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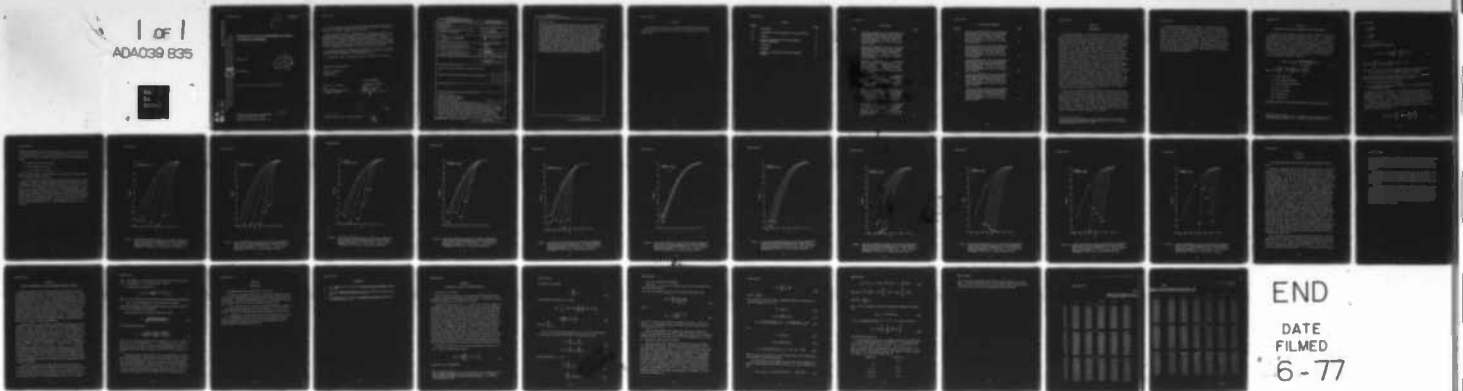
AIR FORCE WEAPONS LAB KIRTLAND AFB N MEX  
TRUNCATION OF THE DISTRIBUTION OF PARTS FAILURES BY SCREENING.(U)  
MAR 77 R D CHRONISTER, R J MAIER  
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# TRUNCATION OF THE DISTRIBUTION OF PARTS FAILURES BY SCREENING

March 1977

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Final Report



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AIR FORCE WEAPONS LABORATORY  
Air Force Systems Command  
Kirtland Air Force Base, NM 87117

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER 14 AFWL-TR-76-247	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) 6 TRUNCATION OF THE DISTRIBUTION OF PARTS FAILURES BY SCREENING	5. TYPE OF REPORT & PERIOD COVERED 9 Final Report	
7. AUTHOR(s) 10 R. D. Chronister, Capt, USAF R. J. Maier	8. CONTRACT OR GRANT NUMBER(s) 16 2097	
9. PERFORMING ORGANIZATION NAME AND ADDRESS Air Force Weapons Laboratory (ELP) Kirtland Air Force Base, NM 87117	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 62601F 88091126	
11. CONTROLLING OFFICE NAME AND ADDRESS Air Force Weapons Laboratory (ELP) Kirtland Air Force Base, NM 87117	12. REPORT DATE 11 Mar 1977	13. NUMBER OF PAGES 36 12 38p
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS. (of this report) UNCLASSIFIED	
15a. DECLASSIFICATION/DOWNGRADING SCHEDULE		
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Hardness assurance Hardness assurance screens Bivariate normal distribution Statistical correlation 013 150 83		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) The value of acceptance test screening as a quality assurance tool is explored from the viewpoint of radiation hardness assurance. Since parts failures that occur due to the application of extreme environments such as radiation can most frequently be described using the normal distribution, the bivariate normal distribution is assumed. Numerical and graphical tabulations of the distribution failures are presented for various screening levels below which all parts are rejected. (over)		

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ABSTRACT (cont'd)

Before applying these tabulations, one must be assured that part failures are normally distributed for the specific application of interest. This must be determined by experiment and analysis. One method of analysis is to show that the screening parameters and failure levels are themselves the result of sums of other variates. Then, using the central limit theorem, the error associated with assuming a normal distribution can be estimated. Experimentally, a population sample is stressed to failure and the distribution of failure levels compared to the normal distribution to estimate the error introduced by assuming normality.

→ In this report the calculations required to evaluate acceptance test screening as a radiation hardness assurance tool are explained and the methods of estimating the error resulting from assuming normal distribution detailed. Examples are given to show that poorly correlated screens are nevertheless useful for radiation hardness assurance when extreme radiation levels are not anticipated and that hardening by screening is unproductive when high radiation levels are anticipated.

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PREFACE

The authors wish to thank Lt. Eric Brown for performing the calculations presented in section II on the H.P. 9830 and plotting the figures also using the H.P. 9830.

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## SECTION I

## INTRODUCTION

The radiation hardness of a given electronics piecepart is a quality which is not exhibited until the part is required to survive in a nuclear environment. Normally, quality assurance preserves the desired electrical and reliability characteristics of the piecepart throughout production. Since radiation hardness quality assurance is not as directly visible as reliability or electrical performance, it is often overlooked when schedules and/or dollars become strained. It is no less important, however. There are two generic methods of radiation hardness quality assurance, process controls and screens. Process controls are used to tighten the piecepart distribution about the mean radiation level at which failure occurs. Screens are used to truncate the low-level radiation tail from the parts failure distribution. Process controls are usually tied to specific processing procedures, none of which are now standard throughout the semiconductor industry. Screens are more general and can be used for a variety of manufacturers products. In addition, screens can provide a rapid feedback channel for information concerning the effectiveness of process controls or the effect of processing changes on radiation hardness. In this report the value of acceptance test screening as a quality assurance tool is explored from the viewpoint of radiation hardness quality assurance. The degree of correlation required between the screen used and radiation failure level is discussed, including the effects of prescreening, cost, radiation specifications, and the acceptable failure level.

Before one can examine the value of acceptance test screening, the piecepart failure distribution must be known or assumed. Since parts failures that occur due to the application of extreme environments such as radiation can most frequently be described using the normal distribution, the bivariate normal distribution is assumed. Thus, parameters that obey the Weibull distribution,\* i.e., wearout, are excluded. The failure level then is the measure of environmental stress at which failure occurs rather than the number of times a stress level is endured or the time over which it is endured.

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\*The Weibull distribution results when the assumed rate of failure under a certain stress can be described as a power of time ( $\alpha t^{\beta-1}$ ,  $t > 0$ ) then the frequency function is  $f(t) = \alpha \beta t^{\beta-1} e^{-\alpha t^{\beta}}$ .

Before using the calculations in this report, one must be assured that part failures are indeed normally distributed for the specific application of interest. This must be determined by experiment and analysis. One method of analysis is to show that the screening parameters and failure levels are themselves the result of sums of other variants. Then, using the central limit theorem, the error associated with assuming a normal distribution can be estimated. Experimentally, a population sample is stressed to failure and the distribution of failure levels compared to the normal distribution to estimate the error introduced by assuming normality. Both techniques of estimating the error are described in the appendix.

## SECTION II

## CALCULATIONS OF THE FAILURE PROBABILITY FOR 100% SCREENING

As discussed in the introduction, the calculations performed in this section assume the bivariate normal distribution (refs. 1 and 2). The bivariate normal distribution describes the case where two variables (i.e., screening parameter and failure level) are individually normally distributed and in the joint probability,  $p(x, y)$ , the variables are not independent, i.e., that  $x$  is large (or small) changes the mean of the  $y$  distribution. (Note that the log normal\* distribution often reported for radiation failure probability (ref. 3) is a subset of bivariate normal distributions where  $\log X$  is substituted for  $X$ .) The frequency distribution is given by

$$\phi(x', y', \rho, \sigma_x, \sigma_y) = \frac{\exp[-K/2(1-\rho^2)]}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} dx'dy' \quad (1)$$

$$\text{where } K = \frac{(x' - m_x)^2}{\sigma_x^2} - \frac{2\rho(x' - m_x)(y' - m_y)}{\sigma_x\sigma_y} + \frac{(y' - m_y)^2}{\sigma_y^2}$$

$x'$  is the failure level

$y'$  is the screening parameter

$\rho$  is the correlation coefficient

$\sigma_x^2$  is the  $x'$  variance

$\sigma_y^2$  is the  $y'$  variance

$m_x$  is the mean of  $x'$

$m_y$  is the mean of  $y'$

The failure probability calculations were done using standard variates, i.e.,

---

\*The log-normal distribution (ref. 3) frequently occurs whenever the physical parameter cannot be negative, e.g., particle size and biological dose.

$$x = \frac{(x' - m_x)}{\sigma_x}$$

$$y = \frac{(y' - m_y)}{\sigma_y}$$

$$K = x^2 - 2\rho xy - y^2$$

The failure probability calculated is

$$P(X, H, \rho) = \frac{1}{1-Q(H)} \int_{-\infty}^X dx \int_H^{\infty} dy \phi(x, H, \rho) \quad (2)$$

where  $Q(H) = \int_{-\infty}^H dy \exp(-y^2/2) / \sqrt{2\pi} = \Phi(H) = 1 - P(H)$

Note:  $\phi(x)$  or  $\phi(x, y, \rho)$  will always represent the normal frequency distribution and  $\Phi(X)$  or  $\Phi(X, Y, \rho)$  the normal cumulative distribution.

$P(H)$  is the fraction of the population left after screening, i.e., discarding all parts with standardized screening parameters less than  $H$ .

$H$  is the standardized screening parameter

$X$  is the specified standardized failure level, i.e., the level below which the system must not fail.

The integral,  $P(X, H, \rho)$ , corresponds to the probability that an element in the bivariate distribution is both above  $H$ , the screening level, and below  $X$ , the specified failure level. It is necessary to divide by  $P(H)$  to correct this to the probability that  $X$  is below  $X_s$  for the population of parts after screening, i.e., there are now fewer parts. There are expansions for the integral in equation (2) but these converge slowly for values of  $\rho$  near one. Noting that the first integral can be done, i.e.,

$$P(X, H, \rho)P(H) = \int_{-\infty}^X dx \phi(x) P\left(\frac{H - \rho x}{\sqrt{1 - \rho^2}}\right) \quad (3)$$

the remaining integral can be integrated numerically. The results compare with using the expansion for  $P(X, H, \rho)$  in Hermite polynomials for  $\rho = 0.25$  and  $0.5$  within about 1 percent. The numerical calculations were done for the matrix of values

$$\rho = 0.25, 0.5, 0.75, 1, 0.95, 0.99$$

$$H = -1.28, -0.67, 0, 0.67, 1.28$$

$$X = -5.5, -0.5, -4.5, \dots, 0, 0.5, \dots, 2$$

and are listed in table 1. Also in table 1, the values of  $Q(X)$  are listed for each of the above failure levels ( $X$ ).

The results of these calculations are also graphically presented in figures 1 through 11. Figures 1 through 5 present the data for different correlation coefficients given the percentage of parts eliminated by the screen,  $H = -1.28, -0.67, 0, 0.67, 1.28$  correspond to 10, 25, 50, 75, 90 percent discard. Figures 1 through 5 are intended to answer the question, if a certain percentage of the parts are to be screened out how good a screen is needed to obtain the required failure probability. Figures 6 through 11 present the same data for different  $H$ 's given the correlation coefficient. Each figure also has a plot of  $Q(X)$ , which is both the probability for  $\rho = 0$  and the probability for  $H = -\infty$ , i.e., no screen.

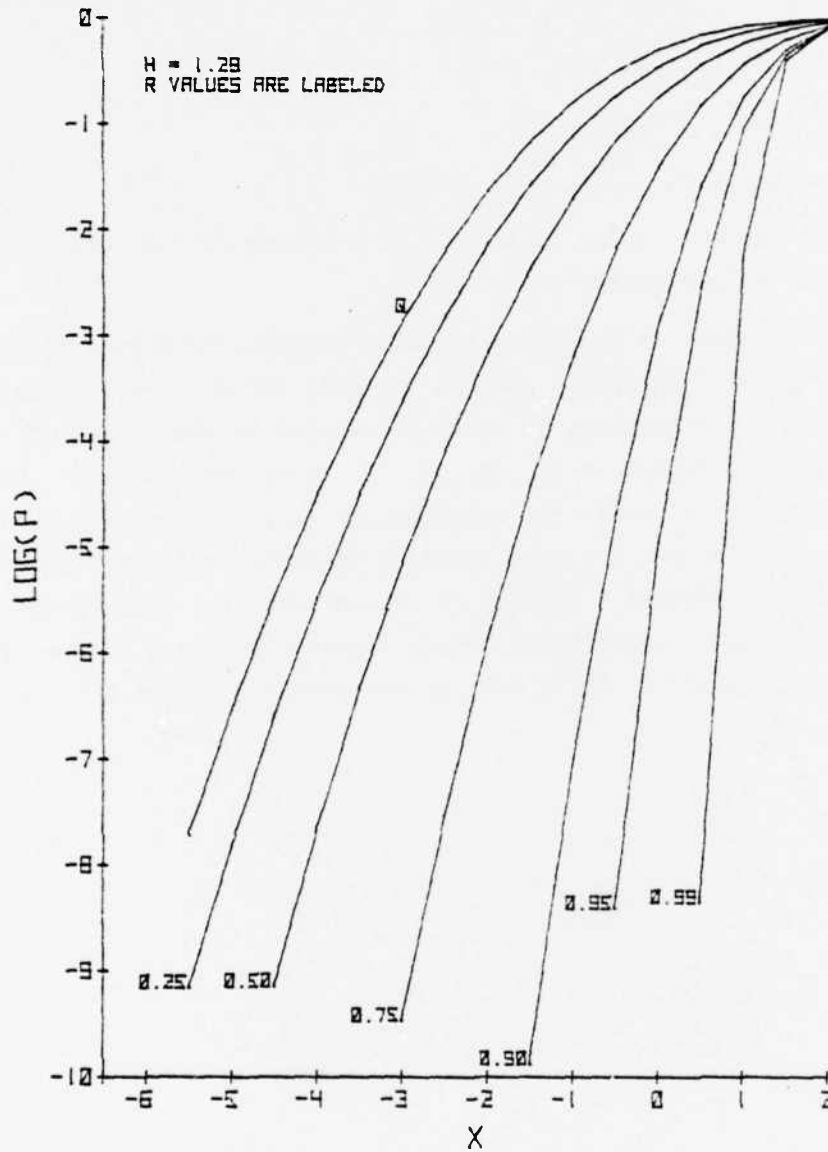


Figure 1. Log Failure Probability (Log P) as a Function of Standardized Failure Level (X) After Screening With a Screen Correlated With a Failure Level With Coefficient (R). (Q(H)) Parts Were Rejected by Screening. H = -1.28, -.67, 0, .67, and 1.28 Correspond to 10, 25, 50, 75, and 90 Percent Discard.

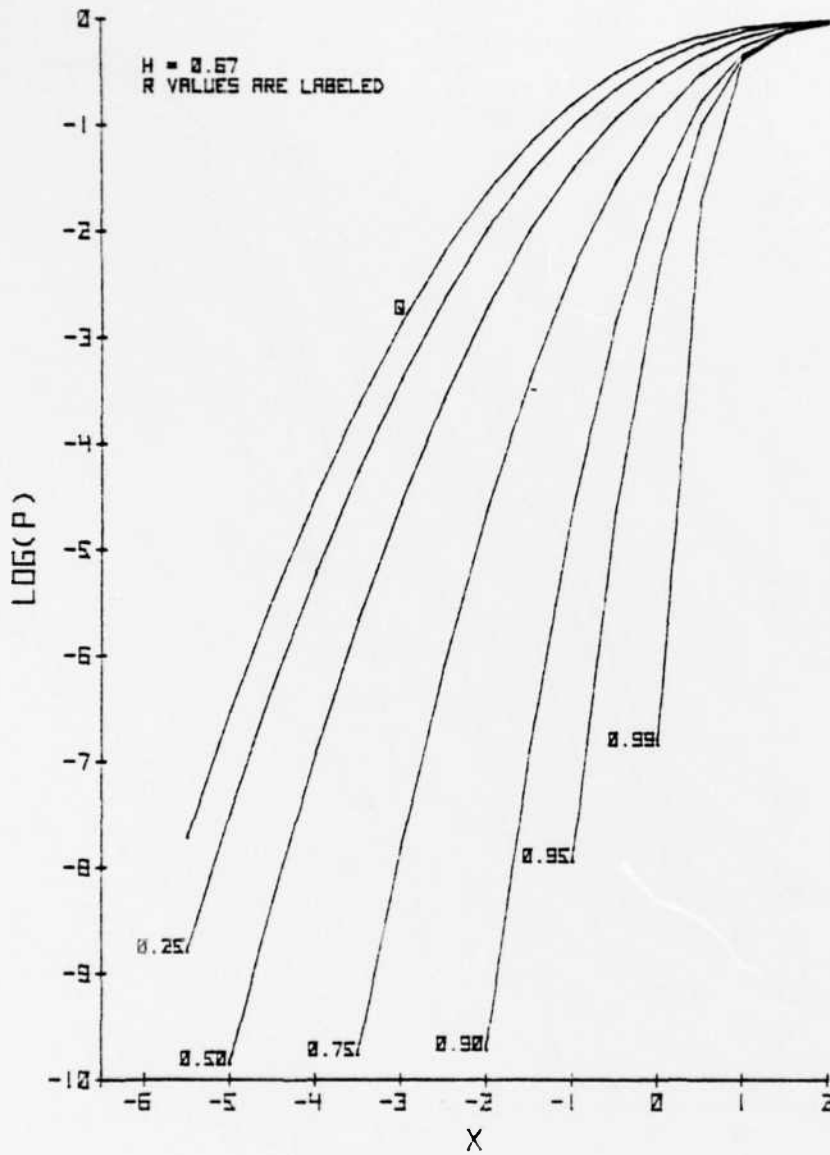


Figure 2. Log Failure Probability (Log P) as a Function of Standardized Failure Level (X) After Screening With a Screen Correlated With a Failure Level With Coefficient (R). (Q(H)) Parts Were Rejected by Screening. H = -1.28, -.67, 0, .67, and 1.28 Correspond to 10, 25, 50, 75, and 90 Percent Discard.

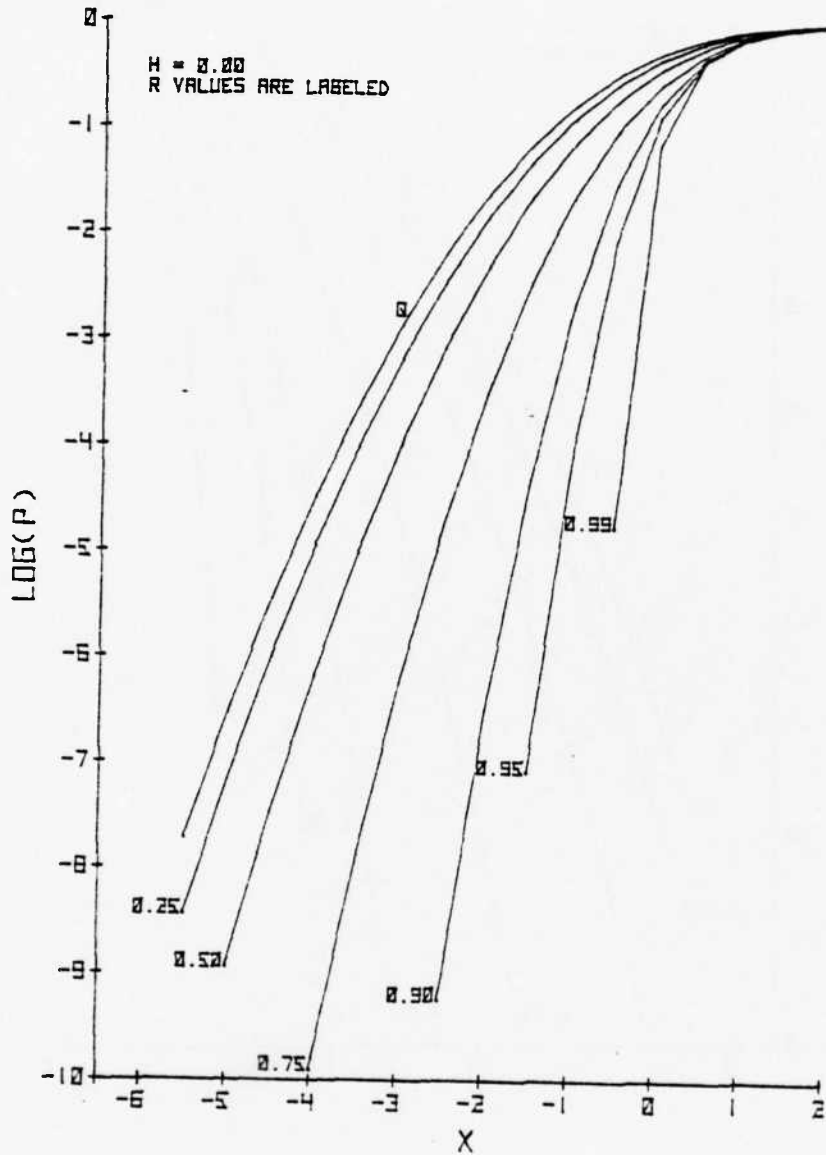


Figure 3. Log Failure Probability (Log P) as a Function of Standardized Failure Level (X) After Screening With a Screen Correlated With a Failure Level With Coefficient (R). (Q(H)) Parts Were Rejected by Screening.  $H = -1.28, -.67, 0, .67,$  and  $1.28$  Correspond to 10, 25, 50, 75, and 90 Percent Discard.

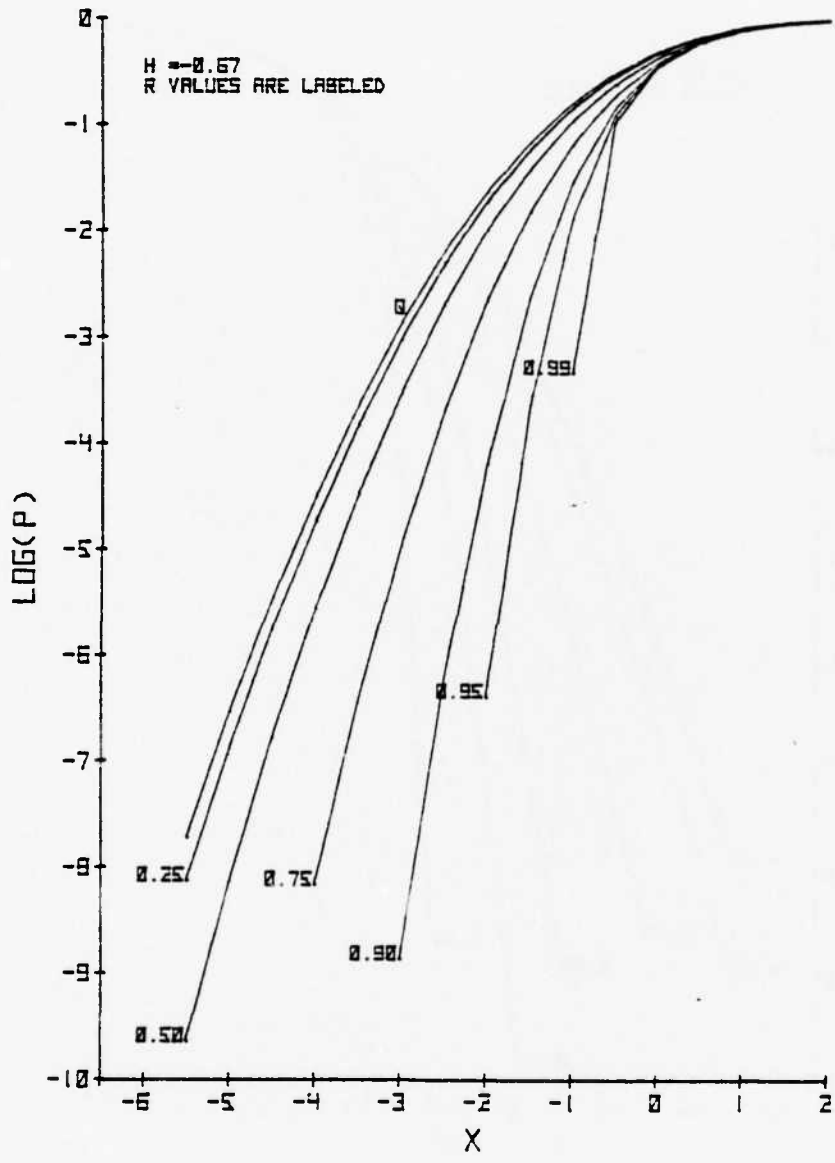


Figure 4. Log Failure Probability (Log P) as a Function of Standardized Failure Level (X) After Screening With a Screen Correlated With a Failure Level With Coefficient (R). (Q(H)) Parts Were Rejected by Screening. H = -1.28, -.67, 0, .67, and 1.28 Correspond to 10, 25, 50, 75, and 90 Percent Discard.

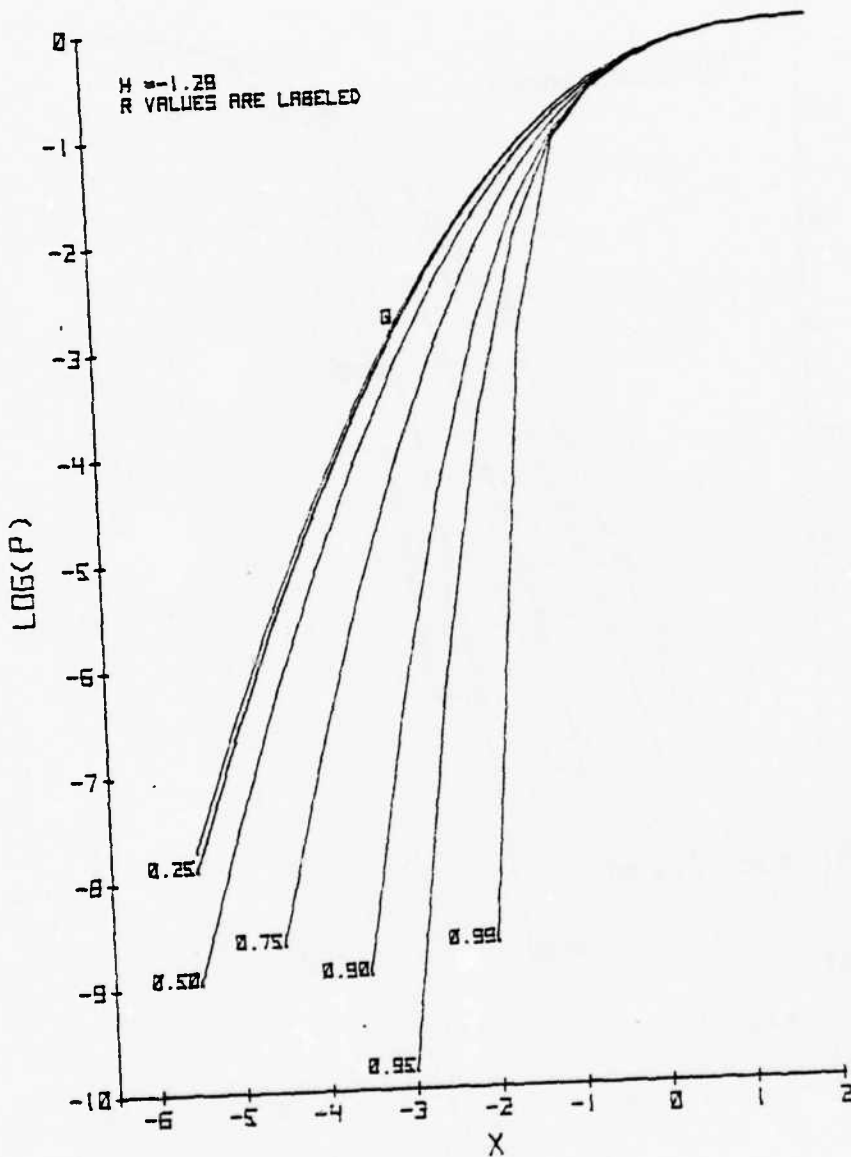


Figure 5. Log Failure Probability (Log P) as a Function of Standardized Failure Level (X) After Screening With a Screen Correlated With a Failure Level With Coefficient (R). (Q(H)) Parts Were Rejected by Screening. H = -1.28, -.67, 0, .67, and 1.28 Correspond to 10, 25, 50, 75, and 90 Percent Discard.

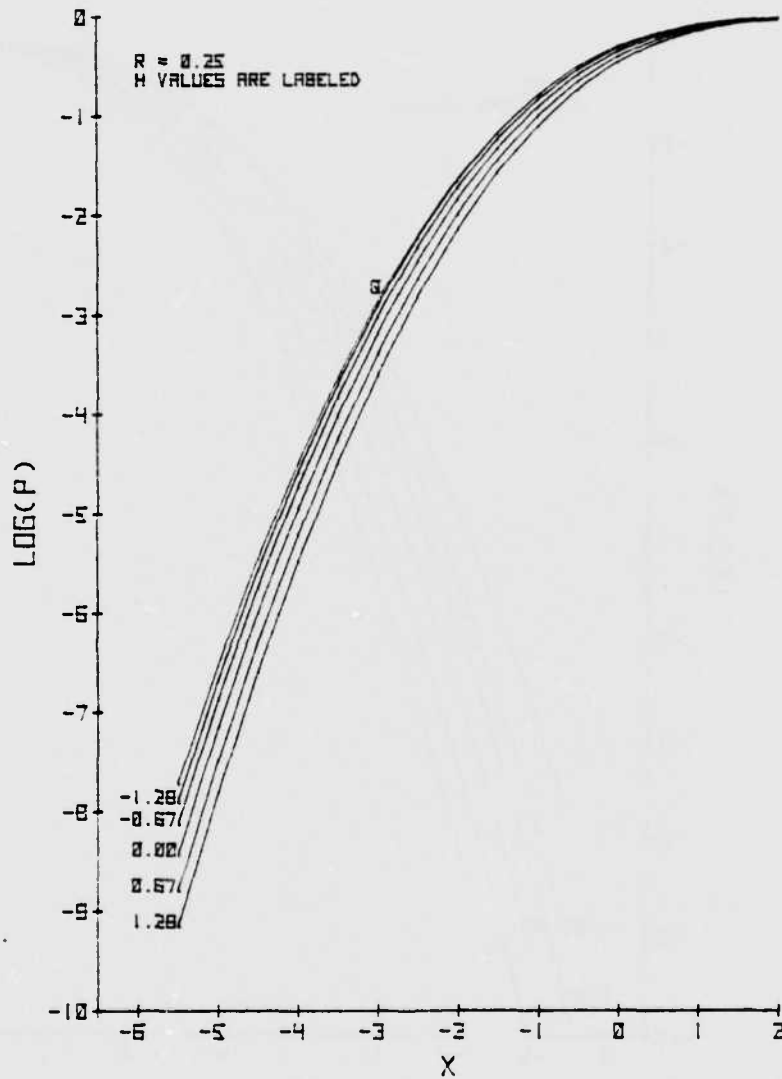


Figure 6. Log Failure Probability (Log P) as a Function of Standardized Failure Level (X) After Screening With a Screen Correlated With a Failure Level With Coefficient (R). (Q(H)) Parts Were Rejected by Screening. H = -1.28, -.67, 0, .67, and 1.28 Correspond to 10, 25, 50, 75, and 90 Percent Discard.

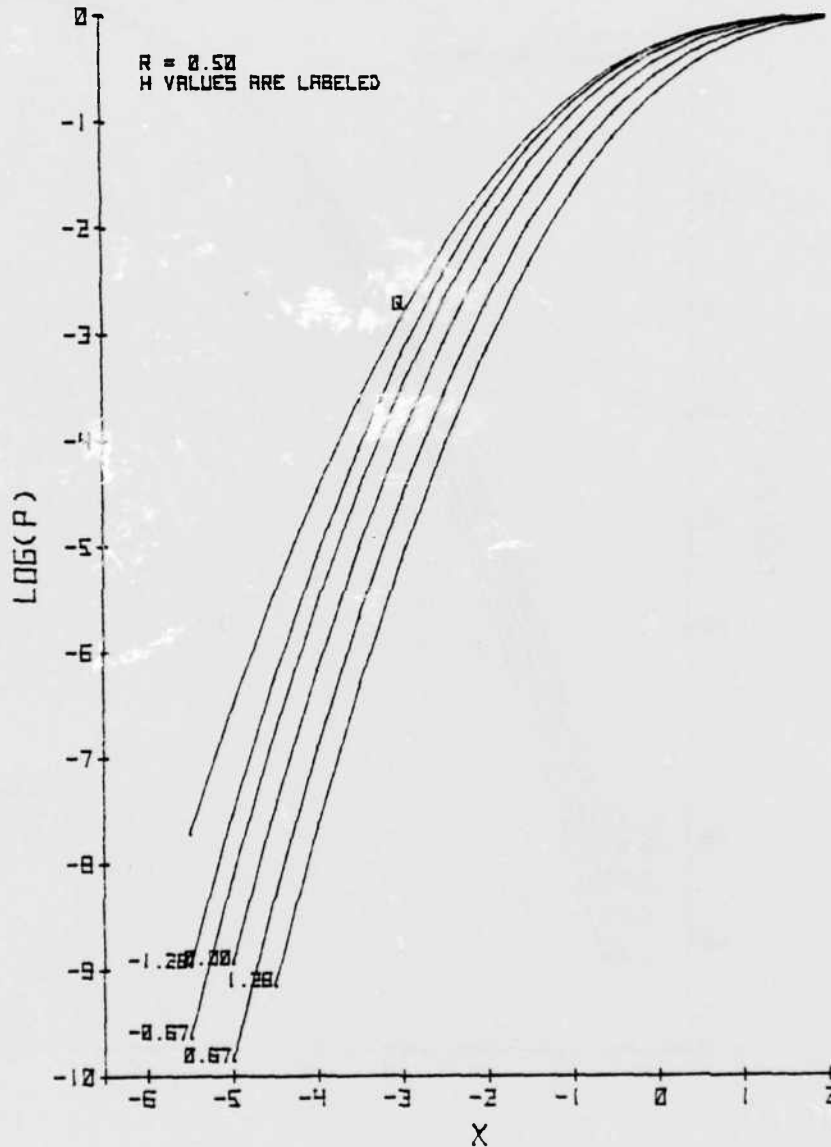


Figure 7. Log Failure Probability (Log P) as a Function of Standardized Failure Level (X) After Screening With a Screen Correlated With a Failure Level With Coefficient (R). (Q(H)) Parts Were Rejected by Screening. H = -1.28, -.67, 0, .67, and 1.28 Correspond to 10, 25, 50, 75, and 90 Percent Discard.

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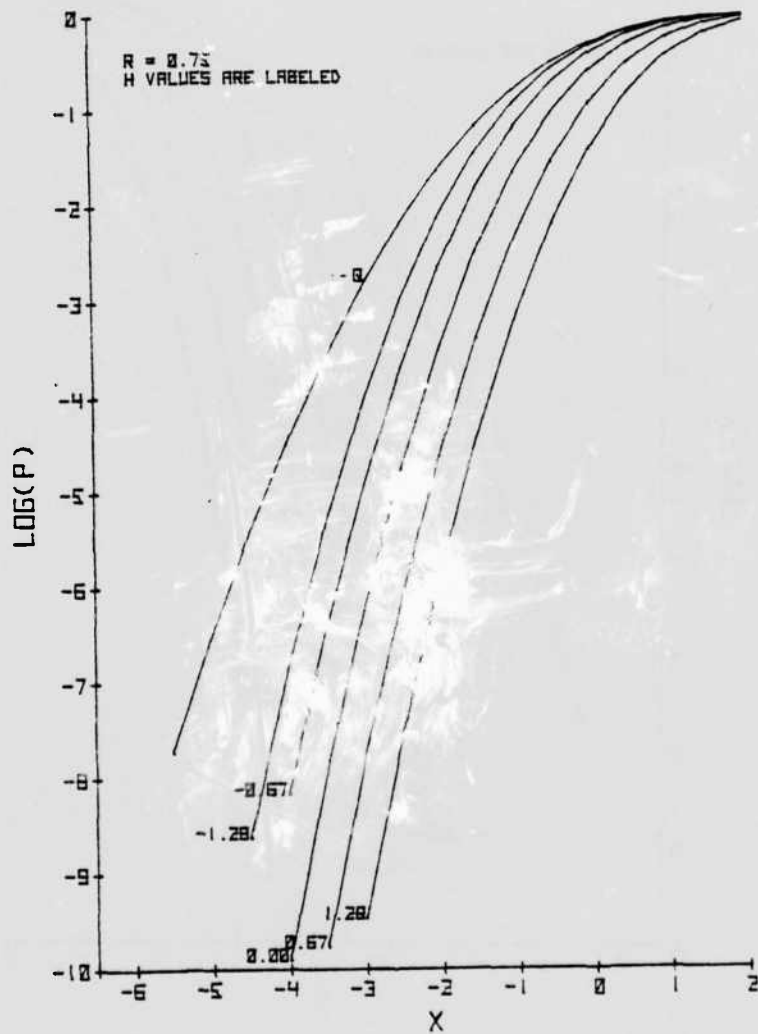


Figure 8. Log Failure Probability (Log P) as a Function of Standardized Failure Level (X) After Screening With a Screen Correlated With a Failure Level With Coefficient (R). (Q(H)) Parts Were Rejected by Screening.  $H = -1.28, -0.67, 0, 0.67,$  and  $1.28$  Correspond to 10, 25, 50, 75, and 90 Percent Discard.

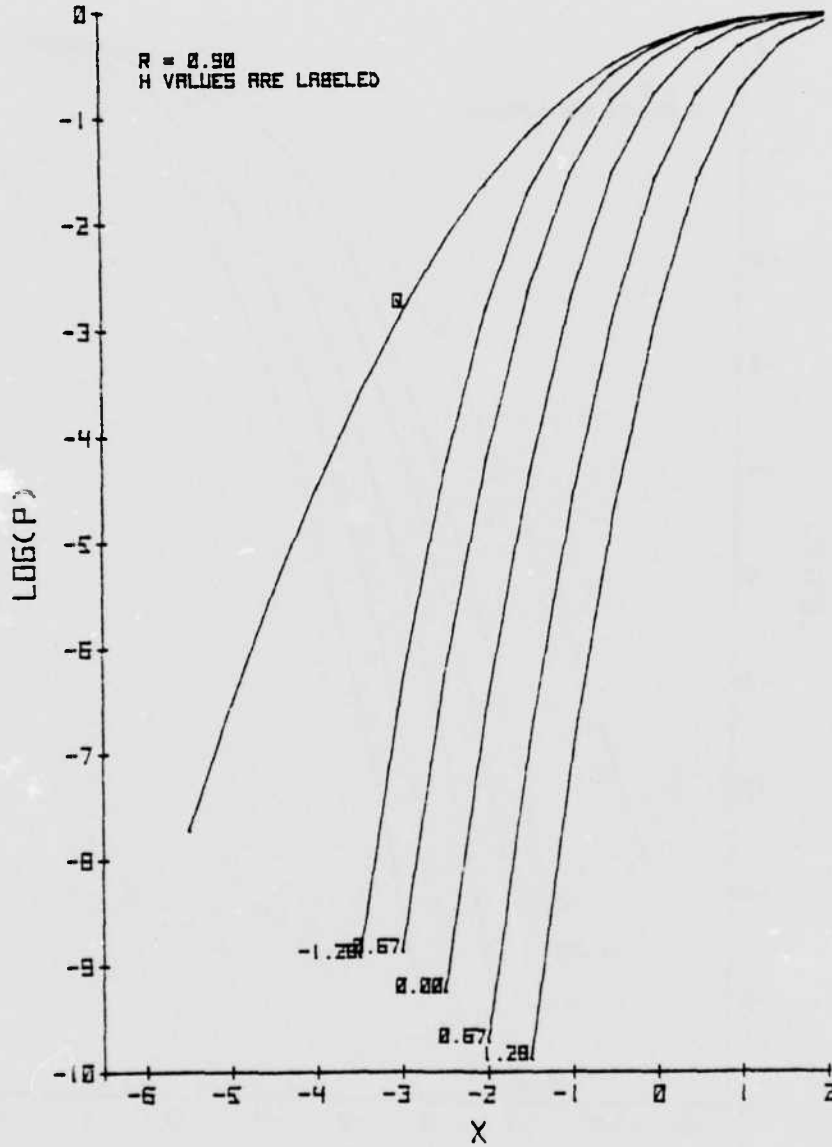


Figure 9. Log Failure Probability (Log P) as a Function of Standardized Failure Level (X) After Screening With a Screen Correlated With a Failure Level With Coefficient (R). (Q(H)) Parts Were Rejected by Screening. H = -1.28, -.67, 0, .67, and 1.28 Correspond to 10, 25, 50, 75, and 90 Percent Discard.

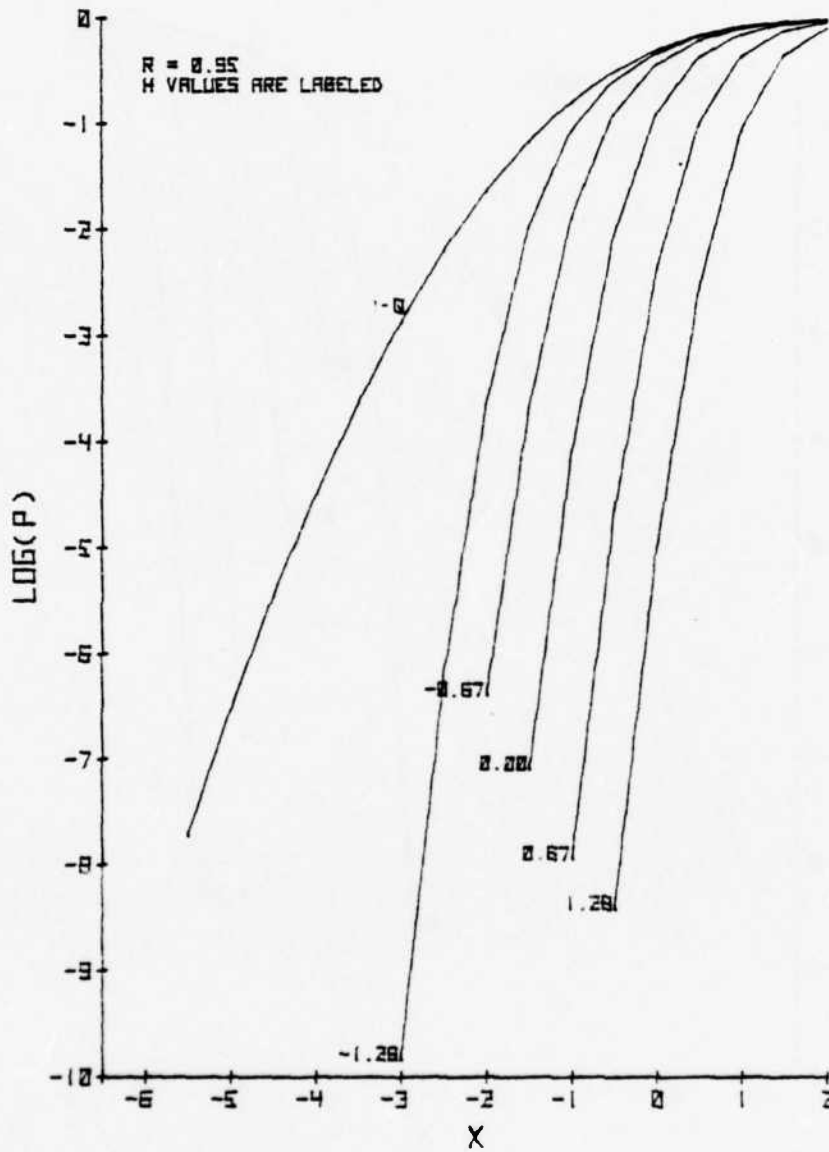


Figure 10. Log Failure Probability (Log P) as a Function of Standardized Failure Level (X) After Screening With a Screen Correlated With a Failure Level With Coefficient (R). (Q(H)) Parts Were Rejected by Screening. H = -1.28, -.67, 0, .67, and 1.28 Correspond to 10, 25, 50, 75, and 90 Percent Discard.

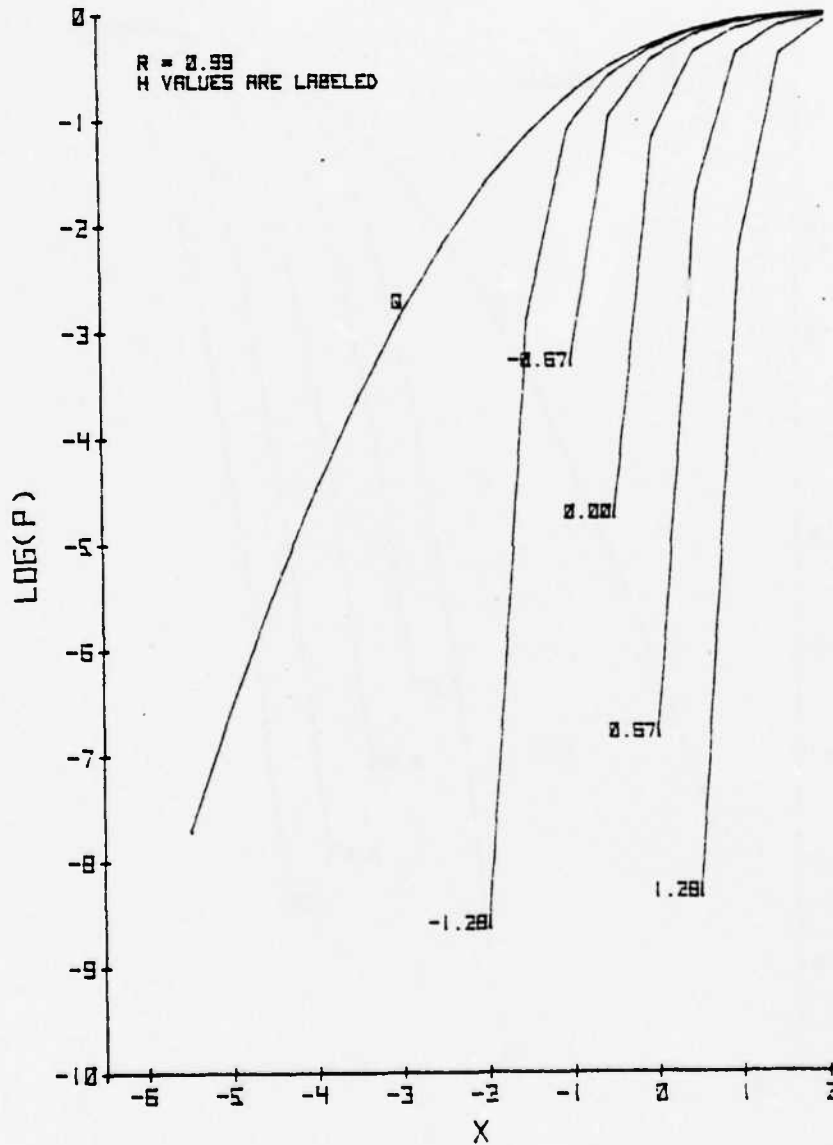


Figure 11. Log Failure Probability ( $\text{Log } P$ ) as a Function of Standardized Failure Level ( $X$ ) After Screening With a Screen Correlated With a Failure Level With Coefficient ( $R$ ). ( $Q(H)$ ) Parts Were Rejected by Screening.  $H = -1.28, -.67, 0, .67, \text{ and } 1.28$  Correspond to 10, 25, 50, 75, and 90 Percent Discard.

## SECTION III

## EXAMPLES

A few examples showing the use of these graphs are appropriate at this point.

Consider a system where there are  $10^4$  mission critical parts and only one system per 100 is permitted to fail at a certain radiation level or,  $P(X, H, \rho) = 10^{-6}$  and  $\log_{10} P = -6$ . If the part failures due to radiation are distributed log-normally with  $\sigma = 1/2 \log_{10} 10$  and the mean failure level is  $\log_{10} 100$  above the system radiation specifications, then the standardized failure level  $X$  is  $(-\log_{10} 100) / (\frac{1}{2} \log_{10} 10) = -4$ . Now if it were economically feasible to discard 50 percent of the parts, then from figure 3 we find that a screen that has a correlation coefficient of only 0.5 is all that is needed. A screen with a correlation coefficient of 0.5 is not really a well correlated screen at all. If, however, only a 10 percent parts discard is economically acceptable, then from figure 5 we find that a screen with a correlation coefficient of about 0.65 - 0.70 is needed. Similarly, if the system developers can afford to discard 90 percent of the parts (figure 1), a screen with a correlation coefficient of only 0.30 is acceptable. Two conclusions are immediately obvious. First, practically any measurement that is correlated with the radiation failure level can be useful as a radiation hardness quality assurance acceptance test screen. Second, almost trivially, a system developer with a tight budget requires better correlated screens. Examination of figures 1 through 5 also yields the somewhat trivial observation that as the allowable failure probability decreases the screen correlation coefficient must improve for a given parts discard level. However, even for a failure probability of one part in  $10^{10}$  with a 50 percent parts discard the screen correlation coefficient must be only 0.75. This again supports the first conclusion. Perfectly correlated screens are not required for systems with radiation specifications well below the mean parts failure level.

Next, consider the case where the best available screen has a correlation coefficient of 0.75 (figure 8). The parts failure specification is again one part in  $10^6$  and  $\sigma = 1/2$ . However, now the average parts failure level is only a factor of 10 above the system radiation specification. Then,  $\log_{10} P = -6$  and for log normally distributed parts  $X = -2$ . From figure 8, we find that 90

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percent of the parts must be discarded to meet the parts failure specification. Thus, even for reasonably well correlated screens, if the system radiation specification is close to the average parts failure level, a large percentage of parts must be discarded and screening for radiation hardness assurance is expensive.

Now carefully considering figures 1 and 5 we see that even with a screening correlation of 0.9 to 0.99 increasing the number discarded from 10 to 90 percent only moves the  $10^{-6}$  failure point 1.5 standard deviations. Thus, unless the standard deviation is large, radiation hardening by screening is not cost effective.

In the above illustrations to demonstrate the use of the graphs, several somewhat obvious and trivial conclusions are drawn. However, two important general conclusions were identified. First, a parameter does not have to be highly correlated with radiation failure to be useful as a radiation hardness assurance screen. Second, radiation hardening (as opposed to hardness assurance) by screening to identify only the hardest parts seems useful only in desperate last-resort situations.

## SECTION IV

## EFFECT OF PRESCREENING ON RADIATION HARDNESS ASSURANCE SCREENING

During the production of any electronic system, the pieceparts will be screened for desired electrical and reliability characteristics prior to radiation hardness assurance screening. This happens when the semiconductor processor decides which parts are to be sold. In selecting the pieceparts to be sold, he first probes each circuit on a wafer and inks those circuits which are functionally deficient. He may also screen the packaged parts to determine if packaging has reduced the circuit performance below specifications. Some of the tests which the semiconductor processor uses may also be radiation hardness assurance screens with some level of correlation. Thus, the semiconductor processor's screening may change the distribution of parts in an undetectable way which could destroy the statistical information needed to evaluate the effectiveness of a radiation hardness acceptance test.

Acceptance tests for system pieceparts are generally developed in the following way. First, the designer of a system with a radiation hardness specification will prepare a qualified parts list. This list is made by selecting a good set of pieceparts which fulfill the system design needs. A sample of the parts is then purchased for radiation and other testing. The resulting data is then used to specify the acceptance tests to be used. A problem associated with this method occurs when the effectiveness of a particular measurement as a radiation hardness screen must be determined. The parts sample is purchased and subjected to radiation tests as described. However, prior to these tests the semiconductor manufacturer has already applied acceptance test screening. The correlation between the radiation hardness screening measurements and the irradiation test is determined as a measure of the efficiency of the screen on a pre-screened population. Here, it will be shown that the information needed is seriously degraded by buying parts that have been pre-screened by the semiconductor manufacturer.

To show this consider the case where the semiconductor manufacturer is screening on a parameter which has correlation  $\rho$  with radiation susceptibility, e.g.,  $h_{fe}$  in bipolar transistors is correlated to both susceptibility to neutron and total dose effects, likewise fanout in TTL gates. Next, as the radiation hardness acceptance test, the same parameter is used only to a higher

level. The designer, or screen developer takes the sample and does a correlation. The mathematics of this situation are as follows.

The distribution of the parts buy is

$$p(x,y,\rho') = \frac{\phi(x,y,\rho)}{P(h)} \quad \text{for } \begin{matrix} -\infty < x < \infty \\ h < y < \infty \end{matrix} \quad (4)$$

where  $h$  is the  $y$  acceptance test level applied by the semiconductor manufacturer

$x$  and  $y$  are the standardized failure and screening parameters

given this distribution a correlation coefficient can be computed in terms of the  $\rho$ , and  $L$  of the original distribution.

From the definition of correlation coefficient, namely,

$$\rho' = \frac{E[(x-E(x))(y-E(y))]}{\sqrt{E[(x-E(x))^2]E[(y-E(y))^2]}} \quad (5)$$

the correlation coefficient is

$$\rho'/\rho = \sqrt{\frac{P^2(h) - [hP(h) + \phi(h)]\phi(h)}{P^2(h) - \rho^2[hP(h) + \phi(h)]\phi(h)}} \quad (6)$$

For  $L = 0$  this can be evaluated with a hand calculator, i.e., for  $\rho = 0.25, 0.5,$  and  $0.75, \rho'/\rho = 0.62, 0.66,$  and  $0.75.$   $\rho'$  can be evaluated with a sample as small as 100 parts. However, if the parts are prescreened then a false  $\rho'$  is obtained causing belief that the screen is less efficient than is really the case.

The second question is how many parts must be tested before a truncation can be made statistically significant and thus be detected? Thirdly, how many parts must be tested before the correct  $\rho$ , which will require determining  $h$ , can be accurately determined. The analysis involved in answering these questions is very involved and goes beyond the intent of this report. However, it is clear, as shown in the appendix, we are investigating the tail of the distribution which requires a very large number of parts to achieve accuracy.

SECTION V  
CONCLUSIONS

The following conclusions were derived in this report.

1. Perfectly correlated screens are not required for hardness assurance of systems which have radiation specifications well below the mean piece parts failure level. Practically any measurement which is correlated with the piece part radiation level can be useful as a hardness assurance screen.

2. The use of screening as a radiation hardening technique (as opposed to a hardness assurance technique) is not very effective, is very costly, and should be used only in situations where time for hardening is not available.

3. Prescreening by the piece parts manufacturer changes the parts distribution. If prescreened parts are used in a test to determine if a certain screening parameter is useful, the test may be biased toward indicating a smaller correlation coefficient than is real.

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APPENDIX A  
EVALUATING THE EFFECT OF ASSUMING NORMALITY

I. The Central Limit Theorem

Given the results of Section II for the normal distribution the problem of showing that the normal distribution applies remains. One method is to show approximate normality via the central limit theorem.

The central limit theorem (refs. 1, 2, and 3) states that the probability distribution of a variate, which is itself a sum of other variates, approaches the normal distribution as the number of other variates in the sum approaches infinity provided certain restrictions on the distributions of the other variates are met. This theorem also holds for the approach to the log-normal distribution for the log of variates which are the result of products of other variates.\* An example of the sum is the value of a resistor is the sum of the resistance of the segments of the length of resistive material which makes up that resistor. An example of log-normal is the gain of an amplifier is the product of the gain of each stage which in turn is a product of the current gain of each transistor times the load. The current gain of a transistor is approximately the product of the inverse of the emitter efficiency, base transport factor, and collector efficiency. Here the certain restrictions on the distributions will be derived so that the analyst can examine his other variates (log emitter efficiency, etc.) and see if they satisfy these restrictions.

Consider the set of  $N$  distribution functions  $F_j'(x_j)$  with frequency functions  $f'(x_i)$  and characteristic function

$$\psi_j'(t) = \exp \left[ \sum_{v=1}^{\infty} \kappa_{jv}' (it)^v / v! \right] \quad (A1)$$

Assuming the  $x_j$  are independent.

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\*The log-normal distribution (ref. 2) is the result of the substitution of the log of a physical parameter into the normal distribution. It frequently occurs whenever the physical parameter cannot be negative, e.g., particle size and biological dose.

We construct the variable

$$\omega = \sum_{j=1}^N x_j \quad (A2)$$

the characteristic function of  $\omega$  is then

$$\begin{aligned} \psi(t) &= \prod_{j=1}^N \psi_j^i(t) = \exp \left[ \sum_j \sum_v \kappa_{jv}^i (it)^v / v! \right] \\ &= \exp \left[ \sum \kappa_v^N (it)^v / v! \right] \end{aligned} \quad (A3)$$

where  $\kappa_v^N = \sum_j \kappa_{jv}^i$

The  $\kappa_v$ 's are called the semi-invariants (ref. 3) and all functions  $F(x)$  with the same  $\kappa_v$ 's are the same function. Specifically, the mean

$$\mu_\omega = \kappa_1^N = \sum_j \kappa_{j1}^i = \sum_j \mu_j^i \quad (A4)$$

$$\sigma_\omega^2 = \kappa_2^N = \sum_j \kappa_{j2}^i = \sum_j \sigma_j^{i2} \quad (A5)$$

Define shape factor  $\lambda_v = \kappa_v / \sigma^v$

$$\lambda_v^N = \sum_j \kappa_{jv}^i / \sigma_\omega^v \quad (A6)$$

$$= \sum_j \lambda_{jv}^i \left( \sigma_j^i / \sigma_\omega \right)^v \quad (A7)$$

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$[\sigma_j'/\sigma_\omega]$  is in the order of  $[1/\sqrt{N}]$

Note (refs. 1, 2, and 3): all functions with all the shape factors the same have the same shape.

If we add a distribution  $F_{N+1}(x_{N+1})$  to the N other distributions, then

$$\lambda_{\nu}^{N+1} = \frac{\lambda_{\nu}^N \sigma_{\omega}^{\nu} + \lambda_{N+1, \nu}^1 \sigma_{N+1}^{\nu}}{\sigma_{\omega}^{\nu} + \nu \sigma_{N+1}^{\nu-1} \sigma_{\omega}^{\nu-1}} \quad (A8)$$

and if

$$\lambda_{N+1, \nu}^1 < \nu \left( \frac{\sigma_{\omega}}{\sigma_{N+1}} \right)^{\nu-1} \lambda_{\nu}^N \quad (A9)$$

then  $\lambda_{\nu}^{N+1}$  will be less than  $\lambda_{\nu}^N$  and will approach zero as  $N \rightarrow \infty$ . Thus  $F(\omega)$  approaches  $\phi(\omega)$ , the normal distribution function, as all the shape factors  $\lambda_{\nu}$  of  $\phi(\omega)$  are zero for  $\nu \geq 3$ .

Then to use the central limit theorem, the analyst must consider the various variates which make up his variate and be sure that no one variate has standard deviation which dominates the sum  $\kappa_{\nu}^{(N)}$  and no pathological  $\lambda_{j\nu}$ 's exist.

## II. Experimental Verification of Normality

There are several ways of checking to see how well a population sample fits the normal distribution. The simplest hand calculator method is to compare estimates of the moments (refs. 1, 2, and 3) with those of the normal distribution. A method which estimates the error of assuming a normal distribution involves regression analysis using Hermite polynomials. All these methods require a population sample. This sample must be obtained independent of screening by the manufacturer. Any screening will distort the distribution. As such a sample is required to evaluate the correlation coefficient anyway, this isn't as much extra expense as might be thought. The moment calculation is based on the mathematical fact (refs. 1, 2, and 3) that the variates are random and unbiased (as for example, by manufacturer's screening). The estimate of central moments is

$$m_v = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^v \quad (\text{A10})$$

where  $\bar{x} = \frac{1}{n} \sum x_i$

This estimator is, however, biased. Unbiased estimators of the moments and the semi-invariants are (ref. 2)

$$M_2 = \frac{n}{n-1} m_2 \quad (\text{A11})$$

$$M_3 = \frac{n^2}{(n-1)(n-2)} m_3 \quad (\text{A12})$$

$$M_4 = \frac{n(n^2 - 2n + 3)}{(n-1)(n-2)(n-3)} m_4 - \frac{3n(2n-3)}{(n-1)(n-2)(n-3)} m_2^2 \quad (\text{A13})$$

and

$$K_2 = \frac{n}{(n-1)} m_2 \quad (\text{A14})$$

$$K_3 = \frac{n^2}{(n-1)(n-2)} m_3 \quad (\text{A15})$$

$$K_4 = \frac{n^3}{(n-1)(n-2)(n-3)} [(n+1)m_4 - 3(n-1)m_2^2] \quad (\text{A16})$$

Then if  $K_3$  and  $K_4$ , the invariants, are much smaller than  $K_2$  this indicates that the distribution function may be normal.

Any function which can be expanded in a power series can be expanded in a series of the derivatives of the normal functions, i.e., the distribution function

$$F(X) = \phi(X) + C_1 \phi^{(1)}(X) + \frac{C_2}{2} \phi^{(2)}(X) + \dots + \frac{C_n}{n!} \phi^{(n)}(X) \quad (\text{A17})$$

$$f(x) = \phi(x) + c_1 \phi^{(1)}(x) + \frac{c_2}{2} \phi^{(2)}(x) + \dots + \frac{c_n}{n!} \phi^{(n)}(x) \quad (A18)$$

where  $\phi(x) = e^{-x^2/2} / \sqrt{-2\pi}$ ,  $\phi(X) = \int_{-\infty}^X dx \phi(x)$ ,  $F(X) = \int_{-\infty}^X dx f(x)$

and  $\phi^{(n)}(x) = \frac{d^n}{dx^n} \phi(x)$

Note:  $\phi^{(n)}(x)$  is related to one form of the Hermite orthogonal polynomials, i.e.,

$$H_n(x) = (-1)^n \phi^{(n)}(x) / \phi(x) \quad (A19)$$

If  $x$  is a standardized variable, i.e.,  $x = (x' - \mu) / \sigma$ , then  $C_1 = C_2 = 0$  and

$$F(X) = \phi(X) \left\{ 1 + \sum_{v=3}^n \frac{C_v}{v!} H_{v-1}(X) \right\} \quad (A20)$$

An equivalent form gives  $f(x)$ .

All the information contained in the sample can be used by reordering the  $X_i$  in order of the algebraic value of  $X_i$  then  $F(X_i) = i/n$  and equation (A20) can be solved for by least squares regression. Then the factor in the { } braces in equation (A20) is an estimate of the error in using table 1. Further analysis can be done where the central limit theorem is known to be working. There, the  $C_v$  should be of order (ref. 2).

Subscript $v$	Order of $C_v$
3	$n^{-1/2}$
4, 6	$n^{-1}$
5, 7, 9	$n^{-3/2}$
8, 10, 12	$n^{-2}$
11, 13, 15	$n^{-5/2}$

The  $C_v$  can thusly be examined to see if they are getting smaller in the right order. If so and if the central limit theorem applies then the expansion equation (A20) can be used as an error estimate with great confidence.

PROBABILITY OF FAILURE (P) GIVEN TH  
THE SCREENING PARAMETER AND THE FR

	-1.2816	-0.6745	H VALUES 0.0000	0.6745	1.2816
			R = 0.25		
X					
-5.5	1.24E-08	7.46E-09	3.65E-09	1.58E-09	6.88E-10
-5.0	2.03E-07	1.29E-07	6.78E-08	3.17E-08	1.48E-08
-4.5	2.59E-06	1.74E-06	9.77E-07	4.92E-07	2.47E-07
-4.0	2.57E-05	1.82E-05	1.09E-05	5.91E-06	3.18E-06
-3.5	1.99E-04	1.48E-04	9.46E-05	5.51E-05	3.18E-05
-3.0	1.21E-03	9.38E-04	6.39E-04	3.99E-04	2.46E-04
-2.5	5.75E-03	4.67E-03	3.38E-03	2.26E-03	1.49E-03
-2.0	2.17E-02	1.83E-02	1.40E-02	1.00E-02	7.03E-03
-1.5	6.50E-02	5.69E-02	4.60E-02	3.49E-02	2.61E-02
-1.0	1.57E-01	1.42E-01	1.20E-01	9.68E-02	7.68E-02
-0.5	3.08E-01	2.86E-01	2.54E-01	2.16E-01	1.81E-01
0.0	5.01E-01	4.78E-01	4.40E-01	3.92E-01	3.45E-01
0.5	6.94E-01	6.74E-01	6.40E-01	5.94E-01	5.46E-01
1.0	8.44E-01	8.31E-01	8.07E-01	7.74E-01	7.35E-01
1.5	9.35E-01	9.29E-01	9.16E-01	8.97E-01	8.73E-01
2.0	9.78E-01	9.76E-01	9.70E-01	9.62E-01	9.51E-01
			R = 0.5		
X					
-5.5	1.08E-09	2.30E-10	3.01E-11	2.98E-12	3.05E-13
-5.0	2.82E-08	7.17E-09	1.15E-09	1.40E-10	1.73E-11
-4.5	5.41E-07	1.63E-07	3.17E-08	4.75E-09	7.10E-10
-4.0	7.63E-06	2.70E-06	6.40E-07	1.17E-07	2.11E-08
-3.5	7.95E-05	3.29E-05	9.42E-06	2.10E-06	4.55E-07
-3.0	6.16E-04	2.96E-04	1.02E-04	2.76E-05	7.15E-06
-2.5	3.57E-03	1.98E-03	8.00E-04	2.65E-04	8.20E-05
-2.0	1.57E-02	9.86E-03	4.76E-03	1.87E-03	6.89E-04
-1.5	5.28E-02	3.72E-02	2.09E-02	9.79E-03	4.26E-03
-1.0	1.38E-01	1.07E-01	6.95E-02	3.83E-02	1.96E-02
-0.5	2.87E-01	2.42E-01	1.77E-01	1.13E-01	6.73E-02
0.0	4.83E-01	4.34E-01	3.53E-01	2.58E-01	1.76E-01
0.5	6.83E-01	6.43E-01	5.68E-01	4.64E-01	3.57E-01
1.0	8.39E-01	8.14E-01	7.62E-01	6.79E-01	5.78E-01
1.5	9.33E-01	9.22E-01	8.95E-01	8.45E-01	7.76E-01
2.0	9.78E-01	9.74E-01	9.63E-01	9.41E-01	9.06E-01
			R = 0.75		
X					
-5.5	1.55E-13	1.89E-15	6.64E-18	1.14E-20	2.06E-23
-5.0	2.52E-11	5.01E-13	3.07E-15	9.19E-18	2.77E-20
-4.5	2.38E-09	7.67E-11	8.15E-13	4.25E-15	2.12E-17
-4.0	1.31E-07	6.81E-09	1.24E-10	1.13E-12	9.31E-15
-3.5	4.26E-06	3.53E-07	1.10E-08	1.72E-10	2.34E-12
-3.0	8.29E-05	1.07E-05	5.65E-07	1.52E-08	3.37E-10
-2.5	9.83E-04	1.95E-04	1.70E-05	7.75E-07	2.81E-08
-2.0	7.28E-03	2.14E-03	3.04E-04	2.32E-05	1.36E-06
-1.5	3.49E-02	1.46E-02	3.27E-03	4.09E-04	3.82E-05
-1.0	1.13E-01	6.38E-02	2.16E-02	4.32E-03	6.35E-04
-0.5	2.64E-01	1.87E-01	9.03E-02	2.79E-02	6.31E-03
0.0	4.70E-01	3.89E-01	2.50E-01	1.13E-01	3.82E-02
0.5	6.77E-01	6.19E-01	4.88E-01	3.01E-01	1.45E-01
1.0	8.37E-01	8.05E-01	7.22E-01	5.60E-01	3.62E-01
1.5	9.33E-01	9.19E-01	8.81E-01	7.89E-01	6.32E-01
2.0	9.78E-01	9.73E-01	9.60E-01	9.24E-01	9.44E-01

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Table 1

FAILURE (P) GIVEN THE CORRELATION COEFFICIENT (R) OF  
PARAMETER AND THE FRACTION OF REJECTED PARTS (Q(H))

1.2816		-1.2816	-0.6745	H VALUES 0.0000	0.6745	1.2816
	Q			R= 0.9		
6.88E-10	1.90E-08	2.58E-25	8.00E-31	7.87E-38	9.64E-46	1.28E-53
1.48E-08	2.87E-07	1.45E-20	1.85E-25	8.79E-32	5.23E-39	2.90E-46
2.47E-07	3.40E-06	2.26E-16	1.17E-20	2.67E-26	7.71E-33	1.77E-39
3.13E-06	3.17E-05	9.79E-13	2.04E-16	2.22E-21	3.09E-27	2.94E-33
3.18E-05	2.33E-04	1.20E-09	9.80E-13	5.08E-17	3.38E-22	1.03E-27
2.46E-04	1.35E-03	4.25E-07	1.35E-09	3.21E-13	1.01E-17	1.63E-22
1.49E-03	6.21E-03	4.51E-05	5.28E-07	5.70E-10	8.39E-14	5.50E-18
7.03E-03	2.28E-02	1.53E-03	6.14E-05	2.88E-07	1.94E-10	5.11E-14
2.61E-02	6.68E-02	1.83E-02	2.24E-03	4.26E-05	1.27E-07	1.32E-10
7.68E-02	1.59E-01	9.29E-02	2.81E-02	1.93E-03	2.40E-05	9.61E-08
1.81E-01	3.09E-01	2.53E-01	1.42E-01	2.39E-02	1.36E-03	2.62E-05
3.45E-01	5.00E-01	4.67E-01	3.65E-01	1.64E-01	2.48E-02	1.26E-03
5.46E-01	6.91E-01	6.77E-01	6.12E-01	4.36E-01	1.64E-01	2.49E-02
7.35E-01	8.41E-01	8.37E-01	8.04E-01	7.07E-01	4.68E-01	1.73E-01
8.73E-01	9.33E-01	9.33E-01	9.19E-01	8.79E-01	7.62E-01	5.63E-01
9.51E-01	9.77E-01	9.78E-01	9.73E-01	9.60E-01	9.20E-01	8.66E-01
	Q			R= 0.95		
3.05E-13	1.90E-08	8.83E-45	3.00E-56	8.09E-71	2.78E-87	8.06E+00
1.73E-11	2.87E-07	9.67E-36	6.19E-46	4.39E-59	3.99E-74	2.91E-89
7.10E-10	3.40E-06	8.34E-28	1.00E-36	1.85E-48	4.44E-62	6.17E-76
2.11E-08	3.17E-05	5.72E-21	1.27E-28	6.12E-39	3.84E-51	1.01E-63
4.55E-07	2.33E-04	3.17E-15	1.28E-21	1.59E-30	2.60E-41	1.30E-52
7.15E-06	1.35E-03	1.46E-10	1.04E-15	3.25E-23	1.37E-32	1.29E-42
8.20E-05	6.21E-03	5.85E-07	6.94E-11	5.32E-17	5.72E-25	1.01E-33
6.89E-04	2.28E-02	2.25E-04	3.98E-07	7.11E-12	1.90E-18	6.30E-26
4.26E-03	6.68E-02	1.02E-02	2.13E-04	8.02E-08	5.10E-13	3.02E-19
1.96E-02	1.59E-01	8.45E-02	1.27E-02	8.14E-05	1.14E-08	1.19E-13
6.73E-02	3.09E-01	2.52E-01	1.22E-01	8.49E-03	2.23E-05	3.84E-09
1.76E-01	5.00E-01	4.67E-01	3.61E-01	1.22E-01	4.22E-03	1.08E-05
3.57E-01	6.91E-01	6.77E-01	6.12E-01	4.22E-01	9.72E-02	2.84E-03
5.78E-01	8.41E-01	8.37E-01	8.04E-01	7.07E-01	4.33E-01	8.57E-02
7.76E-01	9.33E-01	9.33E-01	9.19E-01	8.79E-01	7.58E-01	4.45E-01
9.06E-01	9.77E-01	9.78E-01	9.73E-01	9.60E-01	9.19E-01	7.99E-01
	Q			R= 0.99		
2.06E-23	1.90E-08	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
2.77E-20	2.87E-07	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
2.12E-17	3.40E-06	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
9.31E-15	3.17E-05	1.62E-85	0.00E+00	0.00E+00	0.00E+00	0.00E+00
2.34E-12	2.33E-04	2.37E-58	2.50E-91	0.00E+00	0.00E+00	0.00E+00
3.37E-10	1.35E-03	1.27E-36	4.58E-63	0.00E+00	0.00E+00	0.00E+00
2.81E-08	6.21E-03	2.60E-20	3.06E-40	1.14E-71	0.00E+00	0.00E+00
1.36E-06	2.28E-02	2.25E-09	7.75E-23	5.08E-47	5.90E-81	0.00E+00
3.82E-05	6.68E-02	1.30E-03	8.00E-11	8.42E-28	1.75E-54	7.58E-87
6.35E-04	1.59E-01	7.89E-02	4.60E-04	5.47E-14	1.91E-33	2.96E-59
6.31E-03	3.09E-01	2.51E-01	1.03E-01	1.67E-05	7.97E-18	4.03E-37
3.82E-02	5.00E-01	4.67E-01	3.60E-01	6.69E-02	1.42E-07	2.08E-20
1.45E-01	6.91E-01	6.77E-01	6.12E-01	4.18E-01	1.85E-02	4.40E-09
3.62E-01	8.41E-01	8.37E-01	8.04E-01	7.06E-01	4.13E-01	5.52E-03
6.32E-01	9.33E-01	9.33E-01	9.19E-01	8.79E-01	7.58E-01	3.97E-01
9.44E-01	9.77E-01	9.78E-01	9.73E-01	9.60E-01	9.19E-01	7.99E-01

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