AD-A039 471 NAVAL UNDERWATER SYSTEMS CENTER NEW LONDON CONN NEW --ETC F/G 12/1

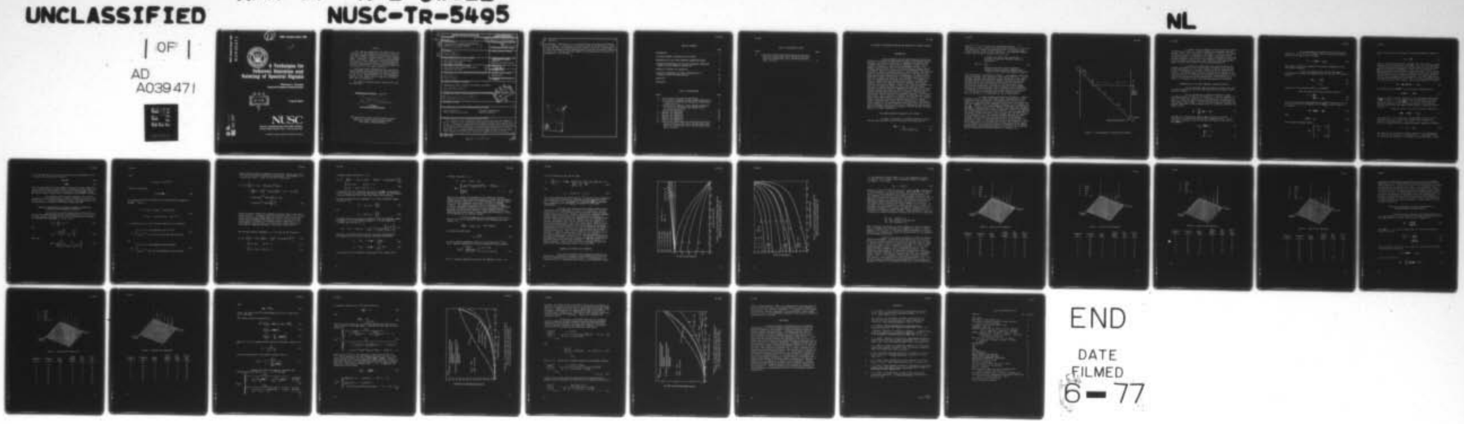
A TECHNIQUE FOR COHERENT DETECTION AND RELATING OF SPECTRAL SIG--ETC(U)

APR 77 N L OWSLEY

UNCLASSIFIED NUSC-TR-5495 NL

| OF |

AD A039471



END

DATE FILMED

6-77

NUSC Technical Report 5495

ADA039471

(12)

NUSC Technical Report 5495

A Technique for Coherent Detection and Relating of Spectral Signals

Norman L. Owsley
Special Projects Department

7 April 1977

NUSC
NAVAL UNDERWATER SYSTEMS CENTER
Newport, Rhode Island • New London, Connecticut

PREFACE

This work was accomplished under NUSC Project No. A-654-00, and Navy Subproject and Task No. SF 11 121 110-15806, "Adaptive Space-Time Processing for Narrowband Passive Sonar," Program Manager, J. Neely (NAVSEA 06H1). Additional support was provided under NUSC Project No. A-654-01, "Automatic Detection and Automatic Classification for Towed Arrays," Program Manager, R. Cockerill (NAVSEA 06H2). The Principal Investigator for both projects is N. L. Owsley (Code 315).

The author wishes to thank Edward Eby, Dave Kennedy, Ronald Kneipfer, and Joseph Wolcin for their discussions pertaining to the work reported herein. Further gratitude is due to Mr. Robert Bettigole for his programming efforts on this project while a cooperative student at NUSC from Yale University.

The Technical Reviewer for this report was Dr. Ding Lee (Code 222).

REVIEWED AND APPROVED: 7 April 1977

R. W. Hasse
Head: Special Projects Department

The author of this report is located at the New London Laboratory, Naval Underwater Systems Center, New London, Connecticut 06320.

(14)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER
NUSC TR-5495

2. GOVT ACCESSION NO.

3. RECIPIENT'S CATALOG NUMBER
(9) Technical rept.

4. TITLE (and Subtitle)
(6) A TECHNIQUE FOR COHERENT DETECTION AND RELATING OF SPECTRAL SIGNALS.

5. TYPE OF REPORT & PERIOD COVERED

6. PERFORMING ORG. REPORT NUMBER

7. AUTHOR(s)
(10) Norman L. Owsley

8. CONTRACT OR GRANT NUMBER(s)

9. PERFORMING ORGANIZATION NAME AND ADDRESS
Naval Underwater Systems Center
New London Laboratory
New London, CT 06320

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
A-654-00
Task No. SF 11 121 110-15806

11. CONTROLLING OFFICE NAME AND ADDRESS
Naval Sea Systems Command (Code 06H2)
Washington, DC 20362

12. REPORT DATE
(11) 7 April 1977

13. NUMBER OF PAGES
31

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)
(12) 35p.

15. SECURITY CLASS. (of this report)
UNCLASSIFIED

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)
Approved for public release; distribution unlimited.
(16) F11121

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)
(17) SF11121110

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)
Coherent detection
Cross frequency correlation
Orthogonal decomposition
Spectral signal set

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)
An important class of spectral signals is that for which the components within a signal set are characterized by complex envelopes which exhibit correlation. A technique is proposed for extraction of such a signal set based on an orthogonal decomposition of the cross (discrete) frequency correlation matrix describing the set in the presence of additive uncorrelated noise. Certain advantages of the technique relative to those based on Fourier analysis with either smoothed estimated power spectrum or minimum variance processing
next page

DD FORM 1 JAN 73 1473

405918

4/B

20. Abstract

cont →

are presented. Specifically, it is illustrated how the detection performance of the technique is established by the total energy in the spectral set rather than by the levels of individual spectral elements in the set. Furthermore, it is shown how minimum variance spectrum analysis actually can suppress components in such a set. Finally, simulated examples are presented which illustrate the performance of the technique.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF ILLUSTRATIONS (CONT)

A TECHNIQUE FOR COHERENT DETECTION AND RELATING OF SPECTRAL SIGNALS

INTRODUCTION

In many frequency and wavenumber analysis problems the objective is to detect multiple spectral terms in simultaneous fashion. Typically, this detection objective is implemented by power spectral estimation followed by a search routine designed to relate detected components according to some a priori determined structure. For example, spectral terms could be related according to harmonic family in frequency analysis or a plane wave arrival structure with possible multipath in spatial wavenumber analysis. Until recently, the detection aspect of this procedure has been performed as if the spectral terms being detected were characterized by random phase. As such, phase information is ignored and detections are performed sequentially on the basis of estimated power spectrum. More recently [1,2,3] sequential adaptive schemes have been considered which exploit stationary phase estimators which yield coherent detection algorithms that exhibit improved detection performance. However, very little consideration has been given to exploiting potential coherence between the complex envelopes of multiple spectral components with the objective of improving both detection and relation. Intuitively, one feels that the best detection scheme should have implicit an estimation algorithm for simultaneous coherent detection and component relating. In the following sections, such an approach to coherent spectral component relation which occurs simultaneously with detection is presented. The proposed scheme is based on a principal component analysis of the random Fourier transformed data vector [4] and more specifically an orthogonal vector analysis of the cross-frequency correlation (CFC) matrix.[5] This analysis decomposes the CFC matrix into its eigenvectors and indicates that the frequency information on harmonic sets with coherent envelopes appears in a single eigenvector.

THE CROSS-FREQUENCY CORRELATION (CFC) MATRIX

Consider the frequency-to-frequency Hermitian correlation matrix $\mathbf{R}$ with term in the i^{th} row and j^{th} column given by

$$[\mathbf{R}]_{ij} = r_{ij} \tag{1}$$

$$= \overline{\chi(\nu_i, p)\chi^*(\nu_j, p)}, \tag{2}$$

where $\overline{(\qquad)}$ is the statistical expectation operator, $(\ )^*$ indicates the matrix complex conjugate transpose and $X(\nu, p)$ is the p-th time sample of a complex envelope function for a narrowband random process centered at the discrete frequency ν. Two important examples of this type of complex envelope time series are

$$X(\nu, p) = \begin{cases} \text{output time series from a phase rate normalized discrete frequency analyzer at frequency } \nu^{6} \\ \text{or} \\ \text{output time series from a beamformer with steering at spatial frequency (wave number) } \nu \text{ for a particular temporal frequency.} \end{cases} \quad (3)$$

The CFC matrix **R** is illustrated in figure 1. The diagonal of **R** is the discrete frequency power spectrum of the random process $X(\nu)$. In most frequency analysis based detection systems, estimates of this power spectrum obtained with appropriate temporal averaging and spectral smoothing is the only information used to detect the presence of possibly multiple spectral components in the interval $[\nu_0, \nu_{N-1}]$. An extraction scheme which uses only the diagonal of **R** is commonly referred to as being incoherent since it ignores both the possibility of nonrandom (or at least short term stable) phase rate in the term $X(\nu_i, p)$ and possible coherence between the "complex envelopes" $X(\nu_i, p)$ and $X(\nu_j, p)$ at frequencies ν_i and ν_j respectively.

In terms of improving detection performance, a logical question to ask is, "What off-diagonal terms in **R** might exist which constitute useful information?" In answering this, consider as an example the spectrum analysis problem where the signal of interest consists of a harmonic set with fundamental frequency ν_i with harmonics at $\{m\nu_i\}_{m=1}^{M}$. Clearly, if the signal component of the phase rate normalized envelopes at each frequency in the set have stable amplitude and phase, the complex envelopes would exhibit correlation. In fact, the amplitude of the off-diagonal term $r_{i,mi}$ in the i-th row would be proportional to the product of the amplitudes of the fundamental and the m-th harmonic. If the fundamental at ν_i has completely random amplitude and phase envelope, then it might still be characteristic of the harmonic envelopes to exhibit random amplitude and phase variations that are proportional, that is, correlated with those of the fundamental. Thus, while there might be no justification for phase detecting any element in the harmonic set individually because of randomness in the complex envelope phase, there might certainly be justification for exploiting finite crosscorrelation between elements of a harmonic set.

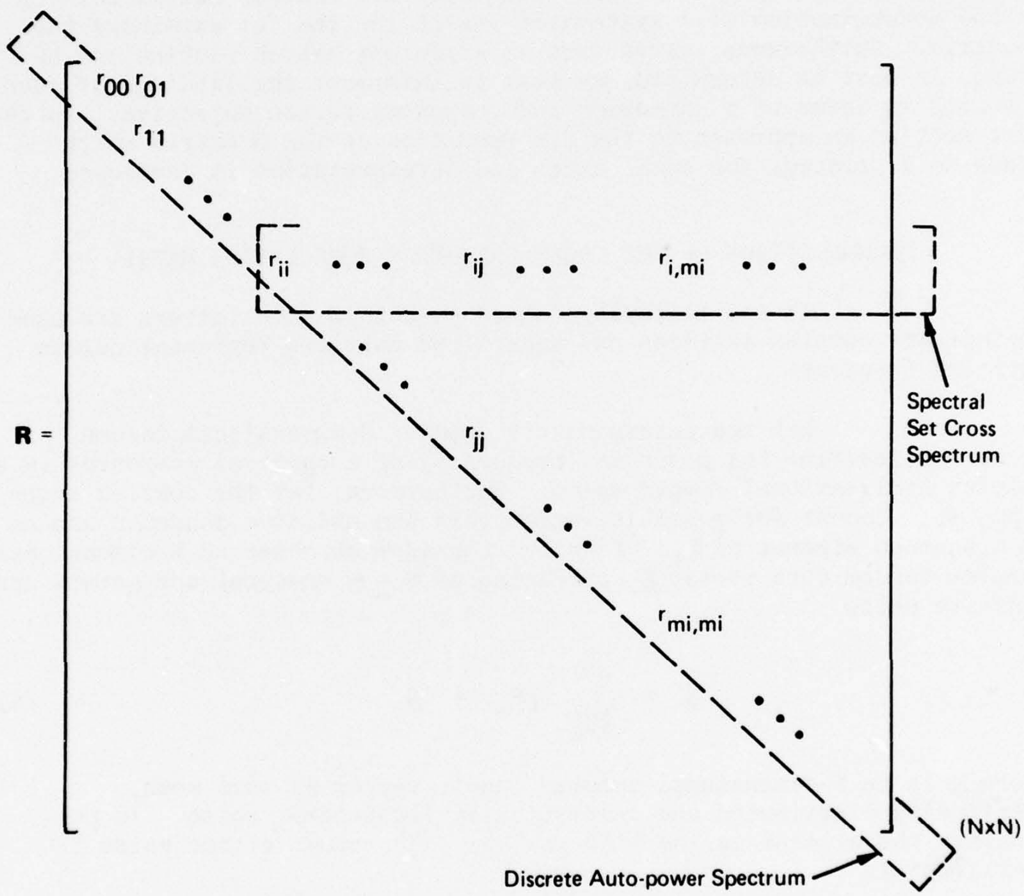


Figure 1. Cross-Frequency Correlation (CFC) Matrix

A somewhat similar argument to the above could be applied to detection of signals in a Doppler compensated wavenumber spectrum. However, rather than a mechanism as structured as a harmonic set leading to envelope correlation, multipath, for example, might well provide spectral components at different arrival angles (wavenumbers) which have not been completely "decorrelated" by propagation through the medium. As such, the complex envelopes would lead to nonzero cross-(spatial) frequency terms off the diagonal in the corresponding **R** matrix.

Accepting the premise that there might be useful off-diagonal information in the cross-frequency correlation matrix **R** leads to the determination of a systematic search routine for examining the R-matrix. Furthermore, given that an efficient search routine can be found, it must be determined how best to interpret the findings of such a search in terms of a detection and component relate objective. In the next section an approach to the decomposition of the **R** matrix which leads to a strategy for both search and interpretation is developed.

DECOMPOSITION OF THE CROSS-FREQUENCY CORRELATION MATRIX

In the following, upper case bold face letters are used to indicate complex matrices and underlined matrices represent column matrices (vectors).

Let the deterministic complex N-dimensional column vector $\underline{\mathbf{E}}_k$ describe the position (frequency) of a spectral component in a complex N-dimensional sample space. Furthermore, let the complex envelope, s_k, account for possible random gain and additive random phase on each nonzero element of $\underline{\mathbf{E}}_k$. Finally, consider an observed N-dimensional complex random data vector $\underline{\mathbf{X}}$ consisting of $K \leq N$ spectral components and additive noise

$$\underline{\mathbf{X}} = \sum_{k=1}^{K} s_k \underline{\mathbf{E}}_k + \underline{\mathbf{N}}, \tag{4}$$

where $\underline{\mathbf{N}}$ is an N-dimensional complex sample vector of zero mean, identically distributed and statistically independent noise. In particular, the element in the i-th row and j-th column of the noise CFC matrix $\overline{\underline{\mathbf{N}}\underline{\mathbf{N}}^*}$ is

$$\sigma_{ij}^2\Big|_N = \left[\overline{\underline{\mathbf{N}}\underline{\mathbf{N}}^*}\right]_{ij} \tag{5}$$

$$= \begin{cases} \sigma_0^2, & i = j \\ 0, & i \neq j \; . \end{cases} \tag{6}$$

It is advantageous to introduce the following matrix quantities: An N (row) by K (column) matrix, $\mathbf{E}$, wherein the spectral location vector $\underline{\mathbf{E}}_k$ is the k-th column,

$$\mathbf{E} = \left[\underline{\mathbf{E}}_1 \underline{\mathbf{E}}_2 \cdots \underline{\mathbf{E}}_k\right] . \quad (7)$$

The columns of $\mathbf{E}$ will be assumed to be linearly independent but not necessarily orthogonal.

A K by K correlation matrix, $\mathbf{P}$, for the complex envelope factor, wherein the element in the i-th row and j-th column is given by

$$[\mathbf{P}]_{ij} = \overline{s_i s_j^*} \quad (8)$$

$$= p_{ij} \quad . \quad (9)$$

Finally, an N by N identity matrix $\mathbf{I}_N$ is defined.

With the above the CFC matrix can be written as

$$R = \overline{\underline{\mathbf{X}}\underline{\mathbf{X}}^*} \quad (10)$$

$$= \mathbf{EPE}^* + \sigma_0^2 \mathbf{I}_N. \quad (11)$$

It is now desirable to orthonormalize the spectral location vectors in terms of a set of new orthonormal vectors $\{\underline{\boldsymbol{\Phi}}_\ell\}_{\ell=1}^K$ where if $\boldsymbol{\Phi}$ is an N by K matrix

$$\boldsymbol{\Phi} = \left[\underline{\boldsymbol{\Phi}}_1 \underline{\boldsymbol{\Phi}}_2 \cdots \underline{\boldsymbol{\Phi}}_K\right] , \quad (12)$$

then

$$\boldsymbol{\Phi}^* \boldsymbol{\Phi} = \mathbf{I}_K. \quad (13)$$

A K by K upper triangular matrix

$$\mathbf{B} = \begin{bmatrix} b_{11} b_{12} & \cdots & b_{1k} \\ & b_{22} & \\ & & \ddots \\ 0 & & b_{kk} \end{bmatrix} \quad (14)$$

can be found using a Gram-Schmidt procedure where $\boldsymbol{\Phi}$ can be expressed as

$$\boldsymbol{\Phi} = \mathbf{EB}. \tag{15}$$

That is, each spectral location vector $\mathbf{E}_k$ can be expressed as a linear combination of the orthonormal vectors $\underline{\Phi}_\ell$. Of course, the orthonormalization procedure presented above could have been used had the vectors $\mathbf{E}_k$ not been linearly independent. In this case, an orthogonal set $\{\underline{\Phi}_\ell\}_{\ell=1}^{L}$ would have been established where the integer $L \leq K$ is called the dimensionality of the signal (sub-)space defined by the set $\{\underline{\mathbf{E}}_k\}_{k=1}^{K}$. For the duration of this discussion, however, the spectral location vectors will be assumed linearly independent, since this ensures that the Gram-Schmidt coefficient matrix, $\mathbf{B}$, is a square, nonsingular matrix for which an inverse, $\mathbf{B}^{-1}$, exists. Using (15), equation (11) becomes

$$\mathbf{R} = \boldsymbol{\Phi}\mathbf{B}^{-1}\mathbf{P}\mathbf{B}^{-1*}\boldsymbol{\Phi}^{*} + \sigma_0^2\mathbf{I}_N. \tag{16}$$

The Hermitian matrix $\mathbf{B}^{-1}\mathbf{P}\mathbf{B}^{-1*}$ is now written in diagonal form as

$$\mathbf{B}^{-1}\mathbf{P}\mathbf{B}^{-1*} = \boldsymbol{\gamma}\boldsymbol{\Sigma}\boldsymbol{\gamma}^{*}, \tag{17}$$

where $\boldsymbol{\gamma}$ is a K by K matrix consisting of the orthonormal eigenvectors of $\mathbf{BPB}^{-1*}$ and $\boldsymbol{\Sigma}$ is a K by K diagonal matrix consisting of the eigenvalues $\sigma_1^2 \geq \sigma_2^2 \geq \ldots \geq \sigma_K^2$ of $\mathbf{BPB}^{-1*}$. It is worth noting that (16) represents a coupling of the information contained in the spectral position vectors with the complex envelope factor correlation characteristics.

Using (17) in (16) and post-multiplying by $\boldsymbol{\Phi}\boldsymbol{\gamma}$ yields

$$\mathbf{R}\boldsymbol{\Phi}\boldsymbol{\gamma} = \boldsymbol{\Phi}\boldsymbol{\gamma}\left[\boldsymbol{\Sigma} + \sigma_0^2\mathbf{I}_K\right]. \tag{18}$$

Observing (18), it is evident that the matrix $\boldsymbol{\Phi}\boldsymbol{\gamma}$ contains K of the N eigenvectors of the CFC matrix $\mathbf{R}$ as columns with corresponding positive, real eigenvalues $\lambda_1 = \sigma_1^2 + \sigma_0^2 \geq \lambda_2 = \sigma_2^2 + \sigma_0^2 \geq \ldots \geq \lambda_K = \sigma_K^2 + \sigma_0^2 \geq \sigma_0^2$ given as the corresponding diagonal elements of the matrix

$$\boldsymbol{\lambda} = \boldsymbol{\Sigma} + \sigma_0^2\mathbf{I}_K. \tag{19}$$

The remaining N-K eigenvalues of $\mathbf{R}$ are identically σ_0^2 corresponding to a root of the CFC matrix of multiplicity N-K. The elements of $\boldsymbol{\lambda}$ in (19) have been assumed to be arranged in decreasing order. Finally, it is seen that the N by K matrix of eigenvectors corresponding to the K largest eigenvalues of $\mathbf{R}$ is given by

$$\mathbf{M} = \boldsymbol{\Phi}\gamma \tag{20}$$

$$= \mathbf{EB}\gamma. \tag{21}$$

Thus, the eigenvectors (columns of $\mathbf{M}$) are seen to be linear combinations of the spectral position vectors where the combination coefficients are functions of the inner products of the position vectors $\underline{\mathbf{E}}_i^* \underline{\mathbf{E}}_j$ through $\mathbf{B}$ and the complex envelope factor correlation characteristics through γ.

Some examples of the results obtained in this section are given in the following section. Specific attention is given to the case of two (K = 2) spectral components because more complex spectra can be considered as extensions of this basic signal spectrum.

ORTHOGONAL DECOMPOSITION OF THE CROSS-FREQUENCY CORRELATION MATRIX FOR AN M COMPONENT SPECTRAL SET

For simplicity, let the observed data vector consist of two spectral components plus additive uncorrelated noise. If the notation $a_{ij} = \underline{\mathbf{E}}_i^* \underline{\mathbf{E}}_j$ is introduced, then a two-step Gram-Schmidt procedure to orthogonalize $\mathbf{E} = [\underline{\mathbf{E}}_1 \, \underline{\mathbf{E}}_2]$ as in (15) yields

$$b_{11} = \frac{1}{a_{11}^{1/2}}, \; b_{21} = 0, \tag{22}$$

and

$$b_{12} = -\frac{a_{12}/a_{11}^{1/2}}{\Delta}, \; b_{22} = \frac{a_{11}^{1/2}}{\Delta}, \tag{23}$$

such that

$$\mathbf{B}^{-1} = \frac{1}{a_{11}^{1/2}} \begin{bmatrix} a_{11} & a_{12} \\ 0 & \left(a_{11}a_{12} - |a_{12}|^2\right)^{1/2} \end{bmatrix}, \tag{24}$$

where

$$\Delta = \left(a_{11}a_{22} - |a_{12}|^2\right)^{1/2} .$$

Therefore, the matrix

$$\mathbf{C} = \mathbf{B}^{-1}\mathbf{P}\mathbf{B}^{-1*} \qquad (25)$$

has eigenvalues given by the roots of the characteristic equation of $\mathbf{B}^{-1}\mathbf{P}\mathbf{B}^{-1*}$, which is

$$\sigma^4 - (a_{11}p_{11} + a_{22}p_{22} + 2\,\mathrm{Re}(p_{12}a_{12}^*))\sigma^2 + (a_{11}a_{22} - |a_{12}|^2)\,(p_{11}p_{22} - |p_{12}|^2) = 0. \qquad (26)$$

As a simplification, let the following definitions be introduced

$$a_{ij} = \begin{cases} G_i \text{ for } i=j \text{ (intracomponent spectral gain)} \\ (G_iG_j)^{1/2}\,\alpha \text{ for } i\neq j \text{ (intercomponent spectral gain)} \end{cases} \qquad (27)$$

and

$$p_{ij} = \begin{cases} S_i \text{ for } i=j \text{ (intracomponent spectral power)} \\ (S_iS_j)^{1/2}\,\beta \text{ for } i\neq j \text{ (intercomponent spectral power)} \end{cases} \qquad (28)$$

where α shall be called the spectral overlap factor ($|\alpha| \leq 1$) and β shall be called the envelope correlation factor ($|\beta| \leq 1$). The parameter G is a scalar gain factor. Using (27) and (28) in (26) gives

$$\sigma_1^2,\ \sigma_2^2 = \frac{1}{2}\Bigg[G_1S_1 + G_2S_2 + 2(G_1S_1G_2S_2)^{1/2}\,\mathrm{Re}(\alpha\beta^*) \pm \Bigg(\Big[G_1S_1 - G_2S_2\Big]^2 + 4G_1S_1G_2S_2\Big[|\alpha|^2 + |\beta|^2 - |\alpha|^2|\beta|^2\Big] + 4(G_1S_1G_2S_2)^{1/2}\,\mathrm{Re}(\alpha\beta^*)\Big[G_1S_1 + G_2S_2 + (G_1S_1G_2S_2)^{1/2}\,\mathrm{Re}(\alpha\beta^*)\Big]\Bigg)^{1/2}\Bigg]. \tag{29}$$

Notice the duality between the envelope correlation factor β and spectral overlap factor α. Two special cases with respect to (29) are of interest in that they give the extreme values of σ_1^2 and σ_2^2 where, in particular, either the intercomponent spectral gain factor, α, is zero due to the noncoincidence in frequency of elemental spectral components, or the intercomponent spectral power is zero due to the lack of coherence between spectral component envelopes. Accordingly, there results:

Noncoincident spectral components (α = 0 for $\mathbf{E}1$ and $\mathbf{E}2$ orthogonal):

$$\sigma_1^2,\ \sigma_2^2 = \frac{1}{2}\left[G_1S_1 + G_2S_2 \pm \left(\left[G_1S_1 - G_2S_2\right]^2 + 4G_1S_1G_2S_2|\beta|^2\right)^{1/2}\right] \tag{30}$$

$$= \begin{cases} G_1S_1,\ G_2S_2 & \text{for } |\beta| = 0 \quad (31) \\ G_1S_1 + G_2S_2,\ 0 & \text{for } |\beta| = 1. \quad (32) \end{cases}$$

Incoherent complex envelopes ($\beta = 0$);

$$\sigma_1^2, \sigma_2^2 = \frac{1}{2}\left[G_1S_1 + G_2S_2 \pm ([G_1S_1 - G_2S_2]^2 + 4G_1S_1G_2S_2|\alpha|^2)^{1/2}\right] \tag{33}$$

$$= \begin{cases} G_1S_1, \; G_2S_2 & \text{for } |\alpha| = 0 \\ G_1S_1 + G_2S_2, \; 0 & \text{for } |\alpha| = 1 \, . \end{cases} \tag{34}$$

An expression for the eigenvectors $\underline{\gamma}_1$ and $\underline{\gamma}_2$ of $\mathbf{B}^{-1}\mathbf{P}\mathbf{B}^{-1*}$ given general α and β would not be particularly informative; however, for the bounding conditions underlying (30) to (34) there results the following:

For noncoincident spectral components ($\alpha = 0$) with incoherent complex envelopes ($\beta = 0$)

$$\sigma_1^2 = G_1S_1, \; \underline{\gamma}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \tag{35}$$

and

$$\sigma_2^2 = G_2S_2, \; \underline{\gamma}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{36}$$

For <u>either</u> noncoincident spectral components (α=0) with coherent complex envelopes ($\beta = 1$) <u>or</u> incoherent envelopes ($\beta = 0$) with completely coincident spectral sets ($\alpha = 1$)

$$\sigma_1^2 = G_1S_1 + G_2S_2, \; \underline{\gamma}_1 = \frac{1}{[G_1S_1 + G_2S_2]^{1/2}} \begin{bmatrix} (G_1S_1)^{1/2} \\ (G_2S_2)^{1/2} \end{bmatrix}. \tag{37}$$

Using (20) in conjunction with (35) and (36) gives the eigenvalues and eigenvectors of the cross-frequency correlation matrix $\mathbf{R}$ as

$$\lambda_1 = G_1S_1 + \sigma_0^2, \; \underline{\mathbf{M}}_1 = \frac{1}{G_1^{1/2}}\underline{\mathbf{E}}_1 \, , \tag{38}$$

$$\lambda_2 = G_2G_2 + \sigma_0^2, \; \underline{\mathbf{M}}_2 = \frac{1}{G_2^{1/2}}\underline{\mathbf{E}}_2 \, , \tag{39}$$

and similarly for the conditions stipulated for (37), there results (note $\mathbf{E}_1 = \mathbf{E}_2$ given $\alpha = 1$)

$$\lambda_1 = G_1 S_1 + G_2 S_2 + \sigma_0^2$$

$$\mathbf{M}_1 = \begin{cases} \dfrac{1}{[S_1 G_1 + S_2 G_2]^{1/2}} \left(S_1^{1/2} \mathbf{E}_1 + S_2^{1/2} \mathbf{E}_2 \right), & \alpha = 0 \\ \dfrac{1}{G_1^{1/2}} \mathbf{E}_1, & \alpha = 1 . \end{cases} \qquad (40)$$

Thus, at one extreme, the occurrence of two spectrally disjoint components in the same eigenvalue (eigenvector) stems from correlated complex envelope factors and at the other extreme, the complete separation of disjoint components occurs due to the lack of envelope coherence. The significant result from the preceding development is that by orthogonal decomposition of the cross-frequency correlation matrix **R**, simultaneous extraction of envelope coherent spectral components can be achieved, because a linear combination of the component position vectors forms a single eigenvector of **R** which, in turn, can be examined for its spectral content.

To further emphasize the significance of (19) and (21), consider the case of K spectral components with orthogonal position vectors, i.e.,

$$\mathbf{E}_i^* \mathbf{E}_j = G_i \delta_{ij} \; (\delta_{ij} = \text{dirac delta}), \qquad (41)$$

and complex envelope power

$$S_i = \overline{|s_i|^2} \; . \qquad (42)$$

Let the K spectral components consist of L sets where the ℓ^{-th} set $\{i \varepsilon \Omega_\ell\}$ contains components for which the magnitude squared coherence

$$|C_{ij}|^2 = \frac{|\overline{s_i s_j^*}|^2}{\overline{|s_i|^2}\;\overline{|s_j|^2}} = \begin{cases} 1 \text{ for } i,j \varepsilon \Omega_\ell \\ 0 \text{ for } i \varepsilon \Omega_\ell \text{ and } j \varepsilon \bar{\Omega}_\ell \end{cases}, \qquad (43)$$

that is, envelope coherence exists only for components within a set.

It can be shown from (19) and (21) that

$$\lambda_\ell = \sum_{i\epsilon\Omega_\ell} G_i S_i + \sigma_0^2, \mathbf{M}_\ell = \frac{1}{\left[\sum_{i\epsilon\Omega_\ell} G_i S_i\right]^{1/2}} \sum_{i\epsilon\Omega_\ell} (G_i S_i)^{1/2} \mathbf{E}_i \text{ for } \ell \leq L \quad (44)$$

and

$$\lambda_\ell = \sigma_0^2 \text{ for } N - L < \ell \leq N. \quad (45)$$

Thus, it is suggested that detecting the ℓ^{-th} spectral set be accomplished by comparing an estimated λ_ℓ to σ_0^2 and relating components within the ℓ-th spectral set be accomplished by interrogating $\mathbf{M}_\ell$ to determine which components of the set $\{\mathbf{E}_k\}_{k=1}^{K}$ are present in the linear combinations.

The above case pertains to the situation where each of the spectral sets is disjoint, i.e., $\Omega_k \cap \Omega_\ell = 0$ with $\ell \neq k$. For the case $\Omega_k \cap \Omega_\ell \neq 0$, which includes partially coherent complex envelopes, leakage between sets occurs. However, inclusion of a spectral component within a given set will tend to be in proportion to the amount of coherence which exists between that component and the components within the set. Returning to the case of 2 spectral sets, it is useful to consider the behavior of σ_1^2, σ_2^2 for $|\alpha|$ and $|\beta|$ in the interval (0, 1). This consideration assumes α and β to be real valued. (See Cox[7] for a discussion of beamformer mismatch which can be related to the spatial interpretation of the variable α.) N.L. Owsley[8] addresses the problem of signal suppression in noise canceling array processors due to signal and noise correlation. This type of suppression can be related to the variable β. Figure 2 presents plots of σ_1^2 (————) and σ_2^2 (--------) for various values of G_1S_1 and G_2S_2 versus envelope coherence β for a specified intercomponent spectral gain, α. The transfer of energy from one eigenvalue to another as the two spectral components become either spectrally disjoint or envelope incoherent, as the case may be, is readily apparent. Figure 3 gives the ratio of eigenvalues σ_1^2/σ_2^2 (dB) versus β for fixed α and three values of G_1S_1/G_2S_2 (dB).

EXAMPLES OF SPECTRAL SET SEPARATION

This section presents several examples of spectral set extraction by the method of orthogonal decomposition of the CFC matrix which was presented in the previous sections. The presentation is in terms of an N by N, 2-dimensional square array O_{ij} with eigenvalue number

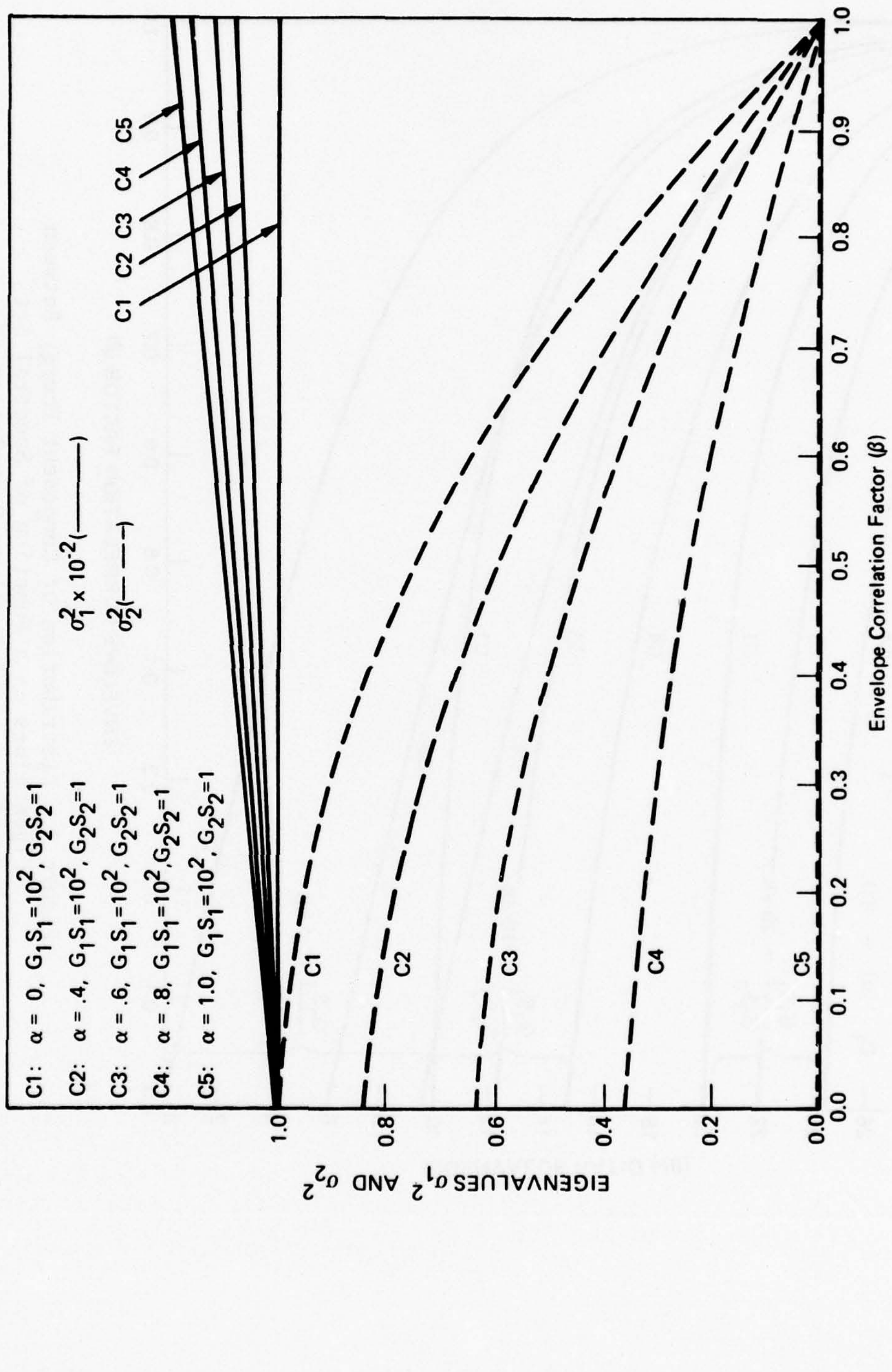


Figure 2. Distribution of Component Energy Between Eigenvalues as a Function of Spectral Set Overlap (α) and Envelope Correlation (β)

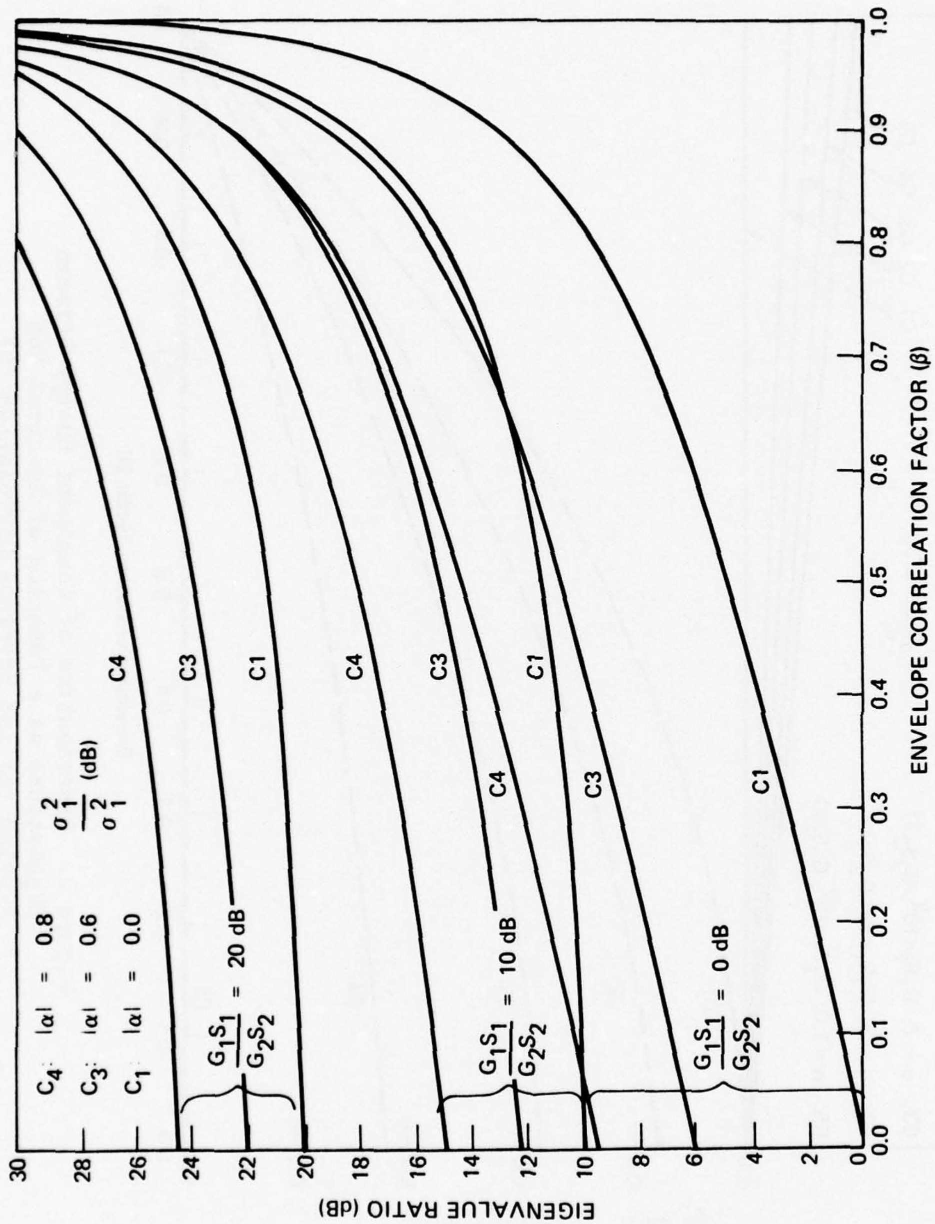


Figure 3. Distribution of Component Energy Between Eigenvalues as a Function of Spectral Set Overlap (α) and Envelope Correlation (β)

(i) and eigenvector element number (j) as the independent variables. The spectral level plotted in the 3-dimensional format of figure 4, for example, is the value

$$O_{ij} = \lambda_i |m_{ij}|^2, \tag{46}$$

where m_{ij} is the j-th element in the N by 1 eigenvector $\mathbf{M}_i$ corresponding to the eigenvalue λ_i of the CFC matrix $\mathbf{R}$. Thus, since the i-th eigenvector is a linear combination of the spectral position vectors for all components with spectral overlap and/or envelope correlation, the term $|m_{ij}|^2$ will give an indication of the presence of energy in the i-th spectral set at the j-th frequency cell. The eigenvalue λ_i weights the normalized component energy by the total energy in the harmonic set. The examples which follow are described as labeled on each figure. A CFC matrix for N=64 frequencies is treated in each case. Figures 4 to 8 present cases where the CFC matrix for spectral components with specified phase randomness is stipulated analytically and formulated for simulation on a general purpose computer. Phase randomness is specified in terms of a uniform distribution of phase over one of three intervals:

$(0^\circ, 0^\circ)$	coherent (C)
$(0^\circ, 180^\circ)$	partially coherent (PC)
$(0^\circ, 360^\circ)$	incoherent (I).

When a component is indicated as being a member of a specific harmonic set, this implies that the phases are coherent with respect to other members of the set, that is, the phase difference between the component envelopes is a constant even though the absolute phase of each component may be randomly distributed.

Figures 4 and 5 give an example of a three set harmonic extraction operation. All parameters for the two runs are identical, except for a relative decrease of 6 dB for the results shown in figure 5. Notice that the results are substantially the same except for some artifacts in figure 5 due to operation at low SNR, i.e., small eigenvalue magnitudes. Figure 6 has the same parameters as the case of figure 4 except components 2 and 6 are partially coherent. This coherence of phase leads to the appearance of considerable spectral energy from these components in the first eigenvalue and corresponding vector. Figure 7 illustrates an example of two sets with components which are spectrally coincident but incoherent at frequency cell 30. This results in the appearance of energy in eigenvectors 1 and 3. Also for this example, the partial envelope coherence of the component at frequency 45 results in a spreading of energy over several eigenvectors,

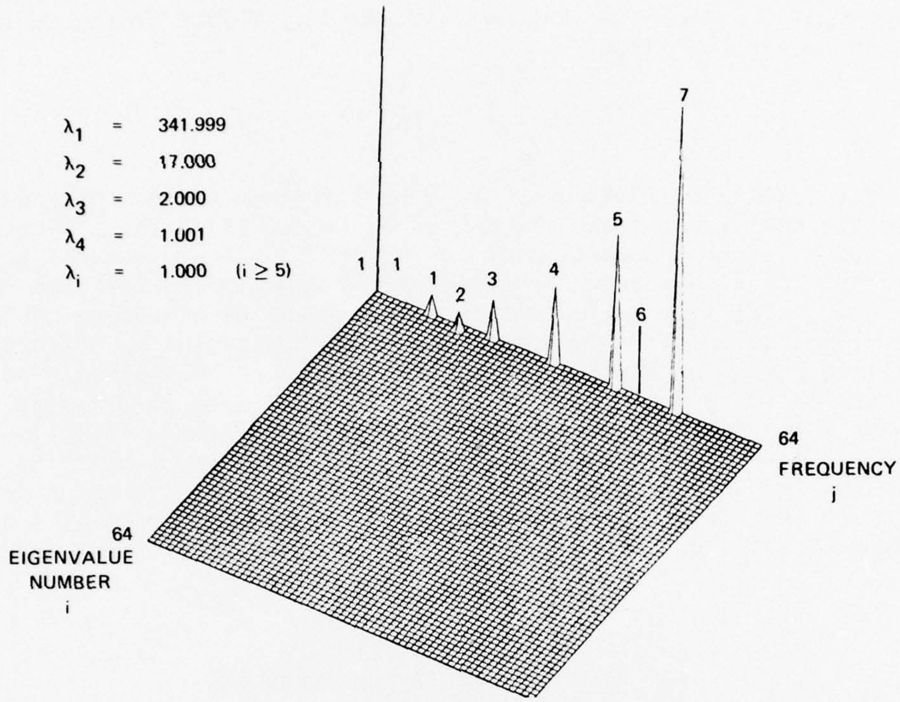


Figure 4. Spectral Set Separation

Component No.	Frequency No.	Amplitude	Phase Randomness	Harmonic Set	SNR in Cell (dB)
1	10	1	C	1	0
2	15	4	I	2	12
3	20	2	C	1	6
4	30	4	C	1	12
5	40	8	C	1	18
6	45	1	I	3	0
7	50	16	C	1	24

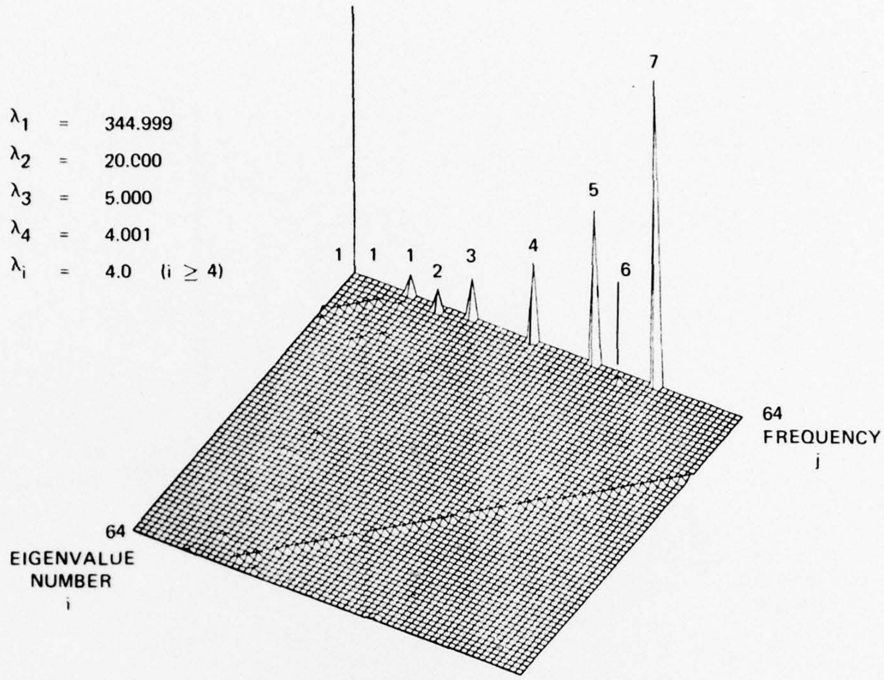


Figure 5. Spectral Set Separation

Component No.	Frequency No.	Amplitude	Phase Randomness	Harmonic Set	SNR in Cell (dB)
1	10	1	C	1	-6
2	15	4	I	2	6
3	20	2	C	1	0
4	30	4	C	1	6
5	40	8	C	1	12
6	45	1	I	3	-6
7	50	16	C	1	18

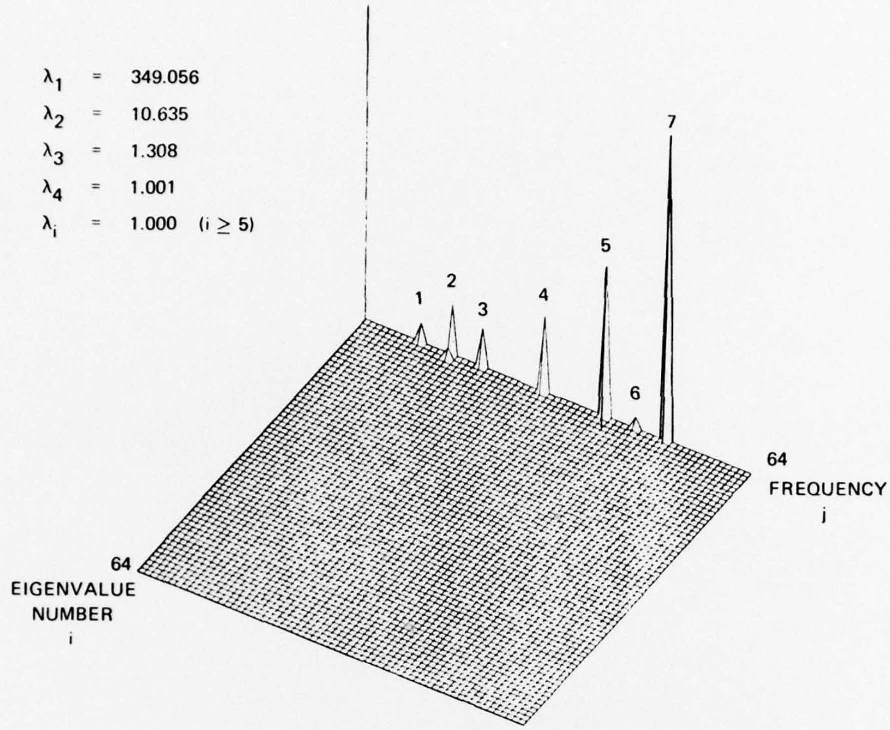


Figure 6. Spectral Set Separation

Component No.	Frequency No.	Amplitude	Phase Randomness	Harmonic Set	SNR in Cell (dB)
1	10	1	C	1	0
2	15	4	P	1,2	12
3	20	2	C	1	6
4	30	4	C	1	12
5	40	8	C	1	18
6	45	1	P	1,2,3	0
7	50	16	C	1	24

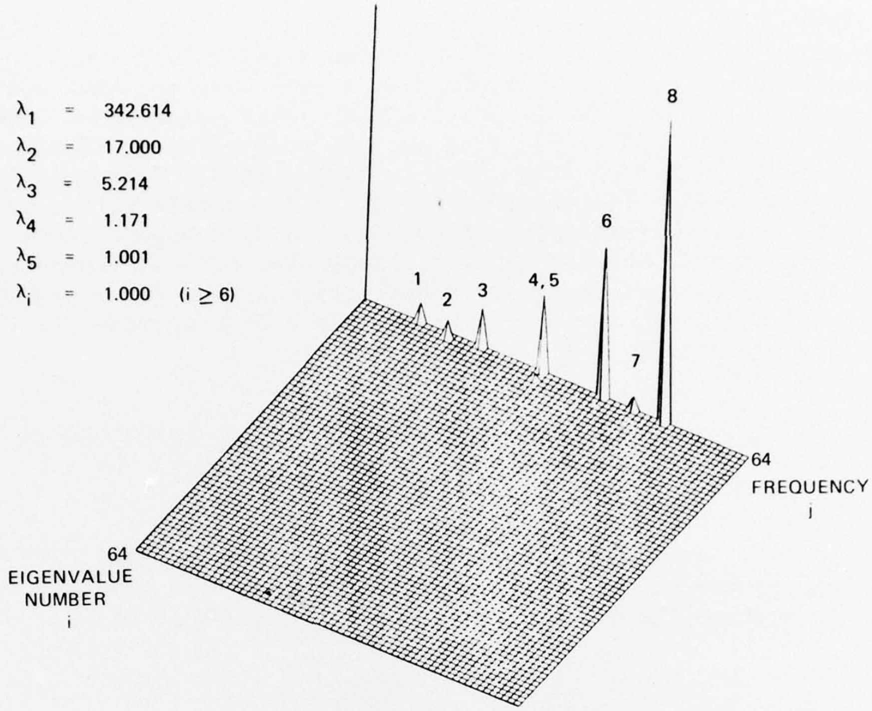


Figure 7. Spectral Set Separation

Component No.	Frequency No.	Amplitude	Phase Randomness	Harmonic Set	SNR in Cell (dB)
1	10	1	C	1	0
2	15	4	I	2	12
3	20	2	C	1	6
4	30	4	C	1	12
5	30	2	I	3	6
6	40	8	C	1	18
7	45	1	P	1,3,4	0
8	50	16	C	1	24

essentially as predicted by the results of the section on ORTHOGONAL DECOMPOSITION OF THE CROSS-FREQUENCY CORRELATION MATRIX FOR AN M COMPONENT SPECTRAL SET. Figure 8 contains 4 sets with spectral coincidence at frequency cell 30 between sets 1 and 4. This coincidence leads to coupling between spectral position vectors as predicted. The results illustrated in figure 9 derive from an actual random frequency "jitter" at cells 15 and 45. The random frequencies are stable with respect to each other and therefore are contained in the same eigenvectors. This would be an example wherein coherent processing of each component individually would be an exercise in futility, whereas joint processing in terms of eigenvectors would yield nearly a 3 dB increase in the corresponding eigenvalue.

RELATION OF ORTHOGONAL CFC MATRIX DECOMPOSITION TO MINIMUM VARIANCE SPECTRUM ANALYSIS

A minimum variance estimate of the complex envelope for a possible component of the random data vector $\underline{\mathbf{X}}$ with spectral position vector $\underline{\mathbf{D}}$ requires the minimum variance filter for $\underline{\mathbf{X}}$ given by

$$\underline{\mathbf{H}} = G \frac{\mathbf{R}^{-1}\underline{\mathbf{D}}}{\underline{\mathbf{D}}^*\mathbf{R}^{-1}\underline{\mathbf{D}}} , \tag{47}$$

where $\underline{\mathbf{D}}^*\underline{\mathbf{D}} = G$. The expected output power for the minimum variance filter is given by

$$\overline{|\underline{y}_{mv}|^2} = \overline{|\underline{\mathbf{H}}^*\underline{\mathbf{X}}|^2} \tag{48}$$

$$= \frac{G^2}{\underline{\mathbf{D}}^*\mathbf{R}^{-1}\underline{\mathbf{D}}} . \tag{49}$$

In the preceding sections the cross-frequency correlation matrix $\mathbf{R}$ has been decomposed according to

$$\mathbf{R} = \mathbf{M}\Sigma\mathbf{M}^* + \sigma_0^2\mathbf{I}_N, \tag{50}$$

which can be written as

$$\mathbf{R} = \sum_{k=1}^{K} \sigma_k^2 \underline{\mathbf{M}}_k \underline{\mathbf{M}}_k^* + \sigma_0^2 \mathbf{I}_N, \tag{51}$$

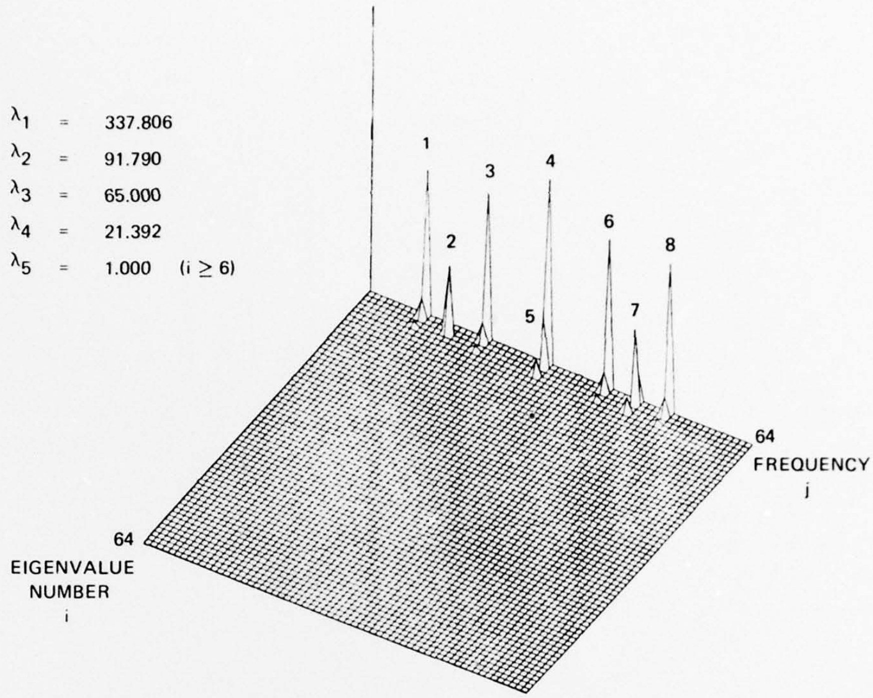


Figure 8. Spectral Set Separation

Component No.	Frequency No.	Amplitude	Phase Randomness	Harmonic Set	SNR in Cell (dB)
1	10	8	C	1	18
2	15	8	I	3	18
3	20	8	C	1	18
4	30	8	C	1	18
5	30	8	I	4	18
6	40	8	C	1	18
7	45	8	I	2	18
8	50	8	C	1	18

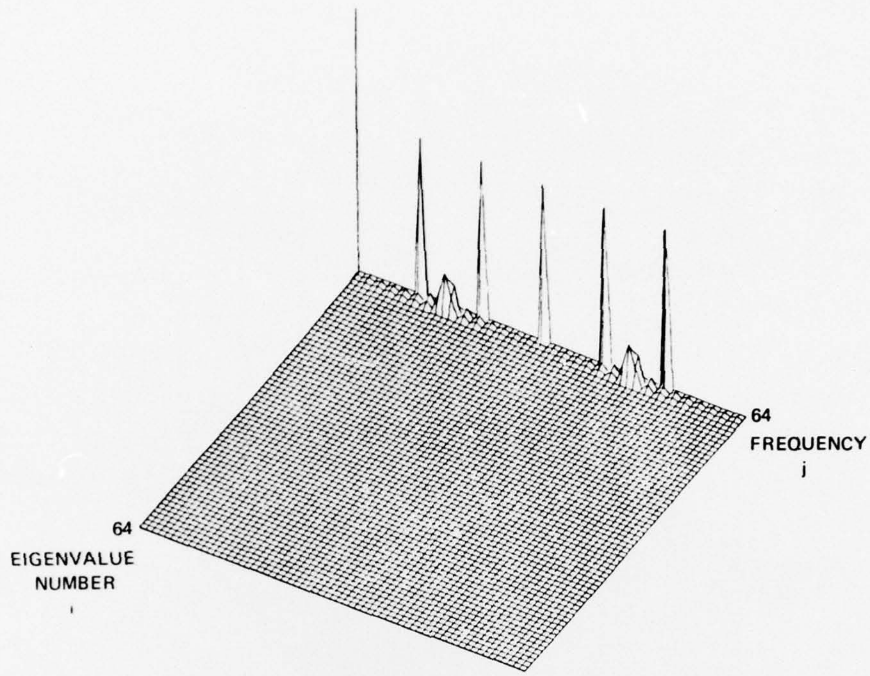


Figure 9. Spectral Set Separation

Component No.	Frequency No.	Amplitude	Phase Randomness	Harmonic Set	SNR in Cell (dB)
1	10	8	C	1	18
2	15	8	I	2	18
3	20	8	C	1	18
4	30	8	C	1	18
5	40	8	C	1	18
6	45	8	I	2	18
7	50	8	C	1	18

where

$$\underline{\mathbf{M}}_k = \mathbf{EB}\gamma_k \cdot \tag{52}$$

(Note: $\mathbf{A}_k$ implies the k-th column and $\mathbf{A}_k$ the k-th row, respectively, of the matrix $\mathbf{A}$.)

The inverse of $\mathbf{R}$ can be expressed as

$$\mathbf{R}^{-1} = \frac{1}{\sigma_0^2}\left[\mathbf{I}_N - \mathbf{M}\Sigma(\Sigma + \sigma_0^2\mathbf{I}_k)^{-1}\mathbf{M}^*\right] \tag{53}$$

$$= \frac{1}{\sigma_0^2}\left[\mathbf{I}_N - \mathbf{M}\Psi\mathbf{M}^*\right] \tag{54}$$

$$= \frac{1}{\sigma_0^2}\left[\mathbf{I}_N - \sum_{k=1}^{K}\Psi_k\underline{\mathbf{M}}_k\underline{\mathbf{M}}_k^*\right], \tag{55}$$

where Ψ is a K by K diagonal matrix with the k-th diagonal element given by

$$\Psi_k = \frac{\sigma_k^2}{\sigma_k^2 + \sigma_0^2} \cdot \tag{56}$$

Thus, the output power of the minimum variance filter is

$$|y_{mv}|^2 = \frac{G^2\sigma_0^2}{G - \sum_{k=1}^{K}\Psi_k|\underline{\mathbf{D}}^*\underline{\mathbf{M}}_k|^2} \cdot \tag{57}$$

Consider the case of two spectral components with orthogonal position vectors, i.e., $\alpha = 0$, (57) becomes

$$|y_{mv}|^2 = \begin{cases} \dfrac{G^2\sigma_0^2}{G - \dfrac{S_1}{GS_1 + \sigma_0^2}|\underline{\mathbf{D}}^*\underline{\mathbf{E}}_1|^2 - \dfrac{S_2}{GS_2 + \sigma_0^2}|\underline{\mathbf{D}}^*\underline{\mathbf{E}}_2|^2} & \text{incoherent envelopes } (\beta = 0) \quad (58) \\[2ex] \dfrac{G^2\sigma_0^2}{G - \dfrac{1}{GS_1 + GS_2 + \sigma_0^2}|\underline{\mathbf{D}}^*(S_1^{1/2}\underline{\mathbf{E}}_1 + S_2^{1/2}\underline{\mathbf{E}}_2)|^2} & \text{coherent envelopes } (\beta = 1) \quad (59) \end{cases}$$

An important special case of the above exists for

$$\underline{\mathbf{E}}_i^{*}\underline{\mathbf{E}}_i = G \tag{60}$$

and

$$\begin{aligned} \underline{\mathbf{D}}^{*}\underline{\mathbf{E}}_i &= G\rho_i \\ &= G|\rho_i|e^{j\theta_i} \end{aligned} \tag{61}$$

where $0 \le |\rho_i| \le 1$ where $|\rho_i|$ is termed the generalized cosine of the angle between the vectors $\underline{\mathbf{D}}$ and $\underline{\mathbf{E}}_i$. Equations (58) and (59) can now be written as

$$\overline{|y_{mv}|^2} = \begin{cases} \dfrac{G\sigma_0^2}{1 - \dfrac{(S_1|N)}{1 + (S_1 N)}|\rho_1|^2 - \dfrac{(S_2|N)}{1 + (S_2|N)}|\rho_2|^2} & (\beta = 0) \\[2ex] \dfrac{G\sigma_0^2}{1 - \dfrac{1}{1 + (S_1|N) + (S_2|N)}\Big[(S_1|N)|\rho_1|^2 + (S_1|N)|\rho_2|^2 + 2(S_1|N)^{1/2}(S_2|N)^{1/2}|\rho_1||\rho_2| \cos(\theta_1 - \theta_2)\Big]} & (\beta = 1) \end{cases} \tag{62}$$

where $(S_i/N) = GS_i/\sigma_0^2$ is the postfilter signal-to-noise ratio for $\beta = 1$ in the absence of any other component and where $(\theta_1 - \theta_2)$ is the phase difference between the filter outputs $\underline{\mathbf{D}}^{*}\underline{\mathbf{E}}_1$ and $\underline{\mathbf{D}}^{*}\underline{\mathbf{E}}_2$. Figures 9 and 10 present plots of the ML beamformer output power for two spectral components, where the two components exhibit both incoherent envelopes (MVI, ———) and coherent envelopes (MVC, ------). In addition, the conventional filter output

$$\overline{|y_C|^2} = \overline{|\underline{\mathbf{D}}^{*}\underline{\mathbf{X}}|^2}\ , \tag{63}$$

$$\overline{|y_C|^2} = \begin{cases} G\sigma_0^2\big[(S_1|N)|\rho_1|^2 + (S_2|N)|\rho_2|^2 + 1\big] & (\beta = 0) \\ G\sigma_0^2\big[(S_1|N)|\rho_1|^2 + (S_2|N)|\rho_2|^2 \\ \quad + 2(S_1|N)^{1/2}(S_2|N)^{1/2}|\rho_1||\rho_2| \cos(\theta_1 - \theta_2) + 1\big] & (\beta = 1) \end{cases} \tag{64}$$

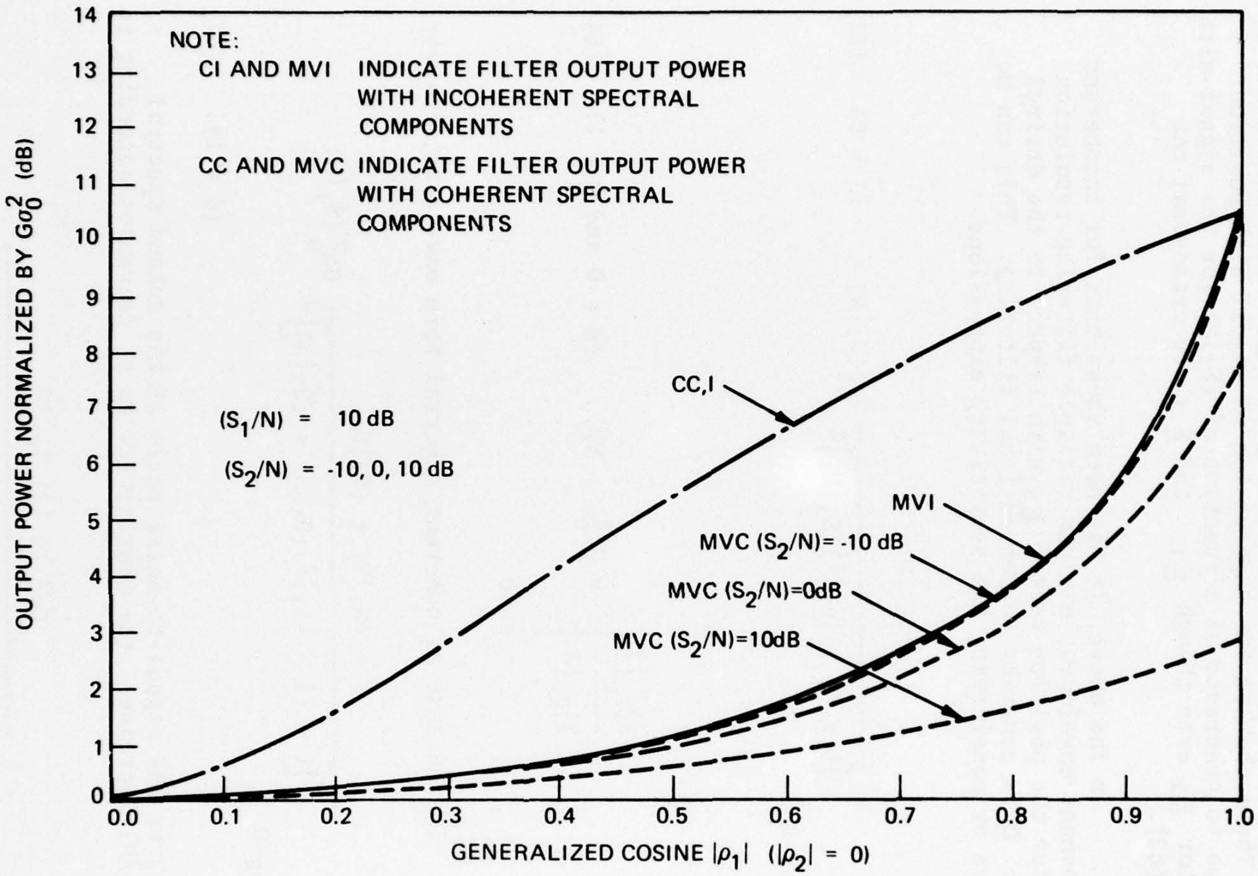


Figure 10. Filter Output Power Versus the Generalized Cosine $|\rho_1|$ Between a Spectral Term Vector and the Filter Look Vector for Conventional (CI,C) and Minimum Variance (MVI,C) Filters

is given. In figures 10 and 11 the filter output power is plotted as a function of the generalized cosine $|\rho_1|$ of the angle between $\mathbf{D}^*$ and $\mathbf{E}_1$ and $|\rho_2|$ is assumed to be zero, that is, $\mathbf{D}$ and $\mathbf{E}_2$ are considered to be orthogonal. The mechanism for suppression of signal with position vector $\mathbf{E}_1$ due to coherence as a function of $(S_2|N)$ for the signal with position vector $\mathbf{E}_2$ even though $\mathbf{E}_1$ and $\mathbf{E}_2$ are orthogonal can observed in (62).

In the above, it has been shown that for incoherent spectral component envelopes, minimum variance filtering resolution sensitivity for the position vector $\mathbf{E}_1$ with respect to the desired filter vector $\mathbf{D}$ is superior to conventional filtering. This can be easily be seen by considering the sensitivity expressions

$$\left.\frac{\partial \overline{|y_{MV}|^2}}{\partial|\rho_1|^2}\right|_{\rho_2=0} = \frac{1 + (S_1|N)}{\left[1 + (1 - |\rho_1|^2)(S_1|N)\right]^2} G\sigma_0^2(S_1|N) \qquad (\beta = 0) \qquad (65)$$

and

$$\left.\frac{\partial |y_c|^2}{\partial|\rho_1|^2}\right|_{\rho_2=0} = G\sigma_0^2(S_1|N), \qquad (\beta = 0 \text{ and } \beta = 1) \qquad (66)$$

for $|\rho_2| = 0$. In the case of coherent spectral term envelopes, however,

$$\left.\frac{\partial |y_{MV}|^2}{\partial|\rho_1|^2}\right|_{\rho_2=0} = \frac{1 + (S_1|N) + (S_2|N)}{\left[1 + (1 - |\rho_1|^2)(S_1|N) + (S_2|N)\right]^2} G\sigma_0^2 (S_1|N)$$

$$(\beta = 1). \qquad (67)$$

In which case, as the signal-to-noise ratio of the second spectral component (S_2/N) increases, the sensitivity to s_1 decreases according to

$$\left.\frac{\partial |y_{MV}|^2}{\partial(S_2/N)}\right|_{\rho_2 = 0} = - \frac{G\sigma_0^2 (S_1|N) |\rho_1|^2}{\left[1 + (1 - |\rho_1|^2)(S_1|N) + (S_2|N)\right]^2} \qquad (\beta = 1) \qquad (68)$$

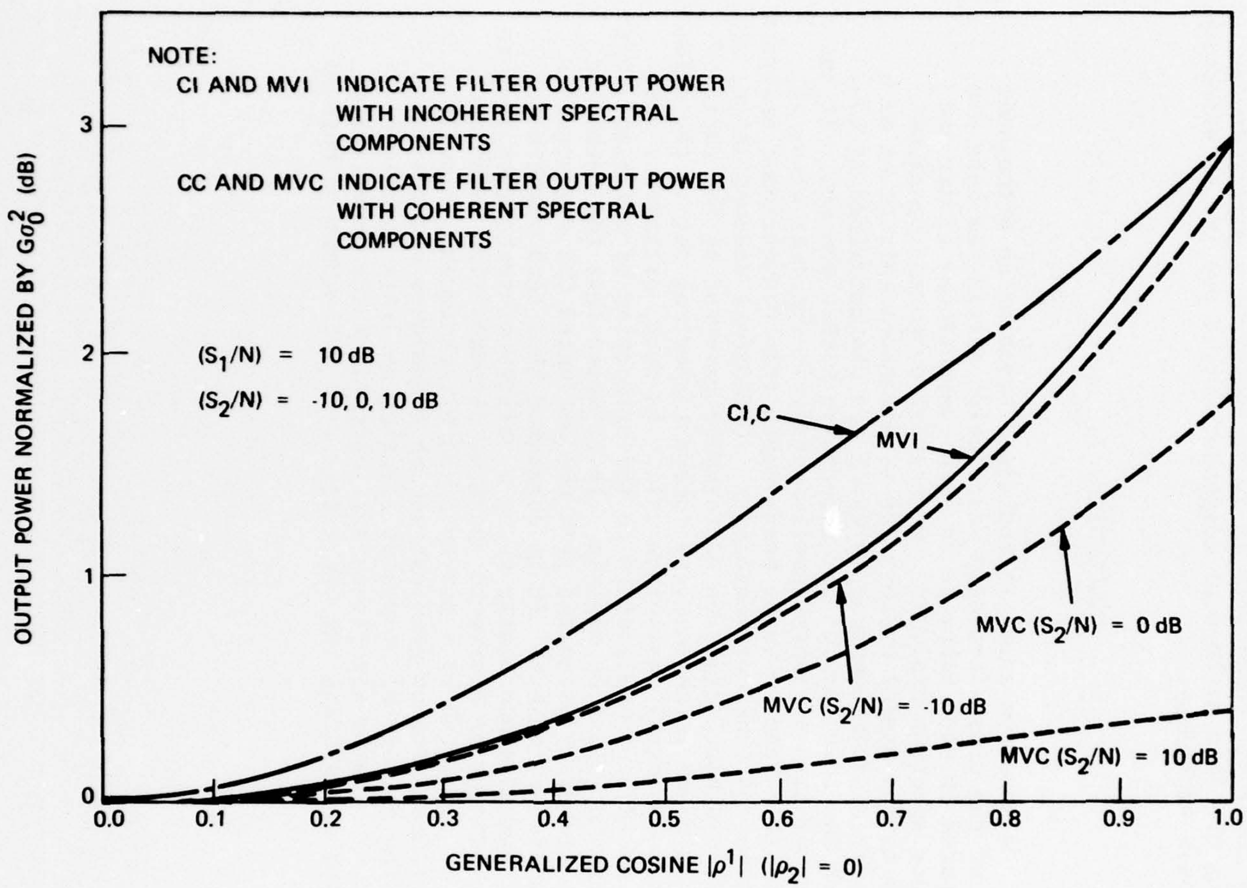


Figure 11. Filter Output Power Versus the Generalized Cosine $|\rho_1|$ Between a Spectral Term Vector and the Filter Look Vector for Conventional (CI,C) and Minimum Variance (MVI,C) Filters

which is always negative. Thus, it is apparent that the application of maximum likelihood filtering to coherent envelope spectral components in noise can give poor results. Furthermore, if a spectrum rich in coherent (and partially coherent) components is to be analyzed, a direct decomposition of the CFC into orthogonal components could provide an effective spectrum analysis technique.

CONCLUSION

A spectrum analysis technique based on an orthogonal decomposition of the cross-frequency correlation matrix has been proposed and compared in some respects to both conventional linear and minimum variance spectrum analysis. Orthogonal component spectrum analysis exhibits the useful property of simultaneous extraction of a set of spectrally disjoint components which are characterized by coherence between the envelopes of the components within the set. It has been shown that minimum variance spectrum analysis for this class of signals can actually lead to worse performance than conventional analysis due to suppression of coherent components. Orthogonal decomposition, on the other hand, exhibits detection performance governed by the ratio of the total signal power in an envelope coherent spectral set to the noise level in a single spectral resolution cell. As such, orthogonal spectrum analysis would have application to the extraction of a limited, but perhaps interesting, class of signals. It is noted that the method of estimating the frequency parameter from the estimated CFC eigenvectors has not been addressed herein. It is suggested that each eigenvector could be interrogated sequentially using a variety of techniques such as either maximum entropy or Pisarenko spectral decomposition techniques. Reference (9) is suggested as a survey of such techniques. Finally, adaptive algorithms based on gradient search techniques are available for realizing the orthogonal component spectrum analysis technique described herein.[10] These algorithms will circumvent the necessity of first estimating and storing the CFC matrix with subsequent orthogonal decomposition.

REFERENCES

1. R. A. Roberts, "On the Detection of Signals Known Except for Phase," IEEE Transactions on Information Theory, vol. IT-11, no. 1, January 1965, p. 76.

2. N. L. Owsley, "The Parametric Sequential Classification of Spectral Patterns with Application to Signal Detection and Extraction," IEEE EASCON Proceedings, 1968, pp. 585-592.

3. A. H. Nuttall, Detection Capabilities of Several Phase-Processing Receivers, NUSC Technical Report 4529, 13 July 1973.

4. M. Okamoto, "Optimality of Principal Components," in Multivariate Analysis Proceedings of International Symposium, Editor P. R. Krishnaiah, Academic Press, New York, 1969, pp. 673-685.

5. N. L. Owsley, "Spectral Set Extraction," Proceedings of the NATO Advanced Study Institute on Signal Processing, vol. 4, Paper 42, August 1976. Also, NUSC Reprint Report 5537, 29 September 1976.

6. N. L. Owsley, "A Note on a Desampling Technique for Correlating Proportional Bandwidth Processes," NUSC, New London, Connecticut, April 1976 (unpublished notes), Appendix A.

7. H. Cox, "Resolving Power and Sensitivity to Mismatch of Optimum Array Processors," in Signal Processing, Academic Press, New York, 1973.

8. N. L. Owsley, "Noise Cancellation in the Presence of Correlated Signal and Noise," NUSC Technical Report 4639, 11 January 1974.

9. O. L. Frost, "Power Spectrum Estimation," Proceedings of the NATO Advanced Study Institute on Signal Processing, vol. 1, paper 7, Portovenere, Italy, August 1976.

10. N. L. Owsley, "A Recent Trend in Adaptive Array Processing: Constrained Adaptation," in Signal Processing, Academic Press, New York, 1973.

INITIAL DISTRIBUTION LIST

Addressee	No. of Copies
CNO, OP-095	1
CNM, MAT-03, -03L, ASW-14, -24	4
NRL (Adams, Heitmeyer) Underwater Sound Reference Division (2), W. Morrough)	5
Strategic Systems Project Office, PM-1	2
NAVOCEANO, Code 7200	1
NAVELEX, Code 00, 03, 320 (CDR A. Miller), PME-124 (Dr. D. Hyde, CDR C. Robbins, J. Hussman)	6
NAVSEA, SEA-06H, -06H1 (J. Neely), -06H1-3, -06H2 (CMDR Bolka, D. Early, E. Landers, C. Tolsdorf, R. Cockerill), SEA-09G3 (4)	12
NOSC (2), (Dr. R. Smith, C.O., T.D., W. Marsh, R. Trueblood, F. Romeo, R. Bolam, J. McCool, W. Hodgkiss	11
NAVAIRDEVCEN	1
NELC	1
NUC	1
NISC	1
NAVPGSCOL	1
DDC, ALEXANDRIA	12
MARINE PHYSICAL LAB, SCRIPPS	1
WEAPON SYSTEM EVALUATION GROUP	1
WOODS HOLE OCEANOGRAPHIC INSTITUTION	1
ARPA (CAPT H. Cox, R. Cook)	2
ASN R&D (H. Sonnemann)	1
APL Johns Hopkins University (Dr. G. Smith)	1
TRW Inc., Mclean, VA (W. Richter)	1
NADC (T. Costaldi)	1
Duke University, Dept. of Electrical Engineering (Prof. L. Nolte) Duke Station, N.C.	1
MIT Lincoln Laboratory (R. T. Lacoss), Lexington, MA	1
Stanford University, Department of Electrical Engineering (Prof. B. Widrow), Palo Alto, Calif	1
Yale University (P. Schultheiss)	1
SACLANT ASW Research Centre	1