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TE-MODE SOLUTIONS FOR DIELECTRIC SLAB CENTER LOADED RIDGED WAVE--ETC(U)

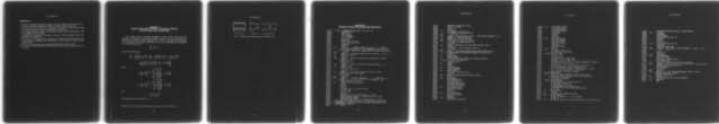
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NRL Report 8105

TE- Mode Solutions for Dielectric Slab Center Loaded Ridged Waveguide

CHARLES W. YOUNG, JR.

*Microwave Techniques Branch
Electronics Technology Division*

April 29, 1977



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TE-MODE SOLUTIONS FOR DIELECTRIC-SLAB CENTER-LOADED RIDGED WAVEGUIDE

INTRODUCTION

Applications exist which require dielectric or ferrite slab center loaded rectangular waveguide to be used in conjunction with ridged waveguide. Transitions can be made with stepped matching transformers; these transformers are appropriate sections of a composite of the two different waveguide configurations. The objective of this report is to present an analysis of such dielectric slab center loaded ridged waveguide and to provide a method of obtaining equations for the TE_{n0} propagation characteristics, thus facilitating transformer design.

BACKGROUND

Because higher order modes can easily cause mismatch and transmission loss spikes, waveguide operation is generally limited to a frequency band where only the principal mode may propagate. Conventional rectangular waveguide has a theoretical two-to-one, or single octave, principal-mode-only frequency bandwidth; in practice, the useable bandwidth is less because of large attenuation near the cut-off frequency.

Ridged waveguide, particularly double-ridged waveguide, is commonly used when larger bandwidths are required at high power levels. A frequency range of more than four to one between the cut-off frequencies of the TE_{10} and TE_{20} modes can easily be obtained [1,2] using double-ridged waveguide. Similar bandwidths can be obtained with dielectric slab center loaded rectangular waveguide [3,4,5]. Both ridged and slab loaded waveguide achieve broad bandwidths by adding large capacitance to the dominant mode while only slightly affecting the capacitance of the next higher order mode.

Ferrite toroidal phase shifters also can be designed for operation in excess of one octave. Because of the small gap spacing of ridged waveguide, the phase shifters are generally made in rectangular waveguide. Dielectric slab center loaded rectangular waveguide would be readily compatible with the ferrite toroidal phase shifter, but it is not a commonly used transmission line. Since ridged waveguide is commonly used, it would be desirable to have compatibility, i.e., matching transitions, between ridged waveguide and ferrite toroidal loaded rectangular waveguide. Dielectric loaded tapered transitions are possible, but the fabrication would be very difficult. Also, a quasi-Tchebycheff transformer design should give better matching for given length transitions. The latter approach requires transformer sections of dielectric loaded ridged waveguide, but analysis of this type of transmission line is not currently available in the literature. The analysis in this report employs an equivalent transmission line circuit for the transverse component of the propagating electromagnetic wave to derive solutions for the TE_{n0} propagation constants in such waveguide.

Manuscript submitted January 27, 1977.

ANALYSIS

TE-mode solutions for the dielectric slab center loaded rectangular waveguide of Fig. 1a can be derived by using ABCD matrices [6] or by using an equivalent transmission line circuit for the crossguide component of the electromagnetic wave. The homogeneous double-ridged waveguide of Fig. 1b has been analyzed [1,2] by using the latter method in conjunction with the equivalent discontinuity susceptance due to the height change at the ridge wall.

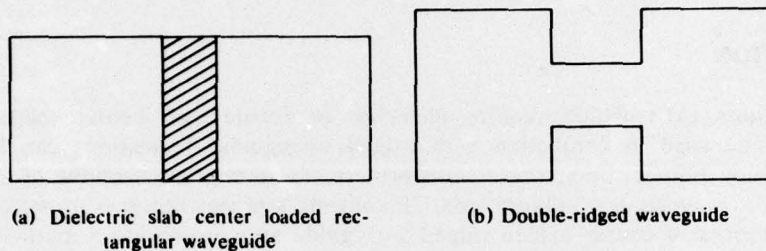


Fig. 1 — Broadband waveguide cross sections

For the dielectric slab center loaded double ridged waveguide of Fig. 2, the analysis is similar to that for the homogeneous case, with an extra section incorporated in the equivalent transmission line circuit. The dimensions referred to in all subsequent calculations are those shown in Fig. 2. For simplicity, this report will consider only the case for TE_{n0} modes and will assume that the transmission line is lossless, i.e., perfectly conducting waveguide walls and a dielectric loss tangent of zero. Axial symmetry will also be assumed.

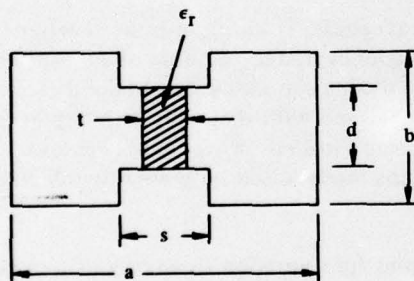
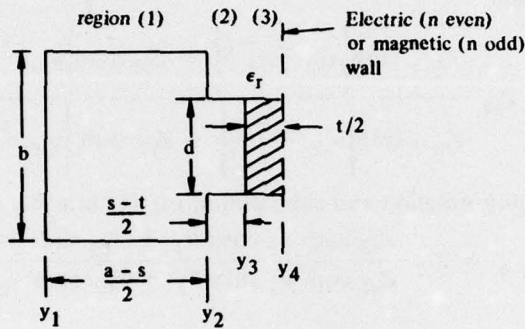


Fig. 2 — Cross section of dielectric slab center loaded, double-ridged waveguide

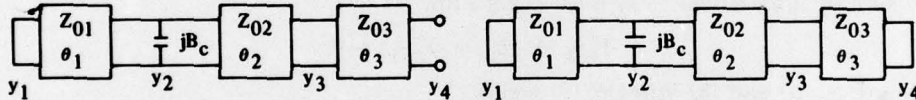
Cohn's article on ridged waveguide [2] points out that for the homogeneous case (i.e. $\epsilon_r = 1$) the cross section may be treated at the cut-off frequency by assuming that it is an infinitely wide, composite, parallel strip transmission line short-circuited at two points. The resultant electromagnetic field may be considered as an electromagnetic wave traveling from side to side without longitudinal propagation. The resonant conditions can then be solved for the cut-off frequencies of the different TE_{n0} modes.

A similar argument holds for the inhomogeneous case. In addition, the longitudinal propagation constant may be treated as the unknown quantity, and solutions at any frequency may be obtained by separating the wave vector in each region into its transverse and longitudinal components. Since the waveguide configuration is symmetrical, the resonance condition for the transverse wave component will result in an infinite (zero) impedance at the center for n odd (even). Half of a cross section is shown in Fig. 3a, and the equivalent transmission line circuits

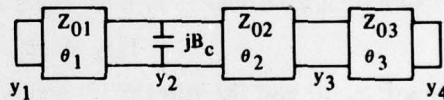
for the transverse wave are shown in Fig. 3b for n odd and Fig. 3c for n even. Since the equivalent circuit is a composite, dissipationless, passive line matched at both ends, it is matched at all points. Therefore, the sum of the admittances at the plane y_2 of the effective lumped capacitance due to the ridge wall must equal zero. Within each region, where the regions are shown in Fig. 3a, Z_{0i} is the characteristic impedance, $Y_{0i} = 1/Z_{0i}$ is the characteristic admittance, and θ_i is the transverse electrical length; θ_i is equal to the product of the physical transverse dimension of the region and γ_{yi} , the complex transverse propagation constant. Since all regions are lossless, γ_{yi} , and therefore θ_i , will be purely real or purely imaginary.



(a) Half cross section



(b) Equivalent transmission line circuit for n odd



(c) Equivalent transmission line circuit for n even

Fig. 3 — Half waveguide cross section and equivalent transmission line circuits for transverse wave

The reflected impedance Z presented by a load impedance Z_L terminating a transmission line of characteristic impedance Z_0 with propagation constant γ and length w is [7]

$$Z = Z_0 \frac{(Z_L + Z_0)e^{\gamma w} + (Z_L - Z_0)e^{-\gamma w}}{(Z_L + Z_0)e^{\gamma w} - (Z_L - Z_0)e^{-\gamma w}} \quad (1)$$

The short circuit at y_1 in Fig. 3b will be reflected back to y_2 as Z_{1-2} where

$$Z_{1-2} = Z_{01} \tanh \left[\gamma_{y1} \frac{a-s}{2} \right] \quad (2)$$

or

$$Y_{1-2} = \gamma_{01} \coth \left[\gamma_{y1} \frac{a-s}{2} \right] \quad (3)$$

The open circuit at y_4 will reflect back to y_3 as Z_{4-3} with

$$Z_{4-3} = Z_{03} \coth \left[\gamma_{y3} \frac{l}{2} \right]. \quad (4)$$

Equation (1) can be rewritten in the form

$$Z = Z_0 \frac{Z_L \cosh \gamma w + Z_0 \sinh \gamma w}{Z_L \sinh \gamma w + Z_0 \cosh \gamma w}. \quad (5)$$

Since Z_{4-3} terminates region 2,

$$Z_{4-2} = Z_{02} \frac{Z_{4-3} \cosh \left[\gamma_{y2} \frac{s-l}{2} \right] + Z_{02} \sinh \left[\gamma_{y2} \frac{s-l}{2} \right]}{Z_{4-3} \sinh \left[\gamma_{y2} \frac{s-l}{2} \right] + Z_{02} \cosh \left[\gamma_{y2} \frac{s-l}{2} \right]}. \quad (6)$$

Using $\theta_i = \gamma_{yi} w_i$ to simplify notation and substituting Eq. (4) into Eq. (6) yields

$$Z_{4-2} = Z_{02} \frac{Z_{03} \coth \theta_3 \cosh \theta_2 + Z_{02} \sinh \theta_2}{Z_{03} \coth \theta_3 \sinh \theta_2 + Z_{02} \cosh \theta_2} \quad (7)$$

or

$$Y_{4-2} = Y_{02} \frac{Z_{03} \coth \theta_3 \sinh \theta_2 + Z_{02} \cosh \theta_2}{Z_{03} \coth \theta_3 \cosh \theta_2 + Z_{02} \sinh \theta_2}. \quad (8)$$

Since the sum of admittances at y_2 must equal zero,

$$Y_{1-2} + jB_c + Y_{4-2} = 0. \quad (9)$$

Substituting Eqs. (3) and (8) into Eq. (9) yields

$$Y_{01} \coth \theta_1 + jB_c + Y_{02} \frac{Z_{03} \coth \theta_3 \sinh \theta_2 + Z_{02} \cosh \theta_2}{Z_{03} \coth \theta_3 \cosh \theta_2 + Z_{02} \sinh \theta_2} = 0 \quad (10)$$

or

$$\coth \theta_1 + j \frac{B_c}{Y_{01}} + \frac{Y_{02}}{Y_{01}} \frac{\coth \theta_3 \sinh \theta_2 + \frac{Z_{02}}{Z_{03}} \cosh \theta_2}{\coth \theta_3 \cosh \theta_2 + \frac{Z_{02}}{Z_{03}} \sinh \theta_2} = 0. \quad (11)$$

Since region 1 and region 2 have the same propagation constant, $\gamma_{y1} = \gamma_{y2}$, the impedances are proportional to the heights:

$$\frac{Z_{02}}{Z_{01}} = \frac{Y_{01}}{Y_{02}} = \frac{d}{b}. \quad (12)$$

Regions 2 and 3 have equal heights, and since the transverse wave is *TE*, the impedance ratio is

$$\frac{Z_{02}}{Z_{03}} = \frac{\gamma_{y3}}{\gamma_{y2}}. \quad (13)$$

The left side of Eq. (11) may be rewritten as a single fraction. All terms in the denominator are finite, so the numerator may be equated to zero. The resultant expression is

$$\left[\frac{b}{d} \sinh \theta_1 \right] [\gamma_{y2} \cosh \theta_3 \sinh \theta_2 + \gamma_{y3} \sinh \theta_3 \cosh \theta_2] + \left[\cosh \theta_1 + j \frac{B_c}{Y_{01}} \sinh \theta_1 \right] \times [\gamma_{y2} \cosh \theta_3 \cosh \theta_2 + \gamma_{y3} \sinh \theta_3 \sinh \theta_2] = 0. \quad (14)$$

Within each region,

$$\gamma_{xi}^2 + \gamma_{yi}^2 + \gamma_{zi}^2 = -\omega^2 \mu_0 \epsilon_i \quad (15)$$

where

$$\begin{aligned} \epsilon_i &= \epsilon_0 \text{ for } i = 1, 2 \\ &= \epsilon_r \epsilon_0 \text{ for } i = 3. \end{aligned}$$

For *TE* modes, $\gamma_{xi} = 0$ for all regions and $\gamma_{zi} = j\beta$ for all regions; β is the longitudinal propagation constant (above cutoff) for the waveguide configuration. Substituting

$$\begin{aligned} \gamma_{yi} &= \sqrt{\beta^2 - \omega^2 \mu_0 \epsilon_i} \text{ for } \omega^2 \mu_0 \epsilon_i < \beta^2 \\ &= j\sqrt{\omega^2 \mu_0 \epsilon_i - \beta^2} \text{ for } \omega^2 \mu_0 \epsilon_i \geq \beta^2 \end{aligned} \quad (16)$$

and

$$\theta_i = \gamma_{yi} w_i$$

with

$$\begin{aligned} w_1 &= 1/2 (a - s) \\ w_2 &= 1/2 (s - t) \\ w_3 &= 1/2 t \end{aligned} \quad (17)$$

into Eq. (14) yields the transcendental equation in β that must be solved for *TE*_{*n*0} (*n* odd) modes. The smallest root of Eq. (14) is the *TE*₁₀ solution, the next root the *TE*₃₀ solution, etc.

For *TE*_{*n*0} (*n* even) modes, the analysis starts with the equivalent transmission line circuit of Fig. 3b and proceeds in a manner similar to the case for *n* odd. The resultant transcendental equation is

$$\left[\frac{b}{d} \sinh \theta_1 \right] [\gamma_{y2} \sinh \theta_3 \sinh \theta_2 + \gamma_{y3} \cosh \theta_3 \cosh \theta_2] + \left[\cosh \theta_1 + j \frac{B_c}{Y_{01}} \sinh \theta_1 \right] \times [\gamma_{y2} \sinh \theta_3 \cosh \theta_2 + \gamma_{y3} \cosh \theta_3 \sinh \theta_2] = 0 \quad (18)$$

with Eqs. (16) and (17) being applicable.

If $\epsilon_r = 1$, it is straightforward to show that Eqs. (14) and (18) reduce to the expressions for the odd and even mode cutoff frequencies, respectively, for double-ridged waveguide [1,2]. Also, if $b = d$, B_c equals zero and Eqs. (14) and (18) result in expressions for the odd- and even-mode propagation constants of dielectric slab center loaded rectangular waveguide identical to those obtained by use of ABCD matrices [6].

The discontinuity-susceptance term B_c/Y_{01} is obtained from the *Waveguide Handbook* [8]. Appendix A gives the necessary equations for calculating B_c/Y_{01} in terms of the waveguide dimensions (from Fig. 2) and the effective wavelength λ_g . Note that λ_g is the wavelength of the wave component which is incident normal to the height change. Therefore λ_g of Appendix A is the wavelength of the transverse wave in regions 1 and 2, namely λ_{y1} .

For standard (i.e. air filled) double-ridged waveguide $\epsilon_r = 1$; thus

$$\gamma_{y1} = \gamma_{y2} = \gamma_{y3} = j\beta_y$$

and $\lambda_{yi} = 2\pi/\beta_y$ is a real constant for a given configuration. However, for the general case $\epsilon_r > 1$ and

$$\gamma_{y1} = \gamma_{y2} \neq \gamma_{y3}$$

with the result that the values of γ_{yi} that satisfy Eq. (14) or (18), subject to (16) and (17), are no longer constant but depend on the frequency. This is of course to be expected, since for any nonhomogeneous waveguide a $1/\sqrt{1 - (f_{c/f})^2}$ term no longer describes the dispersive nature. However, there is another problem because of the inhomogeneity. At all frequencies above cutoff, the transverse wave propagation constant in the dielectric region will be entirely imaginary; i.e.,

$$\gamma_{y3} = j\beta_{y3} \text{ for } \omega > \omega_c.$$

However, there is a critical frequency ω_{crit} (ω_{crit} is greater than the cutoff frequency ω_c , how much greater depends upon the degree of dielectric loading) such that for frequencies greater than ω_{crit} the transverse propagation constant in regions 1 and 2 is real, that is,

$$\gamma_{y1} = \gamma_{y2} = \alpha_{y1} \text{ for } \omega > \omega_{crit}.$$

When $\omega > \omega_{crit}$, the transverse "wave" in these regions is no longer a resonant traveling wave but rather the fields are decaying exponentially away from the dielectric region, and the concept of wavelength in the region of the discontinuity is not meaningful. The expression for the B_c/Y_{01} term from Ref. 8 is no longer applicable; indeed, the validity of the equivalent transmission line circuit for the waveguide height change (a shunt susceptance at the junction of two transmission lines of unequal characteristic impedance) is questionable for operation below cutoff. Also, the calculation for B_c/Y_{01} is based on a model which assumes that the waveguide extends to infinity in both directions away from the height discontinuity; in practice, the assumption is valid if additional mismatches are far enough removed from the height discontinuity so that the local fields have decayed to small proportions. These local fields are the evanescent modes of the fringing fields caused by the height discontinuity, and they decay very rapidly.

Future investigation is planned to model an equivalent circuit of the waveguide height change to include operation below as well as above the cut-off frequency, and to include the proximity effects of waveguide walls and dielectric center loading. However, for this report the following two engineering assumptions are made:

1. The B_c/Y_{01} term can be neglected for frequencies below the critical frequency. Since

$$\frac{B_c}{Y_{01}} \rightarrow 0 \text{ as } \omega \rightarrow \omega_{crit}^{(+)}$$

and for $\omega < \omega_{crit}$ the fields of the transverse wave are decaying exponentially in the region of the height discontinuity, a small shunt susceptance term will have only a minor effect on the solution for β . Equations (14) and (18) are transcendental equations and must be solved by some algorithm using trial values of β . If a trial value of β yields an imaginary transverse propagation constant in region 1, the B_c/Y_{01} term is calculated with

$$\gamma_{y1} = j\beta_{y1} \text{ and } \lambda_{y1} = \frac{2\pi}{\beta_{y1}}$$

If the trial value of β results in γ_{y1} real, the B_c/Y_{01} term is neglected, i.e. set equal to zero, in Eqs. (14) and (18).

2. Proximity effects can be neglected in the calculation of B_c/Y_{01} .

Although the validity of these two assumptions may be questioned from a rigorous theoretical aspect, the close agreement between calculated and measured values of β for different configurations (shown in Figs. 4 and 5) indicates that both assumptions result in accuracy sufficient for most practical applications.

A listing of a computer program to solve for the principal (TE_{10}) mode propagation constant of dielectric slab center loaded double-ridged waveguide is given in Appendix B.

All discussions and calculations thus far have assumed a double-ridged waveguide configuration. For the asymmetric or single-ridged waveguide configurations shown in Fig. 6, Eqs. (14) and (18) remain valid; however, the expression for B_c/Y_{01} must have λ_{y1} replaced by $1/2 \lambda_{y1}$.

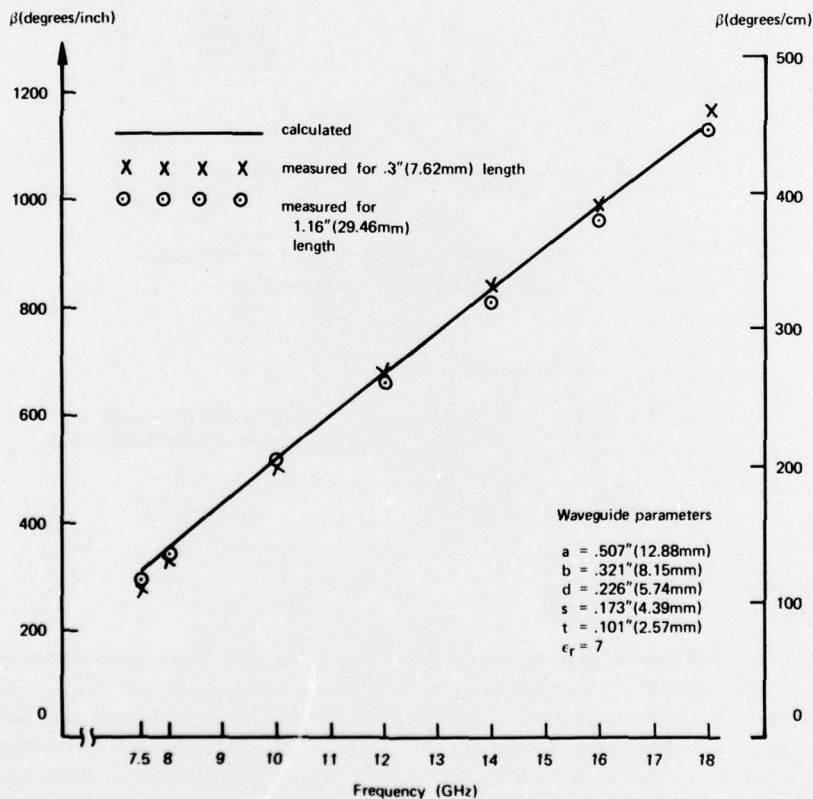


Fig. 4 - Calculated and measured values of β for dielectric slab loaded double-ridged waveguide

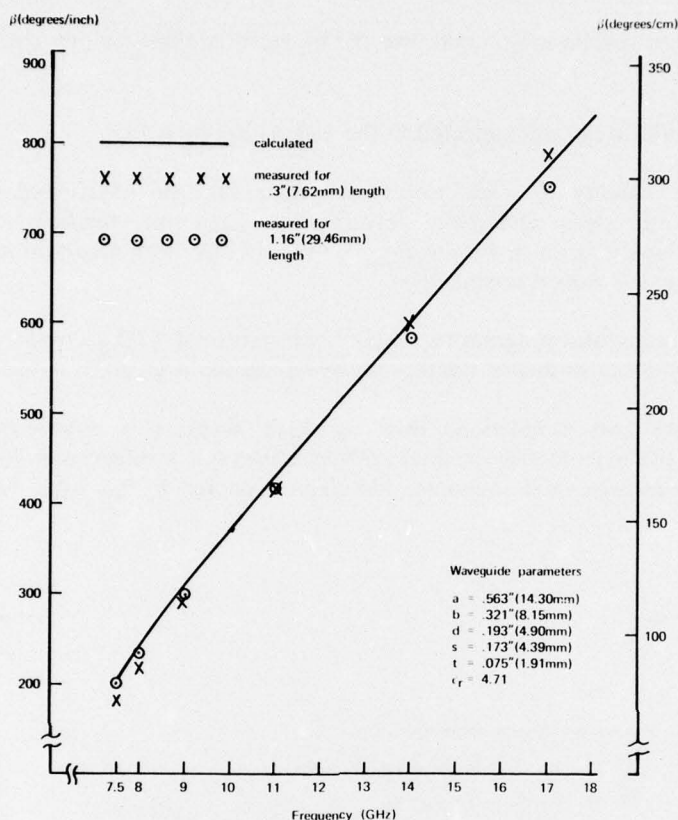


Fig. 5 — Calculated and measured values of β for dielectric slab loaded doubled-ridged waveguide

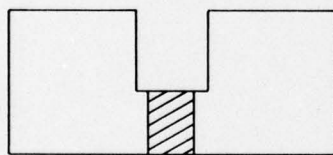


Fig. 6 — Cross section of dielectric slab center loaded single-ridged waveguide

CONCLUSION

Based on the equivalent transmission line circuit for the transverse component of the propagating electromagnetic wave, expressions have been derived for the TE_{n0} mode propagation constants of a dielectric slab center loaded ridged waveguide configuration. These expressions are transcendental equations involving the propagation constant, but they can readily be solved with a computer. Based on the agreement between calculated and measured data, certain assumptions made in the derivation appear valid. The analysis should prove useful in designing transformers to match ridged waveguide to dielectric or ferrite slab center loaded rectangular waveguide.

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Appendix A EQUIVALENT CIRCUIT FOR A CHANGE IN HEIGHT OF RECTANGULAR WAVEGUIDE

For a height change of rectangular waveguide as shown in Figs. A1a and A1b, the equivalent circuit given by the *Waveguide Handbook** is shown in Fig. A1c. The characteristic admittances of the different height waveguides are Y_0 and Y_0' , T is the effective terminal plane, B_c is the effective shunt capacitive susceptance, and λ_g is the wavelength of the propagating wave. The admittance ratio is

$$\frac{Y_0}{Y_0'} = \frac{d}{b} = \alpha$$

and at the terminal plane T

$$\frac{B_c}{Y_0} = \frac{2b}{\lambda_g} \left\{ \ln \left(\frac{1 - \alpha^2}{4\alpha} \right) + \frac{1}{2} \left(\alpha + \frac{1}{\alpha} \right) \ln \left(\frac{1 + \alpha}{1 - \alpha} \right) + 2 \frac{A + A' + 2C}{AA' - C^2} + \left(\frac{b}{4\lambda_g} \right)^2 \left(\frac{1 - \alpha}{1 + \alpha} \right)^{4\alpha} \left(\frac{5\alpha^2 - 1}{1 - \alpha^2} + \frac{4}{3} \frac{\alpha^2 C}{A} \right) \right\}$$

where

$$A = \left(\frac{1 + \alpha}{1 - \alpha} \right)^{2\alpha} \frac{1 + \sqrt{1 - \left(\frac{b}{\lambda_g} \right)^2}}{1 - \sqrt{1 - \left(\frac{b}{\lambda_g} \right)^2}} - \frac{1 + 3\alpha^2}{1 - \alpha^2}$$

$$A' = \left(\frac{1 + \alpha}{1 - \alpha} \right)^{2/\alpha} \frac{1 + \sqrt{1 - \left(\frac{d}{\lambda_g} \right)^2}}{1 - \sqrt{1 - \left(\frac{d}{\lambda_g} \right)^2}} + \frac{3 + \alpha^2}{1 - \alpha^2}$$

and

$$C = \left(\frac{4\alpha}{1 - \alpha^2} \right)^2$$

The equivalent circuit is valid for $b/\lambda_g < 1$.

*N. Marcuvitz, *Waveguide Handbook*, MIT Radiation Laboratory Series, McGraw-Hill, New York, 1951.

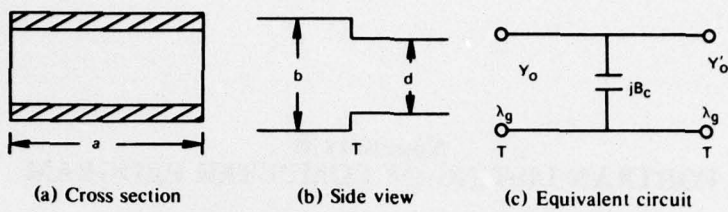


Fig. A-1 — Height change of rectangular waveguide and equivalent circuit

Appendix B
FORTRAN LISTING OF COMPUTER PROGRAM

```

00100 C THIS IS PROGRAM DRWGDL.FOR - CWY -OCT 75
00200 INTEGER RIK
00300 REAL KXAIR
00400 PI=3.1415927
00500 C=2.997925E+08
00600 R1=39.37008
00700 R2=2.0*R1
00800 RRMDI=180.0/(PI*R1)
00900 C1=(2.0E+09*PI/C)**2
01000 NEWRUN=0
01100 TYPE 600
01200 600 FORMAT (//// PROGRAM DRWGDL/CWY/OCT 75/// COMPUTES
01300 1 TE10 CUTOFF FREQUENCIES AND PROPAGATION CONSTANTS OF//
01400 2 / SYMMETRIC DIELECTRIC LOADED DOUBLE RIDGED WAVEGUIDE/)
01500 105 TYPE 605
01600 605 FORMAT(/// WAVEGUIDE DIMENSIONS IN INCHES - A,B,D,S: '$)
01700 READ(5,*)A,B,D,S
01800 106 TYPE 615
01900 615 FORMAT (// RELATIVE DIELECTRIC CONSTANT OF CENTER
02000 1 LOADING = '$)
02100 READ(5,*)EPSR
02200 TYPE 625
02300 625 FORMAT (// WIDTH IN INCHES OF CENTER LOADING = '$)
02400 ACCEPT 630,T
02500 630 FORMAT (F8.3)
02600 IF(T.LT.S)GO TO 108
02700 TYPE 631
02800 631 FORMAT (// DIELECTRIC WIDTH MUST BE LESS THAN
02900 1 RIDGE WIDTH ---- TRY AGAIN/)
03000 GO TO 105
03100 108 TYPE 606
03200 606 FORMAT (/// DRWGDL PARAMETERS ----- DIMENSIONS IN
03300 1 INCHES//8X/ A/9X/B/9X/D/9X/S/12X/T/6X4HEPS//1X$)
03400 TYPE 607,A,B,D,S,T,EPSP
03500 607 FORMAT (4F10.4,F13.4,F10.3)
03600 R=D/B
03700 RS=R**2
03800 IFR=0
03900 IF (ABS(R-1.0).LT.1.0E-06) IFR=1
04000 W1=(A-S)/R2
04100 W2=(S-T)/R2
04200 W3=T/R2
04300 CEREST=1.+(1./R-1.)*COS(PI*(A-S)/(2.*A))
04400 CLREST=CEREST+(EPSR-1.)/R*COS(PI*(A-T)/(2.*A))
04500 EDCTRY=CLREST/CEREST
04600 ALCEST=A*CLREST*(R+(1.-R)*SIN(PI*(A-S)/(2.*A)))
04700 C THE ABOVE FOUR QUANTITIES ARE TO BE USED FOR CALCULATING
04800 C APPROXIMATE (STARTING VALUES) OF CUTOFF FREQUENCIES AND
04900 C PROPAGATION CONSTANTS
05000 IBC=1

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05100      FREQ=C*R1/(ALCEST*2.0E+09)
05200      DELBY=0.31*FREQ
05300      BY=0.0
05400      GO TO 112
05500  109   IF (NEWRUN.LT.2) GO TO 210
05600      IF (FSTART.GT.FCGHZ) GO TO 220
05700  210   TYPE 635
05800  635   FORMAT (// FREQUENCIES IN GHZ - START, STOP, INCREMENT: ($)
05900      READ (5, *) FSTART, FSTOP, DELF
06000  640   FORMAT (F9.3, 1X, F9.3, 1X, F9.3)
06100  220   IF (FSTART.LT.1.0E-13) GO TO 180
06200      IF (FSTART.GT.FCGHZ) GO TO 230
06300      TYPE 645
06400  645   FORMAT (// FREQUENCY MUST BE GREATER THAN CUTOFF)
06500      GO TO 210
06600      IF (FSTOP.LT.1.0E-13) FSTOP=FSTART-1.0
06700      TYPE 655
06800  655   FORMAT (//4X4HFREQ08X4HBETA9X3HGWL7X5HRATIO8X5HKXAIR/
06900  1 5X3HGHZ6X6HDEG/IN6X6INCHES4X8HGWL/FSWL7X6HR OR I/)
07000  110   IFREQ=0
07100      FREQ=FSTART
07200  111   IFREQ=IFREQ+1
07300      BY=PI*2.E+09/C*SQRT (EDCTRY*(FREQ**2+FCGHZ**2))
07400  C THIS IS A FIRST TRY FOR BETA
07500      DELBY=-0.31*BY
07600  112   ICROSS=0
07700      ITAN=0
07800      IBTRY=0
07900  115   C1F=C1*FREQ**2
08000      C1FEP=C1F*EPSR
08100  120   IBTRY=IBTRY+1
08200      IF (IBTRY.LT.26) GO TO 122
08300      TYPE 705
08400  705   FORMAT (// MORE THAN 25 TRIES AT ROOT)
08500      GO TO 170
08600  122   BYSQ=BY**2
08700      GX3SQ=C1FEP-BYSQ
08800      GX2SQ=C1F-BYSQ
08900      GX3=SQRT (ABS (GX3SQ))
09000      GX2=SQRT (ABS (GX2SQ))
09100      IF (GX3SQ) 130, 132, 132
09200  130   CHS3=SINH (GX3*M3)
09300      CHC3=COSH (GX3*M3)
09400      IRGX3=1
09500      GO TO 134
09600  132   CHS3=SIN (GX3*M3)
09700      CHC3=COS (GX3*M3)
09800      IRGX3=-1
09900  134   CONTINUE
10000      IF (GX2SQ) 136, 138, 138

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10100 136  CHS2=SINH(GX2*W2)
10200      CHC2=COSH(GX2*W2)
10300      CHS1=SINH(GX2*W1)
10400      CHC1=COSH(GX2*W1)
10500      KXAIR=GX2
10600      RIK=1HR
10700      IRGX2=1
10800      GO TO 140
10900 138  CHS2=SIN(GX2*W2)
11000      CHC2=COS(GX2*W2)
11100      CHS1=SIN(GX2*W1)
11200      CHC1=COS(GX2*W1)
11300      IRGX2=-1
11400      KXAIR=GX2*PRMDI
11500      RIK=1HI
11600 140  B0Y=0.0
11700      IF(IFR.EQ.1)GO TO 153
11800      IF(IRGX2.EQ.1)GO TO 153
11900  C CALCULATE B/Y TERM
12000      P=(1+R)/(1-R)
12100      GL=2.0*PI/GX2
12200      P2=SQRT(1.0-(B/(R1*GL))**2)
12300      P3=SQRT(1.0-(D/(R1*GL))**2)
12400      PA=P**2*(2.0*R)*(1.0+P2)/(1.0-P2)-(1.0+3.0*RS)/(1.0-RS)
12500      PAP=P**2*(2.0/R)*(1.0+P3)/(1.0-P3)+(3.0+RS)/(1.0-RS)
12600      PC=((4.0*R)/(1.0-RS))**2
12700      PT1=ALOG((1.0-RS)/(4.0*R)*P**2*(0.5*(R+1.0/R)))
12800      PT2=2.0*(PA+PAP+2.0*PC)/(PA+PAP-PC**2)
12900      PT3=(B/(R1*4.0*GL))**2*(1.0/P)**2*(4.0*R)**2*(5.0*RS
13000 1 -1.0)/(1.0-RS)+4.0*RS*PC/(3.0*PA)**2
13100      B0Y=2.0*B*(PT1+PT2+PT3)/(R1*GL)
13200  C CALCULATE F (BETA)
13300 153  FBETA=R*(-B0Y*CHS1+CHC1)*(GX2/GX3*CHC3*CHC2
13400 1 +IRGX3*CHS3*CHS2)+IRGX2*CHS1*(GX2/GX3*CHC3*CHS2
13500 2 +IRGX2*IRGX3*CHS3*CHC2)
13600      IF(IBC.EQ.1)BY=FREQ
13700  C ROOT SEARCH ROUTINE
13800      IF(ABS(FBETA).LT.1.0E-08)GO TO 170
13900      IF(IBTRY.EQ.1)GO TO 163
14000      IF(ITAN.EQ.1)GO TO 164
14100      IF(FBETA*FBOLD.LT.0.0)GO TO 161
14200      IF(ABS(FBETA).GT.ABS(FBOLD))DELBY=-DELBY
14300      GO TO 162
14400 161  DELBY=-DELBY
14500      ICROSS=1
14600 162  IF(ICROSS.EQ.1)DELBY=0.5*DELBY
14700 163  BYNEW=BY+DELBY
14800      IF(ABS((BY-BYNEW)/BY).LT.0.1)ITAN=1
14900      GO TO 166
15000 164  IF(ABS(BY-BYOLD).LT.1.E-05.AND.FBETA.LT.1.E-06)GO TO 170

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15100          BYNEW=BY-FBETA*(BY-BYOLD)/(FBETA-FBOLD)
15200 166      BYOLD=BY
15300          BY=BYNEW
15400          FBOLD=FBETA
15500          IF(IBC.EQ.2)GO TO 120
15600          FREQ=BY
15700          BY=0.0
15800          GO TO 115
15900 170      IF(IBC.EQ.2)GO TO 175
16000          FCGHZ=BY
16100          TYPE 658,FCGHZ,BOY
16200 658      FORMAT(// TE10 MODE CUTOFF FREQUENCY IN GHZ = 'F7.4'
16300 1         B/Y = 'F7.3)
16400          IBC=2
16500          GO TO 109
16600 173      CONTINUE
16700 175      BYDI=BY*RRMDI
16800          GML=360.0/BYDI
16900          FSWL=R1*C/(FREQ*1.0E+09)
17000          RGLFS=GML/FSWL
17100 177      TYPE 660,FREQ,BYDI,GML,RGLFS,KXAIR,RIK
17200 660      FORMAT (1X,F7.3,3X,F9.2,3X,F9.4,4X,F8.4,3X,F8.2,1X,A1)
17300          IF(FREQ.GE.FSTOP)GO TO 180
17400          FREQ=FREQ+DELF
17500          GO TO 111
17600 180      TYPE 665
17700 665      FORMAT (/// WISH NEW PARAMETERS? NONE=0, ALL=1,
17800 1 CENTER LOADING=2, FREQ=3      ($)
17900          ACCEPT 670,NEWRUN
18000 670      FORMAT (I1)
18100          GO TO(199,105,106,210,180)NEWRUN+1
18200 199      CONTINUE
18300          END

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