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SOLAR ILLUMINATION CALCULATOR

by

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A model of the earth's orbital relationship to the sun is developed. Equations for computing the solar declination, the equation of time, sunrise, sunset, the starting and ending times of the twilights, and the solar elevation and azimuth angles are developed. An explanation of the meaning of the cosine of the Greenwich Hour Angle exceeding ± 1.0 is given. The accuracy of the equations is discussed. The equations presented in this report were incorporated into a computer program written at USAFETAC for use by the World Wide Military Command and Control System (WWMCCS).

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Preface

USAFETAC prepared this report to document the Solar Illumination Calculator (SIC) computer program developed by USAFETAC and currently being used by USAFETAC and the Prototype World Wide Military Command and Control System (WWMCCS) Intercomputer Network (PWIN). This report documents the equations used by the SIC.

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SOLAR ILLUMINATION CALCULATOR

Introduction

The basic assumption used in writing the Solar Illumination Calculator (SIC) was that sunrise, sunset, and twilight times could be accurately calculated for any point on the surface of the earth if the latitude and longitude of that point and the solar declination for the given day were known. Work commenced on the SIC in late February 1976. It soon became evident that the SIC was a much more complex project than was originally envisioned. This explanation will aid the user in understanding the limitations of the SIC, how to use it, and answer some questions as to how the equations were derived.

Solar Declination

In order to make the SIC more flexible, the author decided to model the sun's apparent daily change in declination, thus removing the problem of creating and maintaining a large data file. It was thought, at first, that the declination of the sun, with respect to a viewer located on the earth's surface, could be approximated by a simple harmonic equation of the form

$$y = A \text{ Sin } (2\pi\, T) \tag{1}$$

where A represents the maximum declination of the sun and T is a time factor expressing the fractional value of orbital completion. The value of A, called the obliquity of the ecliptic, can be obtained from the equation

$$A = 23^\circ 27' 8''.26 - 0''.4684\ (t-1900) \tag{2}$$

where t = the year. A natural starting point for T is the March equinox. Therefore, T can be expressed as the time (in days) since the last March equinox divided by the length of one tropical year (365.24 days). The apparent daily solar declination can be expressed as:

$$\text{DECLINATION} = A \text{ Sin } \left(2\pi \frac{T_o}{365.24}\right) \tag{3}$$

There are always 365.24 days between March equinoxes. When a leap year occurs, the March equinox occurs one (actually closer to 0.75) day earlier than on the previous year. For example, the March equinox for 1975 occurred at 0600Z on 21 March, while the March equinox for 1976 occurred at 1150Z on 20 March. Note that the time of occurrence of the next March equinox can be approximated by adding 0.2422 day to the last March equinox and subtracting one if the year is a leap year.

Compared with data extracted from The Nautical Almanac [6], the declination values obtained with Equation (3) are close, but not exact. The major cause of error is the varying orbital velocity of the earth. Kepler's laws of planetary motions explain this effect. However, if the earth's orbit is viewed as consisting of four seasons of varying lengths, with the length of each season fixed, a set of four simple harmonic equations can be derived.

$$\text{DECLINATION} = A \text{ Sin } \left(\frac{\pi}{2} \frac{T_o}{\text{SPRING}}\right) \tag{4}$$

for period from March equinox to June solstice;

$$\text{DECLINATION} = \text{A Cos}\left(\frac{\pi}{2}\,\frac{T_s}{\text{SUMMER}}\right) \tag{5}$$

for period from June solstice to September equinox;

$$\text{DECLINATION} = -\ \text{A Sin}\left(\frac{\pi}{2}\,\frac{T_f}{\text{AUTUMN}}\right) \tag{6}$$

for period from September equinox to December solstice;

$$\text{DECLINATION} = \text{A Sin}\left(2\pi\,\frac{T_o}{365.24}\right) \tag{7}$$

for period from December solstice to March equinox;

where T_o = time since March equinox

SPRING = 92.78 (number of days in Northern Hemisphere spring season)

$T_s = T_o$ - SPRING (number of days since summer started)

SUMMER = 93.64 (number of days in Northern Hemisphere summer season)

$T_f = T_o$ - (summer and spring) (number of days since autumn started)

AUTUMN = 89.83 (number of days in Northern Hemisphere autumn)

The maximum error found in checking the results of these equations against data for the 1975-1976 time frame is 32 minutes of arc. Expected error rates are 14.8, 3.0, 17.7, and 7.7 minutes of arc for the spring, summer, autumn, and winter seasons, respectively.

Equation of Time

An additional factor that must be considered is the equation of time (EOT). Basically stated, the EOT is the difference between apparent time and mean time. An in-depth explanation of this phenomenon can be found in any introduction to astronomy text such as that by Russell [3]. The EOT results from two effects; the obliquity of the ecliptic, and the eccentricity of the earth's orbit.

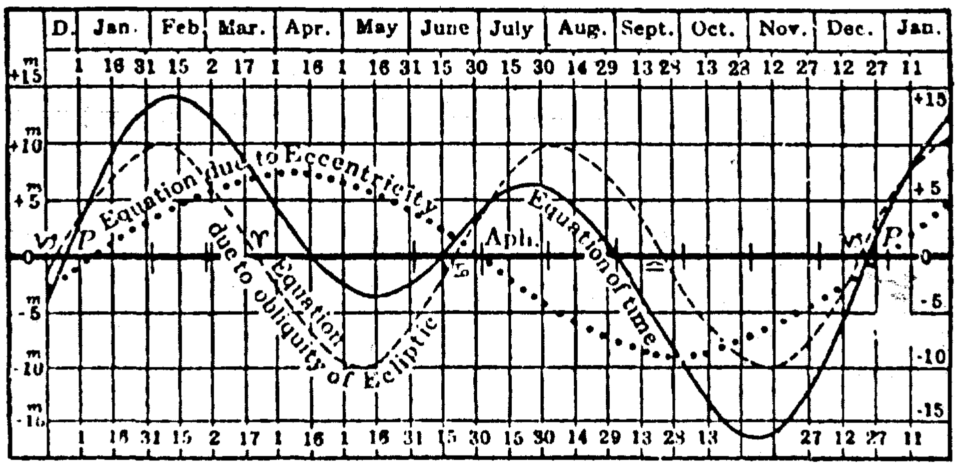

Figure 1. The Equation of Time
(From Russell [3] p. 147).

The maximum effect of the obliquity term is 10 minutes of time, and of the eccentricity term, 7-3/4 minutes of time. Note that both terms approach zero at about the times of the solstices (for the eccentricity term, it is actually perigee and aphelion, which is on the order of 11 days after the solstice). Using the winter solstice and perigee as a starting point in time, two simple harmonic equations can be derived, which, when combined, mathematically measure this phenomenon. The EOT reduces to the form:

$$EOT = 10 \text{ Sin } \left(2\pi \frac{T_{sws}}{182.62}\right) + 7.75 \text{ Sin } \left[\pi \frac{(T_{sws} - 11)}{182.62}\right] \quad (8)$$

where T_{sws} = time (in days since last winter solstice, and 182.62 = ½ tropical year.

Results from this equation are in close agreement with data extracted from The Nautical Almanac [6]. A maximum error of 1 minute of time occurs in the March and the late September through early October time periods. Soule [4] argued that the EOT at both sunrise and sunset should be computed to improve accuracy. USAFETAC adopted this convention.

Semiduration

The semiduration of a phenomenon in terms of the cosine of the Greenwich Hour Angle (GHA) is computed using a simplified version of the time-sight formula as presented by Bowditch [1].

$$\text{Cos (GHA)} = \left[- \frac{\text{Sin (Alpha)}}{\text{Cos (Latitude)} * \text{Cos (Declination)}}\right]$$

$$- \text{Tan (Latitude)} * \text{Tan (Declination)} \quad (9)$$

Alpha has a value of 0.833 degrees for sunrise/sunset, 6 degrees, 12 degrees, and 18 degrees for civil, nautical, and astronomical twilight, respectively, when computing data at sea level and with no terrain effect. These are standard, defined values. The value, 0.833 degrees, is arrived at by adding the solar diameter, assumed to be fixed at 16 minutes, to an altitude correction factor for refraction of 34 minutes. No methods are available to account for a changing atmospheric refractive index caused by temperature and/or pressure changes. The value of alpha can be changed for increases in altitude. This correction is made solely for sunrise/sunset and civil twilight calculations. The equations used to compute the new value of alpha are based on data extracted from The Air Almanac [5].

In computing semiduration there are times when the Cosine (GHA) exceeds ± 1. When this happens, it indicates nonoccurrence of the phenomenon. Consider the following examples:

- Cos (GHA) = 1.1   phenomenon = sunrise

  This indicates that the sun will not rise, because the sun is always below the horizon (winter case).

- Cos (GHA) = - 1.1   phenomenon = sunrise

  This indicates that the sun will not rise, because the sun is always above the horizon. In addition, the cosine values for civil, nautical, and astronomical twilight will also be less than -1.1 (absolute value will be greater than 1.1). But, since the sun doesn't set, these phenomena will also not occur (summer case).

In the more general sense, there are two possibilities when computing semiduration; the possibility that the phenomenon does not occur, and the possibility that the phenomenon occurs at least once during the day. In the former, there are two cases, the winter case and the summer case. In the latter, there are three cases; the normal case, the spring case, and the autumn case. All cases are based on Northern Hemisphere seasons. In the winter case, the Cos (GHA) value is always greater than 1.0. The sun is always below the horizon, hence no sunrise or sunset, and the twilights may or may not occur. In the summer case, the Cos (GHA) value is always less than -1.0. The sun is always above the horizon, hence no sunrise or sunset, and the twilights never occur. In the normal case, the Cos (GHA) value always lies in the range of equal to or greater than -1.0 and equal to or less than 1.0, hence the sun rises and sets, and the twilights occur. In the spring case the Cos (GHA) value for the beginning time of some phenomenon is always in the range of values for the normal case, but the Cos (GHA) value for the ending time is less than -1.0 (the phenomenon begins but does not end before the day is over). The autumn case is the reverse of the spring case (the phenomenon is occurring at 0000Z, but ends before 2400Z).

Half Phenomenon Length

Once the semiduration is computed, the SIC program computes half phenomenon length (HDL). If the Cos (GHA) has an absolute value greater than 1.0, the HDL is set to zero and a flag is set in the program. If the Cos (GHA) value lies in the normal case range, the arcos of the GHA is taken. This value is then multiplied by 57.2958 to convert a radian value to degrees and then divided by 15 to convert a degree value to an xx.yyyy hour format. At this point HDL has a value greater than 0.0, but less than 12.0.

Starting and Ending Times

Phenomena starting and ending times can now be computed. If the HDL has a value of 0., no computation is done. If the HDL has a non-zero value, starting and/or ending phenomena times are computed using the following formulas:

$$ST = 12. + EOTR - HDL\ (I) + ADJUST \tag{10}$$

where ST = starting time

12. = local noon

EOTR = equation of time at sunrise

HDL (I) = half phenomenon length starting

ADJUST = time adjustment factor for being East or West of Greenwich Meridian ($\pm$ Longitude/15)

$$ET = 12. + EOTS + HDL\ (I) + ADJUST \tag{11}$$

where ET = ending time

12. = local noon

EOTS = equation of time at sunset

HDL (I) = half phenomenon length ending time

ADJUST = same as above

Table 1 contains the maximum error in minutes of time for values computed at Greenwich. Since all times are based on GMT, ST and ET values lie in the range of -24.0 to +48.0. A value of less than 0. indicates that the phenomenon starts/ends on the previous Greenwich Day, while a value of greater than 24.0 indicates that the phenomenon starts/ends on the following Greenwich Day. Table 2 contains an example.

Table 1. Maximum Timing Errors at Greenwich. (Unless otherwise stated, all latitudes are given in degrees N and S of the equator.)

	SUNRISE/SUNSET AND CIVIL TWILIGHT	NAUTICAL TWILIGHT
JAN	1 minute to 40° 2 minutes above 40°	3 minutes above 40°
FEB	1 minute to 40° 2 minutes above 40°	4 minutes above 40°
MAR	1 minute to 50° 3 minutes above 50°	4 minutes above 50°
APR	1 minute to 30° 2 minutes to 40° 4 minutes to 50° 6 minutes to 70°	8 minutes above 50°
MAY	1 minute to 40° 2 minutes above 40° 17 minutes at 70°	8 minutes above 50°
JUN	1 minute to 40° 2 minutes above 40°	2 minutes
JUL	1 minute	1 minute
AUG	1 minute to 60° 3 minutes at 70°	8 minutes at 70°
SEP	1 minute to 60° 2 minutes below 30°S 3 minutes at 60°S and at 70°N	22 minutes at 70°
OCT	1 minute to 20° 2 minutes above 20°	5 minutes above 20°
NOV	1 minute to 30° 2 minutes to 40° 3 minutes above 40°	5 minutes above 40°
DEC	1 minute to 50° 2 minutes above 50°	4 minutes above 50°

NOTE: Difference between computed SIC values and data extracted from The Nautical Almanac [6] using 1975 and 1976 as base years. Data were checked from 60°S to 70°N in 10° increments. Notice the values for April and November when the equation of time was off by 1 minute. The large error in nautical twilight during September was due to an autumn case occurrence — the SIC produced a normal case, i.e., starting time of 0022 GMT.

Table 2. Example SIC Results for 5 January 1975.

	TIME (GMT) 20°N LAT/120°E LONG	TIME (GMT) 20°N LAT/120°W LONG
Astronomical Twilight	-0242 (42118)	1318 (51318)
Nautical Twilight	-0215 (42145)	1345 (51345)
Civil Twilight	-0148 (42212)	1412 (51412)
Sunrise	-0124 (42336)	1436 (51436)
Sunset	0934 (50934)	2534 (60134)
Civil Twilight	0958 (50958)	2558 (60158)
Nautical Twilight	1026 (51026)	2626 (60226)
Astronomical Twilight	1053 (51053)	2653 (60253)

NOTE: The numbers in parentheses are in a DDHHMM (day, hour, minute) format and they are printed out by the printing routine of the SIC. Therefore, at 20°N latitude, 120°W longitude sunset occurred at 0134 GMT on 6 January (1734 local on 5 January), Elevation and Azimuth

Elevation and azimuth angles are computed using the following equations (from Soule [4]):

$$\text{ELEVAT} = \text{ARSIN}\ [\text{SIN (STALAT)} \times \text{SIN (DEC)} + \text{COS (STALAT)} \times \text{COS (DEC)} \times \text{DOS (LHA)}] \tag{12}$$

where ELEVAT = Elevation

STALAT = Station latitude

DEC = Declination

LHA = Local Hour Angle (assumed to be LHA at sunrise, unless specified otherwise through parameter HOUR)

$$\text{AZIMUT} = \text{ARSIN}\left[\frac{\text{SIN (STALAT)} \times \text{SIN (LHA)}}{\text{COS (ELEVAT)}}\right] \tag{13}$$

where AZIMUT = Azimuth and STALAT, LHA, and ELEVAT are the same as above. (If LHA at sunrise is used, ELEVAT = 0.0.) Elevation and azimuth angles are not computed for the winter case (when the sun is never above the horizon), or for hours that are either earlier than sunrise or later than sunset. Computer-derived values for azimuth are expressed in terms of the principle sine value. In order to maintain standard references: North ~~is~~ zero degrees; East is 90 degrees; South is 180 degrees; West is 270 degrees. The following equations were taken from Dave, Halpern, and Myers [2].

$$\text{AZIMUTH} = \text{PI} - \text{AZIMUT} \tag{14}$$

if the condition Cos (LHA) $\leq$ Tan (DEC)/Tan (STALAT) is true. Otherwise

$$\text{AZIMUTH} = 2\times\text{PI} - \text{AZIMUT} \tag{15}$$

where LHA, DEC, STALAT, and AZIMUT are the same as above, PI = 3.1416, and AZIMUTH is true AZIMUTH. (In the subroutines no distinction is made between AZIMUTH and AZIMUT.) When the SIC ran on the MAC Information Management System Honeywell 6080 computer, library differences between IBM and Honeywell resulted in the sun rising in the west and setting in the east. Therefore, the azimuth Equations (14) and (15) were changed to:

$$\text{AZIMUTH} = \text{PI} - \text{AZIMUT} \tag{16}$$

if STALAT/DEC is greater than 1. Otherwise

$$\text{AZIMUTH} = \text{AZIMUT} \tag{17}$$

The sunrise azimuth angle results are in close agreement with values computed using a Hewlett-Packard 65 calculator. (The H-P 65 azimuth values agree to within ± 0.01 degrees with those calculated by the National Severe Storms Laboratory celestial positions program.) At present the USAFETAC has no method for verifying the accuracy of the elevation angle or the azimuth angle at some time other than sunrise/sunset. (While sunset is not computed directly it can be found by subtracting the sunrise azimuth value from 360 degrees.) It should also be noted that solar declination is assumed to be a constant in computing the elevation angle. This assumption is valid because the maximum rate of change for solar declination is 1 minute of arc per hour.

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