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WEIGHTED LINEAR REGRESSION FOR TWO VARIABLES.(U)
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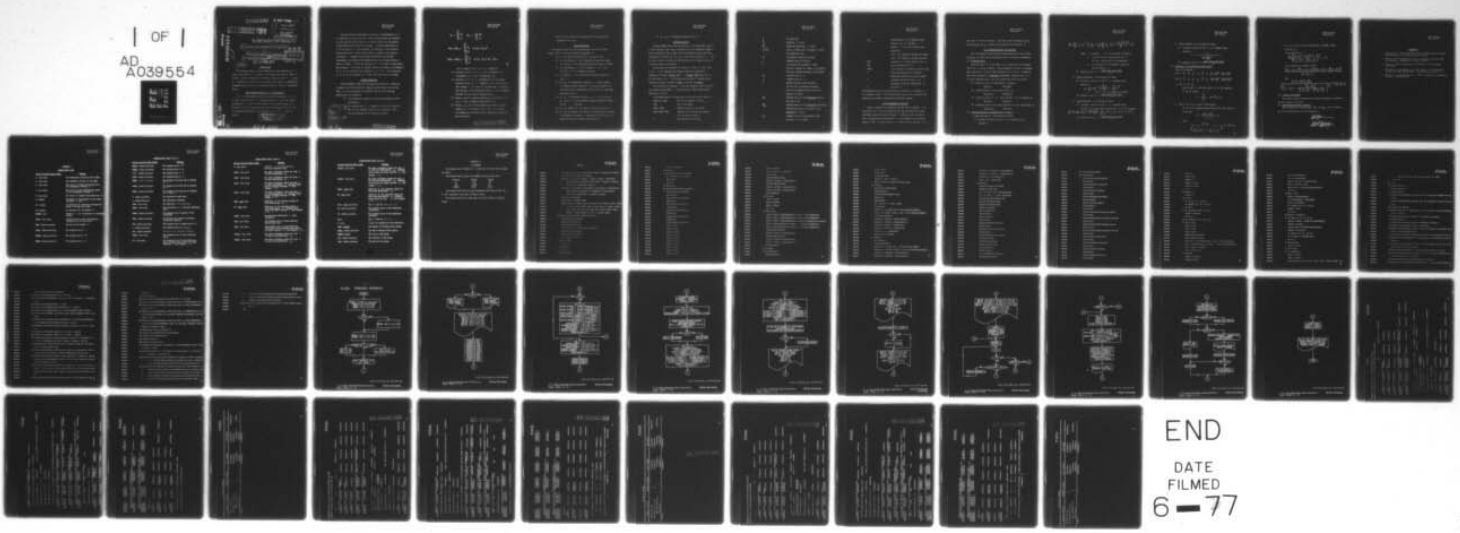

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(6) WEIGHTED LINEAR REGRESSION FOR TWO VARIABLES.

by

(10) John Skory and Robert C. Jennings

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INTRODUCTION

The primary purpose of linear regression is the prediction of Y from a given value of X by means of the regression equation Y=BX or Y=A+BX , and the estimation of confidence limits for true Y , A and B . Regression analysis for two variables may be made on a weighted or unweighted basis with the line passing through the origin or through the means.

NON-CONSTANT VARIANCE OF Y AND WEIGHTING

If Y is the dependent variable, the linear regression of Y on X may be calculated if X is measured without error and each single measurement of Y has the same variance over the whole range of X . When this is not the case deviations from the regression line will not be normally distributed and the prediction of Y from X will be suspect.

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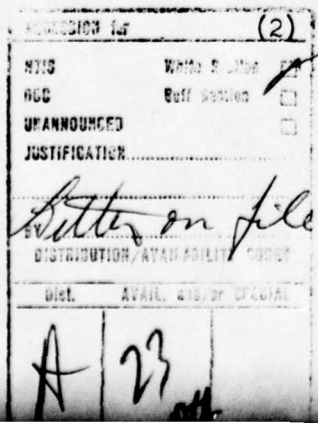
When the variance requirement is not met, a transformation of Y to logarithms or square root, etc., may be tried whereby the variances at the lower end of the scale of X will be found to be non-significantly different from those at the upper. If repeat measurements of Y at one value of X are available, the reciprocal of the variance derived from such a cluster may be used as a weight in the analysis. If variable Y comprises both single measurements and averaged measurements, the number of measurements that were averaged should be used as a weight. If both the variances of Y and number of measurements per averaged Y are known the weights would be the number of measurements of Y at X_i divided by the variance of Y at X_i. If the variance of Y at X_i is a function of X then X_i^{-2} may be used as a weight.

PROGRAM EXTENSIONS

Program REGRESS (S1281) was developed and written by J. Skory and Z. Ungar at the United Aircraft Corporate Systems Center for the IBM 1800. It has been adapted to the UNIVAC 1108 with the following modifications and additions:

(1) the maximum input sample size has been increased to 2000 observations.

(2) the weighted mean $\bar{X}$, weighted corrected sum of squares SSX, and the weighted corrected sum of products SSXY, are now evaluated by the recursion formulas:



$$\bar{X}_k = \frac{\sum_{k-1} W_i}{\sum_{k} W_i} \bar{X}_{k-1} + \frac{W_k X_k}{\sum_{k} W_i}$$

$$SSX_k = SSX_{k-1} + \frac{\sum_{k-1} W_i}{\sum_{k} W_i} W_k (X_k - \bar{X}_{k-1})^2$$

$$SSXY_k = SSXY_{k-1} + \frac{\sum_{k-1} W_i}{\sum_{k} W_i} W_k (X_k - \bar{X}_{k-1}) (Y_k - \bar{Y}_{k-1})$$

with k interated from 1 to n for a sample of n

(3) Confidence limits for true Y are calculated at the experimental X_i and post experimental X_i .

(4) Effective N at X_i is calculated for use in finding tolerance limits for Y . Effective N is a value such that average Y at a given X is predicted as accurately from the point on the regression line as if N observations had been actually made at that specified X .

(5) Linear calibration formulas are given for prediction of X from Y with confidence limits for true X .

(6) Formulas are given for the statistics necessary for the comparison of 2 regression lines. Tests are provided for homogeneity of variances about the lines, parallelism and coincidence.

(The last three items are calculated by the user after the program has been run.)

INPUT INFORMATION

The program reads all the following input data from the card reader (unit 3): (For I/O formats see Appendix A).

(1) n the number of data points in the input sample.

(2) IH an indicator, = 1 for regression through the origin (Y = BX) or = 2 for regression through the means (Y =A+BX).

(3) IW an indicator, = 1 if all weights = 1 , or = 2 if $W_i = X_i^{-2}$, or = 3 if weights are given.

(4) t Student's t for calculating confidence limits for true Y .

(5) $X_1 \ldots X_n$ the independent variable at n points equally or unequally spaced along the X axis and assumed to be controlled by the experimenter without error (or only with negligible error).

(6) $Y_1 \ldots Y_n$ the dependent variable. Y_i is an associate of X_i and if Y is an individual single measurement $\mathrm{Var}(Y_1) = \mathrm{Var}(Y_2) = \ldots = \mathrm{Var}(Y_n)$.

(7) $W_1 \ldots W_n$ weights for Y_i given as input only if IH = 3.

(8) $P_1 \ldots P_{10}$ ten points of X chosen post experimentally at which it is desired to predict Y and estimate its error as an exploratory step for future experimentation.

(9) $Q_1 \ldots Q_{10}$ ten weights associated with P_i .

COMPUTER ANALYSIS

Program REGRESS first sorts the points of the independent variable X into ascending order using a modified Shell sort embodied in SØRT, a Fortran subroutine (NUSL Program 1525) programmed by Lesick and Shores and modified to retain the Y_i and W_i associated with each X_i . This is done in order to obtain the within groups sums of squares for use in a one-way analysis of variance.

The program then evaluates the equation Y=A+BX or Y=BX where A and B are weighted least squares estimates determined by minimizing the sums $\Sigma W_i(Y_i-\hat{Y}_i)^2$ or $\Sigma W_i(Z_i-\hat{Z}_i)^2$ where the Z_i are translates of Y_i . The program thus performs a weighted linear regression analysis through the means or through the origin.

The deviations of observed Y from the regression line are tested for normality using WTEST , a Fortran function (NUSL Program S1640), (References 2 and 3).

The program writes all output data on the line printer (Unit 4):

SUMWX and SUMWY	sum of the weighted X_i and Y_i
SUMW	sum of the weights
AVERX, AVERY	weighted average of X_i and Y_i
SUMDX SUMDY SUMXY	weighted sum of squares and products about the mean or origin
B	the regression coefficient

A	the intercept
$\hat{Y}_i$	predicted Y vector
$\hat{Y}_{ei}$	predicted exploratory Y vector
$(Y_i-\hat{Y}_i)$	vector of deviations of observed Y from the regression line
R^2	correlation coefficient squared
S	standard error of estimate
F	a variance ratio testing if B=R=0 F(1,n-1), through the origin, is evaluated F(1,n-2), through the means, is evaluated
S_B	standard error of B
S_A	standard error of A
F	another variance ratio testing if A=0 (for case of through the means). F(1,n-2) is evaulated with 1 and (n-2) degrees of freedom
$S_{\hat{Y}_i}$	standard error of each predicted point on the line at X_i
$S_{\hat{Y}_{ei}}$	standard error of each predicted exploratory (postulated) point on the line at post experimental $X_i(=P_i)$
S_{Y_i}	standard error of each weighted newly observed Y or $(Y_i-\hat{Y}_i)$

$S_{Y_{ei}}$	standard error of each weighted newly observed Y_{ei} or $(Y_{ei}-\hat{Y}_{ei})$
t	Student's t for each deviation from regression: t(n-1) for regression through the origin t(n-2) for regression through the means
VARY	variance of Y within groups at one X
SSB	sum of squares due to B
SSDEV	sum of squares of deviations from the line
U	normal distribution test statistic (applicable only when all W_i are equal)
CL	confidence limits for true Y at experimental and post experimental X_i
N	effective N

The mathematical theory and derivation is presented, for example, in K.A. Brownlee (1960). In the calculations, many of the statistics are evaluated by alternate algebraically equivalent methods.

POST EXPERIMENTAL ESTIMATION

After an experiment is completed one may wish to estimate Y at some points of X(=P) not already tried in the experiment and obtain confidence limits for these estimates. In addition, one may wish to know how many repeat measurements, Q , are required at a specified value of X(=P) so that the error of a weighted newly observed Y is

less than θ at 95% confidence. This may be done by making $P_1=P_2=P_3$ and varying Q_1, Q_2, Q_3 widely and plotting the error against Q.

POST COMPUTER-ANALYSIS CALCULATIONS

From the computer output further calculations may be made by desk calculator to obtain the following five types of estimated information.

(1) Tolerance Limits

Using effective N in the manner of (4) (page 498) the tolerance limits for Y may be estimated. The following is the type of statement that would result; There is 95% confidence that, at the X specified, 99% of the population of individual measurements (observations) of Y, in future experimentation, will fall between the limits calculated.

(a) Tolerance limits at experimental X_i equals

$$\hat{Y}_i \pm (ru)_i S \qquad \text{(Using } \hat{N}_i\text{)}$$

(b) Tolerance limits at post experimental X_i

$$\hat{Y}_{ei} \pm (ru)_{ei} S \qquad \text{(Using } \hat{N}_{ei}\text{)}$$

(c) Tolerance limits for W_i newly observed Y at experimental X_i

$$\hat{Y}_i \pm (ru)_i S \qquad \text{(Using } N_i\text{)}$$

(d) Tolerance limits for Q_i newly observed Y at post experimental X_i

$$\hat{Y}_{ei} \pm (ru)_{ei} S \qquad \text{(Using } N_{ei}\text{)}$$

(2) Linear Calibration: Prediction of X from Y

A. Sample Unweighted: Line Through the Means

(1) Confidence limits for true X at a weighted newly observed Y

$$\bar{X} + \frac{Y-\bar{Y}}{B} \pm t F^{-.5} \left(\left(\frac{1}{W} + \frac{1}{n} \right) \sum_{i}^{n} (X_i - \bar{X})^2 + \frac{(Y-\bar{Y})^2}{B^2} \right)^{1/2} = X$$

where t = Student's t for $(n-2)$ degrees of freedom

F comes from test of significance for B.

If $F < 1$ results may not be meaningful.

W is the number of Y's averaged to give the new observation Y.

(2) Standard error of $X = \dfrac{\text{upper limit-lower limit}}{2t}$

B. Sample Unweighted: Line Through the Origin

(1) Confidence limits for true X at a weighted newly observed Y

$$X = \frac{Y}{B} \pm t F^{-.5} \left(\frac{\sum^{n} X_i^2}{W} + \frac{Y^2}{B^2} \right)^{1/2}$$

with t at $(n-1)$ degrees of freedom.

(2) Standard error of $X = \dfrac{\text{upper limit-lower limit}}{2t}$

C. Sample Weighted: Line Through the Means

(1) Confidence limits for true X at a weighted newly observed Y are obtained by solving for X in the quadratic:

$$\left(Y - \bar{Y} - B(X - \bar{X})\right)^2 = \frac{t^2 B^2}{F} \left(\left(\frac{X^2}{W} + \frac{1}{\sum^{n} W_i} \right) \sum^{n} W_i (X_i - \bar{X})^2 + (X - \bar{X})^2 \right)$$

(2) Standard error of $X = \dfrac{\text{upper limit-lower limit}}{2t}$

D. Sample Weighted: Line Through the Origin

(1) Confidence limits for true X at a weighted newly observed Y

$$X = \frac{Y}{B \pm tS\left(\frac{1}{W} + \frac{1}{n}\right)^{\frac{1}{2}}}$$

(2) Standard error of $X = \dfrac{\text{upper limit} - \text{lower limit}}{2t}$

(3) Comparison of Two Regression Lines ($W_i = 1$)

Given the estimates:

Line 1: $\bar{X}_1$ $\bar{Y}_1$ B_1 n_1 S_1^2 f_1 $\Sigma(X-\bar{X}_1)^2$ $\Sigma(Y-\bar{Y}_1)^2$

Line 2: $\bar{X}_2$ $\bar{Y}_2$ B_2 n_2 S_2^2 f_2 $\Sigma(X-\bar{X}_2)^2$ $\Sigma(Y-\bar{Y}_2)^2$

A. Test If $S_1^2 = S_2^2$

Make 2-sided F test with larger S^2 in the numerator.

If S_1^2 is larger:

$$F = \frac{S_1^2}{S_2^2} \quad \text{with } (f_1, f_2)$$

B. Test If $B_1 = B_2$ (A Test of Parallelism)

If S_1^2 is not significantly different from S_2^2 they are pooled and

$$S^2 = \frac{f_1 S_1^2 + f_2 S_2^2}{f_1 + f_2}$$

Then

$$\frac{B_1 - B_2}{S\left(\frac{1}{\Sigma(X-\bar{X}_1)^2} + \frac{1}{\Sigma(X-\bar{X}_2)^2}\right)^{\frac{1}{2}}} = t(f_1+f_2)$$

C. Test If the Two Lines Are Coincidental (if $\bar{X}_1=\bar{X}_2$; $\bar{Y}_1=\bar{Y}_2$; given $B_1= B_2$)

Pooled estimate of B

$$\frac{B_1\sum^{n_1}(X_i-\bar{X}_1)^2 + B_2\sum^{n_2}(X_i-\bar{X}_2)^2}{\sum^{n_1}(X_i-\bar{X}_1)^2 + \sum^{n_2}(X_i-\bar{X}_2)^2} = B$$

Pooled variance of estimate = S^2 =

$$\frac{\sum^{n_1}(Y_i-\bar{Y}_1)^2 + \sum^{n_2}(Y_i-\bar{Y}_2)^2 - B^2\left(\sum^{n_1}(X_i-\bar{X}_1)^2 + \sum^{n_2}(X_i-\bar{X}_2)^2\right)}{f_1+f_2+1}$$

Then

$$\frac{\bar{Y}_1-\bar{Y}_2-B(\bar{X}_1-\bar{X}_2)}{S\left(\frac{1}{n_1}+\frac{1}{n_2}+\frac{(\bar{X}_1-\bar{X}_2)^2}{\sum^{n_1}(X_i-\bar{X}_1)^2+\sum^{n_2}(X_i-\bar{X}_2)^2}\right)^{1/2}} = t(f_1+f_2+1)$$

(4) Analysis of Variance

The computer output contains the necessary information to perform a one-way analysis of variance.

(5) Simultaneous Tolerance Intervals

These intervals are calculated as $\hat{Y}_{ei} \pm c^{**}S_{\hat{Y}_{ei}}$ with c** tabled and its use explained in 4.7.4 of Reference 5.

John Skory
JOHN SKORY

Robert C Jennings
ROBERT C. JENNINGS

REFERENCES

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2. Shapiro, S.S., and Wilk, M.B., 1965. An Analysis of Variance Test for Normality. Biometrika, 52, 3 and 4, 591-611.

3. Skory, J., 1969. Variance Test for Normality. NUSL Tech Memo 2211-343-69.

4. Weissberg, A., and Beatty, G.H., 1960. Tables of Tolerance-Limit Factors for Normal Distributions. Technometrics Vol 2, No. 4.

5. Hillier, F.S., and Lieberman, G.J., 1968. Introduction to Operations Research. Holden-Day, Inc.

APPENDIX A

NOMENCLATURE TABLE

Fortran Variable Name & Type	Meaning
X real array	The independent variable of the input.
Y real array	The dependent variable of the input.
W real array	The vector of weights associated with the independent variable X .
P real array	The vector of post-experimental points of the independent variable X .
Q real array	The vector of weights associated with P .
N integer	The number of observations in the input variables X and Y .
IH integer	An indicator for regression through the means or through the origin.
IW integer	An indicator for the weights W .
STUDNT real	Student's t for calculation of confidence levels.
TITLE real array	An array used to store the Hollerith characters of the title card.
SUMW double precision	The sum of the weights W .
SUMWX double precision	The weighted sum of X .
SUMWX2 double precision	The weighted sum of X^2 .
SUMWY double precision	The weighted sum of Y .

NOMENCLATURE TABLE (Cont'd)

Fortran Variable Name & Type	Meaning
SUMWY2 double precision	The weighted sum of Y^2 .
SUMWXY double precision	The weighted sum of XY .
AVERX double precision	The weighted mean of X .
AVERY double precision	The weighted mean of Y .
SUMDX double precision	The weighted corrected sum of squares of X .
SUMDY double precision	The weighted corrected sum of squares of Y .
SUMXY double precision	The weighted corrected sum of products of X and Y .
B double precision	The regression coefficient.
A double precision	The regression intercept.
YHAT real array	The predicted Y of regression.
REGRY real array	The deviations of Y from the regression line.
SUMRS double precision	The weighted sum of squares of the deviations.
RSQD double precision	The multiple and simple correlation coefficient squared.
SSB double precision	The weighted sum of squares due to B .
S double precision	The standard error of estimate.
AW1 double precision	Multiplier for intercept variance.
SYHAT real array	The standard error of each predicted point.
SY real array	The standard error of each predicted point assuming these points are newly observed Y with weights = W .

NOMENCLATURE TABLE (Cont'd)

Fortran Variable Name & Type	Meaning
T real array	Student's t for each deviation from the regression line.
CLY1H real array	The upper confidence limits for true Y at each experimental X_i .
CLY1L real array	The lower confidence limits for true Y at each experimental X_i .
CLY2H real array	The upper confidence limits for true Y at each experimental X_i assuming these points are newly observed Y with weights = W .
CLY2L real array	The lower confidence limits for true Y at each experimental X_i assuming these points are newly observed Y with weights = W .
NHAT real array	Effective N for tolerance limits at each experimental X_i .
NI real array	Effective N for tolerance limits at each experimental X_i assuming these points are newly observed Y with weights = W .
YSUBE real array	The predicted exploratory Y after regression.
SYHE real array	The standard error of each predicted exploratory point.
SEYE real array	The standard error of each predicted exploratory point assuming these points are newly observed Y_e with weights = Q .
CLYE1H real array	The upper confidence limits for true Y at each post-experimental X_{ei} .
CLYE1L real array	The lower confidence limits for true Y at each post-experimental X_{ei} .

NOMENCLATURE TABLE (Cont'd)

Fortran Variable Name & Type	Meaning
CLYE2H real array	The upper confidence limits for true Y at each post-experimental X_{ei} assuming these points are newly observed Y_e with weights = Q .
CLYE2L real array	The lower confidence limits for true Y at each post-experimental X_{ei} assuming these points are newly observed Y_e with weights = Q .
NEHAT real array	Effective N for tolerance limits for each post-experimental X_{ei} .
NE real array	Effective N for tolerance limits for each post-experimental X_{ei} assuming these points are new Y_e with weights = Q .
F12B double precision	The F test for R = B = 0 .
SB double precision	The standard error of the regression coefficient.
SA double precision	The standard error of the regression intercept.
F12A	The F test for A = 0 .
U real	W-test for normality of the deviations.
SUMN integer	The degrees of freedom within groups.
SUMSQ double precision	The sum of squares within groups.
NGRØUP integer	The size of each group.
SSY double precision	The variance of each group.
YBAR double precision	The mean of each group.

APPENDIX B

I/O FORMATS

The program reads (TITLE(1), 1 = 1,14) from the first card in 13A6, A2 format.

The program reads N,IH,IW, and STUDNT from the second card:

Variable	Columns	Format
N	1-5	I5
IH	6-10	I5
IW	11-15	I5
STUDNT	16-30	E15.8

The program then reads the input variables X,Y,W (only if IW = 3), P, and Q separately from cards in 5E15.8 format.

The program writes all output data on the line printer in 7E17.8 format.

APPENDIX C

```
000001            C     REGRESS--PROGRAM FOR TWO-VARIABLE WEIGHTED REGRESSION (THROUGH
000002            C     THE ORIGIN AND THROUGH THE MEANS)
000003                  DIMENSION TITLE(14), X(2000), Y(2000), W(2000), YHAT(2000),
000004                 1     YSUBE(2000), RESRY(2000), SYHAT(2000), SY(2000), T(2000),
000005                 2     P(10), G(10), SEYE(10), SYHE(10)
000006                  DIMENSION CLY1H(2000), CLY1L(2000), CLYE1H(2000), CLYE1L(2000),
000007                 1     CLY2H(2000), CLY2L(2000), CLYE2H(2000), CLYE2L(2000),
000008                 2     NHAT(2000), NEHAT(2000), NI(2000), NE(2000)
000009                  REAL NHAT,NI,NEHAT,NE
000010                  INTEGER SUMN, DEGFB1, DEGFC
000011                  DOUBLE PRECISION SUMW, SUMWX, SUMWX2, SUMWY, SUMWY2, SUMXY, SUMWXY
000012                 1     , SUMDX, SUMDY, SUMRS, RSQD, S, SA, SB, A, B, AVERX, AVERY,
000013                 2     F12A, F12B, AW1, SZ, SSB, SUMY, SSY, SUMSQ, YBAR, MNSQB,
000014                 3     MNSQB1, MNSQB2, MNSQC, B1, F2, C, BBSMSQ, SMGRPY
000015           1000  READ (3,700,END=999) (TITLE(I),I=1,14)
000016                  READ(3,5)N,IW,IX,STUDNT
000017                  READ(3,6)(X(I),I=1,N)
000018                  READ(3,6)(Y(I),I=1,N)
000019                  IF ( IW .EQ. 3 )  READ(3,6) (W(I),I=1,N)
000020                  READ(3,6)(P(I),I=1,10)
000021                  READ(3,6)(G(I),I=1,10)
000022                  GO TO (15,16,17), IW
000023               15 DO 18 I=1,N
000024                  W(I) = 1.0
000025               18 CONTINUE
000026                  GO TO 17
000027               16 DO 19 I=1,N
```

```
000028              W(I) = X(I)**(-2)
000029           19 CONTINUE
000030           17 CALL SORT3(X,Y,W,1,N,,TRUE.)
000031              IF ( IH .EQ. 2 )  GO TO 27
000032              WRITE(4,67)
000033              GO TO 28
000034           27 WRITE(4,7)
000035           28 WRITE(4,800)(TITLE(I),I=1,14)
000036              WRITE(4,12)N,IH,IW,STUDNT
000037              WRITE (4,23)
000038              WRITE (4,50) (X(I),I=1,N)
000039              WRITE(4,8)
000040              WRITE(4,50)(Y(I),I=1,N)
000041              WRITE(4,24)
000042              WRITE(4,50)(W(I),I=1,N)
000043              WRITE(4,21)
000044              WRITE(4,50)(P(I),I=1,10)
000045              WRITE(4,25)
000046              WRITE(4,50)(Q(I),I=1,10)
000047              SUMW = 0.000
000048              SUMWX = 0.000
000049              SUMWX2 = 0.000
000050              SUMWY = 0.000
000051              SUMWY2 = 0.000
000052              SUMWXY = 0.000
000053              AVERX = 0.000
000054              AVERY = 0.000
000055              SUMDX = 0.000
000056              SUMDY = 0.000
```

```
000057              SUMXY = 0.000
000058              IF ( M .EQ. 2 )  GO TO 31
000059              DO 29 I=1,N
000060              SUMWX=SUMWX+W(I)*X(I)
000061              SUMWX2 = SUMWX2 + W(I)*X(I)*X(I)
000062              SUMWY=SUMWY+W(I)*Y(I)
000063              SUMWY2 = SUMWY2 + W(I)*Y(I)*Y(I)
000064              SUMWXY = SUMWXY + W(I)*X(I)*Y(I)
000065              SUMW=SUMW+W(I)
000066           29 CONTINUE
000067              SUMDX = SUMWX2
000068              SUMDY = SUMWY2
000069              SUMXY = SUMWXY
000070              GO TO 32
000071           31 DO 26 I=1,N
000072              SUMDX = SUMDX + SUMW/(SUMW+W(I)) * W(I) * (X(I)-AVERX)**2
000073              SUMDY = SUMDY + SUMW/(SUMW+W(I)) * W(I) * (Y(I)-AVERY)**2
000074              SUMXY = SUMXY + SUMW/(SUMW+W(I)) * W(I)*(X(I)-AVERX)*(Y(I)-AVERY)
000075              AVERX = SUMW/(SUMW+W(I)) * AVERX  +  W(I)*X(I)/(SUMW+W(I))
000076              AVERY = SUMW/(SUMW+W(I)) * AVERY  +  W(I)*Y(I)/(SUMW+W(I))
000077              SUMWX=SUMWX+W(I)*X(I)
000078              SUMWX2 = SUMWX2 + W(I)*X(I)*X(I)
000079              SUMWY=SUMWY+W(I)*Y(I)
000080              SUMWY2 = SUMWY2 + W(I)*Y(I)*Y(I)
000081              SUMWXY = SUMWXY + W(I)*X(I)*Y(I)
000082              SUMW=SUMW+W(I)
000083           26 CONTINUE
000084           32 B=SUMXY/SUMDX
000085              A=AVERY-B*AVERX
```

```
000086                SUMRS = 0.000
000087                DO 15 I=1,N
000088                YHAT(I)=A+B*X(I)
000089                REGRY(I) = Y(I) - YHAT(I)
000090                SUMRS = SUMRS + W(I)*(Y(I)-YHAT(I))**2
000091             15 CONTINUE
000092                RSQD=(SUMDY-SUMRS)/SUMDY
000093                SSD=RSQD*SUMDY
000094                S2=(1.000-RSQD)*SUMDY/(N-IH)
000095                S=SQRT(S2)
000096                AW1=1.0/SUMW
000097                IF ( IH .EQ. 1 )  AW1 = 0.000
000098                DO 41 I=1,N
000099                SYHAT(I) = S * SQRT( AW1 + (X(I)-AVERX)**2/SUMDX )
000100                SY(I) = S * SQRT( 1./W(I) + AW1 + (X(I)-AVERX)**2/SUMDX )
000101                T(I)=REGRY(I)/SY(I)
000102                CLY1H(I) = YHAT(I) + STUDNT*SYHAT(I)
000103                CLY1L(I) = YHAT(I) - STUDNT*SYHAT(I)
000104                CLY2H(I) = YHAT(I) + STUDNT*SY(I)
000105                CLY2L(I) = YHAT(I) - STUDNT*SY(I)
000106                NHAT(I) = S2 / SYHAT(I)**2
000107                NI(I) = S2 / SY(I)**2
000108             41 CONTINUE
000109                DO 46 I=1,10
000110                YSUBE(I)=A+B*P(I)
000111                SYHE(I) = S * SQRT( AW1 + (P(I)-AVERX)**2/SUMDX )
000112                SEYE(I) = S * SQRT( 1./W(I) + AW1 + (P(I)-AVERX)**2/SUMDX )
000113                CLYE1H(I) = YSUBE(I) + STUDNT*SYHE(I)
000114                CLYE1L(I) = YSUBE(I) - STUDNT*SYHE(I)
```

```
000115              CLYE2H(I) = YSUBE(I) + STUDNT*SEYE(I)
000116              CLYE2L(I) = YSUBE(I) - STUDNT*SEYE(I)
000117              NEHAT(I) = S2 / SYHE(I)**2
000118              NE(I) = S2 / SEYE(I)**2
000119           46 CONTINUE
000120              F12B=(N-IH)*RSQD/(1.0-RSQD)
000121              SB=ABS(B)/SQRT(F12B)
000122              SA = S * SQRT( AW1 + AVERX**2/SUMDX )
000123              IF ( IH .EQ. 2 )  F12A = A**2/SA**2
000124              WRITE(4,102)SUMWX,SUMWY,SUMW
000125              WRITE(4,103)AVERX,AVERY
000126              WRITE(4,104)SUMDX,SUMDY
000127              WRITE(4,105)SUMXY
000128              WRITE(4,106)B,SSB
000129              WRITE(4,107)A
000130              WRITE(4,126)A,B
000131              WRITE(4,108)
000132              WRITE(4,50)(YHAT(I),I=1,N)
000133              WRITE(4,109)
000134              WRITE(4,50)(YSUBE(I),I=1,10)
000135              WRITE(4,110)
000136              WRITE(4,50)(REGRY(I),I=1,N)
000137              WRITE(4,111)RSQD
000138              WRITE(4,112)S,SUMRS
000139              WRITE(4,113)IH,F12B
000140              WRITE(4,114)SB
000141              WRITE(4,115)SA
000142              U = WTEST(REGRY,N,.FALSE.)
000143              IF(IH-2)144,147,147
```

```
000144            147 WRITE(4,110)IH,F12A
000145            144 WRITE(4,117)
000146                WRITE(4,50)(SYHAT(I),I=1,N)
000147                WRITE(4,118)
000148                WRITE(4,50)(SYME(I),I=1,10)
000149                WRITE(4,119)
000150                WRITE(4,50)(SY(I),I=1,N)
000151                WRITE(4,125)
000152                WRITE(4,50)(SEYE(I),I=1,10)
000153                WRITE(4,121)IH
000154                WRITE(4,51)(T(I),I=1,N)
000155                WRITE(4,122) U
000156                WRITE(4,123)
000157                WRITE(4,53)(CLY1H(I),CLY1L(I),I=1,N)
000158                WRITE(4,124)
000159                WRITE(4,53)(CLYE1H(I),CLYE1L(I),I=1,10)
000160                WRITE(4,126)
000161                WRITE(4,53)(CLY2H(I),CLY2L(I),I=1,N)
000162                WRITE(4,127)
000163                WRITE(4,53)(CLYE2H(I),CLYE2L(I),I=1,10)
000164                WRITE(4,130)
000165                WRITE(4,50)(NHAT(I),I=1,N)
000166                WRITE(4,131)
000167                WRITE(4,50)(NEHAT(I),I=1,10)
000168                WRITE(4,132)
000169                WRITE(4,50)(N1(I),I=1,N)
000170                WRITE(4,133)
000171                WRITE(4,50)(NE(I),I=1,10)
000172                WRITE(4,134)
```

```
000173                SUMW = 0
000174                SUMSQ = 0.0
000175                NUMGRP = 0
000176                SMGRPY = 0.0D0
000177                BGSMSQ = 0.0D0
000178                I = 1
000179             39 NGROUP = 1
000180                ISTART = I
000181             36 I = I+1
000182                IF (I .LE. N)  GO TO 35
000183                IF (NGROUP .GT. 1)  GO TO 40
000184                GO TO 42
000185             35 IF (X(ISTART)-X(I)) 34, ,34
000186                NGROUP = NGROUP+1
000187                GO TO 36
000188             34 IF (NGROUP .LE. 1)  GO TO 39
000189             40 SSY = 0.0D0
000190                YBAR = 0.0D0
000191                SUMY = 0.0D0
000192                SUMW = 0.0D0
000193                ISTOP=ISTART+NGROUP-1
000194                DO 38 J=ISTART,ISTOP
000195                SSY = SSY + SUMW/(SUMW+W(J)) * W(J) * (Y(J)-YBAR)**2
000196                YBAR = SUMW/(SUMW+W(J)) * YBAR  +  W(J)*Y(J)/(SUMW+W(J))
000197                SUMY = SUMY + W(J)*Y(J)
000198                SUMW = SUMW + W(J)
000199             38 CONTINUE
000200                NUMGRP = NUMGRP+1
000201                SUMSQ=SUMSQ+SSY
```

```
000202              SSY = SSY/(NGROUP-1)
000203              SUMN = SUMN+NGROUP-1
000204              BGSMSQ = BGSMSQ + SUMY**2/NGROUP
000205              SMGRPY = SMGRPY + SUMY
000206              WRITE(4,135) X(ISTART),NGROUP,SUMY,SSY
000207              GO TO 39
000208           42 NMIH = N-IH
000209              MNSQB = SUMRS/ABS(NMIH)
000210              B1 = SNGL(SUMRS) - SNGL(SUMSQ)
000211              DEGFB1 = NMIH-SUMN
000212              IF (DEGFB1 .NE. 0)  GO TO 45
000213              MNSQB1 = 0.0D0
000214              GO TO 48
000215           45 MNSQB1 = B1/DEGFB1
000216           48 IF (SUMN .EQ. 0)  GO TO 43
000217              BGSMSQ = BGSMSQ - SMGRPY**2/(SUMN+NUMGRP)
000218              NUMGRP = NUMGRP-1
000219              WRITE(4,136) SUMN,SUMSQ,NUMGRP,BGSMSQ
000220              MNSQB2 = SUMSQ/SUMN
000221              IF (DEGFB1 .EQ. 0)  GO TO 47
000222              F2 = B1*SUMN / (SUMSQ*DEGFB1)
000223              GO TO 44
000224           43 WRITE(4,138)
000225              MNSQB2 = 0.0D0
000226           47 F2 = 0.0D0
000227           44 C = SSB+SUMRS
000228              DEGFC = 1+NMIH
000229              MNSQC = C/DEGFC
000230              WRITE(4,137) SSY, SSB, F12B, NMIH, SUMRS, MNSQB, DEGFB1, B1,
```

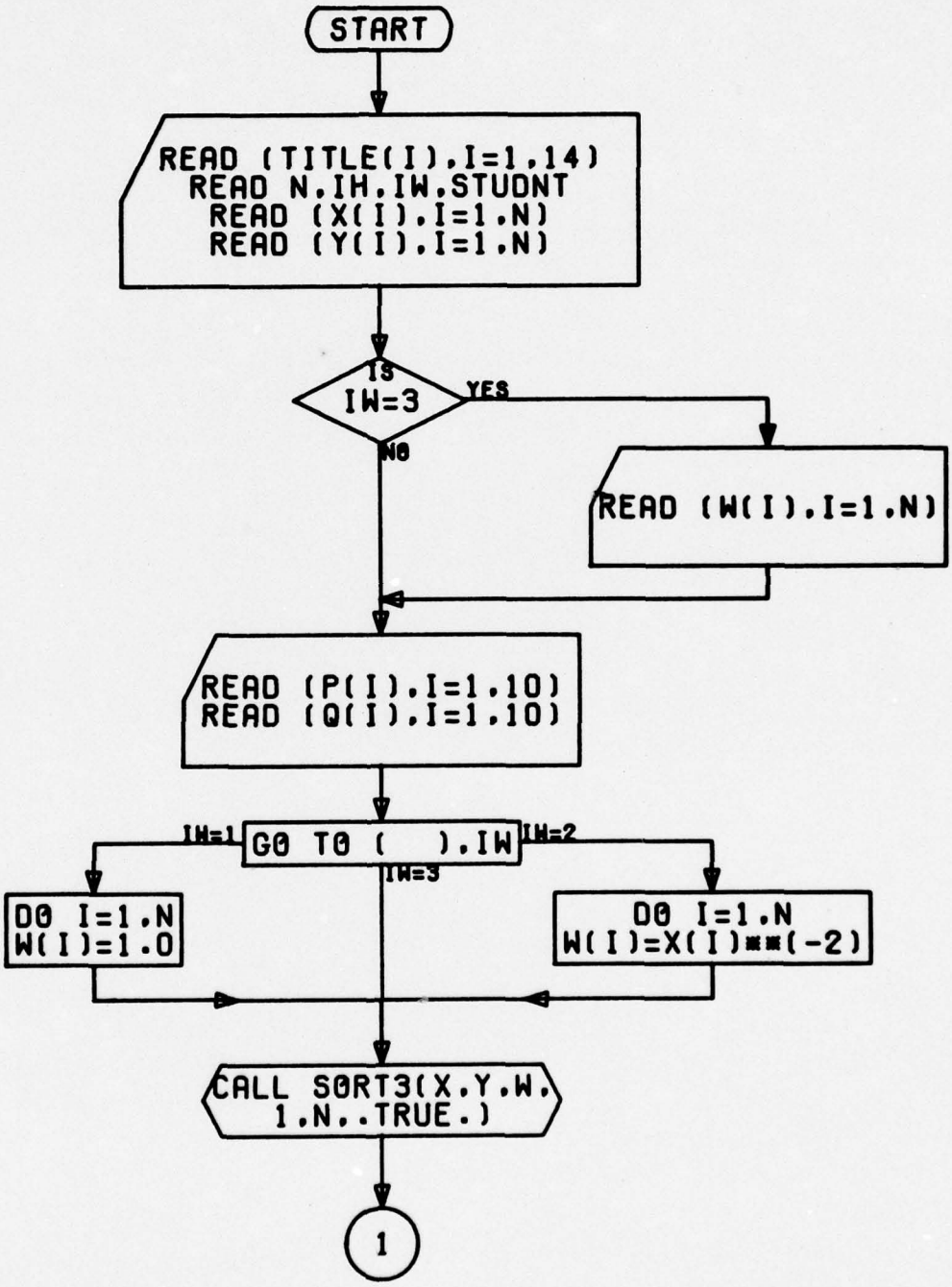
```
000231              1      MNSQB1, F2, SUMN, SUMSQ, MNSQB2, DEGFC, C, MNSQC
000232               GO TO 1000
000233            999  STOP
000234             700 FORMAT(13A6,A2)
000235               5 FORMAT(3I5,E15.8)
000236               6 FORMAT(5E15.8)
000237              67 FORMAT(60H1REGRESSION ANALYSIS FOR TWO VARIABLES--(THROUGH THE ORI
000238                1GIN))
000239               7 FORMAT(59H1REGRESSION ANALYSIS FOR TWO VARIABLES--(THROUGH THE MEA
000240                1NS))
000241             800 FORMAT(1H0,13A6,A2)
000242              12 FORMAT(//10X,4HN = ,I5,10X,5H1M = ,I5,10X,5H1W = ,I5,10X,14HSTUDEN
000243                1T'S T = ,E15.8)
000244              23 FORMAT(//10X,37HINPUT VECTOR X (INDEPENDENT VARIABLE))
000245              50 FORMAT(/E17.8)
000246               8 FORMAT(//10X,35HINPUT VECTOR Y (DEPENDENT VARIABLE))
000247              24 FORMAT(//10X,24HINPUT VECTOR W (WEIGHTS))
000248              21 FORMAT(//10X,78H1NPUT VECTOR P (POST EXPERIMENTAL EXPLORATORY POIN
000249                1TS OF X VARIABLE-ALWAYS TEN))
000250              25 FORMAT(//10X,48HPOST EXPERIMENTAL WEIGHTS W ASSOCIATED WITH P(I))
000251              52 FORMAT(//10X,50HSTUDENT'S T FOR CALCULATION OF CONFIDENCE LEVELS =
000252                1,E14.8)
000253             102 FORMAT(////10X,15HSUM OF WX(I) = ,E15.8,5X,15HSUM OF WY(I) = ,E15.
000254                1 8,5X,14HSUM OF W(I) = ,E15.8)
000255             103 FORMAT(//10X,12HMEAN OF X = ,E15.8,24X,12HMEAN OF Y = ,E15.8)
000256             104 FORMAT(//10X,29HSS OF DEV. FROM MEAN OF WX = ,E15.8,//10X,29HSS OF D
000257                1EV. FROM MEAN OF WY = ,E15.8)
000258             105 FORMAT(//10X,24HSUM OF CROSS-PRODUCTS = ,E15.8)
000259             106 FORMAT(//10X,29HREGRESSION COEFFICIENT = B = ,E15.8,10X,32HSSB = S
```

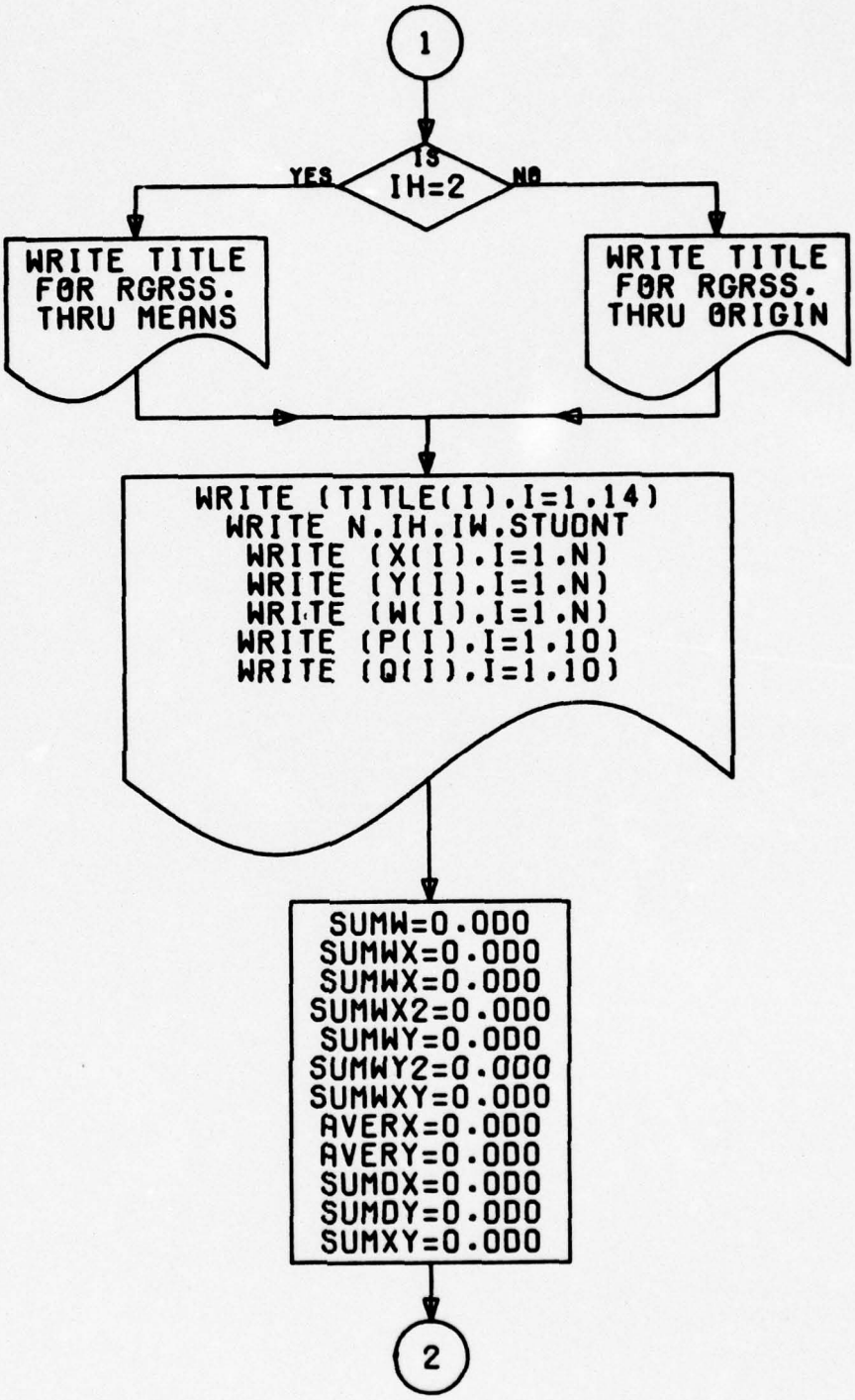
```
000260            1UM OF SQUARES DUE TO B = ,E15.8)
000261          107 FORMAT(//10X,16HINTERCEPT = A = ,E15.8)
000262          120 FORMAT(//10X,27HEQUATION = YHAT = A + BX = ,E15.8,5H  +  ,E15.8,2H
000263            1 X)
000264          108 FORMAT(//10X,18HPREDICTED Y = YHAT)
000265          109 FORMAT(//10X,33HPREDICTED EXPLORATORY Y = Y SUB E)
000266          110 FORMAT(//10X,45HDEVIATIONS OF Y FROM REGRESSION LINE (Y-YHAT))
000267          111 FORMAT(//10X,47HMULT. AND SIMPLE CORR. COEFF. SQUARED = R**2 = ,E1
000268            15.8)
000269          112 FORMAT(//10X,29HSTD. ERROR OF ESTIMATE = S = ,E15.8,10X,46HSSDEV =
000270            1 SUM SQUARES OF DEVIATIONS FROM LINE = ,E15.8)
000271          113 FORMAT(//10X,6HF(1,N-,I1,1H),1X,26HAS A TEST IF R = B = 0. = ,E15.
000272            15)
000273          114 FORMAT(//10X,28HSTD. ERROR OF B = S SUB B = ,E15.8)
000274          115 FORMAT(//10X,28HSTD. ERROR OF A = S SUB A = ,E15.8)
000275          116 FORMAT(//10X,6HF(1,N-,I1,1H),1X,82HAS A TEST IF A (INTERCEPT) = 0.
000276            1 OR THAT REGRESSION LINE GOES THROUGH THE ORIGIN = ,E15.8)
000277          117 FORMAT(//10X,76HSTD. ERROR OF EACH PREDICTED POINT (ON THE LINE),(
000278            1YHAT) AT EXPERIMENTAL X(I))
000279          118 FORMAT(//10X,107HSTD. ERROR OF EACH PREDICTED EXPLORATORY POINT (O
000280            1N THE LINE),(YHAT SUB E) AT POST EXPERIMENTAL X(I) OR P(I))
000281          119 FORMAT(//10X,61HSTD. ERROR OF EACH OBSERVED Y, (Y-YHAT) AT EXPERIM
000282            1ENTAL X(I),//10X,70HASSUMING THESE Y(I) ARE NEWLY OBSERVED INDIVIDU
000283            2ALS WITH WEIGHTS = W(I))
000284          125 FORMAT(//10X,104HSTD. ERROR OF EACH OBSERVED Y SUB E OR OF EACH (Y
000285            1 SUB E - YHAT SUB E) AT POST EXPERIMENTAL X(I) OR P(I),//10X,70HASS
000286            2UMING THESE Y(I) ARE NEWLY OBSERVED INDIVIDUALS WITH WEIGHTS = Q(I
000287            3))
000288          121 FORMAT(//10X,4HT(N-,I1,1H),1X,41HAS TEST OF EACH DEVIATION FROM RE
```

```
000289              1GRESSION)
000290           51 FORMAT(7F15.3)
000291          122 FORMAT(//10X,30HNORMALIZATION STATISTIC = U = ,E15.8)
000292          123 FORMAT(//10X,49HCONFIDENCE LIMITS FOR TRUE Y AT EXPERIMENTAL X(I))
000293          124 FORMAT(//10X,62HCONFIDENCE LIMITS FOR TRUE Y AT POST EXPERIMENTAL
000294              1X(I) OR P(I))
000295          126 FORMAT(//10X,49HCONFIDENCE LIMITS FOR TRUE Y AT EXPERIMENTAL X(I)/
000296              110X,70HASSUMING THESE Y(I) ARE NEWLY OBSERVED INDIVIDUALS WITH WEI
000297              2GHTS = W(I))
000298          127 FORMAT(//10X,62HCONFIDENCE LIMITS FOR TRUE Y AT POST EXPERIMENTAL
000299              1X(I) OR P(I)/10X,70HASSUMING THESE Y(I) ARE NEWLY OBSERVED INDIVID
000300              2UALS WITH WEIGHTS = Q(I))
000301           53 FORMAT(3(E17.8,2H ,,E17.8,2H ))
000302          130 FORMAT(//10X,11HEFFECTIVE N//10X,4HNHAT)
000303          131 FORMAT(//10X,10HNHAT SUB E)
000304          132 FORMAT(//10X,1HN)
000305          133 FORMAT(//10X,7HN SUB E)
000306          134 FORMAT(///10X,64HVARY = VARIANCE OF Y WITHIN GROUPS AT ONE X, GROU
000307              1P SIZE, Y TOTAL)
000308          135 FORMAT(//T16,'AT X = ',E13.6,6X,'N =',I4,6X,'SUM Y = ',E13.8,6X,
000309              1     'VARIANCE = ',E13.8,' ')
000310          136 FORMAT(//T11,'WITHIN GROUPS DEGREES OF FREEDOM AND SUM OF SQUARES'
000311              1     ,'  *  BETWEEN GROUPS DEGREES OF FREEDOM AND SUM OF SQUARES'/
000312              2     T64,'*'/28X,I5,14X,E13.8,T64,'*',20X,I5,14X,E13.8/T64,'*')
000313          137 FORMAT(1H1T26,'ANALYSIS OF VARIANCE'/1X,131('*')//8X,'SOURCE OF ',
000314              1     'VARIATION',11X,'DEGREES OF FREEDOM',9X,'SUM OF SQUARES',15X,
000315              2      'MEAN SQUARE',15X,'F'/1X,131('*')//T6,'SLOPE OF THE LINE',
000316              3     21X,'1',20X,E13.8,14X,E13.8,9X,E13.8//T6,'DEVIATIONS FROM ',
000317              4     'THE LINE',10X,I5,20X,E13.8,14X,E13.8//T12,'GROUP MEANS ',
```

```
000318            5     'FROM LINE',10X,I5,20X,E13.8,14X,E13.8,9X,E13.8//T12,
000319            6     'WITHIN GROUPS',18X,I5,20X,E13.8,14X,E13.8/1X,131('-')/T6,
000320            7     'TOTALS',20X,I5,20X,E13.8,14X,E13.8)
000321          138 FORMAT(//T11,'THERE ARE NO REPLICATES OF Y AT ANY SINGLE VALUE ',
000322            1     'OF X')
000323              END
```

S1281 PROGRAM REGRESS





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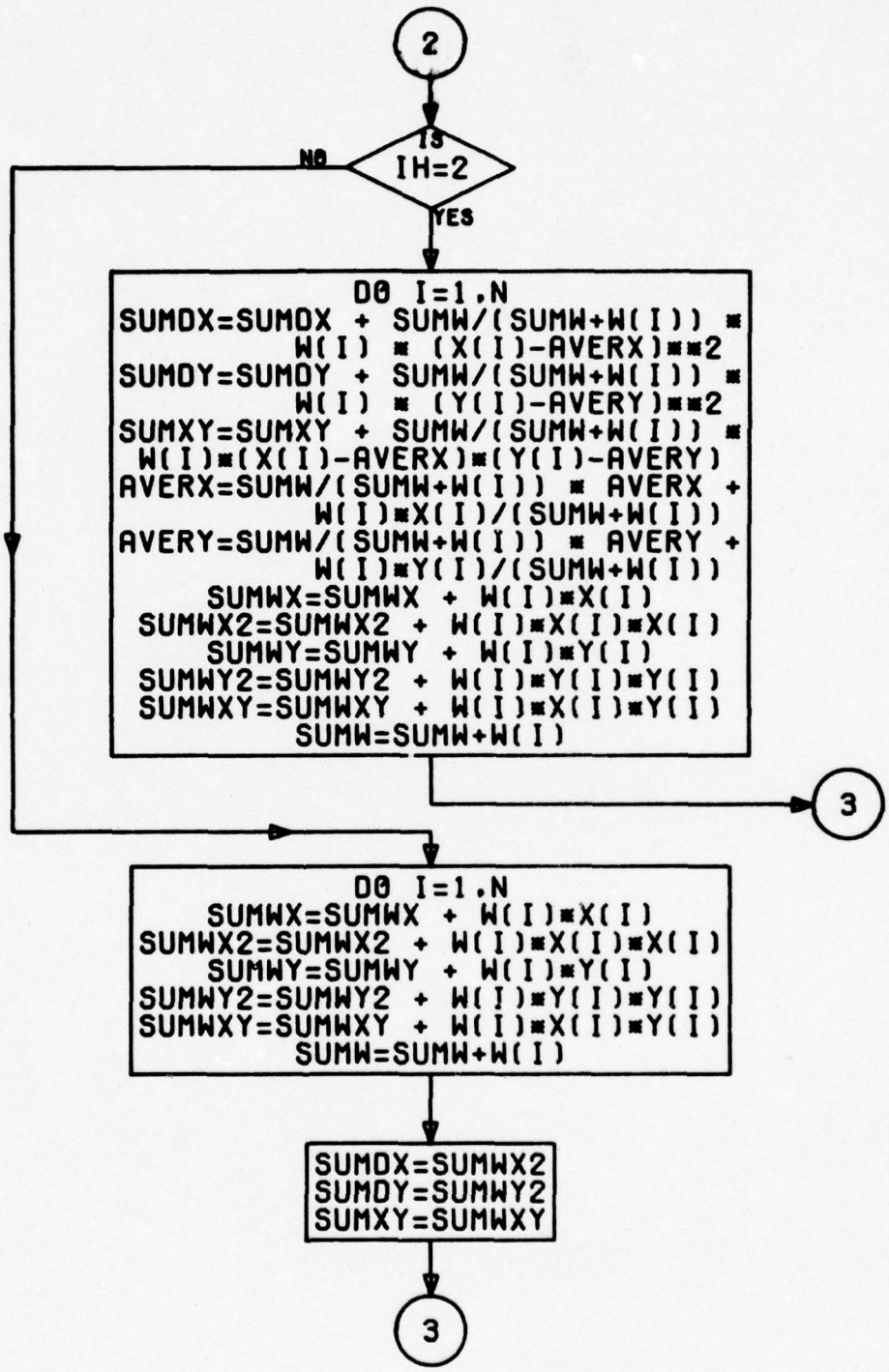
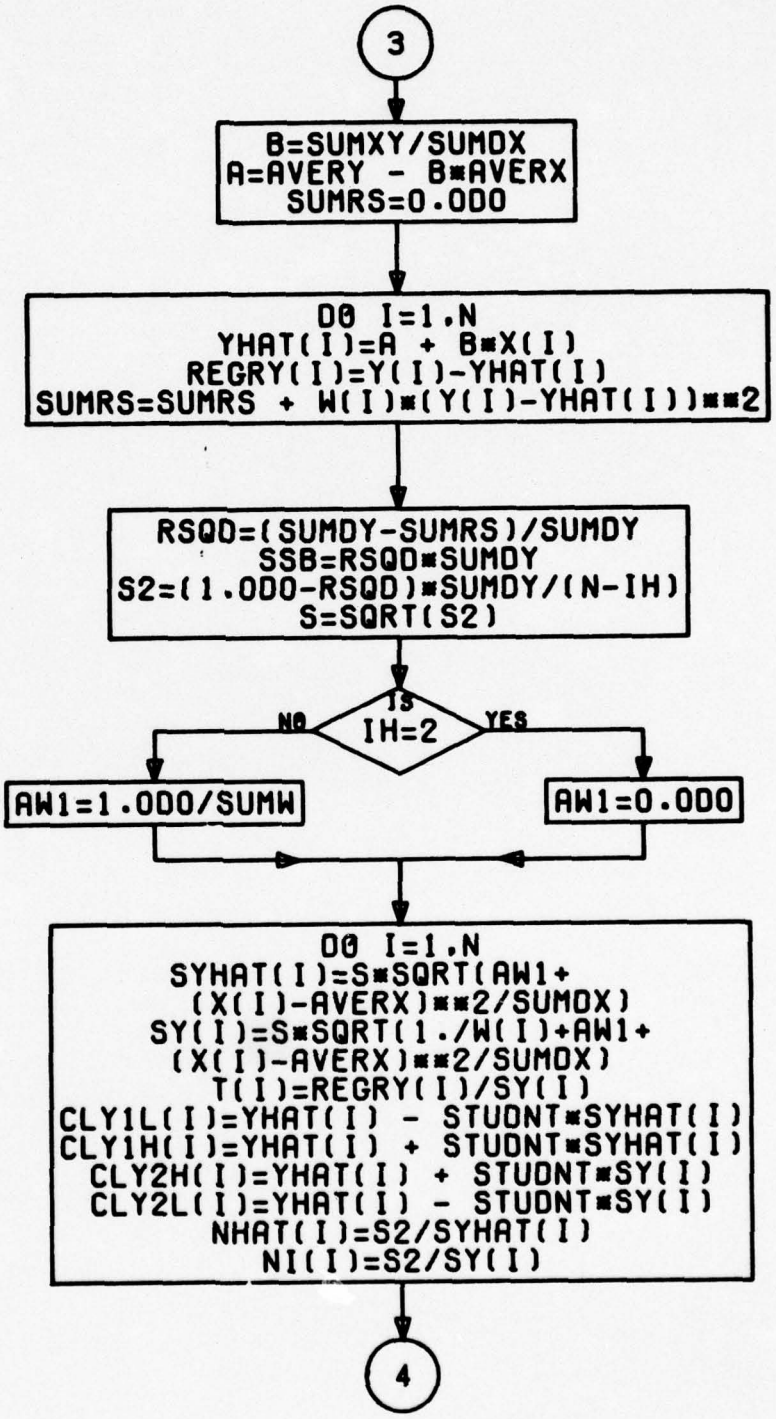
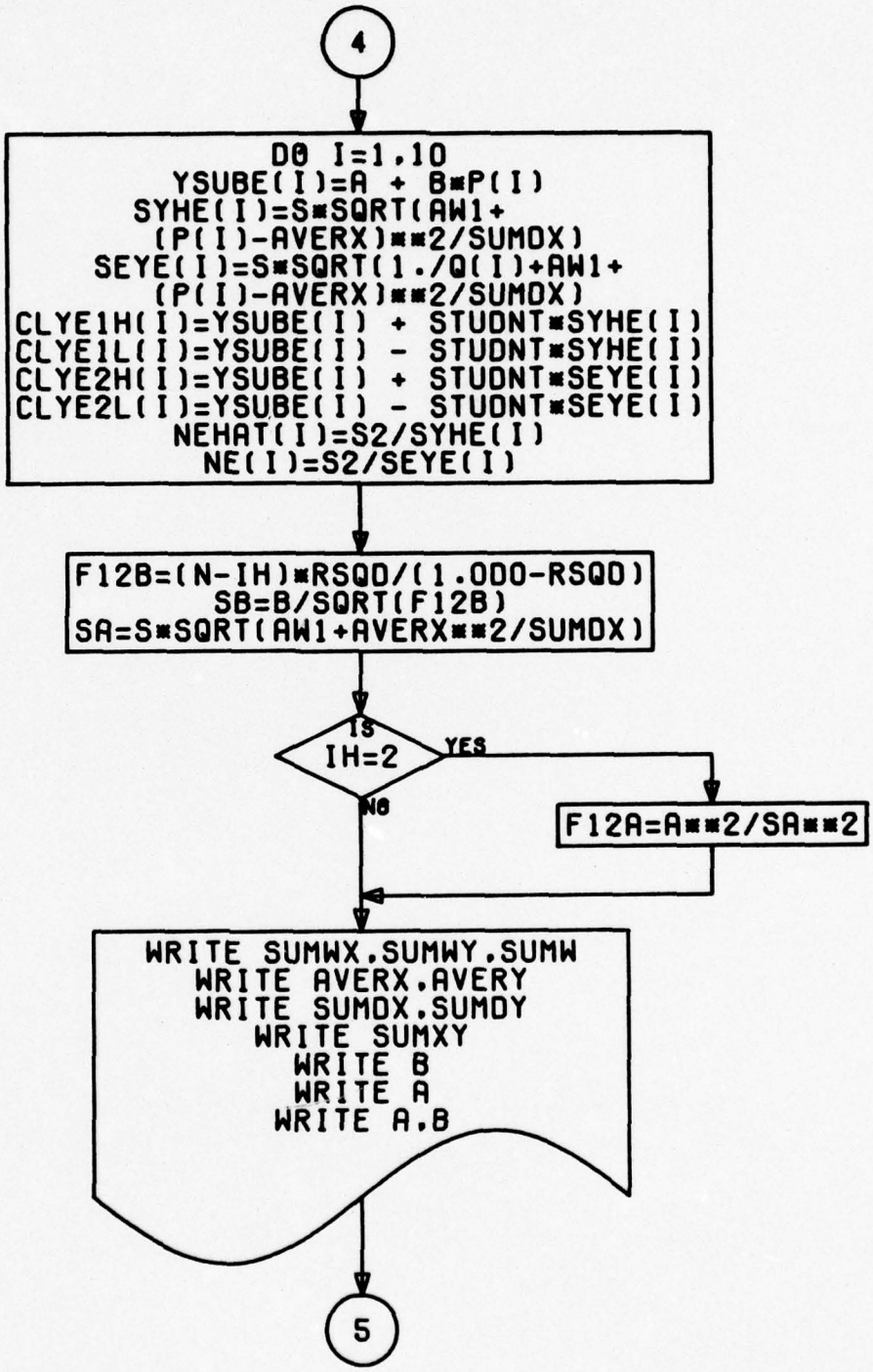
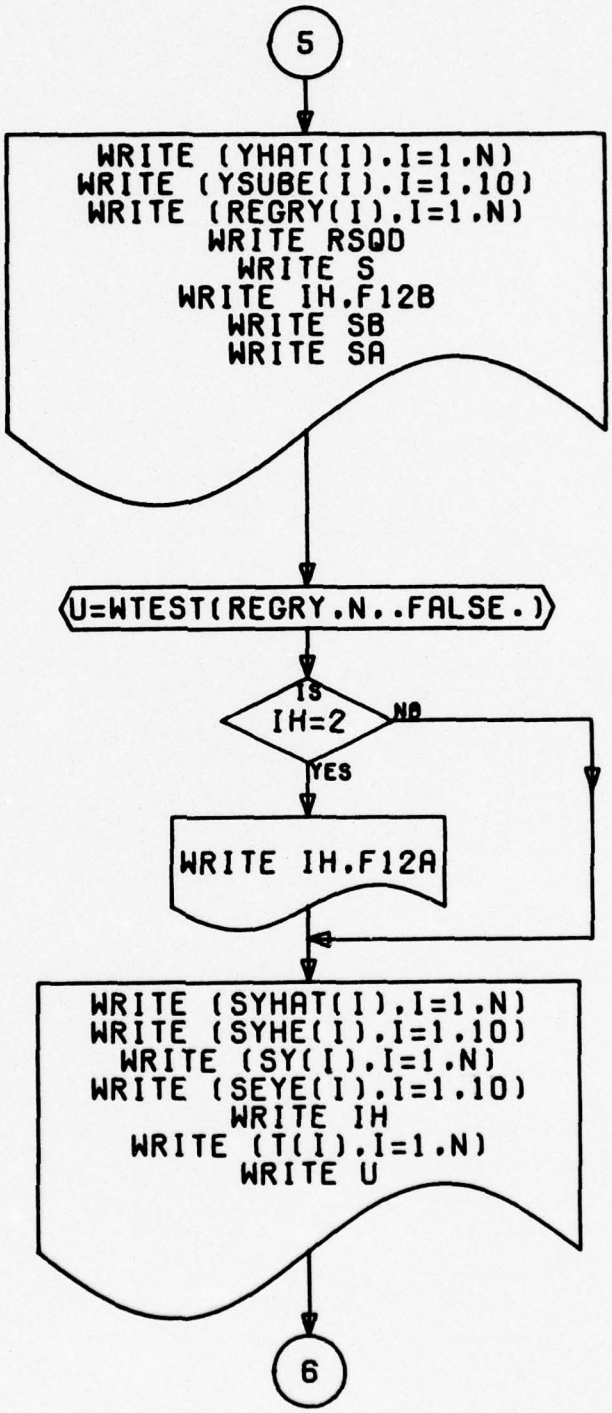
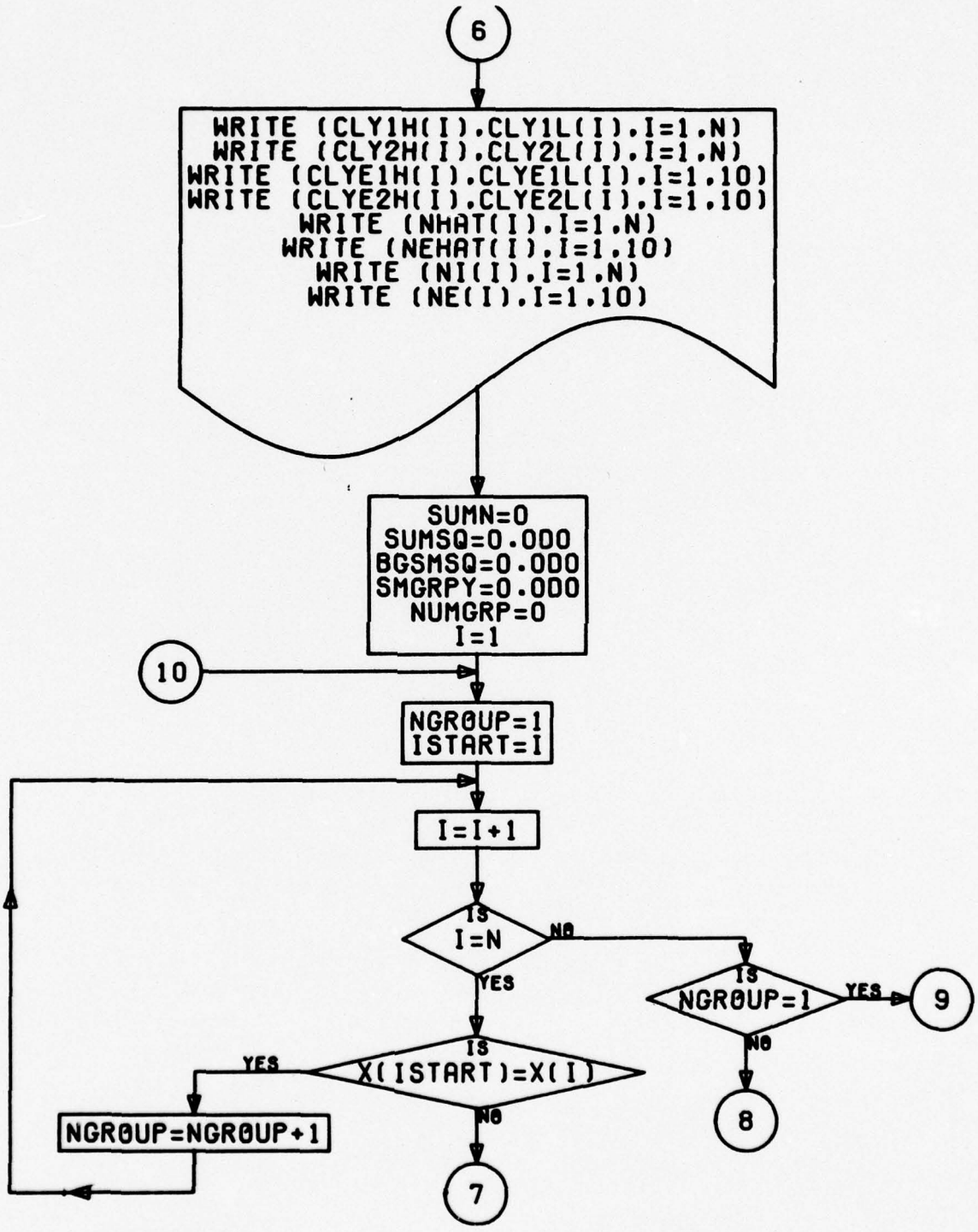

2
IS
IH=2
NO
YES
DO I=1,N
SUMDX=SUMDX + SUMW/(SUMW+W(I)) *
W(I) * (X(I)-AVERX)**2
SUMDY=SUMDY + SUMW/(SUMW+W(I)) *
W(I) * (Y(I)-AVERY)**2
SUMXY=SUMXY + SUMW/(SUMW+W(I)) *
W(I)*(X(I)-AVERX)*(Y(I)-AVERY)
AVERX=SUMW/(SUMW+W(I)) * AVERX +
W(I)*X(I)/(SUMW+W(I))
AVERY=SUMW/(SUMW+W(I)) * AVERY +
W(I)*Y(I)/(SUMW+W(I))
SUMWX=SUMWX + W(I)*X(I)
SUMWX2=SUMWX2 + W(I)*X(I)*X(I)
SUMWY=SUMWY + W(I)*Y(I)
SUMWY2=SUMWY2 + W(I)*Y(I)*Y(I)
SUMWXY=SUMWXY + W(I)*X(I)*Y(I)
SUMW=SUMW+W(I)
3
DO I=1,N
SUMWX=SUMWX + W(I)*X(I)
SUMWX2=SUMWX2 + W(I)*X(I)*X(I)
SUMWY=SUMWY + W(I)*Y(I)
SUMWY2=SUMWY2 + W(I)*Y(I)*Y(I)
SUMWXY=SUMWXY + W(I)*X(I)*Y(I)
SUMW=SUMW+W(I)
SUMDX=SUMWX2
SUMDY=SUMWY2
SUMXY=SUMWXY
3


3
B=SUMXY/SUMDX
A=AVERY - B*AVERX
SUMRS=0.000
DO I=1,N
YHAT(I)=A + B*X(I)
REGRY(I)=Y(I)-YHAT(I)
SUMRS=SUMRS + W(I)*(Y(I)-YHAT(I))**2
RSQD=(SUMDY-SUMRS)/SUMDY
SSB=RSQD*SUMDY
S2=(1.000-RSQD)*SUMDY/(N-IH)
S=SQRT(S2)
IS
IH=2
NO
YES
AW1=1.000/SUMW
AW1=0.000
DO I=1,N
SYHAT(I)=S*SQRT(AW1+
(X(I)-AVERX)**2/SUMDX)
SY(I)=S*SQRT(1./W(I)+AW1+
(X(I)-AVERX)**2/SUMDX)
T(I)=REGRY(I)/SY(I)
CLY1L(I)=YHAT(I) - STUDNT*SYHAT(I)
CLY1H(I)=YHAT(I) + STUDNT*SYHAT(I)
CLY2H(I)=YHAT(I) + STUDNT*SY(I)
CLY2L(I)=YHAT(I) - STUDNT*SY(I)
NHAT(I)=S2/SYHAT(I)
NI(I)=S2/SY(I)
4


4
DO I=1,10
YSUBE(I)=A + B*P(I)
SYHE(I)=S*SQRT(AW1+
(P(I)-AVERX)**2/SUMDX)
SEYE(I)=S*SQRT(1./Q(I)+AW1+
(P(I)-AVERX)**2/SUMDX)
CLYE1H(I)=YSUBE(I) + STUDNT*SYHE(I)
CLYE1L(I)=YSUBE(I) - STUDNT*SYHE(I)
CLYE2H(I)=YSUBE(I) + STUDNT*SEYE(I)
CLYE2L(I)=YSUBE(I) - STUDNT*SEYE(I)
NEHAT(I)=S2/SYHE(I)
NE(I)=S2/SEYE(I)
F12B=(N-IH)*RSQD/(1.0D0-RSQD)
SB=B/SQRT(F12B)
SA=S*SQRT(AW1+AVERX**2/SUMDX)
IS
IH=2
YES
NO
F12A=A**2/SA**2
WRITE SUMWX,SUMWY,SUMW
WRITE AVERX,AVERY
WRITE SUMDX,SUMDY
WRITE SUMXY
WRITE B
WRITE A
WRITE A,B
5


5
WRITE (YHAT(I).I=1.N)
WRITE (YSUBE(I).I=1.10)
WRITE (REGRY(I).I=1.N)
WRITE RSQD
WRITE S
WRITE IH.F12B
WRITE SB
WRITE SA
U=WTEST(REGRY.N..FALSE.)
IS
IH=2
NO
YES
WRITE IH.F12A
WRITE (SYHAT(I).I=1.N)
WRITE (SYHE(I).I=1.10)
WRITE (SY(I).I=1.N)
WRITE (SEYE(I).I=1.10)
WRITE IH
WRITE (T(I).I=1.N)
WRITE U
6



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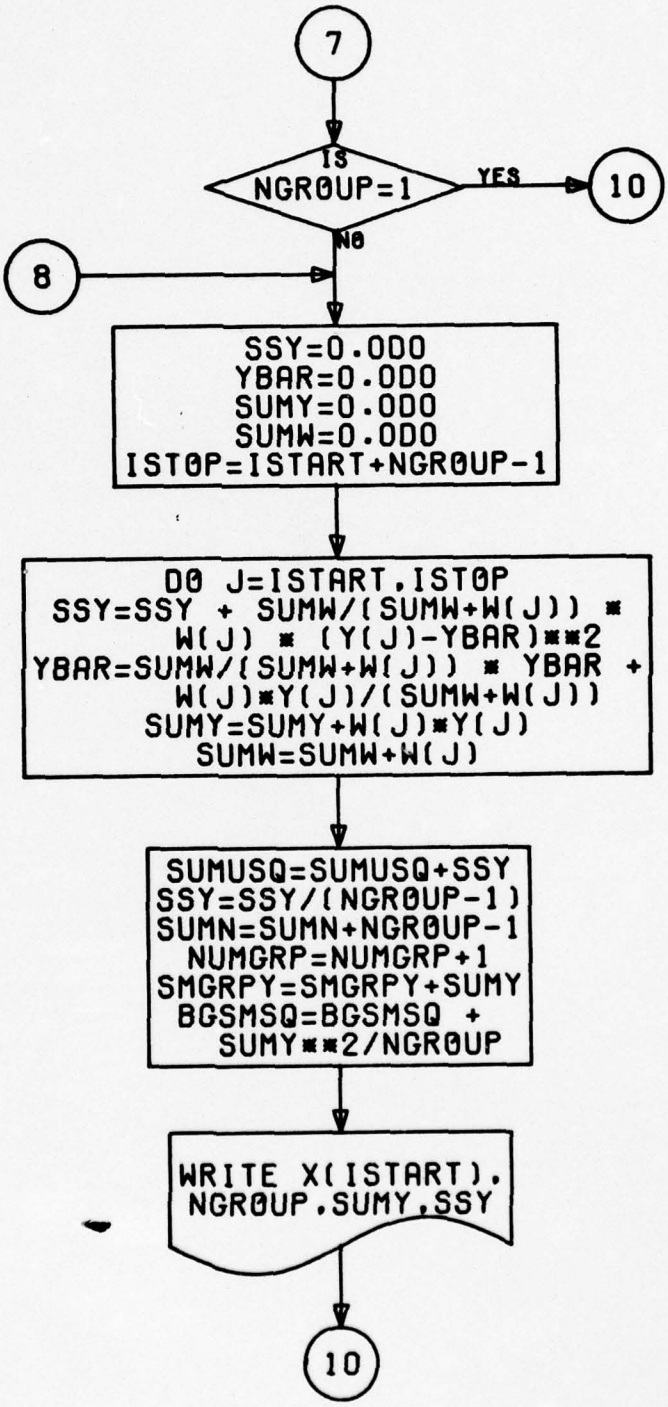
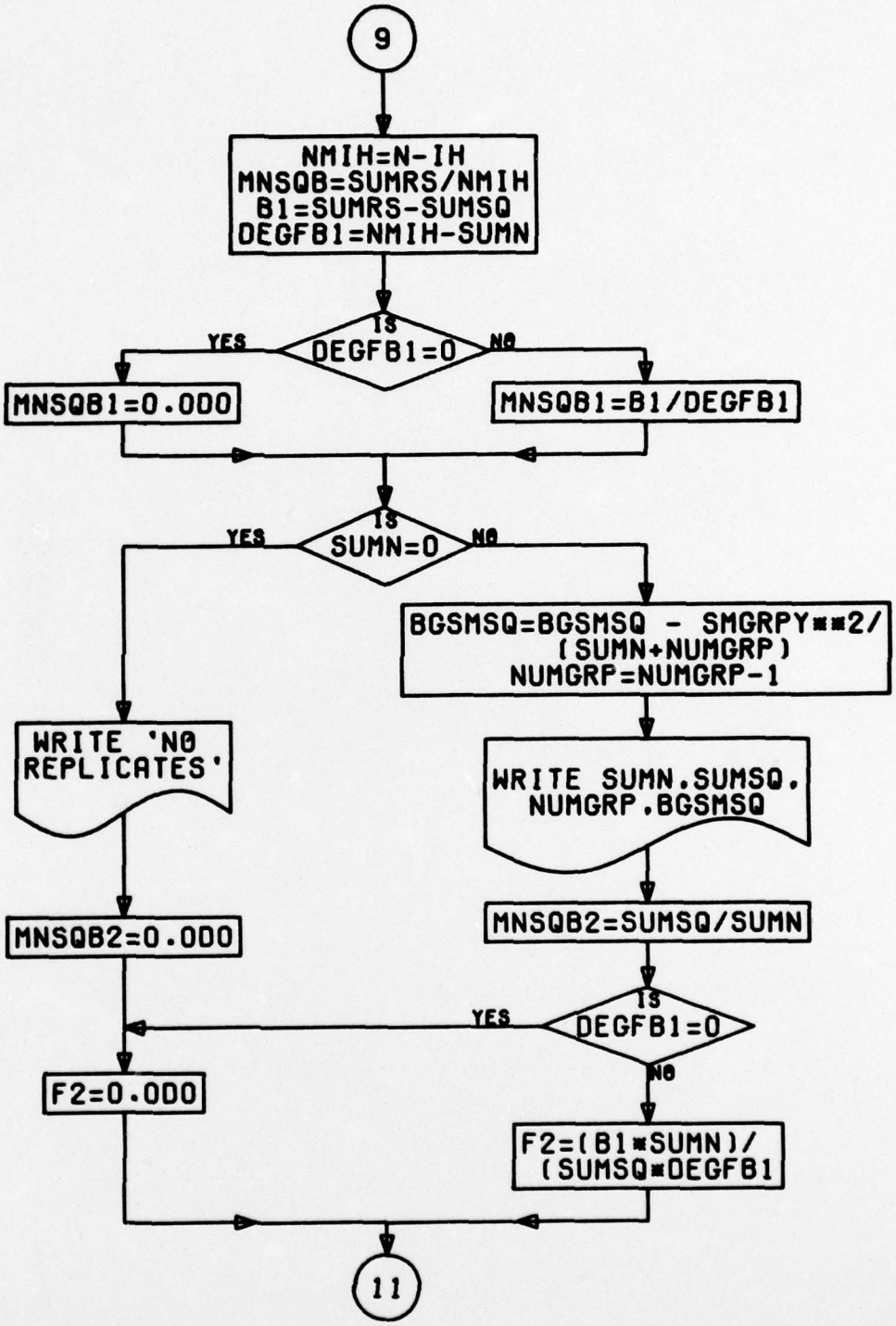

7
IS
NGROUP=1
YES
10
NO
8
SSY=0.0D0
YBAR=0.0D0
SUMY=0.0D0
SUMW=0.0D0
ISTOP=ISTART+NGROUP-1
DO J=ISTART,ISTOP
SSY=SSY + SUMW/(SUMW+W(J)) *
W(J) * (Y(J)-YBAR)**2
YBAR=SUMW/(SUMW+W(J)) * YBAR +
W(J)*Y(J)/(SUMW+W(J))
SUMY=SUMY+W(J)*Y(J)
SUMW=SUMW+W(J)
SUMUSQ=SUMUSQ+SSY
SSY=SSY/(NGROUP-1)
SUMN=SUMN+NGROUP-1
NUMGRP=NUMGRP+1
SMGRPY=SMGRPY+SUMY
BGSMSQ=BGSMSQ +
SUMY**2/NGROUP
WRITE X(ISTART),
NGROUP,SUMY,SSY
10



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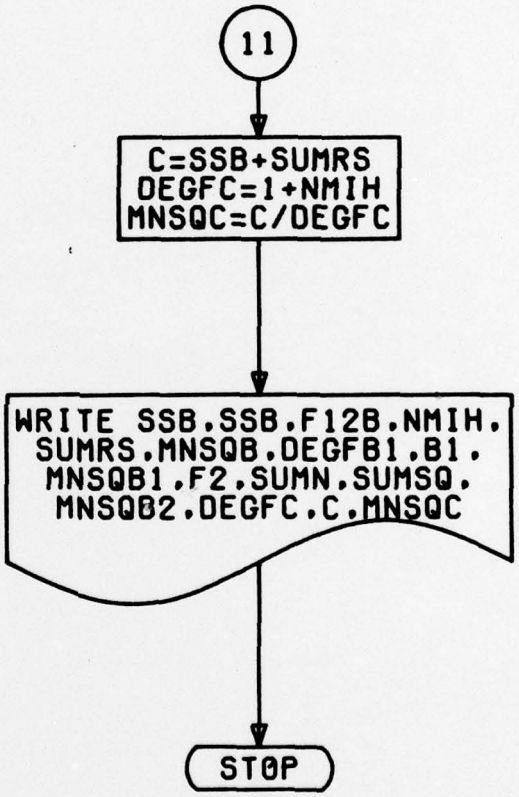
Official Photograph



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REGRESSION ANALYSIS FOR TWO VARIABLES--(THROUGH THE MEANS)

EXAMPLE FOR PROGRAM REGRESS FROM BROWNLEE PAGE 324 NO GROUP VARIANCE

N = 5 M = 2 NW = 3 STUDENT'S T = .45410000+01

INPUT VECTOR X (INDEPENDENT VARIABLE)
.26000000+02 .30500000+02 .40500000+02 .46800000+02 .57800000+02

INPUT VECTOR Y (DEPENDENT VARIABLE)
.37650000+01 .39100000+01 .85500000+01 .10630000+02 .11940000+02

INPUT VECTOR W (WEIGHTS)
.20000000+01 .20000000+01 .10000000+01 .10000000+01 .10000000+01

INPUT VECTOR P (POST EXPERIMENTAL EXPLORATORY POINTS OF X VARIABLE-ALWAYS TEN)
.10000000+02 .15000000+02 .50000000+02 .60000000+02 .65000000+02 .70000000+02 .75000000+02
.80000000+02 .80000000+02 .80000000+02

POST EXPERIMENTAL WEIGHTS W ASSOCIATED WITH P(I)
.10000000+02 .10000000+02 .10000000+02 .10000000+02 .10000000+02 .20000000+02 .30000000+02
.40000000+02 .10000000+03 .10000000+04

SUM OF WX(I) = .24910000+03 SUM OF WY(I) = .49710000+02 SUM OF W(I) = .70000000+01

MEAN OF X = .35585714+02 MEAN OF Y = .71014286+01

SS OF DEV. FROM MEAN OF WX = .11991286+04
SS OF DEV. FROM MEAN OF WY = .65023636+02

SUM OF CROSS-PRODUCTS = .27746015+03

REGRESSION COEFFICIENT = B = .23140149+00 SSB = SUM OF SQUARES DUE TO B = .64209320+02

INTERCEPT = A = -.11331589+01

EQUATION = YHAT = A + BX = -.11331589+01 + .23140149+00 X

PREDICTED Y = YHAT
.36105717+01 .59245667+01 .82386016+01 .10159234+02 .12241847+02

PREDICTED EXPLORATORY Y = Y SUB E
.11000500+01 .23378033+01 .10436916+02 .12750931+02 .13907938+02 .15064946+02 .16221953+02
.17378960+02 .17378960+02 .17378960+02

```
     DEVIATIONS OF Y FROM REGRESSION LINE (Y-YHAT)
.1744282E+00     -.4145866E+00      .3113983E+00      .4707659E+00     -.3018473E+00

     MULT. AND SIMPLE CORR. COEFF. SQUARED = R**2 =     .9874766E+00

     STD. ERROR OF ESTIMATE = S =     .5209977E+00          SSDEV = SUM SQUARES OF DEVIATIONS FROM LINE =     .8143160E+00

     F(1,N-2) AS A TEST IF R = B = 0. =     .2365518E+03

     STD. ERROR OF B = S SUB B =     .1504537E-01

     STD. ERROR OF A = S SUB A =     .5704651E+00

     F(1,N-2) AS A TEST IF A (INTERCEPT) = 0. OR THAT REGRESSION LINE GOES THROUGH THE ORIGIN =     .3945693E+01

     STD. ERROR OF EACH PREDICTED POINT (ON THE LINE),(YHAT) AT EXPERIMENTAL X(I)
.3004869E+00      .2112822E+00      .2103418E+00      .2798283E+00      .3879193E+00

     STD. ERROR OF EACH PREDICTED EXPLORATORY POINT (ON THE LINE),(YHAT SUB E) AT POST EXPERIMENTAL X(I) OR P(I)
.4526897E+00      .3670194E+00      .2929314E+00      .4167762E+00      .4843826E+00      .5539573E+00      .6248432E+00
.6966402E+00      .8966402E+00      .6906402E+00

     STD. ERROR OF EACH OBSERVED Y, (Y-YHAT) AT EXPERIMENTAL X(I),
     ASSUMING THESE Y(I) ARE NEWLY OBSERVED INDIVIDUALS WITH WEIGHTS = W(I)
.4754069E+00      .4246775E+00      .5618561E+00      .5913903E+00      .6495537E+00

     STD. ERROR OF EACH OBSERVED Y SUB E OR OF EACH (Y SUB E - YHAT SUB E) AT POST EXPERIMENTAL X(I) OR P(I),
     ASSUMING THESE Y(I) ARE NEWLY OBSERVED INDIVIDUALS WITH WEIGHTS = Q(I)
.4627145E+00      .4023023E+00      .3300843E+00      .4481588E+00      .5116349E+00      .5660747E+00      .6320419E+00
.7014938E+00      .8965857E+00      .6908350E+00

     T(N-2) AS TEST OF EACH DEVIATION FROM REGRESSION
      .367           -.976            .554             .796            -.465

     NORMALIZATION STATISTIC = U =     .9030503E+00

     CONFIDENCE LIMITS FOR TRUE Y AT EXPERIMENTAL X(I)
.4973003E+01 ,     .2246003E+01 ;     .6883928E+01 ,     .4965244E+01 ;     .9193763E+01 ,     .7283439E+01 ;
.1142993E+02 ,     .8886533E+01 ;     .1400338E+02 ,     .1048030E+02 ;

     CONFIDENCE LIMITS FOR TRUE Y AT POST EXPERIMENTAL X(I) OR P(I)
.5144037E+01 ,    -.7826256E+00 ;     .4004498E+01 ,     .6712281E+00 ;     .1176711E+02 ,     .9106714E+01 ;
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.14643512+02 ,     .10858350+02 ;     .16107519+02 ,     .11708357+02 ;     .17580466+02 ,     .12549425+02 ;
.19059360+02 ,     .13364540+02 ;     .20542404+02 ,     .14215517+02 ;     .20542404+02 ,     .14215517+02 ;
.20542404+02 ,     .14215517+02 ;

     CONFIDENCE LIMITS FOR TRUE Y AT EXPERIMENTAL X(I)
     ASSUMING THESE Y(I) ARE NEWLY OBSERVED INDIVIDUALS WITH WEIGHTS = W(I)
.57693946+01 ,     .14517488+01 ;     .78530475+01 ,     .39961258+01 ;     .10789990+02 ,     .56872128+01 ;
.12844736+02 ,     .74737504+01 ;     .15191471+02 ,     .92922237+01 ;

     CONFIDENCE LIMITS FOR TRUE Y AT POST EXPERIMENTAL X(I) OR P(I)
     ASSUMING THESE Y(I) ARE NEWLY OBSERVED INDIVIDUALS WITH WEIGHTS = Q(I)
.32820427+01 ,    -.92033058+00 ;     .41647163+01 ,     .51100871+00 ;     .11963075+02 ,     .89107568+01 ;
.14786020+02 ,     .10715042+02 ;     .16231272+02 ,     .11584604+02 ;     .17635491+02 ,     .12494400+02 ;
.19092055+02 ,     .13351651+02 ;     .20564444+02 ,     .14193477+02 ;     .20551238+02 ,     .14206683+02 ;
.20543288+02 ,     .14214633+02 ;

     EFFECTIVE N

     NMAT
.30062174+01      .30617436+01      .61350846+01      .34664767+01      .18036015+01

     NMAT SUB E
.14318475+01      .20130857+01      .31632957+01      .15626647+01      .11568969+01      .88454331+00      .69523165+00
.55931256+00      .55931256+00      .55931256+00

     N
.12009936+01      .15050573+01      .85984750+00      .77610988+00      .64334137+00

     N SUB E
.12677845+01      .18877129+01      .24031163+01      .13514745+01      .10369343+01      .84707938+00      .67948499+00
.55139965+00      .55821186+00      .55899991+00

     VART = VARIANCE OF Y WITHIN GROUPS AT ONE X, GROUP SIZE, Y TOTAL

     THERE ARE NO REPLICATES OF Y AT ANY SINGLE VALUE OF X
```

ANALYSIS OF VARIANCE

SOURCE OF VARIATION	DEGREES OF FREEDOM	SUM OF SQUARES	MEAN SQUARE	F
SLOPE OF THE LINE	1	.64209320+02	.64209320+02	.23655185+03
DEVIATIONS FROM THE LINE	3	.81431600+00	.27143867+00	
GROUP MEANS FROM LINE	3	.81431600+00	.27143867+00	.00000000
WITHIN GROUPS	0	.00000000	.00000000	
TOTALS	4	.65023636+02	.16255909+02	

REGRESSION ANALYSIS FOR TWO VARIABLES--(THROUGH THE MEANS)

EXAMPLE FOR PROGRAM REGRESS FROM BROWNLEE PAGE 324

N = 7 IM = 2 IW = 1 STUDENT'S T = .33650000+01

INPUT VECTOR X (INDEPENDENT VARIABLE)

.20500000+02	.20500000+02	.30500000+02	.30500000+02	.40500000+02	.48800000+02	.57800000+02

INPUT VECTOR Y (DEPENDENT VARIABLE)

.39200000+01	.36500000+01	.58200000+01	.52000000+01	.85500000+01	.10630000+02	.11940000+02

INPUT VECTOR W (WEIGHTS)

.10000000+01	.10000000+01	.10000000+01	.10000000+01	.10000000+01	.10000000+01	.10000000+01

INPUT VECTOR P (POST EXPERIMENTAL EXPLORATORY POINTS OF X VARIABLE-ALWAYS TEN)

.10000000+02	.15000000+02	.50000000+02	.60000000+02	.65000000+02	.70000000+02	.75000000+02
.80000000+02	.80000000+02	.80000000+02				

POST EXPERIMENTAL WEIGHTS W ASSOCIATED WITH P(I)

.10000000+02	.10000000+02	.10000000+02	.10000000+02	.10000000+02	.20000000+02	.30000000+02
.40000000+02	.10000000+03	.10000000+04				

SUM OF WX(I) = .24910000+03 SUM OF WY(I) = .49710000+02 SUM OF W(I) = .70000000+01

MEAN OF X = .35585714+02 MEAN OF Y = .71014286+01

SS OF DEV. FROM MEAN OF WX = .11991286+04
SS OF DEV. FROM MEAN OF WY = .65252286+02

SUM OF CROSS-PRODUCTS = .27746015+03

REGRESSION COEFFICIENT = B = .23140149+00 SSB = SUM OF SQUARES DUE TO B = .64209320+02

INTERCEPT = A = -.11331589+01

EQUATION = YHAT = A + BX = -.11331589+01 + .23140149+00 X

PREDICTED Y = YHAT

.36105717+01	.36105717+01	.59245867+01	.59245867+01	.82386016+01	.10159234+02	.12241847+02

PREDICTED EXPLORATORY Y = Y SUB E

.11800501+01	.23378035+01	.10436916+02	.12750931+02	.13907938+02	.15064946+02	.16221953+02
.17378960+02	.17378960+02	.17378960+02				

```
     DEVIATIONS OF Y FROM REGRESSION LINE (Y-YHAT)
.3094282?+00      .3942826+-01     -.10458666+00     -.72458667+00      .31139839+00      .47076595+00     -.3018734+00

     MULT. AND SIMPLE CORR. COEFF. SQUARED = R**2 =     .98401641+00

     STD. ERROR OF ESTIMATE = S =     .45672005+00          SSDEV = SUM SQUARES OF DEVIATIONS FROM LINE =     .10429660+01

     F(1,N-2) AS A TEST IF R = B = 0. =     .30782077+03

     STD. ERROR OF B = S SUB B =     .13189162-01

     STD. ERROR OF A = S SUB A =     .50008445+00

     F(1,N-2) AS A TEST IF A (INTERCEPT) = 0. OR THAT REGRESSION LINE GOES THROUGH THE ORIGIN =     .51344615+01

     STD. ERROR OF EACH PREDICTED POINT (ON THE LINE),(YHAT) AT EXPERIMENTAL X(I)
.2634140?+00      .2634140?+00      .18519769+00      .18519769+00      .18439103+00      .24530473+00      .34006012+00

     STD. ERROR OF EACH PREDICTED EXPLORATORY POINT (ON THE LINE),(YHAT SUB E) AT POST EXPERIMENTAL X(I) OR P(I)
.3790439?+00      .3217367?+00      .25679122+00      .36535678+00      .42462225+00      .48561325+00      .54775368+00
.6106927?+00      .6106927?+00      .6106927?+00

     STD. ERROR OF EACH OBSERVED Y (Y-YHAT) AT EXPERIMENTAL X(I),
     ASSUMING THESE Y(I) ARE NEWLY OBSERVED INDIVIDUALS WITH WEIGHTS = W(I)
.5272365?+00      .5272365?+00      .4928402?+00      .4928402?+00      .4925375?+00      .5184280?+00      .5694155?+00

     STD. ERROR OF EACH OBSERVED Y SUB E OR OF EACH (Y SUB E - YHAT SUB E) AT POST EXPERIMENTAL X(I) OR P(I),
     ASSUMING THESE Y(I) ARE NEWLY OBSERVED INDIVIDUALS WITH WEIGHTS = Q(I)
.4050274?+00      .3520665?+00      .2946201?+00      .3928675?+00      .4485124?+00      .4962357?+00      .5540642?+00
.6149475?+00      .6123962?+00      .6108635?+00

     T(N-2) AS TEST OF EACH DEVIATION FROM REGRESSION
      .587              .075             -.212            -1.470             .632              .908             -.530

     NORMALIZATION STATISTIC = U =     .9377154?+00

     CONFIDENCE LIMITS FOR TRUE Y AT EXPERIMENTAL X(I)
.4496961?+01 ,    .2724161?+01 ;    .4496961?+01 ,    .2724161?+01 ;    .6547777?+01 ,    .5301395?+01 ;
.6547777?+01 ,    .5301395?+01 ;    .8859077?+01 ,    .7618125?+01 ;    .1098468?+02 ,    .9333783?+01 ;
.1338615?+02 ,    .1109754?+02 ;

     CONFIDENCE LIMITS FOR TRUE Y AT POST EXPERIMENTAL X(I) OR P(I)
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.24563389+01 ,    -.94626614-01 ;     .34205142+01 ,     .12552128+01 ;     .11301018+02 ,     .95728134+01 ;
.13980356+02 ,     .11521505+02 ;     .15336792+02 ,     .12479084+02 ;     .16699034+02 ,     .13430857+02 ;
.18065144+02 ,     .14378762+02 ;     .19433941+02 ,     .15323979+02 ;     .19433941+02 ,     .15323979+02 ;
.19433941+02 ,     .15323979+02 ;


     CONFIDENCE LIMITS FOR TRUE Y AT EXPERIMENTAL X(I)
     ASSUMING THESE Y(I) ARE NEWLY OBSERVED INDIVIDUALS WITH WEIGHTS = W(I)
.53847293+01 ,     .18364141+01 ;     .53847293+01 ,     .18364141+01 ;     .75829939+01 ,     .42661794+01 ;
.75829939+01 ,     .42661794+01 ;     .98959904+01 ,     .65812128+01 ;     .11903744+02 ,     .84147239+01 ;
.14157931+02 ,     .10325764+02 ;


     CONFIDENCE LIMITS FOR TRUE Y AT POST EXPERIMENTAL X(I) OR P(I)
     ASSUMING THESE Y(I) ARE NEWLY OBSERVED INDIVIDUALS WITH WEIGHTS = Q(I)
.25457924+01 ,    -.18408632+00 ;     .35245952+01 ,     .11511338+01 ;     .11428313+02 ,     .94455189+01 ;
.14072930+02 ,     .11428931+02 ;     .15417182+02 ,     .12398694+02 ;     .16734779+02 ,     .13395112+02 ;
.18086379+02 ,     .14357527+02 ;     .19448259+02 ,     .15309662+02 ;     .19439680+02 ,     .15318240+02 ;
.19435516+02 ,     .15323405+02 ;


     EFFECTIVE N

     NHAT
.30062174+01      .30062174+01      .60817429+01      .60817429+01      .61350846+01      .34664767+01      .18038015+01

     NHAT SUB E
.14518475+01      .20150857+01      .31032957+01      .15626647+01      .11568969+01      .88454330+00      .69523164+00
.55931257+00      .55931257+00      .55931257+00

     N
.75036790+00      .75036790+00      .85879182+00      .85879182+00      .85984750+00      .77610988+00      .64334137+00

     N SUB E
.12677646+01      .16771297+01      .24031163+01      .13514745+01      .10369343+01      .84707937+00      .67948500+00
.55159969+00      .55620166+00      .55899991+00


     VART = VARIANCE OF Y WITHIN GROUPS AT ONE X, GROUP SIZE, Y TOTAL

          AT X =  .20500000+02      N =   2      SUM Y =  .75700000+01      VARIANCE =  .36450003-01 ;

          AT X =  .30500000+02      N =   2      SUM Y =  .11020000+02      VARIANCE =  .19220000+00 ;

     WITHIN GROUPS DEGREES OF FREEDOM AND SUM OF SQUARES  *  BETWEEN GROUPS DEGREES OF FREEDOM AND SUM OF SQUARES
                                                          *
                    2                .22865001+00         *                    1                .29756249+01
                                                          *
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ANALYSIS OF VARIANCE

SOURCE OF VARIATION	DEGREES OF FREEDOM	SUM OF SQUARES	MEAN SQUARE	F
SLOPE OF THE LINE	1	.64209320+02	.64209320+02	.30782077+03
DEVIATIONS FROM THE LINE	5	.10429660+01	.20859320+00	
GROUP MEANS FROM LINE	3	.81431599+00	.27143866+00	.23742721+01
WITHIN GROUPS	2	.22865001+00	.11432500+00	
TOTALS	6	.65252286+02	.10875381+02	

REGRESSION ANALYSIS FOR TWO VARIABLES--(THROUGH THE ORIGIN)

NEW TEST CASE FOR WEIGHTED REGRESSION WITH GROUPS

N = 6 IM = 1 IW = 3 STUDENT'S T = .20000000+01

INPUT VECTOR X (INDEPENDENT VARIABLE)
.10000000+02 .10000000+02 .20000000+02 .20000000+02 .20000000+02 .40000000+02

INPUT VECTOR Y (DEPENDENT VARIABLE)
.20000000+01 .50000000+01 .70000000+01 .60000000+01 .40000000+01 .80000000+01

INPUT VECTOR W (WEIGHTS)
.10000000+01 .20000000+01 .10000000+01 .30000000+01 .20000000+01 .10000000+01

INPUT VECTOR P (POST EXPERIMENTAL EXPLORATORY POINTS OF X VARIABLE-ALWAYS TEN)
.50000000+02 .55000000+02 .60000000+02 .65000000+02 .70000000+02 .75000000+02 .80000000+02
.85000000+02 .90000000+02 .95000000+02

POST EXPERIMENTAL WEIGHTS G ASSOCIATED WITH P(I)
.10000000+02 .15000000+02 .20000000+02 .25000000+02 .30000000+02 .35000000+02 .40000000+02
.45000000+02 .50000000+02 .55000000+02

SUM OF WX(I) = .19000000+03 SUM OF WY(I) = .53000000+02 SUM OF W(I) = .10000000+02

MEAN OF X = .00000000 MEAN OF Y = .00000000

SS OF DEV. FROM MEAN OF WX = .43000000+04
SS OF DEV. FROM MEAN OF WY = .30700000+03

SUM OF CROSS-PRODUCTS = .11000000+04

REGRESSION COEFFICIENT = B = .25581395+00 SSB = SUM OF SQUARES DUE TO B = .28139535+03

INTERCEPT = A = .00000000

EQUATION = YHAT = A + BX = .00000000 + .25581395+00 X

PREDICTED Y = YHAT
.25581395+01 .25581395+01 .51162791+01 .51162791+01 .51162791+01 .10232558+02

PREDICTED EXPLORATORY Y = Y SUB E
.12790698+02 .14069767+02 .15348837+02 .16627907+02 .17906977+02 .19186046+02 .20465116+02
.21744186+02 .23023256+02 .24302325+02

DEVIATIONS OF Y FROM REGRESSION LINE (Y-YHAT)
-.55813953+00 .24418605+01 .18837209+01 .88372093+00 -.11162791+01 -.22325581+01

MULT. AND SIMPLE CORR. COEFF. SQUARED = R**2 = .91659723+00

STD. ERROR OF ESTIMATE = S = .22629472+01 SSDEV = SUM SQUARES OF DEVIATIONS FROM LINE = .25604651+02

F(1,N-1) AS A TEST IF R = B = 0. = .54950045+02

STD. ERROR OF b = S SUB b = .54509622-01

STD. ERROR OF A = S SUB A = .00000000

STD. ERROR OF EACH PREDICTED POINT (ON THE LINE),(YHAT) AT EXPERIMENTAL X(I)
.34509622+00 .34509622+00 .69019244+00 .69019244+00 .69019244+00 .13803849+01

STD. ERROR OF EACH PREDICTED EXPLORATORY POINT (ON THE LINE),(YHAT SUB E) AT POST EXPERIMENTAL X(I) OR P(I)
.17254811+01 .18960292+01 .20705773+01 .22431254+01 .24156735+01 .25882216+01 .27607698+01
.29333178+01 .31058660+01 .32784141+01

STD. ERROR OF EACH OBSERVED Y, (Y-YHAT) AT EXPERIMENTAL X(I),
ASSUMING THESE Y(I) ARE NEWLY OBSERVED INDIVIDUALS WITH WEIGHTS = W(I)
.22691093+01 .16369351+01 .23658605+01 .14776137+01 .17426505+01 .26507343+01

STD. ERROR OF EACH OBSERVED Y SUB E OR OF EACH (Y SUB E - YHAT SUB E) AT POST EXPERIMENTAL X(I) OR P(I),
ASSUMING THESE Y(I) ARE NEWLY OBSERVED INDIVIDUALS WITH WEIGHTS = Q(I)
.18679677+01 .19659280+01 .21315105+01 .22883288+01 .24507501+01 .26163340+01 .27838593+01
.29520517+01 .31223104+01 .32925836+01

T(N-1) AS TEST OF EACH DEVIATION FROM REGRESSION
-.244 1.492 .796 .598 -.641 -.842

NORMALIZATION STATISTIC = U = .95420499+00

CONFIDENCE LIMITS FOR TRUE Y AT EXPERIMENTAL X(I)
.32483320+01 , .18679471+01 ; .32483320+01 , .18679471+01 ; .64966639+01 , .37358942+01 ;
.64966639+01 , .37358942+01 ; .64966639+01 , .37358942+01 ; .12993328+02 , .7471784+01 ;

CONFIDENCE LIMITS FOR TRUE Y AT POST EXPERIMENTAL X(I) OR P(I)
.16241660+02 , .93397354+01 ; .17865826+02 , .10273709+02 ; .19489992+02 , .11207683+02 ;
.21114157+02 , .12141656+02 ; .22738324+02 , .13075630+02 ; .24362489+02 , .14009603+02 ;
.25986656+02 , .14943577+02 ; .27610821+02 , .15877550+02 ; .29234987+02 , .16811524+02 ;
.30859154+02 , .17745497+02 ;

CONFIDENCE LIMITS FOR TRUE Y AT EXPERIMENTAL X(I)
ASSUMING THESE Y(I) ARE NEWLY OBSERVED INDIVIDUALS WITH WEIGHTS = W(I)

.71363582+01 ,	-.20200792+01 ;	.58320097+01 ,	-.71573064+00 ;	.98479999+01 ,	.38455814+00 ;
.86715065+01 ,	.21610516+01 ;	.86015800+01 ,	.16309781+01 ;	.15534027+02 ,	.49310895+01 ;

CONFIDENCE LIMITS FOR TRUE Y AT POST EXPERIMENTAL X(I) OR P(I)
ASSUMING THESE Y(I) ARE NEWLY OBSERVED INDIVIDUALS WITH WEIGHTS = Q(I)

.16526073+02 ,	.90547222+01 ;	.18041623+02 ,	.10097911+02 ;	.19611858+02 ,	.11085816+02 ;
.21204564+02 ,	.12051249+02 ;	.22808477+02 ,	.13005476+02 ;	.24416714+02 ,	.13953378+02 ;
.26032635+02 ,	.14897598+02 ;	.27649469+02 ,	.15838883+02 ;	.29267876+02 ,	.16778635+02 ;
.30887493+02 ,	.17717158+02 ;				

EFFECTIVE N

NHAT

.43000000+02	.43000000+02	.10750000+02	.10750000+02	.10750000+02	.26875000+01

NHAT SUB E

.17200000+01	.14214876+01	.11944445+01	.10177515+01	.87755103+00	.76444445+00	.67187500+00
.59515572+00	.53086420+00	.47645430+00				

N

.97727273+00	.19111111+01	.91489363+00	.23454545+01	.16862745+01	.72881357+00

N SUB E

.14675766+01	.12984399+01	.11271298+01	.97793952+00	.85261071+00	.74810489+00	.66077603+00
.58738711+00	.52526707+00	.47236231+00				

VARY = VARIANCE OF Y WITHIN GROUPS AT ONE X, GROUP SIZE, Y TOTAL

AT X = .10000000+02 N = 2 SUM Y = .12000000+02 VARIANCE = .60000000+01 ;

AT X = .20000000+02 N = 3 SUM Y = .33000000+02 VARIANCE = .37500000+01 ;

WITHIN GROUPS DEGREES OF FREEDOM AND SUM OF SQUARES	*	BETWEEN GROUPS DEGREES OF FREEDOM AND SUM OF SQUARES
3 .13500000+02	*	1 .30000000+02
	*	

ANALYSIS OF VARIANCE

SOURCE OF VARIATION	DEGREES OF FREEDOM	SUM OF SQUARES	MEAN SQUARE	F
SLOPE OF THE LINE	1	.28139535+03	.28139535+03	.54950045+02
DEVIATIONS FROM THE LINE	5	.25604651+02	.51209302+01	
GROUP MEANS FROM LINE	2	.12104651+02	.60523255+01	.13449612+01
WITHIN GROUPS	3	.13500000+02	.45000000+01	
TOTALS	6	.30700000+03	.51166667+02	