

AD-A040 330

NORTH CAROLINA UNIV AT CHAPEL HILL DEPT OF STATISTICS
APPROXIMATE RELATIONSHIPS AMONG SOME ESTIMATORS OF MORTALITY PR--ETC(U)
DEC 76 N L JOHNSON

F/G 12/1

DAAG29-74-C-0030

UNCLASSIFIED

MIMEO SER-1098

ARO-11959.15-M

NL

| OF |
AD
A040330



END

DATE
FILMED
7-77

Q60-11959.15-0n

(12) J

APPROXIMATE RELATIONSHIPS AMONG SOME
ESTIMATORS OF MORTALITY PROBABILITIES

TECHNICAL REPORT

NORMAN L. JOHNSON

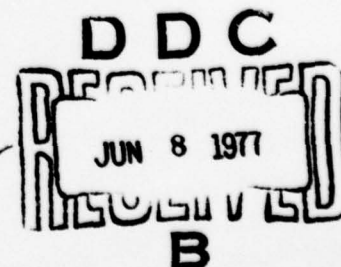
DECEMBER, 1976

U. S. ARMY RESEARCH OFFICE
RESEARCH TRIANGLE PARK, NORTH CAROLINA

DAI 629-74-C-0030

DEPARTMENT OF STATISTICS
UNIVERSITY OF NORTH CAROLINA AT CHAPEL HILL
CHAPEL HILL, NORTH CAROLINA 27514

APPROVED FOR PUBLIC RELEASE
DISTRIBUTION UNLIMITED



AD A 040330

UNCLASSIFIED

(14) Mimeo Ser-1098

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) ⑥ Approximate Relationships Among Some Estimators of Mortality Probabilities.		5. TYPE OF REPORT & PERIOD COVERED ④ TECHNICAL rept.
7. AUTHOR(s) ⑩ No Y. L. / Johnson		6. PERFORMING ORG. REPORT NUMBER Mimeo Series No. 1098
		8. CONTRACT OR GRANT NUMBER(s) ⑮ DAK G29-74-C-0030
9. PERFORMING ORGANIZATION NAME AND ADDRESS Department of Statistics University of North Carolina Chapel Hill, North Carolina 27514		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Army Research Office Box 1221 Research Triangle Park, NC 27709		12. REPORT DATE ⑪ December, 1975
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		13. NUMBER OF PAGES 5 ⑫ 7p.
		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for Public Release: Distribution Unlimited ⑮ ARO		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report) ⑮ 11959.15-M		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Mortality investigations, approximations		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) Some approximate relations among estimators of probabilities of death are described. 182850 B.		

APPROXIMATE RELATIONSHIPS AMONG SOME
ESTIMATORS OF MORTALITY PROBABILITIES

by

N.L. Johnson*

University of North Carolina at Chapel Hill

1. Introduction

Kuzma [1967] and Drolette [1975] have compared by means of quite extensive numerical calculations, several estimators of q , the probability of death in an interval (given alive at the beginning of the interval). It is the purpose of this note to give some approximate algebraic relations among some of these estimators. These confirm, in general terms, the conclusions reached from the numerical investigations. They also provide quite good approximations (to four decimal places when $q < 0.15$, apparently) to two rather complicated (in appearance) estimators, in terms of estimators of simpler form. This also gives an improved appreciation of the nature of the more complicated estimators.

2. Notation

We use the following notation (as in Elandt-Johnson [1976]).

- N = number alive at beginning of interval
- d = number of deaths in the interval
- c = number "due to be withdrawn" in the interval
- d' = number of deaths among these c individuals
- w = number actually withdrawn.

It will be assumed that there are no "lost to follow up," or new entrants during the interval so that $w+d' = c$; $d' \leq d \leq N - w$.

*This research was supported by the Army Research Office, under Contract
DAH G29-74-C-0030.

White Section	☒
Buff Section	☐
ANNOUNCED	☐
NOTIFICATION	☐
DISTRIBUTION/AVAILABILITY CODES	
DISC	AVAIL. SPC/OF SPECIAL
A	

The estimators to be discussed are:

$$\hat{q}^{(2)} = \{N' - (N'^2 - 2Nd)^{1/2}\}N^{-1} \quad (\text{see Elveback [1958]}) \quad (1)$$

where $N' = N - \frac{1}{2}w + \frac{1}{2}d$,

$$\hat{q}^{(3)} = d(N - \frac{1}{2}w)^{-1} \quad (\text{the "actuarial" estimator}), \quad (2)$$

$$\hat{q}^{(4)} = 1 - \frac{1}{4}(2N-c)^{-2} \{d'^2 + 4(2N-c)(2N-w-2d)\}^{1/2} d'^2 \quad (\text{Chiang [1961]}) , \quad (3)$$

$$\hat{q}^{(5)} = d(N - \frac{1}{2}c)^{-1} \quad (\text{see Elveback [1958]}). \quad (4)$$

The estimators $\hat{q}^{(3)}$ and $\hat{q}^{(5)}$ are much simpler in form than $\hat{q}^{(2)}$ and $\hat{q}^{(4)}$.

3. The Approximations

Using the method outlined in the Appendix, the following approximations are obtained.

$$\hat{q}^{(2)} \doteq \frac{1}{2} \left[\left(1 + \frac{1}{2}\hat{q}^{(3)}\right)^{-1} + \left\{1 + \left(\frac{1}{2} - R\right)\hat{q}^{(3)}\right\}^{-1} \right] \hat{q}^{(3)} \quad (5)$$

where

$$R = N/N' = \left(\frac{N}{N - \frac{1}{2}w} \right) \left(1 + \frac{1}{2}\hat{q}^{(3)}\right)^{-1} \quad (6)$$

$$\hat{q}^{(4)} \doteq \left\{1 - \frac{1}{4}(d'/d)\hat{q}^{(5)}\right\} \hat{q}^{(5)} . \quad (7)$$

Results of some numerical calculations illustrating the accuracy of these approximations, using data from Table 2 of Chiang [1961] are presented in Table 1.

For larger $\hat{q}$, (5) may be improved by addition of $\frac{1}{8}\hat{q}^{(3)}\{\hat{q}^{(3)}(1 + \frac{1}{2}\hat{q}^{(3)})^{-1}\}^3$, but values of $\hat{q}$ ($>.20$, say) for which this is of any importance are so large that validity of assumptions made in derivation of formulae become of overriding importance (and usually uncertain validity).

Table 1.

Approximations to various estimators of q

Interval	$\hat{q}^{(2)}$	Approx. (5)	$\hat{q}^{(4)}$	Approx. (7)	$\hat{q}^{(3)}$	$\hat{q}^{(5)}$
0-1	.2434	.2429	.2425	.2426	.2417	.2436
1-2	.1822	.1821	.1814	.1815	.1810	.1819
2-3	.1033	.1033	.1030	.1030	.1028	.1032
3-4	.0860	.0860	.0853	.0858	.0856	.0859
4-5	.0643	.0643	.0641	.0641	.0641	.0642
5-6	.0584	.0584	.0582	.0582	.0582	.0583
6-7	.0438	.0438	.0438	.0438	.0437	.0438
7-8	.0433	.0433	.0432	.0432	.0431	.0433
8-9	.0337	.0337	.0337	.0337	.0337	.0337
9-10	.0466	.0466	.0465	.04655*	.0464	.0467
10-11	.0439	.0439	.0438	.0438	.0437	.0440
11-12	.0513	.0513	.0511	.0511	.0508	.0513

* - exactly

Except for the approximation to $\hat{q}^{(2)}$ in interval 0-1, there is never more than 0.0001 difference between approximation and calculated value (to 4 decimal places). In most cases the values agree to 4 decimal places.

It is interesting to note that for these data $\hat{q}^{(5)}$ happens to be quite a good approximator for $\hat{q}^{(2)}$.

4. Discussion

From (7) it is clear that the approximate value for $\hat{q}^{(4)}$ is always less than $\hat{q}^{(5)}$. This is consistent with numerical results obtained by Drolette [1975]. She also finds that $\hat{q}^{(2)}$ is greater than $\hat{q}^{(3)}$. To see that this is in accordance with our formulae, we note that the right hand side of (5) is

greater than $\hat{q}^{(3)}$ if and only if $R > (1 + \frac{1}{2}\hat{q}^{(3)})(1 + \hat{q}^{(3)})^{-1}$. From the definition of R in (6), it follows that $R \geq (1 + \frac{1}{2}\hat{q}^{(3)})^{-1}$ and this is less than $(1 + \frac{1}{2}\hat{q}^{(3)})$, so it is possible for the approximate value of $\hat{q}^{(2)}$ to be less than $\hat{q}^{(3)}$, though the least possible value (when $w = 0$) is

$$\begin{aligned} \hat{q}^{(3)} [1 - (\hat{q}^{(3)})^3 \{8 + 4\hat{q}^{(3)} + 2(\hat{q}^{(3)})^2 + (\hat{q}^{(3)})^3\}^{-1}] \\ > \hat{q}^{(3)} \{1 - \frac{1}{8}(\hat{q}^{(3)})^3\}. \end{aligned}$$

While the labor of calculating $\hat{q}^{(2)}$ and $\hat{q}^{(4)}$ is not great, the approximations put forward here provide some insight into the nature of these estimators, as well as quite simple formulae for approximate calculation.

In view of the small differences usually found among the numerical values given by the different formulae, and the usually dubious nature of assumptions involved in setting up the likelihood functions underlying $\hat{q}^{(2)}$ and $\hat{q}^{(4)}$, there are good grounds for using simpler formulae (like $\hat{q}^{(3)}$ or $\hat{q}^{(5)}$) except in very special cases.

APPENDIX

For $|t| < 1$, we have the expansion

$$\begin{aligned} (1-2t)^{1/2} &= 1 - t - \frac{1}{2}t^2 - \frac{1}{2}t^3 - \frac{5}{8}t^4 - \frac{7}{8}t^5 - \dots \\ &= 1 - \frac{1}{2}t - \frac{1}{2}(t + t^2 + t^3 + t^4 + t^5) - \frac{1}{8}t^4 - \frac{3}{8}t^5 - \dots \\ &\doteq 1 - \frac{1}{2}t - \frac{1}{2}t(1-t)^{-1}. \end{aligned}$$

So

$$1 - (1-2t)^{1/2} \doteq \frac{1}{2}t[1 + (1-t)^{-1}].$$

This approximation is used in

$$\hat{q}^{(2)} = (N'/N)[1 - \{1 - 2(N/N')^2d\}^{1/2}]$$

to yield (5).

Similarly, we write (remembering $d' = c-w$)

$$\hat{q}^{(4)} = 1 - \frac{1}{4(2N-c)^2} [4(2N-c)^2 (1 - \frac{2d-d'}{2N-c}) + 2d'^2 - 4d'(2N-c) \{1 - \frac{2d-d'}{2N-c} + \frac{d'^2}{4(2N-c)^2}\}^{1/2}]$$

and expand the square root, to yield (7).

REFERENCES

- Chiang, C.L. [1961]. A stochastic study of the life table and its applications: III. The follow-up studies with the considerations of competing risk, *Biometrics*, 17, 57-78.
- Drolette, M.E. [1975]. The effect of incomplete follow-up, *Biometrics*, 31, 135-144.
- Elandt-Johnson, R.C. [1976]. Various estimators of conditional probabilities of death in follow-up studies: Summary of results, *J. Chron. Dis.*, 29, to appear.
- Elveback, L. [1958]. Actuarial estimation of survivorship in chronic disease, *J. Amer. Statist. Assoc.*, 53, 420-440.
- Kuzma, J.W. [1967]. A comparison of two life table models. *Biometrics*, 23, 51-64.