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APPLICATION OF THE RESPONSE PROBABILITY DENSITY
FUNCTION TECHNIQUE TO BIODYNAMIC MODELS

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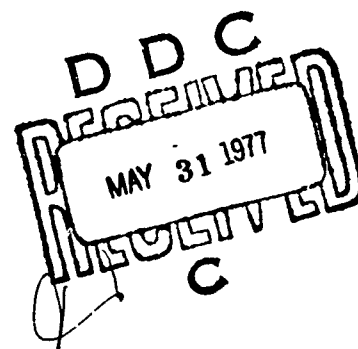
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FEDERAL AVIATION ADMINISTRATION



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Final Report



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16. Abstract A method has been developed, which we call the "response probability density function technique," which has applications to predicting probability of injury in a wide range of biodynamic situations. The method, which was developed in connection with sonic boom damage prediction, utilizes the probability density function of the excitation force and the probability density function of the sensitivity of the material being acted upon. The method is especially simple to use when both these probability density functions are lognormal. Studies thus far have shown that the stresses from sonic booms, as well as the strengths of glass and mortars, are distributed lognormally. Some biodynamic processes also have lognormal distributions, and are therefore amenable to modeling by this technique. In particular this paper discusses the application of the response probability density function technique to the analysis of the thoracic response to air blast and the prediction of skull fracture from head impact.					
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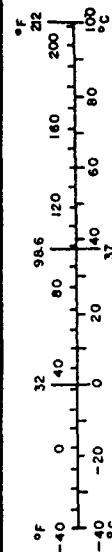
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METRIC CONVERSION FACTORS

Approximate Conversions to Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
LENGTH				
in	inches	2.5	centimeters	cm
ft	feet	30	meters	m
yd	yards	0.9	kilometers	km
mi	miles	1.6		
AREA				
in ²	square inches	6.5	square centimeters	cm ²
ft ²	square feet	0.09	square meters	m ²
yd ²	square yards	0.8	square meters	m ²
mi ²	square miles	2.6	square kilometers	km ²
	acres	0.4	hectares	ha
MASS (weight)				
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2000 lb)	0.9	tonnes	t
VOLUME				
teaspoon	teaspoons	5	milliliters	ml
fl oz	fluid ounces	15	milliliters	ml
c	cups	30	milliliters	ml
pt	pints	0.24	liters	l
qt	quarts	0.47	liters	l
gal	gallons	0.95	liters	l
ft ³	cubic feet	3.8	liters	l
yd ³	cubic yards	0.03	cubic meters	m ³
		0.76	cubic meters	m ³
TEMPERATURE (exact)				
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C

Symbol	When You Know	Multiply by	To Find	Symbol
LENGTH				
mm	millimeters	0.04	inches	in
cm	centimeters	0.4	inches	in
m	meters	3.3	feet	ft
km	kilometers	1.1	yards	yd
		0.6	miles	mi
AREA				
cm ²	square centimeters	0.16	square inches	in ²
m ²	square meters	1.2	square yards	yd ²
km ²	square kilometers	0.4	square miles	mi ²
ha	hectares (10,000 m ²)	2.5	acres	
MASS (weight)				
g	grams	0.035	ounces	oz
kg	kilograms	2.2	pounds	lb
t	tonnes (1000 kg)	1.1	short tons	
VOLUME				
ml	milliliters	0.03	fluid ounces	fl oz
l	liters	2.1	pints	pt
l	liters	1.06	quarts	qt
l	liters	0.26	gallons	gal
m ³	cubic meters	35	cubic feet	ft ³
m ³	cubic meters	1.3	cubic yards	yd ³
TEMPERATURE (exact)				
°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature	°F



*In 1.2.54 (reaching) for other exact conversions and more detailed tables, see NBS Misc. Publ. 286, Units of Weights and Measures, Price \$2.25. SD Catalog No. C13 10 286

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INTRODUCTION

The practical prediction of the effects on man of various hostile environments involves a statistical approach. The large variation of human susceptibilities and the wide range of exposures dictate that predictions be quoted on the basis of probability of injury, rather than as flat statements of certainty. This paper suggests that a technique which has been used to good advantage in finding the probability of damage to structures (1-6) can also be used for biodynamic prediction.

The method which we call the response probability density function technique utilizes the probability density function (pdf) of the excitation from the environment and the pdf of the sensitivity of man. In cases where these pdf's are log-normal (the distribution of the logarithms is the familiar bell curve), then the results come out particularly simple. In discussing the use of the method, we will first go through the equations of the step-by-step procedure involved. We will then illustrate how the technique was used for analysis of structural damage from sonic boom. Finally we will extend the technique to biodynamic uses, discussing its application to biodynamic situations, particularly blast lethality and skull fracture.

RESPONSE PDF TECHNIQUE

Use of the response pdf technique involves the following steps:

1. Express the response R , which is being sought, as the quotient of a sensitivity s and an excitation e .

$$R = s/e \quad [1]$$

2. Express both the excitation and the sensitivity as the product of statistically independent factors.

$$e = \prod_{i=1}^m e_i \quad [2]$$

$$s = \prod_{j=1}^n s_j \quad [3]$$

3. Then the logarithm of the excitation and sensitivity are the sums of the logarithms of their respective factors.

$$\log_{10} e = \sum_{i=1}^m \log_{10} e_i \quad [4]$$

$$\log_{10} s = \sum_{j=1}^n \log_{10} s_j \quad [5]$$

4. Sample each of the factors and take the logarithm of each reading. Verify that to a reasonable approximation the distribution of the logarithms is normal

and, hence, the distribution of the original factors is lognormal. Deterministic factors have delta functions as their pdf's and do not affect the shape of the combined pdf.

5. Compute the mean and variance for each distribution of the logarithm of a factor.
6. Compute the mean of the logarithm of the response as the difference of the sensitivity factor logarithm means less the excitation factor logarithm means.

$$E (\log_{10} R) = E \left[\log_{10} (s/e) \right] \quad [6]$$

$$E (\log_{10} R) = \sum_{j=1}^n E(\log_{10} s_j) - \sum_{i=1}^m E(\log_{10} e_i) \quad [7]$$

7. Compute the variance of the logarithm of the response as the sum of the variances of the logarithms of the excitation and sensitivity factors.

$$\text{Var} (\log_{10} R) = \sum_{i=1}^m \text{Var} (\log_{10} e_i) + \sum_{j=1}^n \text{Var} (\log_{10} s_j) \quad [8]$$

8. Find the resulting probability for the normal pdf of $\log_{10} R$ from a standard table from

$$z = E (\log_{10} R) / \left[\text{Var} (\log_{10} R) \right]^{1/2} \quad [9]$$

SONIC BOOM STRUCTURAL DAMAGE

The response probability density function technique has been used for predicting sonic boom structural damage, where R is an effective factor of safety N_e , s is a material strength σ_G , and e is an imposed stress σ_m .

For this case Equation [1] takes the form

$$N_e = \sigma_G / \sigma_m \quad [10]$$

Correlation analysis from sonic boom test data showed that the stress could be expressed as the following product,

$$\sigma_m = p_o (p_f/p_o) (p_e/p_f) F(DAF) \quad [11]$$

where p_o is a calculated sonic boom overpressure, p_f is an overpressure measured in the field, p_e is an overpressure measured outside a structure, F is a stress factor, and DAF is the dynamic amplification factor. The form of Equation [11] was chosen because it expresses σ_m as a product of calculated values and ratios of measured values. The nominal pressure p_o is calculated from the aircraft parameters and F is a calculated value based on the geometry of the structural element being subjected to sonic boom loading; the other expressions in parentheses are measured values.

The sensitivity represented by the strength of the material is likewise represented by a similar equation

$$\sigma_G = p_G F \quad [12]$$

where p_G is the breaking pressure.

Taking logarithms of Equations [11] and [12] as indicated in Equations [4] and [5] we obtain:

$$\log_{10}\sigma_m = \log_{10}p_o + \log_{10}(p_f/p_o) + \log_{10}(p_e/p_f) \quad [13]$$

$$+ \log_{10}F + \log_{10}(DAF),$$

$$\log_{10}\sigma_G = \log_{10}F + \log_{10}p_G. \quad [14]$$

The factors of the above equations were sampled and the logarithms shown indeed had normal pdf's. The mean and variance of each factor was then determined. By Equation [7] and [8] we thus have

$$E \log_{10}N_e = E \log_{10}p_G - \log_{10}p_o - E \log_{10}(p_f/p_o) \quad [15]$$

$$- E \log_{10}(p_e/p_f) - E \log_{10}(DAF) .$$

and

$$\text{Var}(\log_{10}N_e) = \text{Var} \log_{10}(p_f/p_o) + \text{Var} \log_{10}(p_e/p_f) \quad [16]$$

$$= \text{Var} \log_{10}(DAF) + \text{Var} \log_{10}(p_G)$$

The pdf's of the terms involved are illustrated in Figure 1.

Then finding the parameter z from Equation [9]

$$z = E(\log_{10}N_e) \left[\text{Var}(\log_{10}N_e) \right]^{-1/2} \quad [17]$$

we can look up the probability of breakage from a table of the normal probability density function. Looking up z in the table corresponds to finding the area shown in Figure 2. The

shaded area shown to the left of zero corresponds to a negative logarithm, indicating the strength is less than the stress, and the structural element fails.

Using the above analysis the effects of sonic boom on glass, plaster, brick, and bric-a-brac were calculated and probability of breakage was plotted against p_0 . The results for various window configurations are shown in Figure 3 and the breakage probability curves of various materials are combined in Figure 4.

AIR BLAST EXPOSURE

Exposure to air blast can occur as the result of detonation of an explosive. In such cases the most susceptible part of the human body is the lung, which is damaged by the sharp rise in pressure. Shardin, in the 1940's (7) hypothesized that there might be similar damage patterns for windows exposed to blast and for mammals exposed to blast. Indeed in the 1960's Bowen and coworkers at the Lovelace Foundation modeled the thorax as a resonant cavity with appropriate masses and stiffnesses (8). This model explained the effects of varying the duration of the blast which had been observed; namely at short durations the response varied with the impulse while at long durations the response varied directly with the overpressure. This effect is expected under the dynamic loading characteristics of the thorax-abdomen system, which has a resonance of close to 60 Hz.

Bowen and coworkers performed extensive experiments on animal sensitivity to blast using shock tubes and concluded that the air blast severity s_a varied with the reflected overpressure p_r (psi) and the duration T (msec) according to the expression

$$s_a = \frac{p_r}{61.5 + 415.7T^{-1.064}} \quad [18]$$

Note from this expression that for long durations the second term in parentheses will become very small and s_a will be directly proportional to overpressure. For short durations, however, the second term in parentheses will dominate and s_a will be dependent on impulse. Equation [18] merely reflects the fact that man's lungs behave as a resonant system and act differently for excitations above and below resonance.

Another important observation by Bowen et al was that the pdf's of the Equation [18] values to produce animal mortality were lognormal. Thus it is apparent that the response pdf technique can be used to predict lethal probabilities from air blast. Indeed when Bowen's mortality curves are plotted versus his parameter on lognormal paper as shown in Figure 5 they plot as straight lines.

For the case of long duration blast Equation [18] takes the form

$$s_a = F_\ell p_\ell \quad [19]$$

where F_ℓ is a constant applying to long duration blasts and p_ℓ is the overpressure for long duration blast mortality. Note that this is exactly the same form as Equation [12] above.

For comparison purpose we have plotted mortality curves for man on the same plot as breakage curves for windows (see Figure 6). Note that the prediction curves for windows slope upward more sharply than the curves for man. This is because the window breakage model also takes into account variations in sonic boom overpressure due to the attenuation of the sonic boom in the atmosphere and reflections from buildings and other obstructions, while the data on man comes from extrapolations of controlled laboratory experiments. Thus we will expect the higher standard deviation on the windows which can be observed in Figure 6. To account for these additional sources of variation an excitation equation like Equation [11] must be used for blast excitations of man. The equation presented below incorporates the propagation of the blast wave

$$e_a = p_o \left(\frac{p_f}{p_o} \right) \left(\frac{p_r}{p_f} \right) \quad [20]$$

In this expression p_f represents the measured field overpressure and p_o the predicted overpressure based on blast theory. The expression for the response then becomes

$$R_a = F_\ell p_\ell / p_o \left(\frac{p_f}{p_o} \right) \left(\frac{p_r}{p_f} \right) \quad [21]$$

Using this form with the response pdf technique will result in a greater predicted variation in the effects on man for a given predicted blast overpressure p_o . This will account for persons who suffer the effects of blast focusing by being located near a corner or those who are shielded from the blast by being located behind an obstacle. The net effect will be to make the human blast effect curves in Figure 8 slope upward more sharply, so that they are more parallel to the window curves.

For the case of short duration air blasts the excitation becomes an impulse rather than just an overpressure and the equations become those shown below:

$$s_a = F_s p_s T_s \quad [22]$$

$$e_a = p_o t_o \frac{p_f}{p_o} \frac{t_f}{t_o} \frac{p_r}{p_f} \frac{T}{t_f} \quad [23]$$

$$R_a = \frac{F_s p_s T_s}{p_o t_o \left(\frac{p_f}{p_o} \right) \left(\frac{t_f}{t_o} \right) \left(\frac{p_r}{p_f} \right) \left(\frac{T}{t_f} \right)} \quad [24]$$

where F_s is a constant sensitivity factor for short duration blasts, p_s is the overpressure in the laboratory situation, T_s is the duration in the laboratory situation, t_o is the predicted duration, t_f is the duration measured in the field, and T is the duration measured at the man.

Using forms such as Equation [21] and Equation [24] requires knowledge of the distribution of overpressures and durations of blast waves, so that lognormality can be confirmed and the means and variances calculated. As of this writing such distribution data has not yet become available, but we would expect the distributions would look very much like those for sonic booms.

SKULL FRACTURE

In recent years there has been much research on skull fracture, particularly in relation to automobile accidents. The typical use which has been modeled is that of a driver's head striking the windshield with a given amount of acceleration. Research with cadavers and dummies has indicated that both the acceleration and its duration are key parameters. Gadd observed that the best fit of data to a straight-line

injury tolerance curve is normally obtained if the loading as well as time duration are plotted logarithmically (10). This tends to indicate that like the cases of sonic booms on windows and air blast injury, the case of skull fracture is lognormal, and hence solvable by the response pdf technique.

Recently the National Highway Traffic Safety Administration has promulgated Motor Vehicle Safety Standard 209, which provides an acceleration severity index for scaling the danger of head injury. This index, which we designate here as s_h , depends on the time history of the acceleration as follows:

$$s_h = \left[\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} a dt \right]^{2.5} (t_2 - t_1) \quad [25]$$

In the above expression, a is the resultant acceleration expressed as a multiple of g and t_1 and t_2 are two points in time during the crash. The regulation specifies that in crash tests with dummies the above expression shall not exceed 1000 where a is measured at the center of gravity of the head.

Note that for the case of constant acceleration Equation [25] reduces to

$$s_h = a^{2.5} (t_2 - t_1) \quad [26]$$

If we designate $(t_2 - t_1)$ by T_c , the critical acceleration for skull fracture by a_c , and a proportionality constant by F_h we have

$$s_h = F_h T_c a_c^{2.5} \quad [27]$$

as the severity index at fracture. Similarly for the excitation, we can come up with a form similar to Equation [24]

$$e_h = a_o^{2.5} t_o \left(\frac{a_f}{a_o} \right)^{2.5} \left(\frac{t_f}{t_o} \right) \left(\frac{a_h}{a_f} \right)^{2.5} \left(\frac{t_h}{t_f} \right) \quad [28]$$

where a_o is the predicted acceleration, t_o is the predicted duration, a_f is the measured acceleration of the vehicle in the field, t_f is the measured duration in the field, a_h is the measured acceleration at the head, and t_h is the measured duration at the head.

It must be emphasized that the possible use of the response pdf technique for head injury prediction is merely a hypothesis at this point. However, if all the random variable factors in Equations [27] and [28] turned out to be lognormal

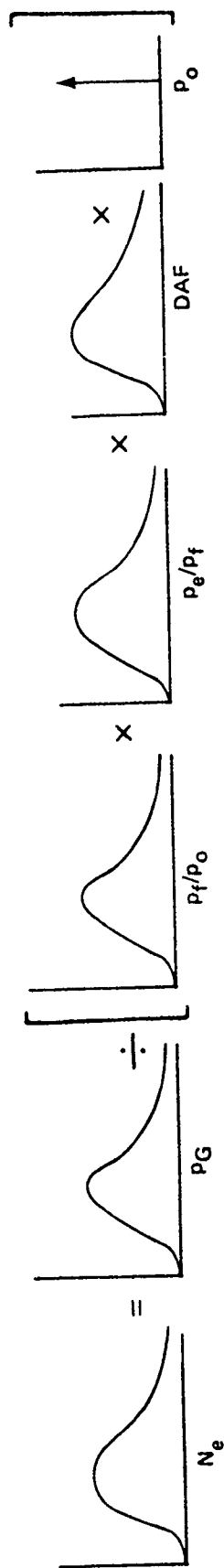
then the response

$$R_h = \frac{F_h T_c a_c^{2.5}}{a_o^{2.5} t_o \left(\frac{a_f}{a_o} \right)^{2.5} \left(\frac{t_f}{t_o} \right) \left(\frac{a_h}{a_f} \right)^{2.5} \left(\frac{t_h}{t_f} \right)} \quad [29]$$

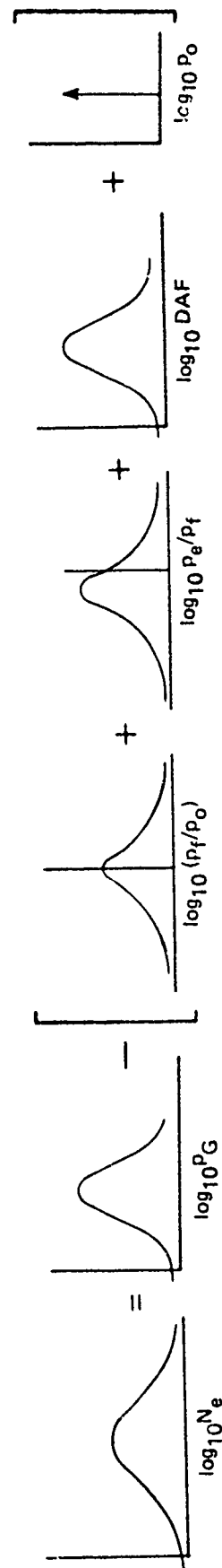
can be used for determining the probability of skull fracture in a given accident situation. There is, however, no available data to indicate whether the pdf's are indeed lognormal. This represents an area for possible further research.

SUMMARY

The use of the response pdf technique for biodynamic models has been discussed. The technique can be used directly in predicting the probability of lethality for air blast exposure, since the pdf of lethal overpressures is lognormal. Use of the response pdf technique for other injury predictions, such as skull fracture in automobile accidents, seems promising, if sufficient data to establish lognormality can be obtained.



(a) N_e is the product of lognormal random variables



(b) $\log_{10} N_e$ is the sum of normal random variables

Figure 1. Development of Probability Density Function of $\log_{10} N_e$

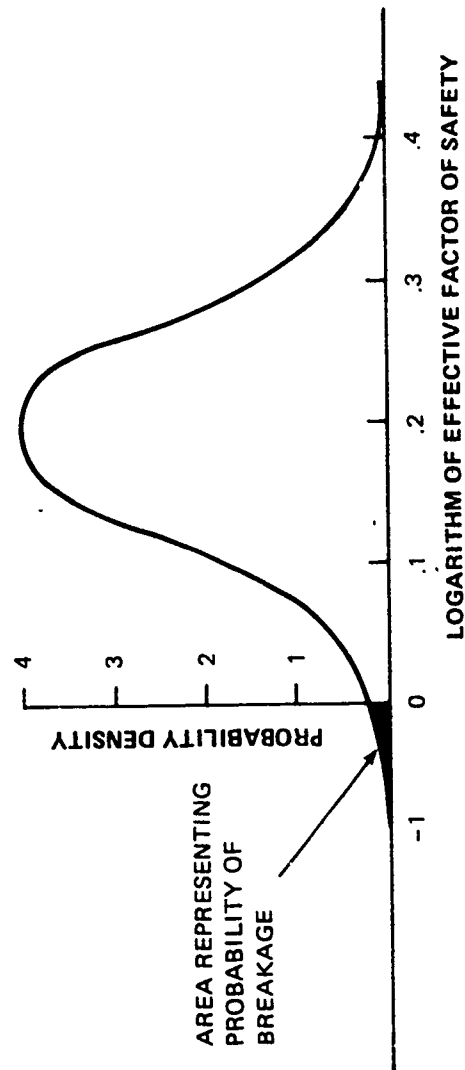


Figure 2. Probability Density Function of the Logarithm of the Effective Factor of Safety

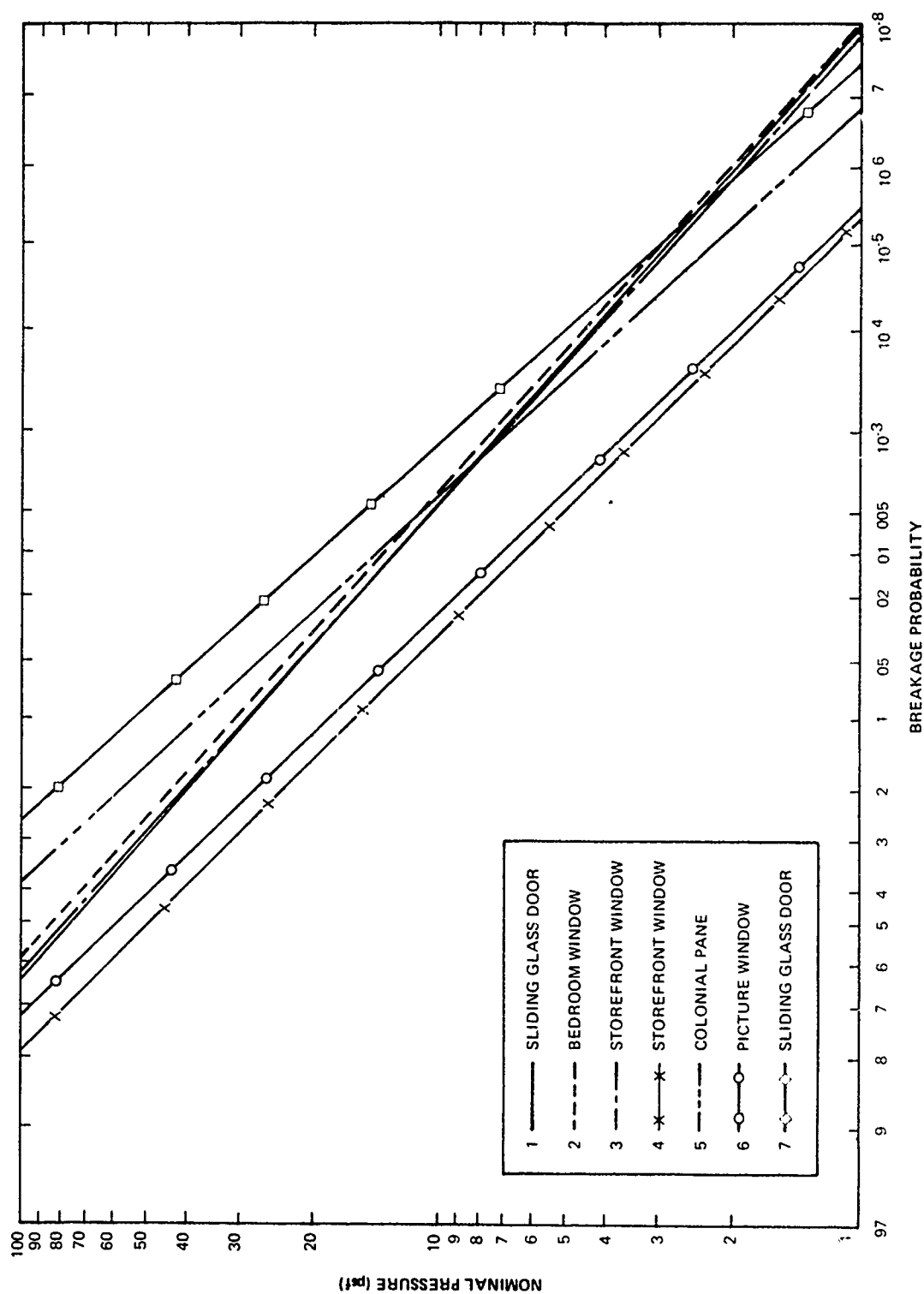


Figure 3. Breakage Probabilities for Windows of Various Types

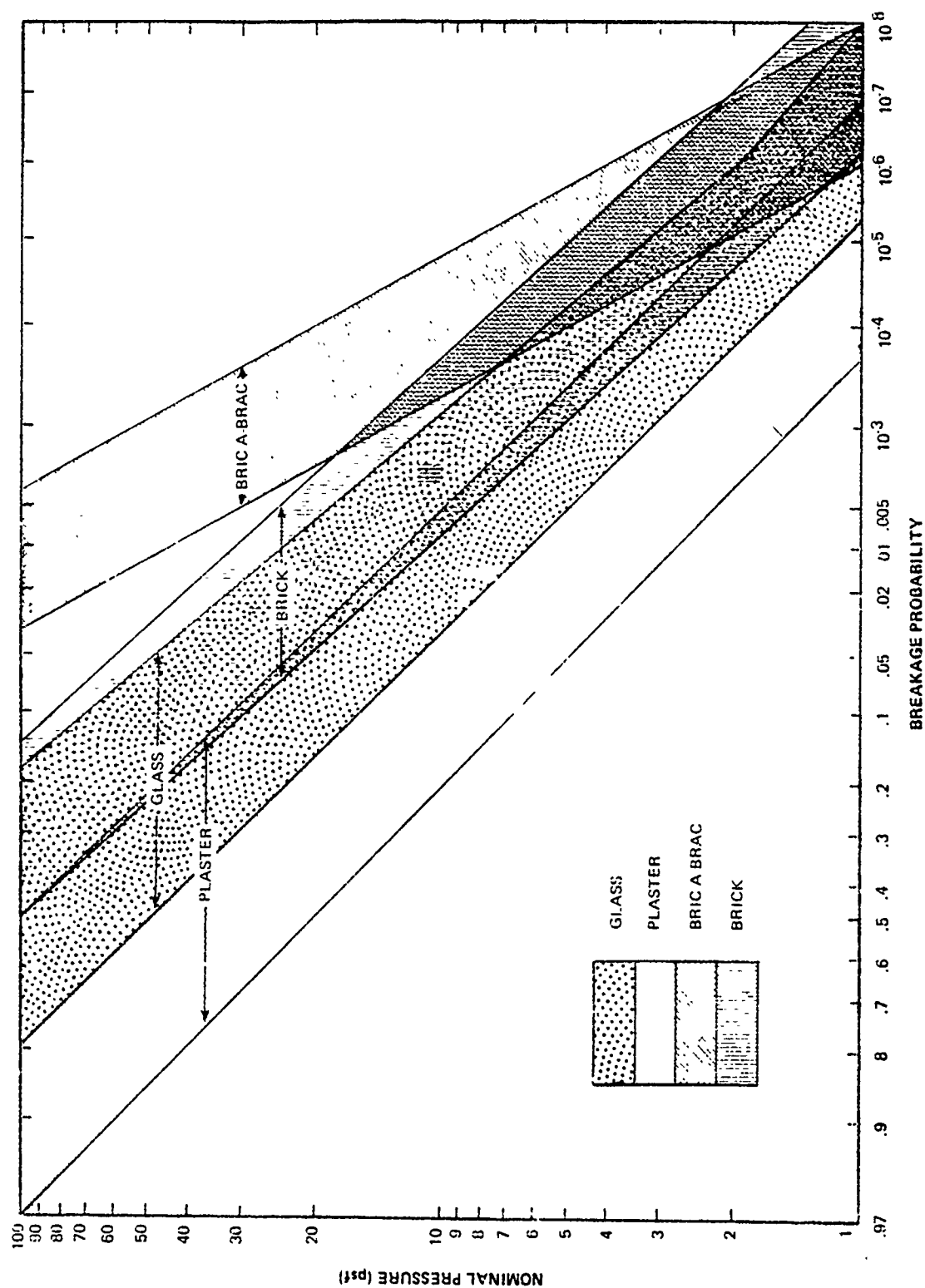


Figure 4. Breakage Probabilities for Glass, Plaster, Bric-a-Brac, and Free-Standing Brick Walls

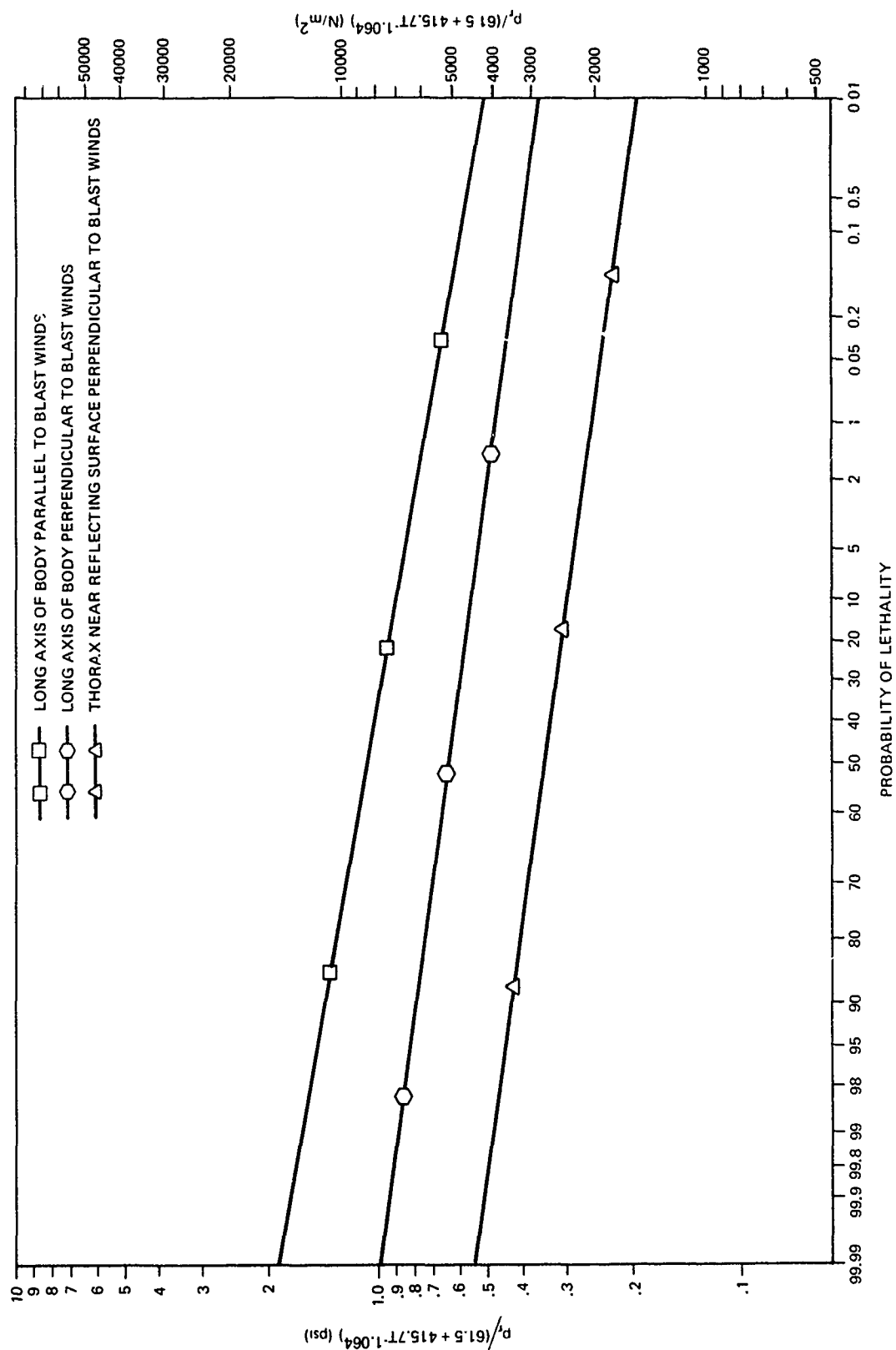


Figure. 5. Probability of Lethality vs Air Blast Severity

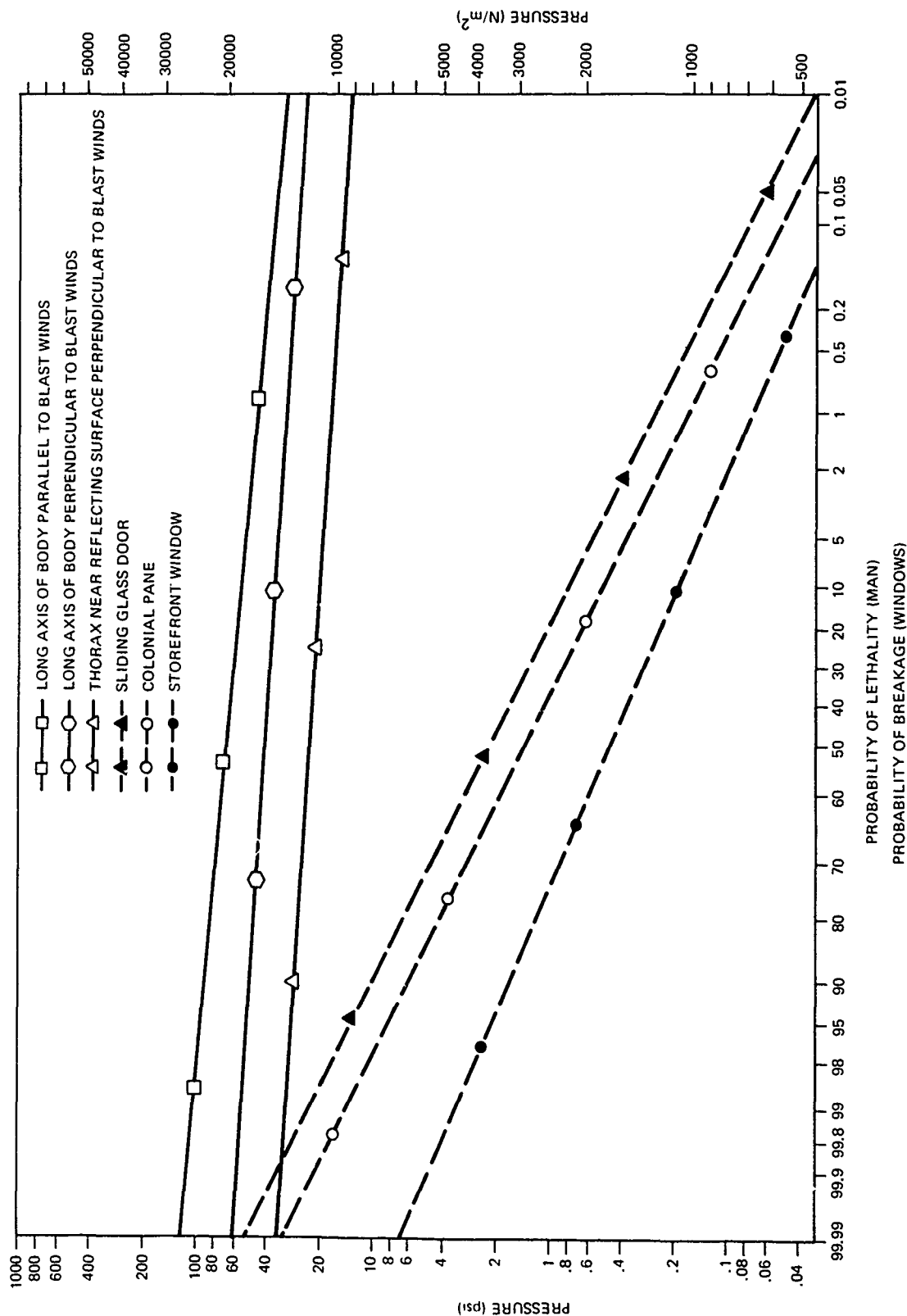


Figure 6. Probability of Lethality for Man Exposed to Air Blast and Probability of Breakage for Windows Exposed to Sonic Boom as a Function of Overpressure

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