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This memorandum reviews the basics of stereoscopic computer vision and describes ideas developed to use run-length encoded images for such work. A brief bibliography of scene analysis and image coding work is included.

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SIGNAL PROCESSING LABORATORY

REPORT #15

Progress in Stereoscopic Image Analysis (January - July 1976)

by

J.N. England

July 1976

NORTH CAROLINA STATE UNIVERSITY
ELECTRICAL ENGINEERING DEPARTMENT

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INTRODUCTION

This research project is concerned with a primary goal of analyzing scenes in terms of three-dimensional object descriptions. Stereo pairs of images are to be used to determine x, y and z coordinates of surfaces within the scene. The three-dimensional surfaces are then to be analyzed as solid objects.

The research thrust so far has been primarily in the area of methods of determination of the x, y and z coordinates from two stereo images. This report contains a brief review of some previous work in computer depth determination using stereopsis, a review of this project's efforts currently, and an outline of problem areas to be investigated.

It should be noted that the amount of work done by other researchers in the fields of image processing, image coding, image registration and scene analysis is tremendous. Much of this project's effort so far has been in reviewing other research and in analyzing the problem. As with most research projects, once the broad base of knowledge is established a great deal of progress can be made. The next six months of this project should be highly productive in terms of solutions to the problems involved in computer binocular vision.

I. Review

The basic procedure involved in any stereopsis situation is to locate corresponding points within the two images and through reasonably straightforward trigonometry determine the x, y and z locations of the point.

Quite obviously, this procedure cannot be performed for every point within the scene. Locating a point in the second image which corresponds to a point in the first image is only possible when some structure exists around the area of the point in question. Uniformly shaded areas within images typically represent flat surfaces, or curved surfaces containing no sharp irregularities. A point located within such an area cannot be distinguished from other points within the area — no match is possible.

A procedure for depth determination, then, usually starts by finding a subarea about a point in one image which has a high likelihood of being matched with a subarea in the second image. Subareas with a variance exceeding some threshold may be used as a target for matching. This measure can lead to false expectations and consequent wasted effort when the image contains regions that are textured (hence, with high subarea variance) but of uniform texture. Another strategy involves finding edges (directed variance) in the image and using those subareas containing edges as targets for matching. Care must be taken, however, to use subareas which contain more than one edge (hopefully with the edges not oriented in the same direction). This is necessary to avoid trying to match a target with the multitude of subareas within the second image which may lie upon a common edge (see Fig. 1). Feature selection, then, is an important first step in the matching process. Selection of edge intersections (corners) as the location of target areas can eliminate much wasted effort.

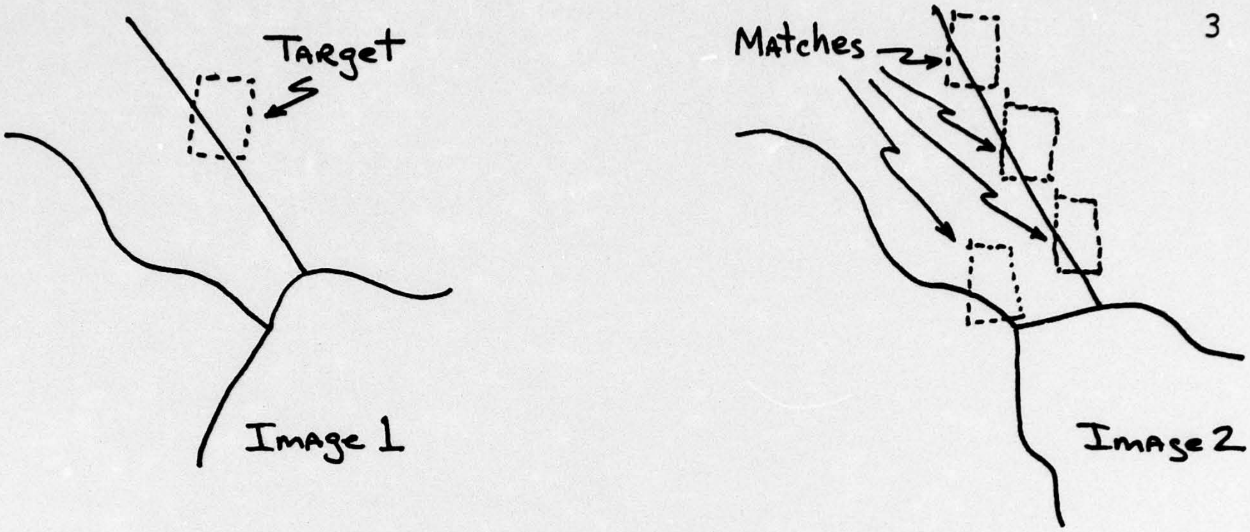

Fig. 1. Target chosen showing several possible matches within second image.

Normalized cross-correlation is probably the most widely used measure of match between sub-areas contained in two images. Rather than attempt to match a target area with subareas centered about every point in the second image, various heuristics are used to narrow the search considerably. A correlation may be attempted on a coarse grid of points in the second image. Those attempts which show promise (correlation measure above some threshold) are then subjected to a hill-climbing procedure to determine local maxima. The largest of these local maxima is then selected as a match. Another heuristic of value lies in assuming that once a match for a target has been found, matches for neighboring targets may be found in the neighborhood of the first match.

The process of determining a measure of match at a particular point may be aided in several ways. One involves using a threshold, not for the entire subarea calculation, but as a running threshold throughout the process of correlation over the area. The assumption in this case is that a true match will be indicated by a large majority of elements within a subarea matching corresponding elements within the second subarea. If several elements (chosen randomly or in a specific sampling sequence) differ significantly from their corresponding elements a match is unlikely; the procedure should be terminated and re-initiated at a different location.

A second method of speeding correlation lies in speeding computation. The use of the Fast Fourier Transform (FFT) to perform the cross-correlation can be beneficial.

II. Concept Development

Image analysis is often very costly both in terms of time required for computation and in amount of needed storage. Disk and tape storage are generally used for retaining images, with consequent limitations on access speed and convenience. Anything approaching real time image processing is only achieved on dedicated hardware systems or on very large expensive dedicated computer systems (ILLIAC, etc.).

Researchers interested in image coding and transmission have developed a number of methods for reducing the amount of data required to describe an image. The goal of those researchers has generally been the presentation of an image which is acceptable to a human viewer. The structure which humans perceive in images (and by which they judge acceptability) is precisely the structure often needed in artificial vision-image analysis. It is reasonable then to look to the field of image coding for methods of data compaction which may be of assistance in image analysis.

Fourier transform coding of images has been used in image analysis for some time. Features may be extracted from the Fourier transform plane for use in pattern recognition systems. However, analysis techniques such as contour following, region growing, corner recognition, etc. are not at all amenable to operations within the Fourier transform domain. In addition, a heavy computational penalty may be incurred because of the necessity for complex arithmetic, even though FFT techniques alleviate part of the burden.

There are other transforms used for image coding which are not amenable to analysis techniques applied to scenes of solid objects. One which may be useful, however, is the Walsh/Hadamard transform. Some work in pattern recognition has involved feature extraction in the Hadamard domain. This research project is currently investigating methods of identification in the Hadamard transfer domain of areas which are suitable for use as targets in the image matching process. The Hadamard transform is particularly attractive in terms of computational ease, since it requires no multiplication, only real additions, and a fast Hadamard algorithm exists.

This project is also currently evaluating the use of the Hadamard transform in the matching process itself. Multiplying the Hadamard transform of a target subarea by the transform of a subarea in the second image yields the transform of the logical cross-correlation of the two areas. This cross-correlation will be used as a measure of match with large computational savings over FFT or lagged-product procedures.

Another technique used in image coding which is under investigation in this project is that of run-length coding. Simply described, the grey value of a run (horizontal or vertical) of similar pixels is coded along with the length of the run. Thus, two words take the place of many words representing individual pixels. Since the termination of a run depends upon the difference between two elements exceeding some threshold, there is an added benefit in terms of image analysis, that of edge detection. The location of a run length start corresponds to an abrupt change in image characteristics — precisely the measure used to define an edge. This gain in information with a decrease in storage space has evidently not been explored in other research in image analysis. This project is engaged in examining efficient analysis techniques in a run-length coded data base. Contour following and region growing appear particularly simple in this instance. Data base organization for horizontal and vertical run length coding is being explored. The use of run-lengths in addition to gray values as a measure of texture also will be investigated in the future. The run length encoding process is particularly amenable to fairly simple hardware implementation.

Currently under investigation are methods of increasing matching efficiency in the stereopsis system by using run-length encoded data. The correlation of encoded data can be performed very efficiently. A new measure of match is also provided by correlating the run lengths themselves. This is a measure of structural match independent of shading.

In addition to performing image analysis within the run-length encoded data, it is useful, as mentioned before, to analyze the Fourier, Hadamard or other transform of the image. This research project is developing algorithms for obtaining Hadamard and Fourier transforms from the run-length encoded data. The results should be very efficient computationally.

III. Hardware and Software Development

A two-camera television interface with 256x256 pixels per frame is being designed and will shortly be constructed in the Signal Processing Laboratory. A fast buffer memory system has been designed so that a 1024 pixel window from a TV frame can be acquired during one frame interval (1/60 second). A Direct Memory Access (DMA) system is being added to the Adage AGT-30 within the laboratory. The fast TV buffer will be tied to this DMA to enable very fast access to 1024 pixels in either TV image. The location of the window is specified (under program control) as centered around any arbitrary pixel location. To provide flexibility, the addressing system for the buffer memory is designed so that the window may be configured as any rectangular shape, from 4x256 pixels to 256x4 pixels.

The input to the buffer system may be either direct TV data (8 bits of grey level) or from a run length encoder which stores grey level and run length in consecutive locations.

Some basic software for image analysis has been developed so far. Until the television interface is completed, input is from magnetic tapes of images (obtained from the Image Processing Institute of the University of Southern California). Routines to evaluate the effectiveness of different criteria for run-length encoding have been developed. Various gradient and thresholding measures have been implemented. The image derivative can be taken with direction indicated by color in the displayed result. Software for x-direction run-length encoding has been developed, as well as a statistical package for evaluating storage reductions achieved by various methods of encoding. Current work is on developing two-axis coding, and on correlation of run-length coded data.

FUTURE WORK

Briefly, the project objectives for the future are as follows:

(1) Develop 2-axis run-length coding techniques (area coding).

(2) Develop a fast Hadamard transform using encoded data. Likewise for Fourier transform.

(3) Use FHT to perform subarea correlation.

(4) Develop feature extraction system using Hadamard transform.

(5) Develop routines for obtaining 3-D data once subarea matches are known.

(6) Develop surface growing techniques for 3-D data.

(7) Develop object extraction and analysis system.

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