Report No. FAA-RD-75-108

(12)

FAA LIGHTNING PROTECTION STUDY: LIGHTNING - INDUCED TRANSIENTS ON BURIED SHIELDED TRANSMISSION LINES

John D. Nordgard

Chin-Lin Chen

June 1975

Final Report

Prepared for

U.S. DEPARTMENT OF TRANSPORTATION
FEDERAL AVIATION ADMINISTRATION
Systems Research & Development Service
Washington, D.C. 20590

NOTICE

This document is disseminated under the sponsorship of the Department of Transportation in the interest of infomation exchange. The United States Government assumes no liability for its contents or use thereof.

Technical Report Documentation Page

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
FAA-RD-75-108		

4. Title and Subtitle
FAA Lightning Protection Study: Lightning - Induced Transients on Buried Shielded Transmission Lines

5. Report Date
June 1975

6. Performing Organization Code
RADC/RBC

7. Author(s)
John D. Nordgard, Chin-Lin Chen

8. Performing Organization Report No.

9. Performing Organization Name and Address
Department of Defense
U.S. Air Force
Rome Air Development Center
Rome, New York 13441

10. Work Unit No. (TRAIS)
9567 0006

11. Contract or Grant No.
DOT-FA72WAI-356

12. Sponsoring Agency Name and Address
Department of Transportation
Federal Aviation Administration
System Research and Development Service
Washington, D. C. 20590

13. Type of Report and Period Covered
Final Report, June 1974-June 1975

14. Sponsoring Agency Code
ARD-350 (215-607)

15. Supplementary Notes
Prepared by Post Doctoral Program, Rome Air Development Center

16. Abstract

This report is primarily concerned with the analysis of induced transient current and voltage pulses on buried shielded transmission lines, due to earth conduction effects of nearby lightning discharges. Two basic analytical methods are presented in this report to describe the various kinds of coupling mechanisms between a lightning discharge to ground and an earth-return transmission line. The transmission line is assumed to be a long straight horizontal coaxial cable with an inner shield and an outer armor, terminated on both ends with typical communication equipment load impedances. The general case is considered here, in which the outermost conductor is not necessarily in perfect contact with the conducting earth but has a contact impedance with the earth, as in cables with an outer dielectric covering for corrosion or water protection. Both direct strikes to the cable via arcing from the terminal ground point of the lightning channel to the outer conductor of the coaxial cable and indirect strikes to the cable via conductive coupling mechanisms through the earth are considered.

17. Key Words

Lightning, Transmission Lines, Induced Transient

18. Distribution Statement
Document is available to the public through the National Technical Information Service, Springfield, Virginia 22151.

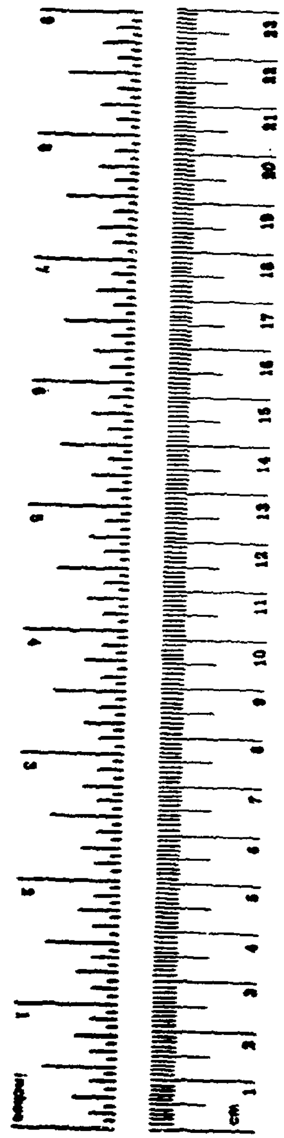
19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price
Unclassified	Unclassified	68	

METRIC CONVERSION FACTORS

Approximate Conversions to Metric Measures

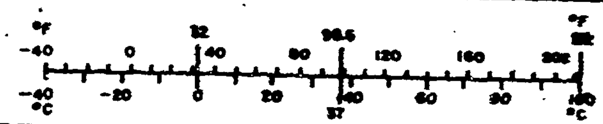
Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
in	inches	*2.5	centimeters	cm
ft	feet	30	centimeters	cm
yd	yards	0.9	meters	m
mi	miles	1.6	kilometers	km
		AREA		
in^2	square inches	6.5	square centimeters	cm^2
ft^2	square feet	0.09	square meters	m^2
yd^2	square yards	0.8	square meters	m^2
mi^2	square miles	2.6	square kilometers	km^2
	acres	0.4	hectares	ha
		MASS (weight)		
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2000 lb)	0.9	tonnes	t
		VOLUME		
tsp	teaspoons	5	milliliters	ml
Tbsp	tablespoons	15	milliliters	ml
fl oz	fluid ounces	30	milliliters	ml
c	cups	0.24	liters	l
pt	pints	0.47	liters	l
qt	quarts	0.95	liters	l
gal	gallons	3.8	liters	l
ft^3	cubic feet	0.03	cubic meters	m^3
yd^3	cubic yards	0.76	cubic meters	m^3
		TEMPERATURE (exact)		
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C

*1 in = 2.54 (exactly). For other exact conversions and more detailed tables, see NBS Misc. Publ. 286, Units of Weights and Measures, Price $2.25, SD Catalog No. C13.10:286.



Approximate Conversions from Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
mm	millimeters	0.04	inches	
cm	centimeters	0.4	inches	
m	meters	3.3	feet	
m	meters	1.1	yards	
km	kilometers	0.6	miles	
		AREA		
cm^2	square centimeters	0.16	square inches	
m^2	square meters	1.2	square yards	
km^2	square kilometers	0.4	square miles	
ha	hectares (10,000 m^2)	2.5	acres	
		MASS (weight)		
g	grams	0.035	ounces	
kg	kilograms	2.2	pounds	
t	tonnes (1000 kg)	1.1	short tons	
		VOLUME		
ml	milliliters	0.03	fluid ounces	
l	liters	2.1	pints	
l	liters	1.06	quarts	
l	liters	0.26	gallons	
m^3	cubic meters	35	cubic feet	
m^3	cubic meters	1.3	cubic yards	
		TEMPERATURE (exact)		
°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature	



FOREWORD

The Post-Doctoral Program at Rome Air Development Center is pursued via Project 9567 under the direction of Dr. W. W. Everett, Jr. The Post-Doctoral Program is a cooperative venture between RADC and the participating universities: Syracuse University (Department of Electrical and Computer Engineering, the U. S. Air Force Academy (Department of Electrical Engineering), Cornell University (School of Electrical Engineering), Purdue University (School of Electrical Engineering), University of Kentucky (Department of Electrical Engineering), Georgia Institute of Technology (School of Electrical Engineering), Clarkson College of Technology (Department of Electrical Engineering), State University of New York at Buffalo (Department of Electrical Engineering), Florida Technological University (Department of Electrical Engineering), Florida Institute of Technology (College of Engineering), Air Force Institute of Technology (Department of Electrical Engineering), and the University of Adelaide (Department of Electrical Engineering), in South Australia. The Post-Doctoral Program provides, via contract, the opportunity for faculty and visiting faculty at the participating universities to spend a year full time on exploratory development and operational problem-solving efforts with the post-doctorals splitting their time between RADC (or the ultimate customer) and the educational institutions.

This effort was conducted via RADC Job Order No. 9567 0006 for the Federal Aviation Administration. Mr. Fred Sakate was the FAA focal point, and he participated closely in the technical coordination meetings and cable testing sessions.

BIOGRAPHIES

John D. Nordgard is an Associate Professor in the School of Electrical Engineering, Georgia Institute of Technology. He received the B.E.E. (1966) from Georgia Institute of Technology and the M.S. (1967) and Ph.D. (1969) degrees in Applied Physics from California Institute of Technology. He has worked as Electronic Engineer for the Charleston Naval Shipyard and as Associate Engineer for the Jet Propulsion Laboratory. He spent one year at the University of Oslo doing research on the polar ionosphere. He joined the Georgia Tech faculty in 1970 where he is engaged in teaching electromagnetic field theory and doing research in related areas. His research interest includes the areas of moving media, heterogeneous media, plasma physics, scattering, high field emission, illumination, and multi-conductor transmission media. His memberships include Tau Beta Pi, Eta Kappa Nu, Phi Kappa Phi, Sigma Xi, IEEE, and IES.

Chin-Lin Chen was born in Honan, China, on March 27, 1937. He received the B.S.E.E. degree from National Taiwan University, Taipei, Taiwan, China, in 1958, the M.S. degree from North Dakota State University, Fargo, N.D., in 1961, and the Ph.D. degree from Harvard University, Cambridge, Mass., in 1965.

He served as a teaching assistant at North Dakota State University from 1959 to 1961 and as a research assistant and Teaching Fellow at Harvard University from 1962 to 1965. From 1965 to 1966, he was a Research Fellow in the Division of Engineering and Applied Physics, Harvard University. In 1966, he joined the School of Electrical Engineering, Purdue University, West Lafayette, IN, as an assistant professor. He is now an associate professor of Electrical Engineering. His research interests are in the areas of scattering and diffraction of electromagnetic waves, antenna radiation, surface acoustic wave devices, ultrasonics, transient protection devices, and the application of interactive computer graphics in electrical engineering education.

Dr. Chen is a member of Sigma Xi, Eta Kappa Nu, and IEEE and the American Society for Engineering Education. Since 1966, he has been a reviewer for the IEEE Transactions on Antennas and Propagations.

ACKNOWLEDGEMENT

The authors wish to acknowledge the contributions of Mr. Fred Sakate of FAA and Dr. W. W. Everett, Jr., of the RADC Post-Doctoral Program in formulating the program approach and for their guidance throughout the course of this study. They are indebted to Dr. B. J. Leon and Mr. Warren Peele of Purdue University for their support and encouragement. To all other members of the Post-Doctoral Program team on the FAA Lightning Protection Study, and in particular to Mr. R. M. Cosel (F.I.T.), Mr. Keith Huddleston (Georgia Tech), Dr. R. Larson (Georgia Tech), and Dr. A. W. Revay, Jr. (F.I.T.), the authors would like to express their appreciation for their contributions and assistance.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

CHAPTER 1

Introduction

Earth conduction problems are encountered in various communication and power system circuits in connection with strong electromagnetic disturbances caused by nearby lightning discharges. Excessive interference in conventional exposed metallic communication lines is indicated unless adequate protection measures are provided. This may require extra electromagnetic shielding of certain important communication lines and associated buildings housing sensitive equipment and/or the installation of protective devices on certain communication equipment.

This study is primarily concerned with the theoretical analysis of the resultant circuit disturbances caused by earth conduction effects of lightning discharges. Two basic analytical models are presented in this study to describe the various kinds of coupling mechanisms between a lightning discharge and an earth-return transmission line.

The theoretical study described in this report is part of a larger study program to provide protection for communication electronics equipment against transient electromagnetic disturbances. Current and voltage pulses are induced in cables running between buildings or equipment enclosures. These currents and voltages are then coupled into the terminal equipment. The electromagnetic disturbances may be the result of nearby lightning activities or man-made electromagnetic pulses. This larger study, known as the FAA Lightning Protection Study, has been performed by the Post-Doctoral Program through several of its member universities for the Federal Aviation Administration. The institutions include the Air Force Institute of Technology, Florida Institute of Technology, Georgia Institute of Technology, and Purdue University. The individual participants in the FAA Lightning Protection Study are listed in Appendix D.

1.1 FAA Lightning Protection Study

Increasing use of solid state, integrated circuit electronics in FAA communication and control equipment means that reliance on the over-voltage protection adequate for higher voltage electron tube and discrete transistor circuitry would be inadequate. The over-voltage protection of carbon blocks, in the several hundred volt range, and neon bulbs, with long, relatively high inductance leads in the 40 - 100 volt range, is not adequate for the solid state circuits which operate at lower voltage levels (presently down to 5 volts).

The first phase of the program is an overall study and consists of three technical tasks: (1) the determination of the voltage and current levels likely to be conducted to FAA equipment; (2) the determination of the susceptibility levels of FAA Instrument Landing System [AN(GRN-27(V)]; and (3) the determination of lightning protective devices that are available to reduce the levels of (1) to those permitted by (2). These three tasks have been performed in parallel with close interaction and are essentially completed[(1),(2),(3)]. Appendix D lists the schools having primary responsibility for each of the tasks. This report is the result of the work done under the first technical task. Only the theoretical foundation and analysis are presented in this report; detailed numerical calculations are to be reported in a companion report.

1.2 Lightning Induced Transients on Buried Cables

Numerous interference and protection problems are encountered in the development and operation of extensive communication and power systems. These problems are caused by the internal coupling of such systems with each other and by the external presence of the earth which, in some measure, is involved as a return conductor. The earth also serves as a return conductor for lightning currents, which often occasion disturbances in communication and power circuits. Lightning disturbances are largely atmospheric phenomena governed by the physical properties of the air. However, the behavior and effects of the lightning near the surface of the ground in communication and power systems are primarily earth conduction problems caused by the finite conductivity of the earth. Therefore, problems arise both in communication and power system circuits concerning the protection of transmission lines and associated equipment against interference and possible breakdown caused by excessive voltage or current surges caused by lightning discharges.

To deal adequately with such problems, it is necessary to consider theoretical solutions to the basic problem in which the earth, as well as conducting current paths, are involved in the lightning discharge. The analysis of such problems is inherently more complicated than the problem of completely metallic circuits embedded in an insulating medium, since the great extent of the earth necessitates the use of electromagnetic field theory, rather than conventional transmission line or circuit theory, in the solution of most aspects of the problem. It is necessary to restrict the analysis to fairly simple fundamental cases, in which simplified models of the earth, cable, and lightning channel geometries are used, on account of the complexities that would otherwise arise. Therefore, ionization effects caused by high induced voltages or electrolytic actions are not considered. Also, the heterogeneous character of the earth as a conductor and an electrolyte are not considered. Furthermore, the extremely variable nature of the lightning currents and voltages are not considered; however, typical average values of the lightning channel parameters are used.

1.3 Overview

This report is organized as follows:

First, the basic equations which govern the behavior of the electromagnetic disturbances caused by earth conduction effects of lightning discharges are listed in Chapter 2, along with the definitions of the scalar, vector, and Hertz potentials, which will be used to determine the form of the solutions. Then, the fields due to an electric dipole in free space are given in Chapter 3. These free space dipole fields are then generalized to determine the fields of a vertical or horizontally oriented dipole above a flat earth. A knowledge of these fields is essential for determining the mutual coupling between the dipole-like lightning channel and a buried wire. Next, the actual current induced on a buried wire is determined approximately (in Chapter 4) and exactly (in Chapter 5) from a knowledge of the mutual coupling impedances, which were determined previously in Chapter 3. In both the approximate and the exact formulations, the results are presented in the form of an equivalent distributed transmission line model of the coupling phenomena. In Chapter 4, the induced electric field intensity along the outer conductor of the cable is also determined for a lightning stroke to ground. In Chapter 5, the characteristic equation of the transmission line is solved, via integral transform techniques, to determine the characteristic values of the transmission line, e.g., the propagation constant and associated propagation modes.

The resulting induced current and voltage surges on the outer armour of the cable is thereby determined for a direct strike via arcing to the cable and for an indirect strike via conductive energization through the earth. Once the induced current and voltage surges are known on the outer armour of the cable, the voltage and current standing waves on the transmission line can be determined by finding, and integrating over the length of the line, the Green's Functions for a distributed voltage or current source, as is done in Chapter 6. Next, the induced current and voltage surges on the center conductor of the coaxial cable, resulting from the penetration of the lightning induced discharge through the outer armour and the inner shield of the coaxial cable are determined via the impedance transfer functions, which are developed in Chapter 7. Some of the details of the above analysis are presented in the Appendices.

CHAPTER 2

Basic Equations

This chapter of the study lists the basic equations which govern the behavior of the resultant electromagnetic disturbances caused by earth conduction effects of lightning discharges. For a complete exposition of the subject matter and for other aspects of the subject matter than are of primary concern here, reference is made to the literature on electromagnetic theory, transmission line and circuit theory.

2.1 Maxwell's Equations

We start with the time dependent Maxwell's equations for the field vectors $\vec{\mathcal{E}}$, $\vec{\mathcal{H}}$, $\vec{\mathcal{D}}$, and $\vec{\mathcal{B}}$.

$$-\nabla\times\vec{\mathcal{E}}(\vec{r},t) = \frac{\partial}{\partial t}\vec{\mathcal{B}}(\vec{r},t) \tag{2.1}$$

$$\nabla\times\vec{\mathcal{H}}(\vec{r},t) = \vec{\mathcal{J}}_s(\vec{r},t) + \vec{\mathcal{J}}(\vec{r},t) + \frac{\partial}{\partial t}\vec{\mathcal{D}}(\vec{r},t) \tag{2.2}$$

$$\nabla\cdot\vec{\mathcal{B}}(\vec{r},t) = 0 \tag{2.3}$$

$$\nabla\cdot\vec{\mathcal{D}}(\vec{r},t) = \rho_s(\vec{r},t) \tag{2.4}$$

where ρ_s and $\vec{\mathcal{J}}_s$ are respectively the source charge and current densities and $\vec{\mathcal{J}}$ is the conduction current density. Obviously, we will be dealing with four different media: air, earth, conducting wires, and insulating layers. Each medium is characterized by its permeability, permittivity, and conductivity: μ_i, ε_i, and σ_i, where i stands for either a(air), e(earth), c(conductor), or i(insulator). The constitutive relations in each medium are

$$\vec{\mathcal{B}}(\vec{r},t) = \mu_i\vec{\mathcal{H}}(\vec{r},t) \tag{2.5}$$

$$\vec{\mathcal{D}}(\vec{r},t) = \varepsilon_i\vec{\mathcal{E}}(\vec{r},t) \tag{2.6}$$

$$\vec{\mathcal{J}}(\vec{r},t) = \sigma_i\vec{\mathcal{E}}(\vec{r},t) \tag{2.7}$$

It is desirable to transform equations (2.1) - (2.7) to the "frequency" domain. For this purpose, we **define** the Fourier transform pairs between t and ω:

$$\vec{F}(\vec{r},t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \vec{F}(\vec{r},\omega) e^{+j\omega t} \, d\omega \tag{2.8}$$

$$\vec{F}(\vec{r},\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \vec{F}(\vec{r},t) e^{-j\omega t} \, dt \tag{2.8'}$$

where $\vec{F}$ and $\vec{F}$ are any of the field variables or sources defined above and $j = \sqrt{-1}$. Therefore, in the "frequency" domain, the equations become

$$\nabla \times \vec{E} = -j\omega\vec{B} \tag{2.1'}$$

$$\nabla \times \vec{H} = \vec{J}_s + \vec{J} + j\omega\vec{D} \tag{2.2'}$$

$$\nabla \cdot \vec{B} = 0 \tag{2.3'}$$

$$\nabla \cdot \vec{D} = \rho_s \tag{2.4'}$$

and

$$\vec{B} = \mu_i \vec{H} \tag{2.5'}$$

$$\vec{D} = \varepsilon_i \vec{E} \tag{2.6'}$$

$$\vec{J} = \sigma_i \vec{E} \tag{2.7'}$$

where the arguments $\vec{r}$ and ω are dropped, for simplicity. While the source terms ρ_s and $\vec{J}_s$ are arbitrary, they are related by the conservation of charge, i.e.

$$\nabla \cdot (\vec{J}_s + \vec{J}) = -j\omega\rho_s \tag{2.9}$$

Making use of the conservation of charge (2.9) and the **constitutive relations** (2.5') - (2.7'), the flux densities $\vec{D}$ and $\vec{B}$ can be eliminated in favor of the field intensities $\vec{E}$ and $\vec{H}$, i.e.

$$\nabla \times \vec{E} = -j\omega\tilde{\mu}_i \, \vec{H} \tag{2.1''}$$

$$\nabla \times \vec{H} = +j\omega\tilde{\varepsilon}_i \, \vec{E} + \vec{J}_s \tag{2.2''}$$

$$\nabla \cdot \vec{H} = 0 \tag{2.3''}$$

$$\nabla \cdot \vec{E} = -\frac{1}{j\omega\tilde{\varepsilon}_i} \, \nabla \cdot \vec{J}_s \tag{2.4''}$$

where the complex permeability and complex permittivity are defined by

$$\tilde{\mu}_i = \mu_i$$

$$\tilde{\varepsilon}_i = \varepsilon_i - j\frac{\sigma i}{\omega}$$

are complex quantities.

2.2 Hertz Vectors

Hertz showed that under ordinary conditions, the field intensities $\vec{E}$ and $\vec{H}$ are derivable from a single vector function $\vec{\pi}$, known as the Hertz vector. It can be shown, by direct substitution, that equations (2.1'') - (2.4'') are satisfied by writing

$$\vec{E} = k_i^2 \vec{\pi} + \nabla(\nabla\cdot\vec{\pi}) \tag{2.10}$$

$$\vec{H} = j\omega\tilde{\varepsilon}_i \nabla\times\vec{\pi} \tag{2.11}$$

with the Hertz vector $\vec{\pi}$ specified by the differential equation

$$\nabla^2\vec{\pi} + k_i^2\vec{\pi} = -\vec{J}_s/(j\omega\tilde{\varepsilon}_i) \tag{2.12}$$

where

$$k_i^2 = (\beta_i + j\alpha_i)^2 = \omega^2\tilde{\mu}_i\tilde{\varepsilon}_i$$

with $\alpha \geq 0$, $\beta \geq 0$.

It is recalled that the general solution of an inhomogeneous linear differential equation consists of two parts: a homogeneous solution and a particular solution. One possible way to express the particular solution is

$$-\frac{1}{4\pi j\omega\tilde{\varepsilon}_i}\int_{V'} \vec{J}_s(\vec{r}')\,\frac{e^{-jk_i|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|}\,dv' \tag{2.13}$$

The homogeneous solution can be expressed in various forms. For the problem at hand, it is convenient to express it in terms of the elementary cylindrical wave functions $Z_n^{(4)}$. More specifically, each component of the Hertz vector $\vec{\pi}$ can be written as

$$\sum_{n=-\infty}^{\infty} e^{jn\phi} \int_0^{\infty} C_n(\tau) Z_n(\tau r) \quad e^{\pm z\xi_i} \quad d\tau \tag{2.14}$$

where $\xi_i = \sqrt{\tau^2 - k_i^2}$ and where the dummy variable τ of the integration represents the transverse part of the propagation constant. To insure convergence at $z \to \pm\infty$, we choose Re $\xi_i > 0$.

In equation (2.14), the coefficient $C_n(\tau)$ is left unspecified. To complete the description, it is necessary to state the boundary conditions: the tangential components of the field intensities $\vec{E}$ and $\vec{H}$ are continuous at a boundary free of surface current. The boundary conditions can also be expressed in terms of the Hertz vector $\vec{\pi}$ and its derivatives and will be discussed in detail later.

2.3 Vector and Scalar Potentials

The Hertz vector is also related to the vector and scalar potential $\vec{A}$ and ϕ via the following relations

$$\vec{A} = \mu_i(\sigma_i + j\omega\varepsilon_i)\,\vec{\pi} = j\omega\tilde{\varepsilon}_i\tilde{\mu}_i\vec{\pi} \tag{2.15}$$

$$\phi = -\nabla\cdot\vec{\pi} \tag{2.16}$$

CHAPTER 3

Fields Due To An Electric Dipole

3.1 An Electric Dipole in Free Space

Consider an electric dipole $Id\ell$ surrounded by free space ($\epsilon=\epsilon_0$, $\mu=\mu_0$, $\sigma=0$). For convenience, the coordinate system is so chosen that the dipole is located at the origin and oriented along the z-axis. If the medium is assumed to be infinitely large, then the only condition that the Hertz vector must satisfy is the radiation condition at infinity. Therefore, the Hertz vector is obtained from equation (2.13) by noting that $\vec{J}_s = Id\ell\,\delta(\vec{r}\,')\hat{z}$, i.e.

$$\vec{\pi} = + \hat{z}\; j\frac{Id\ell}{4\pi\omega\epsilon_0} \frac{e^{-jk_0\sqrt{r^2+z^2}}}{\sqrt{r^2+z^2}} \tag{3.1}$$

where

$$k_0 = \omega\sqrt{\mu_0\epsilon_0}.$$

It is instructive and convenient to express equation (3.1) as the superposition of elementary cylindrical waves. For this purpose, use is made of Sommerfeld's formula[5],[6], and we obtain,

$$\vec{\pi} = + \hat{z} j \frac{Id\ell}{4\pi\omega\epsilon_0} \int_0^\infty \frac{\tau}{\xi_0} J_0(\tau r) e^{-|z|\xi_0}\, d\tau \tag{3.2}$$

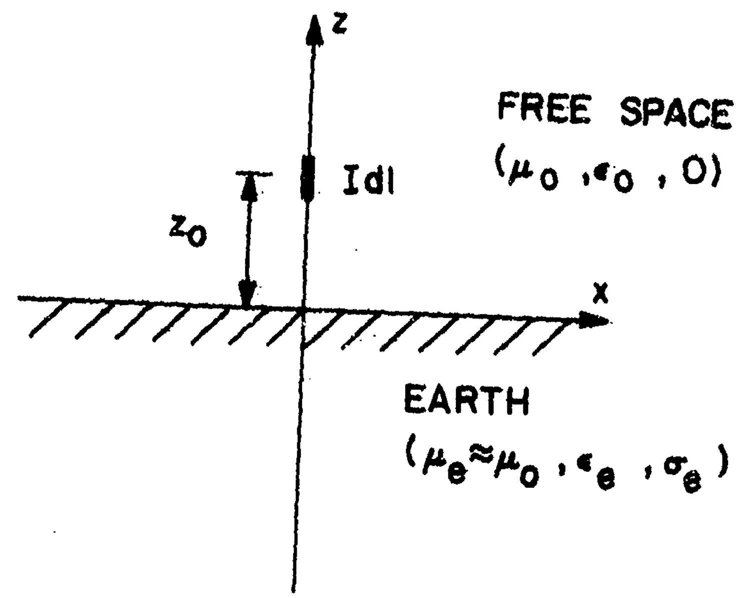
where

$$\xi_0 = \sqrt{\tau^2 - k_0^2}$$

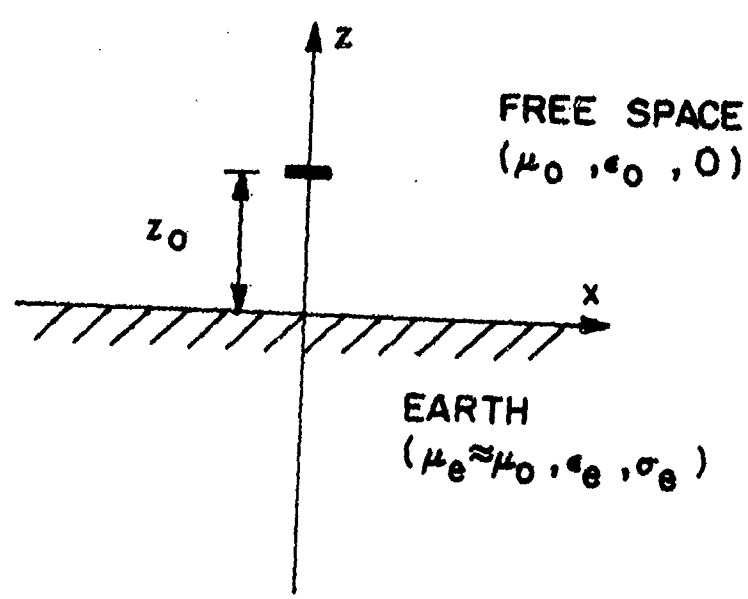
and the branch cut is chosen such that $-\pi/2 \le \arg \xi_0 < \frac{\pi}{2}$. A comparison of equation (3.2) with equation (2.14) shows that only the term independent of angle ($n=0$) is used, as expected from physical considerations.

3.2 A Vertical Electric Deipole Above a Flat Earth

Now consider two regions separated by an interface z=0. The space above the interface is taken to be free space ($\mu=\mu_0$, $\epsilon=\epsilon_0$, $\sigma=0$), while the medium below the interface is taken to be earth ($\mu_e \simeq \mu_0$, ϵ_e, σ_e). Our aim is to derive the expressions for the fields generated by an electric dipole $Id\ell$ oriented along the z-axis and located at $z=z_0$, as shown in Fig. 1a. The procedure is as follows. In the absence of the earth, the field, referred to as the primary field, is due to the dipole alone, and the Hertz potential



(a) A Vertical Dipole



(b) A Horizontal Dipole

Fig. 1. A Dipole above a Flat Earth

is given by equation (3.2), with the dipole shifted from $z=0$ to $z=z_0$. In the presence of the earth, the primary field is partially reflected by the interface and partially transmitted into the earth. It will be demonstrated later that for the geometry under consideration, all the boundary conditions can be satisfied by working with only the z-component of the Hertz vector $\vec{\pi}$, and only the term independent of angle (n=0) in equation (2.14) is needed. Thus, for $z > 0$, the z-component of the Hertz potential can be written as

$$\pi_{za} = + j \frac{Id\ell}{4\pi\omega\varepsilon_0} \int_0^\infty \left[\frac{\tau e^{-|z-z_0|\xi_0}}{\xi_0} + R_{zz}(\tau) e^{-z\xi_0} \right] J_0(\tau r) d\tau \qquad (3.3)$$

and for $z < 0$

$$\pi_{ze} = + j \frac{Id\ell}{4\pi\omega\varepsilon_0} \int_0^\infty T_{zz}(\tau) e^{z\xi_e} J_0(\tau r)\, d\tau \qquad (3.4)$$

where $R_{zz}(\tau)$ and $T_{zz}(\tau)$ are the reflection and transmission coefficients to be determined and $k_e^2 = j\omega\mu_0(\sigma_e + j\omega\varepsilon_e)$

$$\xi_e = \sqrt{\tau^2 - k_e^2}$$

and the branch is chosen such that $-\frac{\pi}{2} \leq \arg\xi_e < \frac{\pi}{2}$.

To determine the reflection and transmission coefficients $R_{zz}(\tau)$ and $T_{zz}(\tau)$, it is necessary to appeal to the boundary conditions. Since $\vec{\pi}_i = \hat{z}\, \pi_{zi}$ and π_{zi} is independent of ϕ, we have, from equations (2.10) and (2.11)

$$\vec{E} = \hat{r}\frac{\partial^2 \pi_{zi}}{\partial r \partial z} + \hat{z}\left(\frac{\partial^2 \pi_{zi}}{\partial z^2} + k_i^2 \pi_{zi}\right) \qquad (3.5)$$

$$\vec{H} = -\,(\sigma_i + j\omega\varepsilon_i) \frac{\partial \pi_{zi}}{\partial r}\, \hat{\phi} \qquad (3.6)$$

Thus, the boundary conditions at $z=0$ require that

$$\frac{\partial^2 \pi_{za}}{\partial r \partial z} = \frac{\partial^2 \pi_{ze}}{\partial r \partial z} \qquad (3.7)$$

$$+ j\omega\varepsilon_0 \frac{\partial \pi_{za}}{\partial r} = (\sigma_e + j\omega\varepsilon_e) \frac{\partial \pi_{ze}}{\partial r} \qquad (3.8)$$

for all values of r. Since equations (3.7) and (3.8) must be valid for all values of r, these two equations can be simplified by integrating with respect to r, i.e.,

$$\frac{\partial \pi_{za}}{\partial z} = \frac{\partial \pi_{ze}}{\partial z} \tag{3.9}$$

$$k_0^2 \pi_{za} = k_e^2 \pi_{ze} \tag{3.10}$$

In equation (3.10), the coefficients are expressed in a convenient form by noting that $\mu_a \simeq \mu_e \simeq \mu_0$. By substituting equations (3.3) and (3.4) into equations (3.9) and (3.10), we obtain

$$R_{zz}(\tau) = \frac{\tau}{\xi_0} \left[1 - \frac{2 k_0^2 \xi_e}{k_e^2 \xi_0 + k_0^2 \xi_e}\right] e^{-z_0 \xi_0} \tag{3.11}$$

$$T_{zz}(\tau) = \frac{2 k_0^2 \tau}{k_e^2 \xi_0 + k_0^2 \xi_e} e^{-z_0 \xi_0} \tag{3.12}$$

Upon substituting equation (3.11) into equation (3.3), and making use of Sommerfeld's formula, we obtain

$$\pi_{za} = + j \frac{I d\ell}{4\pi\omega\varepsilon_0} \left[\frac{e^{-jk_0 R}}{R} + \frac{e^{-jk_0 R'}}{R'} - \Lambda\right] \tag{3.13}$$

where

$$R = \sqrt{r^2 + (z - z_0)^2}$$

$$R' = \sqrt{r^2 + (z + z_0)^2}$$

and

$$\Lambda = 2k_0^2 \int_0^\infty \frac{\tau \xi_e}{\xi_0 [k_e^2 \xi_0 + k_0^2 \xi_e]} e^{-\xi_0 (z + z_0)} J_0(\tau r)\, d\tau \tag{3.14}$$

The field intensities in the air are determined by substituting equation (3.13) into equations (3.5) and (3.6). More specifically, the cylindrical components of the fields, for $z > 0$, are

$$E_r = +j \frac{I d\ell}{4\pi\omega\varepsilon_0} \frac{\partial^2}{\partial r \partial z} Z_t \tag{3.15}$$

$$E_\phi = 0 \tag{3.16}$$

$$E_z = +j \frac{I d\ell}{4\pi\omega\varepsilon_0} \left[Z_\ell + \frac{\partial^2}{\partial z^2} Z_t \right] \tag{3.17}$$

and

$$H_r = 0$$

$$H_\phi = \frac{I d\ell}{4\pi} \frac{\partial}{\partial z} Z_t \tag{3.18}$$

$$H_z = 0 \tag{3.19}$$

where

$$Z_t = \frac{e^{-jk_0 R}}{R} + \frac{e^{-jk_0 R'}}{R'} - \Lambda, \tag{3.20}$$

and

$$Z_\ell = k_0^2 Z_t \tag{3.21}$$

The significance of the terms Z_t and Z_ℓ are discussed in Appendix C.

3.3 Horizontal Electric Dipole above a Flat Earth

Next, consider a horizontal electric dipole $I d\ell$ oriented along the x-axis, as shown in Fig. 1b. If only the x-component of the Hertz vector $\vec{\pi}$, i.e., $\vec{\pi} = \hat{x}\,\pi_x$, were used, the boundary conditions at the interface z=0 cannot be met [7]. In fact, we have to supplement $\hat{x}\pi_x$ with the homogeneous solution as given in equation (2.14) for $\hat{z}\pi_z$. Since the boundary conditions are somewhat complicated, it is convenient to work with Cartesian coordinates first, and then to transform the results into cylindrical coordinates.

Since

$$\vec{\pi}_i = \hat{x}\pi_{xi} + \hat{z}\,\pi_{zi} \tag{3.22}$$

then the rectanqular components of the field intensities $\vec{E}$ and $\vec{H}$ are, from equations (2.10) and (2.11),

$$E_x = k_i^{\,2}\,\pi_{xi} + \frac{\partial^2\pi_{xi}}{\partial x^2} + \frac{\partial^2\pi_{zi}}{\partial x\partial z} \tag{3.23}$$

$$E_y = \frac{\partial^2\pi_{xi}}{\partial x\partial y} + \frac{\partial^2\pi_{zi}}{\partial y\partial z} \tag{3.24}$$

$$E_z = k_i^{\,2}\,\pi_{zi} + \frac{\partial^2\pi_{xi}}{\partial x\partial z} + \frac{\partial^2\pi_{zi}}{\partial z^2} \tag{3.25}$$

and

$$H_x = (\sigma_i + j\omega\epsilon_i)\,\frac{\partial\pi_{zi}}{\partial y} \tag{3.26}$$

$$H_y = (\sigma_i + j\omega\epsilon_i)\left(\frac{\partial\pi_{xi}}{\partial z} - \frac{\partial\pi_{zi}}{\partial x}\right) \tag{3.27}$$

$$H_z = -(\sigma_i + j\omega\epsilon_i)\,\frac{\partial\pi_{xi}}{\partial y} \tag{3.28}$$

Thus the continuity of E_x, E_y, H_x, and H_y at $z=0$ dictates that

$$k_0^{\,2}\pi_{xa} + \frac{\partial}{\partial x}\left(\frac{\partial\pi_{xa}}{\partial x} + \frac{\partial\pi_{za}}{\partial z}\right) = k_e^{\,2}\pi_{xe} + \frac{\partial}{\partial x}\left(\frac{\partial\pi_{xe}}{\partial x} + \frac{\partial\pi_{ze}}{\partial z}\right) \tag{3.29}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial\pi_{xa}}{\partial x} + \frac{\partial\pi_{za}}{\partial z}\right) = \frac{\partial}{\partial y}\left(\frac{\partial\pi_{xe}}{\partial x} + \frac{\partial\pi_{ze}}{\partial z}\right) \tag{3.30}$$

$$k_0^2 \frac{\partial \pi_{za}}{\partial y} = k_e^2 \frac{\partial \pi_{ze}}{\partial y} \tag{3.31}$$

$$k_0^2 \left(\frac{\partial \pi_{xa}}{\partial z} - \frac{\partial \pi_{za}}{\partial x}\right) = k_e^2 \left(\frac{\partial \pi_{xe}}{\partial z} - \frac{\partial \pi_{ze}}{\partial x}\right) \tag{3.32}$$

These equalities must be valid for all values of x and y. Thus, by integrating equations (3.30) and (3.31) with respect to y, we obtain, at z=0,

$$\frac{\partial \pi_{xa}}{\partial x} + \frac{\partial \pi_{za}}{\partial z} = \frac{\partial \pi_{xe}}{\partial x} + \frac{\partial \pi_{ze}}{\partial z} \tag{3.33}$$

$$k_0^2 \pi_{za} = k_e^2 \pi_{ze} \tag{3.34}$$

Making use of these two relations, equations (3.29) and (3.32) can be simplified to

$$k_0^2 \pi_{xa} = k_e^2 \pi_{xe} \tag{3.35}$$

$$k_0^2 \frac{\partial \pi_{xa}}{\partial z} = k_e^2 \frac{\partial \pi_{xe}}{\partial z} \tag{3.36}$$

In the absence of the flat earth, the primary field π_{xa}' is simply, from equation (2.13) and Sommerfeld's formula,

$$\pi_{xa}' = +j\frac{I d\ell}{4\pi\omega\varepsilon_0} \int_0^\infty \frac{\tau}{\xi_0} J_0(\tau r) e^{-|z-z_0|\xi_0} \, d\tau \tag{3.37}$$

In the presence of the flat earth, there also exists the z-components of the Hertz vector in the air and in the earth, i.e. π_{za} and π_{ze}, and the x-components of the Hertz vector in the air and in the earth, i.e. π_{xa} and π_{xe}. The x-component of the Hertz vector in the air π_{xa} includes the primary field π_{xa}', as given in equation (3.37). Before writing down these terms, it is necessary to examine their angular dependence. It is noted that π_{xa} and π_{xe} must satisfy equations (3.35) and (3.36) for z=0 and for all values of $0 \le r \le \infty$, $0 \le \phi \le \pi$. It is also noted that the primary part of π_{xa}', as given by equation (3.37), is independent of ϕ. Thus, in view of the orthogonality properties of $e^{jn\phi}$, $0 \le \phi \le 2\pi$, we conclude that π_{xa} and π_{xe} contain only the terms independent of angle (n=0). Thus, for π_{xa} and π_{xe}, only the n=0 term of equation (2.14) is retained, i.e. for z > 0

$$\pi_{xa} = +j\frac{Id\ell}{4\pi\omega\varepsilon_0}\int_0^{\infty}\left[\frac{\tau}{\xi_0}e^{-|z-z_0|\xi_0} + R_{xx}(\tau)e^{-z\xi_0}\right]J_0(\tau r)\,d\tau \qquad (3.38)$$

$$\pi_{xe} = +j\frac{Id\ell}{4\pi\omega\varepsilon_0}\int_0^{\infty} T_{xx}(\tau)e^{z\xi_e}\, J_0(\tau r)\,d\tau \qquad (3.39)$$

To determine the ϕ dependence of π_{za} and π_{ze}, equation (3.33) is expressed in cylindrical coordinates

$$\cos\phi\frac{\partial\pi_{xa}}{\partial r} + \frac{\partial\pi_{za}}{\partial z} = \cos\phi\frac{\partial\pi_{xe}}{\partial r} + \frac{\partial\pi_{ze}}{\partial z} \qquad (3.33')$$

As π_{xa} and π_{xe} are independent of ϕ and equation (3.33') is valid for all values of $0 \leq \phi \leq 2\pi$, π_{za} and π_{ze} must have $\cos\phi$-type dependence. Thus, in writing down the expressions for π_{ze} and π_{za}, it is only necessary to keep the $n=\pm 1$ terms, i.e.

$$\pi_{za} = +j\frac{Id\ell}{4\pi\omega\varepsilon_0}\left[\int_0^{\infty} R_{xz}(\tau)e^{-z\xi_0}\, J_1(\tau r)d\tau\right]\cos\phi \qquad (3.40)$$

$$\pi_{ze} = +j\frac{Id\ell}{4\pi\omega\varepsilon_0}\left[\int_0^{\infty} T_{xz}(\tau)e^{z\xi_e}\, J_1(\tau r)d\tau\right]\cos\phi \qquad (3.41)$$

Upon substitution of equations (3.38)-(3.41) into equations (3.33'), (3.34)-(3.36), we obtain:

$$T_{xx}(\tau) = 2\frac{k_0^2}{k_e^2}\frac{\tau}{\xi_0 + \xi_e}e^{-z_0\xi_0} \qquad (3.42)$$

$$R_{xx}(\tau) = \frac{\tau}{\xi_0}\left[1 - \frac{2\xi_e}{\xi_0 + \xi_e}\right]e^{-z_0\xi_0} \qquad (3.43)$$

$$T_{xz}(\tau) = -2\,\frac{k_0^2}{k_e^2}\,\frac{(\xi_0 - \xi_e)\tau^2}{k_e^2\xi_0 + k_0^2\xi_e}\,e^{-z_0\xi_0} \tag{3.44}$$

$$R_{xz}(\tau) = -2\,\frac{(\xi_0 - \xi_e)\tau^2}{k_e^2\xi_0 + k_0^2\xi_e}\,e^{-z_0\xi_0}, \tag{3.45}$$

Thus, the Hertz potential due to a horizontal dipole $Id\ell$ oriented along the x-axis and located at $z=z_0$ can be determined by combining equations (3.42) - (3.45) with (3.38) - (3.41). It is noted that π_{xa} can be written in a rather simple form:

$$\pi_{xa} = +j\,\frac{Id\ell}{4\pi\omega\varepsilon_0}\left[\frac{e^{-jk_0R}}{R} + \frac{e^{-jk_0R'}}{R'} - \Lambda'\right], \tag{3.46}$$

where R and R' are defined in Section 3.2 and

$$\Lambda' = 2\int_0^\infty \frac{\xi_e\tau}{\xi_0[\xi_0 + \xi_e]}\,e^{-\xi_0(z + z_0)}\,J_o(\tau r)\,d\tau\ . \tag{3.47}$$

Also

$$\pi_{za} = +j\frac{Id\ell}{4\pi\omega\varepsilon_0}\,\Lambda''\cos\phi \tag{3.48}$$

where

$$\Lambda'' = -2\int_0^\infty \frac{(\xi_0-\xi_e)\tau^2}{k_e^2\xi_0 + k_e^2\xi_e}\,e^{-\xi_0(z + z_0)}\,J_1(\tau r)\,d\tau \tag{3.49}$$

The transverse electric field intensities in the air are determined by substituting equations (3.46) and (3.48) into equations (3.23)-(3.28). More specifically,

$$E_x = Id\ell\,[-Z_t(r) + \frac{\partial^2}{\partial x^2}Z_\ell(r)] \tag{3.50}$$

$$E_y = Id\ell\,[\frac{\partial^2}{\partial x\partial y}\,Z_\ell(r)] \tag{3.51}$$

where

$$Z_t = \frac{1}{2\pi} \frac{1}{j\omega\varepsilon_c} \int_0^\infty d\tau \; J_0(\tau r) \simeq \frac{1}{2\pi} \frac{1}{j\omega\varepsilon_c} \frac{1}{r} \tag{3.52}$$

$$Z_\ell = \frac{1}{2\pi} j\omega\mu_c \int_0^\infty d\tau \frac{\tau}{\tau+\gamma} \; J_0(\tau r) \simeq \frac{1}{2\pi} j\omega\mu_c \frac{1-(1+\gamma r)e^{-\gamma r}}{(\gamma r)^2} \tag{3.53}$$

where $\gamma = jk$.

Again, the significance of the terms Z_t and Z_ℓ are discussed in Appendix C.

CHAPTER 4

Coupling Model (Approximate Solution)

Our next task is to evaluate the current induced on the conductor by the lightning strokes. Practically all buried cables presently in use have some sort of plastic jacket. Even if a bare conductor is used, there will be a layer of oxidation at the surface of the metal wire. In addition, it is likely that a thin layer of air is present between the metal surface (or metal oxide surface) and the soil. In short, we should not expect the bare wire to be in electrical contact with the soil. To be realistic, a lossy dielectric layer is included in our consideration.

Most cables are buried 30''-36'' below the surface of the earth. For most frequencies of practical interest, the current which arrives at the far end of the wire due to a lightning stroke is essentially insensitive to the variation of the depth of the cable, provided that the wire is long and that the depth is deep enough. Thus, in this section, the model depicted in Figure 2b, which is an approximation to the configuration shown in Figure 2a, will be used to derive a set of differential equations governing the current induced on the buried wire.

4.1 Transmission Line Equations

The axial direction of the wire is taken to be the x-axis, (Figure 3). The field at any arbitrary point in the space can be decomposed into two parts. One part is the field produced by the external sources, in the absence of the wire and the insulation layer, and will be designated by a superscript 0; the second part is due to the current induced on the conducting wire, and will be designated by superscript 1.

To find the field produced by the current on the wire, we start with one of the Maxwell's equations (2.1') in integral form:

$$\oint_{C_1} \vec{E}^1 \cdot d\vec{\ell} = -j\omega \int_{S_1} \vec{B}^1 \cdot d\vec{S} \tag{4.1}$$

and choose the contour to be C_1, as shown in Figure 3a. As shown in Figure 3a, the line integral along the path C_1 can be broken into five parts

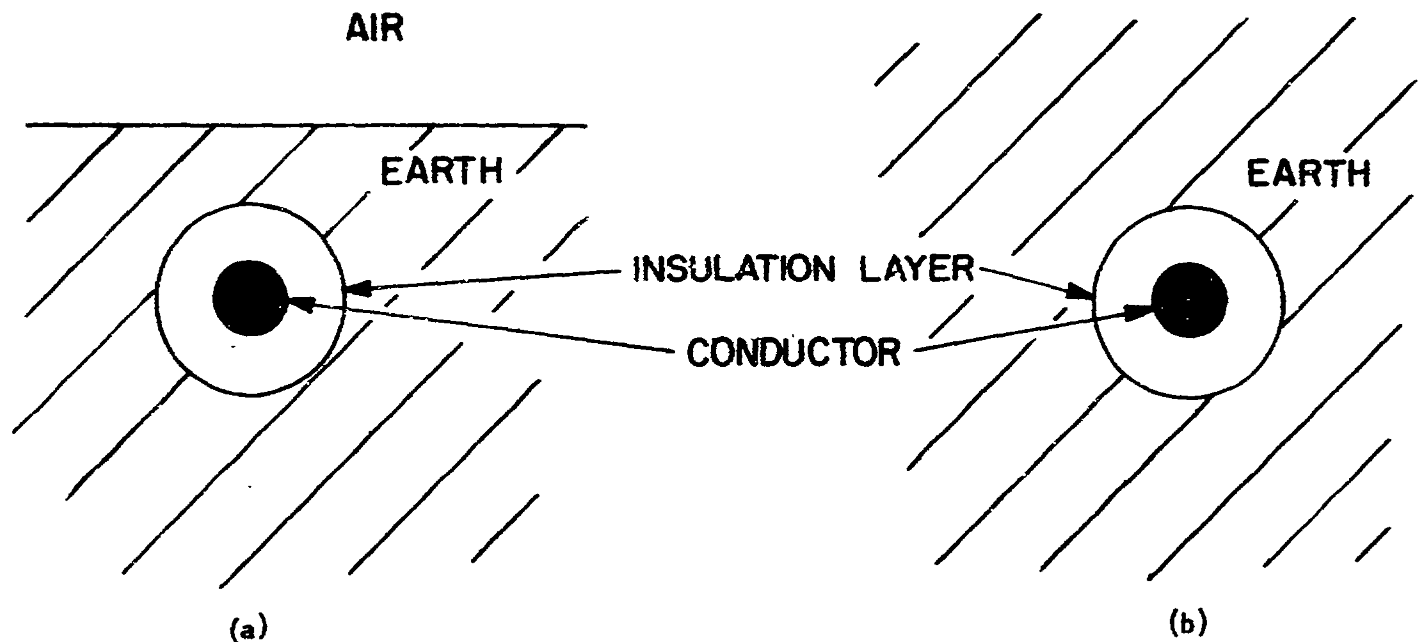


Fig. 2. Buried Cable and Approximate Model for Buried Cable

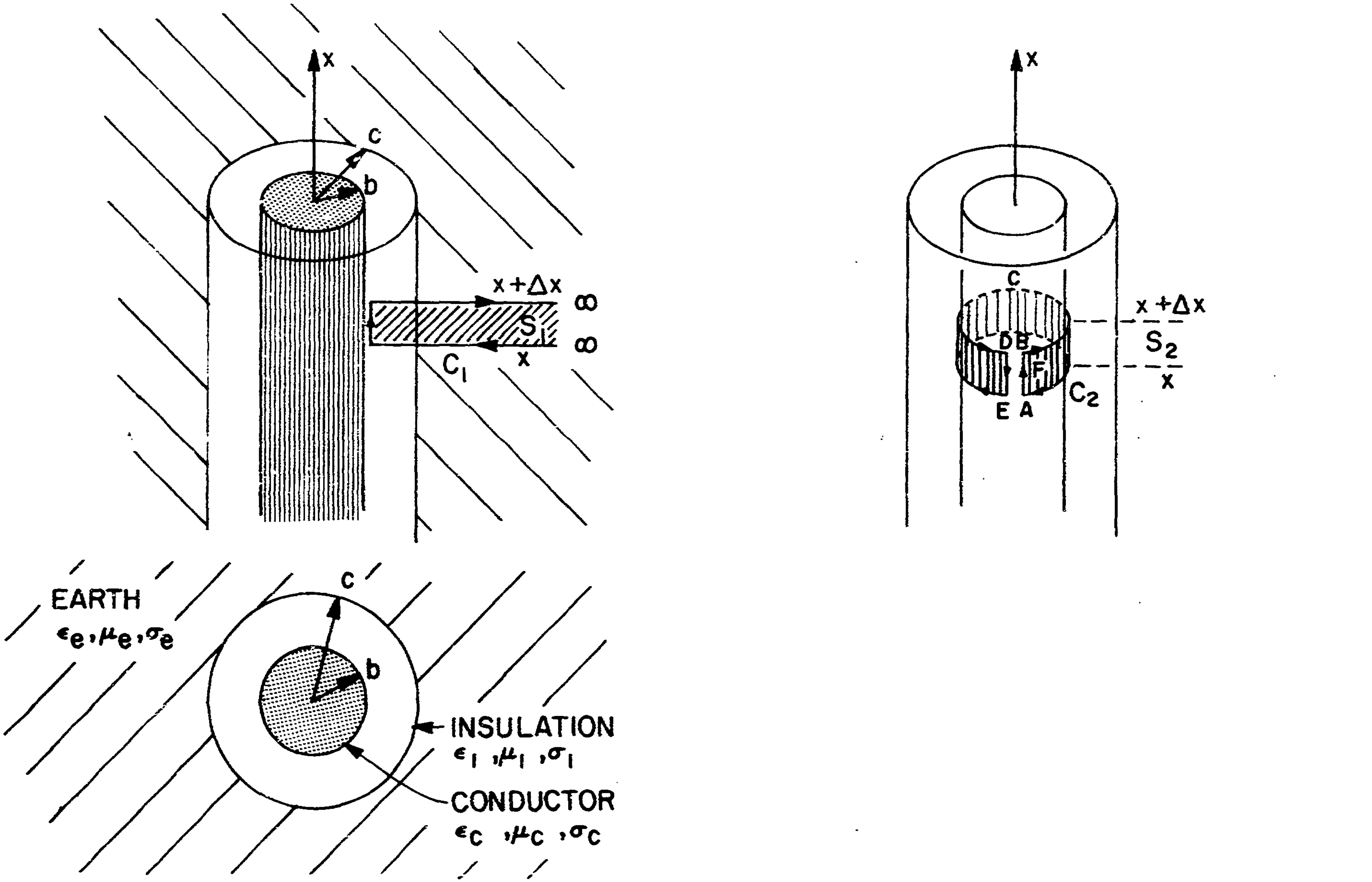


Fig. 3. Geometry of a Buried Cable

$$\int_{\infty}^{c} E_{re}^{1}(r,x)dr + \int_{c}^{b} E_{rl}^{1}(r,x)dr + \int_{x}^{x+\Delta x} E_{xl}^{1}(b,x)dx$$
$$+ \int_{b}^{c} E_{rl}^{1}(r,x+\Delta x)dr + \int_{c}^{\infty} E_{re}^{1}(r,x+\Delta x)dr$$
$$= -j\omega\int_{x}^{x+\Delta x} \left[\int_{b}^{c} B_{\theta l}^{1}(r,x)dr + \int_{c}^{\infty} B_{\theta e}^{1}(r,x)dr\right] dx \tag{4.2}$$

After collecting the terms and noting that

$$\int_{\infty}^{c} E_{re}^{1}(r,x)dr + \int_{c}^{\infty} E_{re}^{1}(r,x+\Delta x)dr \simeq \Delta x \int_{c}^{\infty} \frac{\partial E_{re}^{1}(r,x)}{\partial x} dr$$
$$\int_{c}^{b} E_{ri}^{1}(r,x)dr + \int_{b}^{c} E_{ri}^{1}(r, x+\Delta x)dr \simeq \Delta x \int_{b}^{c} \frac{\partial E_{rl}^{1}(r,x)}{\partial x} dr$$

we have

$$\Delta x \left[\int_{b}^{c} \frac{\partial E_{rl}^{1}(r,x)}{\partial x} dr + \int_{c}^{\infty} \frac{\partial E_{re}^{1}(r,x)}{\partial x} dr + E_{xl}^{1}(b,x)\right]$$

$$= -j\omega\Delta x \left[\int_{b}^{c} B_{\theta l}^{1}(r,x)dr + \int_{c}^{\infty} B_{\theta e}^{1}(r,x)dr\right] \tag{4.3}$$

If a voltage V and a flux Φ are introduced as

$$V(x) = \int_{b}^{c} E_{rl}^{1}(r,x)dr + \int_{c}^{\infty} E_{re}^{1}(r,x)dr \tag{4.4}$$

$$\phi(x) = \int_{b}^{c} B_{\theta l}^{1}(r,x)dr + \int_{c}^{\infty} B_{\theta e}^{1}(r,x)dr \tag{4.5}$$

then the equation (4.6) becomes

$$\frac{dV(x)}{dx} = -j\omega\phi(x) - E_{zl}^{1}(b,x) \tag{4.6}$$

This is one of the desired differential equations.

To find the second differential equation, we examine another one of the Maxwell's equations (2.2') in integral form:

$$\oint_{C_2} \vec{H}^1 \cdot d\vec{\ell} = \int_{S_2} (\sigma_1 + j\omega\varepsilon_1)\vec{E}^1 \cdot d\vec{S} \tag{4.7}$$

and choose the contour to be C_2, as shown in Figure 3b. Clearly, the line integral along path AB and along path DE cancel exactly. When the wire is made of good conductors, i.e., $\sigma_c >> \omega\varepsilon_c$, the displacement current term in the wire may be neglected, then

$$\int_{BCD} \vec{H}^1 \cdot d\vec{\ell} = I_x(x + \Delta x) \tag{4.8}$$

$$\int_{EFA} \vec{H}^1 \cdot d\vec{\ell} = -I_x(x) \tag{4.9}$$

where I_x is the axial current carried by the wire. Substituting these two equations into (4.10), we have

$$\frac{dI_x(x)}{dx} \simeq -(\sigma_1 + j\omega\varepsilon_1)E_{r1}^{\ 1}(b,x)\cdot 2\pi b \tag{4.10}$$

To make (4.9) and (4.11) useful, it is necessary to express V, Φ, $E_{r1}^{\ 1}$, and $E_{x1}^{\ 1}$ in terms of I_x explicitly. For this purpose, three impedances are introduced[8]:

(i) Surface impedance (or skin effect impedance)

$$Z_s = \frac{E_x(b,x)}{I_x(x)} = \frac{E_x^{\ 0}(b,x) + E_x^{\ 1}(b,x)}{I_x(x)} \tag{4.11}$$

(ii) Longitudinal (inductive) impedance

$$Z_L = +j\omega\frac{\Phi(x)}{I_x(x)} \tag{4.12}$$

(iii) Transverse impedance

$$Z_T = \frac{V(x)}{2\pi b(\sigma_1 + j\omega\varepsilon_1)E_{r1}^{\ 1}(b,x)} \tag{4.13}$$

Upon substituting (4.11)-(4.13) into (4.6) and (4.9), these two equations assume the form of the transmission line equations.

$$\frac{dV(x)}{dx} = -I_x(x)\ (Z_L + Z_s) + E_0\ (x) \tag{4.14}$$

$$\frac{dI_x(x)}{dx} = -\frac{1}{Z_T}\ V(x) \tag{4.15}$$

Figure 4 gives a simple and familiar representation for the transmission line equations. As we are mainly interested in the current I_x, it would be desirable to eliminate the voltage V in favor of the current I_x. However, the resulting equation would be extremely complicated as the impedances Z_L, Z_S, and Z_T are also dependent on x. Fortunately, the impedances Z_S, Z_L, and Z_T are approximately constants under certain circumstances, to be discussed in Appendix A. Under the approximation indicated there, these impedances are

$$Z_L = Z_{L1} + Z_{L2} \tag{4.16}$$

$$Z_T = Z_{T1} + Z_{T2} \tag{4.17}$$

The expressions for Z_{L1}, Z_{L2}, Z_{T1}, Z_{T2}, and Z_S are given in Equation (A.32)-(A.36) While these equations are approximate, they are valid for most situations of practical interest. In addition, from the definition of, and the expressions for, the longitudinal and transverse impedance Z_L and Z_T, it is clear that these expressions can be extended to include multi-layer cables. When the impedances Z_S, Z_L, and Z_T are approximately constant, equations (4.14) and (4.15) reduce to the well-known transmission line equations with a distributed source term E_0 (x). The solution to such a system of coupled differential equations can best be expressed in terms of voltage and current Green's functions as discussed in Chapter 6.

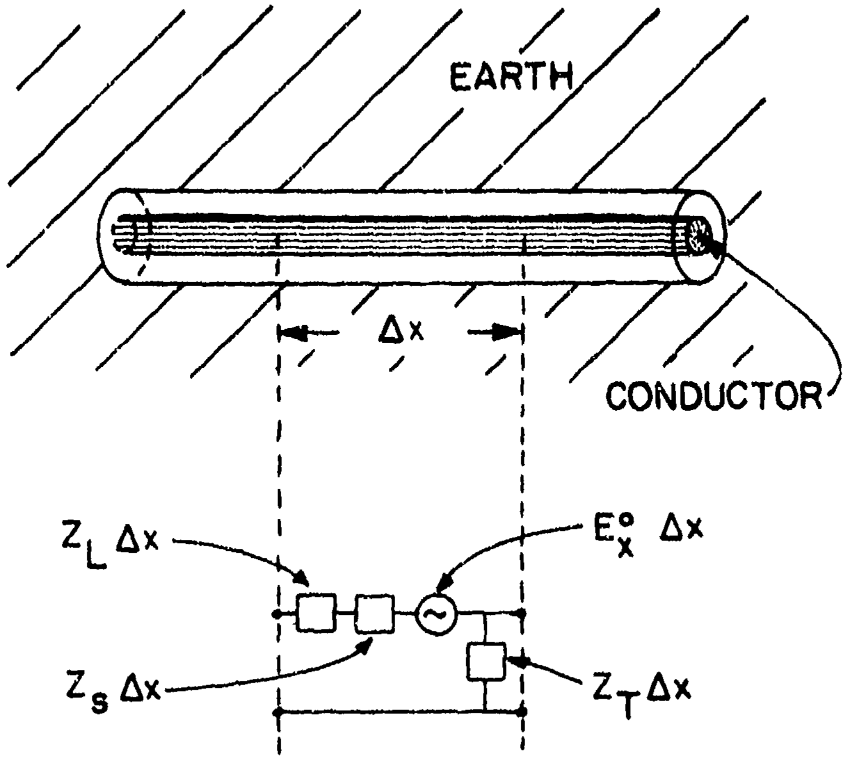


Fig. 4 Transmission Line Model for Buried Cable

4.2 Distributed Fields Due to Lightning Strokes to Ground

We shall use the results just obtained in Chapter 3 to calculate the fields and the potentials of the lightning stroke to ground. The lightning strokes are represented by a current I along the z-axis. The current path is subdivided into small segments dz and the Hertz vector $\vec{\pi}$ due to each segment is given by equations (3.4) and (3.12). Thus, the total Hertz vector, for z < 0, is

$$\pi_{ze} = + j \frac{I}{4\pi\omega\varepsilon_0} \int_0^\infty dz' \int_0^\infty \frac{2k_0^2 \tau}{k_e^2 \xi_0 + k_0^2 \xi_e} e^{-z'\xi_0 + z\xi_e} J_0(\tau r) \, d\tau \tag{4.18}$$

and the scalar potential ϕ_e is, by substituting the above equation into (2.16),

$$\phi_e(r,z) = - j \frac{I k_0^2}{2\pi\kappa\varepsilon_0} \int_0^\infty \frac{\xi_e}{\xi_0} \frac{e^{z\xi_e}}{[k_e^2 \xi_0 + k_0^2 \xi_e]} \tau J_0(\tau r) \, d\tau \tag{4.19}$$

Of particular interest to us is the case of low frequencies where $\omega\varepsilon << \sigma$, and the displacement current term is negligible. Under such conditions, a rather simple approximation for (4.19) can be obtained. By letting $\omega\varepsilon \to 0$, we obtain,

$$\phi_e(r,z) \simeq \frac{I}{2\pi\sigma_e} \int_0^\infty e^{z\tau} J_0(\tau r) \, d\tau$$

The integral can be evaluated [7] and leads to

$$\phi_e(r,z) \simeq \frac{I}{2\pi\sigma_e} \frac{1}{[r^2 + z^2]^{1/2}} \tag{4.20}$$

which is the same as the potential at (r,z) inside or on the surface of the earth when a dc current I is entering at the origin.

In particular, the distributed source $E_0(x)$ is given by

$$E_0(x) = - \frac{d\phi_e}{dX}$$

CHAPTER 5

Coupling Model (Exact Solution)

In order to establish exact formulas for the propagation characteristics on earth return conductors, rigorous theoretical solutions for the propagation of current along an extended conductor in infinite contact with the earth are obtained for the case in which the current enters the earth at an electrode which represents the terminal effects of a lightning channel. The general case is again considered here, in which the conductor is not necessarily in direct contact with the earth, but has a contact impedance with the earth, as in cables with coverings provided for corrosion or mechanical protection or electrical shields.

5.1 Transmission Line Equations

Consider an extended straight conductor of radius ρ_o half buried in the xy plane of a rectangular coordinate system (x,y,z), as shown in Figure 5, and let the x axis extend along the conductor. Let $\phi_c(x)$ and $\phi_e(x,y)$ be the scalar potential in the conductor and in the earth (at the separation distance y), respectively. Also, let $A_c(x)$ and $A_e(x,y)$ be the x components of the vector potentials in the conductor and in the earth (at the separation distance y). respectively. It is assumed that there are no radial variations in the field variables inside the conductor.

The x component of the electric field intensity $E_c(x)$ along the surface of the conductor is

$$E_c(x) = -\frac{d}{dx}\phi_c(x) - j\omega A_c(x) \tag{5.1}$$

and the x component of the electric field intensity $E_e(x,\rho_0)$ in the earth adjacent to the conductor surface at $y=\rho_0$ is

$$E_e(x,\rho_0) = -\frac{d}{dx}\phi_e(x,\rho_0) - j\omega A_e(x,\rho_0) \tag{5.2}$$

Since there is no change in the magnetic flux between the conductor and the earth adjacent to it [10]

$$A_c(x) = A_e(x,\rho_0) \tag{5.3}$$

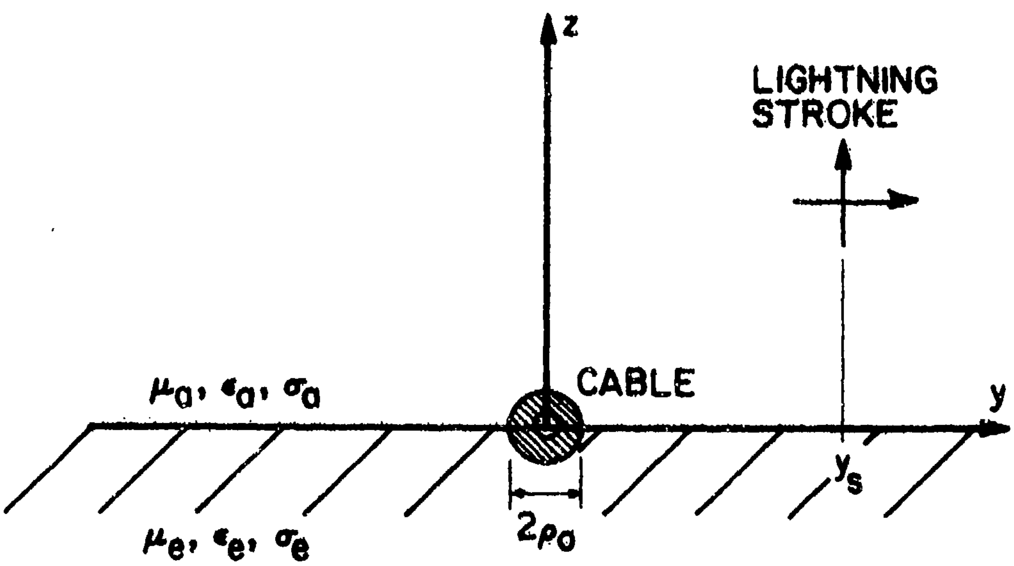


Fig. 5. Lightning Stroke/Cable Geometry

The electric field intensity $E_c(x)$ along the surface of the conductor is also given by

$$E_c(x) = Z_i I_c(x) \tag{5.4}$$

where Z_i is the internal impedance of the conductor per unit length. As shown in Appendix B,

$$Z_i = -\frac{k}{j\omega\varepsilon} \quad \frac{1}{2\pi\rho_0} \quad \frac{J_0(k\rho_0)}{J_1(k\rho_0)}$$

From the above equations

$$E_e(x,\rho_0) - Z_i I_c(x) = \frac{d}{dx}\,[\phi_c(x) - \phi_e(x,\rho_0)] \tag{5.5}$$

The resultant electric field intensities are regarded as the sum of an impressed primary field (denoted by a single prime superscript), due to the lightning channel, and an induced secondary field (denoted by a double prime superscript), due to current in the conductor, i.e., let

$$E_e(x,y) = E_e'(x,y) + E_e''(x,y) \tag{5.6}$$

and

$$\phi_e(x,y) = \phi_e'(x,y) + \phi_e''(x,y) \tag{5.7}$$

$$\phi_c(x) = \phi_c'(x) + \phi_c''(x) \tag{5.8}$$

The induced secondary potential between the conductor and an adjacent point in the earth has the following relation to the leakage current $I_\ell(x)$ where

$$\phi_c''(x) - \phi_e''(x,\rho_0) = I_\ell(x)/\,Y_I \tag{5.9}$$

and

$$I_\ell(x) = -\frac{d}{dx}\,I_c(x) \tag{5.10}$$

and Y_I is the admittance of the conductor insulation per unit length. Also, for a wire of infinite length, the induced secondary electric field intensity due to a current $I_c(x)$ distributed along the entire length of the conductor is

$$E_e''(x,\rho_0) = -\int_{-\infty}^{+\infty} dx' Z_\ell(r_{\rho_0 X})\ I_c(x') + \frac{d}{dx}\int_{-\infty}^{+\infty} dx' Z_t(r_{\rho_0 X})\ \frac{d}{dx'}\ I_c(x') \tag{5.11}$$

where

$$r_{\rho_0\chi} \equiv \sqrt{\rho_0^2 + \chi^2} \tag{5.12}$$

and

$$-\chi \equiv x - x' \tag{5.13}$$

and $Z_\ell(r_{\rho_0\chi})$ is the longitudinal electric field intensity at the surface of the conductor at x, for a unit axial current at x', while $Z_t(r_{\rho_0\chi})$ is the transverse electric field intensity at the surface of the conductor at x, for a unit radial current leaving the conductor at x'.

Therefore, by superposition, the following integro-differential equation is obtained for the current I_c:

$$\frac{d}{dx}\left[\frac{1}{Y_1}\frac{d}{dx} I_c(x) + \int_{-\infty}^{+\infty} dx' Z_t(r_{\rho_0\chi}) \frac{d}{dx'} I_c(x')\right] - Z_1 I_c(x)$$

$$- \int_{-\infty}^{+\infty} dx' Z_\ell(r_{\rho_0\chi}) I_c(x') = -E_0(x) \tag{5.14}$$

where the impressed electric field intensity $E_0(x)$ along the conductor is related to the impressed electric field intensity $E_e'(x,\rho_0)$ in the earth next to the conductor by

$$E_0(x) = -\frac{d}{dx}\phi(x) = E_e'(x,\rho_0) - \frac{d}{dx}[\phi_c'(x) - \phi_e'(x,\rho_0)] \tag{5.15}$$

Equation (5.14) is an exact generalization of the approximate transmission line equations (4.14) and (4.15) derived earlier. Once the current $I_c(x)$ is determined from the above transmission line equation, the conductor potential $\phi_c(x)$ and the potential in the earth $\phi_e(x,y)$ are also determined as integrals over the current. The difference between the impressed potential

$\phi_0(x,y)$ and the resultant potential $\phi_e(x,y)$ in the earth, which is the negative induced secondary earth potential $\phi_e''(x,y)$, is given by [10]

$$\phi_0(x,y) - \phi_c(x,y) = + \int_{-\infty}^{+\infty} dx' Z_t(r_{y\chi}) \frac{d}{dx'} I_c(x') \tag{5.16}$$

Also, the induced secondary potential $\phi_c''(x)$ is greater than the induced secondary potential $\phi_e''(x,y)$ of an adjacent point in the earth by an amount $I_\ell(x)/Y_1$ such that [10]

$$\phi_0(x,\rho_0) - \phi_c(x) = \frac{1}{Y_1} \frac{d}{dx} I_c(x) + \int_{-\infty}^{+\infty} dx' Z_t(r_{\rho_0\chi}) \frac{d}{dx'} I_c(x') \tag{5.17}$$

5.2 Integral Transform Representations

In order to solve the characteristic equation (5.14) for the current $I_c(x)$, a Fourier integral transformation between the space variable x and the wave number variable ζ is introduced, i.e., let

$$\vec{F}(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} d\zeta \vec{f}(\zeta) e^{+j\zeta x} \tag{5.18}$$

$$\vec{f}(\zeta) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dx \vec{F}(x) e^{-j\zeta x} \tag{5.19}$$

where $\vec{F}$ and $\vec{f}$ are any of the field variables or sources defined above. Therefore, in the "wave number" domain, the characteristic equation for the transformed current $i_c(\zeta)$ is

$$\left[\frac{\zeta^2}{Y_1} + \zeta^2 z_t(r_{\rho_0}\zeta) + Z_1 + z_\ell(r_{\rho_0}\zeta)\right] I_c(\zeta) = e_0(\zeta) \tag{5.20}$$

where

$$z_t(r_{\rho_0}\zeta) = \int_{-\infty}^{\infty} d\chi \, Z_t(r_{\rho_0}\chi) e^{+j\zeta\chi} \tag{5.21}$$

$$z_\ell(r_{\rho_0}\zeta) = \int_{-\infty}^{\infty} d\chi \, Z_\ell(r_{\rho_0}\chi) e^{+j\zeta\chi} \tag{5.22}$$

and

$$e_0(\zeta) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dx \, E_0(x) e^{-j\zeta x} \tag{5.23}$$

Therefore

$$I_c(\zeta) = e_0(\zeta) \,/\, \Delta(\zeta) \tag{5.24}$$

where

$$\Delta(\zeta) = \frac{\zeta^2}{Y} + Z \tag{5.25}$$

and

$$Z = Z_1 + z_\ell(r_{\rho_0}\zeta) \tag{5.26}$$

$$\frac{1}{Y} = \frac{1}{Y_1} + z_t(r_{\rho_0}\zeta) \tag{5.27}$$

Therefore, to summarize the preceding derivations, the current $I_c(x)$, the conductor potential $\phi_c(x)$, and the potential in the earth $\phi_e(x,y)$ are given by

$$I_c(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\zeta \, \frac{e_0(\zeta)}{\Delta(\zeta)} e^{+j\zeta x} \tag{5.28}$$

$$\phi_c(x) = \phi_0(x,\rho_0) - \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\zeta \, \zeta \frac{e_0(\zeta)}{\Delta(\zeta)} \left[\frac{1}{Y_1} + z_t(r_{\rho_0}\zeta)\right] e^{+j\zeta x} \tag{5.29}$$

$$\phi_e(x,y) = \phi_0(x,y) - \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\zeta \, \zeta \frac{e_0(\zeta)}{\Delta(\zeta)} z_t(r_{\rho_0}\zeta) e^{+j\zeta x} \tag{5.30}$$

5.3 Propagation Constant

To find the current $I_c(x)$, the conductor potential $\phi_c(x)$, and the earth potential $\phi_e(x,y)$, the following transformations of the transfer and longitudinal impedances are required.

$$z_t(r_{\rho_0\zeta}) = \int_{-\infty}^{+\infty} d\chi Z_t(r_{\rho_0\chi}) e^{+j\zeta\chi} \tag{5.31}$$

$$z_\ell(r_{\rho_0\zeta}) = \int_{-\infty}^{+\infty} d\chi Z_\ell(r_{\rho_0\chi}) e^{+j\zeta\chi} \tag{5.32}$$

where

$$r_{\rho_0\zeta} = \sqrt{\rho_0^{\,2} + \zeta^2} \tag{5.33}$$

$z_\ell(r_{\rho_0\zeta})$ and $z_t(r_{\rho_0\zeta})$ are the longitudinal and the transverse mutual impedances, respectively, of two collinear conductor elements separated by the distance r.

When the earth is uniform and displacement currents in the air are neglected, the functions $z_\ell(r_{\rho_0\chi})$ and $z_t(r_{\rho_0\chi})$ as derived earlier in Chapter 4 for a horizontal dipole, are

$$Z_t(r_{\rho_0\chi}) = \frac{1}{j\omega\tilde{\varepsilon}} \quad \frac{1}{2\pi r_{\rho_0\chi}}$$

$$Z_\ell(r_{\rho_0\chi}) = j\omega\tilde{\mu} \quad \frac{1}{2\pi} \frac{e^{-\gamma r_{\rho_0\chi}}}{r_{\rho_0\chi}} \quad \left[\frac{e^{\gamma r_{\rho_0\chi}} - 1 - \gamma r_{\rho_0\chi}}{(\gamma r_{\rho_0\chi})^2} \right] \tag{5.34}$$

where

$$\gamma = jk = [-\omega^2\tilde{\mu}\tilde{\varepsilon}]^{1/2} = [j\omega\mu \ (\sigma + j\omega\varepsilon)]^{1/2} \tag{5.35}$$

Therefore, the functions $z_t(r_{\rho_0\zeta})$, $z_\ell(r_{\rho_0\zeta})$ are

$$z_t(r_{\rho_0\zeta}) = \frac{1}{\pi} \frac{1}{j\omega\tilde{\varepsilon}} \quad K_0(|\zeta|\rho_0) \tag{5.36}$$

$$z_\ell(r_{\rho_0\zeta}) = \frac{1}{\pi} \frac{j\omega\tilde{\mu}}{\gamma^2\rho_0} \quad \left[|\zeta| K_1(|\zeta|\rho_0) - \sqrt{\gamma^2+\zeta^2} \ K_1(\sqrt{\gamma^2+\zeta^2}\,\rho_0) \right] \tag{5.37}$$

Since the important part of the integration range $z_\ell(r_{\rho_0\zeta})$ and $z_t(r_{\rho_0\zeta})$, as given above, vary nearly logarithmically with ζ, large variations in ζ produce small changes in these functions. For this reason it is permissible to approximate these functions at $\zeta = \zeta_0$, where ζ_0 is a constant so chosen that, in the important part of the integration range,

$$z_t(r_{\rho_0 \zeta}) \simeq z_t(r_{\rho_0 \zeta_0}) \tag{5.38}$$

$$z_\ell(r_{\rho_0 \zeta}) \simeq z_\ell(r_{\rho_0 \zeta_0}) \tag{5.39}$$

With sufficient accuracy for most practical applications ζ_0 can be taken equal to Γ, the propagation constant of the cable, in which case the following transcendental equation for Γ is obtained:

$$\Gamma^2 \left.\frac{1}{Y}\right|_{\zeta=\Gamma} = Z\Big|_{\zeta=\Gamma} \tag{5.40}$$

or

$$\Gamma^2 \left[\frac{1}{Y_l} + \frac{1}{\pi}\frac{1}{j\omega\tilde{\epsilon}} K_0(|\Gamma|\rho_0)\right] = Z_l + \frac{1}{\pi}\frac{j\omega\tilde{\mu}}{\gamma^2\rho_0}\left[|\Gamma| K_1(|\Gamma|\rho_0) - \sqrt{\gamma^2 + \Gamma^2}\, K_1(\sqrt{\gamma^2 + \Gamma^2}\rho_0)\right] \tag{5.41}$$

If $\Gamma\rho_0$ and $\rho_0(\gamma^2 + \Gamma^2)^{\frac{1}{2}}$ are less than 0.01, then

$$z_t(r_{\rho_0 \Gamma}) \simeq \frac{1}{\pi}\frac{1}{j\omega\tilde{\epsilon}} \ln\frac{1.12\ldots}{\Gamma\rho_0} \tag{5.42}$$

$$z_\ell(r_{\rho_0 \Gamma}) \simeq \frac{1}{2\pi} j\omega\tilde{\mu} \ln\frac{1.85\ldots}{\rho_0\sqrt{\gamma^2 + \Gamma^2}} \tag{5.43}$$

and

$$\Gamma^2 \simeq \frac{Z_l + \frac{j\omega\tilde{\mu}}{2\pi}\ln\frac{1.85\ldots}{\rho_0\sqrt{\gamma^2 + \Gamma^2}}}{\frac{1}{Y_L} + \frac{1}{\pi j\omega\tilde{\epsilon}}\ln\frac{1.12\ldots}{\Gamma\rho_0}} \tag{5.44}$$

An approximate solution to the above transcendental equation is, by inspection,

$$\Gamma^2 \simeq \frac{\gamma^2}{2} \tag{5.45}$$

Inserting this value for Γ into the logarithmic terms above, the expression reduces to

$$\Gamma^2 \simeq \frac{1}{2}\gamma^2 \frac{\ln \frac{1.52}{\Gamma\rho_0}}{\ln \frac{1.58}{\Gamma\rho_0}} \tag{5.46}$$

Since the ratio of the above logarithmic terms is practically unity, for all practical purposes

$$\Gamma \simeq \frac{\gamma}{\sqrt{2}} \tag{5.47}$$

5.4 Ground Strokes

The propagation characteristics of the current $I_c(x)$ along the surface of a buried cable due to lightning strokes to ground in the vicinity of the cable are now determined.

5.4.1 Direct Strike (Arcing)

A lightning stroke to ground may arc directly to a buried cable in the vicinity of the base of the lightning channel, in which case, virtually all of the current will enter the sheath near the stroke point.

When a lightning current I_s enters the sheath at the point $x = 0$ and the sheath is assumed to extend indefinitely in opposite directions from this point, the sheath current $I_c(x)$ at the distance x is given approximately by

$$I_c(x) = \frac{I_s}{2} e^{-\Gamma|x|} \tag{5.48}$$

where Γ is the propagation constant of the earth/sheath circuit, as described above.

The induced electric field $E_0(x)$ along the inner surface of the sheath is

$$E_0(x) = \frac{I_s}{2} Z_i \, e^{-\Gamma|x|} \tag{5.49}$$

where Z_i is the internal surface impedance of the conductors per unit length, as developed in Appendix B.

5.4.2 Indirect Strike (Conductive Energization)

Alternatively, a lightning stroke to ground may be too distant from a buried cable in the vicinity of the base of the lightning channel to arc directly to the cable; however, in this case the cable can still be conductivily energized by the current entering the ground at the base of the lightning discharge, which is easily represented mathematically by an electrode near the wire.

When a current I_s enters the earth at an electrode at the separation distance ρ_s from the conductor, the impressed secondary earth potential along the conductor is given by

$$\phi_0\,(x,\rho_0) = Z_t(r_{\rho_s\chi})\; I_s \tag{5.50}$$

where

$$r_{\rho_s\chi} = \sqrt{\rho_s^{\,2} + \chi^2}$$

and

$$-\chi = x - x'$$

Therefore, the impressed electric field intensity along the conductor is then given by

$$E_0(x) = -\frac{d}{dx}\,\phi_0(x,\rho_0) = \int_{-\infty}^{+\infty} \frac{d\zeta}{\sqrt{2\pi}} \left[-\frac{I_s}{\sqrt{2\pi}}\, j\zeta z_t(r_{\rho_s\zeta}) e^{-j\zeta x'} \right] e^{+j\zeta x} \tag{5.51}$$

Therefore,

$$e_0(\zeta) = -\frac{I_s}{\sqrt{2\pi}}\, j\zeta z_t(r_{\rho_s\zeta}) e^{-j\zeta x'} \tag{5.52}$$

The conductor current I_c, the conductor potential ϕ_c, and the earth potential $\phi_e(x,y)$ are then given by

$$I_c(x) = -\frac{I_s}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \frac{d\zeta}{\sqrt{2\pi}}\, j\zeta\, \frac{z_t(r_{\rho_s\zeta})}{\Delta(\zeta)}\, e^{+j\zeta x} \tag{5.53}$$

$$\phi_c(x) = \frac{I_s}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \frac{d\zeta}{\sqrt{2\pi}}\, \frac{Z_I + z_I(r_{\rho\zeta})}{\Delta(\zeta)}\, z_t(r_{\rho_s\zeta}) e^{+j\zeta x} \tag{5.54}$$

$$\phi_e(x,y) = \frac{I_s}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \frac{d\zeta}{\sqrt{2\pi}} \frac{\Delta(\zeta) z_t(r_{\bar{\rho}\zeta}) - \zeta^2 z_t(r_{\rho_s}\zeta) z_t(r_{y\zeta})}{\Delta(\zeta)} e^{+j\zeta x} \qquad (5.55)$$

where

$$\bar{\rho} \equiv \rho_s \pm y \qquad (5.58)$$

Therefore, after substituting the values for the transformed longitudinal impedance $z_\ell(r_{\rho_s}\zeta)$ and the transformed transverse impedance $z_t(r_{\rho_s}\zeta)$, into equations (5.53)-(5.55) the conductor current $I_c(x)$, the conductor potential $\phi_c(x)$, and the earth potential $\phi_e(x,y)$ are approximated by

$$I_c(x) \simeq \frac{I_s}{\sqrt{2\pi}} \frac{Y}{j\omega\tilde{\varepsilon}} \psi(\Gamma\chi, \Gamma\rho_s) \qquad (5.56)$$

$$\phi_c(x) \simeq \frac{I_s}{\sqrt{2\pi}} \Gamma \frac{1}{j\omega\tilde{\varepsilon}} \Phi(\Gamma\chi, \Gamma\rho_s) \qquad (5.57)$$

$$\phi_e(x,y) \simeq \phi_0(x,y) - \frac{I_s}{\sqrt{2\pi}} \Gamma \frac{1}{j\omega\tilde{\varepsilon}} \xi \left[\frac{1}{\Gamma r_{\rho_s}\chi} - \Phi(\Gamma\chi, \Gamma\rho_s) \right] \qquad (5.58)$$

and

$$\xi = \frac{Y_1 K_0(\Gamma y)}{\pi j\omega\tilde{\varepsilon} + Y_1 K_0(\Gamma y)}$$

where

$$\phi(u,v) \equiv \int_{-\infty}^{+\infty} dt \frac{e^{-t}}{\sqrt{t^2 + v^2}}$$

and

$$\psi(u,v) = \frac{e^{-u}\phi(+u,+v) - e^{+u}\phi(-u,+v)}{2}$$

$$\Phi(u,v) = \frac{e^{-u}\phi(+u,+v) + e^{+u}\phi(-u,+v)}{2}$$

The function ϕ is related to the Bessel and Neumann functions as follows (10)

$$\phi(u,v) = J_0(v)\ln\frac{v}{w+u} - \frac{\pi}{2}Y_0(v) + \sum_{\iota=1}^{\infty}\frac{1}{\iota!}A_\iota \qquad (5.59)$$

where

$$w = \sqrt{u^2+v^2}$$

and

$$A_0 = 0$$

$$A_1 = w$$

and

$$A_\iota = \frac{wu^{\iota-1}-(\iota-1)v^2A_{\iota-2}}{\iota} \qquad (\iota = 2,3,4,\ldots)$$

where J_0 and Y_0 denote, respectively, the zero order Bessel and Neumann functions of the complex argument v.

Also

$$K_0(v) = I_0(v)\ln\left(\frac{2}{v}\right) + \sum_{\iota=0}^{\infty}\frac{\left(\frac{v}{2}\right)^{2\iota}}{(\iota!)^2}\psi(\iota) \qquad (5.60)$$

and

$$I_0(v) = J_0(iv)$$

$$\psi(\iota) = \frac{d}{d\iota}\ln\iota! = \frac{(\iota!)'}{\iota!}$$

where I_0 and K_0 are the modified Bessel and Neumann functions, and ψ is the digamma function.

CHAPTER 6

Distributed Source

The voltage and current waves on the outermost conductor of a coaxial transmission line excited by a distributed voltage source due to a lightning discharge as determined either approximately (Chapter 4) or exactly (Chapter 5) are now determined.

6.1 Telegrapher's Equation

The transmission line extends for a length ℓ along the x-axis of a rectangular coordinate system, as shown in Figure 6. The transmission line has a terminating impedance Z^- at x=0 and a terminating impedance Z^+ at $x=\ell$. The transmission line is excited by a distributed voltage source $\mathcal{V}(x')$.

The voltage V(x) and the current I(x) on the transmission line satisfy the coupled wave equations, c.f. equations (4.14) and (4.15),

$$\frac{d}{dx} V = - ZI + \mathcal{V} \tag{6.1}$$

$$\frac{d}{dx} I = - YV + \mathcal{I} \tag{6.2}$$

where

$$Z = Z_L + Z_S,$$

$$Y = 1/Z_T$$

and

$$\mathcal{V} = E_0(x)$$
$$\mathcal{I} = 0^0$$

These coupled wave equations are easily solved with the use of Green's Functions for the voltage and current.

6.2 Green's Functions

The voltage V(x) and the current I(x) on the transmission line due to a distributed voltage source $\mathcal{V}(x')$ are determined by the superposition integrals

$$V(x) = \int_0^\ell dx' G_V(x,x') \mathcal{V}(x')$$

$$I(x) = \int_0^\ell dx' G_I(x,x') \mathcal{V}(x')$$

where the voltage Green's Function $G_V(x,x')$ and the current Green's Function $G_I(x,x')$ satisfy the coupled wave equations

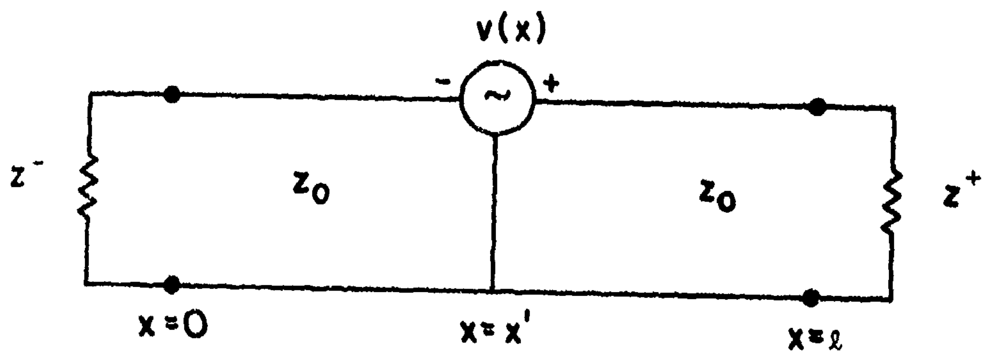


Fig. 6. Transmission Line Model

$$\frac{d}{dx} G_V = - ZG_I + \delta(x-x') \tag{6.5}$$

$$\frac{d}{dx} G_I = -YG_V \tag{6.6}$$

where $\delta(x-x')$ is the Dirac delta function at $x = x'$.

The coupled wave equations (6.5) and (6.6) are solved simultaneously to yield

$$G_V(x,x') = \begin{cases} V^-(e^{+\gamma x} + \Gamma^- e^{-\gamma x}) & (x < x') \\ V^+(e^{-\gamma x} + \Gamma^+ e^{+\gamma x} e^{-2\gamma \ell}) & (x > x') \end{cases} \tag{6.7}$$

$$G_I(x,x') = \begin{cases} -\dfrac{V^-}{Z_c}(e^{+\gamma x} - \Gamma^- e^{-\gamma x}) & (x < x') \\ +\dfrac{V^+}{Z_c}(e^{-\gamma x} - \Gamma^+ e^{+\gamma x} e^{-2\gamma \ell}) & (x > x') \end{cases} \tag{6.8}$$

where the propagation constant γ and the characteristic impedance Z_c are defined by

$$\gamma = \sqrt{YZ}$$

$$Z_c = \sqrt{\frac{Z}{Y}}$$

and the reflection coefficients $\Gamma^\pm$ are defined by

$$\Gamma^- = \frac{Z^- - Z_c}{Z^- + Z_c}$$

$$\Gamma^+ = \frac{Z^+ - Z_c}{Z^+ + Z_c}$$

The constants $V^\pm$ are to be determined from the boundary conditions which relate the continuity of the current at $x = x'$ and the discontinuity of the voltage at $x = x'$, i.e.,

$$G_I(x' + \varepsilon,\ x') - G_I(x' - \varepsilon,\ x') = 0 \tag{6.9}$$

$$G_V(x' + \varepsilon,\ x') - G_V(x' - \varepsilon,\ x') = 1 \tag{6.10}$$

The boundary conditions yield the following values of $V^{\pm}$:

$$V^- = + \frac{e^{-\gamma x'} - \Gamma^+ e^{+\gamma x'} e^{-2\gamma \ell}}{\Delta} \tag{6.11}$$

$$V^+ = - \frac{e^{+\gamma x'} - \Gamma^- e^{-\gamma x'}}{\Delta} \tag{6.12}$$

where

$$\Delta = -2(1 - \Gamma^- \Gamma^+ e^{-2\gamma \ell})$$

Therefore,

$$G_V(x,x') = \begin{cases} -\dfrac{e^{-\gamma x'} - \Gamma^+ e^{+\gamma x'} e^{-2\gamma \ell}}{2(1-\Gamma^-\Gamma^+ e^{-2\gamma \ell})} (e^{+\gamma x} + \Gamma^- e^{-\gamma x}) & (x < x') \\[2ex] +\dfrac{e^{+\gamma x'} - \Gamma^- e^{-\gamma x'}}{2(1-\Gamma^-\Gamma^+ e^{+2\gamma \ell})} (e^{-\gamma x} + \Gamma^+ e^{+\gamma x} e^{-2\gamma \ell}) & (x > x') \end{cases} \tag{6.13}$$

$$G_I(x,x') = \begin{cases} +\dfrac{e^{-\gamma x'} - \Gamma^+ e^{+\gamma x'} e^{-2\gamma \ell}}{2Z_c(1-\Gamma^-\Gamma^+ e^{-2\gamma \ell})} (e^{+\gamma x} - \Gamma^- e^{-\gamma x}) & (x < x') \\[2ex] +\dfrac{e^{+\gamma x'} - \Gamma^- e^{-\gamma x'}}{2Z_c(1-\Gamma^-\Gamma^+ e^{-2\gamma \ell})} (e^{-\gamma x} - \Gamma^+ e^{+\gamma x} e^{-2\gamma \ell}) & (x > x') \end{cases} \tag{6.14}$$

CHAPTER 7

Transfer Functions

Once the induced current and voltage surges on the outer conductor ($\rho = \rho_>$) of a coaxial conductor are known due to a nearby lightning discharge, the induced current and voltage surges on the inner conductor ($\rho = \rho_<$) can be determined via the use of the impedance transfer functions for a coaxial cable, as developed in Appendix B. As shown in the Appendix B,

$$E_x|_{\rho=\rho_>} = Z_{ee} I_{ext} + Z_{ei} I_{int} \tag{7.1}$$

$$E_x|_{\rho=\rho_<} = Z_{ie} I_{ext} + Z_{ii} I_{int} \tag{7.2}$$

where

$$Z_{ee} = \frac{k}{-j\omega\tilde{\varepsilon}} \frac{1}{2\pi\rho_>} \frac{\Delta ee}{\Delta}$$

$$Z_{ei} = \frac{1}{-j\omega\tilde{\varepsilon}} \frac{1}{2\pi} \frac{1}{\rho_<\rho_>} \frac{1}{\Delta}$$

$$Z_{ie} = \frac{1}{-j\omega\tilde{\varepsilon}} \frac{1}{2\pi} \frac{1}{\rho_<\rho_>} \frac{1}{\Delta'}$$

$$Z_{ii} = \frac{k}{-j\omega\tilde{\varepsilon}} \frac{1}{2\pi\rho_<} \frac{\Delta_{ii}}{\Delta'}$$

and

$$\Delta_{ee} = Y_1(k\rho_<)J_0(k\rho_>) + J_1(k\rho_<)Y_0(k\rho_>)$$

$$\Delta_{ii} = Y_1(k\rho_>)J_0(k\rho_<) + J_1(k\rho_>)Y_0(k\rho_<)$$

and

$$\Delta = J_1(k\rho_>)Y_1(k\rho_<) - J_1(k\rho_<)Y_1(k\rho_>)$$

$$\Delta' = J_1(k\rho_<)Y_1(k\rho_>) - J_1(k\rho_>)Y_1(k\rho_<)$$

If the coaxial cable is composed of several concentric shells rather than just one, as depicted above, then the above formulas can be used recursively to obtain the current and voltages on the innermost cylinder due to the current and voltages on the outermost cylinder.

CHAPTER 8

Conclusions

The above theories are all that are necessary to find the induced voltage and current surges on the center conductor of a coaxial conductor resulting from the penetration of the outer armour and the inner shield of the cable of a transient electro-magnetic pulse caused by a nearby lightning discharge to earth.

The resulting equations are being programmed for numerical solution on a digital computer. When the program codes are completed and checked, a parametric study of the results will be undertaken. The results of the parametric study will be presented in a forthcoming report.

REFERENCES

(1) Cosel, R. M., Revay, A.W. Jr., and Stahmann, J., "FAA Lightning Protection Study: Lightning Simulation Test of Communication Cables," Report No. FAA-RD-74-41, January 1974.

(2) Chen, C. L., "FAA Lightning Protection Study: Lightning Protection Devices," Report No. FAA-RD-74-104, April 1974.

(3) Huddleston, G. K., Nordgard, J.D., and Larson, R.W., "FAA Lightning Protection Study: Lightning Protection Requirements for AN/GRN-27(V) Instrument Landing System," Report No. FAA-RD-74-131, April 1974.

(4) Stratton, J.A., Electromagnetic Theory, McGraw-Hill Book Company, Inc. NY 1941.

(5) Magnus, W. and Oberhettinger, F., Formulas and Theorems for the Functions of Mathematical Physics, Chelsea Publication Company, NY 1943, p. 34.

(6) Erdelyi, A., Magnus, W., Oberhettinger, F., Tricomi, F.G., Higher Transcendental Functions, vol. II, McGraw-Hill Book Company, NY 1953, p. 95.

(7) Sommerfeld, A., Partial Differential Equations and Physics, Academic Press, Inc., 1949, p. 257.

(8) Marston, D.R. and Graham, W.R., "Current Induced in Cables in Earth by a Continuous-wave Electromagnetic Field," Technical Report AFWL-TR-65-94, Air Force Weapons Laboratory. Kirtland AFB, New Mexico, May 1966.

(9) Ref. 5, p. 33.

(10) Sunde, E.D., Earth Conduction Effects in Transmission Systems, Dover Publications, Inc. New York, NY 1967.

APPENDIX A

Surface, Longitudinal and Transverse Impedances

In this Appendix, the impedances defined in (4.11), (4.12), and (4.13) are evaluated and the results are listed in (4.19)-(4.23). The derivation is put in an Appendix so that it would not interrupt the continuity and the development of the main text. In additon, because of the geometry involved in the derivation, it is convenient to take a Fourier transform with respect to x, instead of a Hankel transform with respect to r, as is the case in the main text. It is felt that the least confusion is caused by removing the derivation to an Appendix.

A.1 Hertz Vector

The geometry to be considered is a conducting wire of radius b, surrounded by a coaxial insulating layer of thickness c-b and embedded in an extended region as shown in Figure A.1. The axis of the wire is taken to be the x-axis. As demonstrated later, all boundary conditons are satisfied by using the x-component of Hertz vector π_x, alone. In terms of the x component of the Hertz vector π_x, the field intensities $\vec{E}$ and $\vec{H}$ are

$$\vec{E}_i = \hat{r}\,\frac{\partial \pi_{xi}}{\partial r \partial x} + \hat{x}\,\left(\frac{\partial^2 \pi_{xi}}{\partial x^2} + k_i^{\,2} \pi_{xi}\right) \tag{A.1}$$

$$\vec{H}_i = -(\sigma_i + i\omega\varepsilon_i)\,\frac{\partial \pi_{xi}}{\partial r}\,\hat{\phi} \tag{A.2}$$

where π_x must be a solution of

$$\nabla^2 \pi_{xi} + k_i^{\,2} \pi_{xi} = 0 \tag{A.3}$$

The continuity of the tangential components of the magnetic field intensity, $\vec{H}$, at r=b and r=c requires that

PRECEDING PAGE BLANK-NOT FILMED

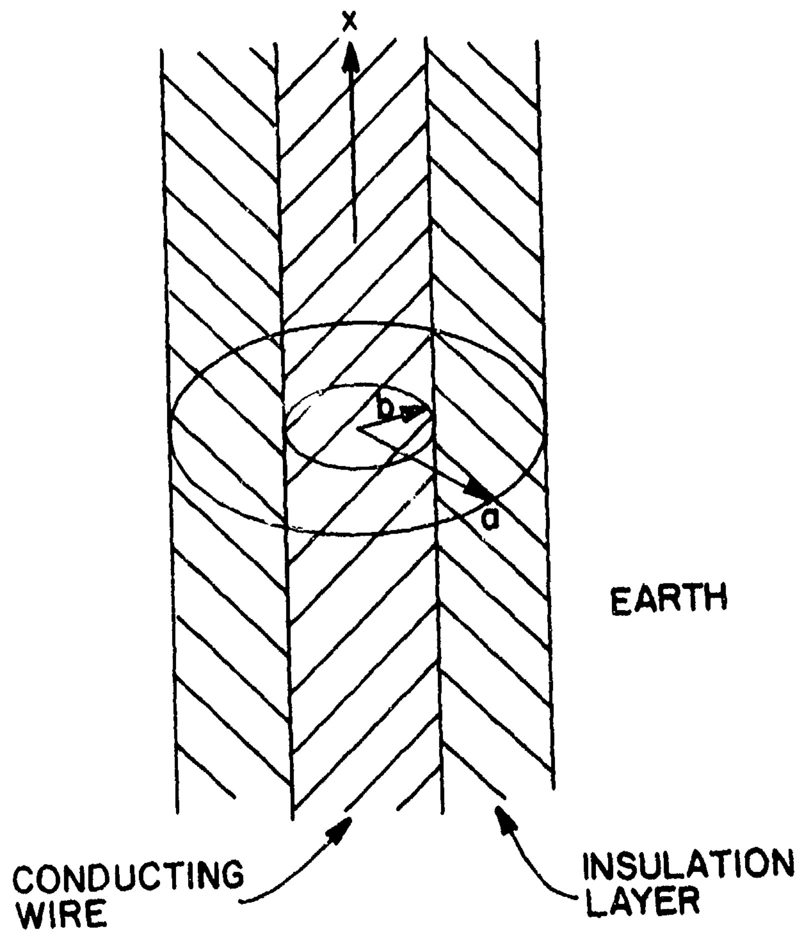


Fig. A.1. Geometry of an Insulated Wire

$$(\sigma_c + i\omega\varepsilon_c)\frac{\partial\pi_{xc}}{\partial r} = (\sigma_I + i\omega\varepsilon_I)\frac{\partial\pi_{xI}}{\partial r} \qquad \text{at } r=b \tag{A.4}$$

$$(\sigma_I + i\omega\varepsilon_I)\frac{\partial\pi_{xI}}{\partial r} = (\sigma_e + i\omega\varepsilon_e)\frac{\partial\pi_{xe}}{\partial r} \qquad \text{at } r=c \tag{A.5}$$

In the above equations, the subscript I stands for the quantities or terms related to the insulating layer. As (A.4) and (A.5) are valid for all values of x, it is obvious that $(\sigma_i + i\omega\varepsilon_i)\frac{\partial^2\pi_{xi}}{\partial r\partial x}$ is also continuous at r=b and r=c, which implies that the continuity of the normal components of the conduction and the displacement currents, $(\sigma_i + i\omega\varepsilon_i)E_{ri}$, is automatically satisfied.

The continuity of the tangential electric field intensity $\vec{E}$ at r=b and r=c requires that

$$\frac{\partial^2{}_{xc}}{\partial x^2} + k_c{}^2\pi_{xc} = \frac{\partial^2\pi_{xI}}{\partial x^2} + k_I{}^2\pi_{xI} \qquad \text{at } r=b \tag{A.6}$$

$$\frac{\partial^2\pi_{xI}}{\partial x^2} + k_I^{2\,2}\pi_{xI} = \frac{\partial^2\pi_{xe}}{\partial x^2} + k_e{}^2\pi_{xe} \qquad \text{at } r=c \tag{A.7}$$

Since the interfaces r=b, $-\infty<x<\infty$, and r=c, $-\infty<x<\infty$, are the entire cylindrical surfaces, it is convenient to use Fourier transform with respect to x,

$$\tilde{\pi}_x(r,\xi,\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\pi_x(r,x,\omega)e^{+j\xi x}dx \tag{A.8}$$

$$\pi_x(r,x,\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\tilde{\pi}_x(r,\xi,\omega)e^{-j\xi x}d\xi \tag{A.9}$$

Applying the Fourier transform to (A.3) and noting that π_x is independent of ϕ, the differential equation for $\tilde{\pi}_x$ is greatly simplified to

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\tilde{\pi}_{xi}}{\partial r}\right) + (k_i{}^2 - \xi^2)\tilde{\pi}_{xi} = 0$$

It is immediately obvious that the solutions for $\tilde{\pi}_x$ are the appropriate cylindrical functions. Thus, for the conducting wire, (r<b), the insulating

region b<r<c, and the earth r>c, the appropriate expressions are, respectively,

$$\pi_{xc}(r,x,\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F_o(\zeta) J_o(\xi_c r) e^{-j\zeta x} d\zeta \quad , \qquad (r < b) \tag{A.10}$$

$$\pi_{x1}(r,x,\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [F_1(\zeta) J_o(\xi_1 r) + F_2(\zeta) Y_o(\xi_1 r)] e^{-j\zeta x} d\zeta, \quad (b<r<c) \tag{A.11}$$

$$\pi_{xe}(r,x,\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F_3(\zeta) H_o^{(2)}(\xi_e r) e^{-j\zeta x} d\zeta \quad , \qquad (r>c) \tag{A.12}$$

where

$$\xi_i = \sqrt{k_i^2 - \zeta^2}.$$

To ensure the convergence of these integrals, the branch $Re(\xi)>0$, is chosen. F_o, F_1, F_2 and F_3 are yet to be determined. In terms of these functions, the axial current I_x carried by the conducting wire can be written as

$$I_x(x,\omega) = 2\pi \int_o^b J_x(r,x,\omega) r dr$$

$$= \sqrt{2\pi}\, b(\sigma_c + i\omega\varepsilon_c) \int_{-\infty}^{\infty} \xi_c F_o(\zeta) J_1(\xi_c b) e^{-j\zeta x} d\zeta \tag{A.13}$$

Similarly V(x), $\Phi(x)$, $E_x(b,x)$, as introduced in Sec. 4.1, can also be expressed in terms of F_o, F_1, F_2, and F_3. Substituting these expressions into (4.11)-(4.13), we obtain

$$Z_{i}(x) = \frac{\int F_o(\zeta) \xi_c^2 J_o(\xi_c b) e^{-j\zeta x} d\zeta}{2\pi b(\sigma_c + j\omega\varepsilon_c) \int F_o(\zeta) \xi_c J_1(\xi_c b) e^{-j\zeta x} d\zeta} \tag{A.14}$$

$$Z_{L1}(x) = \frac{-j\omega\mu_1(\sigma_1 + j\omega\varepsilon_1) \int \{F_1(\zeta)[J_o(\xi_1 c) - J_o(\xi_1 b)] + F_2(\zeta)[Y_o(\xi_1 c) - Y_o(\xi_1 c)]\} e^{-j\zeta x} d\zeta}{2\pi b(\sigma_c + j\omega\varepsilon_c) \int F_o(\zeta) \xi_c J_1(\xi_c b) e^{-j\zeta x} d\zeta} \tag{A.15}$$

$$Z_{Le}(x) = \frac{j\omega\mu_e(\sigma_e + j\omega\varepsilon_e)}{2\pi b(\sigma_c + j\omega\varepsilon_c)} \quad \frac{\int F_3(\zeta) H_o^{(2)}(\xi_e c) e^{-j\zeta x} d\zeta}{\int F_o(\zeta) \xi_c J_1(\xi_c b) e^{-j\zeta x} d\zeta} \tag{A.16}$$

$$Z_{T_1}(x) = \frac{-\int \{F_1(\zeta)[J_0(\xi_1 c)-J_o(\xi_1 b)] + F_2(\zeta)[Y_o(\xi_1 c) - Y_o(\xi_1 c)]\}\zeta e^{-j\zeta x}d\zeta}{2\pi b(\sigma_c+j\omega\epsilon_c)\int\zeta\xi_c F_o(\zeta)J_1(\xi_c b)e^{-j\zeta x}d\zeta} \tag{A.17}$$

$$Z_{T_2}(x) = \frac{\int\zeta F_3(\zeta)H_o^{(2)}(\xi_e c)e^{-j\zeta x}d\zeta}{2\pi b(\sigma_c+j\omega\epsilon_c)\int\zeta\xi_c F_o(\zeta)J_1(\xi_c b)e^{-j\zeta x}d\zeta} \tag{A.18}$$

Strictly speaking, the impedances are functions of x, as exhibited in (A.14)-(A.18). However, only a moderate variation with respect to x over the major portions of the wire is expected, except near the points where I_x varies rapidly. Furthermore, it can be shown that when the fields under consideration are travelling waves, these impedances are truly independent of x. Thus, a good approximation to the impedances can be obtained by considering the natural modes guided by the coaxial configuration.

A.2 Natural Modes

For the natural modes, the external sources or excitations are absent and the fields are completely specified by (A.10)-(A.12). To determine F_o, F_1, F_2, F_3, and the propagation constant, equations (A.10)-(A.12). are required to satisfy the boundary conditions (A.4)-(A.7). The resulting expressions are:

$$(\sigma_c+j\omega\epsilon_c)\xi_c J_1(\xi_c b)F_o(\zeta) - (\sigma_1+j\omega\epsilon_1)\xi_1[J_1(\xi_1 b)F_1(\zeta) + Y_1(\xi_1 b)F_2(\zeta)] = 0 \tag{A.19}$$

$$(\sigma_1+j\omega\epsilon_1)\xi_1[J_1(\xi_1 c)F_1(\zeta) + Y_1(\xi_1 c)F_2(\zeta)] - (\sigma_e+j\omega\epsilon_e)\xi_e H_1^{(2)}(\xi_e c)F_3(\zeta) = 0 \tag{A.20}$$

$$\xi_c^2 J_o(\xi_c b)F_o(\zeta) - \xi_1^2[J_o(\xi_1 b)F_1(\zeta) + Y_o(\xi_1 b)F_2(\zeta)] = 0 \tag{A.21}$$

$$\xi_1^2[J_o(\xi_1 c)F_1(\zeta) + Y_o(\xi_1 c)F_2(\zeta)] - \xi_e^2 H_o^{(2)}(\xi_e c)F_3(\zeta) = 0 \tag{A.22}$$

(A.19)-(A.22) form a set of homogeneous algebraic equations for F_o, F_1, F_2, F_3. For a set of nontrivial solutions to exist, it is necessary that

$$\Delta_1 = \Delta_2 \tag{A.23}$$

where

$$\Delta_1 = \frac{\frac{\sigma_c + j\omega\varepsilon_c}{\sigma_l + j\omega\varepsilon_l}\frac{\xi_l}{\xi_c} J_1(\xi_c b) J_o(\xi_l b) - J_o(\xi_c b) J_1(\xi_l b)}{\frac{\sigma_c + j\omega\varepsilon_c}{\sigma_l + j\omega\varepsilon_l}\frac{\xi_l}{\xi_c} J_1(\xi_c b) Y_o(\xi_l b) - J_o(\xi_c b) Y_1(\xi_l b)} \tag{A.24}$$

$$\Delta_2 = \frac{\frac{\sigma_e + j\omega\varepsilon_e}{\sigma_l + j\omega\varepsilon_l}\frac{\xi_l}{\xi_e} H_1^{(2)}(\xi_e c) J_o(\xi_l c) - H_o^{(2)}(\xi_e c) J_1(\xi_l c)}{\frac{\sigma_e + j\omega\varepsilon_e}{\sigma_l + j\omega\varepsilon_l}\frac{\xi_l}{\xi_e} H_1^{(2)}(\xi_e c) Y_o(\xi_l c) - H_o^{(2)}(\xi_e c) Y_1(\xi_l c)} \tag{A.25}$$

Equation (A.23) is the dispersion relation and is used to solve for the propagation constant. Also obtained are the dependence of F_1, F_2 and F_3 on F_o:

$$\frac{F_1(\xi)}{F_o(\xi)} = \frac{\sigma_c + j\omega\varepsilon_c}{\sigma_l + j\omega\varepsilon_l}\frac{\xi_c}{\xi_l}\frac{J_1(\xi_c b)}{J_1(\xi_l b) - \Gamma Y_1(\xi_l b)} \tag{A.26}$$

$$\frac{F_2(\xi)}{F_o(\xi)} = \frac{-(\sigma_c + j\omega\varepsilon_c)}{\sigma_l + j\omega\varepsilon_l}\frac{\xi_c}{\xi_l}\frac{\Gamma J_1(\xi_c b)}{J_1(\xi_l b) - \Gamma Y_1(\xi_l b)} \tag{A.27}$$

$$\frac{F_3(\xi)}{F_o(\xi)} = \frac{\sigma_c + j\omega\varepsilon_c}{\sigma_l + j\omega\varepsilon_l}\frac{\xi_l \xi_c}{\xi_e^2}\frac{J_1(\xi_c b)[J_o(\xi_l c) - \Gamma Y_o(\xi_l c)]}{H_o^{(2)}(\xi_e c)[J_1(\xi_l b) - \Gamma Y_1(\xi_l b)]} \tag{A.28}$$

where

$$\Gamma = \frac{\frac{\sigma_c + j\omega\varepsilon_c}{\sigma_l + j\omega\varepsilon_l}\frac{\xi_l}{\xi_c} J_1(\xi_c b) J_o(\xi_l b) - J_o(\xi_c b) J_1(\xi_l b)}{\frac{\sigma_c + j\omega\varepsilon_c}{\sigma_l + j\omega\varepsilon_l}\frac{\xi_l}{\xi_c} J_1(\xi_c b) Y_o(\xi_l b) - J_o(\xi_c b) Y_1(\xi_l b)} \tag{A.29}$$

Let the propagation constant determined from (A.23) be denoted by ζ_o, then the current I_x on the wire must be determined by

$$I_x(x,\omega) = I_o e^{-j\zeta_o x} = \int I_o \delta(\zeta-\zeta_o) e^{-j\zeta x} d\zeta \qquad (A.30)$$

where I_o is a constant.

From (A.13) and (A.30), F_o can now be determined,

$$F_o(\zeta) = \frac{I_o \delta(\zeta-\zeta_o)}{\sqrt{2\pi}\, b(\sigma_c+j\omega\varepsilon_c)\ \xi_c\ J_1(\xi_c b)} \qquad (A.31)$$

The evaluation of the integrals in (A.14)-(A.18) are greatly simplified by the presence of the deltat function in (A.31). Thus,

$$Z_s = \left.\frac{\xi_c\ J_o(\xi_c b)}{2\pi b(\sigma_c+j\omega\varepsilon_c)\ \ J_1(\xi_c b)}\right|_{\zeta=\zeta_o} \qquad (A.32)$$

$$Z_{L_1} = \frac{-j\omega\mu_l(\sigma_l+j\omega\varepsilon_l)}{2\pi b(\sigma_c+j\omega\varepsilon_c)} \left.\frac{F_1(\zeta)[J_o(\xi_l c)-J_o(\xi_l b)] + F_2(\zeta)[Y_o(\xi_l c)-Y_o(\xi_l b)]}{\xi_c F_o(\zeta) J_1(\xi_c b)}\right|_{\zeta=\zeta_o} \qquad (A.33)$$

$$Z_{L_2} = \frac{j\omega\mu_e(\sigma_e+j\omega\varepsilon_e)}{2\pi b(\sigma_c+j\omega\varepsilon_c)}\ \frac{F_3(\zeta)}{F_o(\zeta)}\ \left.\frac{H_o^{(2)}(\xi_e c)}{\xi_c J_1(\xi_c b)}\right|_{\zeta=\zeta_o} \qquad (A.34)$$

$$Z_{T_1} = \frac{-1}{2\pi b(\sigma_c+j\omega\varepsilon_c)} \left.\frac{F_1(\zeta)[J_o(\xi_l c)-J_o(\xi_l b)] + F_2(\zeta)[Y_o(\xi_l c)-Y_o(\xi_l b)]}{\xi_c F_o(\zeta) J_1(\xi_c b)}\right|_{\zeta=\zeta_o} \qquad (A.35)$$

$$Z_{T_2} = \frac{1}{2\pi b(\sigma_c+j\omega\varepsilon_c)}\ \frac{F_3(\zeta)}{F_o(\zeta)}\ \left.\frac{H_o^{(2)}(\xi_e c)}{\xi_c J_1(\xi_c b)}\right|_{\zeta=\zeta_o} \qquad (A.36)$$

These expressions are valid in general. They can be simplified for highly conducting wires by replacing the cylindrical functions with their appropriate approximations.

APPENDIX B

Internal and External Impedances

The internal and external impedances of cylindrical conductors are required when the frequency is such that the skin effect is considered.

For a long cylindrical conductor oriented along the x-axis, with internal or external coaxial return, the Hertz potential $\vec{\pi}$ is independent of ϕ and x and has a component in the x-direction only. Therefore, in equation (2.14),

$$n = 0 \tag{B.1}$$

$$\xi = 0 \tag{B.2}$$

Hence

$$\tau = k \tag{B.3}$$

Therefore, the Hertz potential is explicitly chosen to be, $\vec{\pi} = \pi_x \hat{x}$, where

$$\pi_x = c_1 J_o(k\rho) + c_2 Y_o(k\rho) \tag{B.4}$$

where c_1 and c_2 are arbitrary constants to be determined from the boundary conditions. The field intensities are determined by

$$E_x = k^2 \pi_x \tag{B.5}$$

$$H_x = 0 \tag{B.6}$$

and

$$\vec{E}_t = \frac{\partial}{\partial x} \nabla_t \pi_x \tag{B.7}$$

$$\vec{H}_t = -j\omega\tilde{\epsilon}\, \hat{x} \times \nabla_t \pi_x \tag{B.8}$$

Therefore, the only non-zero components of the field intensities are

$$E_x = k^2 [c_1 J_o(k\rho) + c_2 Y_o(k\rho)] \tag{B.9}$$

$$H_\phi = -j\omega\tilde{\epsilon}\, k[c_1 J_1(k\rho) + c_2 Y_1(k\rho)] \tag{B.10}$$

If the conductor has an inside radius $\rho_<$ and an outside radius $\rho_>$, the magnetic field intensity H_ϕ at $\rho = \rho_<$ must be zero, provided the conductor current I_{ext} returns outside of the conductor, and at $\rho = \rho_>$ must equal $I_{ext}/2\pi\rho_>$, i.e., from Ampere's Law

PRECEDING PAGE BLANK-NOT FILMED

$$2\pi\rho_< H_\phi\Big|_{\rho=\rho_<} = 0 \tag{B.11}$$

$$2\pi\rho_> H_\phi\Big|_{\rho=\rho_>} = I_{ext} \tag{B.12}$$

Hence, with the values of H_ϕ from the above equation, the boundary conditions for c_1 and c_2 are

$$c_1 J_1(k\rho_<) - c_2 Y_1(k\rho_<) = 0 \tag{B.13}$$

$$c_1 J_1(k\rho_>) - c_2 Y_1(k\rho_>) = \frac{1}{j\omega\tilde{\varepsilon}\, k}\,\frac{I_{ext}}{2\pi\rho_>} \tag{B.14}$$

which are easily solved for c_1 and c_2 to yield

$$c_1 = \frac{1}{j\omega\tilde{\varepsilon}\, k}\,\frac{I_{ext}}{2\pi\rho_>}\,\frac{Y_1(k\rho_<)}{\Delta} \tag{B.15}$$

$$c_2 = \frac{1}{-j\omega\tilde{\varepsilon}\, k}\,\frac{I_{ext}}{2\pi\rho_>}\,\frac{J_1(k\rho_<)}{\Delta} \tag{B.16}$$

where the determinant Δ is defined by

$$\Delta = J_1(k\rho_>)Y_1(k\rho_<) - J_1(k\rho_<)Y_1(k\rho_>) \tag{B.17}$$

Therefore, the Hertz vector becomes

$$\pi_x = \frac{1}{j\omega\tilde{\varepsilon}\, k}\,\frac{I_{ext}}{2\pi\rho_>}\,\frac{Y_1(k\rho_<)J_o(k\rho) - J_1(k\rho_<)Y_o(k\rho)}{\Delta} \tag{B.18}$$

and the electric field intensity in the x direction becomes

$$E_x = \frac{k}{j\omega\tilde{\varepsilon}}\,\frac{I_{ext}}{2\pi\rho_>}\,\frac{Y_1(k\rho_<)J_o(k\rho) - J_1(k\rho_<)Y_o(k\rho)}{\Delta} \tag{B.19}$$

The ratio of the electric field along the outer surface $\rho = \rho_>$ to the total external current is defined as the external surface impedance with external return Z_{ee}; and, the corresponding ratio of the electric field along the inner surface $\rho = \rho_<$ to the total external current is defined as the internal surface impedance with external return Z_{ie}, i.e.

$$Z_{ee} = \frac{E_x \big|_{\rho=\rho_>}}{I_{ext}} = \frac{k}{j\omega\tilde{\varepsilon}} \quad \frac{1}{2\pi\rho_>} \quad \frac{\Delta_{ee}}{\Delta} \tag{B.20}$$

$$Z_{ie} = \frac{E_x \big|_{\rho=\rho_<}}{I_{ext}} = \frac{1}{-j\omega\varepsilon} \quad \frac{1}{2\pi} \quad \frac{1}{\rho_<\rho_>} \quad \frac{1}{\Delta} \tag{B.21}$$

where

$$\Delta_{ee} = Y_1(k\rho_<)J_o(k\rho_>) - J_1(k\rho_<)Y_o(k\rho_>) \tag{B.22}$$

The following Wronskian relation has been used:

$$-Y_1(k\rho_<)J_o(k\rho_<) + J_1(k\rho_<)Y_o(k\rho_<) = \frac{1}{k\rho_<} \tag{B.23}$$

If the conductor has an inside radius $\rho_<$ and outside radius $\rho_>$, the magnetic field intensity H_ϕ at $\rho=\rho_>$ must be zero, provided the conductor current I_{int} returns inside of the conductor, and at $\rho=\rho_<$ must equal $I_{int}/2\pi\rho_<$, i.e., from Ampere's Law

$$2\pi\rho_> H_\phi \big|_{\rho=\rho_>} = 0 \tag{B.24}$$

$$2\pi\rho_< H_\phi \big|_{\rho=\rho_<} = -I_{int} \tag{B.25}$$

Hence, with the values of H_ϕ from the above equation, the boundary conditions for c_1 and c_2 are

$$c_1 J_1(k\rho_>) - c_2 Y_1(k\rho_>) = 0 \tag{B.26}$$

$$c_1 J_1(k\rho_<) - c_2 Y_1(k\rho_<) = \frac{1}{-j\omega\tilde{\varepsilon}\, k} \quad \frac{I_{int}}{2\pi\rho_<} \tag{B.27}$$

which are easily solved for c_1 and c_2 to yield

$$c_1 = \frac{1}{-j\omega\tilde{\varepsilon}\, k} \quad \frac{I_{int}}{2\pi\rho_<} \quad \frac{Y_1(k\rho_>)}{\Delta'} \tag{B.28}$$

$$c_2 = \frac{1}{j\omega\tilde{\varepsilon}\, k} \quad \frac{I_{int}}{2\pi\rho_<} \quad \frac{J_1(k\rho_>)}{\Delta'} \tag{B.29}$$

where the determinant Δ' is defined by

$$\Delta' = J_1(k\rho_<)Y_1(k\rho_>)-J_1(k\rho_>)Y_1(k\rho_<) \tag{B.30}$$

Therefore, the Hertz vector becomes

$$\pi_x = \frac{1}{j\omega\bar{\varepsilon}\, k}\ \frac{I_{int}}{2\pi\rho_<}\ \frac{Y_1(k\rho_>)J_o(k\rho)-J_1(k\rho_>)Y_o(k\rho)}{\Delta'} \tag{B.31}$$

and the electric field intensity in the x direction becomes

$$E_x = \frac{k}{-j\omega\bar{\varepsilon}}\ \frac{I_{int}}{2\pi\rho_<}\ \frac{Y_1(k\rho_>)J_o(k\rho)-J_1(k\rho_>)Y_o(k\rho)}{\Delta'} \tag{B.32}$$

The ratio of the electric field along the outer surface $\rho=\rho_>$ to the total internal current is defined as the external surface impedance with internal return Z_{ei}; and, the corresponding ratio of the electric field along the inner surface $\rho=\rho_<$ to the total internal current is defined as the internal surface impedance with internal return Z_{ii}, i.e.,

$$Z_{ii} = \frac{E_x|_{\rho=\rho_<}}{I_{int}} = \frac{k}{-j\omega\bar{\varepsilon}}\ \frac{1}{2\pi\rho_<}\ \frac{\Delta'_{ii}}{\Delta'} \tag{B.33}$$

$$Z_{ei} = \frac{E_x|_{\rho=\rho_>}}{I_{int}} = \frac{1}{j\omega\bar{\varepsilon}}\ \frac{1}{2\pi}\ \frac{1}{\rho_<\rho_>}\ \frac{1}{\Delta'} \tag{B.34}$$

where

$$\Delta'_{ii} = Y_1(k\rho_>)J_o(k\rho_<)-J_1(k\rho_>)Y_o(k\rho_<) \tag{B.35}$$

The following Wronskian relation has been used:

$$-Y_1(k\rho_>)J_o(k\rho_>)+J_1(k\rho_>)Y_o(k\rho_>) = \frac{1}{k\rho_>} \tag{B.36}$$

Therefore, by superposition,

$$E_x|_{\rho=\rho_>} = Z_{ee}I_{ext}+Z_{ei}I_{int} \tag{B.37}$$

$$E_x|_{\rho=\rho_<} = Z_{ie}I_{ext}+Z_{ii}I_{int} \tag{B.38}$$

Notice that $\Delta^1 = -\Delta$

Therefore $Z_{ie} = Z_{ei}$

For solid conductors ($\rho_< = 0$), Z_{ee} is usually referred to as the internal impedance Z_i of the conductor, i.e.

$$Z_i \equiv \lim_{\rho_< \to 0} Z_{ee} = \frac{k}{j\omega\varepsilon_c} \frac{1}{2\pi\rho_>} \frac{J_0(k\rho_>)}{J_1(k\rho_>)}$$

When the Bessel and Neumann functions are replaced by their asymptotic expansions for large values of their arguments, the following approximate formulas are obtained, which apply for the type of cylindrical conductors ordinarily encountered.

$$Z_{ee} \simeq \frac{Z_o}{2\pi} \frac{1}{\rho_>} \left[\coth \sigma t - \frac{\pi}{2\sigma} \left(\frac{3}{\rho_<} + \frac{1}{\rho_>} \right) \right] \tag{B.40}$$

$$Z_{ei} \simeq \frac{Z_o}{2\pi} \frac{1}{\sqrt{\rho_< \rho_>}} \operatorname{csch} \sigma t \tag{B.41}$$

$$Z_{ie} \simeq \frac{Z_o}{2\pi} \frac{1}{\sqrt{\rho_< \rho_>}} \operatorname{csch} \sigma t \tag{B.42}$$

$$Z_{ii} \simeq \frac{Z_o}{2\pi} \frac{1}{\rho_<} \left[\coth \sigma t + \frac{\pi}{2\sigma} \left(\frac{3}{\rho_>} + \frac{1}{\rho_<} \right) \right] \tag{B.43}$$

where

$$t = \rho_> - \rho_<$$

$$Z_o = \sqrt{\frac{\mu_c}{\varepsilon_c}}$$

APPENDIX C

Self and Mutual Impedances

The self and mutual impedances of earth return conductors are required between two current carrying paths, such as the paths C and C' as shown in Figure C1.

The fields due to a Hertzian dipole located on the curve C' and oriented in the direction of the z' axis have the form

$$E_z' = I\ell'\left[-Z_\ell(R) + \frac{\partial^2}{\partial z'^2} Z_t(R)\right] \tag{C.1}$$

$$\vec{E}_t' = I\ell' \frac{\partial}{\partial z'} \nabla_{t'} Z_t(R) \tag{C.2}$$

Therefore, the fields due to a continuous distribution of infinitisimal dipoles distributed along the curve C' have the form

$$E_z' = I \int_{a'\,C'}^{b'} d\ell'\left[-Z_\ell(R) + \frac{\partial^2}{\partial z'^2} Z_t(R)\right] \tag{C.3}$$

$$\vec{E}_t' = I \int_{a'\,C'}^{b'} d\ell' \frac{\partial}{\partial z'} \nabla_{t'} Z_t(R) \tag{C.4}$$

The voltage V impressed along the curve C in the same plane as C' is determined by

$$V = -\int_{a\,C}^{b} \vec{d\ell}\cdot\vec{E} = -\int_{a\,C}^{b} dz(\hat{z}\cdot\hat{z}' E_z' + \hat{z}\cdot\vec{E}_t') \tag{C.5}$$

Therefore,

$$V = -I \int_{a\,C}^{b} d\ell \int_{a'\,C'}^{b'} d\ell' \left[-Z_\ell(R)\cos\phi + \frac{\partial^2}{\partial\ell\partial\ell'} Z_t(R)\right] \tag{C.6}$$

since

$$\hat{z}\cdot\hat{z}' = \cos\phi$$

$$\hat{z}\cdot\nabla_{t'} = \frac{\partial}{\partial\ell'}\sin\phi$$

PRECEDING PAGE BLANK-NOT FILMED

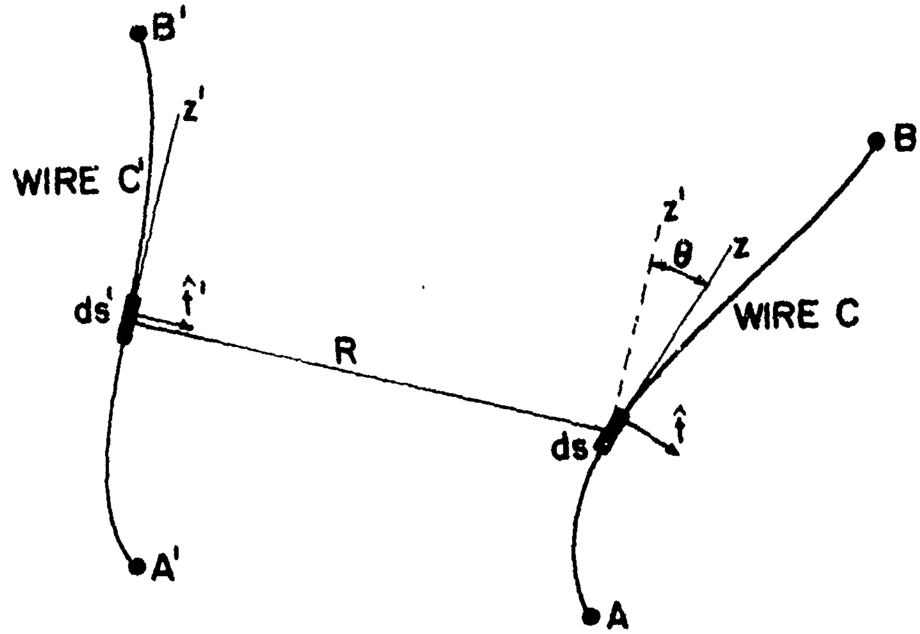


Fig. C.1 Mutual Impedance Between Two Wires C' and C and

$$\frac{\partial}{\partial \ell} = \frac{\partial}{\partial z'} \cos\phi + \frac{\partial}{\partial \ell'} \sin \phi$$

The voltage V is now evaluated as

$$V = I \int_{\substack{a \\ C}}^{b} d\ell \int_{\substack{a' \\ C'}}^{b'} d\ell' \; Z_\ell(R) \cos\phi - I Z_t(R) \Big|_{(b-a)(b'-a')} \tag{C}$$

The mutual impedance between the two current paths C and C' is defined as

$$Z_m = \frac{V}{I} = \int_{\substack{a \\ C}}^{b} d\ell \int_{\substack{a' \\ C'}}^{b'} d\ell' \; Z_\ell(R)\cos\phi - Z_t(R) \Big|_{(b-a)(b'-a')} \tag{C.}$$

The double integral is the mutual impedance between the current paths and represents the longitudinal impedance between the wires, while the second term is independent of the current paths and represents the transverse impedance between the wire terminals through the surrounding medium.

and

$$\frac{\partial}{\partial \ell} = \frac{\partial}{\partial z'} \cos\phi + \frac{\partial}{\partial \ell'} \sin\phi$$

The voltage V is now evaluated as

$$V = I \int_{\substack{a \\ C}}^{b} d\ell \int_{\substack{a' \\ C'}}^{b'} d\ell' \; Z_\ell(R) \cos\phi - I Z_t(R) \Big|_{(b-a)(b'-a')} \tag{C.7}$$

The mutual impedance between the two current paths C and C' is defined as

$$Z_m \equiv \frac{V}{I} = \int_{\substack{a \\ C}}^{b} d\ell \int_{\substack{a' \\ C'}}^{b'} d\ell' \; Z_\ell(R) \cos\phi - Z_t(R) \Big|_{(b-a)(b'-a')} \tag{C.8}$$

The double integral is the mutual impedance between the current paths and represents the longitudinal impedance between the wires, while the second term is independent of the current paths and represents the transverse impedance between the wire terminals through the surrounding medium.

APPENDIX D

FAA Lightning Study Participants

Florida Institute of Technology - Cable Testing

Dr. Andrew W. Revay, Jr.
Richard M. Cosel

Lightning and Transient Research Institute - High Voltage Facility

James C. Stahmann

Purdue University - Protective Devices

Warren Peele (Project Leader)
Dr. Chin-Lin Chen

Georgia Institute of Technology - Equipment Analysis

Keith Huddleston
Dr. Ronal Larson
Dr. John Nordgard

Air Force Institute of Technology - Reliability Aspects

Lt. Col. Jerry L. Hanson
Professor T. L. Regulinski