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DETERMINATION OF THE COMBINED VALUE OF DIVERSE WEAPON SYSTEMS B--ETC(U)

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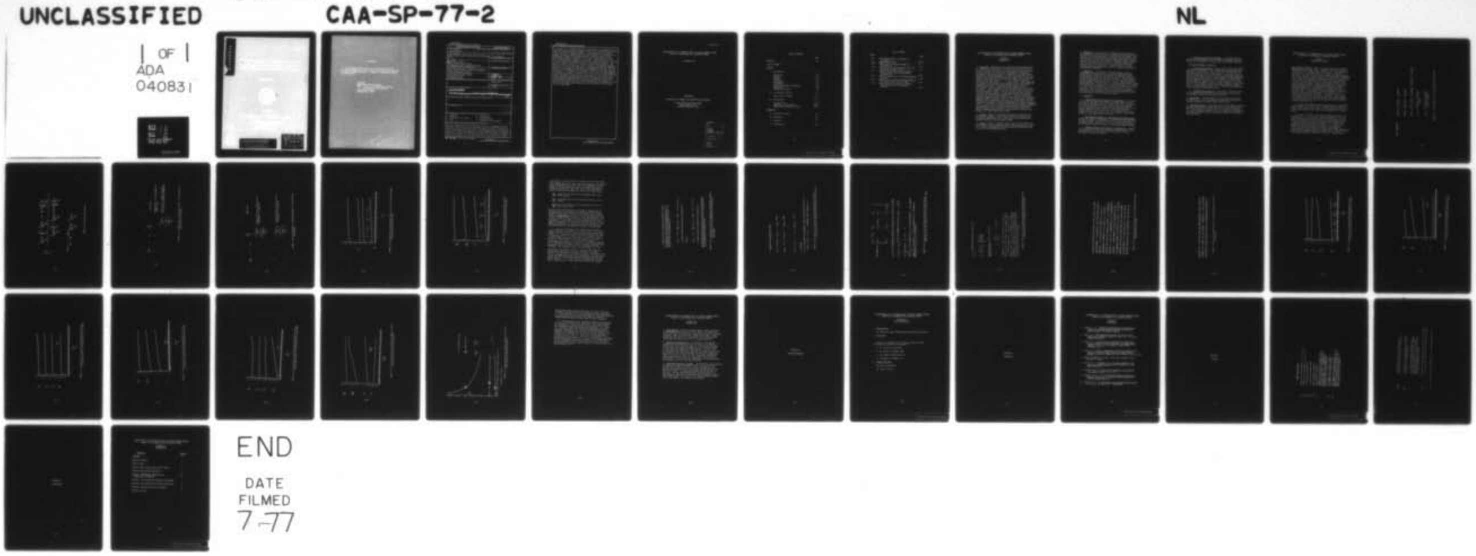

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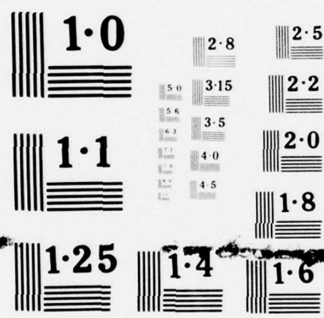

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7. linear weighting
8. homogeneous linear equation

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The paper addresses previous methodologies and results of concepts that have been used in the past by the U.S. and Great Britain for determining the total combined value of diverse weapon types in a weapons system mix. Previous work has all been based on solving a set of linear homogenous equations by means of eigenvalues. Specific methodologies are examined and it is shown that anomalous results are obtained, with regard to the effects of vulnerability. A simple change is introduced, which is shown to give consistent results with regard

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) to vulnerability. A number of Measures of Effectiveness are examined and it is shown that the old and even the modified models with a vulnerability correction suffer from another important standpoint. The results of all the models studied show that if a weapon kills an increasing number of the other side's weapons, the effectiveness decreases and then increases slowly, so that a considerable increase in the number of kills must be inflicted before the effectiveness begins to exceed its initial value. Counterintuitive results such as these present a dilemma for the analyst or decision maker who is depending upon a complex simulation with its many interactions and effects. The specific problem at this point is whether to believe the simulation. Even though the result was explainable from an algebraic or mathematical standpoint, the result in this case was considered to be unacceptable and a new and novel approach to the problem was conceived and is presented in the paper. The new approach avoids dependence on the eigenvalue solution that has been prominent in the past. It is shown to yield acceptable results for the various Measures of Effectiveness studied in the paper. As a result of the new methodology that has been developed for evaluating the effectiveness of a mixed force, certain insights have been obtained and are presented that may apply in many other fields where the eigenvalue method of solution has previously been the only accepted method of solution. By using the new approach, for example, it is shown that more general solutions are obtained and that the old eigenvalue solution falls out as one point in many general and more useful solutions when we use the new method.

CAA-SP-77-2

DETERMINATION OF THE COMBINED VALUE OF DIVERSE WEAPON SYSTEMS BASED ON A NEW GENERALIZED EIGENVALUE CONCEPT

DECEMBER 1976

PREPARED BY

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CHAPTER I
INTRODUCTION

1. Background. Methodologies for determining the combined value of a total force have been in existence and used both in the US and Great Britain since 1968. Mr. Alan Johnsrud, from the Office of the Coordinator of Army Studies (Reference 7) referenced a workshop on the state-of-the-art of mathematics in combat models, sponsored by the Army Mathematics Steering Committee, Jun 73. He referred to a whole body of work by Dare, James, Thrall, Anderson, Spudich, Holter and himself "to show all calculation methods leading to the same results . . . . Thus, there is good reason now to disseminate the procedure and recommend that it (the eigenvalue eigenvector technique) be used for weapon trade-off and cost effectiveness studies. (The COMCAP Study already used it.)" Mr. Farrell's paper (Reference 6) and Mr. Anderson of IDA in 1974 (Reference 1) have pointed out certain problems (but no solutions) with the eigenvalue method. Various versions and applications of a new generalized eigenvalue concept which avoided the problems previously encountered were briefed by Mr. Cohen to the Military Operations Research Society (MORS) and the Army Operations Research Symposium (AORS) (References 3 and 4). Mr. Cohen and Dr. Nussbaum briefed various mathematical applications and implications of the generalized concept to the American Mathematical Society (References 2 and 8). LTC Stebbins, US Army Concepts Analysis Agency, (Reference 9) has applied various versions of the method, including the one developed here to results obtained from the Combat Evaluation Model to demonstrate feasibility and the results of applying the concept to a large complex battle model.

2. Problem. Because of anomalies and conflicts resulting from the eigenvalue method, it has become necessary to develop a new concept and methodology that is sensitive to the contributions and deficiencies of various weapon systems.

3. Purpose. Battle simulations contain the attrition effects of many different kinds of weapon systems. The purpose of this study is to develop a methodology that can evaluate the combined value of a mix of diverse weapon systems.

4. Objectives. It is desired to illuminate the basic problems and anomalies involved with the old methods which have been used in the past and to show the effect of applying the new methods. The algebraic solutions that had been obtained involved many independent variables so that the anomalous interactions among variables had been obscured, and not previously presented, parametrically. Results are included in this study for selected existing and modified models of weapon worth. Important input and output variables have been sorted out by means of analytic and numerical computational procedures.

5. Assumptions. The results of the above process lead to a challenging of the conventional assumptions which have underpinned previous methods for determining the combined value of diverse weapon systems for two opposing forces. The conventional assumption used in the past has been that the value of a weapon system is proportional to the value it kills. The assumption of the new method is that the value of a weapon system is equal (rather than proportional) to the initial value of the system if it did no killing or was not killed plus the value due to the number of kills it causes minus the value of the kills inflicted on it by the other side. As a result of questioning previous assumptions, it has been possible to develop a new concept of weapon worth which radically departs from the eigenvalue concept.

6. Limitations.

a. Varying Value of the Proportionality Factor. As a result of using the old eigenvalue method, the proportionality factor could not be defined to be some constant value, like 1 but had to be solved for mathematically by a complex iterative procedure. This was because the set of equations could only be solved by the use of eigenvalues, which resulted in nonlinear equations. Another limitation was the fact that the proportionality factor varied from one set of input conditions to another, causing many anomalies, and counter-intuitive results.

b. Non Monatomic Behavior of Some Measures of Effectiveness. In addition, some of the Measures of Effectiveness, MOE, decreased and then increased, as the number of kills by a given system, i.e., the effectiveness, increased. It would have been expected that the MOE would continue to increase if the effectiveness increased.

c. Anomalous Behavior With Regard to Vulnerability. Another limitation has been that the results were either insensitive to the vulnerability of the weapon system doing the killing or sensitive in the wrong direction.

d. Effects of Using the New Concept. It is shown that the new concept which departs from reliance on eigenvalues and also corrects for vulnerability displays none of the above limitations.

7. Essential Elements of Analysis

a. Initial Value Concept. The use of the initial value term, discussed in the section titled "Assumptions" changed the entire character of the solution. Before the use of this concept, the resulting equations were all homogeneous in nature and linear in each of the unknown value terms, requiring an eigenvalue solution. The use of the initial value concept resulted in a non homogeneous set of equations and allowed a direct solution of value ratio terms by means of Cramer's rule. The initial value term is the initial value per weapon times the number of weapons. It develops that it is not necessary to know the initial value per weapon since it always appears linearly in the numerator and denominator of the solution and therefore cancels.

b. Vulnerability Correction. The effects of system vulnerability are taken into account by subtracting the value killed of the weapon system that is doing the killing.

8. Methodology. The methodology is fairly simple and involves solving a set of linear non homogeneous equations by means of a standard mathematical method, namely Cramer's rule.

9. Evaluation Criteria. Seven Measures of Effectiveness involving various combinations of Red and Blue value such as value of the two forces, value ratio killed, value of one system to another, etc., are determined parametrically over a wide range of conditions to determine whether there are any inconsistencies or anomalies with the method.

DETERMINATION OF THE COMBINED VALUE OF DIVERSE WEAPON SYSTEMS BASED ON A NEW GENERALIZED EIGENVALUE CONCEPT

CHAPTER II
DISCUSSION OF RESULTS

Description Of Figures. Figure II-1 shows the kinds of equations that one uses in a value model. Appendix C, Glossary, contains a description of the symbols shown on the Figures. For purposes of illustration, it is assumed that a Blue force is fighting a Red force. The Blue force has type 1B weapons and type 2B weapons and the Red force has type 1R weapons. The conventional methodology used in the past was based on the assumption that the total value of a system is proportional to the kills by the type 1B weapon of type 1R weapon times the value of each type 1R weapon. The proportionality constant is shown by the factor 1/K. Because of certain problems with the conventional methodology, (which will be shown subsequently), a vulnerability correction has been introduced which is the second term on the right hand side of the equal sign. The vulnerability term represents a subtraction from the value killed by the amount of the value of the 1B weapons that have been killed by 1R. An equation such as this is written for each type of weapon in the force. Value numbers are solved in terms of ratios and then put into various measures of effectiveness such as fraction of Red value killed, fraction of Blue value killed, etc.

a. Figure II-2 presents a generalized solution of the weapon worth concept in which the value of a weapon system is taken as proportional to the value of what it kills minus the value of the kills of the weapon system itself, which represents its vulnerability.

b. Figure II-3A shows that if the vulnerability term is not considered (i.e., V = 0) in the generalized equations, that the value ratio of the two systems is either insensitive to the respective vulnerabilities or is sensitive in the wrong direction. It was due to this fact that the vulnerability correction was introduced, which corresponds to letting V = 1. Figure II-3B shows that the proper effect is obtained. Unfortunately, Figures II-4A and II-4B show that even with a vulnerability correction that various measures of effectiveness (MOE) decrease and then increase as weapon quality increases, that is, the kills by one weapon of another is allowed to increase. (The abscissa,

BASIC EQUATIONS:

$$V_{1B}n_{1B} = \frac{1}{k} K_{1B,1R}V_{1R} - K_{1R,1B}V_{1B}$$

$$V_{2B}n_{2B} = \frac{1}{k} K_{2B,1R}V_{1R} - K_{1R,2B}V_{2B}$$

$$V_{1R}n_{1R} = \frac{1}{k}\left(K_{1R,1B}V_{1B} + K_{1R,2B}V_{2B}\right) - K_{1B,1R}V_{1R} - K_{2B,1R}V_{1R}$$

$$FRVK = \frac{K_{1B,1R}V_{1R} + K_{2B,1R}V_{1R}}{V_{1R}n_{1R}}$$

$$FBVK = \frac{K_{1R,1B}V_{1B} + K_{1R,2B}V_{2B}}{V_{1B}n_{1B} + V_{2B}n_{2B}}$$

FIGURE II-1, Value Model Based on Vulnerability Correction

$$\frac{V_{2B}}{V_{1B}} = \frac{\frac{K_{2B,1R}}{n_{1R}} \left(1 + \frac{K_{1R,1B}}{n_{1B}} (V - NF.5)\right)^{\frac{n_{1B}}{n_{1R}}}}{\frac{K_{1B,1R}}{n_{1R}} \left(1 + \frac{K_{1R,2B}}{n_{2B}} (V - NF.5)\right)^{\frac{n_{2B}}{n_{1R}}}}$$

LET $n_{1B} = n_{2B}$ AND $\frac{K_{2B,1R}}{n_{1R}} = \frac{K_{1B,1R}}{n_{1R}}$

FIGURE II-2, Generalized Solution

MODEL	V	NF	$\frac{V_{2B}}{V_{1B}}$	CONCLUSION
1	0	0	1	INSENSITIVE, WITH NO VULNERABILITY CORRECTION.
2	0	1	$\frac{\left(1 - \frac{K_{1R,1B}}{n_{1B}}\right)^{\frac{1}{2}}}{\left(1 - \frac{K_{1R,2B}}{n_{2B}}\right)^{\frac{1}{2}}}$	SENSITIVE BUT IN WRONG DIRECTION, WITH NO VULNERABILITY CORRECTION.

FIGURE II-3A, Sensitivity of Eigenvalue Models to Vulnerability
(continued on next page)

MODEL	V	NF	$\frac{V_{2B}}{V_{1B}}$	CONCLUSION
3	1	0	$\dfrac{1 + \dfrac{K_{1R,1B}(1)}{n_{1B}}}{1 + \dfrac{K_{1R,2B}(1)}{n_{2B}}}$	SENSITIVE IN RIGHT DIRECTION, WITH **VULNERABILITY** CORRECTION.
4	1	1	$\dfrac{1 + \dfrac{K_{1R,1B}}{n_{1B}}\dfrac{1}{2}}{1 + \dfrac{K_{1R,2B}}{n_{2B}}\dfrac{1}{2}}$	SENSITIVE IN RIGHT DIRECTION, WITH VULNERABILITY CORRECTION.

FIGURE II-3B, Sensitivity of Eigenvalue Models to Vulnerability (concluded)

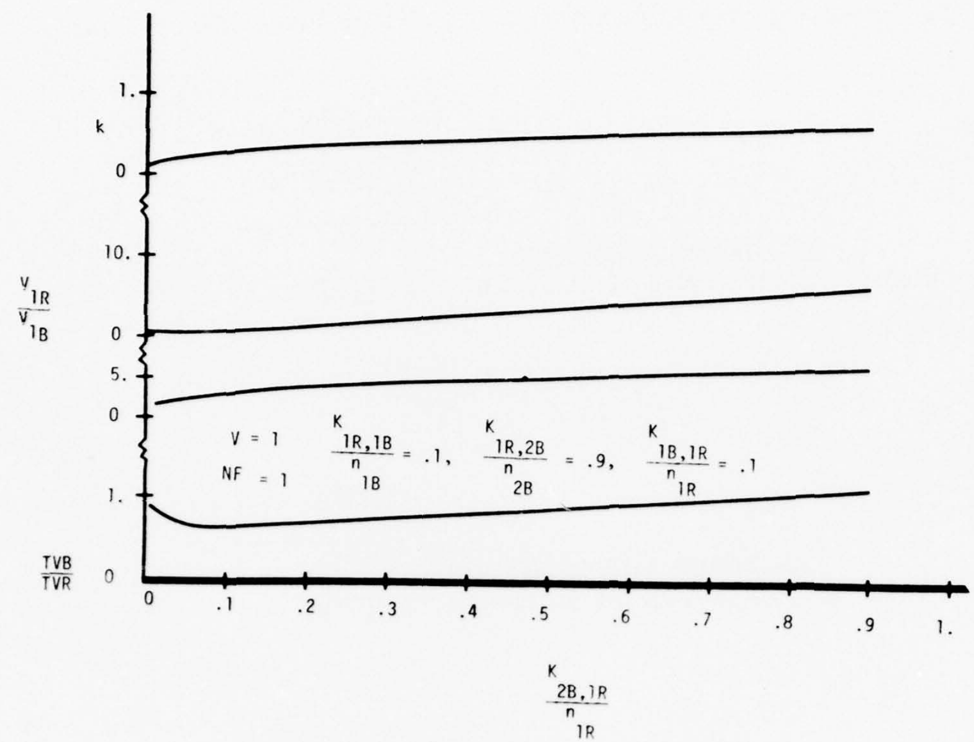

FIGURE II-4A, Sensitivity of Eigenvalue Models to Kills by The Blue System of Red (continued on next page)

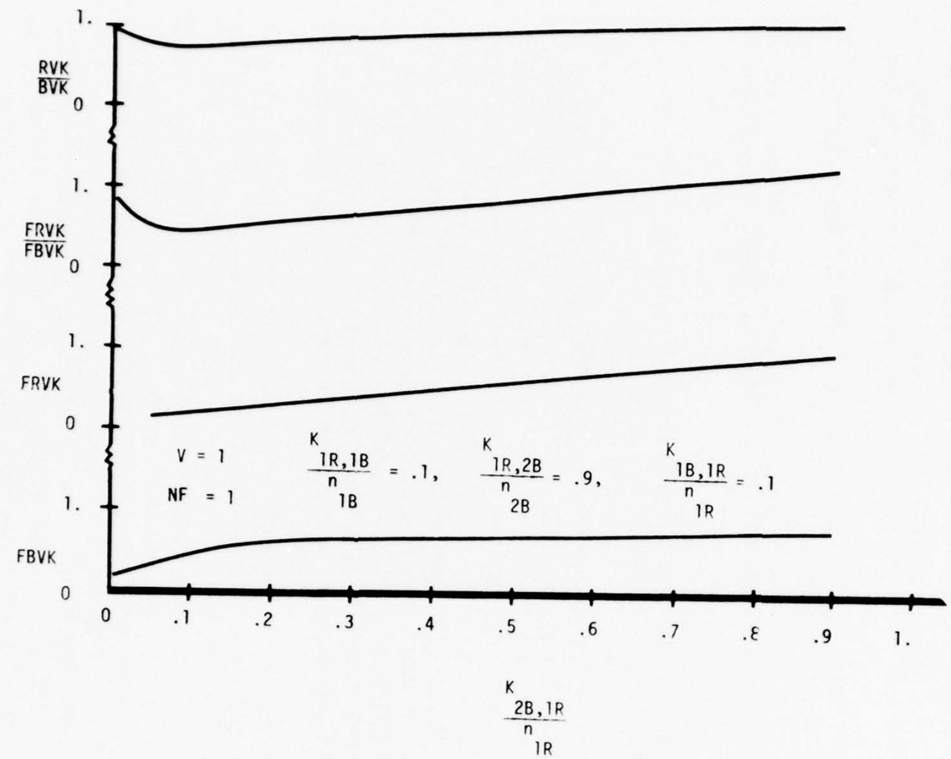

FIGURE II-4B, Sensitivity of Eigenvalue Models to Kills by The Blue System of Red (concluded)

($K_{2B,1R}/n_{1R}$) is actually the total number of kills by the type 2 Blue weapon type of the type 1 Red type divided by the number of type 1 Red weapon types. The quantity in parenthesis may be thought of as the fraction of type 1 Red weapon types that are killed by type 2 Blue.) MOEs that react in this misleading manner for the decision maker are shown in the figures to be:

$\frac{TVB}{TVR}$, total value of Blue forces divided by total value of Red forces;

$\frac{RVK}{BVK}$, total Red value killed divided by total Blue value killed;

$\frac{FRVK}{FBVK}$, fraction of Red value killed divided by fraction of Blue value killed.

Because of the unfortunate dip in the MOE, as weapon quality was increased, it was decided to discard the old conventional assumptions that led to the result that the value of a weapon system is proportional to the value killed (the proportionality "constant" mathematically turned out to be the eigenvalue, and in fact was not a constant at all but varied with each new case considered.) A new concept was then developed which did not depend on eigenvalue. It is based on solving a system of linear homogeneous equations by adding a constant to the right hand side, and solving for the variables in terms of the constant.

c. Figures II-5A through II-6B present the basic theory, solution and physical significance of the results. After using Cramer's rule for solving a linear system of equation by determinants, it is seen from Figures II-5C and II-5D that the result is independent of "A". In other words, "A" can be as large or as small as desired without effecting the value ratio shown by the x_i/x_j, in the figure. If in the original system of linear equations therefore, the value "A" is allowed to approach 0 it is seen that the system of equations begins to approach the homogeneous set of equations with the right hand side zero, in the limit. The constant k remains in the cofactor A_{ij}. An application of the method results in the above MOE constantly increasing as they should, as weapon quality is increased. Results illustrating this are shown in Figures II-7A through II-7F. Figure II-8 demonstrates that the non-eigenvalue solution is more general than the conventional eigenvalue solution that has been used in the past. It is seen that the non-eigenvalue solution results shown by the x's give smooth curves which pass through

1. GENERALIZED SYSTEM OF EQUATIONS FOR WEAPON WORTH CONCEPT.

THE TOTAL VALUE OF THE i^{th} BLUE WEAPON SYSTEM IS EQUAL TO THE INITIAL VALUE (IF IT DIDN'T KILL AND RECEIVED NO KILLS) + THE CONTRIBUTION TO BLUE VALUE DUE TO RED VALUE KILLED - THE VALUE OF ALL THE iB TYPE WEAPONS KILLED.

$$V_{iB}a'_{iB} = b_{iB}A + \frac{1}{k}\sum_{jR} a_{iB,jR}V_{jR} - V_{iB}a''_{iB}V \quad \text{for } jR = 1R,\ 2R,\ \text{etc.}$$

(A SEPARATE EQUATION IS REQUIRED FOR EACH SUBSCRIPT iB).

SIMILARLY,

$$V_{jR}a'_{jR} = b_{jR}A + \frac{1}{k}\sum_{iB} a_{jR,iB}V_{iB} - V_{jR}a''_{jR}V \quad \text{for } iB = 1B,\ 2B,\ \text{etc.}$$

(A SEPARATE EQUATION IS REQUIRED FOR EACH SUBSCRIPT jR).

MULTIPLYING BOTH EQUATIONS THRU BY k AND FACTORING OUT SIMILAR VARIABLES THE ABOVE EQUATIONS ARE THEN REPRESENTED BY A SET OF EQUATIONS OF THE FOLLOWING MORE GENERAL MATHEMATICAL FORM.

FIGURE II-5A, Solution to the Non-Eigenvalue Weapon Worth Concept, With Correction for Vulnerability (continued on next page)

2. GENERALIZED MATHEMATICAL FORM

$$(a'_{11} + ka_{11})x_1 + a_{12}x_2 + \dots a_{1n}x_n = kb_1A$$

$$a_{21}x_1 + (a'_{22} + ka_{22})x_2 + \dots a_{2n}x_n = kb_2A$$

. . . .

$$a_{n1}x_1 + a_{n2}x_2 + \dots (a'_{nn} + ka_{nn})x_n = kb_nA$$

3. SOLUTION:

IF $A \neq 0$ (EPSILON IS ACCEPTABLE), AND AT LEAST ONE OF THE b's $\neq 0$

THEN $x_j = \frac{D_j}{D}$, J = 1,2,... n (CRAMER'S RULE), WHERE $D \neq 0$

FIGURE II-5B, Solution to the Non-Eigenvalue Weapon Worth Concept, With Correction for Vulnerability (continued on next page)

AND WHERE $D \equiv \begin{vmatrix} a'_{11} + ka_{11} & a_{12} \cdots & a_{1n} \\ a_{21} & a'_{22} + ka_{22} \cdots & a_{2n} \\ \cdots & \cdots & \cdots \\ a_{n1} & a'_{n2} \cdots & a'_{nn} + ka_{nn} \end{vmatrix}$

D_j IS THE DETERMINANT OBTAINED BY REPLACING THE RESPECTIVE ELEMENTS

$a_{1j}, a_{2j} \cdots a_{nj}$ IN THE jTH COLUMN OF D BY kb_1A, kb_2A, ... kb_nA OR

$$D_j = \sum_{i=1} A_{ij} kb_i A \quad (j = 1,2, \ldots n)$$

WHERE A_{ij} IS THE COFACTOR OF a_{ij} IN THE DETERMINANT D. THE SINGULAR

VALUES OF k THAT MAKE D = 0 WILL BE REFERRED TO AS EIGENVALUES AND DENOTED AS k_{eiv} IN THIS DISCUSSION.

FIGURE II-5C, Solution to the Non-Eigenvalue Weapon Worth Concept, With Correction for Vulnerability (continued on next page)

IF j' FOR NORMALIZATION PURPOSES IS CHOSEN SO THAT

$D_j' \neq 0$

$$\frac{x_j}{x_{j'}} = \frac{D_j/D}{D_{j'}/D} = \frac{D_j}{D_{j'}} = \frac{\sum_{i=1} A_{ij} k b_i A}{\sum_{i=1} A_{ij'} k b_i A}$$

WHERE A CAN BE ANY VALUE BUT 0, TO AVOID THE INDETERMINATE FORM.

OR $$\frac{x_j}{x_j'} = \frac{\sum_i A_{ij} b_i}{\sum_i A_{ij'} b_i}$$ AND THE RESULT IS

INDEPENDENT OF A, SINCE A HAS BEEN CANCELLED; THUS ALLOWING A NUMERICAL SOLUTION OF THE GENERAL SYSTEM OF EQUATIONS SHOWN IN 2 (& 1) FOR NON EIGENVALUES OF k. THESE VALUES OF k CAN BE ARBITRARILY CLOSE TO k_{eiv}. AS STATED PREVIOUSLY IN "SYMBOLS AND DEFINITIONS", k IS TAKEN EQUAL TO 1 IN THE WEAPON WORTH PROBLEM. THIS IS REQUIRED IN ORDER THAT THE EQUATIONS FOR BLUE AND RED BE DIMENSIONALLY CORRECT AND PHYSICALLY COMPATIBLE.

FIGURE II-5D, Solution to the Non-Eigenvalue Weapon Worth Concept, With Correction for Vulnerability (concluded)

A SOLUTION IS PRESENTED HERE THAT IS NEW, AS BEST THE AUTHOR COULD DETERMINE. THE SOLUTION IS VERY SIMPLE AND PHYSICALLY APPEALING. IT IS BASED UPON THE CONCEPT OF LETTING THE NONHOMOGENEOUS COMPONENT OF THE LINEAR SET OF EQUATIONS APPROACH ZERO AND THUS VIEWING ANY LINEAR SET OF HOMOGENEOUS EQUATIONS AS A LIMIT OF A CORRESPONDING SET OF NONHOMOGENEOUS EQUATIONS. A LARGE NUMBER OF PHYSICAL PROBLEMS HAVE USED THE EIGENVALUE CONCEPT AS A METHOD OF SOLUTION. THE SOLUTION PRESENTED HERE DOES NOT DEPEND UPON THE EIGENVALUE CONCEPT BUT SUPPLEMENTS IT TO THE EXTENT THAT IT FILLS THE VOID OF SOLUTIONS BETWEEN THE VARIOUS EIGENVALUES THAT SATISFY THE

FIGURE II-6A, Physical Significance of the Non-Eigenvalue Solution
(continued on next page)

EQUATION SET. THIS PAPER SHOWS THAT MORE MEANINGFUL RESULTS COULD BE OBTAINED, AS A RESULT OF THIS CONCEPT. POTENTIALLY, IT HAS A VERY WIDE APPLICATION TO MANY PROBLEMS IN PHYSICS, ENGINEERING AND MATHEMATICS.

FIGURE II-6B, Physical Significance of the Non-Eigenvalue Solution (concluded)

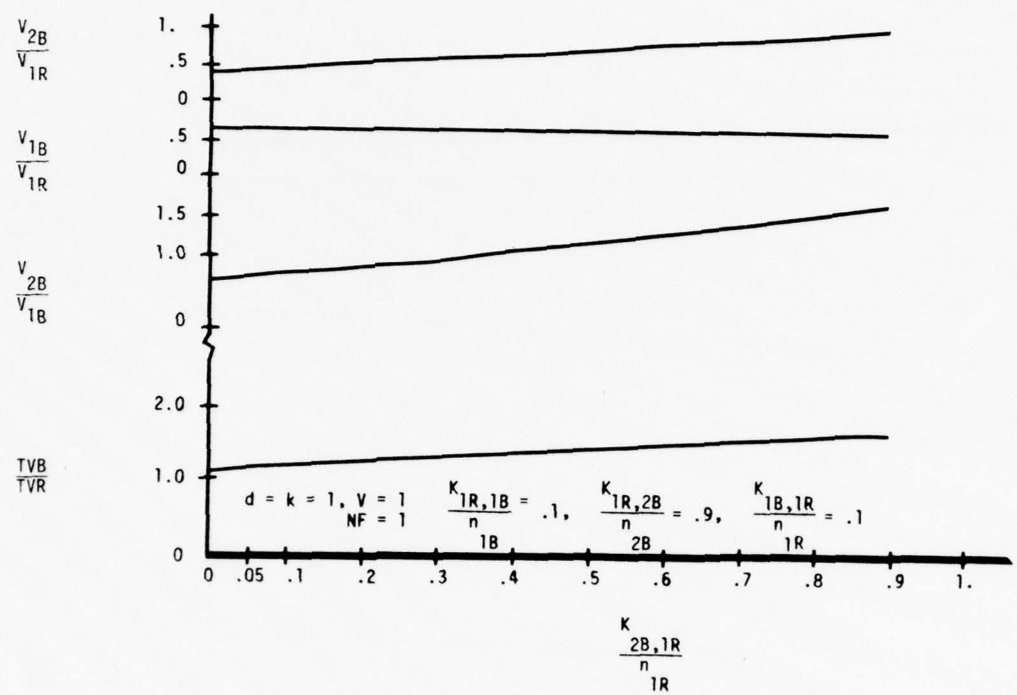

FIGURE II-7A, Sensitivity of the Non-Eigenvalue Model to Kills by the Blue System of Red (continued on next page)

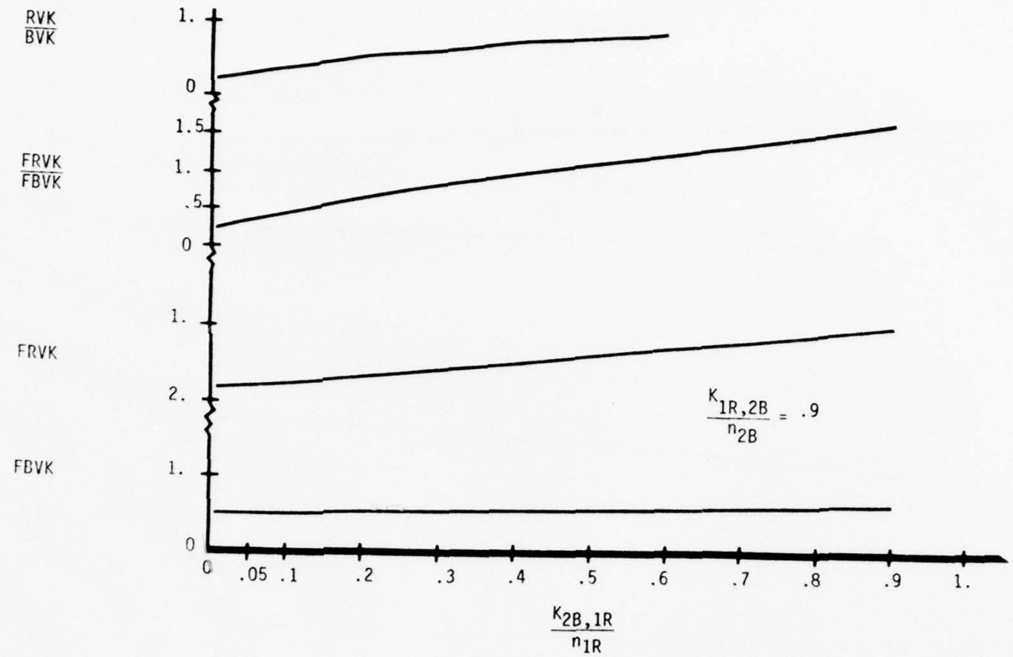

FIGURE II-7B, Sensitivity of the Non-Eigenvalue Model to Kills by the Blue System of Red (continued on next page)

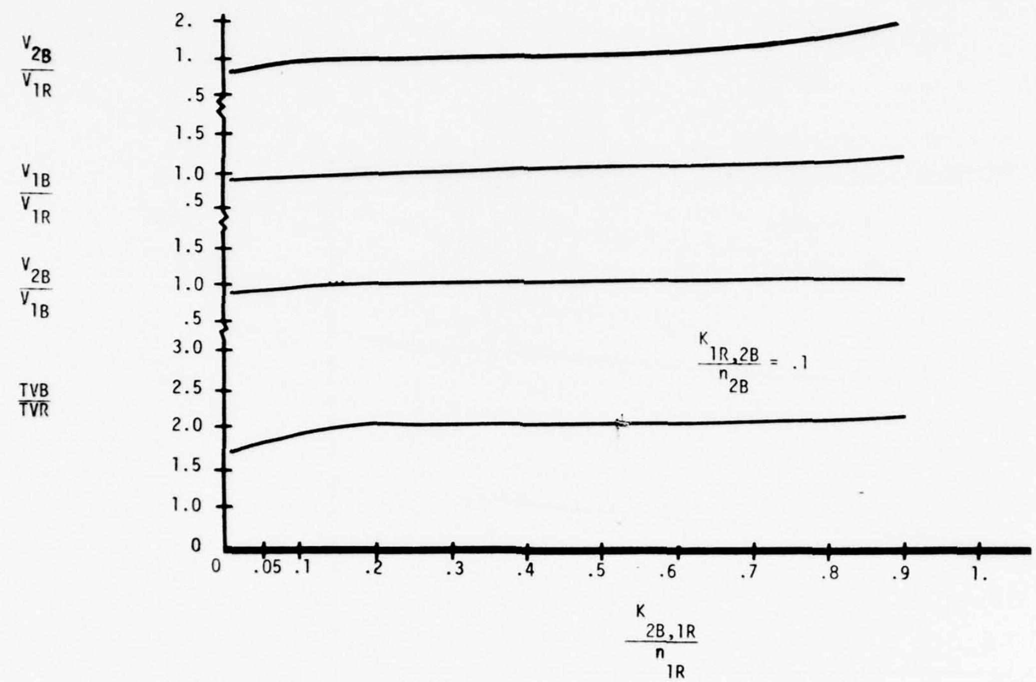

FIGURE II-7C, Sensitivity of the Non-Eigenvalue Model to Kills by the Blue System of Red (continued on next page)

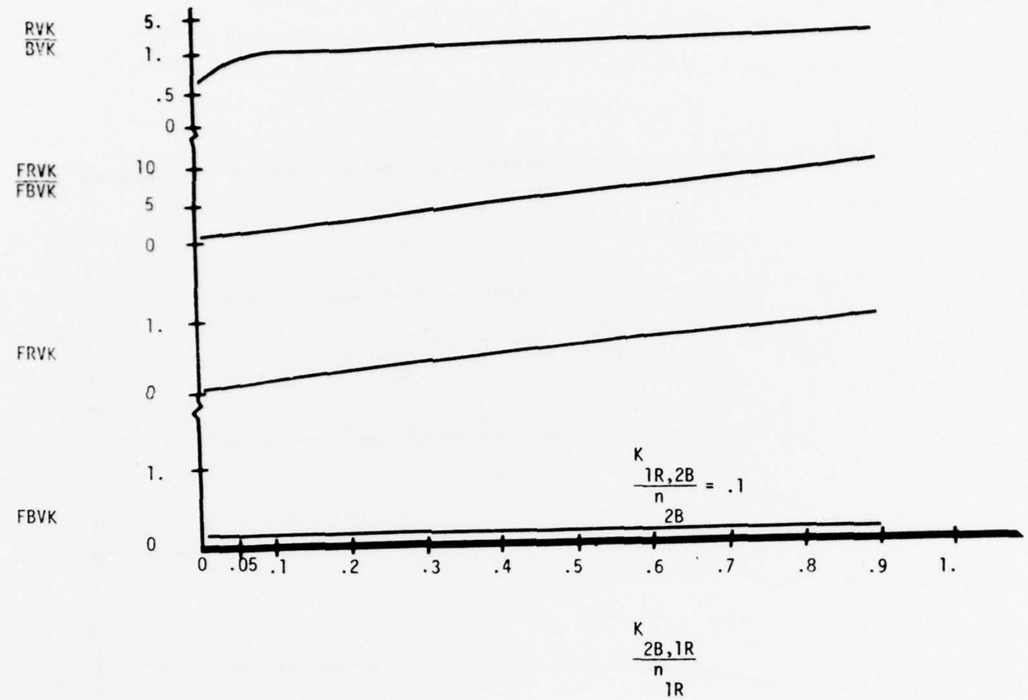

FIGURE II-7D, Sensitivity of the Non-Eigenvalue Model to Kills by the Blue System of Red (continued on next page)

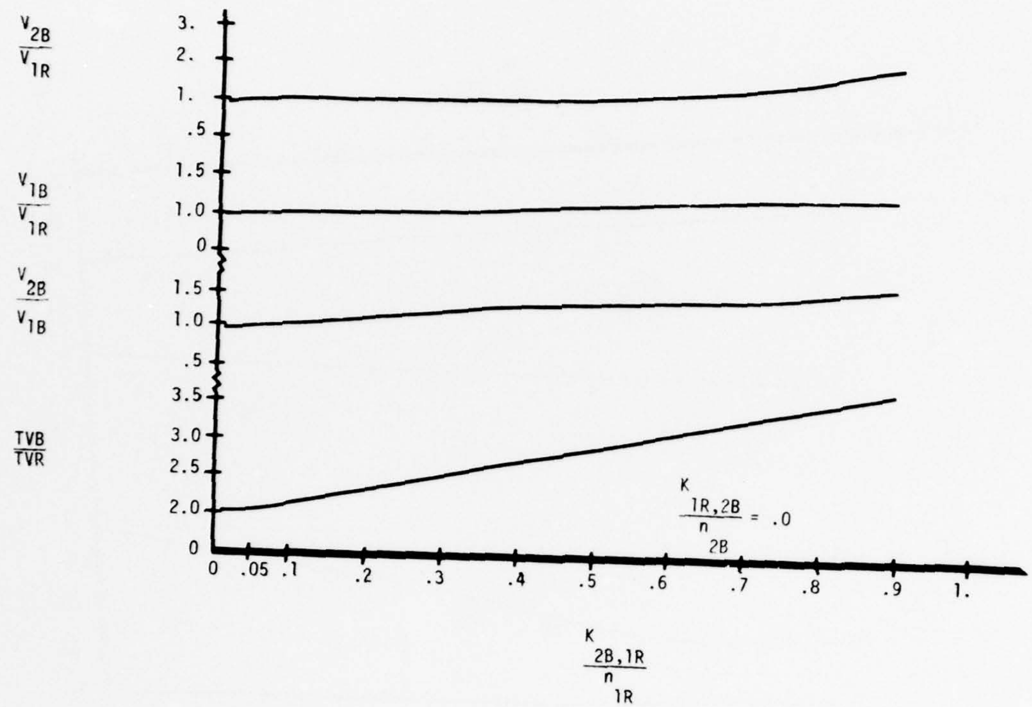

FIGURE II-7E, Sensitivity of the Non-Eigenvalue Model to Kills by the Blue System of Red (continued on next page)

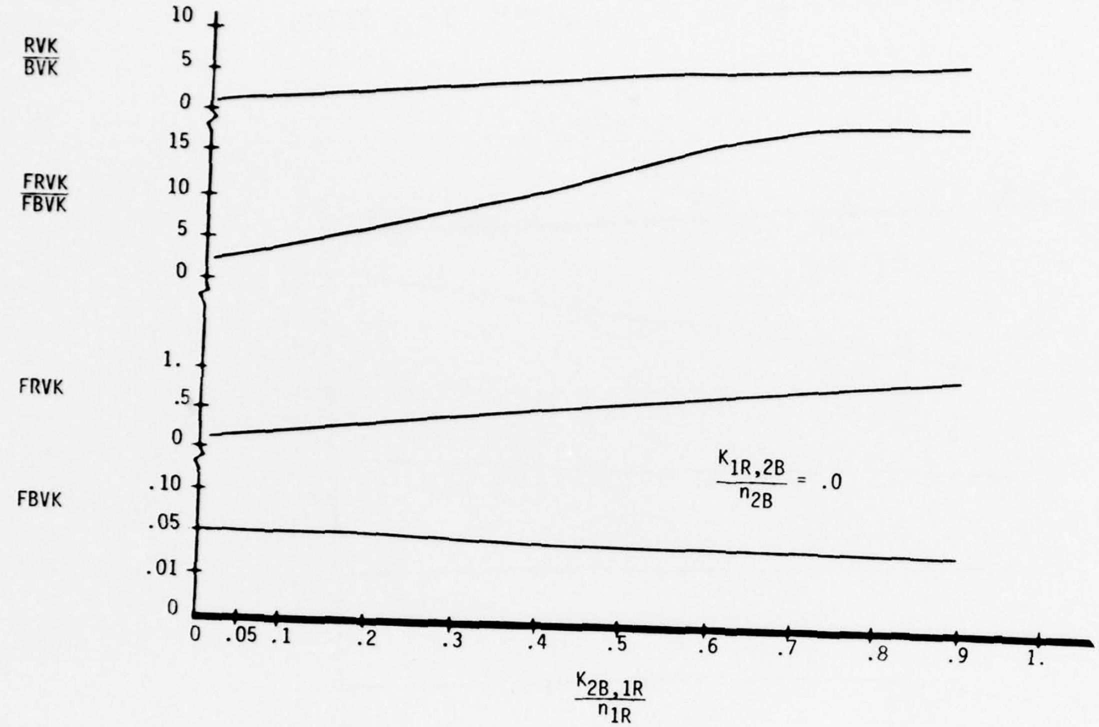

FIGURE II-7F, Sensitivity of the Non-Eigenvalue Model to Kills by the Blue System of Red (concluded)

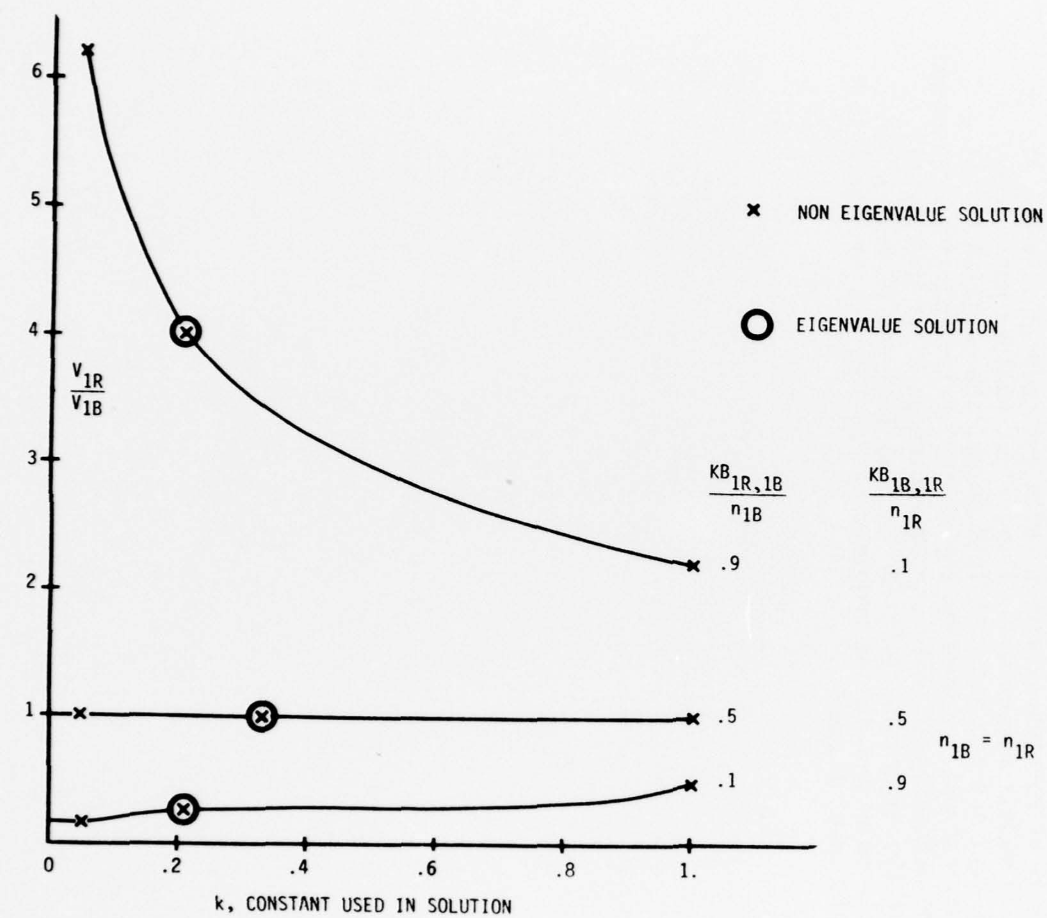

FIGURE II-8, Unification of the Singular Eigenvalue Solution With the Non-Eigenvalue Solutions the discreet eigenvalue solutions shown by circles. The non-eigenvalue solution allowed a comparison of results based upon k = 1 which the physical problem demanded but which the eigenvalue solution could not give because the eigenvalue "constant" changed depending upon the inputs used for each calculation.

d. The coalescing of the eigenvalue concept and the concept of initial value (the A term) presented in this paper indicate that perhaps this concept has much greater applicability than just the weapon system application discussed here. As a result of a literature search that the Defense Documentation Center conducted, based upon the single word eigenvalue (Reference 5) thousands of references were printed out covering diverse fields of basic matrix mathematics, fluid flow, nuclear phenomenon, elasticity, optics, etc. At this point, the question is not a mathematical one but whether any of the various physical phenomenon and the mathematical model can be looked at a bit differently, so that the eigenvalue solution becomes just one of many other possible solutions to the problem, as was the case in Figure II-8, and References 2 and 8.

DETERMINATION OF THE COMBINED VALUE OF DIVERSE WEAPON SYSTEMS BASED ON A NEW GENERALIZED EIGENVALUE CONCEPT

CHAPTER III
OBSERVATIONS

1. Vulnerability. Several eigenvalue models, used in the past, although mathematically correct, were shown to be insensitive to the effects of vulnerability or were sensitive but in the wrong direction. When a vulnerability correction was added, results showed that the effects of vulnerability (kills of the system considered) were sensitive and in the correct direction.

2. Measures of Effectiveness. An increase in kills by the system still gave anomalous effects for some of the measures of effectiveness since they were shown to first decrease and then increase when the eigenvalue method was used. Decisions made on such fallacious models would obviously be in error. Only by eliminating the eigenvalue coefficient of proportionality and going back to basics and saying that the value of a system was equal to the value of what it killed, rather than proportional through the eigenvalue constant, was it possible to obtain a large number of curves which all responded in a more appropriate way insofar as sensitivities were concerned.

3. Generalized Eigenvalue Concept. The blending of the eigenvalue solution into a much more general family of solutions which includes the standard eigenvalue solution indicate that the generalized concept may have a much wider application than this particular study. The eigenvalue concept has had wide use in the past. The new concept potentially has applications in a large range of other physical problems which previously used the eigenvalue concept for a solution, in a manner similar to that used in the study of the combined value problem.

APPENDIX A

STUDY CONTRIBUTORS

DETERMINATION OF THE COMBINED VALUE OF DIVERSE WEAPON SYSTEMS
BASED ON A NEW GENERALIZED EIGENVALUE CONCEPT

APPENDIX A
STUDY CONTRIBUTORS

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2. Study Team

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APPENDIX B

REFERENCES

DETERMINATION OF THE COMBINED VALUE OF DIVERSE WEAPON SYSTEMS BASED ON A NEW GENERALIZED EIGENVALUE CONCEPT

APPENDIX B
REFERENCES

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APPENDIX C

GLOSSARY

SYMBOLS AND DEFINITIONS

iB	REFERS TO THE i'TH BLUE WEAPON, I.E. iB = 1B, 2B, ETC.
jR	REFERS TO THE j'TH RED WEAPON, I.E. jR = 1R, 2R, ETC.
V_{iB}	IS THE VALUE PER iB WEAPON SYSTEM
V_{jR}	IS THE VALUE PER jR WEAPON SYSTEM
$V = 0$	WHEN NO VULNERABILITY CORRECTION
$V = 1$	WITH VULNERABILITY CORRECTION
A	CONSTANT FOR ALL THE SYSTEMS. PHYSICALLY CAN BE THOUGHT OF AS THE VALUE PER WEAPON WHEN IT DOESN'T KILL AND IT IS NOT KILLED. IT MAY BE THOUGHT OF AS AN ASCRIBED VALUE, BUT IT IS SHOWN THAT IT CANCELS OUT FINALLY, FROM ALL THE MEASURES OF EFFECTIVENESS, SINCE THE LATTER ARE RATIOS.
a'_{iB}	MAY BE DEFINED TO BE THE INITIAL, AVERAGE, OR FINAL NUMBER OF iB TYPE WEAPONS SURVIVING.
b_{iB}	MAY BE DEFINED TO BE THE INITIAL, AVERAGE, OR FINAL NUMBER OF iB TYPE WEAPONS SURVIVING. DEFINITION SHOULD AGREE WITH a'_{iB} ABOVE FOR THE WEAPON VALUE PROBLEM. (IN THE GENERALIZED MATHEMATICAL FORM, IT CAN BE INDEPENDENT OF THE OTHER CONSTANTS.)

$a_{iB,jR}$	IS THE NUMBER OF KILLS BY THE iB SYSTEM (1B,2B, ETC.) OF THE jR SYSTEM (1R,2R, ETC.)
a''_{iB}	IS TYPICALLY THE TOTAL NUMBER OF iB WEAPONS KILLED BY ALL THE jR WEAPON TYPES.
k	IS A SPECIFIED CONSTANT APPEARING IN THE GIVEN SYSTEM OF LINEAR EQUATIONS. (IN THE WEAPON VALUE PROBLEM, IT IS 1. IN THE GENERAL MATHEMATICAL FORMULATION, IT MAY BE ANY SPECIFIED CONSTANT VALUE.)
NF = 0	DENOTES THAT INITIAL NUMBER OF WEAPON SYSTEMS IS USED IN THE MODEL.
NF = 1	DENOTES THAT AVERAGE NUMBER OF WEAPON SYSTEMS IS USED IN THE MODEL.

NOTE: THE ABOVE DEFINITIONS APPLY ANALOGOUSLY IF iB IS REPLACED BY jR AND VICE VERSA.

APPENDIX D

DISTRIBUTION

Determination of the Combined Value of Diverse Weapon Systems
Based on A New Generalized Eigenvalue Concept

APPENDIX D
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