ULF GENERATED BY A SATELLITE-EMITTED BEAM

FINAL REPORT

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1.0 INTRODUCTION

The work reported under this contract was motivated by the need to investigate possible LF systems that are capable of providing means to communicate with submerged submarines. Since ULF waves have a large skin depth in sea water, it seems natural to consider them for the stated purpose.

The basic problem encountered in designing any feasible LF communications system has three major elements:

- Generation of ULF signals
- Propagation from source to submerged receiver
- Detection of the signal

A basis for the present work was formed in 1974 when TRW personnel performed preliminary calculations of generation of ULF signals by satellite-launched ion beams. These results indicated that signal levels of some 10 mγ at sea level could be produced by beams of some tens of kilowatts, emitted at 3 to 4 earth radii altitude at the geomagnetic equator. A brief description of this work appears in Proposal No. TRW 26766.000R1 (September 1975) and will not be repeated here.

The model used in the earlier calculations was a "broad" ion beam. The term "broad" means that finite beam radius effects were ignored. Thus, such an idealization, which is mathematically convenient, must lead to larger signals and higher efficiency than the realistic case of a beam of finite radial extent. Since the broad-beam calculation gave encouraging results, it was considered worthwhile to examine the finite-cross-section, or "pencil" beam.

The results of this analysis of a finite-radius beam of ions are presented in Sections 2 through 6 of this report. Section 7 addresses the estimate of spatial or radial spreading of the beam due to electrostatic effects, while Section 8 contains a discussion of current and charge neutralization. Modulation of the beam is addressed in Section 9, and use of heavier ions (other than protons) is treated in Section 10.

Section 11 contains a large number of conclusions drawn by consideration of the results in Sections 5 and 6. Finally, Section 12 contains the recommendations based on conclusions presented in Section 11.

2.0 BASIC EQUATIONS FOR RADIATED FIELDS

Maxwell's equations for a electromagnetic wave field propagating in a magnetized plasma can be combined to produce a single wave equation for the electric field $\hat{\underset{\sim}{E}}\ (\underline{r},t)$

$$\nabla(\nabla \cdot \hat{\underset{\sim}{E}}) - \nabla^2 \hat{\underline{E}} + \partial^2 \hat{\underset{\sim}{E}}/\partial t^2 = - \frac{4\pi}{c^2} \frac{\partial}{\partial t} (\hat{\underset{\sim}{J}}^P + \hat{\underset{\sim}{J}}^B) \tag{1}$$

where $\hat{\underset{\sim}{J}}^P$ is the self-consistent current density which supports the wave in the background plasma, while $\hat{\underset{\sim}{J}}^B$ is the externally supplied beam current density.

We introduce phasor notation, and use the convention

$$E_\nu = \begin{cases} (2)^{-\frac{1}{2}} (E_x - i\nu E_y) & \nu = \pm 1 \\ E_0 & \nu = 0 \end{cases} \tag{2}$$

where $\nu = 1\ (-1)$ corresponds to the left-hand (right-hand) circular polarization with respect to the DC magnetic field $\underset{\sim}{B}_0$, taken to be in the z-direction. The subscript ν on other quantities, such as current density $\hat{J}\nu$, wave number k_ν, etc follows the same convention as defined in equation (2) above.

We next introduce the Fourier-Laplace transform

$$E_\nu\ (\underset{\sim}{k},p) = \int d^3r \int dt\ e^{-(pt + i\underset{\sim}{k} \cdot \underset{\sim}{r})}\ \hat{E}_\nu\ (\underline{r},t) \tag{3}$$

of (3) to (1) yields the algebraic equation

$$(k^2 + p^2/c^2)\ E_\nu - k_\nu\ (\underset{\sim}{k} \cdot \underset{\sim}{E}) = - \frac{4\pi p}{c^2} (J_\nu^P + J_\nu^B) \tag{4}$$

The plasma current is $\underset{\sim}{J}^P = n_o \sum_{\pm} q \underset{\sim}{v}$ where $q = \pm e$ and $\underset{\sim}{v}$ is the self-consistent wave-induced velocity, the summation being made over all charged species present. The velocity $\underline{v}$ is determined by the cold fluid equations.

$$\dot{\underset{\sim}{v}} = \frac{q}{m} \underset{\sim}{E} + \underset{\sim}{v} \times \underset{\sim}{\Omega} \tag{5}$$

where $\underset{\sim}{\Omega} = (qB_o/mc)\hat{\underset{\sim}{z}}$ is the psuedovector gyrofrequency. The transformed solution to (5) is

$$v_\nu = \frac{(q/m)\, E_\nu}{p + i\nu\Omega} \tag{6}$$

We define the quantity q_ν which has dimensions of a wave number, by

$$c^2 q_\nu^2 = p^2 + \sum_{\pm} \frac{p\omega_p^2}{p + i\nu\Omega} \tag{7}$$

where $\omega_p^2 = 4\pi n_o e^2/m$ defines the plasma frequency for species of mass m. Substitution of (6) and (7) into (4) yields the following expression for E_ν:

$$E_\nu = \frac{1}{k^2 + q_\nu^2} \left[k_\nu (\underset{\sim}{k} \cdot \underset{\sim}{E}) - \frac{4\pi p}{c^2} J_\nu^B \right] \tag{8}$$

We can eliminate the quantity $\underset{\sim}{k} \cdot \underset{\sim}{E}$ if we multiply (8) by $k_{-\nu}$ and sum the result over ν to obtain

$$\underset{\sim}{k} \cdot \underset{\sim}{E} = \frac{-\frac{4\pi p}{c^2} \sum_{\mu} \frac{k_{-\mu} J_{\mu}^{B}}{k^2 + q_{\mu}^2}}{1 - \sum_{\mu} \frac{k_{-\mu}\, k_{\mu}}{k^2 + q_{\mu}^2}} = -\frac{4\pi p}{c^2}\, g \tag{9}$$

Elimination of $\underset{\sim}{k} \cdot \underset{\sim}{E}$ between (8) and (9) yields

$$E_{\nu} = \frac{-4\pi p/c^2}{k^2 + q_{\nu}^2} \left(k_{\nu} g + J_{\nu}^{B}\right) \tag{10}$$

The Maxwell's equations relating $\hat{\underset{\sim}{B}} + \hat{\underset{\sim}{E}}$, when transformed, can be manipulated to yield the corresponding expressions for the wave magnetic components

$$B_{\nu} = -\frac{\nu c}{p} \left[k_{o} E_{\nu} - k_{\nu} E_{o} \right] \qquad (\nu = \pm 1)$$

$$B_{o} = \frac{c}{p} \sum_{\mu \neq o} k_{-\mu}\, E_{\mu} \qquad (\nu = 0) \tag{11}$$

3.0 MODEL OF THE ION BEAM

We assume that the ion beam is injected at t=o with a well defined speeds $V_{||}$ (parallel to $\underset{\sim}{B}_o$) and $V_\perp$ (perpendicular to $\underset{\sim}{B}_o$). The ions spiral around $\underset{\sim}{B}_o$ with a Larmor radius $R = V_\perp/\Omega$, while their Lamor phase advances as $\phi = \Lambda_z$ where $\Lambda = V_{||}/\Omega$ is the Larmor wave number. At a time $t > o$ the beam then exists in the region $o \leq z \leq V_{||}t$. In a plane perpendicular to $\underset{\sim}{B}_o$ the beam cannot be assumed to be sharply defined because space-charge and high-frequency electrostatic beam-plasma instabilities must occur. These effects will act to diffuse the beam across magnetic field lines, until the beam density drops to a value comparable to the background density.

To account for this expected radial spread of the guiding centers of ions comprising the beam, we use a Gaussian function centered on the mean gyroradius R and having a spread Δ. The mathematical model of this beam is then given by the distribution function

$$f_B\ (\underline{r}, \underline{v}, t) = \frac{n_B}{v_\perp}\ \delta\ (v_z - V_{||})\ \delta\ (v_\perp - V_\perp)\ \delta\ (\phi - \Lambda z) \cdot (\pi\Delta^2)^{-1} \cdot$$
$$\exp\left[-(\chi - R\cos\phi)^2/\Delta^2 - (y - R\sin\phi)^2/\Delta^2\right] \cdot$$
$$H\ (t)\left[H\ (z) - H\ (z - V_{||}t)\right] \tag{12}$$

where H (t) is the Heaviside unit step distribution, $\delta\ (\chi)$ is the Dirac delta distribution, and n_B is the number of beam ions per unit length (the volumetric number density is n_B/Δ^2).

The beam current density is

$$\underset{\sim}{\hat{j}}^B = e\int d^3v\ \underset{\sim}{v}\ f\ (\underset{\sim}{r}, \underset{\sim}{v}, t)$$

which yields the phasor quantity

$$\hat{J}_\nu^B = n_B \, eV_\nu \, e^{-i\nu\Lambda z} \, \hat{G}\,(\underset{\sim}{r}, t) \tag{13}$$

where

$$\hat{G}\,(\underset{\sim}{r}, t) = (\pi\Delta^2)^{-1} \exp\left[-\,(x-R\cos\phi)^2/\Delta^2 - (y-R\sin\phi)^2/\Delta^2\right] H(t) \left[H(z)-H(z-V_{\parallel}\,t)\right] \tag{14}$$

and $V_o = V_{\parallel}$, $V_{\pm 1} = V_{\perp}$ in (13).

In order to maintain the charge-neutrality of the spacecraft, an electron current must be emitted such that it just cancels the beam ion current. Therefore, in what follows, we assume that the total current parallel to $\underset{\sim}{B}_o$ vanishes, i.e. $J^B_{o\parallel} = 0$.

4.0 HYDROMAGNETIC APPROXIMATION

Under the assumption $J^B_{o||} = 0$ the parallel component of the wave electric field is just

$$E_o = -\,\frac{4\pi p}{c^2}\,\frac{k_o\, g}{k^2 + q_o^{\,2}} \tag{15}$$

For low frequencies compared to the electron plasma frequency, that is $ip \ll \omega_{pe}$, the quantity $q_o \sim \omega_{pe}/c$. Since low frequency waves have very long wavelengths, then $k \ll q_o$ for the hydromagnetic modes ($ip \lesssim \Omega$) of interest to us. If one compares the order of magnitudes of E_o and $E_{\pm 1}$, one finds that E_o is smaller by the ratio $k^2c^2/\omega_{pe}^2 \ll 1$. Therefore, we can justify the neglect of all terms involving E_o in our subsequent calculations.

The polarization E_o is associated with longitudinal electrostatic waves having virtually no magnetic components in cold plasma.

In order to further simplify the calculation, we shall concentrate on waves having frequencies much less than the ion gyrofrequency. In practice, this is expressed by $p^2 << \Omega^2$. Even $|p| \sim \Omega/2$ will satisfy the condition to within a reasonable accuracy. With such an approximation made, the expression (7) for q_ν^2 reduces to

$$q_1^2 = q_{-1}^2 \simeq p^2/c_A^2 \tag{16}$$

where $c_A^2 = B_o^2/4\pi n_o m$ is the Alfven speed. The function g defined by (9) reduces to

$$g \simeq \frac{1}{k_o^2 + p^2/c_A^2} \left[k_x J_x^B + k_y J_y^B \right] \tag{17}$$

which expression can then be used to calculate the components of wave magnetic field.

5.0 TRANSVERSE COMPONENTS OF WAVE MAGNETIC FIELD

The combination of (10), (11), (16) and (17) leads to the following expression for the transverse magnetic field components in the hydro - magnetic approximation:

$$B_\nu = \frac{4\pi\nu/c}{k^2 + p^2/c_A^2} \left[\frac{k_o k_\nu \, (k_x J_x^B + k_y J_y^B)}{k_o^2 + p^2/c_A^2} + J_\nu^B \right] \tag{18}$$

The second term in (18) can be called the "bare current" term. It has the effective plasma dielectric loading of the factor $(k^2 + p^2/c_A^2)^{-1}$, which is independent of the angle between the propagation vector $\underset{\sim}{k}$ and the DC magnetic field $\underset{\sim}{B}_o$. This term corresponds to a part of the magnetosonic (or fast) wave which, in cold plasmas, propagates at all angles to $\underset{\sim}{B}_o$ with the same phase velocity. For this reason we may anticipate that the contribution to B_ν from this term will decay as 1/r, where r is the distance from the beam. This contribution therefore is expected to be small in the radiation zone.

The first term in (18) arises from the divergence of the perpendicular beam current ($\underset{\sim}{k} \cdot \hat{\underset{\sim}{j}}^B$). It has the dielectric loading factor $[(k^2+p^2/c_A^2)\cdot(k_o^2+p^2/c_A^2)]^{-1}$. The individual factor involving k_o represents the transverse Alfven wave, which is guided along the magnetic field. We may anticipate that this term will yield a contribution to B_ν that does not decrease rapidly with distance, at least parallel to $\underset{\sim}{B}_o$. We shall compute the contribution of this term, which we denote as B^1_ν, in the next section.

5.1 CALCULATION OF B^1_ν

If one takes the inverse Fourier-Laplace transform of B^1_ν, equation (18), using also the expressions for beam current of section 3.0 above, one obtains:

$$\hat{B}^1_\nu,\ (\underset{\sim}{r},t) = \frac{4\pi\nu n_B eV_\perp}{c} \int \frac{dp}{2\pi i}\, e^{pt} \int \frac{d^3k}{(2\pi)^3}\ \frac{k_o k_\nu e^{i\underset{\sim}{k}\cdot\underset{\sim}{r}}}{(k^2+p^2/c_A^2)\,(k_o^2+p^2/c_A^2)}\ \cdot$$

$$\int_0^\infty dt' e^{-pt'} \int d^3r' e^{-i\underset{\sim}{k}\cdot\underset{\sim}{r}'}\ [k_x \cos\Lambda z' + k_y \operatorname{Sin}\Lambda z']\ \hat{G}\,(\underset{\sim}{r}',t') \qquad (19)$$

where we have written out the Fourier-Laplace transform of $\underset{\sim}{k}\cdot\hat{\underset{\sim}{j}}^B$ explicitly. The Laplace inversion integral is taken along the standard Bromwich contour, i.e. to the right of all singularities in the complex p-plane.

To facilitate the integrations indicated in (19) we introduce the cylindrical-coordinate representations (with $\underset{\sim}{B}_o$ defining the z-direction)

$$\left.\begin{aligned} k_x &= k_\perp \cos\phi\ , \quad k_y = k_\perp \sin\phi \\ x &= r\cos\Theta\ , \quad y = r\sin\Theta \end{aligned}\right\}$$

$$\underset{\sim}{k}\cdot\underset{\sim}{r} = k_o z + k_\perp r \cos(\Theta-\phi) \qquad (20)$$

Then

$$k_\nu\,[k_x\cos\Lambda z' + k_y\sin\Lambda z'] = \frac{k_\perp^2}{\sqrt{2}}\cos(\Lambda z' - \phi)\, e^{-i\nu\phi}$$

The appearance of the Heaviside unit step functions in the expression for $\hat{G}\,(\underset{\sim}{r},t)$ limits the range of the z' integral to $0 \leq z' \leq V_{11}t'$. The double integration $dt'dz'$ can be integrated by parts to yield

$$\int_0^\infty dt'\, e^{-pt'} \int_0^{V_\parallel t'} dz'\, F(z',x',y') = \frac{V_\parallel}{p} \int_0^\infty dt'\, e^{-pt'}\, F(V_\parallel t',x',y') \tag{21}$$

where F denotes the integrand following d^3r' in (19). The integration over $dx'dy'$ can be performed since the Fourier transform of the Gaussian is

$$(\pi\Delta^2)^{-\frac{1}{2}} \int_{-\infty}^{\infty} dx'\, e^{-ik_x x'}\, e^{-\Delta^{-2}(x'-R\cos\Lambda V_\parallel t')^2} = e^{-k_x^2\Delta^2/4}\, e^{-ik_x R\cos\Omega t'} \tag{22}$$

where we used the identity $\Lambda V_\parallel = \Omega$.

$$\hat{B}_\nu^1(\underset{\sim}{r},t) = \frac{4\pi\nu n_B e V_\perp V_\parallel}{\sqrt{2}\; c} \int \frac{dp\, e^{pt}}{2\pi i p} \int_0^\infty dt'\, e^{-pt'} \int_{-\infty}^{\infty} \frac{k_o\, dk_o}{2\pi} \int_0^\infty \frac{k_\perp^3\, dk_\perp}{(2\pi)} \cdot$$

$$\frac{e^{-k_\perp^2\Delta^2/4}\, e^{ik_o(z - V_\parallel t')}}{(k^2 + p^2/c_A^2)(k_o^2 + p^2/c_A^2)} \int_0^{2\pi} \frac{d\phi}{2\pi}\, e^{-i\nu\phi} \cos(\Omega t' - \phi)\, e^{ik_\perp r\cos(\Theta-\phi) - ik_\perp R\cos(\Omega t' - \phi)} \tag{23}$$

where we have assumed interchangeability of the orders of integration over $\underset{\sim}{k}$ and t'.

To perform the ϕ-integration in (23) we note that

$$k_{\perp}\cos(\Omega t' - \phi)e^{-ik_{\perp}R\cos(\Omega t' - \phi)} = i\frac{\partial}{\partial R}e^{-ik_{\perp}R\cos(\Omega t' - \phi)} \tag{24}$$

and use the well know identity

$$e^{\pm ix\cos\theta} = \sum_{n=-\infty}^{\infty} i^{n} J_{n}(\chi)e^{-in\phi} \tag{25}$$

The ϕ-integration in (23) can then be carried out to give the result

$$i^{\nu+1}e^{-i\nu\theta}\frac{\partial}{\partial R}\sum_{n=-\infty}^{\infty} J_{n+\nu}(k_{\perp}r)\, J_{n}(k_{\perp}R)\, e^{in\psi} \tag{26}$$

where $\psi = \Omega t' - \phi$.

The Gegenbauer summation theorem can be applied to (26).

$$\sum_{n=-\infty}^{\infty} J_{\nu+n}(k_\perp r)\, J_n(k_\perp R) e^{in\psi} = \nu J_\nu(k_\perp \rho) \left[\frac{r-Re^{-i\psi}}{r-Re^{i\psi}} \right]^{\nu/2} \quad (27)$$

where

$$\rho^2 = r^2 + R^2 - 2rR\cos\psi \quad (28a)$$

or

$$\rho^2 = (r-Re^{-i\psi})\,(r-Re^{i\psi}) \quad (28b)$$

The expression for $\hat{B}_\nu^1(\underset{\sim}{r},t)$ can be rewritten as

$$\hat{B}_\nu^1(\underset{\sim}{r},t) = \frac{4\pi\nu n_B e V_\perp V_{||}}{\sqrt{2}\, c}\, i^{\nu+1} e^{-i\nu\theta} \int \frac{dp\, e^{pt}}{2\pi i p} \int_0^\infty dt' e^{-pt'} \int_{-\infty}^{\infty} \frac{dk_o k_o}{2\pi} e^{-ik_o\xi}.$$

$$\frac{\partial}{\partial R} \int_0^\infty \frac{dk_\perp k_\perp^2\, e^{-k_\perp^2 \Delta^2/4}}{2\pi\,(k_o^2+p^2/c_A^2)\,(k^2+p^2 k_A^2)}\, \nu J_\nu(k_\perp \rho) \left[\frac{r-Re^{-i\psi}}{r-Re^{i\psi}} \right]^{\nu/2} \quad (29)$$

where

$$\xi = V_{||} t' - z \quad (30)$$

The quantity ξ defined in (30) will be restricted later to positive values. We note that

$$Q = \frac{\partial}{\partial R} \left[\nu J_\nu(k_\perp \rho) \left(\frac{r-Re^{-i\psi}}{r-Re^{i\psi}} \right)^{\nu/2} \right] = \frac{\partial}{\partial R} \left[\frac{\rho J_\nu(k_\perp \rho)}{r-Re^{i\nu\psi}} \right] \quad (31)$$

5.2 Backward Wave Radiation

A particle beam preferentially radiates hydromagnetic waves propagating in the direction opposite to that of the beam. Therefore, we confine the subsequent analysis to the backward direction, so that in (30) $z = -|z|$ and $\xi = V_{||}t' + |z| \geq 0$. We next interchange the k_o- and the $k_\perp$- integrations in (29), and write the integral over k_o as

$$\int_{-\infty}^{\infty} \frac{dk_o}{2\pi} \frac{k_o e^{-ik_o\xi}}{(k_o^2 + p^2/C_A^2)(k^2+p^2/C_A^2)} = \int_{-\infty}^{\infty} \frac{dk_o}{2\pi} \frac{k_o e^{-ik_o\xi}}{k_\perp^2} \left[\frac{1}{k_o^2+p^2/C_A^2} - \frac{1}{k_o^2+k_\perp^2+p^2/C_A^2} \right] \quad (32)$$

The form of (32) clearly separates the Alfven and magnetosonic contributions. We can evaluate the integral by Cauchy's Theorem, with the closure on an infinite semicircle in the lower half of the complex k_o- plane. This picks up residues at the simple poles $k_o=-ip/C_A$ and $k_o=-i(k_\perp^2+p^2/C_A^2)^{1/2}$, thus producing

$$\int_{-\infty}^{\infty} \frac{dk_o}{2\pi} \left[\cdots\cdots \right] = - \frac{i}{2k_\perp^2} \left[e^{-p\xi/C_A} - e^{-\xi(k_\perp^2 + p^2/C_A^2)^{1/2}} \right] \quad (33)$$

This then leads to the following expression for $\hat{B}_\nu^1(\underset{\sim}{r}, t)$

$$\hat{B}_\nu^1(\underset{\sim}{r},t) = \frac{\nu n_B e V_\perp V_{||}}{\sqrt{2}\, C} \, i^{\nu+1} e^{-i\nu\theta} \int \frac{dp}{2\pi i p} e^{pt} \int_0^\infty dt' e^{-pt'} \int_0^\infty dk_\perp e^{-k_\perp^2\Delta/4} \bullet$$

$$\frac{\partial}{\partial R} \left[\frac{\rho J_\nu(k_\perp \rho)}{r - Re^{i\nu\psi}} \right] \left[e^{-p\xi/C_A} - e^{-\xi(k_\perp^2+p^2/C_A^2)^{1/2}} \right] \quad (34)$$

If the spread in guiding centers $\Delta \to 0$, that is for the case of an infinitely small beam cross-section, the Alfven contribution involving the term $\exp(-p\xi/C_A)$ in (34) would lead to a divergent integral over $k_\perp$. The magnetosonic term involving $\exp\left\{-\xi(k_\perp^2+p^2/C_A^2)^{1/2}\right\}$ leads to a convergent $k_\perp$-integration even in the limit $\Delta \to 0$.

5.2.1 Alfven Contribution

Let us denote by $\hat{B}^{1A}_{\nu}$ the Alfven contribution to (34). The integration over $k_{\perp}$ can be carried out to yield

$$\hat{B}^{1A}_{\nu}(\underset{\sim}{r},t) = \frac{\nu n_B e V_{\perp} V_{||}}{\sqrt{2}\, C}\, i^{\nu+1} e^{-i\nu\theta} \frac{\partial}{\partial R} \int_0^{\infty} dt' \int \frac{dp}{2\pi i}\, e^{p(t-t')-p\xi/C_A} \bullet \left[\frac{1-e^{-p^2/\Delta^2}}{r-Re^{i\nu\psi}}\right] \tag{35}$$

where we have interchanged the order of p- and t'- integrations. By closing the contour of the Laplace inversion integral by an infinite arc in the left-hand p-plane, Cauchy's Theorem yields the expression

$$\hat{B}^{1A}_{\nu}(\underset{\sim}{r},t) = A\frac{\partial}{\partial R} \int_0^{t_o} dt' \frac{1-e^{-\rho^2/\Delta^2}}{r-Re^{i\nu\psi}} H(t-|z|/C_A) \tag{36}$$

where

$$\left.\begin{aligned} A &= \frac{\nu n_B e V_{\perp} V_{||}}{\sqrt{2}\, C}\, i^{\nu} \\ t_o &= \frac{t-|z|/C_A}{1+V_{||}/C_A} \end{aligned}\right\} \tag{37}$$

and we have set the (arbitrary) phase angle $\theta=0$. The unit step function in (36) assures that no wave signal arrives at the point $|z|$ prior to a time $t = |z|/C_A$. With the aid of the identity

$$\frac{\partial}{\partial R}\left[\frac{1-e^{-\rho^2/\Delta^2}}{r-Re^{i\nu\psi}}\right] = \frac{1}{i\nu R}\frac{\partial}{\partial\psi}\left[\frac{1-e^{-\rho^2/\Delta^2}}{r-Re^{i\nu\psi}}\right] - \frac{2e^{-i\nu\psi}e^{-\rho^2/\Delta^2}}{\Delta^2} \tag{38}$$

then (36) may be written

$$\hat{B}^{1A}_{\nu}(\underset{\sim}{r}, t) = \frac{A}{i\nu\Omega R}\left[\frac{1-e^{-\rho^2/\Delta^2}}{r-Re^{i\nu\psi_o}} - \frac{1-e^{-(r-R)^2/\Delta^2}}{r-R}\right] - \frac{2A}{\Omega\Delta^2}\int_0^{\Omega t_o} d\psi e^{-i\nu\psi-\rho^2/\Delta^2} \tag{39}$$

where $\psi_o = \Omega t_o$ and the step function $H(t-|z|/C_A)$ is understood. On the axis of symmetry of the beam, one has $r = 0$, $\rho = R$, so that the Alfven wave magnetic field can be written:

$$\hat{B}_\nu^{1A}(\underset{\sim}{r}, t)\Big|_{r=o} = \frac{A}{i\nu\Omega R^2}\left[\left(e^{-i\nu\Psi_o}-1\right)\left(1-e^{-R^2/\Delta^2}\right) - \frac{2R^2}{\Delta^2} e^{-R^2/\Delta^2}\left(e^{-i\nu\psi_o}-1\right)\right] \quad (40)$$

If $R >> \Delta$, i.e. the mean gyroradius of beam ions is much larger than the spread in guiding centers caused by electrostatic expansive effects on the originally narrow beam, then one has the dominant term

$$\hat{B}_\nu^{1A}(\underset{\sim}{r}, t)\Big|_{r=o} \simeq \frac{A}{i\nu\Omega R^2}\left(e^{-i\nu\psi_o}-1\right) \quad (41)$$

In this limit, the on-axis disturbance consists of a wave contribution oscillating with frequency $\omega_o = \Omega/(1 + V_{\parallel}/C_A)$, and a DC contribution.

At a distance off the axis equal to the mean beam gyroradius, i.e. $r = R$, one has $\rho^2 = 2R^2(1-\cos\psi)$. Then

$$\hat{B}_\nu^{1A}(\underset{\sim}{r}, t)\Big|_{r=R} = \frac{A}{i\nu\Omega R^2}\,\frac{e^{i\nu\psi_o}}{1-e^{i\nu\psi_o}}\left[1-e^{-\frac{2R^2}{\Delta^2}(1-\cos\psi_o)}\right] - \frac{A}{\Omega\Delta^2} e^{-\eta}\frac{\partial}{\partial\eta}\int_0^{\Omega t_o} d\psi e^{\eta\cos\psi} \quad (42)$$

where $\eta=2R^2/\Delta^2$. For $R>>\Delta$, the last term in (42) will dominate. The integrand in this term is periodic, such that the value of the integral accumulates as Ωt_o advances through successive integral multiples of 2π. If we keep only the last term in (42)

$$\hat{B}_\nu^{1A}(\underset{\sim}{r}, t)\Big|_{r=R} \simeq -\frac{2A}{\Omega\Delta^2} e^{-\eta}\frac{\partial}{\partial\eta}\int_0^{\Omega t_o} d\psi e^{\eta\cos\psi} \quad (43)$$

Since we are obviously interested in beams with at least several helical turns, one can tentatively evaluate (43) for $\Omega t_o = 2\pi(N+\epsilon)$, where $N=1,2,3,\ldots$ integer, and $0 < \epsilon < 1$. Then

$$\int_0^{2\pi N+2\pi\epsilon} d\psi e^{\eta\cos\psi} = \sum_{m=0}^{N}\int_0^{2\pi m} d\psi e^{\eta\cos\psi} + \int_{2\pi m}^{2\pi m+2\pi\epsilon} d\psi e^{\eta\cos\psi}$$

$$= N\int_0^{2\pi} d\psi e^{\eta\cos\psi} + \int_0^{2\pi\epsilon} d\psi e^{\eta\cos\psi}$$

$$= 2\pi N I_o(\eta) + \int_0^{2\pi\epsilon} d\psi e^{\eta\cos\psi}$$

where we have used the integral representation of the modified Bessel function $I_0(\eta)$. Thus

$$\hat{B}_{\nu}^{1A}(\underset{\sim}{r},t)\Big|_{r=R} \simeq -\frac{4\pi NA}{\Omega\Delta^2} e^{-\eta}\left[I_1(\eta)+\frac{i}{2\pi N}\frac{\partial}{\partial\eta}\int_0^{2\pi\epsilon} d\psi e^{\eta\cos\psi}\right]$$

For $\eta = R^2/\Delta^2 >> 1$, one has approximately

$$\hat{B}_{\nu}^{1A}(\underset{\sim}{r},t)\Big|_{r=R} \simeq -\frac{A\Omega t_o}{\sqrt{\pi}\Omega\Delta R} \tag{44}$$

At the fixed position $|z|$, for $t > |z|/c_A$, this result (44) represents a pulse whose amplitude grows linearly in time as the number of turns $N = \Omega t/2\pi$ of the beam increases. Also, one notes that the amplitude is larger than that on-axis (r=0) by the factor $R/\Delta >> 1$.

For $r >> R$, $\rho^2/\Delta^2 >> 1$, so that

$$\hat{B}_{\nu}^{1A}(\underset{\sim}{r},t)\Big|_{r>>R} \simeq \frac{A}{i\nu\Omega R^2}\left(e^{i\nu\psi_o}-1\right) \tag{45}$$

Thus, the amplitude of the Alfven pulse outside the cy linder defined by $r = R$ in the backward direction decays as $1/r^2$. We conclude that the largest Alfven response occurs at $r = R$, and is confined to a spatial width about $r = R$ of order Δ.

5.2.2 Magnetosonic Contribution

To estimate the magnetosonic contribution to $\hat{B}_\nu^1(\underset{\sim}{r},t)$, we let $\Delta \to 0$ in equation (34) and carry out the $k_\perp$-integration to obtain

$$\hat{B}_\nu^{1M}(\underset{\sim}{r},t) = -A\frac{\partial}{\partial R}\int \frac{dpe^{pt}}{2\pi ip}\int_0^\infty dt' \frac{e^{-pt'}}{r-Re^{i\nu\psi}}\left[e^{-p\xi/c_A} - \frac{\xi}{(\xi^2+\rho^2)^{\frac{1}{2}}} e^{-\frac{p}{c_A}(\xi^2+\rho^2)^{\frac{1}{2}}}\right] \quad (46)$$

If we confine our attention to the radiation zone $|z| \gg r$, $\xi \gg \rho$, then to the lowest order in ρ/ξ we obtain

$$\hat{B}_\nu^{1M}(\underset{\sim}{r},t) = -\frac{A}{2c_A}\frac{\partial}{\partial R}\int \frac{dpe^{pt}}{2\pi i}\int_0^\infty \frac{dt'}{\xi} e^{-p(t'+\xi/c_A)}(r - Re^{-i\nu\psi}) \quad (47)$$

The derivative with respect to R and the Laplace inversion can be done immediately to obtain

$$\hat{B}_\nu^{1M}(\underset{\sim}{r},t) = \frac{A}{2c_A}\int_0^\infty \frac{dt'}{\xi} e^{-i\nu\psi}\,\delta(t-t'-\xi/C_A)$$

or

$$\hat{B}_\nu^{1M}(\underset{\sim}{r},t) = \frac{A}{2c_A}\frac{e^{-i\nu\psi_o}}{\xi(t_o)} H(t-|z|/C_A) \quad (48)$$

where

$$\xi(t_o) = \frac{V_{||}t + |z|}{1 + V_{||}/c_A}$$

Thus we see that the magnetosonic contribution decays as $1/|z|$ along the axis and is small compared to the Alfven term for $|z| \gg R$.

5.3 BARE CURRENT CONTRIBUTION

We may use the same techniques to evaluate the "bare current" contribution, $\hat{B}_\nu^{(2)}(\underset{\sim}{r}, t)$, represented by the final term in equation (18). This can be reduced to the expression

$$\hat{B}_\nu^{(2)}(\underset{\sim}{r},t) = A\int \frac{dpe^{pt}}{2\pi ip}\int_0^\infty dt' e^{-pt'}\, e^{-i\nu\psi}\, \frac{\xi\left[1+\frac{P}{C_A}(\xi^2+\rho^2)^{1/2}\right]}{(\xi^2+\rho^2)^{3/2}}\, e^{-\frac{P}{C_A}(\xi^2+\rho^2)^{1/2}} \qquad (49)$$

where we assumed $\Delta \to 0$ since no divergence problem existed with the $k_\perp$-integration. For $\xi >> \rho$, the lowest order contribution to (49) is given by

$$\hat{B}_\nu^{(2)}(\underset{\sim}{r},t) = \frac{A}{C_A}\,\frac{e^{-i\nu\psi_o}}{\xi(t_o)}\, H(t - |z|/C_A) \qquad (50)$$

which is simply twice the contribution from $\hat{B}_\nu^{1M}(\underset{\sim}{r},t)$ given by equation (48) above.

5.4 SUMMARY OF RESULTS

The ion beam radiates two types of hydromagnetic signals. The magnetosonic signal spreads out, probably almost isotropically, so that its amplitude decays roughly as the inverse distance away from the beam source.

The Alfven signal is confined approximately to the cylindrical volume $r \leq R$ projected by the beam in $z > o$ into the backward direction $z < o$. This signal contains both left and right hand polarizations. For $|r-R|>\Delta$ and $r < R$, the amplitude of the Alfven contribution is given approximately by equation (41). It is roughly constant across this projected beam cross-section, and it does not decay with axial distance $|z|$ away from the beam source.

For $|r-R|<\Delta$, the amplitude of the Alfven disturbance is given by equation (44), and is an increasing function of the number of turns made by the gyrating ion beam. The total Alfven signal has a sharp peak at $r=R$. This peak results from the anisotropic dielectric properties of the background plasma. Magnetic field lines generated by the gyrating beam current cannot spread out isotropically into the surrounding plasma medium as they would into a surrounding vacuum. The Alfven signal does not transport such field lines in a direction perpendicular to the background DC magnetic field $\underset{\sim}{B}_o$, at least in the approximation of cold, linearized plasma used here; to the contrary, within our approximations, the Alfven signal only propagates information parallel to $\underset{\sim}{B}_o$. This is the physical reason that the Alfven contribution to the beam-generated magnetic flux in the backward direction is confined to a volume $|z|>o$, $r\leq R$ defined by the dimension of the beam ion gyroradius and spread, Δ.

6.0 POWER ESTIMATES

We now may form some rough estimates of the total power radiated by an ion beam. We shall express the results in terms of the total parallel beam current in statamperes (1 amp = 3×10^9 statamp)

$$I_{11} = n_B e V_{11}$$

The Poynting flux is approximately

$$S \simeq c_A \underset{\sim}{B} \cdot \underset{\sim}{B}/4\pi \tag{51}$$

since $|\underset{\sim}{E}| \sim c_A |\underset{\sim}{B}|/c$.

6.1 Magnetosonic Power

For simplicity we shall assume that the magnetosonic contribution spreads out isotropically in the backward hemisphere whose area we approximate as $\sim 2\pi\xi^2(t_o)$. Then we may write the amplitude factor A in the form

$$|A| = I_{11} V_\perp / \sqrt{2} c \tag{52}$$

The time-averaged power follows as

$$P_M \simeq \frac{1}{2c_A} I_{11}^2 V_\perp^2 / c^2 \tag{53}$$

6.2 Alfven Power

We can estimate the powers in the Alfven contribution for $|r - R| > \Delta$ and $|r - R| < \Delta$ separately. For r>R the power is negligible. For $|r - R| > \Delta$, we estimate the effective area as $\sim \pi R^2$, and use (41) to find

$$P_A(|r-R| > \Delta) \simeq \frac{c_A I_{11}^2 V_\perp^2}{2R^2 \Omega^2 c^2} = \frac{c_A I_{11}^2}{2c^2} \tag{54}$$

since $R\Omega = V_\perp$.

For $|r - R| < \Delta$, we assume the effective area $\sim 2\pi R\Delta$, and use (44) to find

$$P_A(|r-R| < \Delta) \simeq \frac{c_A I_{11}^2}{c^2} \frac{(\Omega t_o)^2}{2\pi} \frac{R}{\Delta} \tag{55}$$

Thus, one finds the power in the Alfven signal given by (55) exceeds the total magnetosonic power by a factor $\sim 2\pi(\Omega t_o)^2 R/\Delta$ which can be as large as 1,000. However, (55) is nonoscillatory.

6.3 Power Levels

At ionospheric heights, and at the magnetic equator inside the plasmapause near 3 R_E, the Alfven speed can be about 3×10^7 cm/sec. If we express the current in (MKS) amperes ($I_{11} = 3 \times 10^9 I_A$ where I_A is amperes), the power in both magnetosonic and propagating Alfven waves (for $|r - R| > \Delta$) is

$$P \simeq c_A I_{11}^2/c^2 = 0.03\ I_A^2 \text{ watts} \tag{56}$$

If the ion beam is launched near the geomagnetic equator, then the Larmor radius of a proton beam will be approximately $R \sim c_A/\Omega = 3 \times 10^5$ cm, using $B_o \sim 10^{-2}$ Gauss. The amplitude of the Alfven contribution to the disturbance field would be $B_\nu \sim 0.03\ I_A$ in gammas ($1\gamma = 10^{-5}$ G). The magnetosonic wave would spread out and have a negligible amplitude by the time it reaches the top of the ionosphere. The propagating Alfven wave (not the pulse) should be collimated partially by the magnetic field $\underset{\sim}{B}_o$, and should reach the ionosphere with most of its power. As the beam moves down the magnetic field lines from the equatorial region, the local ion gyrofrequency changes due to increasing geomagnetic field intensity. If a narrow frequency bandwidth is desired, only the spatial region of about $\sim 1R_E$ will be an effective radiation length. Thus, the beam should be turned on for a time $t \lesssim R_E/c_A \sim 20$ sec. The total energy in the Alfven contribution under this condition will be

$$W = P_A t \sim 0.3\ I_A^2 \text{ joules}$$

If the beam is launched in the ionosphere, the magnetosonic wave should couple most effectively into the ionospheric Alfven wave guide. The Alfven wave simply will propagate along the magnetic field and into the E-region. Since the beam will cross the wave guide region in about one second, the total energy would be $\sim 0.02\ I_A^2$ (joules).

Let us suppose that we wish to excite the ionospheric waveguide in such a way that a signal strength or disturbance field $\delta B \simeq 10\ m\gamma = 10^{-7}$ gauss is present in that volume. The total r.m.s. energy present within the guide would be

$$W = \frac{(\delta B)^2}{16\pi} \cdot 4\pi R_E^2 \Delta R \sim 2{,}000 \text{ joules}$$

for $\Delta R \sim 200$ km. In order for the Alfven wave to deliver this energy, the beam current would have to be $I_A \sim 8$ amperes, and this delivered to a potential of perhaps 10 kV. Thus the beam energy would be $\sim$.0 8 to several megajoules, depending upon exact conditions. On the other hand, direct excitation by the magnetosonic wave of a similar level of disturbance would require a beam current $I_A \sim 240$ amperes and a beam energy on the order of 3 or more megajoules.

7.0 BEAM SPREADING

If a well-collimated ion beam at any of the current strengths we have calculated were launched from a spacecraft, either at higher altitudes at the geomagnetic equator, or within the ionspheric Alfven waveguide, the initial beam number density would surely exceed the local plasma electron density by some orders of magnitude.

Two possible mechanisms can be envisaged that would cause a subsequent spatial spreading of such an initially over-dense ion beam.

The first of these is space-charge. If the ion beam is not neutralized immediately by either background plasma electrons, or artificially produced electrons emitted from the spacecraft, space-charge fields of the ions in the beam will cause the beam to spread on a time scale of a few ion plasma periods. Since the ion plasma frequency greatly exceeds the ion gyrofrequency either in the upper ionosphere or in the equatorial magnetosphere at distances 3-4 R_E, then the time scale of the ion plasma period is much shorter than that of the Larmor gyration for beam ions. This implies that the spatial spreading of such beams caused by space-charge forces will occur early in the first helical turn of the gyrating beam.

Even were the beam to be charge-neutralized, it still would constitute an ion current in the background plasma. Such an ion current would certainly excite a variety of beam-driven plasma instabilities. Simple calculations show that the most likely instabilities would be electrostatic modes with frequencies comparable to the ion plasma frequency and wave vectors nearly perpendicular to the local magnetic field $\underset{\sim}{B}_o$ and lead to an effective spread in guiding centers as we assumed in the beam model of equation (12) in section 3.

Such beam spreading probably would continue until the local density of the spread beam decreases to a value roughly comparable to the density of background plasma. A trivial calculation of the spread Δ used in equation (12) et seq. can be made on the basis of such a model. For spread Δ, the beam density (cm^{-3}) is

$$N_B = \frac{n_B}{\pi\Delta^2} = \frac{I_{11}}{eV_{11}\pi\Delta^2} \simeq N_o \quad .$$

If we are interested in geomagnetic equatorial launching of the beam with $I_A \sim 80$ amperes, then $I_{||} \sim 2.4 \times 10^{11}$ e.s.u. If we have $V_{||} \sim 2C_A \sim 10^8$ cm/sec and $N_o \sim 10^2$ cm^{-3} then one calculates

$$\Delta \sim 5 \times 10^4 \text{ cm}$$

$$R \sim 10^6 \text{ cm}$$

$$\Delta/R \sim 0.05 = 1/20$$

Since this value of $\Delta/R << 1$, beam spreading appears unlikely to greatly distort the spatial structure of the beam or disrupt significantly the coherent gyro-motion of the beam. That is, it will not distort the coherence of Lamor phases of beam ions necessary to produce a strong $\underset{\sim}{J}_{\perp}^{B}$, which is the component required to radiate the waves.

8.0 BEAM CHARGE AND CURRENT NEUTRALIZATION

8.1 Charge Neutralization

It is clear a prior that for beam currents required in the previous sections, that is currents of ions on the order of tens of amperes, there will be severe charge-neutralization problems encountered by a spacecraft platform used to launch such beams.

The beam voltages required to produce beam speeds $V_{||}$ of several times the local Alfven speed c_A are tens of kilovolts. Unless negative charge equal to that of the ion beam, i.e.

$$Q = -\int_{o}^{t} I_{||} \, dt$$

is emitted, a spacecraft potential of many kilovolts would ensue.

The local plasma at the geomagnetic equator near $R \sim 3$ to $4 R_E$ cannot supply sufficient ion return current to neutralize the spacecraft charge. For this reason, one would require electron emitters on the spacecraft to supply a sufficient negative current to equalize the negative charge caused by positive ions leaving the spacecraft.

This magnitude of electron current ($I \sim 80$ amps) can be supplied at relatively low voltages from large cathodes. For example large Barium-Strontium-Oxide coated cathodes of dimensions $\sim 3 \times 10^3$ cm^2 have been constructed at TRW for use in laboratory plasma chambers. These cathodes are easily capable of producing electron emission currents of hundreds of amperes at 50-100 volts on the extractor grid.

Thus, charge-neutralization of the spacecraft appears to be within state-of-the-art technology, but it significantly adds to the experimental energy budget. 80 amperes of electron current at $\sim$ 100 volts consumes, even with one-hundred percent efficiency, about 8 kW. This is comparable to the power in the ion beam itself. In addition, one would have to include the heater power for this electron gun, which will be several kW.

8.2 Current Neutralization

It was suggested in section 3.0 above that beam current neutrality ($J_{||}^B = 0$) can be achieved using the same source of electrons as that used to charge-neutralize the spacecraft itself.

The picture painted there is one in which the ion beam is injected at, say, an initial pitch angle of about 45 degrees to $\underset{\sim}{B}_o$, and thus describes a helical current element of mean radius $R \sim 10^5 - 10^6$ cm. The electron emitter furnishing the neutralizing current is then assumed to inject its electrons along magnetic field lines within this cylindrical shell.

However, exact spatial current neutralization will require not only that ion and electron beam currents be equal, but also that beam velocities parallel to $\underset{\sim}{B}_o$ be equal. For 10 keV proton beams, one then would require $\sim$ 5 eV electron beams to match velocity. With reasonably sized cathodes, drawing 80 amperes of electrons at only 5 eV is limited by beam perveance unless large beam cross-sections are employed. Such technological problems are not considered to be beyond the realm of solutions.

Insofar as neutralization by ambient plasma reaction currents is concerned, our formulation of the original problem via equations (1) et seq. contains the quantity $\underset{\sim}{J}^P$, the plasma reaction current density. This self-consistent quantity describes the plasma reaction through the plasma dielectric function, and gives the linearized shielding of the beam charge and current due to the ambient plasma.

8.3 Summary

Thus, the primary problems of neutralization are encountered on the spacecraft itself. These problems must be solved by placing an electron emitter on the craft to ensure zero net charge build-up.

9.0 BEAM MODULATION

Energy requirements for the ion-beam ULF generator operated from a spacecraft located around the geomagnetic equator at some 3-4 R_E are estimated at several megajoules, for operation of neutralized beams at some 80 amperes and 10 kilovolts.

If the beam is launched in a region where $B_o \sim 200\gamma$, then $\Omega/2\pi \sim 3$ Hz and one turn of the beam takes ~ 0.3 sec. For ten turns, one requires 3-second operation, so that beam energy would be ~ 2.4 megajoules.

Modulation of such a beam would not lead to any advantage because of its necessarily short duration.

10.0 HEAVIER ION BEAMS

Again, because the beam energy required is fixed by the Alfven speed in the ambient (assumed proton) plasma, one cannot justify using heavier ions to form the beam. We have found that $V_{||}/c_A \gtrsim 2$ is required, and thus to produce this condition using beam ions of atomic number A simply mandates an increase in beam energy A-times greater than that for a proton beam.

For this reason, use of heavier ion beams is considered to be a distinct disadvantage.

One might argue, on the other hand, that the use of a heavier ion such as Cesium, with a much lower ionization potential, could lead to an offsetting gain in energy saved in producing the ions. However, given even equal ionization efficiencies, the lightest ion of low ionization potential is Lithium at $\Phi_I \sim 5.4$ eV, compared with Hydrogen at 13.6 eV. This ratio is 13.6/5.4 = 2.5 while A=7 for Li. As A increases the disparity between the ratio of ionization potentials to the ratio of required beam energies to produce the specific velocity $V_{||} \gtrsim 2\ c_A$ increases.

Energetics alone militate against the use of heavier ion beams.

11.0 CONCLUSIONS

The following general conclusions are based on the results of this investigation of the efficacy of generating ULF radiation from ion beams emitted from satellites, and based on those conclusions, the recommendations concerning further pursuit of this concept are enumerated in Section 12.

1. The calculation of hydromagnetic radiation from a proton beam injected into a dilute, magnetized background plasma has been carried out in linearized analysis.

2. For such beams emitted near the geomagnetic equator, the spacecraft must reside within the plasmasphere so that sufficient ambient plasma is present to allow coupling of beam-generated ULF into the backward plasma wave at frequencies $\omega < \Omega$ = beam gyro-frequency.

3. Backward-propagating disturbances in the Alfven mode have been found. These have the property of being largely confined to a cylindrical volume $r \leq R$ defined by the mean gyroradius of the ion beam (emitted into $z < 0$).

4. Almost isotropically propagating magnetosonic wave disturbances are also found.

5. For a given beam current (or power) the power in the guided backward Alfven disturbance can exceed the power in the isotropic magnetosonic disturbance by a factor on the order of 1,000.

6. Given a unity efficiency of guidance of the beam-generated Alfven disturbance along geomagnetic field lines down to the ionosphere, we calculate that to produce a ULF disturbance δB of 10 mγ throughout the earth-ionosphere waveguide requires an r.m.s. energy of 2 kilo-joules in the guide, and this translates into a beam energy of the order of using an efficiency factor 2×10^{-3}.

7. Typical operating parameters for such beams are $I_B \sim 20$ amperes, $V_B \sim 10$ kV, $P_B \sim 0.2$ MW, $W_B \sim 2$ MJ.

8. A neutralizing electron emitter is required to prevent the spacecraft from attaining large negative potential. If a low-voltage, high-current electron emitter were used, say $I_e \sim 20$ amperes at $V_e = 5V$, only 100 watts of electron beam power would be expended. Such emitters appear to be within the state-of-the-art. Heater power consumption would have to be added to the power budget, at an expense of about 10 eV per electron.

9. Electrostatic effects can be expected to cause an initially narrow, intense collimated ion beam of small cross-section to expand radially about its mean radius of gyration, $R = V/\Omega$. The spread Δ so produced is estimated on the assumption that the beam will expand until its number density is roughly equal to the background plasma density, and that for the beams considered here, $\Delta/R \sim 0.05$, where $R \sim 3\text{-}10 \times 10^5$ cm. This amount of expansion would have negligible effects on the radiation due to the perpendicular component of beam current density, $\underset{\sim}{J}_{\perp}^{B}$.

10. Plasma reaction currents are included in our calculation. They contribute the plasma dielectric loading that controls the hydromagnetic radiative characteristics of the beam.

11. Because of the energy storage requirements calculated, it is infeasible to consider phase, velocity, or density modulation of such an ion beam.

12. Also because of energy considerations, it appears to be distinctly disadvantageous to use heavier ion beams. The beam velocity is fixed by the magnitude of the Alfven speed in the ambient plasma. Thus, to obtain the necessary beam velocity, the lowest energy requirement is for a proton beam.

13. In any practical consideration of total energy storage requirements for such a spacecraft-borne ion beam, one must include the ionization energy, dissipation in the power supply used to apply accelerating voltage, and energy to produce the neutralizing electron current. The ion beam energy alone is estimated to be $\sim$ 2 MJ. Thus, one could anticipate a 3 MJ storage requirement. At even 0.5 lb per kilojoule, capacitive storage would be on the order of 1,500 lb. Furthermore, even at a recharging rate of 10kW, which represents a solar panel of prohibitive size, the recharging time would be on the order of 5 minutes.

14. The final conclusion, based on this information, is that the production of ULF signals in the frequency range below a few Hertz by means of an ion beam emitted by a spacecraft near the geomagnetic equator at altitudes $\sim$ 3 to 4 R_E has very severe requirements, and at best would supply some sort of "bell-ringer" signal. However, the signature of the signals we have calculated would be difficult to control under varying magnetospheric and ionospheric conditions.

12.0 RECOMMENDATIONS

Based on the foregoing calculations and conclusions which we have drawn, the following recommendations can be made:

1. The concept originally envisaged of an ion beam emitted from a satellite near the geomagnetic equator at distances of some 3 to 4 earth-radii should probably not be pursued further. The results, especially the energy storage requirements, appear to argue against any possible large-scale implementation of such a system. To provide coverage would require several spaced satellites.

2. Even under ideal quiescent magnetospheric or ionospheric conditions, the signals produced would be marginally useful. Given a dynamic magnetosphere-ionosphere system, this concept appears most difficult to utilize in a controlled fashion.

3. Some follow-on studies could be defined to examine related concepts, and to investigate certain features of ion beam dynamics that cannot be calculated with any degree of confidence. Among these are:

    - A study of possible location of the ion beam within the ionosphere waveguide. Since much of the power produced by such a beam appears in the magnetosonic mode directly, and since this is the mode that one assumes produces the leakage field into the earth-ionosphere cavity that appears at the surface, then a calculation placing the beam source in the waveguide is perhaps indicated. This could best be done using numerical analysis and a model of the waveguide.

- A laboratory experiment can be performed in a large plasma facility at TRW to study some features of ion beam dynamics. However, one cannot scale the ULF radiation because the hydromagnetic wavelengths exceed the dimensions of laboratory chambers. Hence, small scale beam dynamics, such as electrostatic stability, expansion, etc., would be the objectives of such a laboratory investigation.

- Numerical modeling of the beam-plasma system, using codes written for that purpose, would be an expensive but perhaps more illuminating way to further the analysis.

- Creation of hydromagnetic disturbances by MPD arcs should also be examined. However, one can size this or any similar concept by some energy considerations. One must always come out with megajoule power supply requirements in all such systems.