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SHIP FLEXURE MEASUREMENT

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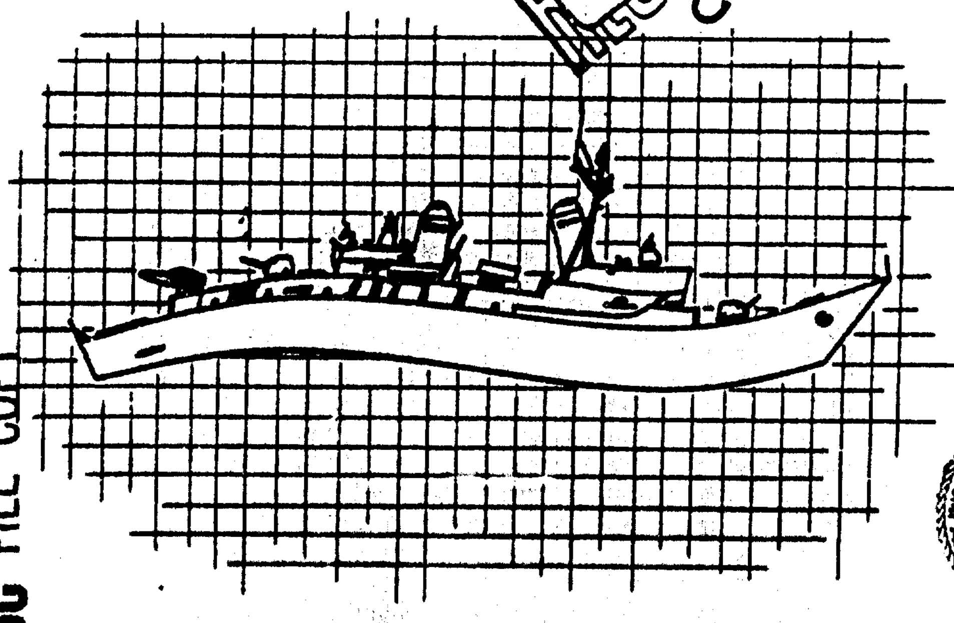

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ABSTRACT

This final report summarizes the results of a recently-concluded dynamic ship flexure measurement program during which data were taken on four naval combatant vessels: USS HORNE (DLG-30), USS REEVES (DLG-24), USS BARNEY (DDG-6), and USS ROBISON (DDG-12).

Four optical tracking systems, each consisting of a biaxial automatic autocollimator, reflector, and laser, were used to obtain the measurements. The data consist of a series of 10- to 90-minute recordings obtained under varying conditions of sea state, ship speed, and ships heading with respect to prevailing wind and wave directions. For each run, amplitude time histories were plotted, flexure frequency spectra were determined, and RMS flexure values were calculated. The data, summarized in tabular form, identify the frequency and magnitude of flexure between weapon system elements, and indicate that, under certain conditions, the flexure magnitude can actually exceed current alignment tolerances. This is particularly true for the aft 5"-54 gun on the DDG-2 class ship.

It is anticipated that the results of this program will be used in conjunction with a tolerance study to develop cost-effective mechanical alignment tolerances compatible with the measured flexure.

Based on the findings, it is recommended that 2 arc-minutes be recognized as the minimum maintainable RPI tolerance. It is also recommended that no further flexure measurements be made on DDG and DLG-class ships unless a specific need arises.

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SECTION 1

INTRODUCTION

1-1. PURPOSE. This final technical report summarizes the results of a recently-concluded ship flexure measurement program under which flexure data were obtained on four Navy combatant ships. The work was performed under authority of NAVSHIPS letter 423E:CJH:aj9780 Ser 1988-423 of 26 June 1972, as part of a program being conducted by the Naval Ship Weapon Systems Engineering Station (NSWSES) to obtain actual data on the flexure characteristics of certain naval combatant vessels. Until this time, the only available substantial flexure data were based on theoretical studies rather than actual measurement. Thus, it became necessary to go to sea to obtain actual dynamic flexure data. Individual reports for each ship tested have been issued under NSWSES Technical Report Numbers 268 (USS HORNE DLG-30), 272 (USS REEVES DLG-24), and 281 (USS BARNEY DDG-6 and USS ROBISON DDG-12). Measurements were attempted in two other ships, USS LOCKWOOD (DE-1064) and USS BAINBRIDGE (DLG(N)-25), in addition to those previously reported.

1-2. Insufficient data were obtained aboard BAINBRIDGE due to inadequate sea states or dynamic conditions and extensive shipboard electrical interference encountered while underway. Inadequate sea states and scheduling limitations imposed as a result of the commitment of USS LOCKWOOD to the HARPOON test program prevented the collection of data sufficient for preparation of a ship flexure report. Enough data were obtained to issue NSWSES Engineering Support Department Interim Report TR 4900-080, at the request of NSWSES, Code 4230, containing information pertinent to the HARPOON test program.

1-3. When subjected to the dynamic conditions of the seaway, a ship must flex to prevent an intolerable buildup of stresses in the structure. This flexure shows up as a relative rotational motion between two points and can result in an unfavorable misalignment between elements located on or between these points. The purpose of the measurements was to determine the relative flexure between weapon system elements in selected Navy ships. It is intended that the results of these measurements will be used in developing or redeveloping error budget analyses for present and future shipboard weapon systems. The results also provide previously unavailable information which could serve as a basis for establishing cost effective mechanical alignment tolerance structures which are compatible with the flexure experienced by operational ships.

SECTION 2

TEST BED DESCRIPTION

2-1. TEST SHIPS AND EQUIPMENT. Of the four ships studied, two were DLGs and two were DDGs, with the following physical characteristics and weapons configurations;

USS HORNE (DLG-30) Class: DLG-28

Physical Characteristics

Displacement		9750 to 8150 tons
Dimensions:	Length	547 feet
	Beam	54 feet
	Draft	28 feet
	Number of Shafts	2
	Number of Blades on Screw	6

Weapons Configuration

Fire Control System MK 76 (2)	Forward
Guided Missile Launcher MK 5	Forward
Fire Control System MK 68	Aft
5"-54 Gun Mount	Aft
3"-50 Gun Mount (2)	Port and Stbd
Torpedo Tubes MK 32 (2)	Port and Stbd

USS REEVES (DLG-24) Class: DLG-16

Physical Characteristics

Displacement		8000 tons
Dimensions:	Length	532 feet
	Beam	54 feet
	Draft	16 feet
	Number of Shafts	2
	Number of Blades on Screw	6

USS REEVES (DLG-24) (Continued)

Weapons Configuration

Fire Control System MK 76 (4)	Forward and Aft
Guided Missile Launcher MK 5 (2)	Forward and Aft
ASROC Launcher	Forward
3"-50 Gun Mount (2)	Port and Stbd
Torpedo Tubes MK 32 (2)	Port and Stbd

USS BARNEY (DDG-6) and USS ROBISON (DDG-12) Class: DDG-2

Physical Characteristics

Displacement		4500 tons
Dimensions:	Length	437 feet
	Beam	47 feet
	Draft	15 feet
	Number of Shafts	2
	Number of Blades on Screw	4

Weapons Configuration

3-D Search Radar AN/SPS-39	Aft
Missile FCS MK 74 (2)	Aft
Guided Missile Launcher MK 8	Aft
ASROC Launcher	Mid
5"-54 Gun Mount (2)	Forward and Aft
Torpedo Tubes MK 32 (2)	Port and Stbd
Gun Fire Control System MK 68	Forward

SECTION 3

FLEXURE MONITORING SYSTEMS

3-1. EQUIPMENT DESCRIPTION

3-1.1. PHYSICAL DESCRIPTION. The flexure monitoring system was composed of two elements: a transmitter-receiver and a mirror. Figure 3-1 is a diagram of the system showing the various components. Locations of the equipment aboard the four ships tested are shown in Table 3-1.

3-1.1.1. Transmitter-Receiver. The transmitter-receiver consisted of a Physitech Model 440 Autocollimator containing an optical head and an electronics package and a Hughes Model 3076H Helium-Neon Laser. The autocollimator optical head was enclosed in one end of a 5-foot long aluminum housing and was focused on a 17-inch square translucent screen located at the other end of the box. The transmitter-receiver electronics package was contained in a location separate from the housing. The laser unit, which was mounted in a sealed tube, received power from a potted power supply and utilized a collimator to minimize divergence of the light beam. The laser assembly was mounted atop the aluminum housing.

3-1.1.2. Mirror. A Davidson Optronics Model D-571 mirror, rigidly attached to the ship, served to reflect the transmitted laser beam back to the screen.

3-1.2. FUNCTIONAL DESCRIPTION. Relative flexure between the mirror and the transmitter-receiver, both of which were rigidly fixed to the ship, was measured by movement of the laser beam. The laser beam was directed toward the mirror and was reflected back to the translucent screen. Ship flexure resulted in a change in the angle between the incident and reflected beams which registered as a shift in beam position on the screen. Thus, any relative angular motion between the elements under observation was manifested by a linear displacement of reflected beam position on the screen. This displacement was, in turn, continually tracked by the transmitter-receiver optical head.

3-2. SYSTEM ACCURACY

3-2.1. AUTOCOLLIMATOR. Although the Physitech Autocollimator is an extremely sensitive device capable of measuring angles within 0.2% of full scale, the accuracy of the measured flexure amplitude for either axis of all systems was considered to be $\pm 6\%$ of the indicated value. This figure resulted from the variable working distance of the systems and from the

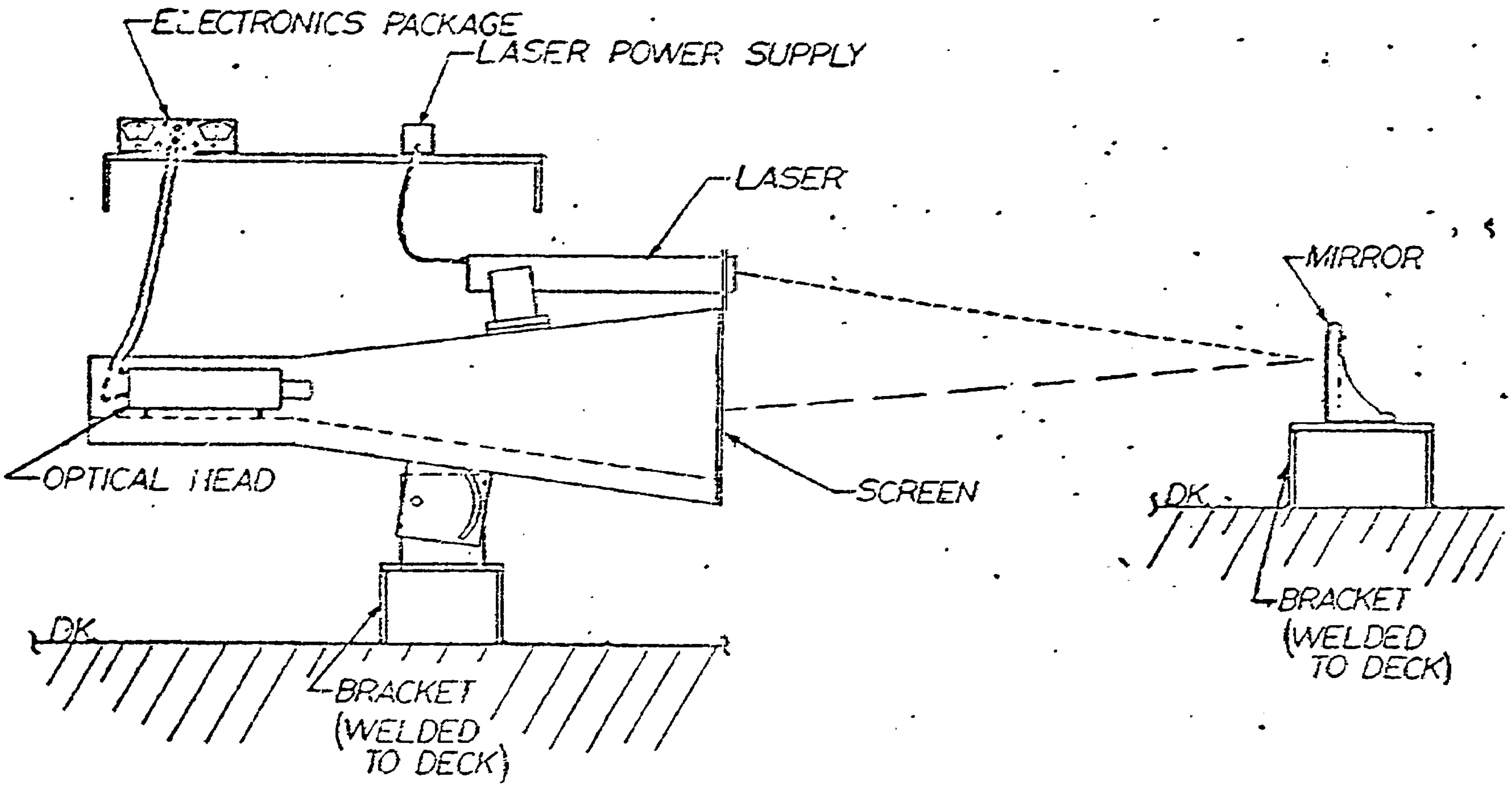

Figure 3-1. Flexure Monitoring System Diagram

inability to optically align the systems perfectly. As the distance between elements of the measuring system decreased, the linear excursion of the laser image on the screen was shortened for the same angular movement. This, in turn, amplified the errors involved in the physical alignment and calibration processes.

3-3. SYSTEM OPERATION

3-3.1. On all ships tested, data runs were conducted under varying environmental conditions. The outputs (2) from each monitoring system were utilized as inputs to a Voltage Controlled Oscillator (VCO) manufactured by EMR. This VCO served to combine the outputs into a single data signal. These data were recorded at 7 1/2 ips on a Genisco 7-channel magnetic tape recorder together with simultaneous recording of voice annotations and a time code emanating from an EECO Time Code Generator. A Sanborne Model 322 Strip Chart Recorder was used to monitor the channels, ensuring that all data were actually being recorded.

Table 3-1. Equipment Locations.

System ID No.	Transmitters Location Frame/Level/ Distance From Centerline	Mirror Location Frame/Level/ Distance from Centerline	Inter-Element Distance
USS HORNE (DLG-30)			
6	67/04/12.7' Stbd	33/01 SPS-48/ 12.7' Stbd	91.2'
4	67/04/0'	91/Platform/0'	62'
1	68/04/14.9' Stbd	104/04/14.9' Stbd	67.8'
2	104/04/14.4' Port	188/01/14.4' Port	210.4'
USS REEVES (DLG-24)			
6	67/04/12.7' Port	34/01 SPS-48/ 12.7' Port	80
4	67/04/0'	95/Platform/0'	76'
1	68/04/10.6' Port	104/04/10.3' Port	77
2	104/04/9.4' Port	183/01/9.4' Port	176
USS BARNEY (DDG-6)			
2	115/03/8' Port	75/04/6'8" Port	94'
3	130/03/7'3" Port	177/01/7'3" Port	96'
5	75/04/5'10" Port	110/39/3'4" Port	77'
USS ROBISON (DDG-12)			
2	133/03/8.3' Port	176/01/8.6' Port	93'
3	118/03/8.3' Port	79/04/8.5' Port	77'

SECTION 4

SHIP FLEXURE AND DATA ANALYSIS

4-1. SHIP FLEXURE. Whenever seakeeping data are analyzed, the fact must be considered that each instant in time is completely unique. Describing the sea can only be done by grouping the factors which determine sea conditions, wave heights or wind speed, into general categories (sea state numbers). The sea states encountered during this measurement program were classified in accordance with the International Sea State Code provided in Appendix B.

4-1.1. FORCING FUNCTIONS. Ship flexure is not so simple a phenomenon that it can be meaningfully discussed without first gaining some understanding of its diverse causes (forcing functions). One criterion which may be used to classify flexure-producing forcing functions is the period of the resulting flexure measured with respect to the "typical" engagement time of a shipboard weapon system. Therefore, the flexure resulting from these forces may be categorized as either long term or short term, with short-term flexure having a period equal to or less than a typical engagement time. Accordingly, flexure-producing forcing functions, and their principal contributors, may be enumerated as follows:

<u>Long Term</u>

1. Temperature Effects
   a. Water
   b. Air
2. Daily Sun Cycles (direct radiation)
3. Loading Changes
   a. Fuel
   b. Ammunition
   c. Stores

<u>Short Term</u>

1. Shipboard Equipment (rotating and reciprocating machinery)

2. Ship's Operations

a. Ship Speed

b. Weapons Launching

c. High-impact Shock

3. Wind-Sea-Ship Interaction

a. Propeller Impulse

b. Sea State

c. Ship Heading to Waves

d. Wind Loading (primarily superstructure)

e. Froude Number (ratio of hydrodynamic forces to gravity forces)

f. Wave Steepness

g. Wave-Length to Ship-Length Ratio

h. Tuning Factor (ratio of the frequency of induced periodic ship motion to the ship's natural frequency response to the motion).

4-1.2. OTHER FACTORS. The validity and extent of application of any conclusions which may be drawn from this study must be evaluated with respect to the aforementioned factors. In particular, it must be remembered that long-term flexure, significant as it may be, was not measured as part of the program. Also, it was possible to categorize and correlate only a few of the many short-term contributors, such as wind velocity, sea state, ship speed, and wave heading. It is important to note that the forcing functions listed are not all independent phenomena. For example, changes in a ship's loading may have a significant effect on its flexural response in a seaway.

4-2. DATA ANALYSIS. The study of a ship's flexural behavior in a seaway becomes, in actual practice, an attempt to describe certain motions which are assumed to be representative of the behavior of the ship. Because of the multitude of random and complex flexure-producing forcing functions, and because of the near-impossibility of deriving an accurate enough transfer function for a ship's hull and superstructure, it is not feasible to describe ship flexure analytically by a single, meaningful, mathematical expression. Since a ship will never repeat a substantial sequence of motion exactly, it is more profitable to try to determine average behavior. This leads to probability theory and statistics.

4-2.1. POWER SPECTRAL DENSITY. One of the most descriptive elements of statistical analysis is the energy spectrum or power spectral density which, in this case, becomes a graph of flexure vs frequency. In order to understand the value and limitations of the power spectrum, it is important to know how it evolves from a particular time history of the raw data to a graph or set of numbers. A review of the supportive theory of spectral analysis is provided in Appendix A.

4-2.2. The data obtained during the flexure program were analyzed by means of a Time/Data 100 Spectrum Analyzer used in conjunction with a PDP-11 Computing System. Initial analysis indicated that the data of interest could best be displayed in power spectral density (PSD) plots covering the 0.1 to 10Hz and the 0.0125 to 1Hz frequency ranges, and in 10-minute amplitude time histories (ATH) of data filtered between 0.0125 and 0.3Hz.

4-2.2.1. Tables 4-1 through 4-4 and 4-7 are power spectral density summaries of the four ships tested. Sumaries of the amplitude time history and summed data for USS REEVES are provided in tables 4-5 and 4-6. Table 4-8 is a summary of summed data for USS HORNE. Environmental data for the four ships are provided in tables 4-9 through 4-12.

4-2.3. RMS VALUES. RMS flexure values were also calculated for each data run. It is generally accepted, in the literature, that the statistics of seakeeping data are Gaussian. This can be verified by the application of the chi-squared test to a distribution of points selected at random from the data record. The Gaussian property is important because linear operations on Gaussian processes are also Gaussian. This property, along with certain other assumptions, permits the development of the tools necessary to analyze ship flexure data. The RMS value is useful because it actually represents the standard deviation of 1 σ level for a Gaussian process. This level represents the highest level of flexure to be expected 68 percent of the time. The 2 σ and 3 σ levels place an upper bound on the flexure 95 percent and 99 percent of the time, respectively. RPI tolerances for flexure measurement paths for the four ships are given in tables 4-13 through 4-15.

TABLE 4-1. USS BARNEY (DDG-6) POWER SPECTRAL DENSITY SUMMARY

Run No.	System No.	RMS[1] arc-sec	VERTICAL AXIS		f[3] Hz	RMS arc-sec	HORIZONTAL AXIS		f Hz	BW[4] Hz
			Max[2]+ arc-sec	Max- arc-sec			Max[2]+ arc-sec	Max- arc-sec		
4	2	15.0			.15	21.3			.25	.1-10
	3	11.0			.20	5.9			.20	
	3	27.5	63	68	.07	5.2	11	17	.07	.0125-1.0
6	3	25.9			.20	7.9			.20	.1-10
	5	80.4	349	207	.18					.0125-1.0
8	3	27.9			.25	12.3			.20	.1-10
	3	28.7	70	62	.20					.0125-1.0
9	3	20.1			.15	9.2			.15	.1-10
10	3	16.3			.20	9.3			.20	
	3	24.1	68	68	.15	10.3	25	32	.15	.0125-1.0
11	3	23.1			.15	7.1			.15	.1-10
	3	41.1	135	135	.10	6.5	18	14	.12	.0125-1.0
12	3	12.6			.20	7.0			.20	.1-10
	3	32.8	72	67	.08					
13	3	16.7			.2, 9.8	8.5			.2, 9.9	

TABLE 4-1. USS BARNEY (DDG-6) POWER SPECTRAL DENSITY SUMMARY (Cont'd)

Run No.	System No.	RMS[1] arc-sec	VERTICAL AXIS Max[2]+ arc-sec	VERTICAL AXIS Max- arc-sec	f^3 Hz	RMS arc-sec	HORIZONTAL AXIS Max+ arc-sec	HORIZONTAL AXIS Max- arc-sec	f Hz	BW[4] Hz
	3	28.2	59	80	.07	7.9	17	21		.0125-1.0
	5	16.9			.20	22.2			.20	.1-10
	5	28.9	93	80	.08					.0125-1.0
14	2	21.9			.15	12.5			.20	.1-10
	2	42.8	138	115	.07	18.6	42	45	.07	.0125-1.0
	3	18.5			.20	8.1			.20	.1-10
	3	38.4	87	120	.07	9.3	27	27	.07	.0125-1.0
	5	19.1			.25	21.2			.25	.1-10
	5	38.4	111	107	.07	28.4	60	85	.06-.11	.0125-1.0
15	3	14.7			.12	11.7			.12	.1-10
	3	41.1	96	127	.05	42.7	88	117	.05	.0125-1.0
16	2	13.9			.12	16.2			.12	.1-10
	2	41.7	84	109	.05	33.8	68	88	.05	.0125-1.0
	3	12.5			.12					.1-10
	3	32.3	85	83	.05					.0125-1.0

TABLE 4-1. USS BARNEY (DDG-6) POWER SPECTRAL DENSITY SUMMARY (Cont'd)

Run No.	System No.	RMS[1] arc-sec	VERTICAL AXIS Max[2]+ arc-sec	VERTICAL AXIS Max- arc-sec	f[3] Hz	RMS arc-sec	HORIZONTAL AXIS Max+ arc-sec	HORIZONTAL AXIS Max- arc-sec	f Hz	BW[4] Hz
	5	11.4			.11	9.5			.20	.1-10
	5	32.7	74	77	.05	16.8	51	49	.05	.0125-1
17*	2	2.0				2.8				.1-10
	3	1.5				2.4				
	5	2.1				3.8				
19	3	11.5			.30	7.1			.30	
	3	21.1	52	55	.04	6.7	35	35	.04	.0125-1.0
	5	6.0			.20	31.0			.30	.1-10
	5	22.2	54	55	.04	16.9	38	46	.04	.0125-1.0
20	3	19.4			.30	10.9			.30	.1-10
	3	37.1	91	117	.05	11.2	24	27	.12,.26	.0125-1.0
	5	21.4			.30					.1-10
	5	48.8	117	142	.05					.0125-1.0
21	3	29.1			.20	21.8			.25	.1-10
	3	42.9	111	133	.06	25.0	53	53	.22	.0125-1.0

TABLE 4-1. USS BARNEY (DDG-6) POWER SPECTRAL DENSITY SUMMARY (Cont'd)

Run No.	System No.	RMS[1] arc-sec	VERTICAL AXIS Max[2]+ arc-sec	Max- arc-sec	f[3] Hz	RMS arc-sec	HORIZONTAL AXIS Max+ arc-sec	Max- arc-sec	f Hz	BW[4] Hz
	5	54.9			.25	18.6			.30	.1-10
	5	79.6	199	177	.05,.21	22.2	53	48	.03-3	.0125-1.0

1. Averaged over all PSD frames
2. Maximum positive and negative valves from last frame ATH
3. Predominant frequency of PSD
4. Data analysis bandwidth

*System Check

TABLE 4-2. USS ROBISON (DDG-12) POWER SPECTRAL DENSITY SUMMARY

Run No.	System No.	RMS arc-sec	VERTICAL AXIS Max+ arc-sec	VERTICAL AXIS Max- arc-sec	f Hz	RMS arc-sec	HORIZONTAL AXIS Max+ arc-sec	HORIZONTAL AXIS Max- arc-sec	f Hz	BW Hz
1	3	48.0			.2	27.5			.2	.1-10
	2	36.7			.2					
		58.4	150	145						.0125-.3
2	3	43.6			.25	29.9			.3	.1-10
	2	39.5			.25					
		51.8	153	145						.0125-.3
3	3	36.4			.3	25.9			.3	.1-10
		57.7	209	182						.0125-.3
	2	52.9			.25					.1-10
		49.5	151	149						.0125-.3
4	3	57.2			.2	18.3			.2	.1-10
	2	44.5			.25					
5	2	40.7			.25					
6	3	15.7			.2	22.6			.15	
	2	14.1			.15					

TABLE 4-2. USS ROBISON (DDG-12) POWER SPECTRAL DENSITY SUMMARY (Cont'd)

Run No.	System No.	VERTICAL AXIS RMS arc-sec			f Hz	HORIZONTAL AXIS RMS arc-sec			f Hz	BW Hz
			Max+ arc-sec	Max- arc-sec			Max+ arc-sec	Max- arc-sec		
7	3	13.5				10.1				
	2	10.4								
9	2	25.0			.2					
10	2	40.1			.2					

TABLE 4-3. USS REEVES (DLG-24) POWER SPECTRAL DENSITY SUMMARY
(0.0125 to 1.0 Hz Frequency Range)

Run No.	System No.	RMS[1] arc-sec	VERTICAL AXIS Max[2]+ arc-sec	VERTICAL AXIS Max- arc-sec	f[3] Hz	RMS arc-sec	HORIZONTAL AXIS Max+ arc-sec	HORIZONTAL AXIS Max- arc-sec	f Hz
2	1	18.3			.01,.19	14.4			.02,.1
	2	60.5			.05-.25	47.2			.1-.24
	4	98.3	303	267	.26	51.0	115	176	.1,.14, .26
4	1	25.3	78	86	.16	26.0	74	78	.11
	4	89.0	221	283	.15,.25	77.8	262	211	.11
	6	44.8	117	136	.15	36.4	107	118	.1-.3
5	4	46.8	123	119	.1-.25	39.4	95	109	.1
	6	57.5	161	141	.08	21.4	65	63	.1-.3
6	4	109.5	321	297	.15,.25	32.4	68	74	.15
	6	68.8	154	186	.15	51.2	96	127	.15-.21
8	4	99.4	264	236	.15,.25	45.1	139	136	.15,.26
	6	64.6	249	196	.15	43.0	115	126	.14-.3
9	1	12.7	39	49	.09	10.7	35	33	.1
	4	27.4	63	60	.13	23.9	59	59	.1-.17

TABLE 4-3. USS REEVES (DLG-24) POWER SPECTRAL DENSITY SUMMARY (Cont'd)

Run No.	System No.	RMS[1] arc-sec	VERTICAL AXIS Max[2]+ arc-sec	Max- arc-sec	f[3] Hz	RMS arc-sec	HORIZONTAL AXIS Max+ arc-sec	Max- arc-sec	f Hz
	6	18.6	50	48	.1-.15	20.4	62	53	.15
10	1	14.0	47	50	.05-.2	9.9	25	33	.08
	4	27.6	70	77	.13	20.6	62	59	.1
	6	24.6	71	57	.06-.09	19.4	59	51	.07-.15

1. Averaged over all PSD frames
2. Maximum positive and negative valves from last frame ATH
3. Predominant frequency of PSD

TABLE 4-4. USS REEVES (DLG-24) POWER SPECTRAL DENSITY SUMMARY
(0.1 to 10 Hz Frequency Range)

Run No.	System No.	RMS arc-sec	VERTICAL AXIS Max+ arc-sec	VERTICAL AXIS Max- arc-sec	f Hz	RMS arc-sec	HORIZONTAL AXIS Max+ arc-sec	HORIZONTAL AXIS Max- arc-sec	f Hz
1	1	19.6			.25	17.0			.2
	2	37.2			.2	25.8			.2
	4	44.3	147	134	.18	36.9	101	104	.15
	6	28.5	38	82	.15	32.7	93	112	.15
2	1	21.2			.15	15.9			.12
	2	32.4			.2	37.9			.15
	4	71.1	174	183	.2	37.3	91	120	.2
	6	29.4	73	93	.2	38.8	118	88	.15
3	1	24.7	63	49	.2	15.9	52	50	.3
	2	39.2	60	48	.15	30.3	51	65	.15
4	1	17.9	47	43	.15	21.0	62	52	.12
	4	63.1	107	135	.2	47.3	92	123	.12
	6	31.5	89	58	.15	28.6	72	75	.3
5	1	13.2	34	32	.12	17.7	48	46	.12
	4	36.9	65	50	.2	42.5	121	118	.12

TABLE 4-4. USS REEVES (DLG-24) POWER SPECTRAL DENSITY SUMMARY (Cont'd)

Run No.	System No.	RMS arc-sec	VERTICAL AXIS		f Hz	RMS arc-sec	HORIZONTAL AXIS		f Hz
			Max+ arc-sec	Max- arc-sec			Max+ arc-sec	Max- arc-sec	
	6	23.5	59	62	.12	21.9	71	52	.2
6	1	27.7	70	79	.15	25.6	79	85	.15
	2	97.3	251	231	.15	41.9	126	113	.2
	4	78.8	135	172	.2	39.2	101	88	.15
	6	43.3	67	90	.15	48.3	102	100	.2
7	1	30.2	70	96	.12	28.2	87	76	.15
	4	100.8	136	129	.2	37.2	80	79	.3
	6	69.6	163	176	.2	61.6	159	210	.2
8	1	19.3	55	48	.2	13.0	35	34	.15
	4	84.5	199	140	.3	41.6	90	77	.2
	6	50.9	154	131	.15	47.3	160	108	.2
9	1	11.9	40	35	.2	12.7	35	35	.1
	4	22.0	60	50	.15	29.9	75	76	.12
	6	18.0	40	30	.12	21.4	39	35	.15
10	1	18.5	35	40	.15	20.2	35	35	.12

TABLE 4-4. USS REEVES (DLG-24) POWER SPECTRAL DENSITY SUMMARY (Cont'd)

Run No.	System No.	RMS arc-sec	VERTICAL AXIS		f Hz	RMS arc-sec	HORIZONTAL AXIS		f Hz
			Max+ arc-sec	Max- arc-sec			Max+ arc-sec	Max- arc-sec	
	4	34.0	70	75	.2	35.2	70	90	.12
	6	27.3	40	60	.15	35.1	65	75	.15

TABLE 4-5. USS REEVES (DLG-24) AMPLITUDE TIME HISTORY SUMMARY
(0.0125 to 0.3 Frequency Range)

Run No.	System No.	RMS arc-sec	VERTICAL AXIS		RMS arc-sec	HORIZONTAL AXIS	
			Max+ arc-sec	Max- arc-sec		Max+ arc-sec	Max- arc-sec
1	1	17.2	44	66	12.0	33	33
	2	64.5	196	101	27.1	111	77
	4	48.8	166	185	39.2	117	94
	6	35.3	121	102	31.8	104	105
2	1	19.5	56	62	12.4	41	38
	2	61.5	198	166	41.0	106	123
	4	83.5	249	227	45.1	150	125
	6	38.4	168	168	44.0	161	161
3	1	24.2	60	78	10.4	44	34
	2	39.7	190	114	34.5	197	90
4	1	23.0	68	70	19.8	58	69
	4	79.6	231	279	71.4	218	185
	6	43.6	116	162	37.7	103	119

TABLE 4-5. USS REEVES (DLG-24) AMPLITUDE TIME HISTORY SUMMARY (Cont'd)

Run No.	System No.	VERTICAL AXIS RMS arc-sec	VERTICAL AXIS Max+ arc-sec	VERTICAL AXIS Max- arc-sec	HORIZONTAL AXIS RMS arc-sec	HORIZONTAL AXIS Max+ arc-sec	HORIZONTAL AXIS Max- arc-sec
5	4	42.3	138	167	71.2	189	189
	6	55.5	167	140	26.5	109	69
6	1	30.9	81	101	10.4	29	27
	2	107.0	308	281	46.5	160	145
	4	107.0	304	362	39.8	144	141
	6	63.1	183	270	58.0	153	189
7	4	111.0	317	303	38.5	104	134
	6	84.7	273	322	57.2	175	168
8	4	90.7	257	253	59.3	188	173
	6	55.7	186	169	44.9	117	127
9	4	25.8	85	72	37.1	123	103
	6	15.5	44	47	23.6	83	73
10	4	24.5	80	67	33.2	109	91
	6	24.0	64	71	38.1	85	85

TABLE 4-6. USS REEVES (DLG-24) SUMMARY OF SUMMED DATA

Run No.	System No.	RMS arc-sec	VERTICAL AXIS Max+ arc-sec	VERTICAL AXIS Max- arc-sec	F Hz	BW* Hz
2	1+2	41.0	118	80	.2	.1-10
2		70.0	221	241	.05-.25	.0125-1.0
2		60.7	170	190		.0125-.3
6		110.8	192	221	.2	.1-10
6		124.0	387	348		.0125-.3
2	1+2-4	72.9	184	198	.3	.1-10
2		77.0	211	290		.0125-.3
6		114.2	195	230	.2	.1-10
6		102.0	328	262		.0125-.3
2	1+2+6	58.2	153	131	.2	.1-10
6		170.0	618	485		.0125-.3

*Data analysis bandwidth

TABLE 4-7. USS HORNE (DLG-30) POWER SPECTRAL DENSITY SUMMARY

Run No.	System No.	RMS arc-sec	f Hz	RMS arc-sec	f Hz	BW Hz
1	1	6.3	.4	12.4	.2	.1-10
	2	54.6	.3	21.9	.35	.1-10
	4	23.9	.3	18.9	.25	.1-10
	6	15.5	.35	23.3	.4	.1-10
2	1	15.7	.2	23.6	.2	.1-10
	2	80.2	.25	54.2	.25	.1-10
	4	40.0	.25	45.3	.2	.1-10
	6	38.9	.3	39.4	.3	.1-10
3	1	7.6	.2	10.0	.2	.1-10
	2	54.9	.2	22.5	.25	.1-10
	4	18.6	.25	30.2	.3	.1-10
	6	28.9	.25	21.2	.25	.1-10
4	1			24.0	.2	.1-10
	2	48.5	.3	39.0	.3	.1-10
	2	54.5	.02	41.7	.02,.2	.0125-1.0
	4	36.2	.25	39.0	.25	.1-10

TABLE 4-7. USS HORNE (DLG-30) POWER SPECTRAL DENSITY SUMMARY (Cont'd)

Run No.	System No.	RMS arc-sec	f Hz	RMS arc-sec	f Hz	BW Hz
	6	24.2	.25	28.5	.3	.1-10
5	1			12.4	.25	.1-10
	2	25.8	.2	20.1	.2	.1-10
	2	46.6	.05-.22			.0125-1.0
	4	21.3	.25	16.9	.2	.1-10
	6	19.3	.3	11.8	.4	.1-10
6	4	26.2	.2	51.3	.2	.1-10
	6	28.5	.2	24.0	.15	.1-10
7	1	20.7	.2	19.8	.2	.1-10
	1	17.0	.02,.1	16.9	.1	.0125-1.0
	4	26.9	.2	40.8	.2	.1-10
	4	34.2	.14	34.5	.11	.0125-1.0
	6	39.0	.2	35.5	.25	.1-10
	6	56.4	.14	38.6	.1	.0125-1.0
8	2	26.2	.2	23.5	.2	.1-10
	4	47.9	.2	54.2	.25	.1-10

TABLE 4-7. USS HORNE (DLG-30) POWER SPECTRAL DENSITY SUMMARY (Cont'd)

Run No.	System No.	RMS arc-sec	f Hz	RMS arc-sec	f Hz	BW Hz
	6	23.6	.2			.1-10
9	2	12.2	.3	11.3	.3	.1-10
	4	15.0	.2	22.3	.2	.1-10
	6	9.2	.25	7.2	.25	.1-10
	6	30.1	.06	18.5	.06	.0125-1.0

TABLE 4-8. USS HORNE (DLG-30) SUMMARY OF SUMMED DATA

Run No.	System No.	RMS arc-sec	VERTICAL AXIS Max+ arc-sec	Max- arc-sec	f Hz	BW Hz
3	2 - 4	38.2			.2	.1-10
3		51.0	191	189		.0125-.3
4		51.3			.2	.1-10
4		61.4	139	160	.05-.25	.0125-1.0
4		58.9	168	168		.0125-.3
6		39.5			.1	.1-10
6		89.7	314	324		.0125-.3
8		33.9			.1	.1-10
8		90.7	271	221		.0125-.3
3	2 + 6	42.2			.2	.1-10
3		50.1	156 195			.0125-.3
4		48.2			.2	.1-10
4		65.1	143	157	.05-.25	.0125-1.0
4		58.5	157	170		.0125-.3
6		41.4			.1	.1-10
6		78.6	305	227		.0125-.3

TABLE 4-9. USS BARNEY (DDG-6) ENVIRONMENTAL DATA

Test Run Number	1	2	3	4	5	6	7
Date	4-25-73	4-25-73	4-26-73	5-14-73	5-15-73	5-15-73	5-15-73
Time	1530	1650	1905	2335	0010	0054	0131
Air Temperature (°F)	51	72	75	79	79	79	79
Water Temperature (°F)	62	65	78	83	83	83	83
Rel Wind Speed (knots)	9	9	15	2.5	5	27	27
Rel Wind Direction (°)	310	320	340	210	063	045	013
Ship's Heading (°)	164	164	168	270	000	045	090
Ship's Heading WRT Waves (°)	030	225	295	180	90	045	000
Ship Speed (knots)	14	14	18	13	13	13	13
Shaft Speed (RPM)	120	120	144	104	104	104	104
Average Roll (°)	1-2	2-3	2-3	2-3	2-3	2-3	2-3
Average Pitch (°)	1	1-2	1-2	1-2	1-2	1-2	1-2
Sea State	1	1	1	2	2	2	2

Ship Heading WRT Waves: 0° to 30° head seas, 30° to 60° bow seas, 60° to 120° beam seas, 120° to 150° quartering seas, 150° to 180° following seas (symmetric about ships center line)

TABLE 4-9. USS BARNEY (DDG-6) ENVIRONMENTAL DATA (Cont'd)

Test Run Number	8	9	10	11	12	13	14
Date	5-15-73	5-15-73	5-15-73	5-15-73	5-15-73	5-15-73	5-15-73
Time	0200	0235	0305	0347	0430	2235	2345
Air Temperature (°F)	79	79	79	79	79	80	80
Water Temperature (°F)	83	83	83	83	83	84	84
Rel Wind Speed (knots)	42	22	22	13	10	7	7
Rel Wind Direction (°)	020	030	0	245	300	067	067
Ship's Heading (°)	090	090	135	225	225	330	330
Ship's Heading WRT Waves (°)	000	000	315	135	135	135	135
Ship Speed (knots)	18	5	5	5	13	20	20
Shaft Speed (RPM)	144	040	040	040	104	160	160
Average Roll (°)	2-3	2-3	15	2-3	2-3	10	10
Average Pitch (°)	1-2	1-2	1-2	1-2	1-2	2-3	2-3
Sea State	2	2	2	2	2	3	3

Ship Heading WRT Waves: 0° to 30° head seas, 30° to 60° bow seas, 60° to 120° beam seas, 120° to 150° quartering seas, 150° to 180° following seas (symmetric about ships center line)

TABLE 4-9. USS BARNEY (DDG-6) ENVIRONMENTAL DATA (Cont'd)

Test Run Number	15	16	17	18	19	20	21
Date	5-16-73	5-17-73	5-17-73	5-17-73	5-17-73	5-18-73	5-18-73
Time	2215	0050	1610		1037	0045	0235
Air Temperature (°F)	78	78	*	71	71	71	74
Water Temperature (°F)	80	80	*	74	74	76	76
Rel Wind Speed (knots)	02	02	*	14	14	10	22
Rel Wind Direction (°)	020	320	*	300	300	270	290
Ship's Heading (°)	333	333	*	333	333	333	320
Ship's Heading WRT Waves (°)	135	135	*	270	270	270	315
Ship Speed (knots)	20	20	*	20	20	20	20
Shaft Speed (RPM)	160	160	*	160	160	160	160
Average Roll (°)	2-3	2-3	*	1-2	1-2	15	15
Average Pitch (°)	1-2	1-2	*	1-2	1-2	2-3	2-3
Sea State	2	2	*	1-2	1-2	3	4

*Noise Run Test

Ship Heading WRT Waves: 0° to 30° head seas, 30° to 60° bow seas, 60° to 120° beam seas, 120° to 150° quartering seas, 150° to 180° following seas (symmetric about ships center line)

TABLE 4-10. USS ROBISON (DDG-12) ENVIRONMENTAL DATA

Test Run Number	1	2	3	4	5
Date	4/14/72	4/14/72	4/14/72	4/14/72	4/14/72
Time	0855	1050	1118	1140	1405
Air Temperature (°F)	57	57	57	57	57
Water Temperature (°F)	60	60	60	60	60
Rel Wind Speed (knots)	25	20	30	30	15
Rel Wind Direction (°)	165	165	165	140	210
Ship's Heading (°)	220	220	222	220	026
Ship's Heading WRT Waves (°)	330	330	320	330	140
Ship Speed Knots	20	15	23	23	11
Shaft Speed (RPM)	160	110	165	160	100
Average Roll (°)	2	2-4	2-4	2-4	2-4
Average Pitch (°)	1	1	1	1	1
Sea State	1	1-2	1-2	1-2	1-2

Ship Heading WRT Waves: 0° to 30° head seas, 30° to 60° bow seas, 60° to 120° beam seas, 120° to 150° quartering seas, 150° to 180° following seas (symmetric about ships center line)

TABLE 4-10. USS ROBISON (DDG-12) ENVIRONMENTAL DATA (Cont'd)

Test Run Number	6	7	8	9	10
Date	4/14/72	4/14/72	4/14/72	4/14/72	4/14/72
Time	1501	0900	1425	1245	1310
Air Temperature (°F)	57	60	60	62	62
Water Temperature (°F)	60	60	60	60	60
Rel Wind Speed (knots)	35	22	30	7	10
Rel Wind Direction (°)	230	140	200	230	170
Ship's Heading (°)	051	305	142	182	270
Ship's Heading WRT Waves (°)	160	040	180	70	160
Ship Speed (knots)	24	17.5	22.5	3	10
Shaft Speed (RPM)	165	144	165	030	070
Average Roll (°)	1-2	2-3	1-2	2	1-2
Average Pitch (°)	1	1	1	1	1
Sea State	1-2	1-2	1-2	1	1

Ship Heading WRT Waves: 0° to 30° head seas, 30° to 60° bow seas, 60° to 120° beam seas, 120° to 150° quartering seas, 150° to 180° following seas (symmetric about ships center line)

TABLE 4-11. USS REEVES (DLG-24) ENVIRONMENTAL DATA

Test Run Number	1	2	3	4	5
Date	6-26-72	6-26-72	6-27-72	6-27-72	6-27-72
Time	1900	2200	1150	1933	2000
Air Temperature (°F)	79	74	79	75	75
Water Temperature (°F)	79	79	79	79	79
Rel Wind Speed (knots)	10.5	23	10	22	10
Rel Wind Direction (°)	239	045	260	315	275
Ship's Heading (°)	189	330	180	135	180
Ship's Heading WRT Waves (°)	225	045	240	310	260
Ship Speed (knots)	15	15	15	15	15
Shaft Speed (RPM)	140	129	129	129	129
Average Roll (°)	2-4	3	10	10-15	6
Average Pitch (°)	1-2	2	2	1	2
Sea State	2	3	4	3	3

Ship Heading WRT Waves: 0° to 30° head seas, 30° to 60° bow seas, 60° to 120° beam seas, 120° to 150° quartering seas, 150° to 180° following seas (symmetric about ships center line)

TABLE 4-11. USS REEVES (DLG-24) ENVIRONMENTAL DATA (Cont'd)

Test Run Number	6	7	8	9	10
Date	6-27-72	6-27-72	6-27-72	6-28-72	6-28-72
Time	2030	2050	2115	1940	2100
Air Temperature (°F)	74	75	74	74	74
Water Temperature (°F)	79	79	79	79	79
Rel Wind Speed (knots)	30	30	20	9	4
Rel Wind Direction (°)	350	025	050	160	015
Ship's Heading (°)	090	045	000	270	270
Ship's Heading WRT Waves (°)	350	020	050	310	310
Ship Speed (knots)	15	15	15	5	2
Shaft Speed (RPM)	129	129	129	042	191
Average Roll (°)	2-3	6	8	3	5
Average Pitch (°)	4	4	3	1	2
Sea State	3	3	3	2	2

Ship Heading WRT Waves: 0° to 30° head seas, 30° to 60° bow seas, 60° to 120° beam seas, 120° to 150° quartering seas, 150° to 180° following seas (symmetric about ships center line)

TABLE 4-12. USS HORNE (DLG-30) ENVIRONMENTAL DATA

Test Run Number	1	2	3	4	5	6	7	8	9
Date	7/5	7/6	7/8	7/8	7/10	7/11	7/11	7/13	7/14
Time	1915	2150	1900	2210	2200	2300	2345	2345	2230
Air Temperature (°F)	80	79	76	76	74	70	71	66	62
Water Temperature (°F)	80	79	75	75	70	65	65	57	52
Rel Wind Speed (knots)	35	35	28	25	17	25	25	16	6
Rel Wind Direction (°)	000	330	345	030	340	240	340	330	035
Ship's Heading (°)	340	125	120	035	332	045	270	045	115
Ship's Heading WRT Waves (°)	045	330	340	055	330	N/A	040	000	180
Ship Speed (knots)	27	15	10	13	17	13	10	16	14
Shaft Speed (RPM)	189	103	66	90	117	90	69	117	107
Average Roll (°)	2	5	3.5	4	2	13-15	10	5	1
Average Pitch (°)	1.5	3-4	2	2-3	1-2	2	3	1	<1
Sea State	2	3	2	2	1	Swells	Swells	2	1

Ship Heading WRT Waves: 0° to 30° head seas, 30° to 60° bow seas, 60° to 120° beam seas, 120° to 150° quartering seas, 150° to 180° following seas (symmetric about ship's center line)

TABLE 4-13. USS BARNEY (DDG-6) AND USS ROBISON (DDG-12) RPI TOLERANCES FOR FLEXURE MEASUREMENT PATHS

Reference Element		Tolerance* (arc/min)	System No's.	
			ROBISON	BARNEY
Director MK 68 to	Director MK 73	$\pm 3'$	3	2
Director MK 68 to	Launcher MK 8	$\pm 5'$	2 + 3	2 + 3
Director MK 68 to	Radar AN/SPS-39	$\pm 10'$	-	5
Director MK 68 to	5"-54 gun mounts	$\pm 2'$	2 + 3	2 + 3 (aft)

*Tolerance values obtained from NAVSHIPS 378-0382, Shipyard Weapon System Alignment Manual.

TABLE 4-14. USS REEVES (DLG-24)
RPI TOLERANCES FOR FLEXURE MEASUREMENT PATHS

Weapon System Elements			Tolerance* (arc-minutes)	Measuring Systems
FWD director	to	FWD launcher	±3	6
FWD director	to	FWD gyro	±3	6
FWD director	to	AFT missile battery	-	1 + 2
FWD director	to	AFT gyro	±3'	1 + 2
FWD director	to	3-D radar	±2	4
3-D radar	to	AFT missile battery	±5	1 + 2 - 4
FWD gyro	to	AFT gyro	-	1 + 2 + 6
FWD gyro	to	AFT missile battery	±3'	1 + 2 + 6
FWD launcher	to	AFT missile battery	-	1 + 2 + 6

*Tolerance values obtained for NAVSHIPS 378-0382, Shipyard Weapon System Alignment Manual.

TABLE 4-15. USS HORNE (DLG-30)
RPI TOLERANCES FOR FLEXURE MEASUREMENT PATHS

Weapon System Elements			Tolerance **	System No's
FWD director	to	FWD launcher	±3'	6
FWD director	to	FWD gyro or mean plane	±3'	6
FWD director	to	AFT gyro or mean plane		2 + 1*
FWD director	to	3-D radar	±2'	4
FWD director	to	GFCS MK 68	-	2 + 1*
GFCS MK 68	to	Gun	±2'	
GFCS MK 68	to	3-D radar	-	2 + 1 - 4*
GFCS MK 68	to	FWD gyro or mean plane	-	2 + 1*
GFCS MK 68	to	AFT gyro or mean plane		

*Note: Total flexure over the path measured by systems 1 and 2 was essentially that measured by system 2 alone, as system 1 had very low RMS levels.

**Tolerance values obtained from NAVSHIPS 378-0382, Shipyard Weapon System Alignment Manual.

SECTION 5

RESULTS AND DISCUSSION

5-1. TOLERANCES. The data contained in this report verify the existence of dynamic flexure between weapon system elements. However, at no time did a measured RMS (1 σ) flexure level exceed a specified weapon system tolerance. There were occasions when certain RPI tolerances were exceeded at the 2- or 3-σ level, however, but there were so few such instances that they were not considered to be detrimental to the successful operation of the TARTAR or TERRIER weapons systems. The only possible area of concern might lie with the aft 5"-54 gun in the DDG-2 Class where the 2 arc-minute operational tolerance between it and the MK 68 director (forward) was exceeded at the 2-σ and 3-σ levels in sea states greater than 1.

5-2. CYCLIC VARIATIONS. Although this report is not all-inclusive, it does indicate the relative magnitude and frequency of flexure to be expected in the types of seas for which measurements were obtained. Dynamic flexure tends to be cyclic in nature, with the length of a cycle or period ranging between .5 and 80 seconds. The predominant period appears to be about 5 seconds in length. This cyclic variation was measured about a mean which remained essentially constant for the length of each data run, indicating that dynamic flexure tends to be elastic in nature (deformations are not permanent).

5-3. VARIABLES. Because of the many variables involved, it is almost impossible to identify numbers which are universally representative of flexure magnitudes. In spite of this, a critical examination of data summary tables in Section 4 indicates that occasionally the measured RMS level approaches 2 arc-minutes and quite often this number is exceeded at the 2-σ level of confidence. The implication of this is that 2 arc-minutes appears to be the minimum alignment tolerance which is realistically maintainable. Also, 2 arc-minutes should be a good figure to use as representative of the contribution of dynamic flexure to an error budget analysis, although there might be occasions where individual circumstances would dictate a different criterion.

5-3.1. This would not necessarily apply to weapon elements located in very close proximity on the same deck, as tighter tolerances could probably then be maintained. It should be remembered that limits inferred from statistical data are not absolute, and can only be interpreted in light of the various conditions of the test (ship class, sea state, etc.,) and in regard to the required confidence level (1 σ, 2 σ, 3 σ). Long term flexure resulting from redistribution of a ship's static load, as reported in NSWSES Technical Report No. 277 Static Flexure Measurements, USS STEIN (DE-1065), also indicates that 2 arc-minutes is a minimum realistic tolerance.

SECTION 6

CONCLUSIONS

6-1. Based on the analyses and results described in Section 4 of this report, it is concluded that:

a. It is important that consideration be given to the effects of dynamic flexure when specifying roller path inclination (RPI) and foundation machining tolerances.

b. Mechanical devices such as leveling rings or electronic devices or software corrections should be utilized extensively in the design of new equipment, and serious consideration should be given when corrections are required on existing equipment.

c. Data obtained in the dynamic flexure measurements program are considered to be sufficiently comprehensive and representative of flexure in DDG and DLG class ships. However, there may be a need to take measurements on newer type ships, particular hydrofoil and surface-effect vessels.

SECTION 7

RECOMMENDATIONS

7-1. Based on the preceding conclusions, it is recommended that:

a. Existing tolerances be re-evaluated under considerations of dynamic flexure when roller path inclination (RPI) and foundation machining tolerances are specified. This process will eliminate costly machining to obtain tolerances which are not realistically maintainable.

b. RPI tolerances less than 2 arc-minutes not be invoked in any new construction or modifications to existing systems.

c. Where RPI tolerances are in the 2- to 6-minute range, a means be provided for correcting out-of-tolerance inclination without re-machining the element foundation.

d. On the basis of sufficiently comprehensive and representative data, no additional measurements be taken on DDG and DLG class ships, unless specific need arises.

APPENDIX A

STATISTICAL ANALYSIS REVIEW

The following discussion is neither intended to be rigorous nor comprehensive. Its purpose is simply to recall certain of the more important concepts relative to statistical analysis which may have been forgotten through disuse.

Important Concepts

Given a random variable $\underline{x}$ and a real number x, the probability that $\underline{x} \leq x$ is denoted by $P\{\underline{x} \leq x\}$.

Distribution Function: The distribution function of the random variable $\underline{x}$ is denoted by $F_{\underline{x}}(x)$ and is defined for any x from $-\infty$ to ∞ by $F_{\underline{x}}(x) = P\{\underline{x} \leq x\}$.

Density Function: The density function of the random variable $\underline{x}$ is denoted by $f_{\underline{x}}(x)$ and is given by $f_{\underline{x}}(x) = \dfrac{d\,F_{\underline{x}}(x)}{dx}$; i.e., it is the derivative of the distribution function.

It will be stated without proof that the following certain useful properties hold:

a. $F_{\underline{x}}(x)$ is nondecreasing (monotonicity); that is, $F_{\underline{x}}(x + h) \geq F_{\underline{x}}(x)$ if $h > 0$. This implies that $F(x) \geq 0$.

b. $\lim\limits_{x \to -\infty} F_{\underline{x}}(x) = 0$; $\lim\limits_{x \to \infty} F_{\underline{x}}(x) = 1$

c. $0 \leq F_{\underline{x}}(x) \leq 1$

d. $\int_{-\infty}^{\infty} F(x)dx = F_{\underline{x}}(\infty) - F_{\underline{x}}(-\infty) = 1$

e. $$F_{\underline{x}}(x_2) - F_{\underline{x}}(x_1) = \int_{x_1}^{x_2} f(x)dx$$
$$= P\{x_1 \leq \underline{x} \leq x_2\}$$

Expected Value of $\underline{x}$: denoted by $E[\underline{x}]$ where $E[\underline{x}] = \int_{-\infty}^{\infty} x f(x) dx$

this is also known as the mean of $\underline{x}$

Variance of $\underline{x}$: The mean m of a random variable locates the center of gravity of $F(x)$. Another important parameter is its variance or disposition σ^2, defined by $\sigma^2 = E[(\underline{x} - m)^2] = \int_{-\infty}^{\infty} (x-m)^2 F(x) dx$

This quantity equals the moment of inertia of the probability masses and gives some idea about their concentration near m. Its positive square root is called the standard deviation. Note that

$$\sigma^2 = E[\underline{x}^2 - 2\underline{x}m + m^2] = E[x^2] - 2mE[\underline{x}] + m^2 = E[\underline{x}^2] - m^2$$

which leads to the important relationship

$$\sigma^2 = E[\underline{x}^2] - E^2[\underline{x}]$$

Up to this point, $\underline{x}$ has been assumed to be a random variable which takes on particular values for specific outcomes z_i of an experiment, $\underline{x}(z_i)$ representing the value of the random variable $\underline{x}$ for the particular outcome z_i. Now, if $\underline{x}$ is allowed to depend not only upon the outcomes z_i, but also upon time, then $\underline{x}$ becomes a random process and is written $\underline{x}(z_i, t)$. This implies that $\underline{x}$ represents four different things:

1. a family of time functions (t and z variables)

2. a single time function (t variable, z fixed)

3. a random variable (t fixed, z variable)

4. a single number (t fixed, z fixed)

For the purposes of this discussion $x(z_i, t)$ will be written as $\underline{x}(t)$. The Expected Value or Mean of a random process $\underline{x}(t)$ is given by the following:

$$E[\underline{x}(t)] = \int_{-\infty}^{\infty} x\, f(x;t) dx = m(t)$$

Note that, in general, this is a function of time.

The Autocorrelation $R(t_1, t_2)$ of a process $\underline{x}(t)$ is given by:

$$R(t_1, t_2) = E[\underline{x}(t_1)\underline{x}(t_2)] = \int_{-\infty}^{\infty} x_1 x_2 f(x_1, x_2, t_1, t_2) dx_1\, dx_2$$

and is a function of t_1 and t_2

The Autovariance $C(t_1, t_2)$ of $\underline{x}(t)$ is simply the autocorrelation with the mean value removed:

$$C(t_1, t_2) = E\{[\underline{x}(t_1) - m(t_1)]\, [\underline{x}(t_2) - m(t_2)]\}$$

The time dependence of these functions may be removed if the process $\underline{x}(t)$ is wide sense stationary. In this case,

$$E[\underline{x}(t)] = m, \text{ a constant}$$

Let $\tau = t_1 - t_2$, then the autocorrelation becomes:

$$R(\tau) = E[\underline{x}(t + \tau)\underline{x}(t)]$$

which depends only on the time difference $t_1 - t_2$. A key point here is that, from the practical point of view, it is not necessary that a process be stationary for all time but only for some observation interval which is long enough to be suitable for a given problem.

If all the statistics of a process can be determined from a single observation, then that process is said to be ergodic. Since the various statistical parameters are expressed as time averages, the above is often stated as follows: $\underline{x}(t)$ is ergodic if time averages of sample functions of the process can be used as approximations to the corresponding ensemble averages. The time average $A[\underline{x}(t)]$ of the process $\underline{x}(t)$ is given by

$$A[\underline{x}(t)] = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \underline{x}(t)dt$$

The ergodic theorem for $E[\underline{x}(t)]$ follows:

$$\lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \underline{x}(t)dt = E[\underline{x}(t)] = m$$

if and only if

$$\lim_{T\to\infty} \frac{1}{T} \int_{0}^{2T} (1 - \frac{\tau}{2T}) \; [R(\tau) - m^2]d\tau = 0$$

This condition generally applies to ship motion records. Therefore, any seakeeping record, assumed to be a stationary random process, may be analyzed to determine the behavior of all seakeeping records obtained in the same way, and so may be used to predict general ship performance for a given set of conditions.

The power spectrum or spectral density $S(\omega)$ of a process $\underline{x}(t)$ is the Fourier transform of its autocorrelation:

$$S(\omega) = \int_{-\infty}^{\infty} e^{-j\omega t} R(\tau)d\tau$$

If $\underline{x}(t)$ is ergodic, then the power spectral density $S(\omega)$ is related to a time average by the following:

$$\lim_{T\to\infty} E[\frac{1}{2T} X_T(\omega) \; X_T(-\omega)] = \lim_{T\to\infty} S_T(\omega)$$
$$= S(\omega)$$

where $X_T(\omega) = \int_{-T}^{T} x(t) e^{-j\omega t} dt$

The average power in the process $\overline{x^2}$ is equal to the integral of the power spectral density which is the area under the spectral curve:

$$\overline{x^2} = \frac{1}{2\pi} \int_{-\infty}^{\infty} S(\omega) d\omega = R(0)$$

If $x(t)$ is a voltage, then $S(\omega)$ has the units of volts^2 per hertz and its integral (above) leads to the mean-square value. If $x(t)$ is associated with a one-ohm resistance, then $\overline{x^2}$ is just the average power dissipated in that resistance. The spectral density, $S(\omega)$, can then be interpreted as the average power associated with a bandwidth of one hertz centered at $\omega/2\pi$ hertz.

The average power in $S(\omega)$ between frequencies F_1 and F_2 is given by

$$\overline{x^2} = B \int_{F_1}^{F_2} S(F) dF \qquad B = F_2 - F_1$$

Reliability of Power Spectrum Estimates

The final results are estimates of the spectral density distributed according to frequency. Estimates can be in error, and the important part of this method of analysis is that it also yields a statement of how much in error the estimates can be.

When certain parameters are known, such as the resolution of the spectrum and the number of sample points contained in the record from which the spectrum was computed, then confidence limits may be obtained which define the probable ensemble spread and consequently qualify the usefulness of the spectrum as a descriptive tool. The narrower the confidence limits, the more reliable the spectrum. Reliability can be increased by giving up resolution or by including more sample points in the record from which the spectrum was computed. Of course, if an infinite number of points could be included, then the spectrum would no longer be an estimate, but would be a true representation.

For further information see:

Athanasios Papoulis, Probability, Random Variables, and Stochastic Processes. New York: McGraw-Hill, 1965.

John B. Thomas, An Introduction to Statistical Communication Theory. New York: John Wiley & Sons, Inc., 1969.

George R. Cooper, Methods of Signal and System Analysis. New York: Holt, Rinehart and Winston, Inc., 1967.

APPENDIX B

International Sea State Code

Sea State Number	Term	Wave Height
0	glassy	0
1	rippled	0-1
2	smooth	1-2
3	slight	2-4
4	moderate	4-8
5	rough	8-13
6	very rough	13-20
7	high	20-30
8	very high	30-45
9	phenomenal	over 45

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