta$ is a known forcing function generated by the inertial navigator therefore a new observation can be defined as

$$\begin{aligned}Z^*(k) &= \Delta\theta M(k) - \Delta\theta(k) \\ &= \theta I_2(k) + EO + G(k)\end{aligned} \qquad (36)$$

thus the discrete state equations are

$$\begin{aligned}\theta I_1(k+1) &= \theta I_1(k) + \theta I_2(k) + \Delta\theta(k) \\ \theta I_2(k+1) &= \theta I_2(k) \\ Z^*(k) &= \theta I_2(k) + EO + G(k)\end{aligned} \qquad (37)$$

Now if the constant bias term EO is added to the $\theta I_2(k)$ and redefined as

$$\theta I_2'(k) = \theta I_2(k) + EO \qquad (38)$$

Then it can be seen that the $\Theta I_2(\kappa)$ term takes on the significance of an estimated drift and the state equations then assume the form

$$\begin{aligned} \Theta I_1(k+1) &= \Theta I_1(k) + \Theta I_2'(k) + \Delta\theta(k) \\ \Theta I_2(k+1) &= \Theta I_2'(k) \\ Z^*(k) &= \Theta I_2'(k) + G(k) \end{aligned} \qquad (39)$$

Where g(k) is white noise and the state equations are now in standard form for Kalman filtering.

It was necessary to add one more state per gyro simulated but this brings the total state vector to only 12 which is still a net savings in state variables.

A block diagram of the algorithm is given in fig.09.

D. KALMAN FILTER RESULTS

The general Kalman equations of part B above were applied to the angular state variables in an off-line calculation by applying pessimistic estimates of initial condition variance such that

$$\sigma^2_{\theta_1(0)} = 1.00 \times 10^{-03}$$

(40)

$$\sigma^2_{\theta_2(0)} = 1.000 \times 10^{-03}$$

and the variance on the measurement noise to be

$$\sigma^2(\varphi) = 1.000 \times 10^{-03}$$

The resulting gains were found to be simple time

functions such that at time $T=K\Delta t$

$$G_1(k) = 1-(2/k+1)$$

$$G_2(k) = 1/k+1 \qquad (41)$$

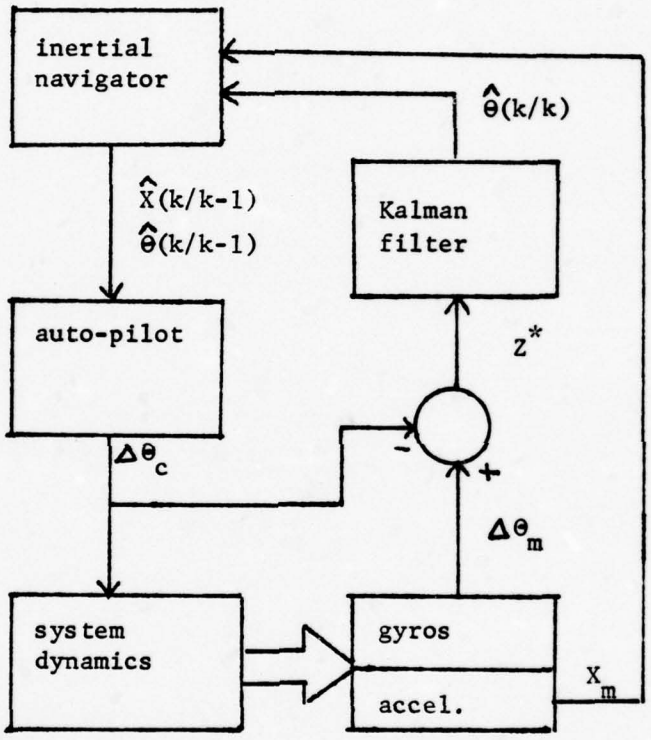


Figure 9 - ALGORITHM FOR PROPOSED KALMAN FILTER IMPLEMENTATION

V. ANALYSIS OF RESULTS

A. VERIFICATION OF RESULTS

Due to the high degree of simplification of plant dynamics, the basic plant model is felt to be weak. The linear dynamics applied to the given g profile produced a trajectory similar to the expected flight profile of the ALVRJ but (due to its simplified nature) with several defects. The descent produced a net increase in forward velocity after level off which would not be the case for a true non-linear model. This defect can be accommodated by the inertial system and hence was not felt to be detrimental to the purposes of this study.

Further work in this area would be recommended in order that basic defects in the missile such as thrust mis-alignment, process noise in control actuators and sensor misalignment be introduced into the simulation . A better tracking scheme could easily be instituted such that the guidance algorithm could institute a more efficient trajectory.

It was felt that for purposes of this study the plant model provided a reasonable approach to ALVRJ simulation. The time of flight and velocity profile are within the same general order of magnitude as the more complex simulations provided by ref.1 and correspond with physical intuition as to proper missile performance.

B. VERIFICATION OF ATIGS SIMULATION

It was felt that the simulation of the ATIGS would be accurate for purposes of this study if the error growth rate and the standard deviation growth rate incurred by the model were close to the observed quantities set forth in ref.1 and ref.3. Reference 3 stated that observed drift in the ATIGS test unit was approximately 1 nm /hr in ground test and 4 nm /hr. in airborne test. The simulation results show a numerical average drift of 3.35 nm /hr. which was felt to be in the range of the actual system. Ref.1 indicates that the simulation reported therein had a cross-range standard deviation growth rate of 1400 ft./min. between the first and second reset positions. The simulation within this study had a cross-range standard deviation growth rate of 1459.4 ft./min.

From the above correlation in performance the study proceeded under the assumption that the simulated model of the ATIGS would provide a reasonable background for analysis of position reset and Kalman filter performance in an actual installation.

C. EFFECT OF POSITION RESET ON SIMULATED PERFORMANCE

The unfiltered position reset feature of this simulation had the expected result of drastically decreasing the variance at mid-course termination. The pure inertial navigator must contend with both initial condition errors and integrated boost phase errors which combine to produce large scale variance at mid-course termination. The initial position reset occurs after boost is complete and thus virtually eliminates the initial condition and boost effect

on position. However, as expected, the velocity and angular errors of boost and initial condition still have effects on the final value of position at mid-course termination. The basic ATIGS navigator without position reset was found to have a radial uncertainty of 1608.6 ft. at mid-course termination (see table3). The addition of position reset to the basic model was found to reduce this uncertainty to 466.9 ft. However these figures reflect the inherent accuracy of the inertial navigator with the very low error terms in the sensors. If the noise terms in the inertial navigator are allowed to have their standard deviations increased by a factor of 5, thus simulating a very noisy inertial navigator, the basic ATIGS model without position reset demonstrates an uncertainty of 8646.5 ft., and after the addition of position reset,4128.0 ft. Thus it can be seen that,even with a position reset very close to mid-course termination, large uncertainty of position can accumulate due to the magnitude of the velocity errors which accrue throughout the flight.

D. EFFECT OF ADDITION OF LINEAR SUBOPTIMAL KALMAN FILTER

The Kalman filter proposed in this report had the effect of decreasing the variance of simulation behavior along the flight path for all tested situations (see table 2) . If cross-range standard deviations is taken as the criterion for performance quality, it can be seen that the filtered performance is readily superior at all noise levels. The "normal" noise level exhibits a radial uncertainty of 248.03 ft. at mid-course termination and the X5 noise level exhibits a radial uncertainty of 1083.1 ft. The fundamental reason for this increase in accuracy is shown in table 4, where the velocity errors at final update position are compared. The filtered velocity estimates, even at the

higher noise levels, are such that mid-course termination position is within much more reasonable bounds.

The overall effect of the Kalman filter proposed in this study then is to reduce the end point variance of missile position at mid-course termination in a significant manner. The unfiltered updates of position are seen to be valuable when used in conjunction with the Kalman filter but do not insure adequate missile performance if high noise levels are encountered throughout the flight.

RUN TYPE	NOISE	D-RANGE ERROR GROWTH	C-RANGE ERROR GROWTH	1-sigma ERROR GROWTH D-RANGE	1-sigma ERROR GROWTH C-RANGE
ATIGS	NORMAL	2.73 nm/hr	1.449 nm/hr	4016 ft/min	1459.4 ft/min
ATIGS	X5	68.64 nm/hr	107.9 nm/hr	14727 ft/min	10728 ft/min
ATIGS & MICRAD	NORMAL	1.849 nm/hr	1.52 nm/hr	4439.8 ft/min	1995.96 ft/min
ATIGS & MICRAD	X5	82.634 nm/hr	83.6 nm/hr	17986 ft/min	10240 ft/min
ATIGS MICRAD & KALMAN	NORMAL	.439 nm/hr	.87 nm/hr	3736 ft/min	664.59 ft/min
ATIGS MICRAD & KALMAN	X5	21.56 nm/hr	14.373 nm/hr	18836 ft/min	3053 ft/min

TABLE 2-SIMULATION PERFORMANCE WITH TYPE OF INSTALLATION

RUN TYPE	NOISE	MEAN FINAL D-RANGE POSITION 1-sigma	MEAN FINAL C-RANGE POSITION 1-sigma	MEAN FINAL D-RANGE ERROR 1-sigma	MEAN FINAL C-RANGE ERROR 1-sigma
ATIGS	NORMAL	240880 ft 1536.7 ft	217.72 ft 475.4 ft	6029.4 ft 3801.2 ft	816.9 ft 2964 ft
ATIGS	X5	241930 ft 8633.9 ft	413.39 ft 466 ft	29370 ft 18514 ft	1532.1 ft 12891 ft
ATIGS & MICRAD	NORMAL	239,610 ft 407.7 ft	74.17 ft 227.7 ft	307.4 ft 246.8 ft	97.5 ft 222.4 ft
ATIGS & MICRAD	X5	238180 ft 3564.7 ft	269.72 ft 2081.7 ft	2716 ft 1590.8 ft	293 ft 1107.5 ft
ATIGS MICRAD & KALMAN	NORMAL	240000 ft 171.5 ft	63.0 ft 194 ft	239.77 ft 238.5 ft	34.036 ft 68.1 ft
ATIGS MICRAD & KALMAN	X5	240540 ft 671.2 ft	61.62 ft 1018.9 ft	1900.7 ft 950.7 ft	59.37 ft 518.9 ft

Figure 11 - TABLE 3-FINAL POINT PERFORMANCE WITH TYPE OF INSTALLATION

type run	noise level	velocity error std. deviation
ATIGS	NORMAL	110.4 ft/sec
ATIGS	X5	562.47 ft/sec
ATIGS & MICRAD	NORMAL	104.65 ft/sec
ATIGS & MICRAD	X5	642 ft/sec
ATIGS, MICRAD & KALMAN	NORMAL	62.8 ft/sec
ATIGS, MICRAD & KALMAN	X5	262.47 ft/sec

Figure 12 - TABLE 4-VELOCITY ERROR OF TYPE OF INSTALLATION AT FINAL CHECKPOINT

APPENDIX A

RESULTS OF ATIGS SIMULATION WITHOUT FILTERING OR POSITION UPDATE

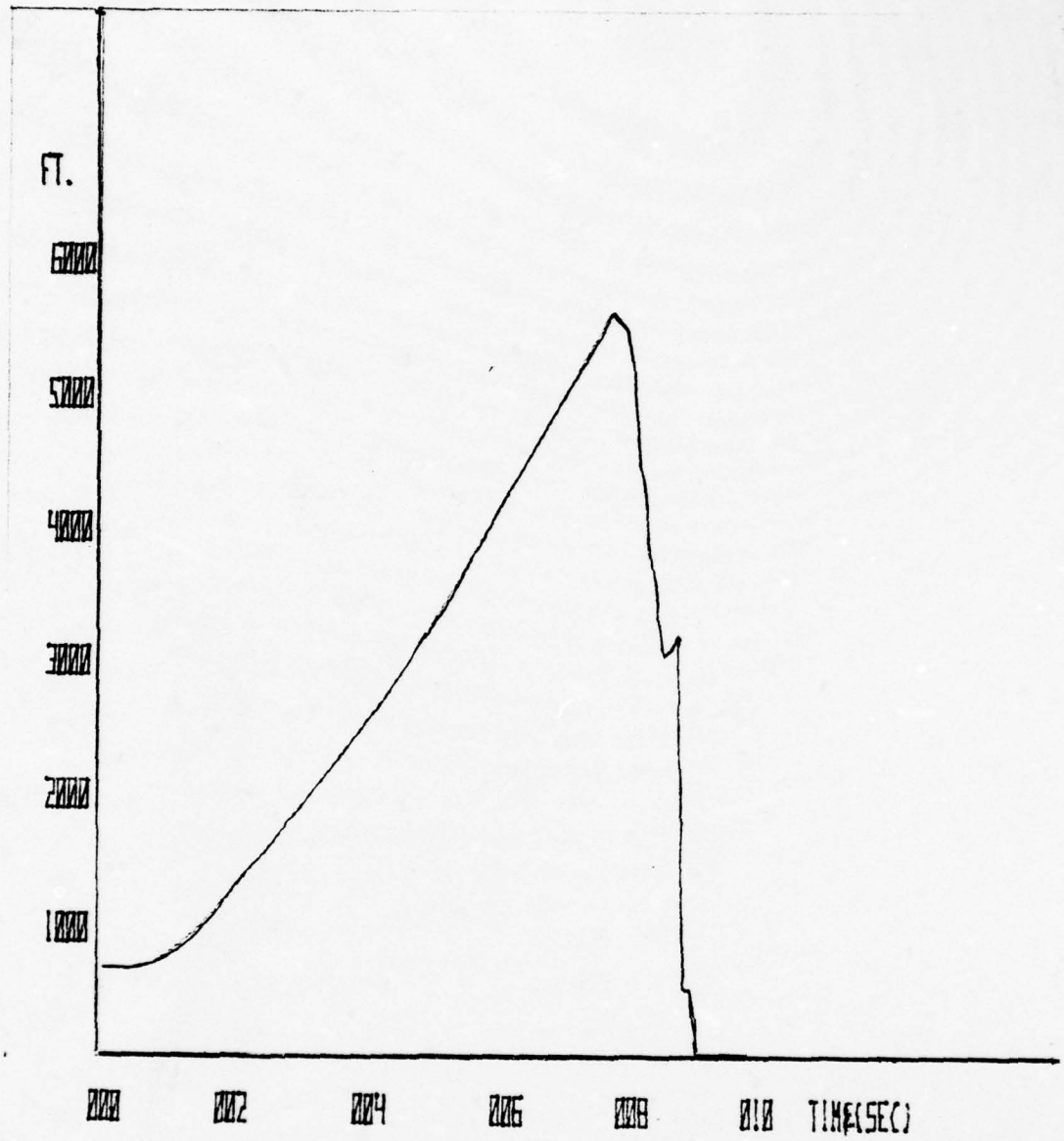


Figure 13 - SQRT OF DOWN RANGE VARIANCE (ATIGS ONLY)

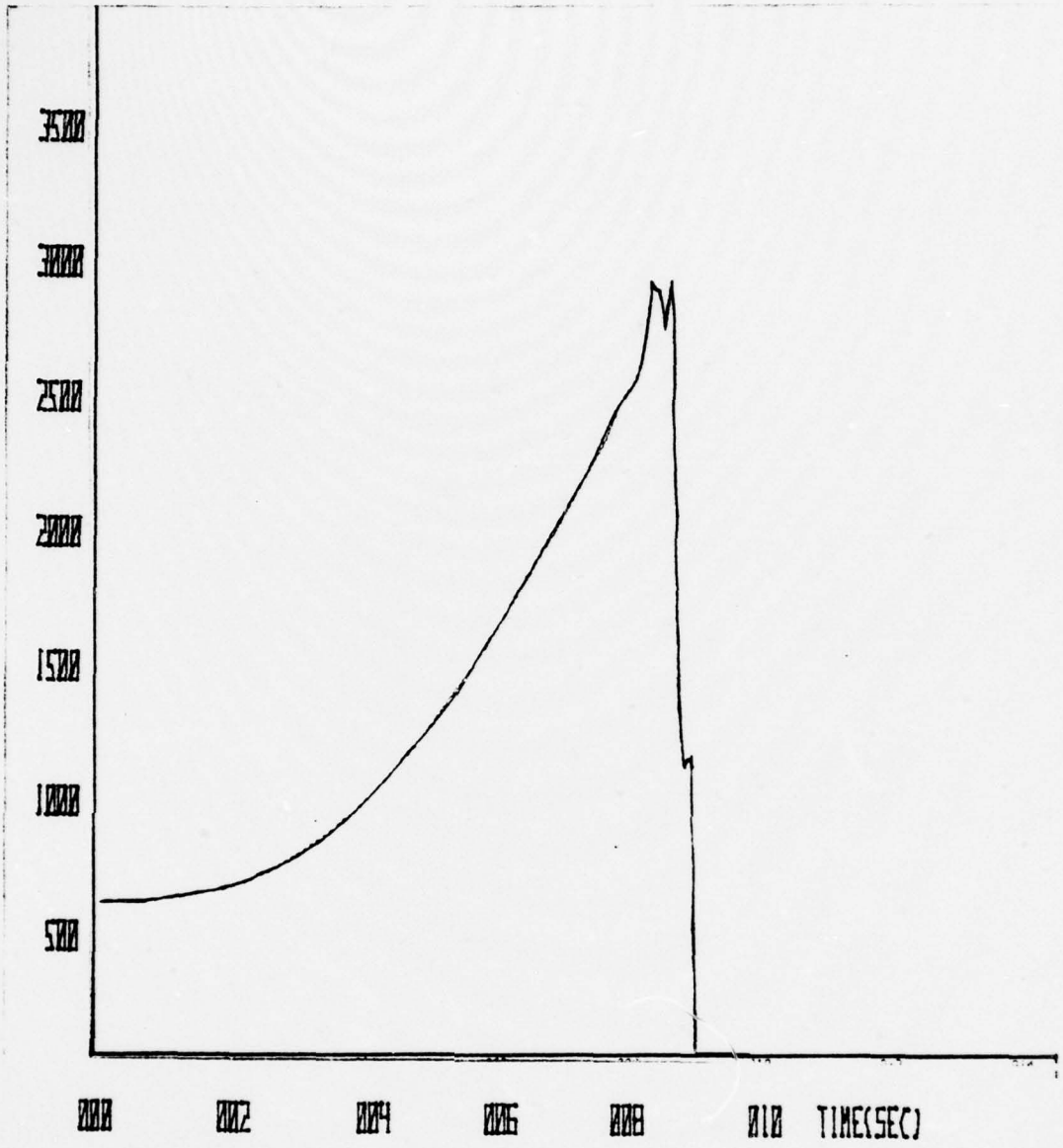


Figure 14 - SQRT OF CROSS RANGE VARIANCE (ATIGS ONLY)

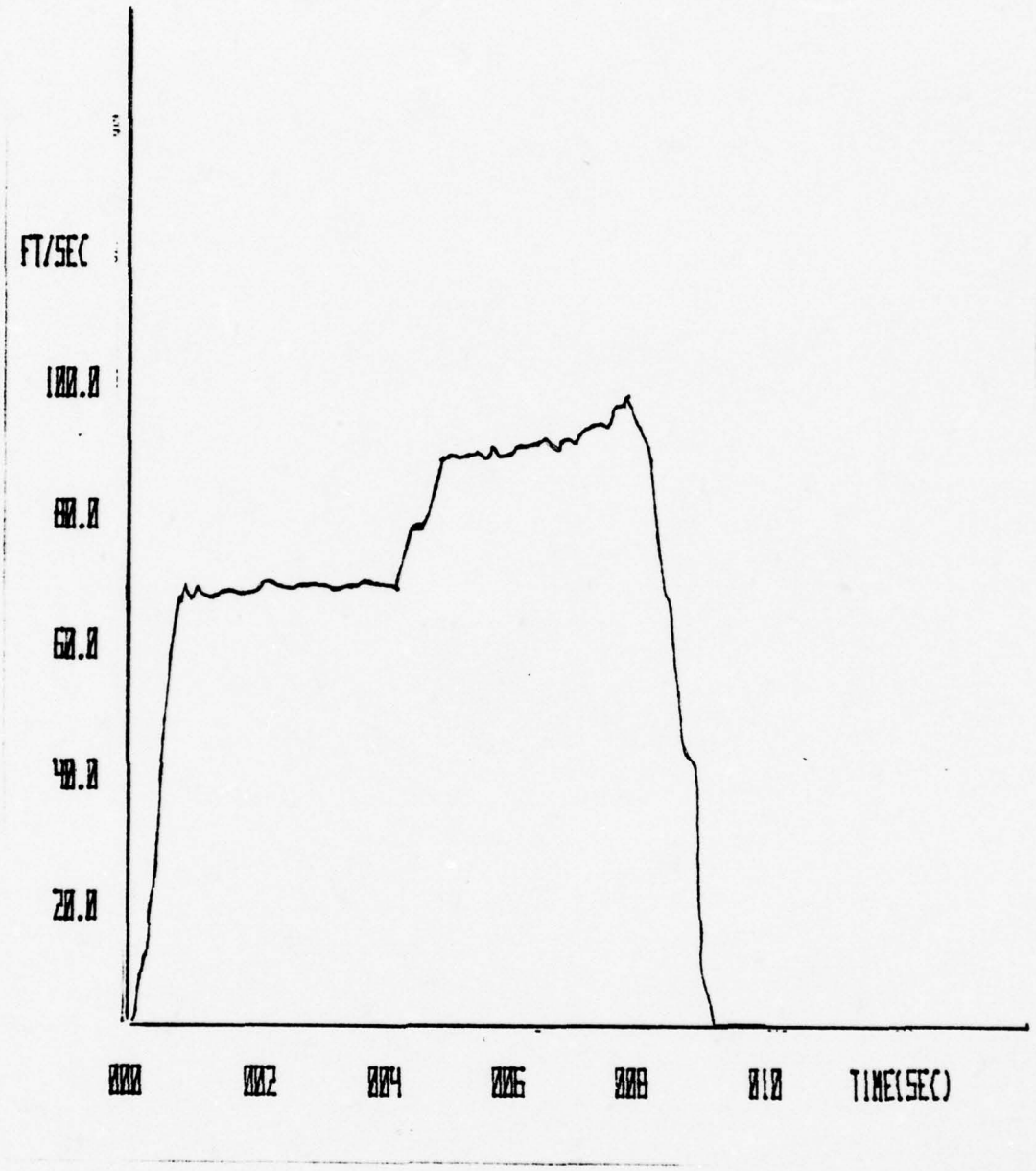


Figure 15 - SQRT• OF DOWN RANGE VELOCITY VARIANCE ‹ATIGS ONLY›

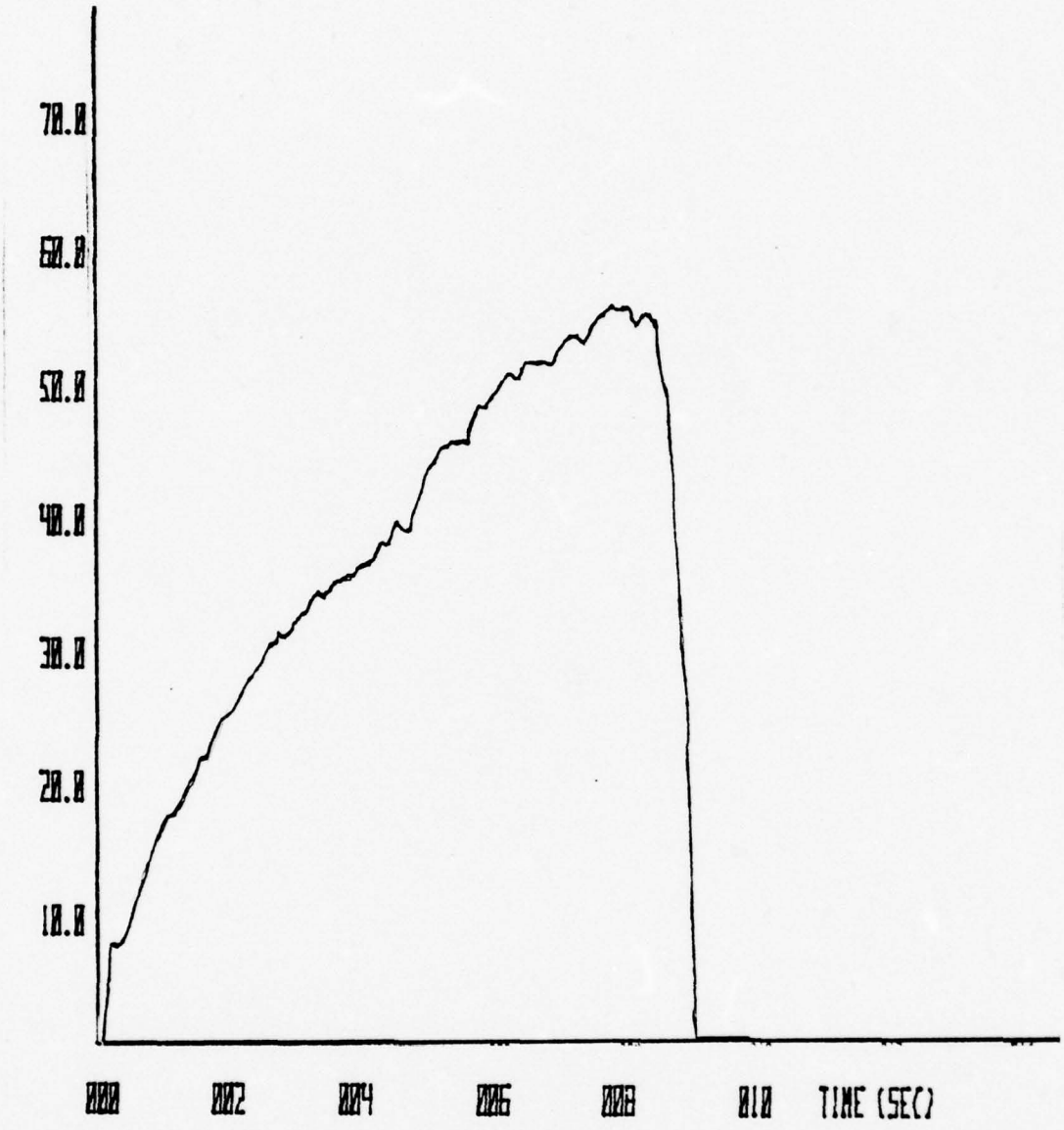


Figure 16 - SQRT OF CROSSRANGE VELOCITY VARIANCE (ATIGS ONLY)

APPENDIX B

RESULT OF ATIGS SIMULATION WITH POSITION RESET ONLY

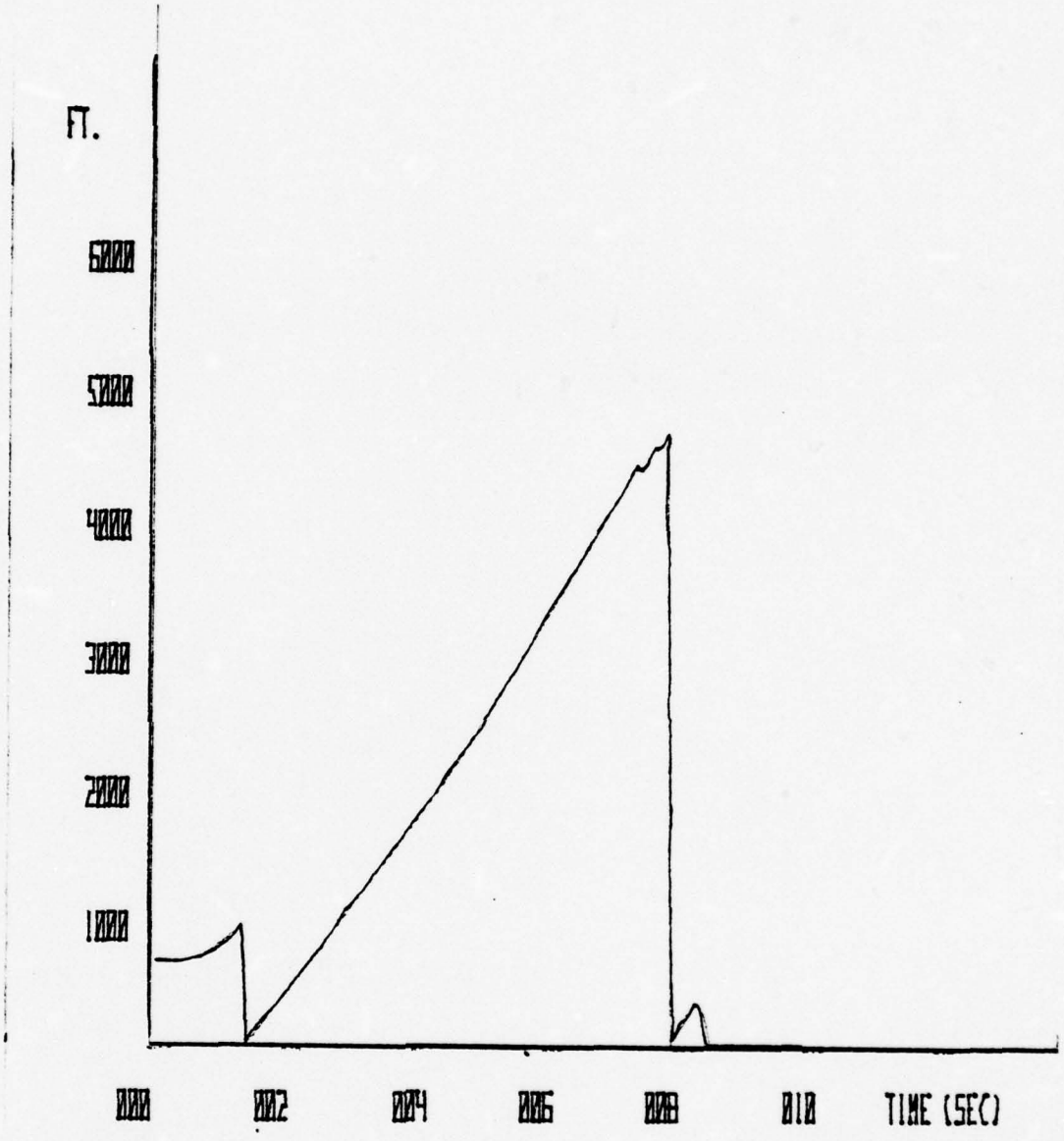


Figure 17 - SQRT OF DOWN RANGE VARIANCE ('ATIGS WITH POSIT RESET')

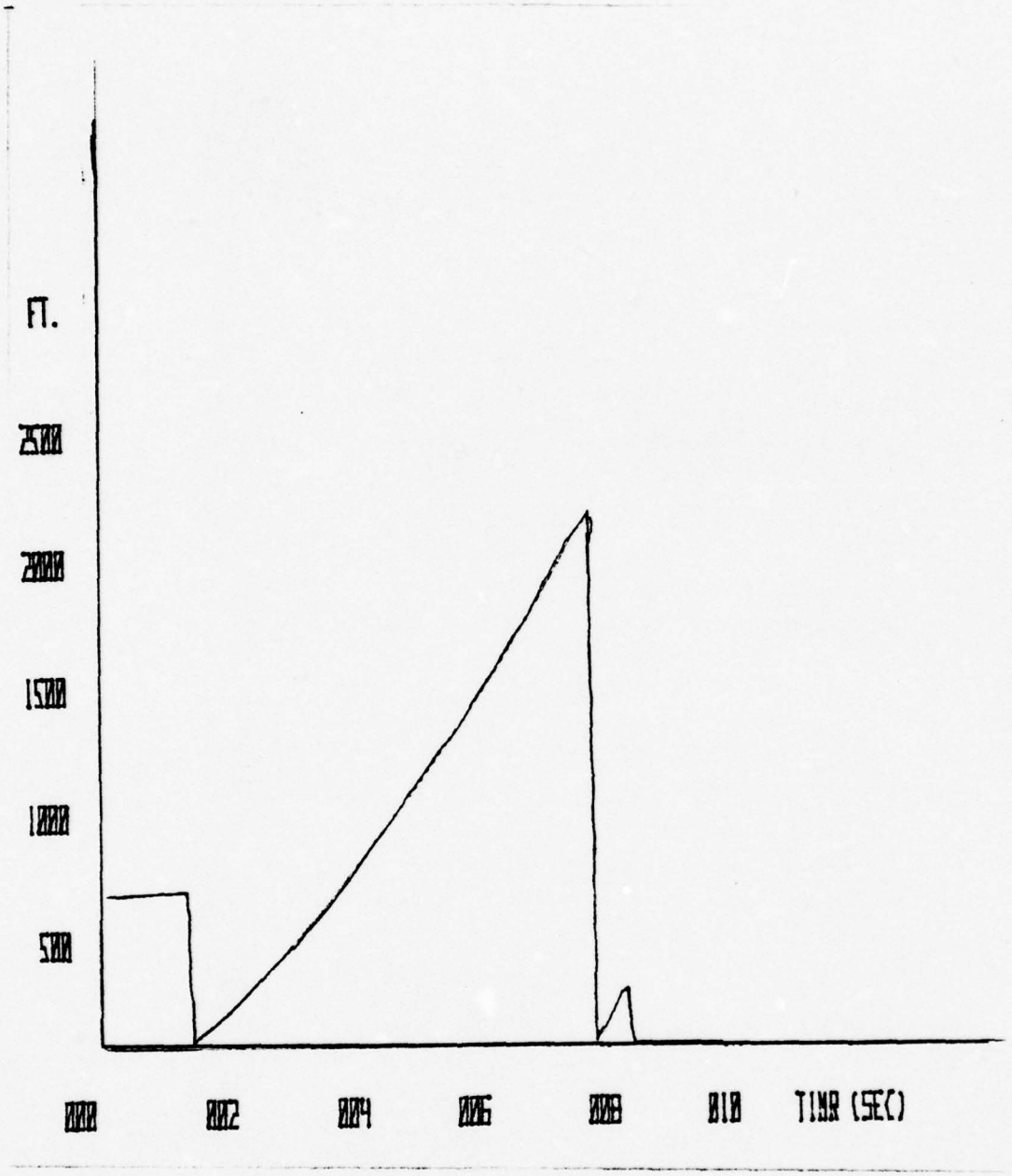


Figure 18 - SQRT OF CROSS RANGE VARIANCE (ATIGS WITH POSIT RESET)

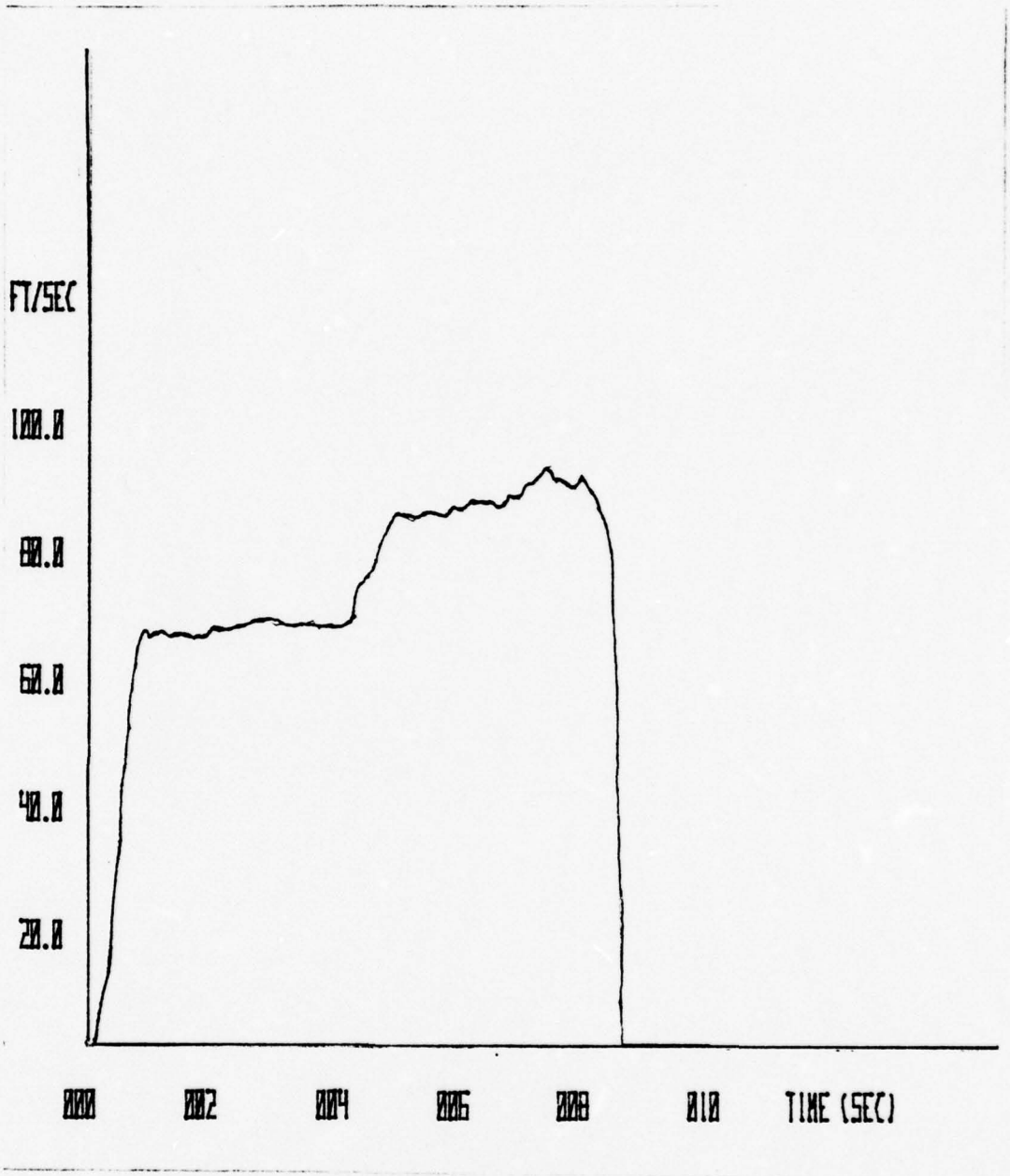


Figure 19 - SQRT OF DOWNRANGE VELOCITY VARIANCE ('ATIGS WITH POSIT RESET')

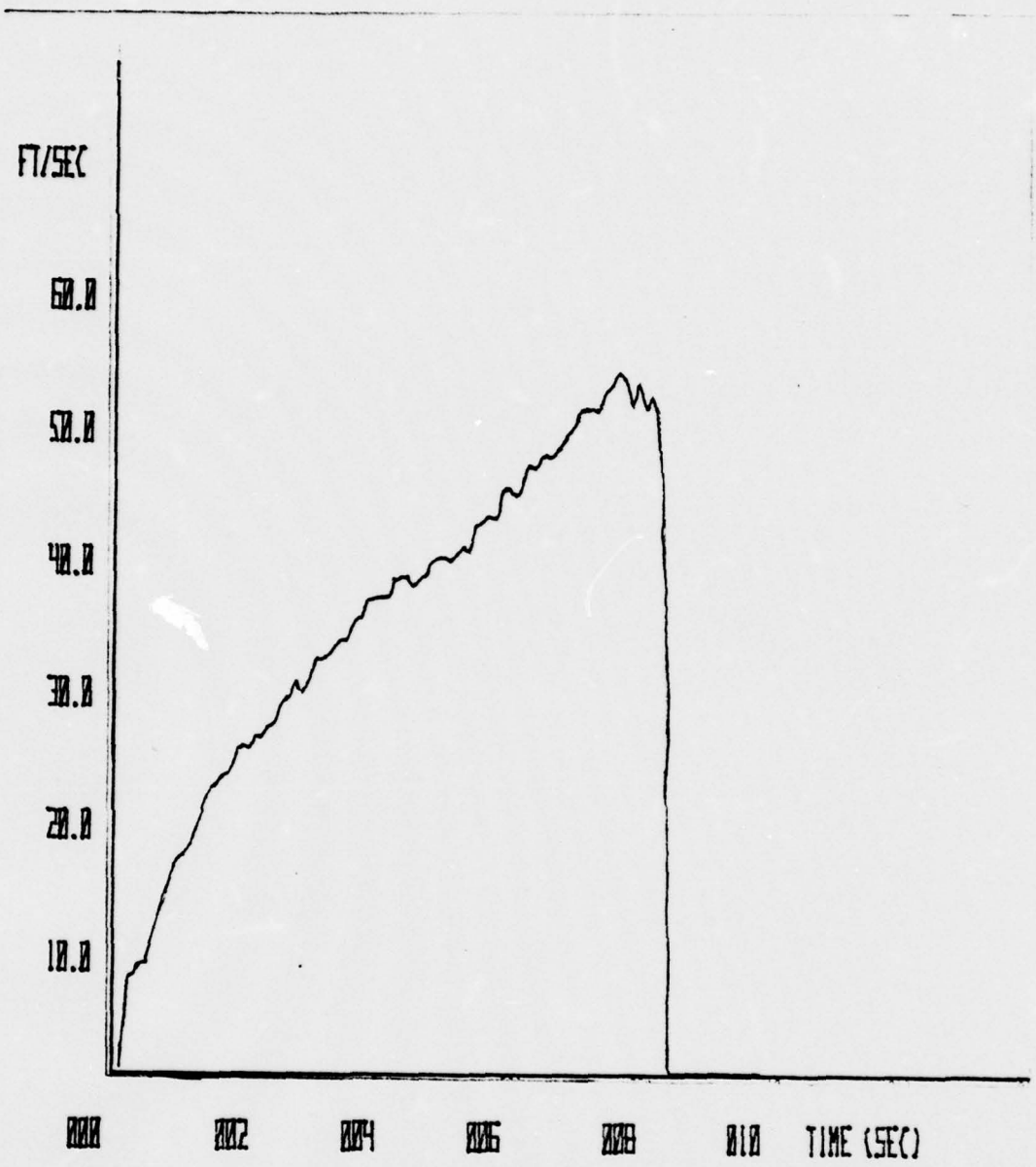


Figure 20 - SQRT OF CROSS RANGE VELOCITY VARIANCE (ATIGS WITH POSIT RESET)

```
THE NO. OF GYROS SIMULATED          3        THE NO. OF          ACCEL. SIMULATED          3        SIZE OF THE ENSEMBLE                200
SIGED =0.330000E-04    SIGW =0.200000E-02    SIGK =0.250000E-01    SIGEG =0.644000E-03    SIGKG =0.330000E-01    SIGT =0.582000E-03
THE UNCERTAINTY IN THE POSITION MEASUREMENT ISO.700000E 01FEET-RADIAL
```

TIME		MEAN OF TRACK	VAR OF TRACK	MEAN OF ERROR	VAR OF ERROR
1	X(1)	-0.638470 02	0.405150 06	-0.638470 02	0.405150 06
	X(2)	0.700000 03	0.102220 00	0.300000 02	0.187750-03
	X(3)	-0.232040 03	0.364500 06	-0.232040 03	0.364500 06
	X(4)	0.0	0.0	-0.300000 02	0.187750-03
	X(5)	0.350000 05	0.255550 03	0.0	0.0
	X(6)	0.0	0.0	0.0	0.0
2	X(1)	0.638270 03	0.405380 06	-0.640290 02	0.406060 06
	X(2)	0.764400 03	0.121900 00	0.297980 02	0.470810 02
	X(3)	-0.201520 03	0.364180 06	-0.231610 03	0.364260 06
	X(4)	0.333980-02	0.151320-02	-0.293720 02	0.501740 02
	X(5)	0.350000 05	0.255550 03	0.729140-02	0.445240 00
	X(6)	0.0	0.0	0.145830-01	0.178100 01
3	X(1)	0.146930 04	0.405800 06	-0.644710 02	0.408310 06
	X(2)	0.957600 03	0.191300 00	0.293560 02	0.120970 03
	X(3)	-0.172260 03	0.364080 06	-0.231490 03	0.364550 06
	X(4)	0.133590-01	0.242110-01	-0.296840 02	0.573580 02
	X(5)	0.350000 05	0.255550 03	-0.407340-01	0.486460 01
	X(6)	0.0	0.0	-0.110630 00	0.411220 01
4	X(1)	0.255800 04	0.405340 06	-0.652690 02	0.410530 06
	X(2)	0.127960 04	0.341580 00	0.287150 02	0.424810 03
	X(3)	-0.142140 03	0.364100 06	-0.230790 03	0.365370 06
	X(4)	0.300580-01	0.122570 00	-0.291330 02	0.674950 02
	X(5)	0.350000 05	0.255550 03	-0.272120 00	0.184670 02
	X(6)	0.0	0.0	-0.352140 00	0.829830 01
5	X(1)	0.403350 04	0.404510 06	-0.664220 02	0.414370 06
	X(2)	0.173040 04	0.624660 00	0.279190 02	0.117030 04
	X(3)	-0.111600 03	0.364660 06	-0.229170 03	0.367150 06
	X(4)	0.534360-01	0.387380 00	-0.286060 02	0.744430 02
	X(5)	0.350000 05	0.255550 03	-0.589180 00	0.502010 02
	X(6)	0.0	0.0	-0.281970 00	0.145920 02
6	X(1)	0.596000 04	0.404770 06	-0.684140 02	0.424770 06
	X(2)	0.218120 04	0.792520 00	0.268450 02	0.234090 04
	X(3)	-0.821350 02	0.365230 06	-0.228580 03	0.369800 06
	X(4)	-0.256770-01	0.552200 01	-0.291160 02	0.113980 03
	X(5)	0.350000 05	0.255550 03	-0.841830 00	0.111090 03
	X(6)	0.0	0.0	-0.223340 00	0.229390 02
7	X(1)	0.827200 04	0.404430 06	-0.725350 02	0.442920 06
	X(2)	0.250320 04	0.130720 01	0.252270 02	0.351820 04
	X(3)	-0.526020 02	0.363530 06	-0.228000 03	0.371340 06
	X(4)	0.105350 00	0.445850 02	-0.287220 02	0.146570 03
	X(5)	0.350000 05	0.255550 03	-0.115790 01	0.219940 03
	X(6)	0.0	0.0	-0.408870 00	0.393750 02
8	X(1)	0.108420 05	0.404020 06	-0.772950 02	0.470640 06
	X(2)	0.269640 04	0.151700 01	0.250050 02	0.427450 04
	X(3)	-0.223300 02	0.359780 06	-0.226620 03	0.373120 06
	X(4)	0.163120 00	0.107410 03	-0.287830 02	0.184000 03
	X(5)	0.350000 05	0.255550 03	-0.164620 01	0.415290 03
	X(6)	0.0	0.0	-0.567720 00	0.683270 02
9	X(1)	0.135400 05	0.403300 06	-0.828710 02	0.507980 06
	X(2)	0.276080 04	0.159080 01	0.244140 02	0.451430 04
	X(3)	0.751150 01	0.354050 06	-0.225460 03	0.374460 06
	X(4)	-0.386370 00	0.151440 03	-0.288320 02	0.242590 03
	X(5)	0.350000 05	0.255550 03	-0.211050 01	0.750980 03
	X(6)	0.0	0.0	-0.360820 00	0.100540 03

TIME		MEAN OF TRACK	VAR OF TRACK	MEAN OF ERROR	VAR OF ERROR
10	X(1)	0.162550 05	0.402730 06	-0.882870 02	0.554220 06
	X(2)	0.272860 04	0.155490 01	0.247610 02	0.435590 04
	X(3)	0.373390 02	0.347690 06	-0.223770 03	0.375950 06
	X(4)	-0.570360 00	0.200250 03	-0.283810 02	0.277950 03
	X(5)	0.350000 05	0.255550 03	-0.241200 01	0.128960 04
	X(6)	0.0	0.0	-0.242200 00	0.130100 03
11	X(1)	0.189540 05	0.402380 06	-0.933980 02	0.609750 06
	X(2)	0.272860 04	0.155570 01	0.248390 02	0.442800 04
	X(3)	0.662590 02	0.341430 06	-0.222880 03	0.378400 06
	X(4)	-0.599070 00	0.220790 03	-0.289290 02	0.295690 03
	X(5)	0.350000 05	0.255550 03	-0.247700 01	0.210860 04
	X(6)	0.0	0.0	0.112150 00	0.160780 03
12	X(1)	0.216520 05	0.403330 06	-0.987780 02	0.676970 06
	X(2)	0.272860 04	0.155640 01	0.248250 02	0.448710 04
	X(3)	0.962310 02	0.335270 06	-0.220830 03	0.381300 06
	X(4)	-0.522420 00	0.264780 03	-0.279560 02	0.319100 03
	X(5)	0.350000 05	0.255550 03	-0.237090 01	0.326170 04
	X(6)	0.0	0.0	0.100120 00	0.192680 03
13	X(1)	0.243510 05	0.402890 06	-0.103470 03	0.750310 06
	X(2)	0.272860 04	0.155760 01	0.251480 02	0.434260 04
	X(3)	0.125560 03	0.329620 06	-0.219220 03	0.384990 06
	X(4)	-0.815840 00	0.288130 03	-0.288120 02	0.370220 03
	X(5)	0.350000 05	0.255550 03	-0.218950 01	0.481360 04
	X(6)	0.0	0.0	0.262660 00	0.227230 03
14	X(1)	0.270500 05	0.403460 06	-0.108780 03	0.832670 06
	X(2)	0.272860 04	0.155840 01	0.245540 02	0.435970 04
	X(3)	0.153590 03	0.325390 06	-0.218880 03	0.390140 06
	X(4)	-0.151100 01	0.319170 03	-0.296940 02	0.437770 03
	X(5)	0.350000 05	0.255550 03	-0.194500 01	0.679190 04
	X(6)	0.0	0.0	0.226290 00	0.238460 03
15	X(1)	0.297480 05	0.403700 06	-0.105840 00	0.471970 02
	X(2)	0.272860 04	0.156200 01	0.243580 02	0.440220 04
	X(3)	0.181580 03	0.321080 06	0.104180 01	0.510090 02
	X(4)	-0.244880 01	0.367920 03	-0.300890 02	0.488020 03
	X(5)	0.350000 05	0.255550 03	-0.168530 01	0.928400 04
	X(6)	0.0	0.0	0.293150 00	0.293090 03
16	X(1)	0.324460 05	0.403750 06	-0.552110 01	0.450450 04
	X(2)	0.272860 04	0.156470 01	0.247190 02	0.434630 04
	X(3)	0.208820 03	0.317100 06	0.129350 01	0.576390 03
	X(4)	-0.313670 01	0.398200 03	-0.294870 02	0.504130 03
	X(5)	0.350000 05	0.255550 03	-0.150300 01	0.123860 05
	X(6)	0.0	0.0	0.715150-01	0.314050 03
17	X(1)	0.351460 05	0.403310 06	-0.997610 01	0.175180 05
	X(2)	0.272860 04	0.156640 01	0.252320 02	0.433940 04
	X(3)	0.236250 03	0.313830 06	0.205640 01	0.211690 04
	X(4)	-0.230980 01	0.481430 03	-0.292760 02	0.531830 03
	X(5)	0.350000 05	0.255550 03	-0.101990 01	0.161130 05
	X(6)	0.0	0.0	0.894640 00	0.348450 03
18	X(1)	0.378440 05	0.403640 06	-0.149840 02	0.390460 05
	X(2)	0.272860 04	0.156660 01	0.246890 02	0.433990 04
	X(3)	0.263020 03	0.310540 06	0.151890 01	0.465990 04
	X(4)	-0.257280 01	0.482640 03	-0.302300 02	0.555830 03
	X(5)	0.350000 05	0.255550 03	-0.236950 00	0.205310 05
	X(6)	0.0	0.0	0.671280 00	0.380510 03
19	X(1)	0.405430 05	0.404400 06	-0.200050 02	0.697200 05
	X(2)	0.272860 04	0.156730 01	0.248580 02	0.444580 04
	X(3)	0.290390 03	0.308730 06	0.195330 01	0.835710 04
	X(4)	-0.302100 01	0.477730 03	-0.292370 02	0.626260 03
	X(5)	0.350000 05	0.255550 03	0.329740 00	0.257160 05
	X(6)	0.0	0.0	0.462100 00	0.416370 03

TIME		MEAN OF TRACK	VAR OF TRACK	MEAN OF ERROR	VAR OF ERROR
20	X(1)	0.432410 C5	0.404990 06	-0.266240 02	0.109490 06
	X(2)	0.272860 C4	0.156940 01	0.237210 02	0.454210 04
	X(3)	0.317180 C3	0.307490 06	0.291270 01	0.133830 05
	X(4)	-0.339630 01	0.517940 03	-0.288450 02	0.647930 03
	X(5)	0.345260 C5	0.691450 05	-0.199630 01	0.324430 05
	X(6)	-0.947160 C3	0.275580 06	-0.511410 01	0.201690 04
21	X(1)	0.459350 05	0.404590 06	-0.336480 02	0.157620 06
	X(2)	0.272860 C4	0.157170 01	0.238320 02	0.448170 04
	X(3)	0.343540 C3	0.307290 06	0.387210 01	0.194300 05
	X(4)	-0.374820 C1	0.559470 03	-0.290930 02	0.643610 03
	X(5)	0.334340 05	0.275880 06	-0.994400 01	0.429920 05
	X(6)	-0.123800 04	0.253040 01	-0.108810 02	0.237540 04
22	X(1)	0.486380 05	0.404570 06	-0.383450 02	0.214370 06
	X(2)	0.272860 C4	0.157620 01	0.253310 02	0.452760 04
	X(3)	0.369650 03	0.308020 06	0.477450 01	0.267950 05
	X(4)	-0.419010 C1	0.574420 03	-0.292670 02	0.690800 03
	X(5)	0.321560 C5	0.276000 06	-0.209020 02	0.579580 05
	X(6)	-0.123800 C4	0.253040 01	-0.109340 02	0.237390 04
23	X(1)	0.513370 C5	0.404620 06	-0.425980 02	0.280560 06
	X(2)	0.272860 C4	0.158020 01	0.251020 02	0.454800 04
	X(3)	0.394790 C3	0.308780 06	0.486130 01	0.351820 05
	X(4)	-0.477210 01	0.592420 03	-0.298080 02	0.696040 03
	X(5)	0.309580 C5	0.276130 06	-0.318580 02	0.775650 05
	X(6)	-0.123800 C4	0.253040 01	-0.109780 02	0.244690 04
24	X(1)	0.540340 05	0.404950 06	-0.475990 02	0.355560 06
	X(2)	0.272860 C4	0.157850 01	0.246090 02	0.456300 04
	X(3)	0.420160 C3	0.310300 06	0.575020 01	0.447240 05
	X(4)	-0.538180 01	0.589500 03	-0.292960 02	0.723700 03
	X(5)	0.297200 C5	0.276260 06	-0.429520 02	0.102160 06
	X(6)	-0.123800 C4	0.253040 01	-0.112090 02	0.245770 04
25	X(1)	0.567340 05	0.404960 06	-0.530400 02	0.439830 06
	X(2)	0.272860 C4	0.158010 01	0.244470 02	0.459270 04
	X(3)	0.444680 C3	0.313600 06	0.617890 01	0.559280 05
	X(4)	-0.588500 C1	0.618970 03	-0.301540 02	0.752610 03
	X(5)	0.284820 C5	0.276390 06	-0.542040 02	0.131760 06
	X(6)	-0.123800 C4	0.253040 01	-0.112960 02	0.248020 04
26	X(1)	0.594330 05	0.405040 06	-0.589170 02	0.534060 06
	X(2)	0.272860 C4	0.158060 01	0.241260 02	0.466140 04
	X(3)	0.468520 03	0.317340 06	0.591490 01	0.689630 05
	X(4)	-0.652440 C1	0.637760 03	-0.306630 02	0.831560 03
	X(5)	0.272440 C5	0.276530 06	-0.656400 02	0.166250 06
	X(6)	-0.123800 C4	0.253040 01	-0.115750 02	0.250440 04
27	X(1)	0.621310 C5	0.405490 06	-0.647900 02	0.638300 06
	X(2)	0.272860 04	0.158450 01	0.240990 02	0.468000 04
	X(3)	0.491580 C3	0.322260 06	0.491480 01	0.837590 05
	X(4)	-0.714090 01	0.666380 03	-0.309320 02	0.856930 03
	X(5)	0.260060 C5	0.276670 06	-0.774290 02	0.205970 06
	X(6)	-0.123800 C4	0.253040 01	-0.120050 02	0.259420 04
28	X(1)	0.648250 05	0.405530 06	-0.715500 02	0.751410 06
	X(2)	0.272860 C4	0.158120 01	0.235430 02	0.468380 04
	X(3)	0.514670 C3	0.328120 06	0.456600 01	0.100520 06
	X(4)	-0.742300 C1	0.666180 03	-0.305070 02	0.940830 03
	X(5)	0.247670 C5	0.276820 06	-0.893940 02	0.251090 06
	X(6)	-0.123800 C4	0.253040 01	-0.119250 02	0.262050 04
29	X(1)	0.675280 C5	0.405600 06	-0.778970 02	0.874750 06
	X(2)	0.272860 C4	0.158400 01	0.239490 02	0.474870 04
	X(3)	0.537210 03	0.334500 06	0.416820 01	0.118200 06
	X(4)	-0.806C30 C1	0.697710 03	-0.308500 02	0.877460 03
	X(5)	0.235250 C5	0.276980 06	-0.101380 03	0.301570 06
	X(6)	-0.123800 C4	0.253040 01	-0.120500 02	0.265420 04

TIME		MEAN OF TRACK	VAR OF TRACK	MEAN OF ERROR	VAR OF ERROR
30	X(1)	0.702260 05	0.406460 06	-0.844740 02	0.100620 07
	X(2)	0.272860 04	0.158640 01	0.235340 02	0.462170 04
	X(3)	0.558520 03	0.343150 06	0.307230 01	0.137540 06
	X(4)	-0.875940 01	0.695960 03	-0.307990 02	0.933590 03
	X(5)	0.222510 05	0.277140 06	-0.113620 03	0.357610 06
	X(6)	-0.123800 04	0.253040 01	-0.124370 02	0.272820 04
31	X(1)	0.729240 05	0.406990 06	-0.920060 02	0.114610 07
	X(2)	0.272860 04	0.158660 01	0.228500 02	0.462690 04
	X(3)	0.579400 03	0.353510 06	0.178020 01	0.160010 06
	X(4)	-0.904100 01	0.713830 03	-0.313280 02	0.104190 04
	X(5)	0.210530 05	0.277300 06	-0.126370 03	0.419250 06
	X(6)	-0.123800 04	0.253040 01	-0.130600 02	0.278310 04
32	X(1)	0.756220 05	0.408420 06	-0.992970 02	0.129530 07
	X(2)	0.272860 04	0.159240 01	0.232740 02	0.461250 04
	X(3)	0.600280 03	0.365260 06	0.110450 01	0.184250 06
	X(4)	-0.992730 01	0.746670 03	-0.307540 02	0.103840 04
	X(5)	0.198150 05	0.277470 06	-0.139350 03	0.486550 06
	X(6)	-0.123800 04	0.253040 01	-0.128880 02	0.283010 04
33	X(1)	0.783210 05	0.409430 06	-0.106070 03	0.145310 07
	X(2)	0.272860 04	0.159850 01	0.235430 02	0.458610 04
	X(3)	0.620070 03	0.377420 06	-0.240650-01	0.209580 06
	X(4)	-0.108740 02	0.754580 03	-0.318920 02	0.106440 04
	X(5)	0.185770 05	0.277650 06	-0.152100 03	0.559690 06
	X(6)	-0.123800 04	0.253040 01	-0.126270 02	0.288180 04
34	X(1)	0.810190 05	0.409820 06	-0.112550 03	0.161950 07
	X(2)	0.272860 04	0.160270 01	0.236070 02	0.459870 04
	X(3)	0.638630 03	0.391840 06	-0.185430 01	0.237470 06
	X(4)	-0.123100 02	0.760890 03	-0.320800 02	0.110050 04
	X(5)	0.173390 05	0.277830 06	-0.164780 03	0.638340 06
	X(6)	-0.123800 04	0.253040 01	-0.127230 02	0.286910 04
35	X(1)	0.837180 05	0.409700 06	-0.118800 03	0.179590 07
	X(2)	0.272860 04	0.160780 01	0.238260 02	0.463190 04
	X(3)	0.656030 03	0.407230 06	-0.420130 01	0.267420 06
	X(4)	-0.127540 02	0.777550 03	-0.324730 02	0.113890 04
	X(5)	0.161010 05	0.278010 06	-0.177350 03	0.722610 06
	X(6)	-0.123800 04	0.253040 01	-0.124240 02	0.293740 04
36	X(1)	0.864170 05	0.410200 06	-0.124400 03	0.198400 07
	X(2)	0.272860 04	0.160750 01	0.241710 02	0.471540 04
	X(3)	0.672700 03	0.424490 06	-0.658760 01	0.299130 06
	X(4)	-0.137130 02	0.804840 03	-0.321110 02	0.114360 04
	X(5)	0.148630 05	0.278200 06	-0.189760 03	0.812720 06
	X(6)	-0.123800 04	0.253040 01	-0.123970 02	0.295230 04
37	X(1)	0.891160 05	0.410270 06	-0.129510 03	0.217680 07
	X(2)	0.272860 04	0.160700 01	0.243940 02	0.452970 04
	X(3)	0.689480 03	0.442990 06	-0.824710 01	0.334190 06
	X(4)	-0.136160 02	0.801440 03	-0.321150 02	0.123440 04
	X(5)	0.136250 05	0.278400 06	-0.202160 03	0.908760 06
	X(6)	-0.123800 04	0.253040 01	-0.123940 02	0.301270 04
38	X(1)	0.918140 05	0.410240 06	-0.136290 03	0.237940 07
	X(2)	0.272860 04	0.160900 01	0.231390 02	0.460650 04
	X(3)	0.705560 03	0.462440 06	-0.108090 02	0.371820 06
	X(4)	-0.144190 02	0.790430 03	-0.332120 02	0.126050 04
	X(5)	0.123870 05	0.278600 06	-0.214660 03	0.101160 07
	X(6)	-0.123800 04	0.253040 01	-0.126030 02	0.308460 04
39	X(1)	0.945130 05	0.410850 06	-0.143100 03	0.259330 07
	X(2)	0.272860 04	0.161050 01	0.233180 02	0.463400 04
	X(3)	0.720410 03	0.483060 06	-0.146690 02	0.414860 06
	X(4)	-0.154220 02	0.809320 03	-0.340590 02	0.135750 04
	X(5)	0.111490 05	0.278800 06	-0.227610 03	0.112090 07
	X(6)	-0.123800 04	0.253040 01	-0.133070 02	0.308450 04

TIME		MEAN OF TRACK	VAR OF TRACK	MEAN OF ERROR	VAR OF ERROR
40	X(1)	0.972110 05	0.411270 06	-0.150280 03	0.281430 07
	X(2)	0.272860 04	0.161200 01	0.228580 02	0.458710 04
	X(3)	0.734930 03	0.505090 06	-0.101160 02	0.456610 06
	X(4)	-0.159560 02	0.823130 03	-0.332780 02	0.136700 04
	X(5)	0.991090 04	0.279010 06	-0.241150 03	0.123570 07
	X(6)	-0.123800 04	0.253040 01	-0.137600 02	0.306650 04
41	X(1)	0.999090 05	0.411310 06	-0.157790 03	0.304440 07
	X(2)	0.272860 04	0.161030 01	0.227880 02	0.460100 04
	X(3)	0.749120 03	0.520130 06	-0.213440 02	0.502470 06
	X(4)	-0.161820 02	0.817560 03	-0.336730 02	0.137630 04
	X(5)	0.867280 04	0.279230 06	-0.254900 03	0.135650 07
	X(6)	-0.123800 04	0.253040 01	-0.137420 02	0.310740 04
42	X(1)	0.102610 06	0.409400 06	-0.164810 03	0.328240 07
	X(2)	0.273140 04	0.157390 04	0.227310 02	0.466010 04
	X(3)	0.762700 03	0.551710 06	-0.247270 02	0.549860 06
	X(4)	-0.171170 02	0.813800 03	-0.335450 02	0.137260 04
	X(5)	0.743790 04	0.279390 06	-0.269130 03	0.148190 07
	X(6)	-0.123150 04	0.761480 04	-0.147290 02	0.306460 04
43	X(1)	0.105320 06	0.411660 06	-0.173440 03	0.354210 07
	X(2)	0.274550 04	0.919570 04	0.202900 02	0.539910 04
	X(3)	0.775400 03	0.578990 06	-0.283500 02	0.602070 06
	X(4)	-0.171670 02	0.823630 03	-0.333860 02	0.149000 04
	X(5)	0.622150 04	0.290730 06	-0.285220 03	0.160450 07
	X(6)	-0.120090 04	0.445860 05	-0.174480 02	0.270940 04
44	X(1)	0.108070 06	0.445120 06	-0.184620 03	0.381830 07
	X(2)	0.281510 04	0.413480 05	0.169320 02	0.557840 04
	X(3)	0.787200 03	0.606490 06	-0.328790 02	0.657450 06
	X(4)	-0.174350 02	0.811330 03	-0.338750 02	0.148790 04
	X(5)	0.509800 04	0.360060 06	-0.304190 03	0.172060 07
	X(6)	-0.104610 04	0.200800 06	-0.204870 02	0.245760 04
45	X(1)	0.110940 06	0.554890 06	-0.199730 03	0.410870 07
	X(2)	0.297860 04	0.779560 05	0.130620 02	0.574030 04
	X(3)	0.798600 03	0.634030 06	-0.377690 02	0.714250 06
	X(4)	-0.182240 02	0.783220 03	-0.343590 02	0.150120 04
	X(5)	0.423140 04	0.672990 06	-0.325440 03	0.183320 07
	X(6)	-0.687030 03	0.378680 06	-0.220170 02	0.242490 04
46	X(1)	0.113960 06	0.765420 06	-0.214360 03	0.441880 07
	X(2)	0.313030 04	0.643200 05	0.174440 02	0.614780 04
	X(3)	0.808990 03	0.660090 06	-0.425480 02	0.772320 06
	X(4)	-0.198900 02	0.742290 03	-0.341050 02	0.144360 04
	X(5)	0.371160 04	0.126350 07	-0.344670 03	0.194340 07
	X(6)	-0.352710 03	0.312440 06	-0.164480 02	0.258650 04
47	X(1)	0.117120 06	0.967280 06	-0.222760 03	0.474110 07
	X(2)	0.324550 04	0.232390 05	0.247390 02	0.658410 04
	X(3)	0.819040 03	0.685680 06	-0.464020 02	0.832440 06
	X(4)	-0.204050 02	0.695010 03	-0.339060 02	0.147270 04
	X(5)	0.348580 04	0.177740 07	-0.357350 03	0.204400 07
	X(6)	-0.988610 02	0.112750 06	-0.890170 01	0.226910 04
48	X(1)	0.120350 06	0.105720 07	-0.227460 03	0.508620 07
	X(2)	0.328200 04	0.466510 04	0.272280 02	0.706850 04
	X(3)	0.827850 03	0.711650 06	-0.505870 02	0.894460 06
	X(4)	-0.218470 02	0.667740 03	-0.342660 02	0.151910 04
	X(5)	0.342720 04	0.200320 07	-0.364150 03	0.212920 07
	X(6)	-0.183760 02	0.226320 05	-0.470460 01	0.181370 04
49	X(1)	0.123610 06	0.107420 07	-0.229350 03	0.545460 07
	X(2)	0.329050 04	0.273110 01	0.290070 02	0.734690 04
	X(3)	0.835890 03	0.737510 06	-0.541870 02	0.961190 06
	X(4)	-0.226140 02	0.695550 03	-0.334710 02	0.158390 04
	X(5)	0.341810 04	0.205450 07	-0.368080 03	0.220640 07
	X(6)	0.198600 00	0.225630 01	-0.315820 01	0.165510 04

TIME		MEAN OF TRACK	VAR OF TRACK	MEAN OF ERROR	VAR OF ERROR
50	X(1)	0.126870 C6	0.107470 07	-0.230730 03	0.584260 07
	X(2)	0.329050 C4	0.274010 01	0.287410 02	0.736220 04
	X(3)	0.842950 C3	0.762310 06	-0.580630 02	0.103000 07
	X(4)	-0.229240 C2	0.698580 03	-0.339440 02	0.160890 04
	X(5)	0.341830 C4	0.205460 07	-0.371140 03	0.228450 07
	X(6)	0.198600 00	0.225630 01	-0.295030 01	0.168540 04
51	X(1)	0.130130 C6	0.107520 07	-0.232430 03	0.624340 07
	X(2)	0.329050 C4	0.274160 01	0.286270 02	0.731840 04
	X(3)	0.849220 C3	0.788650 06	-0.626420 02	0.110170 07
	X(4)	-0.237650 C2	0.703480 03	-0.344370 02	0.163060 04
	X(5)	0.341850 C4	0.205460 07	-0.374280 03	0.236740 07
	X(6)	0.198600 00	0.225630 01	-0.333410 01	0.182370 04
52	X(1)	0.133390 C6	0.107630 07	-0.233600 03	0.665430 07
	X(2)	0.329050 C4	0.280310 01	0.290090 02	0.720860 04
	X(3)	0.854570 C3	0.811390 06	-0.672270 02	0.117370 07
	X(4)	-0.250270 C2	0.627810 03	-0.342250 02	0.160600 04
	X(5)	0.341870 C4	0.205470 07	-0.377260 03	0.245370 07
	X(6)	0.198600 00	0.225630 01	-0.261760 01	0.181390 04
53	X(1)	0.136650 C6	0.107530 07	-0.234710 03	0.708300 07
	X(2)	0.329050 C4	0.280700 01	0.289040 02	0.731900 04
	X(3)	0.859510 C3	0.831770 06	-0.706090 02	0.124860 07
	X(4)	-0.257660 C2	0.618040 03	-0.332120 02	0.163690 04
	X(5)	0.341890 C4	0.205480 07	-0.379550 03	0.254210 07
	X(6)	0.198600 00	0.225630 01	-0.196220 01	0.182090 04
54	X(1)	0.139510 C6	0.107600 07	-0.234800 03	0.752710 07
	X(2)	0.329050 C4	0.280220 01	0.294970 02	0.737560 04
	X(3)	0.863750 C3	0.852040 06	-0.735310 02	0.132850 07
	X(4)	-0.261750 C2	0.619390 03	-0.330500 02	0.169130 04
	X(5)	0.341530 C4	0.204320 07	-0.381580 03	0.263380 07
	X(6)	-0.726600 C1	0.110630 05	-0.210510 01	0.186200 04
55	X(1)	0.143170 C6	0.107730 07	-0.235630 03	0.799040 07
	X(2)	0.329390 C4	0.228150 04	0.287800 02	0.746420 04
	X(3)	0.867380 C3	0.869720 06	-0.766100 02	0.140820 07
	X(4)	-0.266380 C2	0.583620 03	-0.331960 02	0.164680 04
	X(5)	0.341180 C4	0.203710 07	-0.384130 03	0.272860 07
	X(6)	0.200650 C0	0.226500 01	-0.299580 01	0.193350 04
56	X(1)	0.146440 C6	0.108310 07	-0.236730 03	0.846330 07
	X(2)	0.329390 C4	0.228150 04	0.287100 02	0.736820 04
	X(3)	0.870010 C3	0.889410 06	-0.806840 02	0.149350 07
	X(4)	-0.273800 C2	0.586210 03	-0.342250 02	0.183120 04
	X(5)	0.340830 C4	0.204430 07	-0.386960 03	0.282450 07
	X(6)	-0.726510 C1	0.110840 05	-0.266450 01	0.189660 04
57	X(1)	0.149700 C6	0.108900 07	-0.237740 03	0.894760 07
	X(2)	0.329730 C4	0.454230 04	0.285540 02	0.733780 04
	X(3)	0.871880 C3	0.907950 06	-0.844150 02	0.158360 07
	X(4)	-0.289840 C2	0.595720 03	-0.333290 02	0.184660 04
	X(5)	0.339720 C4	0.207220 07	-0.389850 03	0.292350 07
	X(6)	-0.147350 02	0.221330 05	-0.311690 01	0.193760 04
58	X(1)	0.152970 C6	0.109890 07	-0.240040 03	0.945670 07
	X(2)	0.330410 C4	0.901150 04	0.279020 02	0.754260 04
	X(3)	0.872250 C3	0.925890 06	-0.884940 02	0.167590 07
	X(4)	-0.295030 C2	0.562700 03	-0.340610 02	0.188830 04
	X(5)	0.339000 C4	0.209860 07	-0.393290 03	0.302770 07
	X(6)	0.199000 00	0.230830 01	-0.376340 01	0.203500 04
59	X(1)	0.156250 C6	0.112550 07	-0.241820 03	0.998160 07
	X(2)	0.330410 C4	0.901170 04	0.281340 02	0.755750 04
	X(3)	0.871530 C3	0.942270 06	-0.924970 02	0.176990 07
	X(4)	-0.308010 C2	0.561470 03	-0.336150 02	0.186190 04
	X(5)	0.337900 C4	0.211970 07	-0.396610 03	0.313570 07
	X(6)	-0.221970 02	0.329360 05	-0.286860 01	0.208890 04

TIME		MEAN OF TRACK	VAR OF TRACK	MEAN OF ERROR	VAR OF ERROR
60	X(1)	0.15953D C6	0.117010 07	-0.24397D 03	0.10521D 08
	X(2)	0.33142D 04	0.155110 05	0.27976D 02	0.74795D 04
	X(3)	0.87068D C3	0.956800 06	-0.95572D 02	0.18731D 07
	X(4)	-0.32227D C2	0.548930 03	-0.33061D 02	0.20473D 04
	X(5)	0.33605D C4	0.219030 07	-0.39987D 03	0.32497D 07
	X(6)	-0.14735D C2	0.221200 05	-0.36511D 01	0.22524D 04
61	X(1)	0.16281D 06	0.124110 07	-0.24593D 03	0.11079D 08
	X(2)	0.33210D C4	0.197440 05	0.28562D 02	0.77134D 04
	X(3)	0.86757D C3	0.966370 06	-0.99645D 02	0.19808D 07
	X(4)	-0.33098D 02	0.565300 03	-0.34180D 02	0.20743D 04
	X(5)	0.33533D C4	0.223450 07	-0.40328D 03	0.33675D 07
	X(6)	0.19784D C0	0.232730 01	-0.31713D 01	0.21494D 04
62	X(1)	0.16611D 06	0.134750 07	-0.24664D 03	0.11652D 08
	X(2)	0.33210D C4	0.197440 05	0.28922D 02	0.76911D 04
	X(3)	0.86403D 03	0.972280 06	-0.10311D 03	0.20871D 07
	X(4)	-0.34492D C2	0.561260 03	-0.33272D 02	0.20161D 04
	X(5)	0.33466D C4	0.225710 07	-0.40678D 03	0.34890D 07
	X(6)	-0.14733D 02	0.220520 05	-0.38269D 01	0.23075D 04
63	X(1)	0.16940D 06	0.148120 07	-0.24766D 03	0.12242D 08
	X(2)	0.33278D C4	0.238770 05	0.28909D 02	0.77265D 04
	X(3)	0.85919D C3	0.976670 06	-0.10624D 03	0.21991D 07
	X(4)	-0.35665D 02	0.565650 03	-0.33462D 02	0.21036D 04
	X(5)	0.33313D C4	0.231410 07	-0.41055D 03	0.36145D 07
	X(6)	-0.14734D 02	0.221180 05	-0.37280D 01	0.22796D 04
64	X(1)	0.17270D C6	0.164080 07	-0.24837D 03	0.12841D 08
	X(2)	0.33345D C4	0.279230 05	0.29193D 02	0.76757D 04
	X(3)	0.85260D 03	0.980070 06	-0.11046D 03	0.23195D 07
	X(4)	-0.36912D 02	0.584900 03	-0.34375D 02	0.22464D 04
	X(5)	0.33165D C4	0.234490 07	-0.41478D 03	0.37427D 07
	X(6)	-0.14740D 02	0.221610 05	-0.47317D 01	0.23533D 04
65	X(1)	0.17601D C6	0.185110 07	-0.25010D 03	0.13459D 08
	X(2)	0.33413D 04	0.318890 05	0.28293D 02	0.77395D 04
	X(3)	0.84522D C3	0.980480 06	-0.11451D 03	0.24441D 07
	X(4)	-0.38411D 02	0.597490 03	-0.34294D 02	0.22229D 04
	X(5)	0.32980D C4	0.237970 07	-0.41982D 03	0.38779D 07
	X(6)	-0.22202D 02	0.328840 05	-0.53383D 01	0.25318D 04
66	X(1)	0.17932D 06	0.212680 07	-0.25143D 03	0.14079D 08
	X(2)	0.33514D C4	0.376260 05	0.29022D 02	0.76263D 04
	X(3)	0.83616D 03	0.979550 06	-0.11838D 03	0.25739D 07
	X(4)	-0.40562D 02	0.600750 03	-0.34303D 02	0.23232D 04
	X(5)	0.32738D C4	0.245100 07	-0.42435D 03	0.40182D 07
	X(6)	-0.22198D 02	0.329040 05	-0.37221D 01	0.24253D 04
67	X(1)	0.18265D C6	0.245780 07	-0.25172D 03	0.14722D 08
	X(2)	0.33616D 04	0.431670 05	0.29508D 02	0.77986D 04
	X(3)	0.82439D C3	0.974380 06	-0.12341D 03	0.27070D 07
	X(4)	-0.42205D 02	0.654220 03	-0.34968D 02	0.23135D 04
	X(5)	0.32574D 04	0.251600 07	-0.42086D 03	0.41657D 07
	X(6)	-0.14729D C2	0.220550 05	-0.53466D 01	0.27566D 04
68	X(1)	0.18595D C6	0.285080 07	-0.25192D 03	0.15386D 08
	X(2)	0.33694D C4	0.467430 05	0.29812D 02	0.79400D 04
	X(3)	0.81129D C3	0.966630 06	-0.12881D 03	0.28448D 07
	X(4)	-0.43311D 02	0.682250 03	-0.35168D 02	0.23242D 04
	X(5)	0.32464D C4	0.255340 07	-0.43374D 03	0.43204D 07
	X(6)	-0.72629D C1	0.110480 05	-0.43575D 01	0.26427D 04
69	X(1)	0.18934D 06	0.332580 07	-0.25273D 03	0.16061D 08
	X(2)	0.33717D C4	0.484790 05	0.28824D 02	0.78388D 04
	X(3)	0.79704D C3	0.955050 06	-0.13352D 03	0.29972D 07
	X(4)	-0.45328D C2	0.714950 03	-0.34381D 02	0.23723D 04
	X(5)	0.32272D 04	0.262460 07	-0.43825D 03	0.44808D 07
	X(6)	-0.31202D C2	0.487430 05	-0.46729D 01	0.29113D 04

TIME		MEAN OF TRACK	VAR OF TRACK	MEAN OF ERROR	VAR OF ERROR
70	X(1)	0.19268D 06	0.39124D 07	-0.25353D 03	0.16762D 08
	X(2)	0.33859D 04	0.61769D 05	0.29463D 02	0.92198D 04
	X(3)	0.78034D 03	0.93831D 06	-0.13876D 03	0.31360D 07
	X(4)	-0.46956D 02	0.80620D 03	-0.35018D 02	0.24563D 04
	X(5)	0.31957D 04	0.27275D 07	-0.44295D 03	0.46489D 07
	X(6)	-0.23729D 02	0.37862D 05	-0.47166D 01	0.28388D 04
71	X(1)	0.19604D 06	0.46816D 07	-0.25414D 03	0.17485D 08
	X(2)	0.33968D 04	0.73028D 05	0.29344D 02	0.82616D 04
	X(3)	0.76152D 03	0.91718D 06	-0.14436D 03	0.32923D 07
	X(4)	-0.49414D 02	0.91931D 03	-0.34963D 02	0.25596D 04
	X(5)	0.31722D 04	0.28009D 07	-0.44842D 03	0.48239D 07
	X(6)	-0.31202D 02	0.48729D 05	-0.62388D 01	0.30807D 04
72	X(1)	0.19941D 06	0.56681D 07	-0.25527D 03	0.18224D 08
	X(2)	0.34110D 04	0.85566D 05	0.29259D 02	0.84165D 04
	X(3)	0.74092D 03	0.89129D 06	-0.14921D 03	0.34553D 07
	X(4)	-0.51226D 02	0.10717D 04	-0.34181D 02	0.26899D 04
	X(5)	0.31381D 04	0.29165D 07	-0.45507D 03	0.50113D 07
	X(6)	-0.37127D 02	0.54370D 05	-0.70607D 01	0.33077D 04
73	X(1)	0.20280D 06	0.68308D 07	-0.25528D 03	0.18984D 08
	X(2)	0.34279D 04	0.92671D 05	0.30895D 02	0.86818D 04
	X(3)	0.71777D 03	0.85765D 06	-0.15302D 03	0.36218D 07
	X(4)	-0.54962D 02	0.12942D 04	-0.33313D 02	0.26972D 04
	X(5)	0.31121D 04	0.30079D 07	-0.46095D 03	0.52061D 07
	X(6)	-0.14727D 02	0.22045D 05	-0.46902D 01	0.31195D 04
74	X(1)	0.20629D 06	0.81435D 07	-0.25497D 03	0.19768D 08
	X(2)	0.34346D 04	0.95347D 05	0.30149D 02	0.88255D 04
	X(3)	0.69199D 03	0.81562D 06	-0.15565D 03	0.37922D 07
	X(4)	-0.57599D 02	0.16496D 04	-0.32934D 02	0.26758D 04
	X(5)	0.30847D 04	0.30849D 07	-0.46483D 03	0.54037D 07
	X(6)	-0.40208D 02	0.64321D 05	-0.30780D 01	0.31255D 04
75	X(1)	0.20953D 06	0.83896D 07	-0.18589D 03	0.19712D 08
	X(2)	0.34461D 04	0.10599D 06	0.31924D 02	0.83129D 04
	X(3)	0.66569D 03	0.76987D 06	-0.15335D 03	0.39839D 07
	X(4)	-0.60841D 02	0.21368D 04	-0.32101D 02	0.26980D 04
	X(5)	0.30649D 04	0.32165D 07	-0.46204D 03	0.56289D 07
	X(6)	-0.88640D 01	0.16235D 05	-0.32560D 01	0.31932D 04
76	X(1)	0.21295D 06	0.10032D 08	-0.18376D 03	0.20491D 08
	X(2)	0.34501D 04	0.11357D 06	0.32707D 02	0.84054D 04
	X(3)	0.63329D 03	0.71338D 06	-0.15601D 03	0.41645D 07
	X(4)	-0.63526D 02	0.28317D 04	-0.32794D 02	0.28358D 04
	X(5)	0.30561D 04	0.32756D 07	-0.46487D 03	0.58407D 07
	X(6)	-0.88620D 01	0.16187D 05	-0.23966D 01	0.31407D 04
77	X(1)	0.21637D 06	0.11978D 08	-0.18055D 03	0.21283D 08
	X(2)	0.34540D 04	0.12104D 06	0.33235D 02	0.83097D 04
	X(3)	0.59733D 03	0.65229D 06	-0.15850D 03	0.43523D 07
	X(4)	-0.67983D 02	0.38071D 04	-0.31774D 02	0.28990D 04
	X(5)	0.30480D 04	0.33080D 07	-0.46726D 03	0.60589D 07
	X(6)	-0.73176D 01	0.11158D 05	-0.23946D 01	0.32032D 04
78	X(1)	0.21959D 06	0.11287D 08	-0.62979D 02	0.21427D 08
	X(2)	0.34370D 04	0.96726D 05	0.34421D 02	0.81284D 04
	X(3)	0.56556D 03	0.59255D 06	-0.14647D 03	0.44917D 07
	X(4)	-0.71833D 02	0.45884D 04	-0.31407D 02	0.29946D 04
	X(5)	0.30659D 04	0.33131D 07	-0.41775D 03	0.61981D 07
	X(6)	-0.50240D 02	0.81184D 05	-0.12707D 01	0.33552D 04
79	X(1)	0.22282D 06	0.10085D 08	-0.51828D 02	0.22348D 08
	X(2)	0.34458D 04	0.10661D 06	0.34455D 02	0.85058D 04
	X(3)	0.52412D 03	0.52995D 06	-0.14347D 03	0.46802D 07
	X(4)	-0.78346D 02	0.64994D 04	-0.31222D 02	0.29084D 04
	X(5)	0.30632D 04	0.34064D 07	-0.36954D 03	0.62382D 07
	X(6)	-0.43045D 02	0.71282D 05	-0.23221D 01	0.39080D 04

TIME		MEAN OF TRACK	VAR OF TRACK	MEAN OF ERROR	VAR OF ERROR
80	X(1)	0.225910 C6	0.827150 07	0.289120-01	0.614670 02
	X(2)	0.344140 C4	0.112710 06	0.406620 02	0.826020 04
	X(3)	0.489960 03	0.463200 06	0.297560 00	0.540870 02
	X(4)	-0.866640 02	0.633430 04	-0.297380 02	0.269230 04
	X(5)	0.307540 C4	0.346540 07	-0.311370 03	0.634900 07
	X(6)	-0.775660 01	0.118330 05	0.111180 01	0.339500 04
81	X(1)	0.229210 06	0.871880 07	0.105570 02	0.813860 04
	X(2)	0.342980 C4	0.956460 05	0.410650 02	0.802960 04
	X(3)	0.431010 C3	0.402370 06	0.927540 00	0.291620 04
	X(4)	-0.988650 02	0.102000 05	-0.287770 02	0.291270 04
	X(5)	0.311150 04	0.337850 07	-0.263000 03	0.644240 07
	X(6)	-0.951220 C1	0.174270 05	0.223690 01	0.352610 04
82	X(1)	0.232040 C6	0.485970 07	0.146710 02	0.300510 05
	X(2)	0.338560 C4	0.560730 05	0.376290 02	0.759020 04
	X(3)	0.374100 03	0.322690 06	0.478950 01	0.109900 05
	X(4)	-3.113660 03	0.135140 05	-0.280520 02	0.269340 04
	X(5)	0.326820 C4	0.287080 07	-0.401900 02	0.596670 07
	X(6)	-0.168430 C2	0.251380 05	0.772270 01	0.288680 04
83	X(1)	0.234810 C6	0.176510 07	0.124140 02	0.630940 05
	X(2)	0.334250 C4	0.340230 05	0.352010 02	0.707930 04
	X(3)	0.302750 C3	0.216080 06	0.763250 01	0.251960 05
	X(4)	-0.123740 03	0.201270 05	-0.279570 02	0.280620 04
	X(5)	0.346900 C4	0.253820 07	0.279380 03	0.537130 07
	X(6)	-0.907740 C1	0.137570 05	0.132440 02	0.272000 04
84	X(1)	0.237850 C6	0.944410 06	0.218920 02	0.100310 06
	X(2)	0.331240 C4	0.176540 05	0.360850 02	0.644570 04
	X(3)	0.209250 03	0.105700 06	0.102790 02	0.436540 05
	X(4)	-0.148000 C3	0.331500 05	-0.282840 02	0.266710 04
	X(5)	0.353120 C4	0.218470 07	0.405330 03	0.495660 07
	X(6)	-0.198010 02	0.294270 05	0.163680 02	0.259420 04
85	X(1)	0.239610 C6	0.166260 06	0.307370 03	0.609020 05
	X(2)	0.328050 C4	0.955470 03	0.922180 02	0.239820 04
	X(3)	0.741750 02	0.518400 05	-0.975580 02	0.494680 05
	X(4)	-0.168270 C3	0.673910 05	-0.522890 02	0.187150 04
	X(5)	0.237820 04	0.112580 07	-0.116540 04	0.282860 07
	X(6)	0.479920 00	0.240020 01	0.113220 02	0.139810 04

APPENDIX C

RESULTS OF ATIGS SIMULATION WITH POSITION RESET AND KALMAN FILTERING

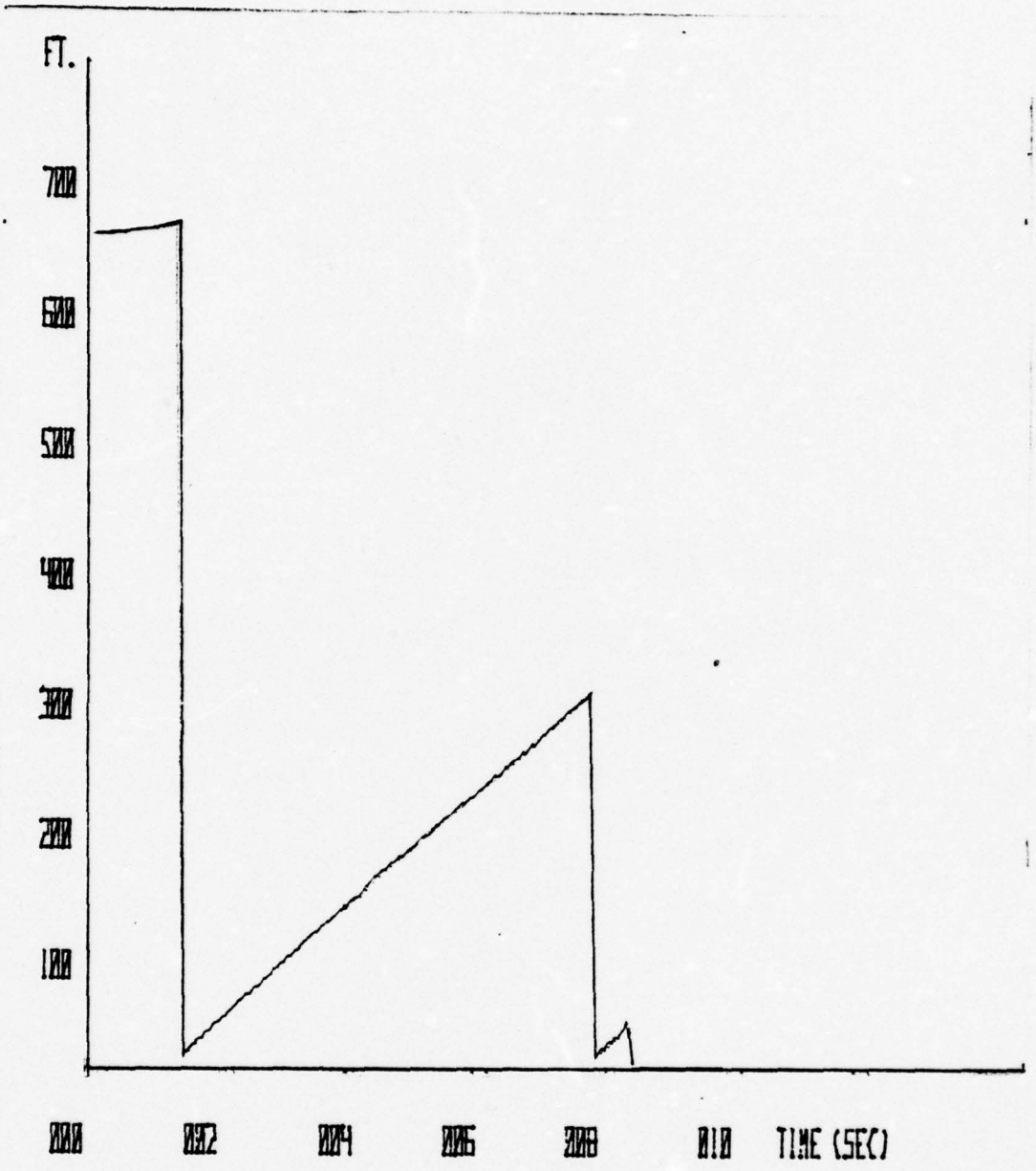


Figure 21 - SQRT OF DOWN RANGE VARIANCE (ATIGS WITH POSITION RESET AND KALMAN FILTERING)

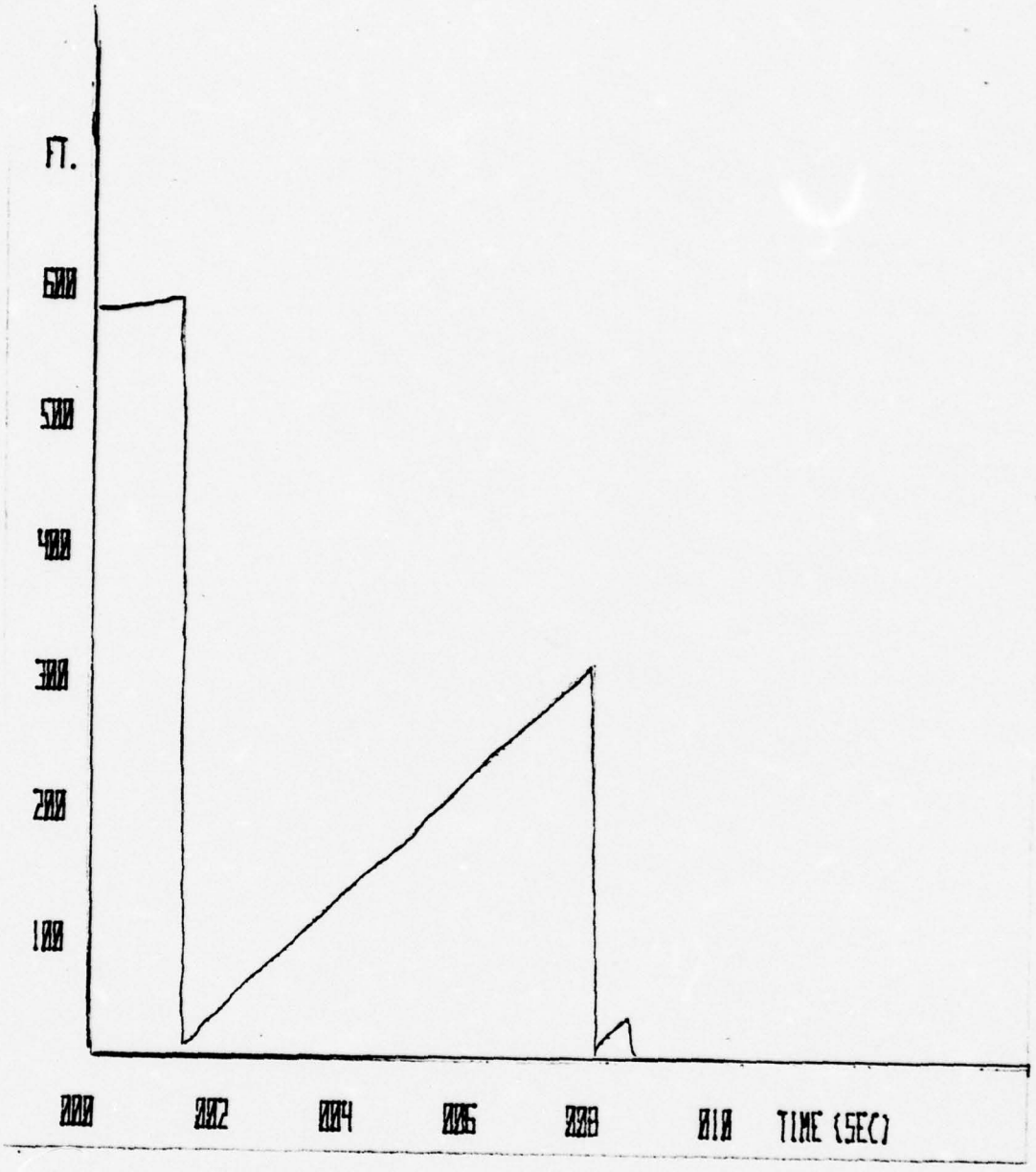


Figure 22 - SQRT OF CROSS RANGE VARIANCE (AIGS WITH POSITION RESET AND KALMAN FILTERING)

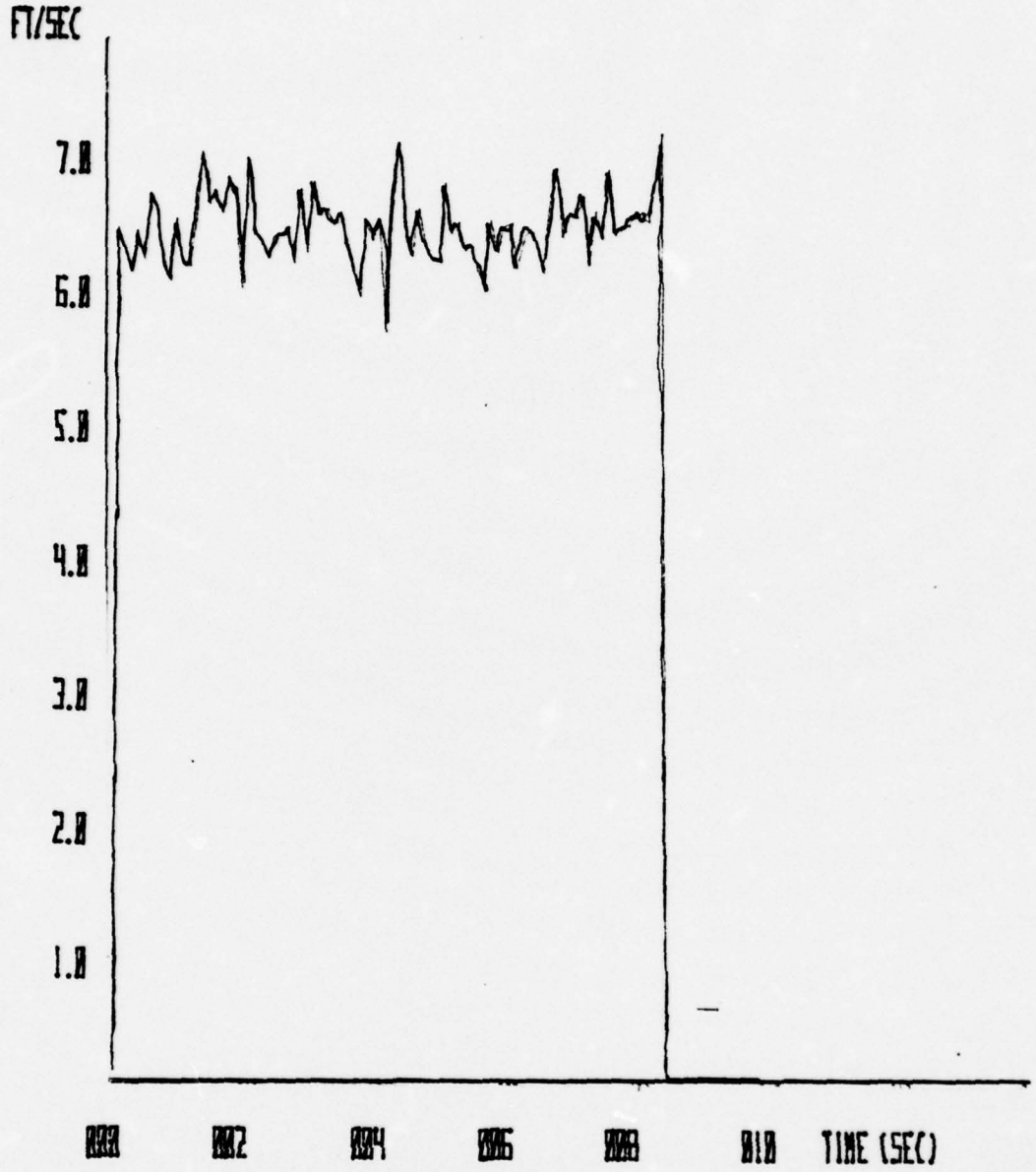


Figure 23 - SQRT• OF DOWN RANGE VELOCITY VARIANCE (ATIGS WITH POSITION RESET AND KALMAN FILTERING)

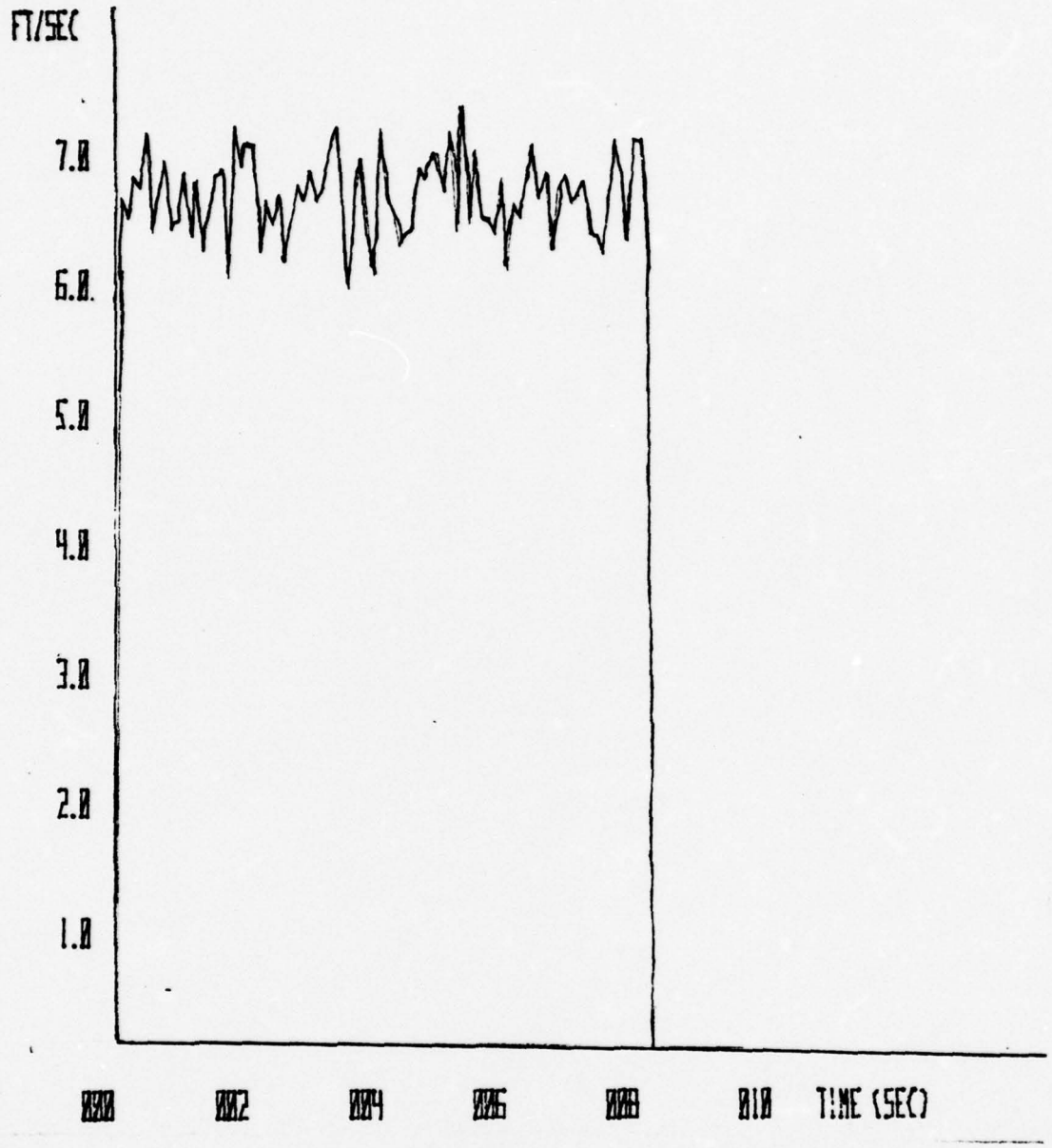


Figure 24 - SQRT• OF CROSS-RANGE VELOCITY VARIANCE (ATIGS WITH POSTION RESET AND KALMAN FILTERING)

APPENDIX D

RESULTS OF ATIGS SIMULATION WITH POSITION RESET AND KALMAN FILTERING AT X5 NOISE LEVEL

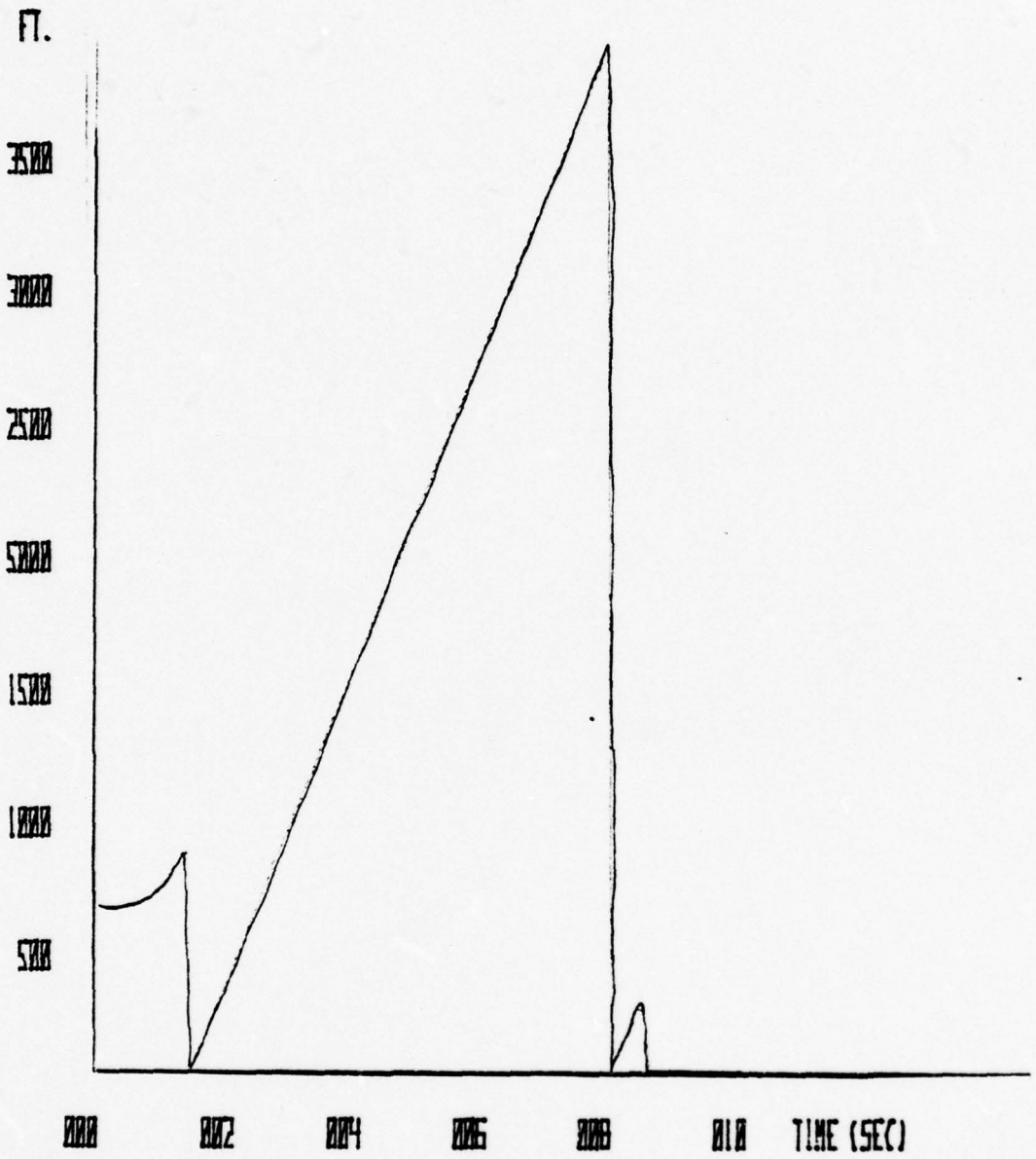


Figure 25 - SQRT• OF DOWN-RANGE VARIANCE ‹ATIGS WITH POSIT RESET AND KALMAN FILTERING AT X5 NOISE LEVEL›

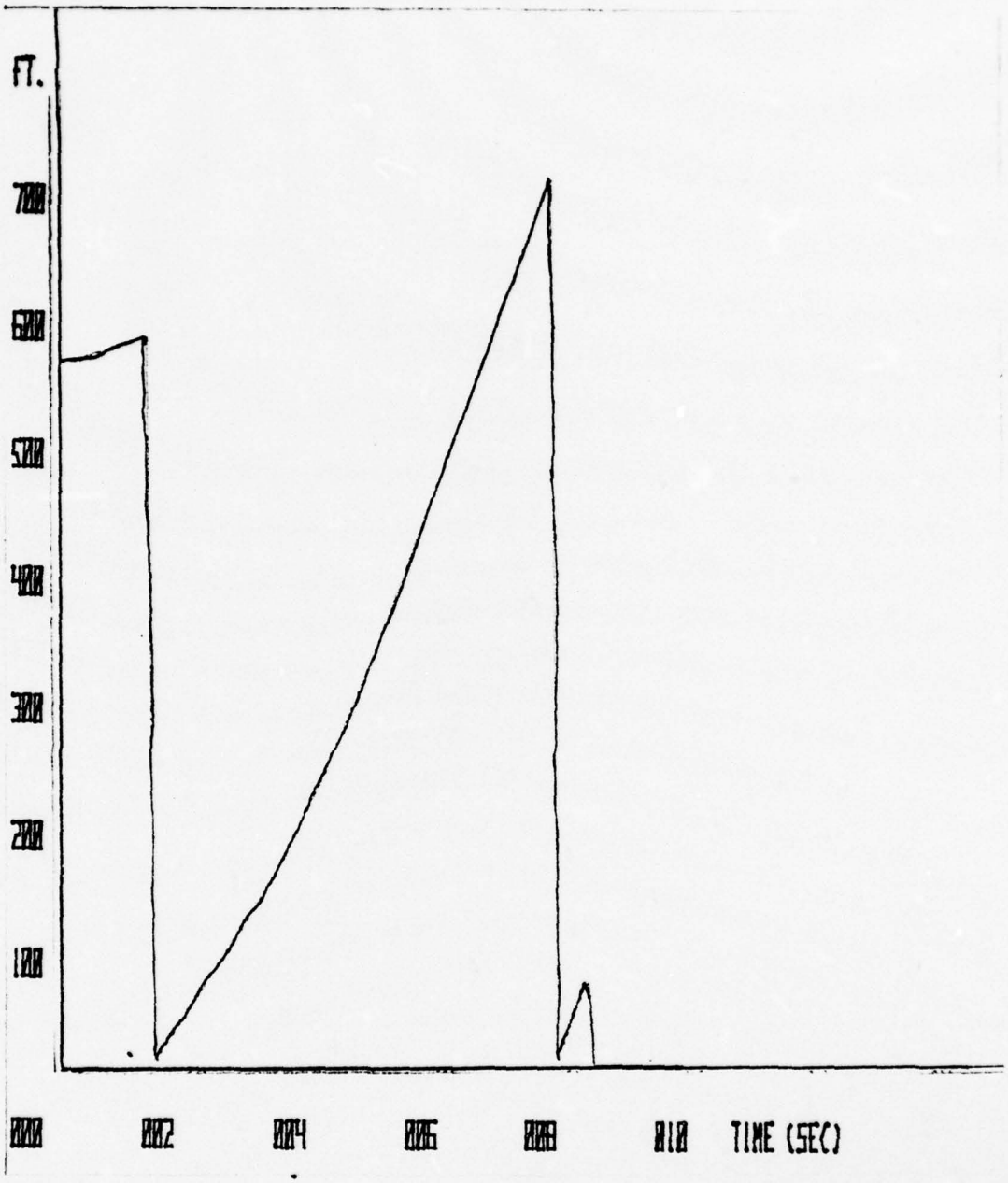


Figure 26 - SQRT• OF CROSS-RANGE VARIANCE ‹ATIGS WITH POSIT RESET AND KALMAN FILTERING AT THE X5 NOISE LEVEL›

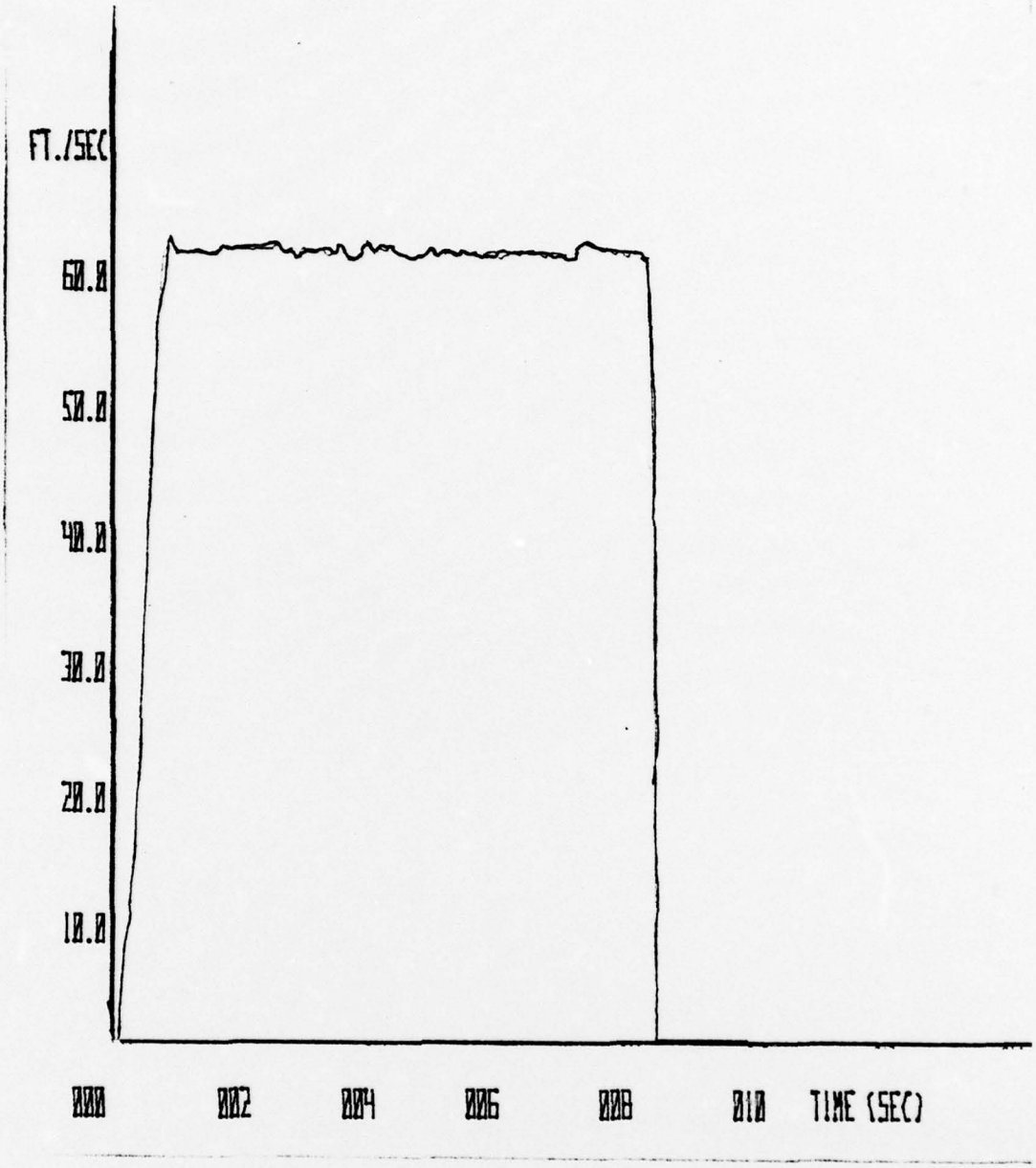


Figure 27 - SQRT• OF DOWN-RANGE VELOCITY VARIANCE (ATIGS WITH POSIT RESET AND KALMAN FILTERING AT THE X5 NOISE LEVEL)

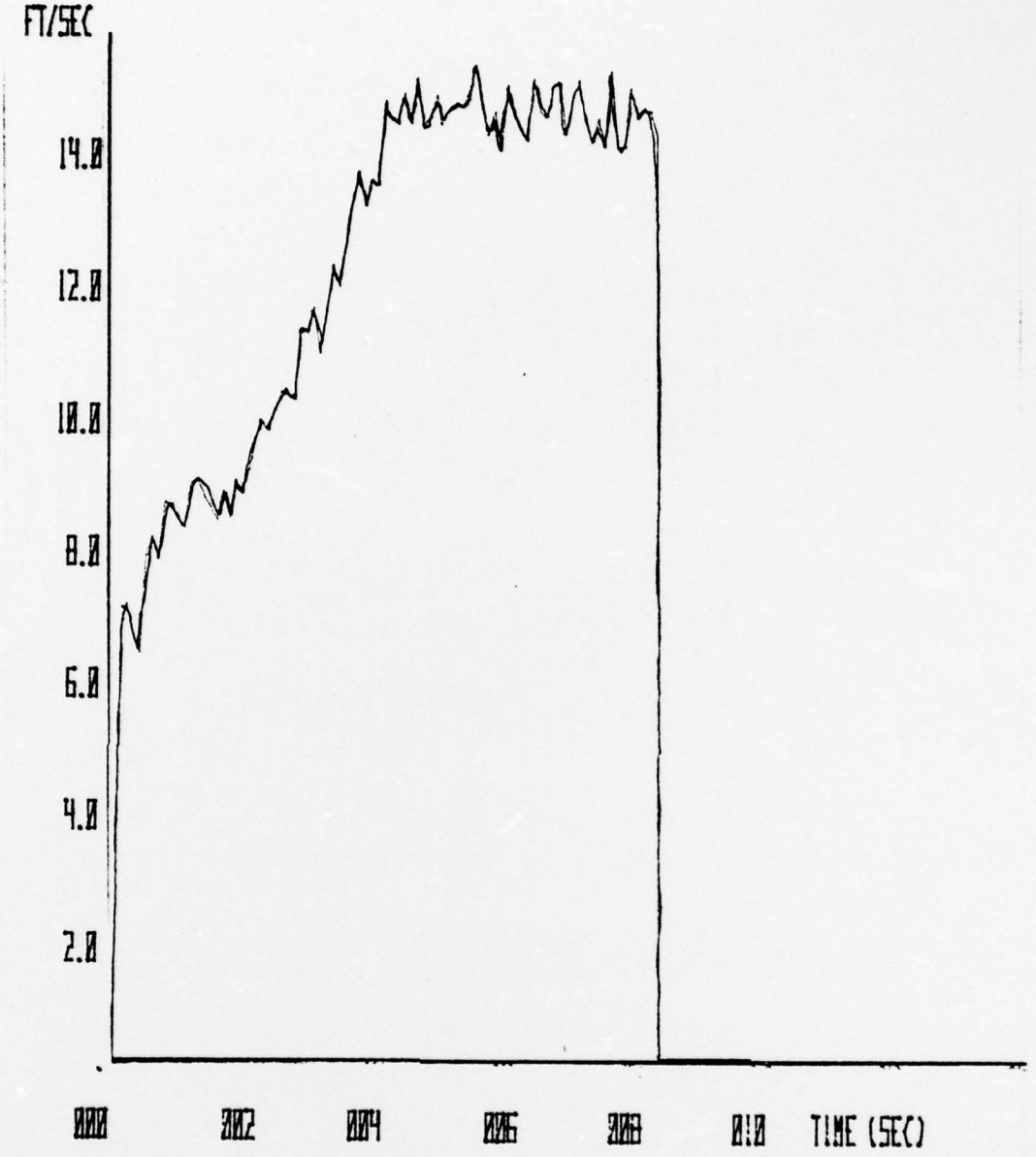


Figure 28 - SQRT• OF CROSS-RANGE VELOCITY VARIANCE ‹ATIGS WITH POSIT RESET AND KALMAN FILTERING AT THE X5 NOISE LEVEL›

APPENDIX E

PARTIAL LISTING OF SYMBOLS AND NOMENCLATURE OF SIMULATION PROGRAM

X(i,j)	i-th missile state at time j
THETA(i,j)	i-th angular state variable at time j
XI(i,j)	estimated i-th state at time j
THETA(i,j)	estimated i-th angular state at time j
XBAR(i,j)	mean of i-th state at time j
XBVAR(i,j)	variance of i-th state at time j
XMEAN(i,j)	error mean of i-th estimated state
XVAR(i,j)	error variance of i-th estimated state
A(j)	thrust acceleration at time j
AWXY AWYY AWYZ AWZZ	
AWXX DELVX DELVY	velocity changes due to angular rotation changes in wind components

BETA1	body referenced accelerations
BETA2	
BETA3	
YIX	inertial referenced accelerations
YIY	
YIZ	
XPOS	micrad sensed positions
YPOS	
G1	Kalman gains for filter
G2	
OMEGAX	extra states for Kalman filter
OMEGAY	
XINT	dummy variable for output
PSI	change in drift angle
XFIN(i)	final value of i-th state per track
NTERM	maximum time steps allowed
VXO	velocity of wind down-range
VYO	velocity of wind cross-range
IX	seed number for random number generators
DTHTAX	change in thetax
dthtxm	measured change in thetax
ZNI(j)	total tracks through time j

XIFIN(i) final value of estimated i-th state per track

XBFIN(i) mean of XFIN(i)

XIBFN(i) mean of XIFIN(i)

XBFV(i) variance of XFIN(i)

N number of gyros simulated

NNA number of accelerometers simulated

IENSB size of ensemble

SIGEO std.deviation of gyro bias

SIGW std.deviation of gyro random walk

SIGK std.deviation of gyro scale factor

SIGEG std.deviation of accelerometer bias

SIGKG std.deviation of accelerometer scale factor

SIGT std.deviation of initial condition on theta

APPENDIX F

SIMULATION PROGRAM

```
// EXEC FORTCLGP,REGION=180K
//FORT.SYSIN DD *
C
C       X(1,J)          THE ACTUAL POSITION ALONG THE DOWNRANGE
C                       AXIS
C       X(2,J)          THE AIRSPEED IN THE DOWNRANGE DIRECTION
C                       NOTE IT IS NOT THE ACTUAL VELOCITY
C       X(3,J)          CROSSRANGE POSITION
C       X(4,J)          CROSS RANGE AIRSPEED
C       XI(K,J)         THE INERTIALLY COMPUTED STATES ALONG THE
C                       DOWN-RANGE/CROSS-RANGE AXIS
C       XBAR            THE MATRIX OF MEAN VALUES OF THE MONTE-
C                       CARLO GENERATED TRACKS OF THE X MATRIX
C       XMEAN           THE MATRIX OF MONTECARLO GENERATED
C                       MEANS OF THE XI MATRIX
C

      DIMENSION NA(3),EOG(3),WG(3),PSI(150),GAM(3),ZNI(150),
      DIMENSION THETA(3,150),THETAI(3,150),G(3),EO(3),ZK(3),
      DIMENSION A(150),XFIN(6),XIFIN(6),YP(150),XINT(150),XP
      REAL*8 XBFIN(6),XIBFN(6),XBFV(6),XIBFV(6),XM1,XM3
      REAL*8 XBAR(6,155),XBVAR(6,155),X(6,155),XI(6,155)
      REAL*8 XMEAN(6,150),XVAR(6,150)
      DIMENSION ERR(6)
C
C     THIS SECTION READS IN THE VARIOUS NECESSARY SPECIFICAT
C       "N "  THE NUMBER OF GYROS INVOLVED PER SIMULATION
C       "NNA" THE NUMBER OF ACCELEROMETERS PER SIMULATION
C       "IENSB"  THE NUMBER OF THE ENSEMBLE
C
      READ(5,300)N,NNA,IENSB
C       "SIGEO" IS THE DIVIATION OF THE GYRO BIAS
C       "SIGW"  IS THE DEVIATION OF THE RANDOM WALK CONSTANT
C               FOR THE GYROS
C       "SIGK"  IS THE DEVIATION OF THE SCALE FACTOR (GYRO)
C       "SIGEG" IS THE DEVIATION OF THE ACCEL. BIAS
C       "SIGKG" IS THE DEVIATION OF THE ACCEL SCALE FACTOR
C       "SIGT"  IS THE DEVIATION OF THE INITIAL CONDITION
C               ON THETA
C
      READ(5,310)SIGEO,SIGW,SIGK,SIGEG,SIGKG,SIGT
C
C       "SIGMIC" IS THE DEVIATION OF THE POSITION MEASUREMEN
C
      READ(5,310)SIGMIC
      WRITE(6,420)N,NNA,IENSB
      WRITE(6,440)SIGEO,SIGW,SIGK,SIGEG,SIGKG,SIGT
      WRITE(6,500)SIGMIC
      CALL OVFLOW
      INDX1=1
      INDX2=1
      INDX3=1
      NDEBG=1
      YP=0.0
      NTERM=100
      IA=1
      IB=3
      IX=11111
      VXO=30.0
      VYO=30.0
      DO 02 J=1,NTERM
      DO 01 K=1,6
      XMEAN(K,J)=0.0
      XVAR(K,J)=0.0
      XBAR(K,J)=0.0
      XBVAR(K,J)=0.0
   01 CONTINUE
   02 CONTINUE
C
C     GENERATE THE THRUST ACCELERATION PROFILE
C
      A(1)=2.0
```

```
      A(2)=6.0
      A(3)=10.0
      A(4)=14.0
      A(5)=14.0
      A(6)=10.0
      A(7)=6.0
      A(8)=2.0
      A(9)=-1.0
      DO 03 I=10,150
   03 A(I)=0.0
      DO 04 J=1,6
      XBFIN(J)=0.0
      XIBFN(J)=0.0
      XBFV(J)=0.0
   04 XIBFV(J)=0.0
      DO 05 I=1,NTERM
      ZNI(I)=0.0
   05 A(I)=A(I)*32.2
C
CC    START THE MONTECARLO SIMULATION
C
      DO 200 NI=1,IENSB
      TIMEX=1.0
      OMEGAY=0.0
      OMEGAX=0.0
C
CC    THE PURPOSE OF THIS SECTION IS TO COMPUTE THE INITIAL
CC    CONDITION FOR THE ACTUAL TRACK OF THE MISSILE.
C
      CALL RANDU(IX,IY,XFL)
      IX=IY
      X(1,1)=XFL*2240.0-1120.0
   10 CALL RANDU(IX,IY,YFL)
      IX=IY
      IF((XFL**2+YFL**2).GT.1.0)GO TO 10
      X(2,1)=700.0
      X(3,1)=YFL*2240.0-1120.0
      X(4,1)=0.0
      X(5,1)=35000.
      X(6,1)=0.0
C     GENERATE THE INITIAL CONDITIONS ON THETA
C
      CALL SNORM(IX,T,N)
      DO 11 J=1,N
      THETAI(J,1)=0.0
   11 THETA(J,1)=T(J)*SIGT
C
CC    THE INITIAL SETTING OF THE INERTIAL NAVIGATOR IS
CC    ZERO POSITION IN DOWN-RANGE AND CROSS-RANGE AND THE
CC    ACTUAL VELOCITY AND ALTITUDE
C
      XI(1,1)=0.0
      XI(2,1)=670.0
      XI(3,1)=0.0
      XI(4,1)=30.0
      XI(5,1)=35000.0
      XI(6,1)=0.0
      DTHTAX=0.0
      DTHTAY=0.0
      DTHTAZ=0.0
C
CC    THE PURPOSE OF THIS SECTION IS TO PRODUCE THE ACTUAL
CC    TRACK OF THE SIMULATED MISSILE FOR COMPARISON WITH
CC    OTHER ESTIMATES OF POSITION
CC    WIND EFFECTS ARE COMPUTED FIRST
C
C     THE INITIAL VALUE OF C IS ALWAYS ZERO
      C=0.0
C
CC    THE VARIOUS RANDOM INPUTS FOR MEASUREMENT DEVICES ARE
CC    GENERATED
C
```

```
C       GENERATE GYRO BIAS  "EO"
        CALL SNORM(IX,EO,N)
        DO 18 KN=1,N
   18 EO(KN)=SIGEO*EO(KN)
C
C
C       GENERATE THE RANDOM SCALE FACTOR "ZK"
        CALL SNORM(IX,ZK,N)
        DO 19 KN=1,N
   19 ZK(KN)=SIGK*ZK(KN)
C
CC      GENERATE RANDOM INPUTS TO ACCELEROMETER PACKAGE
C
C       GENERATE ACCELEROMETER BIAS "EOG"
        CALL SNORM(IX,EOG,NNA)
        DO 20 KN=1,NNA
   20 EOG(KN)=SIGEG*EOG(KN)
C
C       GENERATE ACCELEROMETER SCALE FACTOR "KG"
        CALL SNORM(IX,WG,NNA)
        DO 30 KN=1,NNA
   30 WG(KN)=SIGKG*WG(KN)
        DO 100 J=1,NTERM
        ZNI(J)=ZNI(J)+1.0
        JP1=J+1
        CALL RANDU(IX,IY,VY)
        IX=IY
        CALL RANDU(IX,IY,VX)
        VY=30+(VY*16.67-8.33)
        VX=30+(VX*16.67-8.33)
        IX=IY
        AWXX=X(6,J)*DTHTAX
        AWXY=-X(4,J)*DTHTAY
        AWYY=X(2,J)*DTHTAY
        AWYZ=X(6,J)*DTHTAZ
        AWZZ=-X(4,J)*DTHTAZ
        AWZX=-X(2,J)*DTHTAX
        AY=A(J)*THETA(2,J)
        THETA(1,JP1)=THETA(1,J)+DTHTAX
        THETA(2,JP1)=THETA(2,J)+DTHTAY
        THETA(3,JP1)=THETA(3,J)+DTHTAZ
        X(1,JP1)=X(1,J)+X(2,J)-VX+.5*(A(J)+AWXX+AWXY)
        X(2,JP1)=X(2,J)+A(J)+AWXX+AWXY
        X(3,JP1)=X(3,J)+X(4,J)+.5*(AY+AWYZ+AWYY)+VY
        X(4,JP1)=X(4,J)+AY+AWYZ+AWYY
        X(5,JP1)=X(5,J)+X(6,J)+.5*(AWZX+AWZZ)
        X(6,JP1)=X(6,J)+AWZX+AWZZ
C
CC      GAM IS THE NOISE INPUT FOR EACH ACCELEROMETER
C
        GAM(1)=EOG(1)+WG(1)*A(J)
        GAM(2)=EOG(2)+WG(2)*C
        GAM(3)=EOG(3)
C
CC      GENERATE THE BIAS TERM DUE TO RANDOM WALK
C
        CALL SNORM(IX,G,N)
        DO 50 JI=1,N
   50 G(JI)=SIGW*G(JI)
C
CC      PSI IS THE CHANGE IN THE ANGLE BETWEEN THE COMPUTED
CC      COORDINATE PLANE AND THE ACTUAL COORDINATE PLANE
C
        DO 51 JI=1,N
   51 PSI(JI)=EO(JI)+G(JI)
        DELVX=VXO-VX
        DELVY=VYO-VY
C
CC      COMPUTE INERTIAL POSITION
C
        DTHTXM=DTHTAX+PSI(1)+DTHTAX*ZK(1)
        DTHTYM=DTHTAY+PSI(2)+DTHTAY*ZK(2)
```

```
      DTHTZM=DTHTAZ+PSI(3)+DTHTAZ*ZK(3)
C
CCCC  THE GAINS G1 AND G2 ARE THE KALMAN GAINS GENERATED AS
CCCC  A FUNCTION OF TIME
C
      G1=1.0-2.0/(TIMEX+1.0)
      G2=1.0/(TIMEX+1.0)
C
CCCC  THE COMMANDED HEADING CHANGE IS SUBTRACTED FROM THE
CCCC  OESERVED HEADING CHANGE
C
      DLYJ=DTHTYM-DTHTAY
      DLXJ=DTHTXM-DTHTAX
C
CCC   THE FILTER UPDATE EQUATIONS FOLLOW
C
      THETAI(1,J)=THETAI(1,J)+G1*(DLXJ-OMEGAX)
      OMEGAX=OMEGAX+G2*(DLXJ-OMEGAX)
      THETAI(2,J)=THETAI(2,J)+G1*(DLYJ-OMEGAY)
      OMEGAY=OMEGAY+G2*(DLYJ-OMEGAY)
C
CCCC  THE FILTERED UPDATES ARE USED TO PREDICT THE NEXT
CCCC  STATE IN THE NAVIGATOR
C
      THETAI(1,JP1)=THETAI(1,J)+DTHTAX+OMEGAX
      THETAI(2,JP1)=THETAI(2,J)+DTHTAY+OMEGAY
      THETAI(3,JP1)=THETAI(3,J)+DTHTZM
      TIMEX=TIMEX+1.0
C
CCCCC SENSED ACCELEROMETER INPUTS IN THE BODY AXIS FRAME
C
CCCCC IN THE X(BODY FRAME) DIRECTION
C
      BETA1=A(J)-DELVX+DELVY*THETA(2,J)+GAM(1)
C
CCCC  IN THE Y(BODY FRAME) DIRECTION
C
      BETA2=DELVX*THETA(2,J)+DELVY+GAM(2)
C
CCC   IN THE VERTICAL(BODY FRAME) DIRECTION
C
      BETA3=0.0
C
CCCCC
CCCCC THE PURPOSE OF THIS SECTION IS TO GENERATE THE
CCCCC INERTIAL ESTIMATES OF POSITION BASED ON A PURE
CCCCC INERTIAL COMPUTATION
CCCCC
CCCCC AWXXI IS THE ACCELERATIONS DUE TO HEADING CHANGE
CCCCC AFFECTING THE X DIRECTION FROM THE ANGLE CHANGE THETA
CCCC  X.  SIMILARLY AWXYI IS THE ACCELERATIONS  AFFECTING
      THE X DIRECTION DUE TO THETAY
      AWXXI=XI(6,J)*DTHTAX
      AWXYI=-XI(4,J)*DTHTAY
      AWYYI=XI(2,J)*DTHTAY
      AWYZI=XI(6,J)*DTHTZM
      AWZZI=-XI(4,J)*DTHTZM
      AWZXI=-XI(2,J)*DTHTAX
      DTHTXM=0.0
      DTHTYM=0.0
      DTHTZM=0.0
      DTHTAX=0.0
      DTHTAY=0.0
      DTHTAZ=0.0
C
CCCC  SENSED ACCELERATIONS IN THE BODY FRAME ARE CONVERTED
CCCC  TO THE INERTIAL FRAME.
C
      YIX=BETA1-BETA2*THETAI(2,J)
      YIY=BETA1*THETAI(2,J)+BETA2
      IF(J.EQ.15)GO TO 48
      IF (J.NE.80)GO TO 49
```

```
C
CC      GENERATE THE RANDOM ERROR IN THE POSITION MEASUREMENT
C
   48 CALL SNORM(IX,XPOS,1)
      XPOS=XPOS*SIGMIC
      CALL SNORM(IX,YPOS,1)
      YPOS=YPOS*SIGMIC
      XI(1,J)=XPOS+X(1,J)
      XI(3,J)=YPOS+X(3,J)
   49 CONTINUE
      XI(6,J)=X(6,J)
C
CC      COMPUTE THE INERTIAL ESTIMATES OF POSITION
CC      AND VELOCITY
C
      XI(1,JP1)=XI(1,J)+XI(2,J)+.5*(YIX+AWXXI+AWXYI)
      XI(2,JP1)=XI(2,J)+YIX+AWXXI+AWXYI
      XI(3,JP1)=XI(3,J)+XI(4,J)+.5*(YIY+AWYYI+AWYZI)
      XI(4,JP1)=XI(4,J)+YIY+AWYYI+AWYZI
      XI(5,JP1)=XI(5,J)+XI(6,J)+.5*(AWZZI+AWZXI)
      XI(6,JP1)=XI(6,J)+AWZZI+AWZXI
      VXO=VX
      VYO=VY
      IF (XI(1,JP1).GE.40000.)DTHTAX=.4538-THETA(1,JP1)
      IF (XI(5,JP1).LE.5000.)DTHTAX=-THETA(1,JP1)
      IF(DABS(XI(3,JP1)).LE.150.)GO TO 88
      REM=240000.-XI(1,J)
      IF(REM.LE..001)GO TO 88
      CONTRL=X(3,J)/REM
      DTHTAY=-CONTRL-THETAI(2,JP1)
   88 CONTINUE
C
CC      THIS SECTION GENERATES THE REQUIRED STATISTICS
C
      DO 89 K=1,6
      ERR(K)=X(K,J)-XI(K,J)
      XMEAN(K,J)=XMEAN(K,J)+ERR(K)
      XBAR(K,J)=XBAR(K,J)+X(K,J)
      XBVAR(K,J)=XBVAR(K,J)+X(K,J)**2
      IF(ABS(ERR(K)).LE.0.001)GO TO 89
      XVAR(K,J)=XVAR(K,J)+ERR(K)**2
   89 CONTINUE
      IF(XI(1,J+1).GT.240000.) GO TO 90
      GO TO 100
   90 DO 91 I=1,6
   91 XFIN(I)=X(I,J)
      XIFIN(1)=XI(1,J)
      XIFIN(3)=XI(3,J)
      XIFIN(5)=XI(5,J)
      GO TO 110
  100 CONTINUE
      DO 101 I=1,6
  101 XFIN(I)=X(I,150)
      XIFIN(1)=XI(1,150)
      XIFIN(3)=XI(3,150)
      XIFIN(5)=XI(5,150)
C
CC      THIS SECTION COMPUTES THE FINAL VALUE STATISTICS
C
  110 DO 120 J=1,6
  120 XBFIN(J)=XBFIN(J)+XFIN(J)
      XIBFN(1)=XIBFN(1)+XIFIN(1)
      XIBFN(3)=XIBFN(3)+XIFIN(3)
      XIBFN(5)=XIBFN(5)+XIFIN(5)
      XERR1=XFIN(1)-XIFIN(1)
      XERR3=XFIN(3)-XIFIN(3)
      IF(XERR1.LE..001)GO TO 121
      XBFV(1)=XBFV(1)+XERR1**2
  121 IF(XERR3.LE..001)GO TO 200
      XBFV(3)=XBFV(3)+XERR3**2
  200 CONTINUE
C
```

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