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A SOLUTION OF THE RAYLEIGH SCATTERING PROBLEM FOR PLANE-PARALLE--ETC(U)

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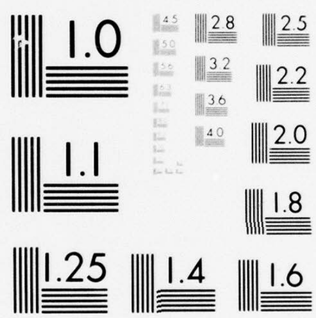
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6 A SOLUTION OF THE RAYLEIGH SCATTERING PROBLEM FOR PLANE-PARALLEL ATMOSPHERES OF LARGE OPTICAL THICKNESS

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PREFACE

This paper represents the culmination of one phase of the research done by Anne Kahle during the twelve years she was on the staff of The Rand Corporation. During a part of this time she was enrolled in the doctoral program at UCLA and this paper is her doctoral thesis. During the period when this work was done there was a close collaboration between Rand scientists and the staff at UCLA.

Although this paper is primarily concerned with the solution of a difficult physical problem, many of the results were used for the solution of operational problems presented to Rand. This paper is being issued in order to make these results more widely available.

SUMMARY

The problem of radiative transfer in a plane-parallel, perfectly scattering atmosphere is described. Chandrasekhar's solution applicable to atmospheres of small and moderate optical thickness is outlined. His solution reduces the problem to that of determining the X-, Y-, K-, and L-functions, the scattering functions. Mullikin has extended this method of solution to atmospheres of large optical thickness.

Sekera and Kahle have used Mullikin's method of solution for calculating the emergent radiation from plane-parallel Rayleigh-scattering atmospheres of large optical thickness. Their numerical results are reproduced here in the Appendix, as tables of scattering functions. The numerical method for determining the intensity and polarization of the radiation emerging from the top and bottom of atmospheres is given, and suggestions for additional uses of the tables are made. Finally, a few examples of representative calculations are presented.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

INTRODUCTION

One of the basic problems of radiative transfer in a planetary atmosphere is that of determining the intensity and polarization of the radiation emerging from the top and bottom of the atmosphere. The logical first step in this problem is the study of scattering in a pure molecular, or Rayleigh scattering atmosphere. One of the most successful and widely used methods is that of Chandrasekhar. His solution for plane-parallel atmospheres involves solutions of nonlinear integral equations for his X- and Y-functions by successive iterations, starting with the solutions for small optical thickness. This method was applied by several workers. However, when they attempted to extend the solutions to atmospheres of large optical thickness ($\tau > 1$) the solutions failed, due to computational instabilities.

Recognizing that in dealing with planetary exploration we might have to deal with atmospheres of large optical thickness, an attempt was made at the Rand Corporation to extend these calculations to large optical thickness by using the solution for $\tau \to \infty$ as a starting value. However, the calculations failed to converge, and it became evident that the solutions were oscillatory. Mullikin made an extensive mathematical study of the nonuniqueness problem and, by going back to the original radiative transfer equations, was able to select the correct solution [Mullikin, 1962a,b,c,d; 1964a,b]. He devised a method of solution which was both unique and computationally desirable as its rate of convergence increased as one looked at larger optical thickness. Carlstedt and Mullikin (1966) computed the X- and Y-functions for large optical thickness for isotropic scattering.

By appropriate modification of their method Sekera and Kahle (1966) were able to use this same method for Rayleigh-scattering atmospheres of large optical thickness. The tables of X- and Y-functions and the related K- and L-functions they derived are given here in the Appendix. These constitute the complete solution to the problem of radiative transfer in a plane-parallel Rayleigh-scattering atmosphere of any optical thickness, to as great an accuracy as desired. Kahle then examined some aspects of the intensity of radiation emerging from Rayleigh scattering atmospheres of large optical thickness [Kahle, 1968a], and also the global radiation fields for the same problem [Kahle, 1968b].

In Chapter I basic definitions are given, and the problem of radiative transfer in a homogeneous plane-parallel Rayleigh-scattering atmosphere is posed. The description of the problem follows that of Chandrasekhar (1950) and is presented here as a review of the concepts and terms necessary for an understanding of the subsequent chapters.

The solution of this problem by Chandrasekhar's method employing X- and Y-functions is discussed in Chapter II. In Section A the solution as developed by Chandrasekhar (1950) is examined. Section B describes Mullikin's extension of this solution to atmospheres of large optical thickness. In Section C the method used by Sekera and Kahle (1966) to calculate the solution for a Rayleigh-scattering atmosphere is described.

In Chapter III, the tables of X- and Y- and K- and L-functions given in the Appendix and their use are described. Section A describes the values available in the tables. The equations required to compute the Stokes parameters from these tables are given in Section B. Other uses for the tables and the accuracy of the tables are discussed in the subsequent sections.

In the final chapter two applications of the tables as solutions to the radiative transfer problem are briefly examined as an illustration of their use.

In retrospect, one might well ask what use can be made of these calculations now. It was recognized at the time the computations were made that the atmospheres of Venus and Jupiter were cloudy. However, it was believed at that time that there well might be extensive molecular atmospheres above the cloud tops, where these computations would be applicable. Care would have to be exercised in this application since the reflection from thick clouds would probably not be perfectly Lambertian. Although we now know that such a molecular atmosphere does not exist above the Venusian cloud tops, these calculations may still be applicable to Jupiter where there appears to be approximately 2 bars of H_2 atmosphere above the cloud tops.

Another reason, at the time, for making these calculations, was the concept that non-molecular effects could be considered as a perturbation on molecular scattering. This argument has its limitations, however, because of possible non-linear interactions between the molecular components of scattering. Subsequently several authors have attempted to solve the problem for cloudy atmospheres, for both homogeneous and non-homogeneous cases. In all such studies, numerical methods of solution have been required. There is always a need for testing such models against a standard calculation, the results of which can be justified on analytic grounds. It is hoped that the present tables will provide such a standard, since the results of these more complicated models should reduce to the results of the Rayleigh-scattering model under the appropriate conditions. This should provide a necessary but by no means sufficient condition that such models should satisfy.

I. THE PROBLEM OF RADIATIVE TRANSFER

In this chapter the problem of radiative transfer in a plane-parallel Rayleigh-scattering atmosphere is outlined. Section A contains definitions of some of the principal quantities used in radiation studies. In Section B the Stokes parameters for polarized light are defined. In Section C the radiative transfer equation, defining the change in the Stokes parameters on interaction of the radiation field with a Rayleigh-scattering atmosphere, is presented. Finally, in Section D, the scattering and transmission matrices are defined, the quantities whose determination will constitute the solution of the problem.

A. BASIC DEFINITIONS

The basic problem of radiative transfer is to determine how a radiation field is altered on passing through and interacting with a medium. If we consider a pencil of radiation of intensity I_ν (in a frequency interval ν to $\nu + d\nu$) propagating through a medium in a specified direction, then we can consider the various ways this intensity will be altered in travelling a distance ds.

The radiation can be weakened by two processes, true absorption by the medium, and scattering of the radiation into another direction. In practice, both of these processes are combined into a mass attenuation coefficient, defined as κ_ν in

$$dI_\nu = - \kappa_\nu \rho I_\nu ds \qquad \text{(I-1)}$$

where ρ is the mass density of the medium. This attenuation coefficient can be due to true absorption only, scattering only, or a combination of both. The case of scattering only will be considered first. If dm is the mass of the element of the medium under consideration, then the rate of scattering of energy out of the element is

$$\kappa_\nu I_\nu \, dm \, d\nu \, d\omega \tag{I-2}$$

where $d\omega$ is the solid angle of the incident pencil of radiation. The rate at which energy is scattered into a given solid angle $d\omega'$ can be found if we specify the phase function for scattering, $p(\cos\theta)$ where θ is the angle between the incident radiation and $d\omega'$. This rate is

$$\kappa_\nu I_\nu p(\cos\theta) \frac{d\omega'}{4\pi} \, dm \, d\nu \, d\omega \tag{I-3}$$

The total amount scattered will thus be

$$\kappa_\nu I_\nu \, dm \, d_\nu \, d\omega \int p(\cos\theta) \, \frac{d\omega'}{4\pi} \tag{I-4}$$

which shows, by comparison with Equation I-2, that we must normalize the phase function such that

$$\int p(\cos\theta) \, \frac{d\omega'}{4\pi} = 1 \tag{I-5}$$

To account for absorption we use the same expression (I-4), only now do not require that the phase function be normalized. Instead, we have

$$\int p(\cos\ \theta)\ \frac{d\omega'}{4\pi} = \omega_0 \leq 1 \qquad \text{(I-6)}$$

This defines ω_0, the <u>albedo for single scattering</u>. This quantity is a measure of how much radiation has been scattered, while $(1 - \omega_0)$ is the amount absorbed. When $\omega_0 = 1$ we have <u>perfect scattering</u>.

The phase function $p(\cos\ \theta)$ depends upon the type of scattering process involved. The simplest, $p(\cos\ \theta)$ = constant, is for <u>isotropic scattering</u>. The phase function for <u>Rayleigh scattering</u>, of great interest in atmospheric problems being appropriate for molecular scattering of visible radiation (or any scattering by dielectric particles which are small compared to the wavelength of light both outside and inside the particle [Van de Hulst, 1957]), is given by

$$p(\cos\ \theta) = \frac{3}{4}\ (1 + \cos^2\theta) \qquad \text{(I-7)}$$

Rayleigh scattering is perfect scattering, i.e.,

$$\int p(\cos\ \theta)\ \frac{d\omega'}{4\pi} = \int \frac{3}{4}\ (1 + \cos^2\theta)\ \frac{d\omega'}{4\pi} = \omega_0 = 1 \qquad \text{(I-8)}$$

As with the weakening of our pencil of radiation in traversing an element of mass, there are two similar processes enhancing the radiation: scattering of radiation into the direction of the pencil and emission by the mass element. Again, the two are treated together, this time with a single emission coefficient j_ν defined such that the element of mass dm emits into the solid angle $d\omega$ an amount of radiation in the frequency range ν to $\nu + d\nu$,

$$j_\nu \, dm \, d\omega \, d\nu \tag{I-9}$$

in unit time.

The contribution to this emission due to scattering from a direction (θ',ϕ') into the direction θ,ϕ will be (cf. Equation I-3)

$$\kappa_\nu \, dm \, d\nu \, d\omega \, p(\theta,\phi:\theta'\phi') I_\nu(\theta',\phi') \frac{\sin\theta' \, d\theta' \, d\phi'}{4\pi} \tag{I-10}$$

Integrating over all incoming angles, the emission coefficient due to scattering only is

$$j_\nu(\theta,\phi) = \kappa_\nu \frac{1}{4\pi} \int_0^\pi \int_0^{2\pi} p(\theta,\phi:\theta,\phi') I_\nu(\theta',\phi') \sin\theta' \, d\theta' \, d\phi'$$

(I-11)

In general, of course, there will be true emission also, but we will not concern ourselves with that in this paper. The source function is defined as

$$J_\nu \equiv \frac{j_\nu}{\kappa_\nu} \tag{I-12}$$

For our case of scattering only

$$J_\nu(\theta,\phi) = \frac{1}{4\pi} \int_0^{\pi} \int_0^{2\pi} p(\theta,\phi:\theta',\phi') I_\nu(\theta',\phi') \sin\theta' \, d\theta' \, d\phi' \tag{I-13}$$

The basic equation of radiative transfer sums up the contribution of the various processes which we have just discussed

$$\frac{dI_\nu}{ds} = -\kappa_\nu \rho I_\nu + j_\nu \rho \tag{I-14}$$

or

$$-\frac{1}{\kappa_\nu \rho} \frac{dI_\nu}{ds} = I_\nu - J_\nu \tag{I-15}$$

The source function J_ν is usually an integral function of the intensity I_ν (see Equation (I-13)) so this equation is usually an integro-differential equation.

$$-\frac{1}{\kappa_\nu \rho}\frac{dI_\nu}{ds} = I_\nu - \frac{1}{4\pi}\int_0^{\pi}\int_0^{2\pi} p(\theta,\phi:\theta',\phi')I_\nu(\theta',\phi')\sin\theta'\,d\theta'\,d\phi' \tag{I-16}$$

It is the solution of this equation, for the geometry of a homogeneous plane-parallel Rayleigh-scattering atmosphere that is the subject of this study.

A homogeneous plane-parallel atmosphere is defined to be an atmosphere which is stratified in plane parallel layers so that the only variation of atmospheric properties is in the vertical direction. This variation can be incorporated into the normal optical thickness, τ, generally measured inward from the top of the atmosphere (considered to extend to infinity).

$$\tau = \int_z^{\infty} \kappa\rho\, dz \tag{I-17}$$

The radiative transfer equation then becomes

$$\mu \frac{dI_\nu}{d\tau}(\tau,\mu,\phi) = I_\nu(\tau,\mu,\phi) - J_\nu(\tau,\mu,\phi) \qquad \text{(I-18)}$$

where $\mu = \cos\theta$.

B. STOKES PARAMETERS FOR POLARIZED LIGHT

In order to describe completely the nature of the radiation field in the atmosphere we must look not only at the intensity of the light, but also at the state of polarization. The system chosen by Chandrasekhar (1950) and his many followers to describe the polarization in the radiative transfer problem is that of using the Stokes parameters. These were first introduced by Stokes in 1852 [Shurcliff, 1962]. These parameters are best suited to both the theoretical and experimental description of incoherent, partially polarized light which is the natural occurrence when sunlight interacts with planetary atmospheres [Deirmendjian, 1969]. All four parameters have the same physical dimensions, the parameters for coincident streams are additive, and they are relatively easily measured experimentally. The radiative transfer solutions of Chandrasekhar for the intensity of the radiation can, in general, be applied to the Stokes parameters by merely substituting the appropriate matrices for the phase functions and other quantities.

The polarization of electromagnetic radiation is traditionally described in terms of the electric vector. If a_ℓ and a_r are the scalar components of the vector electric field in two mutually perpendicular directions, perpendicular to the direction of propagation, then the Stokes parameters are defined as

$$I = \{a_\ell^2 + a_r^2\} = I_\ell + I_r$$

$$Q = \{a_\ell^2 - a_r^2\} = I_\ell - I_r$$

$$U = \{2a_\ell a_r \cos \gamma\} \qquad \text{(I-19)}$$

$$V = \{2a_\ell a_r \sin \gamma\}$$

where γ is difference of the phases $\varepsilon_\ell - \varepsilon_r$. The brackets indicate a time average over an appropriate time length. All these Stokes parameters have the dimension of intensity.

The electric vector traces an ellipse, and it can be shown that the principal axes of the ellipse are in directions making angles χ and $\chi + \frac{\pi}{2}$ with the direction ℓ where

$$\tan 2\chi = \frac{U}{Q} \qquad \text{(I-20)}$$

Also, the ratio of the major and minor axes of the ellipse is tan β with the sign of β determining the direction of rotation of the electric vector and

$$\sin 2\beta = \frac{V}{(Q^2 + U^2 + V^2)^{1/2}} \qquad \text{(I-21)}$$

Sometimes I_ℓ and I_r are used in place of the first two Stokes parameters, I and Q .

For completely polarized light, $I^2 = Q^2 + U^2 + V^2$. For partially polarized light $I^2 > Q^2 + U^2 + V^2$ and the percent of polarization

$$\mathcal{P} = \frac{100(Q^2 + U^2 + V^2)}{I^2}^{1/2} \qquad \text{(I-22)}$$

For natural, or unpolarized, light $Q = U = V = 0$. For linearly polarized light, $V = 0$.

If $\vec{I} = (I_\ell, I_r, U, V)$, then a rotation of axes through an angle ϕ will subject $\vec{I}$ to a linear transformation.

$$\overleftrightarrow{L}(\phi) = \begin{pmatrix} \cos^2\phi & \sin^2\phi & 1/2 \sin 2\phi & 0 \\ \sin^2\phi & \cos^2\phi & -1/2 \sin 2\phi & 0 \\ -\sin 2\phi & \sin 2\phi & \cos 2\phi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \text{(I-23)}$$

C. RAYLEIGH SCATTERING

The Rayleigh scattering phase function $p = \frac{3}{4}(1 + \cos^2\theta)$, which was mentioned in Section A, is appropriate for natural light, but when considering polarized light we need a more complete description of the scattering process. For polarized light characterized by the vector

$$\vec{I} = (I_\ell, I_r, U, V) \qquad \text{(I-24)}$$

which is incident on a single particle, the scattered light in the direction θ will be given by

$$\left(\sigma \frac{d\omega'}{4\pi}\right) \overleftrightarrow{R} \; \vec{I} \, d\omega \qquad \text{(I-25)}$$

upon Rayleigh scattering [Chandrasekhar, 1950]. In this expression σ is the scattering coefficient per particle given by

$$\sigma = \frac{128\pi^5}{3\lambda^4} \, \alpha^2 \qquad \text{(I-26)}$$

for Rayleigh scattering where α is the particle polarizability (to be defined later) and λ is the wavelength of the incident light. Also, $\overleftrightarrow{R}$ in Equation (I-25) is the phase matrix for Rayleigh scattering, defined as

$$\overleftrightarrow{R} = \frac{3}{2} \begin{pmatrix} \cos^2\theta & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & \cos\theta \end{pmatrix} \qquad \text{(I-27)}$$

For natural incident light, with $\vec{I} = (\frac{1}{2} I, \frac{1}{2} I, 0, 0)$, $\overleftrightarrow{R} \vec{I}$ reduces to $\frac{3}{4}(1 + \cos^2\theta) I$, equivalent to the Rayleigh phase function given earlier.

So far, looking at a single particle, our coordinate system has been determined by the angle of incidence and the scattering angle. As we go to the more general problem with many particles, we must transform each scattering event to a chosen coordinate system applicable to the problem. This can be done by use of the rotation transformation $\overleftrightarrow{L}(\phi)$ given in Section B, Equation (I-23). The phase matrix $\overleftrightarrow{P}$ will be given by

$$\overleftrightarrow{P}(\theta, \phi; \theta', \phi') = \overleftrightarrow{L}(\pi - i_2)\,\overleftrightarrow{R}(\cos\Theta)\,\overleftrightarrow{L}(-i_1) \qquad \text{(I-28)}$$

where i_2 and i_1 are angles relating the incident and scattering directions to the chosen coordinate system. It can be shown [Chandrasekhar, 1950] that $\overleftrightarrow{P}$ can then be written as the sum of three terms, an azimuth independent term $\overleftrightarrow{P}^{(0)}$; a term dependent upon $(\phi' - \phi)$, $\overleftrightarrow{P}^{(1)}$; and a term dependent upon $2(\phi' - \phi)$, $\overleftrightarrow{P}^{(2)}$.

For incident natural light, rather than incident arbitrarily polarized light, the problem reduces somewhat. The resulting scattered radiation will be partially plane-polarized. Further scattering will change the angle of the plane of polarization and the degree of polarization, but it will remain plane-polarized. Mathematically, we can say that the matrix $\overleftrightarrow{P}$ is reducible with respect to V. Therefore, for sunlight (natural light) incident on a Rayleigh-scattering atmosphere we need only consider the three-dimensional Stokes vector and three-dimensional Rayleigh phase matrix.

The phase matrix for Rayleigh-scattering in a plane-parallel atmosphere with incident natural light is thus given by

$$\overleftrightarrow{P}(\mu,\phi;\mu',\phi') = \overleftrightarrow{Q}\Big[\overleftrightarrow{P}^{(0)}(\mu,\mu') + (1-\mu^2)^{1/2}(1-\mu'^2)^{1/2}\,\overleftrightarrow{P}^{(1)}(\mu,\phi;\mu',\phi') + \overleftrightarrow{P}^{(2)}(\mu,\phi;\mu',\phi')\Big] \qquad \text{(I-29)}$$

where $\mu \equiv \cos\theta, \quad \mu' \equiv \cos\theta'$

$$\overleftrightarrow{Q} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} \qquad \text{(I-30)}$$

$$\overleftrightarrow{P}^{(0)}(\mu,\mu') = \frac{3}{4}\begin{pmatrix} 2(1-\mu^2)(1-\mu'^2)+\mu^2\mu'^2 & \mu^2 & 0 \\ \mu'^2 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \text{(I-31)}$$

$$\overleftrightarrow{P}^{(1)}(\mu,\phi;\mu',\phi') = \frac{3}{4}\begin{pmatrix} 4\mu\mu' \cos(\phi' - \phi) & 0 & 2\mu \sin(\phi' - \phi) \\ 0 & 0 & 0 \\ -2\mu' \sin(\phi' - \phi) & 0 & \cos(\phi' - \phi) \end{pmatrix}$$

(I-32)

$$\overleftrightarrow{P}^{(2)}(\mu,\phi;\ \mu',\phi')$$

$$= \frac{3}{4}\begin{pmatrix} \mu^2\mu'^2 \cos 2(\phi' - \phi) & -\mu^2 \cos 2(\phi' - \phi) & \mu^2\mu' \sin 2(\phi' - \phi) \\ -\mu'^2 \cos 2(\phi' - \phi) & \cos 2(\phi' - \phi) & -\mu' \sin 2(\phi' - \phi) \\ -\mu\mu'^2 \sin 2(\phi' - \phi) & \mu \sin 2(\phi' - \phi) & \mu\mu' \cos 2(\phi' - \phi) \end{pmatrix}$$

(I-33)

The mass scattering coefficient κ is given by

$$\kappa = \frac{\sigma}{\rho} N \qquad \text{(I-34)}$$

where σ is the scattering coefficient per particle given at the beginning of this section, ρ is the mass density, and N is the number of particles per unit volume. For molecular scattering the polarizability is

$$\alpha = \frac{n^2 - 1}{4\pi N} \qquad \text{(I-35)}$$

where n is the refractive index of the medium (cf. Equation (I-26)). For Rayleigh scattering, the mass scattering coefficient is thus

$$\kappa = \frac{8\pi^3}{3} \frac{(n^2 - 1)^2}{\lambda^4 N\rho} \tag{I-36}$$

This is the quantity that enters into the calculation of the optical thickness τ .

With the phase function defined above, and τ defined (cf. Equation I-17) in terms of κ given in Equation (I-36), the equation of transfer for a plane-parallel Rayleigh-scattering atmosphere is

$$\mu \frac{d\vec{I}(\tau,\mu,\phi)}{d\tau} = \vec{I}(\tau,\mu,\phi)$$

$$- \frac{1}{4\pi} \int_{-1}^{+1} \int_{0}^{2\pi} \overleftrightarrow{P}(\mu, \phi; \mu', \phi') \vec{I} (\tau, \mu', \phi') d\mu' d\phi' \tag{I-37}$$

D. THE SCATTERING AND TRANSMISSION MATRICES

When considering the radiative transfer through a plane-parallel atmosphere, one of the principal goals is to determine the diffuse radiation emerging from the top and bottom of the atmosphere. It is convenient to explicitly separate the diffuse radiation from the attenuated incident solar radiation. If we assume a parallel beam of sunlight of net flux $\pi\vec{F} = \pi(F_\ell, F_r, F_U)$ incident on a plane parallel atmosphere in the direction $(-\mu_0, \phi_0)$, then the radiative transfer equation can be written

$$\mu\frac{d\vec{I}}{d\tau}(\tau,\mu,\phi) = \vec{I}(\tau,\mu,\phi)$$

$$-\frac{1}{4\pi}\int_{-1}^{+1}\int_{0}^{2\pi}\overleftrightarrow{P}(\mu,\phi;\mu',\phi')\,\vec{I}(\tau,\mu',\phi')\,d\mu'\,d\phi'$$

$$-\frac{1}{4}e^{-\tau/\mu_0}\overleftrightarrow{P}(\mu,\phi;-\mu_0,\phi_0)\vec{F}$$

(I-38)

With the direct solar radiation thus separated from the diffuse reflected and transmitted radiation we can define a scattering matrix $\overleftrightarrow{S}$ and a transmission matrix $\overleftrightarrow{T}$ as follows. Let the optical thickness be measured from the top of the atmosphere; $\tau = 0$ at the top and $\tau = \tau_1$ at the bottom of the atmosphere. Then the reflected intensity is given by

$$\vec{I}(0;+\mu,\phi) = \frac{1}{4\mu}\overleftrightarrow{S}(\tau_1;\mu,\phi;\mu_0,\phi_0)\vec{F} \qquad \text{(I-39)}$$

and the transmitted intensity by

$$\vec{I}(\tau_1; -\mu, \phi) = \frac{1}{4\mu} \overleftrightarrow{T} (\tau_1; \mu, \phi; \mu_0, \phi_0)\vec{F} \qquad \text{(I-40)}$$

The solution of our radiative transfer problem is then to find $\overleftrightarrow{S}$ and $\overleftrightarrow{T}$ for a Rayleigh-scattering atmosphere.

II. SOLUTION OF THE RADIATIVE TRANSFER PROBLEM BY USE OF X- AND Y- FUNCTIONS

In this chapter the method of solution of the radiative transfer problem for a plane-parallel Rayleigh-scattering atmosphere by use of X- and Y- functions is presented. Section A describes Chandrasekhar's method as he developed it. When various attempts at computation of the X- and Y- functions for atmospheres of large optical depth failed, due to numerical instability, Mullikin investigated the problem. He found that the usual reduction of the original radiative transfer problem to the solution of equations for X- and Y- functions resulted in the loss of information, causing nonunique solutions. By adding constraints derived from the original radiative transfer problem, he was able to develop a method to select the correct solution from the family of solutions, and thus extend the X- and Y- function method of solution to all optical thicknesses. This is outlined in Section B. In the final part of the chapter, Section C, the numerical method of calculation of these X- and Y- functions, as developed by Carlstadt and Mullikin (1966) and used by Sekera and Kahle (1966), for the Rayleigh-scattering problem, is outlined.

A. CHANDRASEKHAR'S SOLUTION

Chandrasekhar (1950) has shown that the scattering and transmission matrices, $\overleftrightarrow{S}$ and $\overleftrightarrow{T}$ can be written in terms of pairs of X- and Y- functions. This separates the variables in the problem: where $\overleftrightarrow{S}$ and $\overleftrightarrow{T}$ are functions of $(\tau, \mu, \phi, \mu', \phi')$ the X- and Y- functions depend only on (τ, μ). Four pairs of X- and Y- functions are required for the case of Rayleigh scattering. Isotropic scattering requires only one pair. The X- and Y- functions satisfy a pair of simultaneous integral equations

$$X_i(\mu) = 1 + \mu \int_0^1 \frac{X_i(\mu)X_i(\mu') - Y_i(\mu)Y_i(\mu')}{\mu + \mu'} \psi^{(i)}(\mu')\, d\mu' \tag{II-1}$$

$$Y_i(\mu) = \exp\left(-\frac{\tau}{\mu}\right) + \mu \int_0^1 \frac{Y_i(\mu)X_i(\mu') - X_i(\mu)Y_i(\mu')}{\mu - \mu'} \psi^{(i)}(\mu')\, d\mu' \tag{II-2}$$

where, for Rayleigh scattering, $i = 1, 2, 3, 4$. This notation differs slightly from Chandrasekhar, who used the labels $i = 1, 2, \ell, r$ for these quantities. The functions $\psi^{(i)}$ are the <u>characteristic functions</u> which depend upon the type of scattering process. They are even polynomials in μ satisfying the condition

$$\int_0^1 \psi(\mu)\, d\mu \leq \frac{1}{2} \tag{II-3}$$

For Rayleigh scattering the four characteristic functions are

$$\psi^1(\mu) = \frac{3}{8}(1 - \mu^2)(1 + 2\mu^2)$$

$$\psi^2(\mu) = \frac{3}{16}(1 + \mu^2)^2$$

$$\psi^3(\mu) = \frac{3}{4}(1 - \mu^2) \qquad \text{(II-4)}$$

$$\psi^4(\mu) = \frac{3}{8}(1 - \mu^2)$$

For Rayleigh scattering the scattering and transmission matrices are written in terms of the X- and Y-functions as follows:

$$\overleftrightarrow{S}(\mu, \phi; \mu_0, \phi_0) = \overleftrightarrow{Q}\Big[\frac{3}{4}\overleftrightarrow{S}^{(0)}(\mu; \mu_0)$$

$$+ (1 - \mu^2)^{1/2}(1 - \mu_0^2)^{1/2}\overleftrightarrow{S}^{(1)}(\mu, \phi; \mu_0, \phi_0)$$

$$+ \overleftrightarrow{S}^{(2)}(\mu, \phi; \mu_0, \phi_0)\Big]$$

$$\text{(II-5)}$$

$$\overleftrightarrow{T}(\mu, \phi; \mu_0, \phi_0) = \overleftrightarrow{Q} \left[\frac{3}{4} \overleftrightarrow{T}^{(0)}(\mu; \mu_0) \right.$$

$$+ (1 - \mu^2)^{1/2} (1 - \mu_0^2)^{1/2} \overleftrightarrow{T}^{(1)}(\mu, \phi; \mu_0, \phi_0)$$

$$\left. + \overleftrightarrow{T}^{(2)}(\mu, \phi; \mu_0, \phi_0) \right]$$

(II-6)

where $\overleftrightarrow{Q}$ is as defined in Chapter I (Equation I-30), and, for $i = 1, 2$

$$\left(\frac{1}{\mu_0} + \frac{1}{\mu}\right) \overleftrightarrow{S}^{(i)} = [X_i(\mu)X_i(\mu_0) - Y_i(\mu)Y_i(\mu_0)] \overleftrightarrow{P}^{(i)}(\mu, \phi; -\mu_0, \phi_0)$$

(II-7)

$$\left(\frac{1}{\mu_0} - \frac{1}{\mu}\right) \overleftrightarrow{T}^{(i)} = [Y_i(\mu)X_i(\mu_0) - X_i(\mu)Y_i(\mu_0)] \overleftrightarrow{P}^{(i)}(-\mu, \phi; -\mu_0, \phi_0)$$

(II-8)

The $\overleftrightarrow{P}^{(i)}$ are the azimuth dependent components for i = 1, 2 of the Rayleigh scattering phase matrix given in Chapter I (Equation I-32 and Equation I-33). The azimuth independent terms are

$$(\frac{1}{\mu_0} + \frac{1}{\mu})\overleftrightarrow{S}^{(0)}(\mu;\mu_0)$$

$$= \begin{pmatrix} \Psi(\mu) & \sqrt{2}\phi(\mu) & 0 \\ \chi(\mu) & \sqrt{2}\,\zeta(\mu) & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \Psi(\mu_0) & \chi(\mu_0) & 0 \\ \sqrt{2}\phi(\mu) & \sqrt{2}\,\zeta(\mu_0) & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$- \begin{pmatrix} \xi(\mu) & \sqrt{2}\eta(\mu) & 0 \\ \sigma(\mu) & \sqrt{2}\,\theta(\mu) & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \xi(\mu_0) & \sigma(\mu_0) & 0 \\ \sqrt{2}\eta(\mu_0) & \sqrt{2}\,\theta(\mu_0) & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

(II-9)

and

$$(\frac{1}{\mu_0} - \frac{1}{\mu})\overleftrightarrow{T}^{(0)}(\mu;\mu_0)$$

$$
= \begin{pmatrix} \xi(\mu) & \sqrt{2}\,\eta(\mu) & 0 \\ \sigma(\mu) & \sqrt{2}\,\theta(\mu) & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \psi(\mu_0) & \chi(\mu_0) & 0 \\ \sqrt{2}\,\phi(\mu_0) & \sqrt{2}\,\zeta(\mu_0) & 0 \\ 0 & 0 & 0 \end{pmatrix}
$$

$$
- \begin{pmatrix} \Psi(\mu) & \sqrt{2}\,\phi(\mu) & 0 \\ \chi(\mu) & \sqrt{2}\,\zeta(\mu) & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \xi(\mu_0) & \sigma(\mu_0) & 0 \\ \sqrt{2}\,\eta(\mu_0) & \sqrt{2}\,\theta(\mu_0) & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \text{(II-10)}
$$

where the Ψ, ϕ, χ, ζ, ξ, η, σ, and θ are combinations of the X_3-, X_4-, Y_3-, and Y_4-functions and their moments. One of the differences between Chandrasekhar's analysis and Mullikin's is in the exact form of the combination of these functions. Chandrasekhar's form of these expressions can be found in Chandrasekhar (1950) and in Chandrasekhar and Elbert (1954). Mullikin's will be given later in Equation (II-40).

Given these expressions for $\overleftrightarrow{S}$ and $\overleftrightarrow{T}$ in terms of X- and Y-functions, Equation (II-2) for the X- and Y-functions will thus constitute the desired solution of the radiative transfer problem. Chandrasekhar (1950) developed an iterative method for their solution, the solution for small optical thickness as a first approximation.

A problem arises for the case when

$$\int_0^1 \psi(\mu)\, d\mu = 1/2 \qquad \text{(II-11)}$$

When this equality holds, Chandrasekhar recognized that solutions of Equations (II-1 and II-2) for the X- and Y- functions are no longer unique. For Rayleigh scattering this condition occurs for $\psi^{(3)}$ (Chandrasekhar's ψ_ℓ). For this "conservative" case the solution is given by a two-parameter family. Chandrasekhar defined his <u>standard solutions</u> by including the additional constraints

$$\int_0^1 X(\mu)\psi(\mu)\, d\mu = 0$$

$$\int_0^1 Y(\mu)\psi(\mu)\, d\mu = 0 \qquad \text{(II-12)}$$

which he determined by a consideration of the flux equations at the boundaries of the atmosphere.

Since Chandrasekhar (1950) first developed his solution to this problem, there have been several publications showing different aspects of the solution for small and moderate optical thickness $(\tau \leq 1)$. Tables of the X- and Y- functions have been calculated by Sekera and Blanch (1952), and Sekera and Ashburn (1953). Chandrasekhar and Elbert (1954) and Coulson et al. (1960), carried the solution one step further and published tables of the Stokes parameters for radiation emerging from the bottom and top of the atmosphere. Sekera (1957) has illustrated graphically many of the features of the polarization of the downward radiation. Coulson (1959) has shown graphs of the upward intensity and polarization.

B. THE SOLUTION FOR LARGE OPTICAL THICKNESS AND UNIQUENESS CONSIDERATIONS

Chandrasekhar attempted to extend his calculation of the solution to larger optical thickness $(\tau > 1)$ but was restricted by computational instabilities. A similar attempt by Blanch was restricted by computer size. This type of extension was later attempted by Sekera using the solution for $\tau \to \infty$ as a first approximation rather than that for small τ . However, some of the functions failed to converge satisfactorily even after many iterations and it was noted that the iterated values were of an oscillatory nature. Thus it was apparent that modifications had to be made to Chandrasekhar's method of solution for the extension of results for larger optical thickness.

Mullikin (1962a, b, c, d; 1964a, b; 1965) undertook an extensive study of the uniqueness problem. He determined that the difficulty arose in the usual heuristic reduction from the transfer equations to the equations for X- and Y- functions. This process ignored some equations and this loss of information introduced extraneous solutions. This had been recognized in the case of $\psi^{(3)}(\psi_\ell)$ by Chandrasekhar, but Mullikin realized it also extended to $\psi^{(1)}$ and $\psi^{(2)}$. Mullikin developed additional equations in the form of linear constraints which, along with Chandrasekhar's X- and Y- equations, do select unique $\overleftrightarrow{S}$ and $\overleftrightarrow{T}$ matrices. He determined these, as discussed below, by relating the integral equations back to the original radiative transfer equation. These constraints contain information that is lost in going from the integro-differential equations to the coupled nonlinear equations.

Mullikin first considered the work of Busbridge (1960) who demonstrated the existence of solutions to the X- and Y- equations (Equations (II-1) and (II-2)). Busbridge investigated the auxiliary integral equation

$$J(\chi,\mu) = \exp\left(-\frac{\chi}{\mu}\right) + \int_0^1 \frac{\psi(\nu)}{\nu} \int_0^\tau \exp\left(-\frac{|\chi-y|}{\nu}\right) J(y,\mu)\,dy\,d\nu \tag{II-13}$$

where, for isotropic scattering, J is the source function. She showed that a solution to Equations II-1 and II-2 is given by

$$X_0(\mu) = J(0,\mu)$$

$$Y_0(\mu) = J(\tau,\mu) \qquad \text{(II-14)}$$

and that X_0 and Y_0 are defined for all complex $\mu, |\mu|>0$, to be real and non-negative for $\mu \neq 0$ and to be analytic in the extended complex μ-plane except near $\mu = 0$.

Mullikin (1962a, b, c, d), following her proof of existence of solutions, turned to the question of uniqueness, since he realized that there appeared to be a multiplicity of solutions. He returned to the transport equations (e.g., Equation I-27) and noted that for the physically significant solution to most scattering problems, the X- and Y-functions must have extensions to complex μ as analytic functions in $|\mu|>0$. Following Busbridge, he considered the extension of Equations II-1 and II-2 to the complex plane, by rewriting them in the form

$$X(z)\left[1 - z\int_0^1 \frac{X(\nu)\psi(\nu)}{\nu+z}\, d\nu\right] + Y(z)\, z\int_0^1 \frac{Y(\nu)\psi(\nu)}{\nu+z}\, d\nu = 1 \qquad \text{(II-15)}$$

$$Y(z)\left[1 + z\int_0^1 \frac{X(\nu)\psi(\nu)}{\nu - z}\,d\nu\right] - X(z)z\int_0^1 \frac{Y(\nu)\psi(\nu)}{\nu - z}\,d\nu = e^{-\tau/z}$$

(II-16)

with z any complex number not in the interval [-1, 1]. As a system of linear equations in $X(z)$ and $Y(z)$, these equations have a unique solution where the determinant does not vanish. This determinant is the function

$$\lambda(z) = -\left(1 - z\int_0^1 \frac{X(\nu)\psi(\nu)d\nu}{\nu + z}\right)\left(z\int_0^1 \frac{Y(\nu)\psi(\nu)d\nu}{\nu - z}\right)$$

(II-17)

$$-\left(1 + z\int_0^1 \frac{X(\nu)\psi(\nu)d\nu}{\nu - z}\right)\left(z\int_0^1 \frac{Y(\nu)\psi(\nu)d\nu}{\nu + z}\right)$$

which can be shown to be

$$\lambda(z) = 1 - 2z^2\int_0^1 \frac{\psi(z)}{z^2 - \nu^2}\,d\nu \qquad \text{(II-18)}$$

Equations (II-15) and (II-16) can then be rewritten

$$\lambda(z)X(z) = 1 + z\int_0^1 \frac{X(\nu)\psi(\nu)d\nu}{\nu - z} - e^{-\tau/z} z\int_0^1 \frac{Y(\nu)\psi(\nu)d\nu}{\nu + z} \tag{II-19}$$

$$\lambda(z)Y(z) = e^{-\tau/z}\left(1 - z\int_0^1 \frac{X(\nu)\psi(\nu)d\nu}{\nu + z}\right) + z\int_0^1 \frac{Y(\nu)\psi(\nu)d\nu}{\nu - z} \tag{II-20}$$

Mullikin has shown that these equations define a meromorphic extension of X and Y to the complex domain $|z|>0$, giving functions analytic in $|z|>0$ except for possible poles at the zeros of λ.

Following Busbridge (1960), Mullikin considered the characteristic equation

$$\lambda(z) = 1 - 2z^2 \int_0^1 \frac{\psi(\nu)}{z^2 - \nu^2} d\nu = 0 \tag{II-21}$$

The uniqueness or multiplicity of solutions to the X and Y equations will depend upon the roots of this equation, which will in turn depend upon the scattering characteristic functions $\psi(\nu)$, given for Rayleigh scattering by Equation (II-4). Mullikin showed that four cases can occur

I λ has no zeros

II The only zeros of λ are at $\pm 1/k$ where $0 < k < 1$ (the non-conservative case)

III The only zeros of λ are at $\pm 1/k$ where $k = 0$ (the conservative case)

IV λ has a zero at ± 1 .

Mullikin assumed, for simplicity, that case IV never occurs, which is true for Rayleigh scattering.

For case I, λ with no zeros, Mullikin (1962a) showed that the only bounded solutions to Equations (II-19) and (II-20) (and hence to Equations (II-1) and (II-2)) are given by the X_0 and Y_0 solutions of Busbridge (Equation (II-14)). This is the case for $\psi^{(4)}$ of the Rayleigh scattering problem (see Equation (II-4)).

For case II, the non-conservative case with zeros of λ at $\pm 1/k$, Mullikin (1962a, d) showed that all solutions to Equations (II-1) and (II-2) are given by

$$X(\mu) = \left[1 + \frac{a\alpha\,\mu}{1-k\mu} - \frac{b\beta\mu}{1+k\mu}\right] X_0(\mu) + \left[\frac{a\beta\mu}{1-k\mu} - \frac{b\alpha\,\mu}{1+k\mu}\right] Y_0(\mu) \tag{II-22}$$

$$Y(\mu) = \left[1 - \frac{a\alpha\mu}{1+k\mu} + \frac{b\beta\mu}{1-k\mu}\right] Y_0(\mu) - \left[\frac{a\beta\mu}{1+k\mu} - \frac{b\alpha\mu}{1-k\mu}\right] X_0(\mu)$$

(II-23)

where constants α , β , and k are given by

$$\lambda(\tfrac{1}{k}) = 0 \qquad \text{(II-24)}$$

$$\alpha = 1 - \int_0^1 \frac{X_0(\mu)\psi(\mu)d\mu}{1+k\mu} \qquad \text{(II-25)}$$

$$\beta = \int_0^1 \frac{Y_0(\mu)\psi(\mu)d\mu}{1+k\mu} \qquad \text{(II-26)}$$

where the a and b are constants constrained only by the condition $(a^2 - b^2)(\alpha^2 - \beta^2) - 2\alpha ak - 2\beta bk = 0$, and where X_0 and Y_0 are the solutions of Busbridge (Equation II-14). Thus, we have a family of solutions depending upon the values of the parameters a and b. Mullikin (1962a, d) considered the behavior of these solutions at the zeros of λ to derive the additional constraints,

$$a\rho = 1 - \int_0^1 \frac{X(\nu)\psi(\nu)}{1-k\nu}\,d\nu - e^{-k\tau}\int_0^1 \frac{Y(\nu)\psi(\nu)d\mu}{1+k\nu}$$

(II-27)

$$b\rho e^{k\tau} = 1 - \int_0^1 \frac{X(\nu)\psi(\nu)}{1+k\nu}\,d\nu - e^{-k\tau}\int_0^1 \frac{Y(\nu)\psi(\nu)}{1+k\nu}\,d\nu$$

(II-28)

where

$$\rho = -\ 4k\int_0^1 \left[\frac{\nu}{(1-k\nu)^2}\right]^2 \psi(\nu)d\nu \qquad \text{(II-29)}$$

When there is such a multiplicity of solutions, then one must determine which solution corresponds to the physical problem. For most scattering problems the solution that is analytic for all $\mu > 0$ is the desired solution. This analytic solution can be selected from all the possible solutions by demanding that the system of Equations (II-19) and (II-20) be well behaved at the zeros of λ. Thus when $\lambda \to 0$ the R.H.S. of Equation (II-19) and (II-20) must also go to zero. This imposes the constraints that

$$1 - \int_0^1 \frac{X(\nu)\psi(\nu)}{1-k\nu} d\nu - e^{-k\tau} \int_0^1 \frac{Y(\nu)\psi(\nu)}{1+k\nu} d\nu = 0$$

(II-30)

and

$$1 - \int_0^1 \frac{X(\nu)\psi(\nu)}{1+k\nu} - e^{-k\tau} \int_0^1 \frac{Y(\nu)\psi(\nu)}{1-k\nu} d\nu = 0$$

(II-31)

By comparison with Equation (II-27) and (II-28) we see that this imposes the constraint that $a = b = 0$. Then, by substituting $a = b = 0$ into Equations (II-22) and (II-23), we see that the solutions corresponding to the physical problem in this case are the analytic solutions, X_0 and Y_0 of Busbridge. Case II is encountered in the Rayleigh-scattering problem for $\psi^{(1)}$ and $\psi^{(2)}$ (Equation (II-4)). This non-uniqueness was not recognized previously.

Finally we consider Case III, the conservative case $(\psi^{(3)})$ where $k = 0$ (the zeros of λ are at ∞). Here Mullikin (1962d) showed that all solutions to Equations (II-1) and (II-2) are given by

$$X(\mu) = X_0(\mu) + a\mu(X_0(\mu) + Y_0(\mu))$$

$$+ b\mu[\gamma X_0(\mu) + \mu(X_0(\mu) + Y_0(\mu))] \tag{II-32}$$

$$Y(\mu) = Y_0(\mu) - a\mu(X_0(\mu) + Y_0(\mu))$$

$$- b\mu[\gamma Y_0(\mu) - \mu(X_0(\mu) + Y_0(\mu))] \tag{II-33}$$

where

$$\gamma = \frac{x_1 + y_1}{y_0} \tag{II-34}$$

$$x_n = \int_0^1 \nu^n \psi(\nu) X_0(\nu) d\nu \tag{II-35}$$

$$y_n = \int_0^1 \nu^n \psi(\nu) Y_0(\nu) d\nu \tag{II-36}$$

with $n = 0,1$ and the a and b are constants constrained now by $b(b\gamma^2 + 2a\gamma - 2) = 0$.

Again considering the behavior of the zeros of λ which are now at $z \to \infty$, Mullikin derived the constraints on a and b

$$-2b\int_0^1 \nu^2\psi(\nu)d\nu = 1 - \int_0^1 [X(\nu) + Y(\nu)]\psi(\nu)d\nu \tag{II-37}$$

$$-2a\int_0^1 \nu^2\psi(\nu)d\nu = \tau\int_0^1 Y(\nu)\psi(\nu)d\nu - \int_0^1 [X(\nu) - Y(\nu)]\nu\psi(\nu)d\nu \tag{II-38}$$

Once again the analytic solution will have $a = b = 0$, with the solutions to Equations (II-19) and (II-20) being given by X_0 and Y_0. In this case, however, for Rayleigh scattering, Mullikin (1966) has shown that the analytic solution is not the desired solution. The azimuth-independent solution is not the desired solution. The azimuth-independent parts of the Rayleigh-scattering transmission and scattering matrices are determined from the X_3 and Y_3 solutions. Therefore, Mullikin examined the azimuth-independent part of the source function given by

$$J(\chi,\mu,\mathfrak{z}) = \int_0^1 \int_0^\tau p(\mu,\nu) \exp\left(- \frac{|\chi-y|}{\nu}\right) J(y,\nu,\mathfrak{z}) dy \frac{d\nu}{\nu}$$

$$+ \; p(\mu,\mathfrak{z}) \exp\left(- \frac{\chi}{\mathfrak{z}}\right) \quad .$$

(II-39)

This does not go to zero as $\mathfrak{z} \to \infty$ but depends quadratically on $\mathfrak{z}$. Thus, in Equations (II-32) and (II-33) the a and b must be non-zero. Mullikin (1964) derived linear constraints based on the original transfer equation which ensures the required quadratic behavior of the solutions as $\mathfrak{z} \to \infty$.

Sekera (1966a, b) showed how to express the solution of the Rayleigh-scattering radiative transfer problem in terms of combinations of the analytic X's and Y's. The linear constraints derived by Mullikin are included in the constants used in the combining of the analytic X's and Y's to determine the azimuth-independent part of the scattering and transmission functions. Following Mullikin, Sekera showed the following relationships between the K- and L- functions (analogous to Chandrasekhar's ψ, ξ, ϕ, etc., functions of Equation (II-9) used for the azimuth-independent part of the solution) and the X_3-, X_4-, Y_3-, and Y_4- functions.

$$
\begin{aligned}
K_1(\mu) &\equiv \Psi(\mu) = (c + b\mu)\mu\, X_3(\mu) + (a + b\mu)\mu\, Y_3(\mu) \\
L_1(\mu) &\equiv \xi(\mu) = (-\, a + b\mu)\mu\, X_3(\mu) + (-\, c + b\mu)\mu\, Y_3(\mu) \\
K_2(\mu) &\equiv \phi(\mu) = (1 + c'\mu + b'\mu^2)\, X_3(\mu) + (a' + b'\mu)\mu\, Y_3(\mu) \\
L_2(\mu) &\equiv \eta(\mu) = (-\, a' + b'\mu)\mu\, X_3(\mu) + (1 - c'\mu + b'\mu^2)\, Y_3(\mu) \\
K_3(\mu) &\equiv \chi(\mu) = (1 + e\mu - f\mu^2)\, X_4(\mu) + (-\, g + f\mu)\mu\, Y_4(\mu) \\
L_3(\mu) &\equiv \sigma(\mu) = (g + f\mu)\mu\, X_4(\mu) + (1 - e\mu - f\mu^2)\, Y_4(\mu) \\
2K_4(\mu) &\equiv 2\zeta(\mu) = (c + f'\mu)\mu\, X_4(\mu) + (a - f'\mu)\mu\, Y_4(\mu) \\
2L_4(\mu) &\equiv 2\theta(\mu) = (a + f'\mu)\mu\, X_4(\mu) + (-\, c + f'\mu)\mu\, Y_4(\mu)
\end{aligned}
\qquad \text{(II-40)}
$$

where

$$
b = -\frac{1}{2}\Delta \qquad b' = 2mb
$$

$$
f' = \frac{1}{2}\Delta' \qquad f = 2mf' \qquad \text{(II-41)}
$$

$$
\begin{aligned}
a &= -\, b\Gamma + Gf' \qquad & a' &= -\, b'\Gamma + 2(-\, s + nG)f' \\
c &= b\Gamma + Gf' \qquad & c' &= b'\Gamma + 2(-\, s + nG)f'
\end{aligned}
\qquad \text{(II-42)}
$$

$$e = -2(r + n\Gamma)\,b - Gf \qquad g = -2(r + n\Gamma)\,b + Gf \tag{II-43}$$

$$2\Delta = \frac{1}{m + s\Gamma} \qquad 2\Delta' = \frac{1}{m - rG}$$

$$\Gamma = \frac{\alpha_1 - \alpha_3}{\beta_0 + \beta_2} \qquad G = \frac{2(\beta_1' - \beta_3') - \tau(\beta_0' + \beta_2')}{2(\alpha_0' + \alpha_2')}$$

(II-44)

$$m = \alpha_0\beta_0' - \alpha_1\beta_1' \qquad r = \alpha_0\alpha_1' - \alpha_1\alpha_0'$$

$$n = \alpha_0'\beta_0 - \alpha_1'\beta_1 \qquad s = \beta_0\beta_1' - \beta_1\beta_0$$

$$\alpha_0 = 1 - \frac{3}{4} m_0[P_3] \qquad \beta_0 = 1 - \frac{3}{4} m_0\,[Q_3]$$

$$\alpha_i = \frac{3}{4} m_i\,[P_3] \qquad \beta_i = \frac{3}{4} m_i\,[Q_3] \quad (i = 1, 2 \cdots)$$

$$\alpha_0' = 1 - \frac{3}{8} m_0\,[P_4] \qquad \beta_0' = 1 - \frac{3}{8} m_0[Q_4]$$

$$\alpha_i' = \frac{3}{8} m_i\,[P_4] \qquad \beta_i' = \frac{3}{8} m_i\,[Q_4] \quad (i = 1, 2 \cdots)$$

$$P_i = X_i(\mu) + Y_i(\mu) \qquad Q_i = X_i(\mu) - Y_i(\mu) \quad (i = 3, 4)$$

(II-45)

and $m_i[F]$ stands for the i-th moment of the function F, i.e.,

$$m_i\,[F] = \int_0^1 F(x)\,x^i\,dx \qquad \text{(II-46)}$$

These relationships only hold for the case $\omega_0 = 1$, for perfect Rayleigh scattering with no absorption. For $\omega_0 > 1$, the K- and L-functions cannot be expressed by the simple relationships above (Equation (II-40)). The reduction of the singular integral equations for the K- and L-functions is then much more complicated, and will not be discussed here.

This is the case where Chandrasekhar (1960) recognized the non-uniqueness and specified his "standard" solutions by consideration of the flux equations. If one would like to use Chandrasekhar's solution it is necessary to introduce the moment conditions for the "standard" solutions in the equations above. Since the "standard" solution is defined by the relations

$$\alpha_0 + \alpha_2 = 0\,, \qquad \beta_0 + \beta_2 = 0 \qquad \text{(II-47)}$$

one has to substitute in Equation (II-44) $1/\Gamma = 0$ in order to obtain the corresponding forms of the equations for "standard" solutions. This leads to $\Delta = 0$, $b = 0$, $b' = 0$, but we see that

$$b\Gamma = -\frac{1}{4s} \quad , \quad b'\Gamma = \frac{m}{2s} \qquad \text{(II-48)}$$

Equations (II-5) through (II-10) along with (II-40) thus constitute the complete, unique solution to the Rayleigh-scattering radiative transfer problem for all optical thicknesses.

Mullikin (1962d) also obtained from the constraints he developed a new formulation of the equations that was more suitable for numerical computations. These were particularly useful for atmospheres of large optical thickness. His computational scheme will be outlined in the following section.

Following these theoretical considerations by Mullikin and Sekera, a computer program was developed to compute the X- and Y-functions and the K- and L-functions. Carlstedt and Mullikin (1966) calculated and published tables of X- and Y-functions for the case of isotropic scattering, which has the single characteristic function $\psi = \frac{1}{2}$ and only one set of X- and Y-functions.

Sekera and Kahle (1966) extended the computation to the case of Rayleigh-scattering. They published tables of the X- and Y-functions and the K- and L-functions for a complete range of values of τ and μ. These tables are reproduced here in the Appendix. Their calculation and application are the subject of the remainder of this dissertation.

C. METHOD OF COMPUTING THE X- AND Y-FUNCTIONS

Mullikin (1962c, d) and Carlstedt and Mullikin (1966) have shown how to solve for the X- and Y-functions for the three cases discussed in the previous section. The portion of their discussion relevant to the computer calculation of the X- and Y-functions for Rayleigh scattering is outlined below, in sufficient detail to enable the reader to develop computer programs to perform such calculations.

CASE 1 -- Unique Case ($\psi^{(4)}$)

We will look first at the unique case, λ with no zeros. We define the following functions:

$$\theta(t) = \frac{1}{\pi} \tan^{-1} \left[\frac{\pi t \psi(t)}{\lambda_0(t)}\right] \qquad \text{(II-49)}$$

$$N(z) = \exp \left[\int_0^1 \frac{\theta(t)}{t - z}\right] dt \qquad \text{(II-50)}$$

where

$$\lambda_0(t) = 1 - 2t^2 \int_0^1 \frac{\psi(t') - \psi(t)\, dt'}{t^2 - t'^2} + t\psi(t) \ln \frac{1 - t}{1 + t} \qquad \text{(II-51)}$$

and finally

$$N(0) = \left[1 - 2\int_0^1 \psi(t)\, dt\right]^{-1/2} \qquad \text{(II-52)}$$

Then Chandrasekhar's H-function (Chandrasekhar, 1960) can be shown to be related to the N-function (Mullikin, 1964a)

$$H(z) = \frac{N(0)}{N(-z)} = \left[1 - 2\int_0^1 \psi(t) dt\right]^{-1/2} \exp\left[-\int_0^1 \frac{\theta(t)}{t+z}\right] \qquad \text{(II-53)}$$

Mullikin then shows that the X- and Y-functions can be written for z outside [- 1, 0]

$$X(z) = H(z)\left[1 + f(z) + z\int_0^1 \frac{\psi(t)[f(t) - f(z)]}{H(t)\ \Delta(t)\ (t - z)}\, dt\right] \qquad \text{(II-54)}$$

$$Y(z) = H(z)\left[g(z) + z\int_0^1 \frac{\psi(t)[g(t) - g(z)]}{H(t)\ \Delta(t)\ (t - z)}\, dt\right] \qquad \text{(II-55)}$$

where

$$\Delta(t) = [\lambda_0(t)]^2 + [\pi t \psi(t)]^2 \tag{II-56}$$

and

$$f = \frac{p - q}{2} \qquad g = \frac{p + q}{2} \tag{II-57}$$

$$p(z) = - L(p)(z) + \frac{\exp(- \tau/z)}{H(z)} \tag{II-58}$$

$$q(z) = L(q)(z) + \frac{\exp(- \tau/z)}{H(z)} \tag{II-59}$$

and the operator L is defined as

$$L(p)(z) = \frac{z \exp(- \tau/z)}{H(z)} \int_0^1 \frac{\psi(t)p(t)\,dt}{H(t)\,\Delta(t)\,(t + z)} \tag{II-60}$$

The equations for p and q are Fredholm equations in the interval of interest, and can be solved by iteration. The convergence of these iterations is very rapid, and is faster, the larger the value of τ .

The rest of the problem is, then, just quadratures. This method is good for all values of τ .

CASE 2 -- Non-unique, Non-conservative Case $(\psi^{(1)}, \psi^{(2)})$

For this case the X- and Y-functions can be written as

$$X(z) = H(z) = \left[1 + f(z) + \frac{kzN(0)}{N(1/k)} \frac{f(z) - f(1/k)}{1 - kz} + z \int_0^1 \frac{\psi(t)\left[f(t) - f(z)\right]}{H(t)\,\Delta(t)\,(t - z)}\,dt \right] \qquad \text{(II-61)}$$

$$Y(z) = H(z) \left[g(z) + \frac{kzN(0)}{N(1/k)} \frac{g(z) - g(1/k)}{1 - kz} + z \int_0^1 \frac{\psi(t)\left[g(t) - g(z)\right]}{H(t)\,\Delta(t)\,(t - z)} \right] \qquad \text{(II-62)}$$

where f and g are defined in terms of p and q as before, but p and q are more complicated.

$$p = h_1 - ch_2 \qquad q = h_3 - dh_4 \qquad \text{(II-63)}$$

where

$$
\begin{aligned}
h_1 &= - L(h_1) + \frac{N(- z)}{N(0)} \exp\left(- \frac{\tau}{z}\right) \\
h_2 &= - L(h_2) + zN(- z) \exp\left(- \frac{\tau}{z}\right) \\
h_3 &= L(h_3) + \frac{N(- z)}{N(0)} \exp\left(- \frac{\tau}{z}\right) \\
h_4 &= L(h_4) + zN(- z) \exp\left(- \frac{\tau}{z}\right)
\end{aligned}
\tag{II-64}
$$

and the operator L is now

$$
L(h)(z) = \frac{z \exp(- \tau/z)}{(1 + kz)H(z)} \int_0^1 \frac{\psi(t)(1 - kt)h(t)}{H(t)\,\Delta(t)(t + z)}\, dt \tag{II-65}
$$

These are, again, rapidly converging Fredholm equations.

The constants c and d are

$$
c = \frac{h_1(1/k) + h_1(- 1/k) \exp(- k\tau)}{h_2(1/k) - h_2(- 1/k) \exp(- k\tau)} \tag{II-66}
$$

$$
d = \frac{h_3(1/k) - h_3(- 1/k) \exp(- k\tau)}{h_4(1/k) - h_4(- 1/k) \exp(- k\tau)}
$$

CASE 3 -- Non-unique, Conservative Case ($\psi^{(3)}$)

These results are derived from allowing k to approach zero in the above case. Now

$$X(z) = H(z) \left\{ 1 + [1 - zN(0)]\, f(z) + \frac{N(0)d}{2} z + z \int_0^1 \frac{\psi(t)[f(t) - f(z)]}{H(t)\, \Delta(t)\, (t - z)}\, dt \right\}$$

(II-67)

$$Y(z) = H(z) \left\{ [1 - zN(0)]\, g(z) - \frac{N(0)d}{2} z + z \int_0^1 \frac{\psi(t)[g(t) - g(z)]}{H(t)\, \Delta(t)\, (t - z)}\, dt \right\}$$

(II-68)

where now the constant

$$d = -\frac{2}{N(0)} \frac{1 + M_3}{\tau + 2\left[1 - \int_0^1 \theta(t)\, dt\right] - \frac{2}{N(0)} M_4} \qquad \text{(II-69)}$$

and the constant $c = 0$, and

$$M_i = \int_0^1 \frac{\psi(t)(1 - kt)h_i(t)}{H(t)\,\Delta(t)}\, dt \qquad \text{(II-70)}$$

Thus all the values of the X- and Y-functions have been reduced to the solution of rapidly converging Fredholm equations and quadratures. Once the X_3-, X_4-, Y_3-, and Y_4-functions have been calculated, the K- and L-functions are quickly calculated using Equations II-40 through II-46 in Section B of Chapter II. From these functions, the solution to several problems in radiative transfer can be quickly determined. A description of how these functions can be used is given in the next chapter.

III. DESCRIPTION OF THE TABLES AND THEIR USE

In this chapter the tables of X- and Y- and K- and L-functions which are given in the Appendix are described. The method of finding the Stokes parameters from these functions is written out in detail. Other uses for the tables are also suggested. Finally, the accuracy of the tables is discussed.

A. DESCRIPTION OF THE TABLES

Table 1 in the Appendix contains the X- and Y-functions for the τ values 0.15, 0.25, 0.50, 0.70, 1.0, 2.0, 4.0, 8.0, 16.0, and 100.0. The functions are listed for all μ values from 0.00 to 1.00 in steps of 0.02. Table 2 contains the moments of the X- and Y-functions of order zero through four, for the same τ values as in Table 1.

Tables 3 and 4 give the K- and L-functions and their moments for the same parameters. All values are given to five figures. The decimal part of the number is followed by an E and the power of 10 to which the decimal part should be raised; i.e., 0.17452 E-01 = 0.017452.

For the largest value of the optical thickness given here, $\tau = 100$, all the X- and Y-functions except X_3 and Y_3 have essentially reached their values for an atmosphere of infinite thickness (the X-functions approach the H-functions [Chandrasekhar, 1950] and the Y-functions approach zero). In the conservative case (X_3 and Y_3) the values of the X- and Y-functions approach their limiting values as $1/\tau$, rather than exponentially as in the other cases. Since the values of the K- and L-functions depend upon X_3 and Y_3, they too have not yet reached their value for an infinitely thick atmosphere.

B. CALCULATION OF THE STOKES PARAMETERS

For a plane-parallel atmosphere, Rayleigh-scattering atmosphere, the Stokes parameters $\vec{I} \equiv (I_\ell, I_r, U, V)$ of the radiation emerging from either the top or the bottom can be written as the sum of three terms:

$$\vec{I}(\tau; \mu,\phi) = \vec{I}^{(0)}(\tau; \mu, \mu_0) + \vec{I}^{(1)}(\tau; \mu, \mu_0, \phi - \phi_0) + \vec{I}^{(2)}[\tau; \mu, \mu_0, 2(\phi - \phi_0)] \qquad \text{(III-1)}$$

where, in the same manner as with the scattering and transmission matrices and the Rayleigh-scattering phase function, the first term is azimuth-independent, and the second and third terms contain the elements with the cosine or sine of $(\phi - \phi_0)$ or $2(\phi - \phi_0)$, respectively. For Rayleigh scattering $U^{(0)}$ is zero, as are all three V terms, so that $\vec{I}^{(i)}$ can be reduced to a two- or three-element matrix. In Equation (III-1), (μ, ϕ) and (μ_0, ϕ_0) are the sets of the usual directional parameters of the emerging radiation (μ, ϕ) and of the external parallel solar radiation (μ_0, ϕ_0) irradiating the top of the atmosphere. For the upward radiation, $\tau = 0$ and μ is positive; for the downward radiation, $\tau = \tau_1$ (the total optical thickness of the atmosphere) and μ is negative. If $\pi\vec{F}$ denotes the net flux of the external radiation, then by combining Equations (I-39) and (I-40)(defining $\vec{I}$ in terms of $\overleftrightarrow{S}$ and $\overleftrightarrow{T}$) with Equations (II-5) through (II-10) and (II-40) (defining $\overleftrightarrow{S}$ and $\overleftrightarrow{T}$ in terms of the X-, Y-, K-, and L-functions), the azimuth-independent terms can be written in the form [Chandrasekhar, 1950; Sekera, 1963, 1966b]:

$$\vec{I}^{(0)}(0;\, +\mu,\, -\mu_0) = \begin{pmatrix} I_\ell^{(0)}(0;\, \mu,\, \mu_0) \\ I_r^{(0)}(0;\, \mu,\, \mu_0) \end{pmatrix}$$

$$= \frac{3}{16}\,\frac{\mu_0\omega_0}{\mu_0 + \mu}\,[\overleftrightarrow{K}(\mu)\cdot\tilde{\overleftrightarrow{K}}(\mu_0) - \overleftrightarrow{L}(\mu)\cdot\tilde{\overleftrightarrow{L}}(\mu_0)\cdot\vec{F} \tag{III-2}$$

$$\vec{I}^{(0)}(\tau_1;\, -\mu,\, -\mu_0) = \begin{pmatrix} I_\ell^{(0)}(\tau_1;\, \mu,\, \mu_0) \\ I_r^{(0)}(\tau_1;\, \mu,\, \mu_0) \end{pmatrix}$$

$$= \frac{3}{16}\,\frac{\mu_0\omega_0}{\mu_0 - \mu}\,[\overleftrightarrow{K}(\mu)\cdot\tilde{\overleftrightarrow{L}}(\mu_0) - \overleftrightarrow{L}(\mu)\cdot\tilde{\overleftrightarrow{K}}(\mu_0)]\cdot\vec{F} \tag{III-3}$$

where the two-by-two matrices $\overleftrightarrow{K}$ and $\overleftrightarrow{L}$ have the form

$$\overleftrightarrow{N}(\mu) \equiv \begin{pmatrix} N_1(\mu) , & \sqrt{2}N_2(\mu) \\ N_3(\mu) , & \sqrt{2}N_4(\mu) \end{pmatrix} \quad (N \equiv K, L) \tag{III-4}$$

The functions K_i, L_i are given in Table 3 as functions of μ for a particular value of the parameter τ_1 and for $\omega_0 = 1$.

Assuming the unpolarized external irradiation $F_\ell = F_r = 1/2F_0$, then the azimuth-dependent terms are, for upward radiation,

$$\vec{I}^{(1)}(0; +\mu, -\mu_0, \phi - \phi_0) = \begin{pmatrix} I_\ell^{(1)}(0; +\mu, -\mu_0, \phi - \phi_0) \\ I_r^{(1)}(0; +\mu, -\mu_0, \phi - \phi_0) \\ U^{(1)}(0; +\mu, -\mu_0, \phi - \phi_0) \end{pmatrix}$$

$$= \frac{4C\ \mu_0}{\mu_0 + \mu} (1 - \mu^2)^{1/2} (1 - \mu_0^2)^{1/2} \begin{pmatrix} -\mu \cos (\phi - \phi_0) \\ 0 \\ -\sin (\phi - \phi_0) \end{pmatrix} M^{(1)}(\tau_1; \mu, \mu_0) \tag{III-5}$$

$$\vec{I}^{(2)}[0; +\mu, -\mu_0, 2(\phi - \phi_0)] = \begin{pmatrix} I_\ell^{(2)}(0; +\mu, -\mu_0, 2(\phi - \phi_0)) \\ I_r^{(2)}(0; +\mu, -\mu_0, 2(\phi - \phi_0)) \\ U^{(2)}(0; +\mu, -\mu_0, 2(\phi - \phi_0)) \end{pmatrix}$$

$$= \frac{C}{\mu_0 + \mu} (1 - \mu_0^2) \begin{pmatrix} -\mu^2 \cos 2(\phi - \phi_0) \\ \cos 2(\phi - \phi_0) \\ -2\mu \sin 2(\phi - \phi_0) \end{pmatrix} M^{(2)}(\tau_1; \mu, \mu_0) \qquad \text{(III-6)}$$

where $C = (3/32)\omega_0\mu_0 F_0$

$$M^{(i)}(\tau_1; \mu, \mu_0) = X_i(\mu)\, X_i(\mu_0) - Y_i(\mu) Y_i(\mu_0) \quad (i - 1,2) \qquad \text{(III-7)}$$

For downward radiation,

$$\vec{I}^{(1)}(\tau_1; -\mu, -\mu_0, \phi - \phi_0) = \begin{pmatrix} I_\ell^{(1)}(\tau_1; -\mu, -\mu_0, \phi - \phi_0) \\ I_r^{(1)}(\tau_1; -\mu, -\mu_0, \phi - \phi_0) \\ U^{(1)}(\tau_1; -\mu, -\mu_0, \phi - \phi_0) \end{pmatrix}$$

$$= \frac{4C\mu_0}{\mu_0 - \mu} (1 - \mu^2)^{1/2} (1 - \mu_0^2)^{1/2} \begin{pmatrix} \mu \cos (\phi - \phi_0) \\ 0 \\ - \sin (\phi - \phi_0) \end{pmatrix} W^{(1)}(\tau_1; \mu, \mu_0)$$

(III-8)

$$\vec{I}^{(2)} (\tau_1; -\mu, -\mu_0, 2(\phi - \phi_0)) = \begin{pmatrix} I_\ell^{(2)}(\tau_1; -\mu, -\mu_0, 2(\phi - \phi_0)) \\ I_r^{(2)}(\tau_1; -\mu, -\mu_0, 2(\phi - \phi_0)) \\ U^{(2)}(\tau_1; -\mu, -\mu_0, 2(\phi - \phi_0)) \end{pmatrix}$$

$$= \frac{C}{\mu_0 - \mu} (1 - \mu_0^2) \begin{pmatrix} -\mu^2 \cos 2(\phi - \phi_0) \\ \cos 2(\phi - \phi_0) \\ 2\mu \sin 2(\phi - \phi_0) \end{pmatrix} W^{(2)}(\tau_1; \mu, \mu_0)$$

(III-9)

where

$$W^{(i)}(\tau_1; \mu, \mu_0) = X_i(\mu)\, Y_i(\mu_0) - Y_i(\mu)\, X_i(\mu_0) \quad (i = 1, 2) \tag{III-10}$$

Illustrations of the use of the tables to find the intensity of radiation emerging from the top and bottom of a plane-parallel Rayleigh-scattering atmosphere will be given in Chapter IV, Section A.

C. ADDITIONAL USES OF THE TABLES

The primary purpose of Tables 1 through 4, as discussed above, is to provide numerical values of the functions needed to compute the intensity and polarization parameters of the radiation emerging from the top and from the bottom of a plane-parallel planetary atmosphere with Rayleigh scattering. Equations (III-1) through (III-10) give the computational scheme for this application.

In addition, the tables can be used to compute the following quantities, mentioned in the five headings below.

(a) Chandrasekhar's functions $\gamma_\ell(\tau, \mu)$, $\gamma_r(\tau, \mu)$, $\bar{s}(\tau)$

These are needed to compute the effect of ground reflections governed by Lambert's law on the average intensity and net fluxes of the emerging radiation. Lambert reflection is where the surface reflects unpolarized light uniformly in all directions, independent of the direction and polarization of the incident light. As shown by Sekera (1966b), these functions can be computed from the values of the functions K_i, L_i and their zero and first moments. Using the moment notation of Equation (II-46), we can write [Sekera, 1966b, Eq. (150), p. 51]

$$\gamma_\ell(\mu) = \ell_1[K_1(\tau, \mu) + L_1(\tau, \mu)] + 2\ell_2[K_2(\tau,\mu) + L_2(\tau, \mu)] \tag{III-11}$$

$$\gamma_r(\mu) = \ell_1[K_3(\tau, \mu) + L_3(\tau, \mu)] + 2\ell_2[K_2(\tau,\mu) + L_2(\tau, \mu)] \tag{III-12}$$

$$\overline{s}(\tau) = 1 - \ell_1 m_1[K_1 + K_3 + L_1 + L_3] - 2\ell_2 m_1[K_2 + K_4 + L_2 + L_4) \tag{III-13}$$

where

$$\ell_i = \frac{3}{8} m_0[L_i(\tau, \mu) + L_{i+2}(\tau, \mu)] \quad (i = 1, 2) \tag{III-14}$$

If ground reflection is according to Lambert's law with reflectivity A, the terms that must be added to the intensity vectors $\vec{I}^{(0)}(0; +\mu, -\mu_0)$ and $\vec{I}^{(0)}(\tau_1; -\mu, -\mu_0)$ have the form [Chandrasekhar, 1950 ,p. 279), for upward radiation

$$\vec{I}^{\star}(0; \mu, -\mu_0) = \begin{bmatrix} I_\ell^{\star} \\ \\ I_r^{\star} \end{bmatrix} \tag{III-15}$$

$$= \frac{A\mu_0 F_0}{4[1 - A\bar{s}(\tau)]} [\gamma_\ell(\tau, \mu_0) + \gamma_r (\tau, \mu_0)] \begin{bmatrix} \gamma_\ell(\tau, \mu) \\ \\ \gamma_r(\tau, \mu) \end{bmatrix}$$

and for downward radiation,

$$\vec{I}^{\star}(\gamma_1; -\mu, -\mu_0) = \frac{A\mu_0 F_0}{4[1 - A\bar{s}(\tau)]} [\gamma_\ell(\tau, \mu_0) + \gamma_r(\tau, \mu_0)] \begin{bmatrix} 1 - \gamma_\ell(\tau, \mu) \\ \\ 1 - \gamma_r(\tau, \mu) \end{bmatrix}$$

(III-16)

(b) <u>The average intensity of the emerging radiation</u>

This is defined as

$$J(0, \mu_0) = \frac{1}{4\pi} \int_0^1 \int_0^{2\pi} [I_\ell(0; \mu, \phi) + I_r(0; + \mu, \phi)] d\mu \, d\phi \tag{III-17}$$

or

$$J(\tau_1, \mu_0) = \frac{1}{4\pi} \int_0^1 \int_0^{2\pi} [I_\ell(\tau_1; - \mu, \phi) + I_r(\tau_1; - \mu, \phi)] \, d\mu \, d\phi \tag{III-18}$$

Upon integration with respect to ϕ , these reduce to

$$\frac{1}{2} m_0 [I_\ell^{(0)}(0; + \mu, - \mu_0) + I_r^{(0)}(0; + \mu, - \mu_0)] \tag{III-19}$$

or

$$\frac{1}{2} m_0[I_\ell^{(0)}(\tau_1; -\mu, \mu_0) + I_r^{(0)}(\tau_1; -\mu, -\mu_0)] \qquad \text{(III-20)}$$

respectively. Sekera has shown [1966b, Eq. (140), p. 47] that these expressions can easily be computed in terms of the K- and L-functions

$$J(0, \mu_0) = (F_0/4)[\sum_{n=1}^{4} K_n(\tau_1, \mu_0) - 2] \qquad \text{(III-21)}$$

and

$$J(\tau_1, \mu_0) = (F_0/4)\sum_{n=1}^{4} L_n(\tau_1, \mu_0) - 2e^{-\tau_1/\mu_0}] \qquad \text{(III-22)}$$

(c) Global Radiation, G_d and H_d

The downward global radiation, G_d, is the total radiation reaching a unit surface area of a planet, and includes both the diffuse sky radiation, H_d, and the direct (but attenuated) solar radiation, S. The upward global radiation, G_u, is just equal to the upward diffuse sky radiation, H_u.

Deirmendjian and Sekera (1954) give the following expressions for downward radiation:

$$S = \pi F_0 \mu_0 e^{-\tau/\mu_0} \qquad \text{(III-23)}$$

$$H_d = \pi F_0 \mu_0 \left[\frac{\gamma_\ell(\mu_0) + \gamma_r(\mu_0)}{2(1 - A\bar{s})} - e^{-\tau/\mu_0} \right] \qquad \text{(III-24)}$$

$$G_d = H_d + S = \pi F_0 \mu_0 \left[\frac{\gamma_\ell(\mu_0) + \gamma_r(\mu_0)}{2(1 - A\bar{s})} \right] \qquad \text{(III-25)}$$

The upward radiation, as given by Coulson (1959), is

$$H_u = G_u = \pi F_0 \mu_0 \left[1 - \frac{\gamma_\ell(\mu_0) + \gamma_r(\mu_0)}{2} \; \frac{1 - A}{1 - A\bar{s}} \right] \qquad \text{(III-26)}$$

(d) Effect of ground reflection governed by a general law of reflection

The intensity matrices of the contribution to the emerging radiation from ground reflection governed by a more general law than Lambert's (see (a) above) can be expressed in a rather complicated form containing the reflection and transmission matrices. Hence, after a lengthy reduction, we obtain the expressions for these intensities that contain the functions K_i, L_i (i - 1, 2, 3, 4), as well as X_j, Y_j (j = 1, 2). These expressions for Fresnel's law (specular reflection with partial linear polarization) can, for example, be found in papers by Sekera and Frazer (1953) and Frazer (1965).

(e) Computation of the characteristics of the internal radiation field

The intensity and polarization of the diffuse radiation at any level within the atmosphere can be computed from the principles of invariance that express the intensity vector of the upward and downward radiation in terms of the diffuse reflection and transmission by the layers above and below the reference level. Using the diffuse reflection and transmission matrixes $\overleftrightarrow{S}$ and $\overleftrightarrow{T}$ defined in Equations (I-39) and (I-40) respectively, we have for the upward radiation at the level $\tau(0 < \tau < \tau_1)$

$$\mu\vec{I}(\tau; +\mu, \phi) = \frac{1}{4}\overleftrightarrow{S}(\tau_1 - \tau; \mu, \phi; \mu_0, \phi_0) \cdot \vec{F}e^{-\tau/\mu_0}$$

$$+ \frac{1}{4}\{\overleftrightarrow{S}(\tau_1 - \tau; \mu, \phi; \mu', \phi') \cdot \mu\vec{I}(\tau; -\mu', \phi')\}$$

(III-27)

and for the downward radiation

$$\mu\vec{I}(\tau; -\mu, \phi) = \frac{1}{4}\overleftrightarrow{T}(\tau; \mu, \phi; \mu_0, \phi_0) \cdot \vec{F}$$

$$+ \frac{1}{4}\{\overleftrightarrow{S}(\tau; \mu, \phi; \mu', \phi') \cdot \mu\vec{I}(\tau; +\mu, \phi)\}$$

(III-28)

where the symbol used for hemispherical integration is defined as

$$\{F(\mu', \phi')\} \equiv \frac{1}{4\pi}\int_0^{2\pi}\int_0^1 F(\mu', \phi')\,\frac{d\mu'}{\mu'}\,d\phi' \qquad \text{(III-29)}$$

These relations follow directly from the physical meaning of the diffuse reflection and transmission by the sublayers above and below the reference level. These equations can be expanded as before into two equations for the azimuth-independent terms, and one for each azimuth-dependent term. These integral equations must be solved by successive iteration; to compute these iterations for the azimuth-independent terms, we need the K_i- and L_i-functions for the optical thicknesses τ and $\tau_1 - \tau$, and for the azimuth-dependent terms the functions X_i, Y_i $(i = 1, 2)$.

The foregoing list of possible uses of tabulated functions is by no means complete. The applications mentioned should be regarded as selected examples. One can easily extend this list to include other applications. For example, the tables can be used in problems of mixed scattering in which the phase matrix is the sum of a Rayleigh scattering matrix and an additional matrix representing contributions from isotropic, neutral (molecular anisotropy, resonance scattering), or aerosol (turbid atmosphere) scattering. However, some of these applications cannot be carried out yet, as the explicit formulas required have not been developed.

D. ACCURACY OF THE TABULATED VALUES

There are several relationships that should be satisfied by the functions K_i, L_i, X_i, Y_i $(i - 1, 2, 3, 4)$ or their moments, and we have used them to check the accuracy and internal consistency of the computed values of these functions.

From the physical fact that the neutral lines (i.e., the lines where $I_r = I_\ell$ or $Q = 0$) must pass through the zenith or the nadir at 45^o to the sun's vertical (i.e., Q for $\phi - \phi_0 = 45^o$ must approach zero for $\mu \to 1$), it follows that the K- and L-functions for $\mu = 1$ must satisfy the condition

$$K_1 - K_3 = K_2 - K_4 = L_1 - L_3 = L_2 - L_4 = 0 \qquad \text{(III-30)}$$

It can be seen that all five significant figures of the tabulated values satisfy this condition for every τ. This accuracy may be compared with the accuracy of previous computations [Chandrasekhar and Elbert, 1954; Sekera and Blanch, 1952], as shown in Table A.

The relations for the moments of the K- and L-functions can be derived from the linear constraints on the solution of the integral equation for the azimuth-independent terms of the reflection and transmission matrices [Sekera, 1966b, p. 49]. If we introduce the following abbreviations for the moments

$$k_i^{(n)} = \frac{3}{8} m_n[K_i + K_{i+2}], \quad \ell_i^{(n)} = \frac{3}{8} m_n[L_i + L_{i+2}] \quad (i = 1, 2)$$

(III-31)

then these relations have the form

$$M_1 \equiv k_1^{(0)} + \ell_1^{(0)} = 1 \qquad \text{(III-32)}$$

$$M_2 \equiv 2k_2^{(0)} + 2\ell_2^{(0)} = 1 \qquad \text{(III-33)}$$

$$M_3 \equiv 2[k_1^{(1)} - \ell_1^{(1)}] - \tau[1 - k_1^{(0)} + \ell_1^{(0)}] = 0 \qquad \text{(III-34)}$$

$$M_4 \equiv 4[k_2^{(1)} - \ell_2^{(1)}] - \tau[1 - 2k_2^{(0)} + 2\ell_2^{(0)}] = 0 \qquad \text{(III-35)}$$

The left side of Equations (III-32) through (III-35) are tabulated for different τ values in Table B. The larger deviations from zero for large values of τ result from the loss of significant figures in the checking process rather than from the inaccuracy of the moments. The moments of the X- and Y-functions must satisfy two sets of relations. For the characteristic functions of the form

$$\psi(\mu) = a + b\mu^2 \qquad \text{(III-36)}$$

we have the relation [Chandrasekhar, 1950, p. 189]

$$\alpha_0 - \frac{1}{2}[a(\alpha_0^2 - \beta_0^2) + b(\alpha_1^2 - \beta_1^2)] = 1 \qquad \text{(III-37)}$$

where α_n, β_n stand for the n-th moments of the functions X and Y, respectively. Since $a = - b = 3/4$, for X_3, Y_3 and $a = - b = 3/8$ for X_4, Y_4, we have

$$N_1 \equiv (\alpha_3)_0 - \frac{3}{8} [(\alpha_3)_0^2 - (\beta_3)_0^2 - (\alpha_3)_1^2 + (\beta_3)_1^2] = 1 \qquad \text{(III-38)}$$

$$N_2 \equiv (\alpha_4)_0 - \frac{3}{16} [(\alpha_4)_0^2 - (\beta_4)_0^2 - (\alpha_4)_1^2 + (\beta_4)_1^2] = 1 \qquad \text{(III-39}$$

where

$$(\alpha_i)_n = m_n[X_i], \qquad (\beta_i)_n = m_n[Y_i] \qquad \text{(III-40)}$$

stand for the n-th moment of the functions X_i and Y_i, respectively. The left sides of Equations (III-38) and (III-39) are tabulated in Table C.

Another set of relationships follows from the identity satisfied by any set of the X- and Y-functions [cf. Chandrasekhar, 1950, p. 187; Sekera, 1966b, Equation (54)]. If we use for the modified moments the following notation:

$$x_n^{(i)} = \int_0^1 \psi^{(i)}(t)\, X_i(t)\, t^n\, dt, \quad y_n^{(i)} = \int_0^1 \psi^{(i)}(t)\, Y_i(t)\, t^n\, dt$$

(III-41)

then

$$2x_0^{(i)} - x_0^{(i)^2} + y_0^{(i)^2} = 2\int_0^1 \psi^{(i)}(t)\, dt \qquad \text{(III-42)}$$

The modified moments x_n and y_n can be expressed in terms of the ordinary moments, defined in Equation (II-46) by substituting in Equation (III-41) the expressions for the corresponding functions. In this way we obtain

for X_1: $$x_n = \frac{3}{8}\left[(\alpha_1)_n + (\alpha_1)_{n+2} - 2(\alpha_1)_{n+4}\right], \quad 2\int_0^1 \psi^{(1)} dt = 0.70$$

(III-43)

for X_2: $$x_n = \frac{3}{16}\,[(\alpha_2)_n + 2(\alpha_2)_{n+2} + (\alpha_2)_{n+4}],\quad 2\int_0^1 \psi^{(2)}\,dt = 0.70$$

(III-44)

for X_3: $$x_n = \frac{3}{4}\,[(\alpha_3)_n - (\alpha_3)_{n+2}],\quad 2\int_0^1 \psi^{(3)}\,dt = 1.00$$

(III-45)

for X_4: $$x_n = \frac{3}{8}\,[(\alpha_4)_n - (\alpha_4)_{n+2}],\quad 2\int_0^1 \psi^{(4)}\,dt = 0.50$$

(III-46)

The expressions for y_n are obtained from Equations (III-43) through (III-46) by replacing α by β. Table D gives the values of the left side of Equation (III-42) corresponding to various optical thicknesses for the functions X_i, Y_i $(i = 1, 2, 4)$. For $i = 3$, the linear constraints on X_3 and Y_3 lead to the relationship

$$x_0 + y_0 = 1 \qquad \text{(III-47)}$$

The values of $x_0 + y_0$ for various optical thicknesses are also given in Table D. The relations in Equations (III-38), (III-39), (III-42), and (III-47) allow accuracy checks of the moments of even order up to $\mu = 4$ for X_1, Y_1, X_2, Y_2, and of the order 0, 1, 2, for X_3, Y_3, X_4, Y_4. To check the accuracy of higher moments, one can use the following relations [cf. Chandrasekhar, 1950, p. 188; Sekera, 1966b, Eqs. (88) and (89)]:

$$(1 - x_0)x_2 + y_0 y_2 + \frac{1}{2}(x_1^2 - y_1^2) = \int_0^1 \psi(t)t^2 dt \qquad \text{(III-48)}$$

for X_i, Y_i $(i = 1, 2, 4)$, and

$$(x_2 + y_2)(1 - x_0 + y_0) + (x_1^2 - y_1^2) = \frac{1}{5} \qquad \text{(III-49)}$$

for X_3, Y_3.

Table A

COMPARISON OF THE ACCURACY OF THE K_i- AND L_i-FUNCTIONS FOR $\mu = 1.00$ IN VARIOUS TABLES WITH THE USE OF THE RELATION IN EQ. (III-30)

τ	$K_1 - K_3$	$K_2 - K_4$	$L_1 - L_3$	$L_2 - L_4$	Authors
0.15	-.00011	+.00005	-.00010	-.00017	Chandrasekhar and Elbert, 1954
	.00001	.00000	.00001	-.00000	Sekera and Blanch, 1952
	.00000	.00000	.00000	.00000	Present tables
0.25	-.00011	+.00009	-.00009	-.00016	Chandrasekhar and Elbert, 1954
	-.00006	-.00000	-.00005	+.00000	Sekera and Blanch, 1952
	.00000	.00000	.00000	.00000	Present tables
1.00	-.00967	-.00018	-.00535	+.00179	Chandrasekhar and Elbert, 1954
	-.00054	+.00001	-.00025	-.00016	Sekera and Blanch, 1952
	.00000	.00000	.00000	.00000	Present tables

Table B

ACCURACY CHECK OF THE MOMENTS OF THE K_i- AND L_i-FUNCTIONS BY USE OF THE RELATIONS IN EQS. (III-32), (III-33), (III-34), AND (III-35)

τ	M_1	M_2	M_3	M_4
0.15	1.00000	1.00000	0.00000	-0.00000
0.50	1.00001	1.00000	0.00000	0.00001
1.00	1.00000	0.99999	-0.00000	0.00000
4.00	1.00000	1.00000	-0.00002	-0.00005
16.00	1.00001	0.99998	0.00005	0.00030
100.00	0.99999	0.99999	-0.0010	-0.0015

Table C

ACCURACY CHECK OF THE MOMENTS OF THE FUNCTIONS X_i AND Y_i ($i = 3, 4$) SATISFYING THE RELATIONS IN EQS. (III-38) AND (III-39)

	For $i = 3$:	For $i = 4$:
τ	N_1	N_2
0.15	0.99999	0.99999
0.25	1.00000	1.00001
1.00	0.99999	0.99997
4.00	1.00007	0.99992
16.00	1.00004	1.00024
100.00	1.0000	1.0000

Table D

ACCURACY CHECK OF THE MODIFIED MOMENTS OF THE FUNCTIONS X_i, Y_i (i = 1, 2, 4) FROM THE RELATION IN EQ. (III-42) AND OF THE FUNCTIONS X_3 AND Y_3 FROM THE RELATION IN EQ. (III-47)

	$2x_0 - x_0^2 + y_0^2$			$x_0 + y_0$
τ	$i = 1$	$i = 2$	$i = 4$	$i = 3$
0.15	0.69998	0.70001	0.49999	0.99998
0.50	0.69998	0.70001	0.50001	0.99997
1.00	0.70001	0.70001	0.49998	1.00003
2.00	0.70000	0.70000	0.50001	0.99999
4.00	0.70002	0.70001	0.49998	0.99997
8.00	0.70001	0.69999	0.50002	1.00000
16.00	0.70001	0.69999	0.50002	0.99999
100.00	0.70001	0.70000	0.50002	0.99996
$2\int_0^1 \psi_i(x)\,dx$	0.700000	0.700000	0.500000	1.000000

IV. SOME APPLICATIONS OF THE TABLES

The purpose of this chapter is to briefly illustrate the type of information which can be obtained from the solution to the radiative transfer problem as represented by the tables of X- and Y- and K- and L-functions. A more complete description can be found in the published papers of Kahle (1968a, 1968b).

A. INTENSITY OF EMERGENT RADIATION

In this section we shall examine the characteristics of the intensity of the radiation emerging from the upper and lower boundaries of a plane-parallel atmosphere for a range of external parameters (solar elevations, ground reflectivities, angle of emergence) for the entire domain of the optical thickness. Chandrasekhar and Elbert (1954) and also Coulson et al. (1960), published tables of the intensity and polarization parameters (Stokes parameters) for radiation emerging from the bottom and top of the atmosphere for $\tau \leq 1$. Sekera (1957) has illustrated graphically many of the features of the polarization of the downward radiation, and Coulson (1959) has shown graphs of the upward intensity and polarization for the same optical depths. Dave and Furukawa (1966) have also shown a few of the main features of the intensity and polarization values graphically for larger optical thicknesses ($\tau > 1$). However, the results presented here are the first complete accurate representation of the intensity of radiation emerging from a plane-parallel Rayleigh-scattering atmosphere of large optical thickness. The optical thickness of a Rayleigh atmosphere is strongly dependent upon the wavelength, λ [Deirmendjian, 1955]. The intensity depends upon the optical thickness, τ (and thus indirectly, the wavelength, λ), upon the angles of incident radiation, θ_0 (zenith angle) and ϕ_0 (azimuth angle), upon the angles of emerging radiation, θ, ϕ and upon the ground reflectivity, A. We assume Lambert reflection and allow the reflectivity of the ground to range from A = 0 (total absorption) to A = 1 (total reflection). The effect of ground reflectivity is to add a term to the intensity, I , which consequently changes the percent of polarization, $\mathcal{P}$, but the polarization parameters Q and U are unchanged, since a depolarizing reflection is assumed (cf. Chapter III, Section C). Only the intensity, I , is examined in this section.

An incident radiation of unit intensity per unit area is assumed throughout this study. Figures 1 and 2 illustrate the plane-parallel approximation. The intensity vectors of the scattered radiation emerging from the upper and lower boundaries of the atmosphere for a specific set of parameters, $\tau = 1.0$, $\theta_0 = 53.13^0$ ($\mu_0 \equiv \cos\theta_0 = 0.6$), A = 0, are shown in the principal plane--the plane containing the sun, the point of observation, and the local vertical. The direct solar radiation, which decreases exponentially with optical thickness as the radiation travels through the atmosphere, is not included. Figures 1a and 2a show diagramatically how the directions of the upward

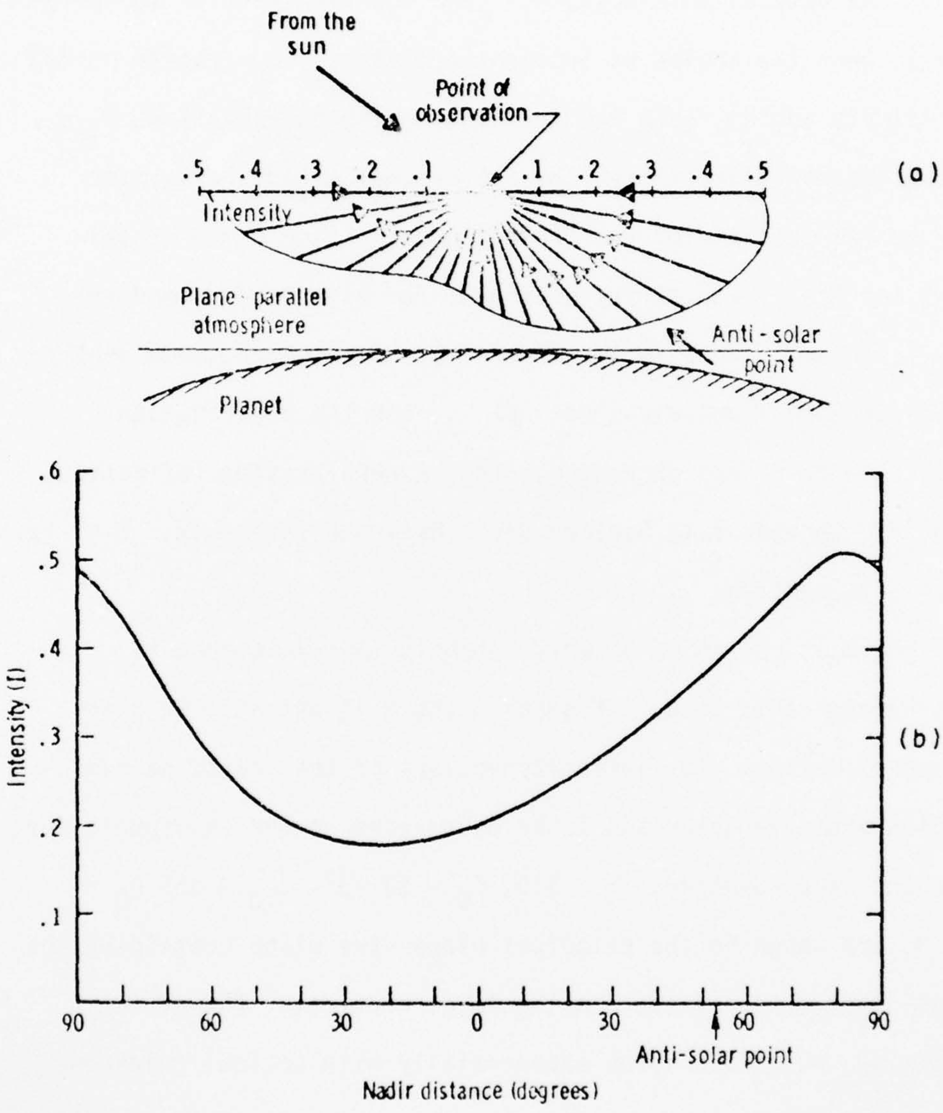

Fig. 1 -- Radiation emerging from the top of the atmosphere with τ = 1.0, θ_0 = 53.13, and A = 0; (a) vector representation, (b) graphical representation.

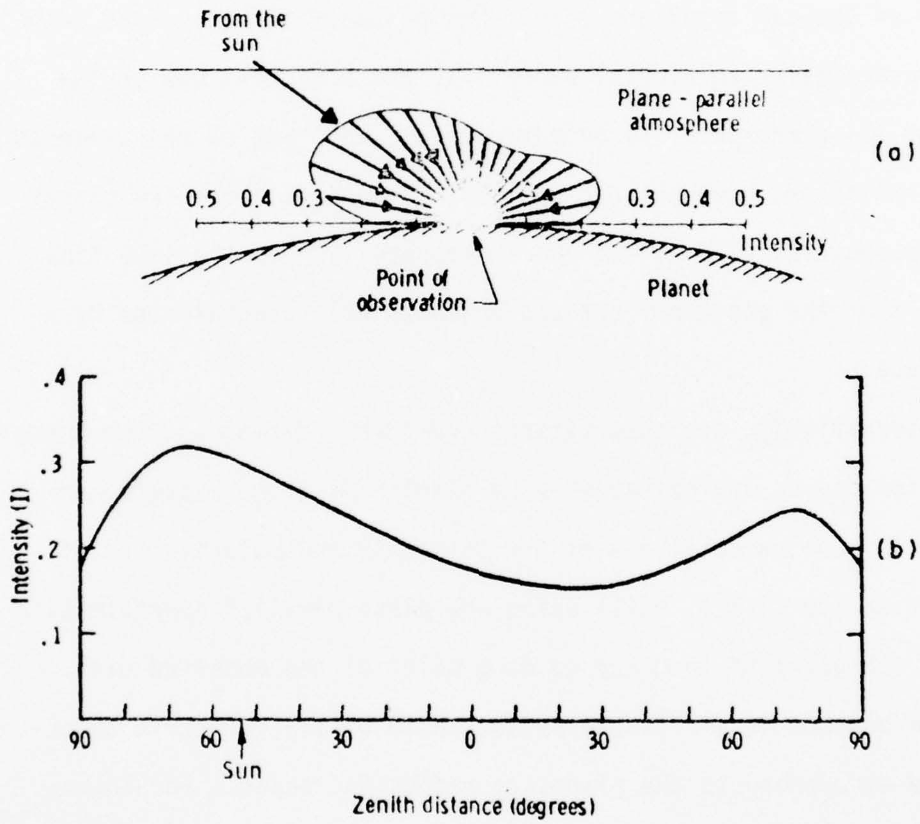

Fig. 2 -- Radiation emerging from the bottom of the atmosphere with $\tau = 1.0$; $\theta_0 = 53.13$, and $A = 0$; (a) vector representation, (b) graphical representation.

and downward emergent radiation are related to the model. Figures 1b and 2b show the same data in a form suitable for comparison and interpretation. The variation with ϕ , the azimuthal angle, will also be considered.

The height of the atmospheric slab in these figures is merely diagramatic. No geometric height is determined since the radiation is a function of optical thickness only. For downward radiation the actual height is practically immaterial except for the effect it has on the validity of the plane-parallel approximation. When making measurements of upward radiation, however, one must be high enough above the planet to be above substantially all of the atmosphere, yet at the same time low enough that the planetary surface below is well approximated by a plane surface.

Alternatively, for observations from great distances, the emerging radiation can be approximated by parallel radiation. Sekera and Viezee (1964) made calculations of the intensity and polarization of such radiation for $\tau \leq 1$, still using the plane-parallel approximation but moving the point of tangency to each point of the observed disk. This method becomes less reliable as the ratio of the geometric thickness of the atmosphere to the planetary radius increases. For intermediate distances, where neither approximation is valid, as for instance at the altitude of the ATS satellites at three earth radii, each case must be calculated with its own particular geometry.

We can investigate how the intensity, I , varies with optical thickness, direction of incident light, and ground reflectivity. We will consider here the downward case, the intensity of the transmitted skylight as would be seen by an observer at the planet's surface, assuming unit incident radiation per unit area, at the top of the atmosphere.

With the sun in the zenith the intensity must, by definition, be symmetric around the local vertical; there is no azimuthal (ϕ) variation. Therefore, only half of the principal plane need be illustrated. Figure 3 shows the variation with both optical thickness, τ, and ground reflectivity, A, for this case of the sun in the zenith. The left half of the diagram presents the intensities for $A = 0$ and the τ values shown; the right half shows the intensities for the same τ values and $A = 0.8$. In both cases we see the maximum intensity in the zenith is at about $\tau = 2$, while at the horizon the maximum is at about $\tau = 1$. For small τ the intensity is greatest at the horizon, but by $\tau = 1$ the intensity becomes greatest at the zenith. Even for $\tau = 100$ there is still a significant amount of radiation that penetrates through the atmosphere to the planet's surface. The effect of a high ground reflectivity is to increase the intensity substantially (by roughly a factor of 2) for all values of τ, and at all zenith angles. For moderate optical thickness ($\tau = 0.5$ to $\tau = 4$) this increase of intensity is considerably greater near the horizon than near the zenith, significantly changing the shape of the intensity curves.

When the sun is not in the zenith, the axial symmetry about the zenith is, of course, lost, the only obvious symmetry remaining being across the principal plane. As an example of moderate solar zenith angle, we will examine the radiation when $\theta_0 = 53.13^0$ ($\mu_0 = 0.6$).

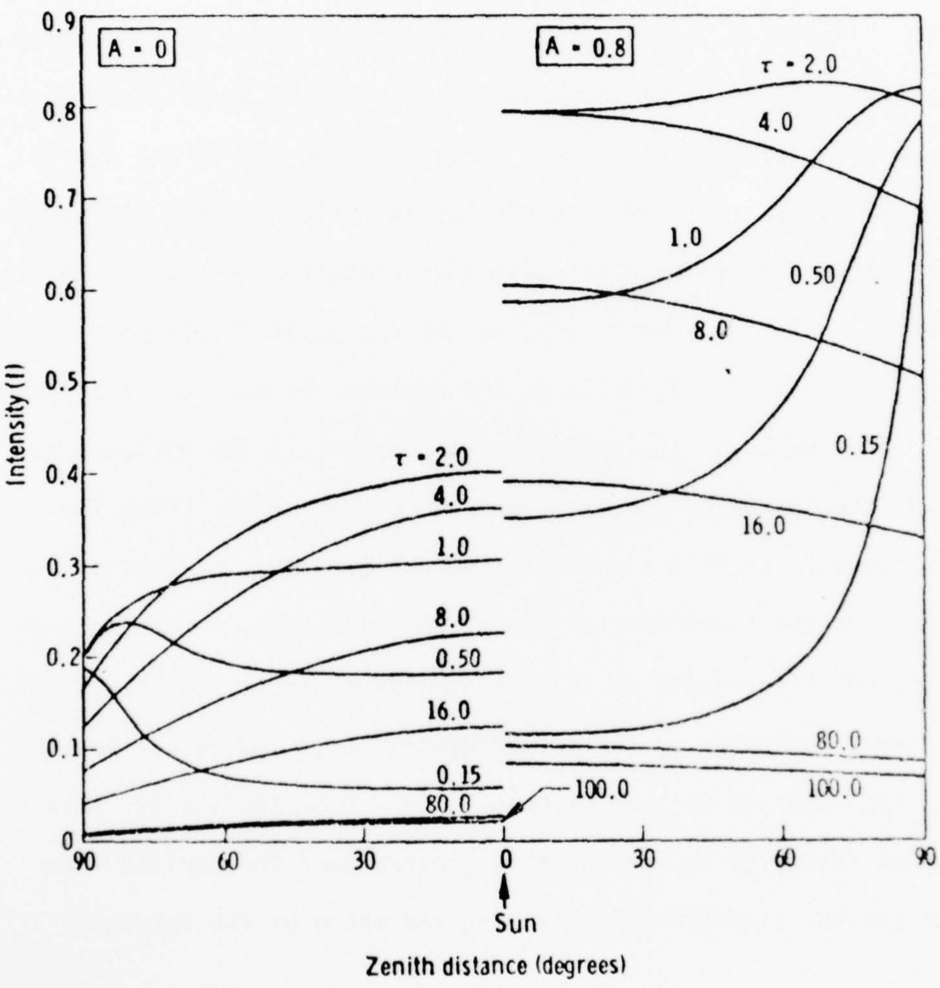

Fig. 3 -- Intensity of downward radiation with sun in the zenith.

The intensity in the principal plane with τ as a parameter is shown in Figure 4a for $A = 0$, and in Figure 4b for $A = 0.8$. Both sets of curves show the same features, differing more in magnitude than in shape, with the high ground reflectivity again almost doubling the intensity. For low optical thickness, the intensity is greatest near the horizon. For increasing optical thickness these two peaks of intensity move toward the zenith until, somewhere between $\tau = 2.0$ and $\tau = 4.0$, the two maxima become one, slightly to the sunward side of the zenith. By $\tau = 8$, all asymmetry from one side of the zenith to the other is essentially gone, with the peak intensity in the zenith. Comparing Figures 3 and 4, we see that the horizon brightening at low τ is more pronounced when the sun is not in the zenith, while the average total intensity is naturally lower.

Figures 5a and 5b show the azimuthal variation of intensity for two of the cases already examined. These figures show a projection of the sky onto a plane; the diameter of the semicircles are of equal zenith distance, and the radial lines are of equal azimuth. Only half the circle is shown since the radiation is symmetrical about the principal plane. The contours are lines of equal intensity. Figure 5a, for $\tau = 1.0$, $\mu_0 = 0.6$ and $A = 0$ (cf Figure 4a), shows how the intensity varies with azimuth when the peak intensity is quite far from the horizon. Figure 5b, for $\tau = 0.15$, $\mu_0 = 0.6$ and $A = 0.8$ (cf Figure 4b) shows the other extreme, where the intensity is greatest on the horizon. Both figures show a slower variation of intensity with azimuth angle than with zenith angle. The peak of intensity in Figure 5a is somewhat broader than it is high. This effect is much more pronounced in Figure 5b. The very bright horizon continues all the way

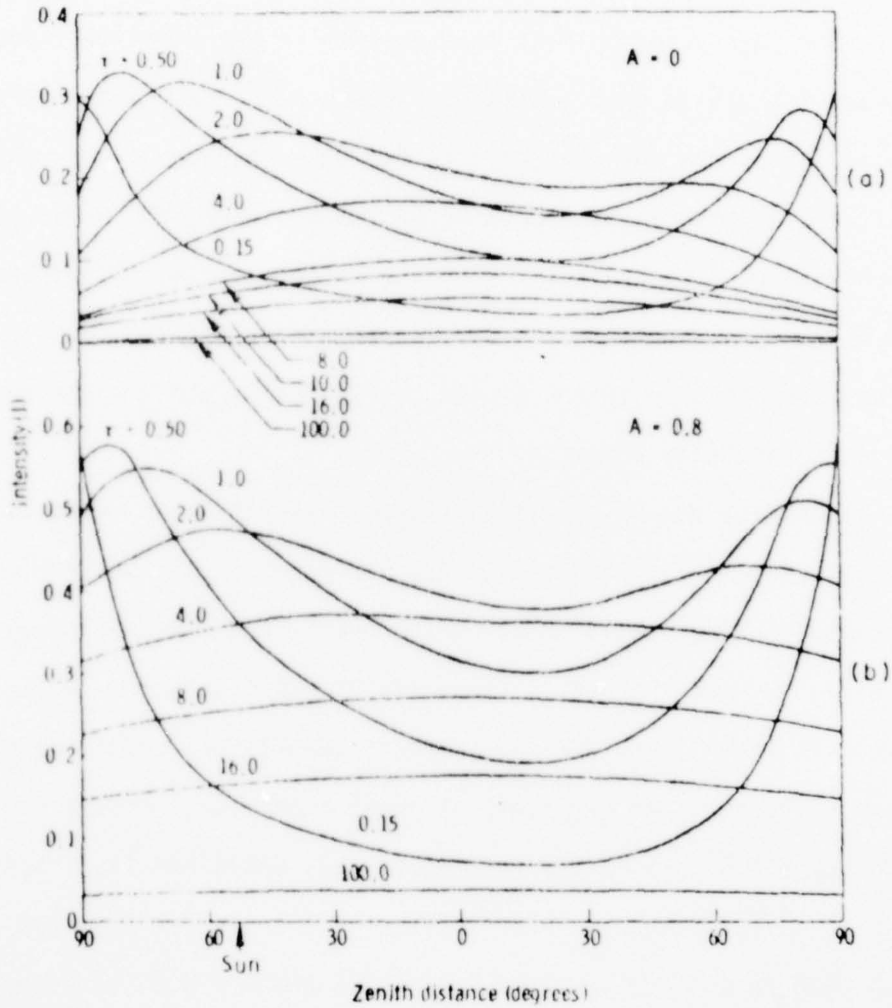

Fig. 4 -- Intensity of downward radiation with solar zenith angle θ_0 = 53.13 deg.

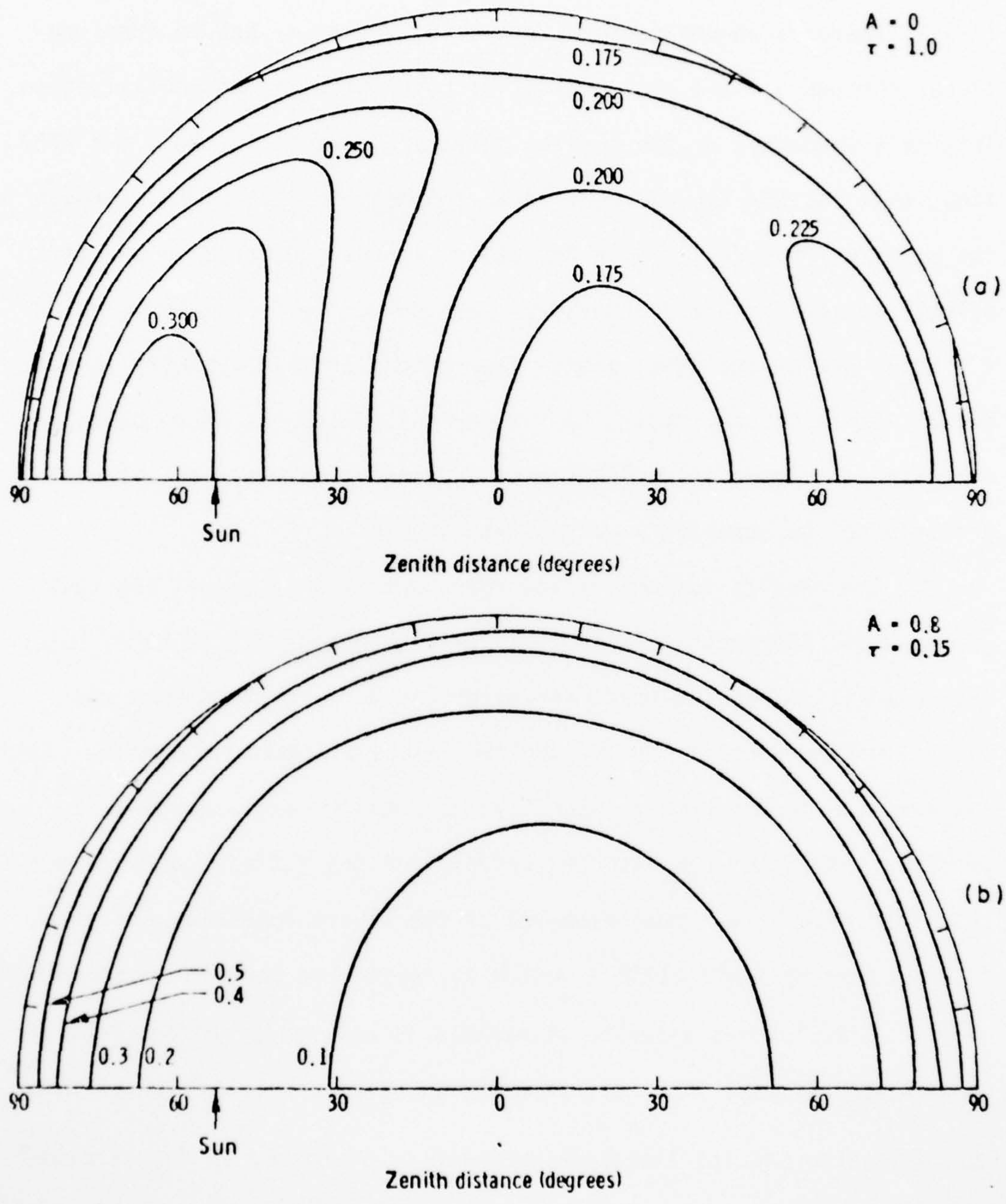

Fig. 5 -- Azimuthal variation of the intensity of downward radiation.

around with only a relatively slight decrease with increasing distance from the principal plane.

There is an unexpected feature in Figures 5a and 5b that continues through all the calculations of both intensity and polarization. This is a symmetry, on the horizon only, around the $\phi = 90^{\circ}$, $\phi = 270^{\circ}$ line, that is, the line perpendicular to the principal plane, through the point of observation. In the other figures one can also see this effect, in that both ends points of each curve ($\phi = 0^{\circ}$ and $\phi = 180^{\circ}$, $\theta = 90^{\circ}$) are at the same value. The reason for this symmetry is not clear. Since it only occurs on the horizon, where the plane-parallel approximation loses its validity for a planetary atmosphere, it will probably not be observed in a real atmosphere.

The results for only a few representative values of the solar zenith angle and ground reflectivity have been illustrated here, but the intensity of the emergent radiation can be determined from the tables in the Appendix for any desired values of these parameters. Similar results for the upward intensity, as would be measured by a low-orbiting satellite, can also be derived from the tables, but will not be illustrated here. Some examples of the upward intensity are given in the paper by Kahle (1968b), which is reproduced here in Appendix B.

A Rayleigh-scattering atmosphere is only an approximation, of course, to any real planetary atmosphere. The applicability of this approximation and its limitations have been discussed in detail elsewhere (see, for example, Chandrasekhar, 1950; Deirmendjian, 1957, 1959; Rozenberg, 1966; Sekera, 1957). It is hoped that even when a real atmosphere differs considerably from a Rayleigh-scattering atmosphere, useful information can be gained by comparing it with, and noting the difference from, the theoretical model.

B. GLOBAL RADIATION

Global radiation, in the sense used here, is the total radiation of a given wavelength reaching a unit surface area of a planet, and includes both the directly transmitted solar radiation and the diffusely scattered radiation from the entire sky. The diffuse part is just the hemispherical integral of the directional intensity illustrated in the previous section. The relative global radiation is usually expressed as a fraction of the radiation incident at the top of the atmosphere within a given spectral range. Satellites now can measure the upward global radiation emerging from a unit surface area at the top of the atmosphere, so we distinguish between upward and downward global radiation.

We assume an incident flux of parallel radiation from the sun of π units per unit area perpendicular to the direction of propagation; i.e., $F_0 = 1$. A unit area of the top of the atmosphere thus receives $\mu_0\pi$ units, where μ_0 is the cosine of the solar zenith-distance θ_0. The global radiation can be found from Equations (III-23) through (III-26).

In Figure 6 we show on a log-log scale the sun radiation S, the downward diffuse sky radiation H_d , and the upward diffuse radiation H_u as a function of optical thickness τ for vertical incidence of solar radiation and no ground reflectivity. The sum of these three components for any given optical thickness must by definition equal the

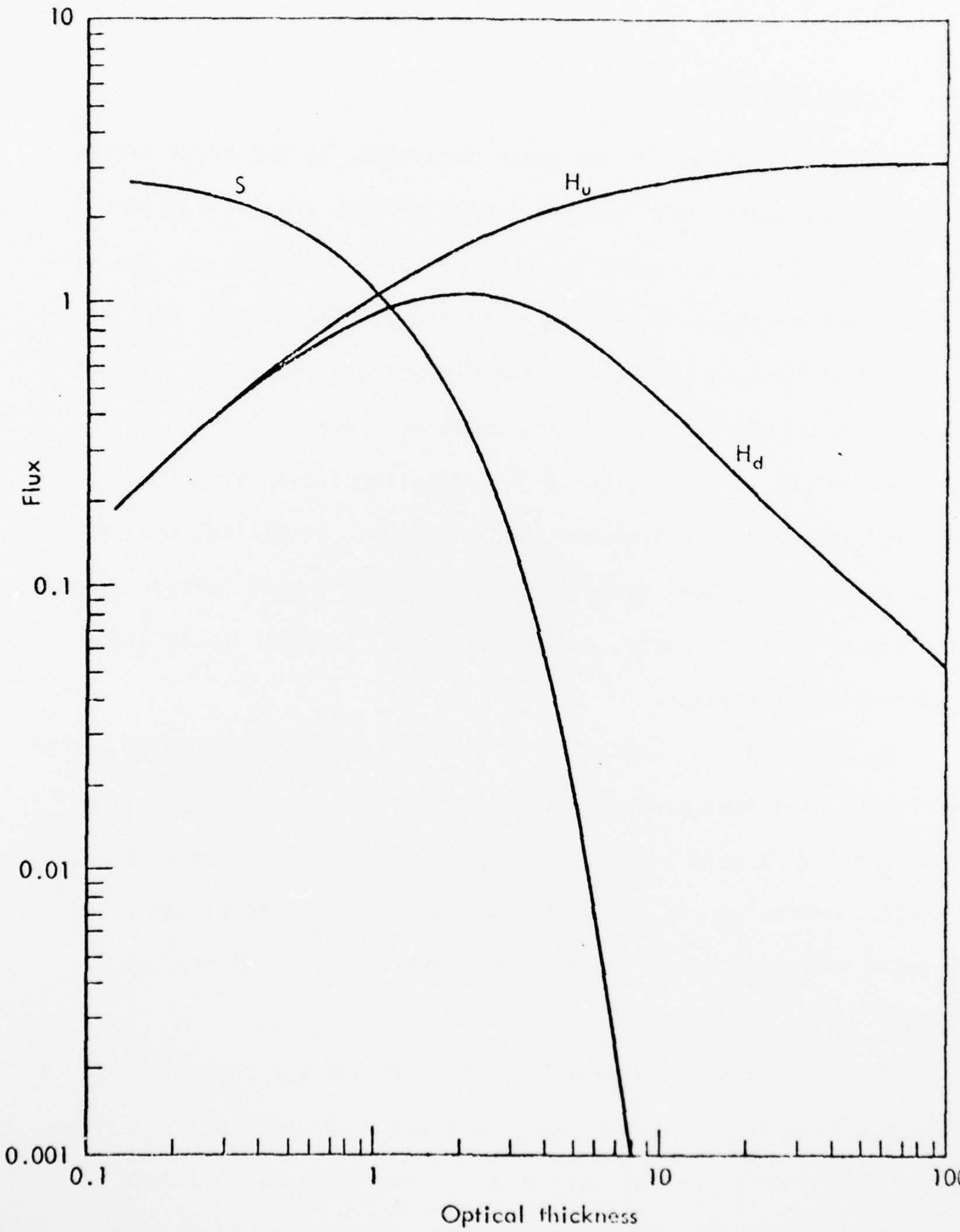

Fig. 6 -- Direct sun radiation S, downward diffuse sky radiation H_d, and upward diffuse sky radiation H_u, for $\theta_0 = 0$, $A = 0$.

incident radiation $\pi F_0 \mu_0$, or in this case just π . For the continuation of these curves in the direction of smaller τ , see Deirmenjian and Sekera (1954). It can be seen that when the optical thickness reaches 4, the direct solar radiation is an order of magnitude smaller than the downward diffuse radiation, and with increasing optical thickness it rapidly becomes negligible. Although the upward radiation is approaching π asymptotically, there is still a measurable amount of downward diffuse radiation (almost 2 percent) at $\tau = 100$.

As with the intensity in the previous section we will examine only a couple of examples of how the global radiation varies with the parameters, solar zenith angle and ground reflectivity. More examples of these variations are given in the paper by Kahle (1968a), and again, of course, calculation over the complete range of parameters is possible from the functions given in the tables of the Appendix.

The variation of the diffuse downward radiation H_d, for an angle of incidence θ_0 and no ground reflection, is shown in Figure 7. This radiation shows an interesting change with solar zenith-distance. The peak flux, at about $\tau = 2$ for vertical incidence, moves to smaller τ with increasing solar zenith-distance, until for $\theta_0 = 88^0$ it has moved below $\tau = 0.15$. The peak is caused by the balance between the increased scattering as path-length through the atmosphere increases, and the reduction of the total light penetrating to lower levels as optical thickness increases.

We next consider the effect of ground reflectivity, again assuming Lambert's law. Figure 8 shows the downward diffuse radiation as a function of τ for various values of A and for $\mu_0 = 1.0$.

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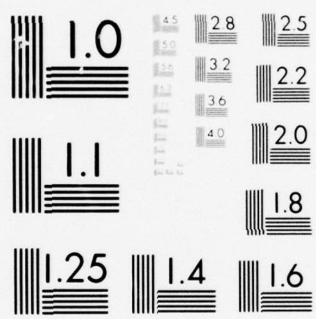

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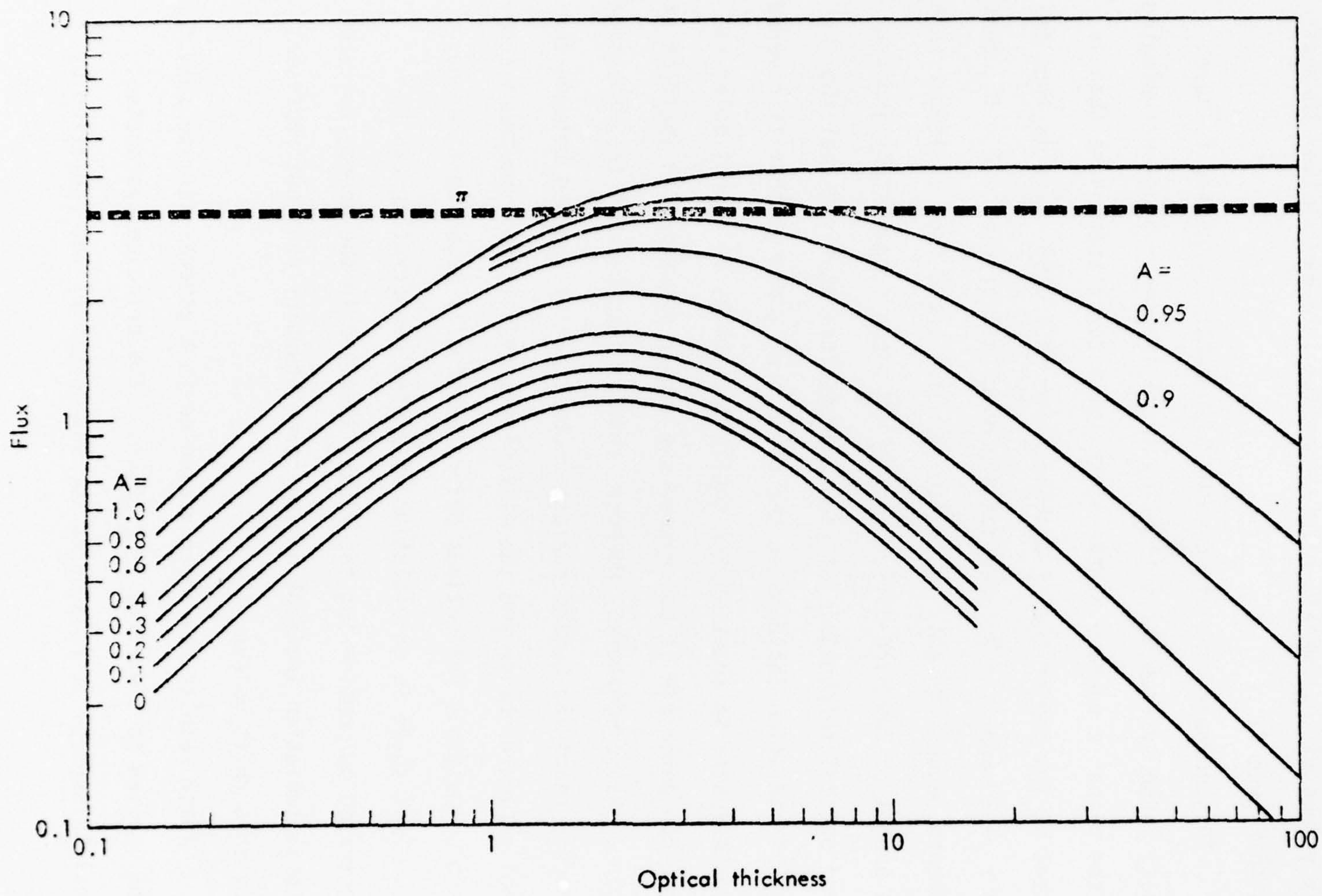

Fig. 8 -- Downward diffuse radiation H_d, for various ground reflectivities, with $\theta_0 = 0$.

(Other values of μ_0 merely reduce the flux in a way similar to that shown in Figure 7.

This downward radiation shows a somewhat unexpected feature: for very large surface reflectivities there is more radiation impinging upon the planet's surface at the bottom of a thick atmosphere than is incident at the top of the atmosphere. For $A = 0.95$ this is true between $\tau = 2$ and $\tau = 7$. For total ground reflectivity, $A = 1$, this phenomenon occurs for all τ larger than 1.5, and for τ larger than 8 the amount of downward diffuse radiation becomes completely independent of optical thickness. It seems paradoxical at first that the amount of radiation incident at the surface of a planet that is covered by an atmosphere so thick that a negligible amount of direct solar radiation can penetrate it can exceed the amount of radiation incident at the top of the atmosphere. However, this is just because the radiation which does reach the bottom surface becomes partly trapped between the perfect reflector below and the diffusively reflecting atmosphere above, undergoing multiple reflections before eventually escaping to space.

That there is no violation of energy conservation can be demonstrated by considering the fluxes involved in the energy balance. The total radiation incident at the lower boundary does not represent a flux through it unless $A = 0$.

These results have been presented in a general form so that they can be applied to a variety of problems. The principal parameter, optical thickness, is related to the wavelength of the radiation by

$$\tau_\lambda = \int_{z=0}^{\infty} \beta_\lambda \, dz \qquad \text{(IV-1)}$$

where z is the vertical coordinate. The extinction coefficient, β_λ, is proportional to λ^{-4}. Thus one could equally well assume that wavelength is the independent variable in the flux curves. For example, the optical thickness of the earth's molecular atmosphere ranges from about $\tau = 0.454$ at $\lambda = 3750\text{Å}$ (blue) through $\tau = 0.15$ for $\lambda = 4950\text{Å}$ (green) to $\tau = 0.0173$ at $\lambda = 8350\text{Å}$ (red) [Deirmendjian, 1955]. In Figure 8 we see that the downward diffuse flux at $\tau = 0.5$ (blue) is several times larger than at $\tau = 0.15$ (green), and from the work of Deirmendjian and Sekera (1954) we see that the flux continues to decrease down into the red. Thus most of the energy of the skylight will be in the blue region of the spectrum. (Deirmendjian and Sekera obtained quantitative results by including the spectral distribution of the incident solar radiation.) If one were next to assume a much thicker atmosphere, several times as thick, optically, as the earth's, then each region of the spectrum would shift to a correspondingly higher optical thickness. This would place the visible part of the radiation beyond the peak of the flux curve in Figure 7. Hence the spectral characteristics of the diffuse sky radiation would be changed, and a much greater portion of the emerging energy would lie in the red.

To make accurate predictions regarding a real planetary atmosphere on the basis of the foregoing analysis, it is necessary to know how the atmosphere departs from idealized conditions. One must consider how closely the atmosphere corresponds to a Rayleigh-scattering atmosphere, and how the reflection characteristics of its lower surface (ground or cloud layer) depart from Lambert's law. Conversely, as more data are gathered on the optical properties of an atmosphere, the discrepancy between the observed and the predicted values will provide information about other physical properties, such as atmospheric turbidity, true absorption and re-emission of light, variations of the composition of the atmosphere with height, and the nature of the ground surface.

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APPENDIX

TABLES OF SCATTERING FUNCTIONS

Table 1 X- AND Y-FUNCTIONS (τ = 0.15)

μ	X_1	Y_1	X_2	Y_2
0.00	0.10000E 01	0.	0.10000E 01	0.
0.02	0.10328E 01	0.15773E-01	0.10209E 01	0.12265E-01
0.04	0.10552E 01	0.55598E-01	0.10361E 01	0.47454E-01
0.06	0.10717E 01	0.13021E 00	0.10477E 01	0.11731E 00
0.08	0.10842E 01	0.21482E 00	0.10566E 01	0.19777E 00
0.10	0.10938E 01	0.29561E 00	0.10634E 01	0.27513E 00
0.12	0.11012E 01	0.36794E 00	0.10687E 01	0.34463E 00
0.14	0.11072E 01	0.43147E 00	0.10730E 01	0.40581E 00
0.16	0.11121E 01	0.48664E 00	0.10766E 01	0.45902E 00
0.18	0.11161E 01	0.53495E 00	0.10795E 01	0.50567E 00
0.20	0.11195E 01	0.57708E 00	0.10820E 01	0.54638E 00
0.22	0.11224E 01	0.61429E 00	0.10841E 01	0.58235E 00
0.24	0.11250E 01	0.64717E 00	0.10859E 01	0.61417E 00
0.26	0.11272E 01	0.67647E 00	0.10875E 01	0.64252E 00
0.28	0.11291E 01	0.70270E 00	0.10889E 01	0.66792E 00
0.30	0.11308E 01	0.72627E 00	0.10901E 01	0.69074E 00
0.32	0.11323E 01	0.74762E 00	0.10912E 01	0.71142E 00
0.34	0.11337E 01	0.76695E 00	0.10922E 01	0.73015E 00
0.36	0.11349E 01	0.78462E 00	0.10931E 01	0.74727E 00
0.38	0.11361E 01	0.80075E 00	0.10940E 01	0.76291E 00
0.40	0.11371E 01	0.81559E 00	0.10947E 01	0.77729E 00
0.42	0.11380E 01	0.82925E 00	0.10954E 01	0.79054E 00
0.44	0.11389E 01	0.84188E 00	0.10960E 01	0.80279E 00
0.46	0.11397E 01	0.85360E 00	0.10966E 01	0.81415E 00
0.48	0.11404E 01	0.86447E 00	0.10971E 01	0.82470E 00
0.50	0.11411E 01	0.87463E 00	0.10976E 01	0.83455E 00
0.52	0.11417E 01	0.88409E 00	0.10981E 01	0.84372E 00
0.54	0.11423E 01	0.89296E 00	0.10985E 01	0.85233E 00
0.56	0.11429E 01	0.90127E 00	0.10989E 01	0.86039E 00
0.58	0.11434E 01	0.90908E 00	0.10993E 01	0.86797E 00
0.60	0.11439E 01	0.91644E 00	0.10996E 01	0.87511E 00
0.62	0.11443E 01	0.92337E 00	0.11000E 01	0.88184E 00
0.64	0.11447E 01	0.92993E 00	0.11003E 01	0.88820E 00
0.66	0.11451E 01	0.93613E 00	0.11006E 01	0.89422E 00
0.68	0.11455E 01	0.94201E 00	0.11009E 01	0.89992E 00
0.70	0.11459E 01	0.94758E 00	0.11011E 01	0.90533E 00
0.72	0.11462E 01	0.95288E 00	0.11014E 01	0.91047E 00
0.74	0.11466E 01	0.95791E 00	0.11016E 01	0.91536E 00
0.76	0.11469E 01	0.96271E 00	0.11018E 01	0.92002E 00
0.78	0.11472E 01	0.96729E 00	0.11020E 01	0.92446E 00
0.80	0.11474E 01	0.97166E 00	0.11022E 01	0.92870E 00
0.82	0.11477E 01	0.97583E 00	0.11024E 01	0.93276E 00
0.84	0.11480E 01	0.97982E 00	0.11026E 01	0.93663E 00
0.86	0.11482E 01	0.98365E 00	0.11028E 01	0.94034E 00
0.88	0.11484E 01	0.98731E 00	0.11030E 01	0.94390E 00
0.90	0.11487E 01	0.99083E 00	0.11031E 01	0.94731E 00
0.92	0.11489E 01	0.99420E 00	0.11033E 01	0.95059E 00
0.94	0.11491E 01	0.99744E 00	0.11034E 01	0.95373E 00
0.96	0.11493E 01	0.10006E 01	0.11036E 01	0.95676E 00
0.98	0.11495E 01	0.10036E 01	0.11037E 01	0.95967E 00
1.00	0.11497E 01	0.10064E 01	0.11039E 01	0.96247E 00

Table 1 (cont.) (τ = 0.15)

μ	X_3	Y_3	X_4	Y_4
0.00	0.10000E 01	0.	0.10000E 01	0.
0.02	0.10629E 01	0.28294E-01	0.10282E 01	0.11967E-01
0.04	0.11053E 01	0.83050E-01	0.10463E 01	0.47891E-01
0.06	0.11368E 01	0.17203E 00	0.10594E 01	0.11911E 00
0.08	0.11603E 01	0.26881E 00	0.10692E 01	0.20099E 00
0.10	0.11783E 01	0.35955E 00	0.10767E 01	0.27962E 00
0.12	0.11924E 01	0.43997E 00	0.10825E 01	0.35022E 00
0.14	0.12036E 01	0.51019E 00	0.10871E 01	0.41234E 00
0.16	0.12128E 01	0.57091E 00	0.10909E 01	0.46635E 00
0.18	0.12204E 01	0.62393E 00	0.10940E 01	0.51368E 00
0.20	0.12268E 01	0.67006E 00	0.10967E 01	0.55499E 00
0.22	0.12323E 01	0.71073E 00	0.10989E 01	0.59149E 00
0.24	0.12371E 01	0.74661E 00	0.11009E 01	0.62375E 00
0.26	0.12412E 01	0.77855E 00	0.11026E 01	0.65251E 00
0.28	0.12448E 01	0.80712E 00	0.11041E 01	0.67827E 00
0.30	0.12480E 01	0.83277E 00	0.11054E 01	0.70141E 00
0.32	0.12509E 01	0.85599E 00	0.11066E 01	0.72239E 00
0.34	0.12535E 01	0.87699E 00	0.11076E 01	0.74137E 00
0.36	0.12558E 01	0.89618E 00	0.11086E 01	0.75873E 00
0.38	0.12579E 01	0.91370E 00	0.11094E 01	0.77459E 00
0.40	0.12598E 01	0.92980E 00	0.11102E 01	0.78917E 00
0.42	0.12616E 01	0.94462E 00	0.11109E 01	0.80260E 00
0.44	0.12632E 01	0.95832E 00	0.11116E 01	0.81502E 00
0.46	0.12647E 01	0.97102E 00	0.11122E 01	0.82654E 00
0.48	0.12661E 01	0.98281E 00	0.11128E 01	0.83723E 00
0.50	0.12674E 01	0.99381E 00	0.11133E 01	0.84721E 00
0.52	0.12685E 01	0.10041E 01	0.11138E 01	0.85651E 00
0.54	0.12696E 01	0.10137E 01	0.11142E 01	0.86524E 00
0.56	0.12707E 01	0.10227E 01	0.11147E 01	0.87341E 00
0.58	0.12716E 01	0.10311E 01	0.11151E 01	0.88109E 00
0.60	0.12726E 01	0.10391E 01	0.11154E 01	0.88833E 00
0.62	0.12734E 01	0.10466E 01	0.11158E 01	0.89515E 00
0.64	0.12742E 01	0.10537E 01	0.11161E 01	0.90160E 00
0.66	0.12750E 01	0.10604E 01	0.11164E 01	0.90770E 00
0.68	0.12757E 01	0.10668E 01	0.11167E 01	0.91348E 00
0.70	0.12764E 01	0.10728E 01	0.11170E 01	0.91896E 00
0.72	0.12770E 01	0.10785E 01	0.11173E 01	0.92417E 00
0.74	0.12776E 01	0.10840E 01	0.11175E 01	0.92913E 00
0.76	0.12782E 01	0.10891E 01	0.11177E 01	0.93385E 00
0.78	0.12787E 01	0.10941E 01	0.11180E 01	0.93835E 00
0.80	0.12793E 01	0.10988E 01	0.11182E 01	0.94265E 00
0.82	0.12798E 01	0.11033E 01	0.11184E 01	0.94676E 00
0.84	0.12803E 01	0.11076E 01	0.11186E 01	0.95068E 00
0.86	0.12807E 01	0.11118E 01	0.11188E 01	0.95445E 00
0.88	0.12811E 01	0.11157E 01	0.11189E 01	0.95805E 00
0.90	0.12816E 01	0.11195E 01	0.11191E 01	0.96151E 00
0.92	0.12820E 01	0.11232E 01	0.11193E 01	0.96483E 00
0.94	0.12824E 01	0.11267E 01	0.11194E 01	0.96802E 00
0.96	0.12827E 01	0.11300E 01	0.11196E 01	0.97108E 00
0.98	0.12831E 01	0.11333E 01	0.11197E 01	0.97403E 00
1.00	0.12834E 01	0.11364E 01	0.11199E 01	0.97687E 00

Table 1 (cont.) (τ = 0.25)

r	X_1	Y_1	X_2	Y_2
0.00	0.10000E 01	0.	0.10000E 01	0.
0.02	0.10336E 01	0.11328E-01	0.10214E 01	0.92481E-02
0.04	0.10573E 01	0.26410E-01	0.10376E 01	0.21516E-01
0.06	0.10767E 01	0.54563E-01	0.10513E 01	0.46150E-01
0.08	0.10929E 01	0.97312E-01	0.10629E 01	0.85119E-01
0.10	0.11064E 01	0.14891E 00	0.10727E 01	0.13303E 00
0.12	0.11176E 01	0.20349E 00	0.10810E 01	0.18420E 00
0.14	0.11271E 01	0.25752E 00	0.10880E 01	0.23513E 00
0.16	0.11352E 01	0.30902E 00	0.10940E 01	0.28386E 00
0.18	0.11421E 01	0.35732E 00	0.10992E 01	0.32968E 00
0.20	0.11481E 01	0.40197E 00	0.11037E 01	0.37212E 00
0.22	0.11534E 01	0.44320E 00	0.11076E 01	0.41138E 00
0.24	0.11580E 01	0.48108E 00	0.11111E 01	0.44748E 00
0.26	0.11621E 01	0.51592E 00	0.11142E 01	0.48072E 00
0.28	0.11658E 01	0.54799E 00	0.11169E 01	0.51134E 00
0.30	0.11691E 01	0.57751E 00	0.11194E 01	0.53953E 00
0.32	0.11721E 01	0.60479E 00	0.11216E 01	0.56562E 00
0.34	0.11748E 01	0.62996E 00	0.11237E 01	0.58969E 00
0.36	0.11772E 01	0.65334E 00	0.11255E 01	0.61205E 00
0.38	0.11795E 01	0.67500E 00	0.11272E 01	0.63279E 00
0.40	0.11815E 01	0.69518E 00	0.11288E 01	0.65211E 00
0.42	0.11834E 01	0.71399E 00	0.11302E 01	0.67012E 00
0.44	0.11852E 01	0.73155E 00	0.11315E 01	0.68695E 00
0.46	0.11868E 01	0.74801E 00	0.11328E 01	0.70272E 00
0.48	0.11883E 01	0.76342E 00	0.11339E 01	0.71749E 00
0.50	0.11897E 01	0.77793E 00	0.11350E 01	0.73139E 00
0.52	0.11910E 01	0.79154E 00	0.11360E 01	0.74445E 00
0.54	0.11922E 01	0.80440E 00	0.11369E 01	0.75678E 00
0.56	0.11934E 01	0.81653E 00	0.11378E 01	0.76841E 00
0.58	0.11945E 01	0.82800E 00	0.11386E 01	0.77941E 00
0.60	0.11955E 01	0.83886E 00	0.11394E 01	0.78983E 00
0.62	0.11965E 01	0.84915E 00	0.11401E 01	0.79970E 00
0.64	0.11974E 01	0.85893E 00	0.11408E 01	0.80908E 00
0.66	0.11982E 01	0.86821E 00	0.11414E 01	0.81799E 00
0.68	0.11990E 01	0.87706E 00	0.11420E 01	0.82648E 00
0.70	0.11998E 01	0.88548E 00	0.11426E 01	0.83456E 00
0.72	0.12005E 01	0.89351E 00	0.11432E 01	0.84228E 00
0.74	0.12012E 01	0.90118E 00	0.11437E 01	0.84964E 00
0.76	0.12019E 01	0.90851E 00	0.11442E 01	0.85668E 00
0.78	0.12025E 01	0.91553E 00	0.11447E 01	0.86341E 00
0.80	0.12031E 01	0.92224E 00	0.11452E 01	0.86986E 00
0.82	0.12037E 01	0.92868E 00	0.11456E 01	0.87604E 00
0.84	0.12043E 01	0.93485E 00	0.11460E 01	0.88197E 00
0.86	0.12048E 01	0.94078E 00	0.11464E 01	0.88766E 00
0.88	0.12053E 01	0.94647E 00	0.11468E 01	0.89313E 00
0.90	0.12058E 01	0.95195E 00	0.11472E 01	0.89839E 00
0.92	0.12063E 01	0.95722E 00	0.11475E 01	0.90345E 00
0.94	0.12067E 01	0.96229E 00	0.11479E 01	0.90832E 00
0.96	0.12071E 01	0.96718E 00	0.11482E 01	0.91302E 00
0.98	0.12076E 01	0.97189E 00	0.11485E 01	0.91755E 00
1.00	0.12080E 01	0.97644E 00	0.11488E 01	0.92192E 00

Table 1 (cont.) (τ = 0.25)

τ	X_3	Y_3	X_4	Y_4
0.00	0.10000E 01	0.	0.10000E 01	0.
0.02	0.10655E 01	0.21693E-01	0.10286E 01	0.77505E-02
0.04	0.11120E 01	0.49194E-01	0.10475E 01	0.18959E-01
0.06	0.11502E 01	0.91230E-01	0.10625E 01	0.43162E-01
0.08	0.11819E 01	0.14791E 00	0.10748E 01	0.82191E-01
0.10	0.12084E 01	0.21255E 00	0.10849E 01	0.13045E 00
0.12	0.12304E 01	0.27891E 00	0.10932E 01	0.18210E 00
0.14	0.12491E 01	0.34343E 00	0.11003E 01	0.23357E 00
0.16	0.12649E 01	0.40418E 00	0.11062E 01	0.28284E 00
0.18	0.12786E 01	0.46070E 00	0.11113E 01	0.32919E 00
0.20	0.12904E 01	0.51261E 00	0.11157E 01	0.37212E 00
0.22	0.13007E 01	0.56033E 00	0.11196E 01	0.41184E 00
0.24	0.13098E 01	0.60400E 00	0.11230E 01	0.44837E 00
0.26	0.13179E 01	0.64405E 00	0.11260E 01	0.48201E 00
0.28	0.13251E 01	0.68082E 00	0.11287E 01	0.51300E 00
0.30	0.13315E 01	0.71458E 00	0.11311E 01	0.54153E 00
0.32	0.13373E 01	0.74574E 00	0.11333E 01	0.56793E 00
0.34	0.13426E 01	0.77444E 00	0.11352E 01	0.59230E 00
0.36	0.13474E 01	0.80105E 00	0.11370E 01	0.61493E 00
0.38	0.13518E 01	0.82569E 00	0.11386E 01	0.63592E 00
0.40	0.13559E 01	0.84862E 00	0.11401E 01	0.65548E 00
0.42	0.13596E 01	0.86996E 00	0.11415E 01	0.67371E 00
0.44	0.13630E 01	0.88988E 00	0.11428E 01	0.69074E 00
0.46	0.13662E 01	0.90853E 00	0.11440E 01	0.70670E 00
0.48	0.13692E 01	0.92598E 00	0.11451E 01	0.72165E 00
0.50	0.13719E 01	0.94239E 00	0.11461E 01	0.73573E 00
0.52	0.13745E 01	0.95780E 00	0.11470E 01	0.74894E 00
0.54	0.13769E 01	0.97233E 00	0.11479E 01	0.76142E 00
0.56	0.13791E 01	0.98603E 00	0.11488E 01	0.77319E 00
0.58	0.13813E 01	0.99899E 00	0.11495E 01	0.78433E 00
0.60	0.13833E 01	0.10112E 01	0.11503E 01	0.79488E 00
0.62	0.13851E 01	0.10229E 01	0.11510E 01	0.80487E 00
0.64	0.13869E 01	0.10339E 01	0.11516E 01	0.81437E 00
0.66	0.13886E 01	0.10444E 01	0.11523E 01	0.82339E 00
0.68	0.13902E 01	0.10543E 01	0.11528E 01	0.83199E 00
0.70	0.13917E 01	0.10638E 01	0.11534E 01	0.84016E 00
0.72	0.13932E 01	0.10729E 01	0.11539E 01	0.84797E 00
0.74	0.13945E 01	0.10815E 01	0.11544E 01	0.85542E 00
0.76	0.13958E 01	0.10898E 01	0.11549E 01	0.86255E 00
0.78	0.13971E 01	0.10977E 01	0.11554E 01	0.86937E 00
0.80	0.13982E 01	0.11052E 01	0.11558E 01	0.87589E 00
0.82	0.13994E 01	0.11125E 01	0.11562E 01	0.88215E 00
0.84	0.14005E 01	0.11194E 01	0.11566E 01	0.88815E 00
0.86	0.14015E 01	0.11261E 01	0.11570E 01	0.89391E 00
0.88	0.14025E 01	0.11325E 01	0.11574E 01	0.89945E 00
0.90	0.14034E 01	0.11386E 01	0.11577E 01	0.90477E 00
0.92	0.14044E 01	0.11446E 01	0.11581E 01	0.90990E 00
0.94	0.14052E 01	0.11503E 01	0.11584E 01	0.91483E 00
0.96	0.14061E 01	0.11558E 01	0.11587E 01	0.91958E 00
0.98	0.14069E 01	0.11611E 01	0.11590E 01	0.92417E 00
1.00	0.14077E 01	0.11662E 01	0.11593E 01	0.92859E 00

Table 1 (cont.) (τ = 0.50)

μ	X_1	Y_1	X_2	Y_2
0.00	0.10000E 01	0.	0.10000E 01	0.
0.02	0.10346E 01	0.71940E-02	0.10221E 01	0.65508E-02
0.04	0.10594E 01	0.15215E-01	0.10391E 01	0.13648E-01
0.06	0.10804E 01	0.24409E-01	0.10540E 01	0.21596E-01
0.08	0.10990E 01	0.35987E-01	0.10675E 01	0.31575E-01
0.10	0.11156E 01	0.51443E-01	0.10798E 01	0.45092E-01
0.12	0.11305E 01	0.71342E-01	0.10910E 01	0.62793E-01
0.14	0.11441E 01	0.95166E-01	0.11013E 01	0.84256E-01
0.16	0.11563E 01	0.12216E 00	0.11107E 01	0.10881E 00
0.18	0.11674E 01	0.15122E 00	0.11192E 01	0.13541E 00
0.20	0.11774E 01	0.18157E 00	0.11271E 01	0.16335E 00
0.22	0.11866E 01	0.21247E 00	0.11342E 01	0.19190E 00
0.24	0.11949E 01	0.24341E 00	0.11408E 01	0.22057E 00
0.26	0.12026E 01	0.27399E 00	0.11468E 01	0.24898E 00
0.28	0.12096E 01	0.30396E 00	0.11524E 01	0.27688E 00
0.30	0.12161E 01	0.33311E 00	0.11575E 01	0.30406E 00
0.32	0.12221E 01	0.36137E 00	0.11622E 01	0.33045E 00
0.34	0.12276E 01	0.38860E 00	0.11666E 01	0.35591E 00
0.36	0.12327E 01	0.41484E 00	0.11707E 01	0.38047E 00
0.38	0.12375E 01	0.44003E 00	0.11745E 01	0.40406E 00
0.40	0.12419E 01	0.46421E 00	0.11780E 01	0.42673E 00
0.42	0.12461E 01	0.48739E 00	0.11813E 01	0.44847E 00
0.44	0.12499E 01	0.50959E 00	0.11845E 01	0.46932E 00
0.46	0.12536E 01	0.53088E 00	0.11874E 01	0.48931E 00
0.48	0.12570E 01	0.55125E 00	0.11901E 01	0.50845E 00
0.50	0.12602E 01	0.57079E 00	0.11927E 01	0.52682E 00
0.52	0.12633E 01	0.58948E 00	0.11951E 01	0.54440E 00
0.54	0.12661E 01	0.60742E 00	0.11974E 01	0.56128E 00
0.56	0.12688E 01	0.62460E 00	0.11996E 01	0.57746E 00
0.58	0.12714E 01	0.64110E 00	0.12017E 01	0.59299E 00
0.60	0.12738E 01	0.65693E 00	0.12036E 01	0.60790E 00
0.62	0.12761E 01	0.67212E 00	0.12055E 01	0.62222E 00
0.64	0.12783E 01	0.68672E 00	0.12073E 01	0.63598E 00
0.66	0.12804E 01	0.70075E 00	0.12089E 01	0.64920E 00
0.68	0.12824E 01	0.71425E 00	0.12105E 01	0.66193E 00
0.70	0.12843E 01	0.72723E 00	0.12121E 01	0.67417E 00
0.72	0.12861E 01	0.73973E 00	0.12135E 01	0.68596E 00
0.74	0.12878E 01	0.75177E 00	0.12149E 01	0.69732E 00
0.76	0.12895E 01	0.76338E 00	0.12163E 01	0.70827E 00
0.78	0.12911E 01	0.77457E 00	0.12176E 01	0.71883E 00
0.80	0.12926E 01	0.78536E 00	0.12188E 01	0.72902E 00
0.82	0.12941E 01	0.79579E 00	0.12200E 01	0.73887E 00
0.84	0.12955E 01	0.80585E 00	0.12211E 01	0.74837E 00
0.86	0.12968E 01	0.81559E 00	0.12222E 01	0.75756E 00
0.88	0.12981E 01	0.82499E 00	0.12233E 01	0.76643E 00
0.90	0.12994E 01	0.83409E 00	0.12243E 01	0.77503E 00
0.92	0.13006E 01	0.84289E 00	0.12252E 01	0.78334E 00
0.94	0.13017E 01	0.85141E 00	0.12262E 01	0.79140E 00
0.96	0.13028E 01	0.85967E 00	0.12271E 01	0.79920E 00
0.98	0.13039E 01	0.86767E 00	0.12280E 01	0.80676E 00
1.00	0.13049E 01	0.87543E 00	0.12288E 01	0.81409E 00

Table 1 (cont.) (τ = 0.50)

μ	X_3	Y_3	X_4	Y_4
0.00	0.10000E 01	0.	0.10000E 01	0.
0.02	0.10693E 01	0.15527E-01	0.10290E 01	0.42028E-02
0.04	0.11202E 01	0.33193E-01	0.10484E 01	0.89608E-02
0.06	0.11640E 01	0.53284E-01	0.10641E 01	0.14636E-01
0.08	0.12032E 01	0.76859E-01	0.10777E 01	0.22494E-01
0.10	0.12385E 01	0.10514E 00	0.10895E 01	0.34114E-01
0.12	0.12706E 01	0.13830E 00	0.11000E 01	0.50169E-01
0.14	0.12997E 01	0.17545E 00	0.11094E 01	0.70225E-01
0.16	0.13260E 01	0.21552E 00	0.11178E 01	0.93591E-01
0.18	0.13501E 01	0.25724E 00	0.11254E 01	0.11919E 00
0.20	0.13719E 01	0.29971E 00	0.11322E 01	0.14628E 00
0.22	0.13918E 01	0.34213E 00	0.11383E 01	0.17411E 00
0.24	0.14100E 01	0.38396E 00	0.11439E 01	0.20216E 00
0.26	0.14266E 01	0.42482E 00	0.11490E 01	0.23005E 00
0.28	0.14420E 01	0.46446E 00	0.11537E 01	0.25750E 00
0.30	0.14561E 01	0.50271E 00	0.11580E 01	0.28429E 00
0.32	0.14691E 01	0.53953E 00	0.11620E 01	0.31034E 00
0.34	0.14812E 01	0.57481E 00	0.11656E 01	0.33550E 00
0.36	0.14924E 01	0.60864E 00	0.11690E 01	0.35980E 00
0.38	0.15028E 01	0.64096E 00	0.11721E 01	0.38317E 00
0.40	0.15125E 01	0.67188E 00	0.11750E 01	0.40564E 00
0.42	0.15216E 01	0.70141E 00	0.11778E 01	0.42720E 00
0.44	0.15301E 01	0.72962E 00	0.11803E 01	0.44789E 00
0.46	0.15380E 01	0.75658E 00	0.11827E 01	0.46774E 00
0.48	0.15455E 01	0.78232E 00	0.11849E 01	0.48676E 00
0.50	0.15526E 01	0.80696E 00	0.11870E 01	0.50502E 00
0.52	0.15592E 01	0.83048E 00	0.11890E 01	0.52250E 00
0.54	0.15655E 01	0.85301E 00	0.11909E 01	0.53929E 00
0.56	0.15714E 01	0.87456E 00	0.11926E 01	0.55538E 00
0.58	0.15770E 01	0.89521E 00	0.11943E 01	0.57084E 00
0.60	0.15823E 01	0.91500E 00	0.11959E 01	0.58568E 00
0.62	0.15874E 01	0.93397E 00	0.11974E 01	0.59993E 00
0.64	0.15922E 01	0.95218E 00	0.11988E 01	0.61364E 00
0.66	0.15968E 01	0.96965E 00	0.12002E 01	0.62680E 00
0.68	0.16011E 01	0.98645E 00	0.12015E 01	0.63949E 00
0.70	0.16053E 01	0.10026E 01	0.12027E 01	0.65168E 00
0.72	0.16093E 01	0.10181E 01	0.12039E 01	0.66344E 00
0.74	0.16131E 01	0.10330E 01	0.12050E 01	0.67476E 00
0.76	0.16167E 01	0.10474E 01	0.12061E 01	0.68567E 00
0.78	0.16202E 01	0.10613E 01	0.12071E 01	0.69621E 00
0.80	0.16235E 01	0.10746E 01	0.12081E 01	0.70637E 00
0.82	0.16267E 01	0.10875E 01	0.12090E 01	0.71618E 00
0.84	0.16298E 01	0.11000E 01	0.12099E 01	0.72566E 00
0.86	0.16328E 01	0.11120E 01	0.12108E 01	0.73483E 00
0.88	0.16356E 01	0.11236E 01	0.12116E 01	0.74368E 00
0.90	0.16383E 01	0.11348E 01	0.12124E 01	0.75226E 00
0.92	0.16410E 01	0.11457E 01	0.12132E 01	0.76055E 00
0.94	0.16435E 01	0.11562E 01	0.12140E 01	0.76859E 00
0.96	0.16460E 01	0.11664E 01	0.12147E 01	0.77637E 00
0.98	0.16483E 01	0.11762E 01	0.12154E 01	0.78392E 00
1.00	0.16506E 01	0.11858E 01	0.12161E 01	0.79124E 00

Table 1 (cont.) (τ = 0.70)

μ	X_1	Y_1	X_2	Y_2
0.00	0.10000E 01	0.	0.10000E 01	0.
0.02	0.10349E 01	0.54237E-02	0.10224E 01	0.52752E-02
0.04	0.10602E 01	0.11411E-01	0.10398E 01	0.10949E-01
0.06	0.10816E 01	0.18009E-01	0.10551E 01	0.17058E-01
0.08	0.11007E 01	0.25385E-01	0.10690E 01	0.23750E-01
0.10	0.11180E 01	0.34033E-01	0.10819E 01	0.31491E-01
0.12	0.11339E 01	0.44589E-01	0.10938E 01	0.40907E-01
0.14	0.11484E 01	0.57409E-01	0.11050E 01	0.52376E-01
0.16	0.11618E 01	0.72692E-01	0.11154E 01	0.66124E-01
0.18	0.11743E 01	0.90147E-01	0.11252E 01	0.81910E-01
0.20	0.11858E 01	0.10958E 00	0.11343E 01	0.99574E-01
0.22	0.11965E 01	0.13051E 00	0.11428E 01	0.11868E 00
0.24	0.12064E 01	0.15262E 00	0.11507E 01	0.13892E 00
0.26	0.12156E 01	0.17552E 00	0.11581E 01	0.15995E 00
0.28	0.12242E 01	0.19893E 00	0.11651E 01	0.18150E 00
0.30	0.12322E 01	0.22259E 00	0.11716E 01	0.20333E 00
0.32	0.12398E 01	0.24629E 00	0.11777E 01	0.22523E 00
0.34	0.12468E 01	0.26986E 00	0.11834E 01	0.24705E 00
0.36	0.12534E 01	0.29319E 00	0.11888E 01	0.26867E 00
0.38	0.12596E 01	0.31616E 00	0.11939E 01	0.28998E 00
0.40	0.12654E 01	0.33870E 00	0.11986E 01	0.31092E 00
0.42	0.12709E 01	0.36077E 00	0.12032E 01	0.33143E 00
0.44	0.12760E 01	0.38231E 00	0.12074E 01	0.35148E 00
0.46	0.12809E 01	0.40331E 00	0.12115E 01	0.37103E 00
0.48	0.12855E 01	0.42374E 00	0.12153E 01	0.39007E 00
0.50	0.12899E 01	0.44362E 00	0.12189E 01	0.40860E 00
0.52	0.12941E 01	0.46291E 00	0.12224E 01	0.42660E 00
0.54	0.12980E 01	0.48165E 00	0.12256E 01	0.44409E 00
0.56	0.13018E 01	0.49983E 00	0.12287E 01	0.46106E 00
0.58	0.13053E 01	0.51746E 00	0.12317E 01	0.47753E 00
0.60	0.13087E 01	0.53456E 00	0.12345E 01	0.49350E 00
0.62	0.13120E 01	0.55113E 00	0.12372E 01	0.50899E 00
0.64	0.13151E 01	0.56720E 00	0.12398E 01	0.52402E 00
0.66	0.13180E 01	0.58277E 00	0.12423E 01	0.53859E 00
0.68	0.13209E 01	0.59788E 00	0.12447E 01	0.55272E 00
0.70	0.13236E 01	0.61251E 00	0.12469E 01	0.56642E 00
0.72	0.13262E 01	0.62671E 00	0.12491E 01	0.57971E 00
0.74	0.13287E 01	0.64048E 00	0.12512E 01	0.59261E 00
0.76	0.13311E 01	0.65383E 00	0.12532E 01	0.60512E 00
0.78	0.13334E 01	0.66679E 00	0.12551E 01	0.61726E 00
0.80	0.13356E 01	0.67937E 00	0.12570E 01	0.62904E 00
0.82	0.13377E 01	0.69158E 00	0.12588E 01	0.64049E 00
0.84	0.13397E 01	0.70343E 00	0.12605E 01	0.65159E 00
0.86	0.13417E 01	0.71494E 00	0.12621E 01	0.66239E 00
0.88	0.13436E 01	0.72612E 00	0.12637E 01	0.67287E 00
0.90	0.13454E 01	0.73699E 00	0.12653E 01	0.68307E 00
0.92	0.13472E 01	0.74755E 00	0.12667E 01	0.69297E 00
0.94	0.13489E 01	0.75782E 00	0.12682E 01	0.70261E 00
0.96	0.13506E 01	0.76781E 00	0.12696E 01	0.71198E 00
0.98	0.13522E 01	0.77753E 00	0.12709E 01	0.72110E 00
1.00	0.13537E 01	0.78699E 00	0.12722E 01	0.72998E 00

Table 1 (cont.) (τ = 0.70)

μ	X_3	Y_3	X_4	Y_4
0.00	0.10000E 01	0.	0.10000E 01	0.
0.02	0.10711E 01	0.13033E-01	0.10291E 01	0.28549E-02
0.04	0.11242E 01	0.27744E-01	0.10486E 01	0.60409E-02
0.06	0.11703E 01	0.44096E-01	0.10645E 01	0.96124E-02
0.08	0.12121E 01	0.62169E-01	0.10782E 01	0.13753E-01
0.10	0.12506E 01	0.82400E-01	0.10904E 01	0.18977E-01
0.12	0.12862E 01	0.10529E 00	0.11012E 01	0.25962E-01
0.14	0.13193E 01	0.13099E 00	0.11111E 01	0.35122E-01
0.16	0.13500E 01	0.15948E 00	0.11200E 01	0.46712E-01
0.18	0.13787E 01	0.19027E 00	0.11282E 01	0.60490E-01
0.20	0.14054E 01	0.22298E 00	0.11357E 01	0.76307E-01
0.22	0.14303E 01	0.25700E 00	0.11426E 01	0.93705E-01
0.24	0.14535E 01	0.29190E 00	0.11489E 01	0.11239E 00
0.26	0.14751E 01	0.32725E 00	0.11548E 01	0.13197E 00
0.28	0.14954E 01	0.36270E 00	0.11603E 01	0.15222E 00
0.30	0.15143E 01	0.39797E 00	0.11653E 01	0.17283E 00
0.32	0.15320E 01	0.43284E 00	0.11701E 01	0.19361E 00
0.34	0.15486E 01	0.46712E 00	0.11745E 01	0.21440E 00
0.36	0.15642E 01	0.50073E 00	0.11786E 01	0.23507E 00
0.38	0.15789E 01	0.53354E 00	0.11824E 01	0.25550E 00
0.40	0.15927E 01	0.56552E 00	0.11860E 01	0.27562E 00
0.42	0.16058E 01	0.59660E 00	0.11894E 01	0.29537E 00
0.44	0.16181E 01	0.62678E 00	0.11926E 01	0.31470E 00
0.46	0.16297E 01	0.65605E 00	0.11957E 01	0.33360E 00
0.48	0.16407E 01	0.68440E 00	0.11985E 01	0.35201E 00
0.50	0.16512E 01	0.71186E 00	0.12012E 01	0.36997E 00
0.52	0.16611E 01	0.73841E 00	0.12037E 01	0.38742E 00
0.54	0.16705E 01	0.76412E 00	0.12062E 01	0.40439E 00
0.56	0.16795E 01	0.78896E 00	0.12085E 01	0.42088E 00
0.58	0.16880E 01	0.81300E 00	0.12106E 01	0.43690E 00
0.60	0.16961E 01	0.83624E 00	0.12127E 01	0.45244E 00
0.62	0.17039E 01	0.85872E 00	0.12147E 01	0.46753E 00
0.64	0.17113E 01	0.88047E 00	0.12166E 01	0.48217E 00
0.66	0.17184E 01	0.90149E 00	0.12184E 01	0.49637E 00
0.68	0.17252E 01	0.92185E 00	0.12201E 01	0.51016E 00
0.70	0.17317E 01	0.94153E 00	0.12218E 01	0.52353E 00
0.72	0.17379E 01	0.96060E 00	0.12233E 01	0.53651E 00
0.74	0.17439E 01	0.97906E 00	0.12248E 01	0.54910E 00
0.76	0.17496E 01	0.99693E 00	0.12263E 01	0.56133E 00
0.78	0.17551E 01	0.10143E 01	0.12277E 01	0.57320E 00
0.80	0.17604E 01	0.10310E 01	0.12290E 01	0.58472E 00
0.82	0.17655E 01	0.10473E 01	0.12303E 01	0.59592E 00
0.84	0.17704E 01	0.10631E 01	0.12316E 01	0.60679E 00
0.86	0.17752E 01	0.10784E 01	0.12327E 01	0.61736E 00
0.88	0.17797E 01	0.10933E 01	0.12339E 01	0.62762E 00
0.90	0.17841E 01	0.11077E 01	0.12350E 01	0.63761E 00
0.92	0.17884E 01	0.11217E 01	0.12361E 01	0.64732E 00
0.94	0.17925E 01	0.11353E 01	0.12371E 01	0.65676E 00
0.96	0.17965E 01	0.11485E 01	0.12381E 01	0.66595E 00
0.98	0.18003E 01	0.11614E 01	0.12391E 01	0.67488E 00
1.00	0.18040E 01	0.11739E 01	0.12400E 01	0.68359E 00

Table 1 (cont.) (τ = 1.0)

μ	X_1	Y_1	X_2	Y_2
0.00	0.10000E 01	0.	0.10000E 01	0.
0.02	0.10352E 01	0.37426E-02	0.10227E 01	0.39609E-02
0.04	0.10608E 01	0.78485E-02	0.10404E 01	0.82030E-02
0.06	0.10826E 01	0.12328E-01	0.10560E 01	0.12737E-01
0.08	0.11021E 01	0.17190E-01	0.10704E 01	0.17566E-01
0.10	0.11198E 01	0.22499E-01	0.10836E 01	0.22743E-01
0.12	0.11361E 01	0.28405E-01	0.10961E 01	0.28400E-01
0.14	0.11513E 01	0.35100E-01	0.11078E 01	0.34719E-01
0.16	0.11654E 01	0.42847E-01	0.11189E 01	0.41950E-01
0.18	0.11787E 01	0.51751E-01	0.11294E 01	0.50205E-01
0.20	0.11911E 01	0.61971E-01	0.11394E 01	0.59642E-01
0.22	0.12028E 01	0.73436E-01	0.11488E 01	0.70212E-01
0.24	0.12138E 01	0.86131E-01	0.11578E 01	0.81909E-01
0.26	0.12242E 01	0.99924E-01	0.11663E 01	0.94621E-01
0.28	0.12340E 01	0.11468E 00	0.11744E 01	0.10823E 00
0.30	0.12433E 01	0.13028E 00	0.11821E 01	0.12262E 00
0.32	0.12521E 01	0.14654E 00	0.11894E 01	0.13764E 00
0.34	0.12604E 01	0.16337E 00	0.11964E 01	0.15319E 00
0.36	0.12683E 01	0.18059E 00	0.12030E 01	0.16912E 00
0.38	0.12758E 01	0.19813E 00	0.12093E 01	0.18535E 00
0.40	0.12829E 01	0.21585E 00	0.12153E 01	0.20177E 00
0.42	0.12897E 01	0.23369E 00	0.12211E 01	0.21831E 00
0.44	0.12961E 01	0.25156E 00	0.12266E 01	0.23488E 00
0.46	0.13023E 01	0.26940E 00	0.12318E 01	0.25144E 00
0.48	0.13081E 01	0.28714E 00	0.12368E 01	0.26791E 00
0.50	0.13137E 01	0.30475E 00	0.12416E 01	0.28428E 00
0.52	0.13191E 01	0.32218E 00	0.12462E 01	0.30048E 00
0.54	0.13242E 01	0.33941E 00	0.12506E 01	0.31650E 00
0.56	0.13291E 01	0.35641E 00	0.12548E 01	0.33231E 00
0.58	0.13337E 01	0.37315E 00	0.12588E 01	0.34790E 00
0.60	0.13382E 01	0.38963E 00	0.12627E 01	0.36324E 00
0.62	0.13425E 01	0.40582E 00	0.12664E 01	0.37832E 00
0.64	0.13466E 01	0.42173E 00	0.12700E 01	0.39314E 00
0.66	0.13506E 01	0.43733E 00	0.12734E 01	0.40768E 00
0.68	0.13544E 01	0.45264E 00	0.12767E 01	0.42195E 00
0.70	0.13581E 01	0.46764E 00	0.12799E 01	0.43594E 00
0.72	0.13616E 01	0.48234E 00	0.12830E 01	0.44965E 00
0.74	0.13650E 01	0.49674E 00	0.12859E 01	0.46308E 00
0.76	0.13683E 01	0.51084E 00	0.12888E 01	0.47623E 00
0.78	0.13714E 01	0.52464E 00	0.12915E 01	0.48911E 00
0.80	0.13745E 01	0.53814E 00	0.12942E 01	0.50172E 00
0.82	0.13774E 01	0.55136E 00	0.12968E 01	0.51406E 00
0.84	0.13803E 01	0.56429E 00	0.12992E 01	0.52614E 00
0.86	0.13830E 01	0.57695E 00	0.13016E 01	0.53796E 00
0.88	0.13857E 01	0.58932E 00	0.13040E 01	0.54952E 00
0.90	0.13882E 01	0.60143E 00	0.13062E 01	0.56083E 00
0.92	0.13907E 01	0.61328E 00	0.13084E 01	0.57190E 00
0.94	0.13931E 01	0.62487E 00	0.13105E 01	0.58273E 00
0.96	0.13955E 01	0.63621E 00	0.13126E 01	0.59333E 00
0.98	0.13977E 01	0.64730E 00	0.13145E 01	0.60370E 00
1.00	0.13999E 01	0.65816E 00	0.13165E 01	0.61385E 00

Table 1 (cont.) (τ = 1.0)

	X_3	Y_3	X_4	Y_4
0.00	0.10000E 01	0.	0.10000E 01	0.
0.02	0.10731E 01	0.10693E-01	0.10291E 01	0.17159E-02
0.04	0.11282E 01	0.22706E-01	0.10488E 01	0.36127E-02
0.06	0.11768E 01	0.35969E-01	0.10648E 01	0.57052E-02
0.08	0.12212E 01	0.50404E-01	0.10786E 01	0.80105E-02
0.10	0.12625E 01	0.66050E-01	0.10908E 01	0.10594E-01
0.12	0.13013E 01	0.83025E-01	0.11018E 01	0.13611E-01
0.14	0.13379E 01	0.10145E 00	0.11119E 01	0.17269E-01
0.16	0.13725E 01	0.12153E 00	0.11210E 01	0.21846E-01
0.18	0.14054E 01	0.14323E 00	0.11295E 01	0.27480E-01
0.20	0.14365E 01	0.16663E 00	0.11373E 01	0.34352E-01
0.22	0.14661E 01	0.19151E 00	0.11446E 01	0.42426E-01
0.24	0.14943E 01	0.21776E 00	0.11514E 01	0.51711E-01
0.26	0.15210E 01	0.24515E 00	0.11578E 01	0.62098E-01
0.28	0.15464E 01	0.27346E 00	0.11637E 01	0.73476E-01
0.30	0.15705E 01	0.30249E 00	0.11693E 01	0.85732E-01
0.32	0.15935E 01	0.33202E 00	0.11746E 01	0.98707E-01
0.34	0.16153E 01	0.36188E 00	0.11795E 01	0.11231E 00
0.36	0.16361E 01	0.39190E 00	0.11842E 01	0.12638E 00
0.38	0.16559E 01	0.42195E 00	0.11886E 01	0.14084E 00
0.40	0.16748E 01	0.45189E 00	0.11928E 01	0.15557E 00
0.42	0.16928E 01	0.48164E 00	0.11967E 01	0.17049E 00
0.44	0.17100E 01	0.51110E 00	0.12004E 01	0.18552E 00
0.46	0.17264E 01	0.54021E 00	0.12040E 01	0.20061E 00
0.48	0.17421E 01	0.56890E 00	0.12074E 01	0.21568E 00
0.50	0.17571E 01	0.59715E 00	0.12106E 01	0.23071E 00
0.52	0.17715E 01	0.62490E 00	0.12137E 01	0.24563E 00
0.54	0.17852E 01	0.65215E 00	0.12166E 01	0.26043E 00
0.56	0.17984E 01	0.67885E 00	0.12194E 01	0.27507E 00
0.58	0.18110E 01	0.70502E 00	0.12220E 01	0.28954E 00
0.60	0.18231E 01	0.73063E 00	0.12246E 01	0.30380E 00
0.62	0.18348E 01	0.75568E 00	0.12270E 01	0.31786E 00
0.64	0.18460E 01	0.78018E 00	0.12294E 01	0.33169E 00
0.66	0.18567E 01	0.80411E 00	0.12316E 01	0.34528E 00
0.68	0.18671E 01	0.82751E 00	0.12338E 01	0.35865E 00
0.70	0.18770E 01	0.85034E 00	0.12358E 01	0.37176E 00
0.72	0.18866E 01	0.87265E 00	0.12378E 01	0.38463E 00
0.74	0.18959E 01	0.89443E 00	0.12397E 01	0.39725E 00
0.76	0.19048E 01	0.91569E 00	0.12416E 01	0.40963E 00
0.78	0.19134E 01	0.93645E 00	0.12433E 01	0.42176E 00
0.80	0.19217E 01	0.95671E 00	0.12451E 01	0.43365E 00
0.82	0.19298E 01	0.97649E 00	0.12467E 01	0.44529E 00
0.84	0.19375E 01	0.99579E 00	0.12483E 01	0.45669E 00
0.86	0.19451E 01	0.10146E 01	0.12498E 01	0.46786E 00
0.88	0.19523E 01	0.10330E 01	0.12513E 01	0.47879E 00
0.90	0.19594E 01	0.10510E 01	0.12527E 01	0.48950E 00
0.92	0.19662E 01	0.10686E 01	0.12541E 01	0.49999E 00
0.94	0.19728E 01	0.10857E 01	0.12555E 01	0.51025E 00
0.96	0.19792E 01	0.11024E 01	0.12568E 01	0.52030E 00
0.98	0.19854E 01	0.11188E 01	0.12580E 01	0.53013E 00
1.00	0.19914E 01	0.11347E 01	0.12592E 01	0.53977E 00

Table 1 (cont.) $(\tau = 2.0)$

μ	X_1	Y_1	X_2	Y_2
0.00	0.10000E 01	0.	0.10000E 01	0.
0.02	0.10355E 01	0.12837E-02	0.10231E 01	0.17191E-02
0.04	0.10613E 01	0.26832E-02	0.10412E 01	0.35525E-02
0.06	0.10835E 01	0.41982E-02	0.10572E 01	0.55019E-02
0.08	0.11033E 01	0.58256E-02	0.10719E 01	0.75640E-02
0.10	0.11214E 01	0.75697E-02	0.10857E 01	0.97425E-02
0.12	0.11381E 01	0.94363E-02	0.10986E 01	0.12041E-01
0.14	0.11537E 01	0.11429E-01	0.11109E 01	0.14463E-01
0.16	0.11683E 01	0.13564E-01	0.11225E 01	0.17019E-01
0.18	0.11821E 01	0.15848E-01	0.11337E 01	0.19715E-01
0.20	0.11952E 01	0.18312E-01	0.11443E 01	0.22576E-01
0.22	0.12075E 01	0.20974E-01	0.11546E 01	0.25616E-01
0.24	0.12193E 01	0.23873E-01	0.11644E 01	0.28869E-01
0.26	0.12306E 01	0.27039E-01	0.11738E 01	0.32361E-01
0.28	0.12413E 01	0.30505E-01	0.11830E 01	0.36121E-01
0.30	0.12516E 01	0.34307E-01	0.11918E 01	0.40181E-01
0.32	0.12614E 01	0.38457E-01	0.12002E 01	0.44552E-01
0.34	0.12708E 01	0.42993E-01	0.12084E 01	0.49269E-01
0.36	0.12799E 01	0.47907E-01	0.12164E 01	0.54323E-01
0.38	0.12886E 01	0.53223E-01	0.12240E 01	0.59738E-01
0.40	0.12970E 01	0.58927E-01	0.12315E 01	0.65500E-01
0.42	0.13051E 01	0.65021E-01	0.12386E 01	0.71612E-01
0.44	0.13129E 01	0.71494E-01	0.12456E 01	0.78063E-01
0.46	0.13204E 01	0.78327E-01	0.12524E 01	0.84837E-01
0.48	0.13276E 01	0.85514E-01	0.12587E 01	0.91929E-01
0.50	0.13346E 01	0.93020E-01	0.12652E 01	0.99306E-01
0.52	0.13414E 01	0.10085E 00	0.12714E 01	0.10697E 00
0.54	0.13479E 01	0.10895E 00	0.12774E 01	0.11488E 00
0.56	0.13542E 01	0.11733E 00	0.12832E 01	0.12304E 00
0.58	0.13603E 01	0.12594E 00	0.12888E 01	0.13140E 00
0.60	0.13663E 01	0.13478E 00	0.12943E 01	0.13997E 00
0.62	0.13720E 01	0.14381E 00	0.12996E 01	0.14871E 00
0.64	0.13776E 01	0.15302E 00	0.13047E 01	0.15761E 00
0.66	0.13829E 01	0.16240E 00	0.13098E 01	0.16665E 00
0.68	0.13882E 01	0.17190E 00	0.13146E 01	0.17581E 00
0.70	0.13932E 01	0.18154E 00	0.13194E 01	0.18507E 00
0.72	0.13982E 01	0.19127E 00	0.13240E 01	0.19443E 00
0.74	0.14029E 01	0.20109E 00	0.13285E 01	0.20385E 00
0.76	0.14076E 01	0.21098E 00	0.13329E 01	0.21334E 00
0.78	0.14121E 01	0.22093E 00	0.13371E 01	0.22288E 00
0.80	0.14165E 01	0.23093E 00	0.13413E 01	0.23245E 00
0.82	0.14207E 01	0.24095E 00	0.13453E 01	0.24204E 00
0.84	0.14249E 01	0.25100E 00	0.13493E 01	0.25165E 00
0.86	0.14289E 01	0.26105E 00	0.13531E 01	0.26126E 00
0.88	0.14328E 01	0.27110E 00	0.13569E 01	0.27086E 00
0.90	0.14367E 01	0.28115E 00	0.13605E 01	0.28045E 00
0.92	0.14404E 01	0.29117E 00	0.13641E 01	0.29002E 00
0.94	0.14440E 01	0.30117E 00	0.13676E 01	0.29956E 00
0.96	0.14476E 01	0.31114E 00	0.13710E 01	0.30907E 00
0.98	0.14510E 01	0.32107E 00	0.13743E 01	0.31853E 00
1.00	0.14544E 01	0.33096E 00	0.13775E 01	0.32795E 00

Table 1 (cont.) (τ = 2.0)

	X_3	Y_3	X_4	Y_4
0.00	0.10000E 01	0.	0.10000E 01	0.
0.02	0.10765E 01	0.69560E-02	0.10292E 01	0.40274E-03
0.04	0.11354E 01	0.14744E-01	0.10488E 01	0.84309E-03
0.06	0.11881E 01	0.23305E-01	0.10649E 01	0.13223E-02
0.08	0.12371E 01	0.32571E-01	0.10788E 01	0.18406E-02
0.10	0.12833E 01	0.42528E-01	0.10911E 01	0.24011E-02
0.12	0.13273E 01	0.53163E-01	0.11022E 01	0.30079E-02
0.14	0.13696E 01	0.64451E-01	0.11123E 01	0.36647E-02
0.16	0.14102E 01	0.76415E-01	0.11215E 01	0.43822E-02
0.18	0.14495E 01	0.89020E-01	0.11301E 01	0.51693E-02
0.20	0.14875E 01	0.10231E 00	0.11381E 01	0.60505E-02
0.22	0.15243E 01	0.11627E 00	0.11456E 01	0.70467E-02
0.24	0.15601E 01	0.13093E 00	0.11526E 01	0.81940E-02
0.26	0.15948E 01	0.14628E 00	0.11591E 01	0.95251E-02
0.28	0.16286E 01	0.16234E 00	0.11653E 01	0.11075E-01
0.30	0.16614E 01	0.17911E 00	0.11712E 01	0.12881E-01
0.32	0.16934E 01	0.19656E 00	0.11768E 01	0.14964E-01
0.34	0.17245E 01	0.21471E 00	0.11820E 01	0.17364E-01
0.36	0.17548E 01	0.23349E 00	0.11870E 01	0.20079E-01
0.38	0.17843E 01	0.25292E 00	0.11918E 01	0.23141E-01
0.40	0.18130E 01	0.27291E 00	0.11964E 01	0.26541E-01
0.42	0.18409E 01	0.29346E 00	0.12007E 01	0.30288E-01
0.44	0.18681E 01	0.31451E 00	0.12049E 01	0.34376E-01
0.46	0.18946E 01	0.33601E 00	0.12089E 01	0.38795E-01
0.48	0.19204E 01	0.35792E 00	0.12127E 01	0.43543E-01
0.50	0.19455E 01	0.38018E 00	0.12164E 01	0.48591E-01
0.52	0.19700E 01	0.40277E 00	0.12199E 01	0.53945E-01
0.54	0.19938E 01	0.42562E 00	0.12233E 01	0.59568E-01
0.56	0.20170E 01	0.44870E 00	0.12265E 01	0.65457E-01
0.58	0.20396E 01	0.47196E 00	0.12296E 01	0.71584E-01
0.60	0.20616E 01	0.49536E 00	0.12326E 01	0.77936E-01
0.62	0.20831E 01	0.51888E 00	0.12356E 01	0.84495E-01
0.64	0.21040E 01	0.54246E 00	0.12384E 01	0.91239E-01
0.66	0.21244E 01	0.56609E 00	0.12411E 01	0.98157E-01
0.68	0.21443E 01	0.58973E 00	0.12437E 01	0.10522E 00
0.70	0.21636E 01	0.61335E 00	0.12462E 01	0.11243E 00
0.72	0.21825E 01	0.63693E 00	0.12487E 01	0.11976E 00
0.74	0.22010E 01	0.66045E 00	0.12510E 01	0.12720E 00
0.76	0.22189E 01	0.68388E 00	0.12533E 01	0.13473E 00
0.78	0.22365E 01	0.70720E 00	0.12555E 01	0.14234E 00
0.80	0.22536E 01	0.73041E 00	0.12577E 01	0.15001E 00
0.82	0.22703E 01	0.75347E 00	0.12598E 01	0.15775E 00
0.84	0.22866E 01	0.77639E 00	0.12618E 01	0.16553E 00
0.86	0.23025E 01	0.79914E 00	0.12637E 01	0.17334E 00
0.88	0.23180E 01	0.82171E 00	0.12657E 01	0.18119E 00
0.90	0.23332E 01	0.84411E 00	0.12675E 01	0.18905E 00
0.92	0.23480E 01	0.86631E 00	0.12693E 01	0.19691E 00
0.94	0.23625E 01	0.88830E 00	0.12710E 01	0.20479E 00
0.96	0.23766E 01	0.91010E 00	0.12727E 01	0.21265E 00
0.98	0.23905E 01	0.93167E 00	0.12744E 01	0.22051E 00
1.00	0.24040E 01	0.95304E 00	0.12760E 01	0.22835E 00

Table 1 (cont.) (τ = 4.0)

μ	X_1	Y_1	X_2	Y_2
0.00	0.10000E 01	0.	0.10000E 01	0.
0.02	0.10355E 01	0.18473E-03	0.10232E 01	0.37148E-03
0.04	0.10614E 01	0.38560E-03	0.10413E 01	0.76708E-03
0.06	0.10836E 01	0.60241E-03	0.10575E 01	0.11870E-02
0.08	0.11035E 01	0.83458E-03	0.10723E 01	0.16304E-02
0.10	0.11216E 01	0.10825E-02	0.10862E 01	0.20978E-02
0.12	0.11384E 01	0.13467E-02	0.10992E 01	0.25900E-02
0.14	0.11541E 01	0.16275E-02	0.11116E 01	0.31068E-02
0.16	0.11687E 01	0.19261E-02	0.11234E 01	0.36503E-02
0.18	0.11826E 01	0.22426E-02	0.11347E 01	0.42198E-02
0.20	0.11957E 01	0.25792E-02	0.11455E 01	0.48180E-02
0.22	0.12082E 01	0.29358E-02	0.11559E 01	0.54447E-02
0.24	0.12200E 01	0.33146E-02	0.11658E 01	0.61021E-02
0.26	0.12314E 01	0.37168E-02	0.11755E 01	0.67912E-02
0.28	0.12422E 01	0.41443E-02	0.11848E 01	0.75138E-02
0.30	0.12526E 01	0.45999E-02	0.11937E 01	0.82726E-02
0.32	0.12625E 01	0.50851E-02	0.12024E 01	0.90685E-02
0.34	0.12721E 01	0.56049E-02	0.12109E 01	0.99066E-02
0.36	0.12813E 01	0.61608E-02	0.12190E 01	0.10788E-01
0.38	0.12901E 01	0.67592E-02	0.12269E 01	0.11717E-01
0.40	0.12987E 01	0.74030E-02	0.12346E 01	0.12698E-01
0.42	0.13069E 01	0.80983E-02	0.12421E 01	0.13736E-01
0.44	0.13143E 01	0.88506E-02	0.12494E 01	0.14834E-01
0.46	0.13226E 01	0.96642E-02	0.12564E 01	0.15997E-01
0.48	0.13301E 01	0.10548E-01	0.12633E 01	0.17233E-01
0.50	0.13373E 01	0.11503E-01	0.12700E 01	0.18542E-01
0.52	0.13443E 01	0.12540E-01	0.12765E 01	0.19934E-01
0.54	0.13511E 01	0.13661E-01	0.12829E 01	0.21410E-01
0.56	0.13576E 01	0.14874E-01	0.12891E 01	0.22976E-01
0.58	0.13640E 01	0.16181E-01	0.12951E 01	0.24635E-01
0.60	0.13702E 01	0.17588E-01	0.13010E 01	0.26392E-01
0.62	0.13763E 01	0.19100E-01	0.13068E 01	0.28249E-01
0.64	0.13821E 01	0.20717E-01	0.13124E 01	0.30209E-01
0.66	0.13878E 01	0.22447E-01	0.13179E 01	0.32276E-01
0.68	0.13934E 01	0.24287E-01	0.13233E 01	0.34448E-01
0.70	0.13988E 01	0.26244E-01	0.13285E 01	0.36733E-01
0.72	0.14040E 01	0.28316E-01	0.13337E 01	0.39124E-01
0.74	0.14091E 01	0.30505E-01	0.13387E 01	0.41628E-01
0.76	0.14141E 01	0.32812E-01	0.13436E 01	0.44242E-01
0.78	0.14190E 01	0.35236E-01	0.13484E 01	0.46965E-01
0.80	0.14237E 01	0.37778E-01	0.13531E 01	0.49799E-01
0.82	0.14284E 01	0.40434E-01	0.13577E 01	0.52739E-01
0.84	0.14329E 01	0.43209E-01	0.13623E 01	0.55790E-01
0.86	0.14373E 01	0.46095E-01	0.13667E 01	0.58943E-01
0.88	0.14416E 01	0.49095E-01	0.13710E 01	0.62202E-01
0.90	0.14458E 01	0.52204E-01	0.13752E 01	0.65562E-01
0.92	0.14500E 01	0.55422E-01	0.13794E 01	0.69022E-01
0.94	0.14540E 01	0.58745E-01	0.13835E 01	0.72578E-01
0.96	0.14579E 01	0.62170E-01	0.13875E 01	0.76229E-01
0.98	0.14618E 01	0.65697E-01	0.13914E 01	0.79972E-01
1.00	0.14655E 01	0.69319E-01	0.13952E 01	0.83802E-01

Table 1 (cont.) (τ = 4.0)

μ	X_3	Y_3	X_4	Y_4
0.00	0.10000E 01	0.	0.10000E 01	0.
0.02	0.10791E 01	0.42104E-02	0.10292E 01	0.32104E-04
0.04	0.11411E 01	0.89213E-02	0.10489E 01	0.67003E-04
0.06	0.11971E 01	0.14096E-01	0.10649E 01	0.10472E-03
0.08	0.12497E 01	0.19692E-01	0.10788E 01	0.14523E-03
0.10	0.12997E 01	0.25700E-01	0.10911E 01	0.18869E-03
0.12	0.13479E 01	0.32111E-01	0.11022E 01	0.23526E-03
0.14	0.13944E 01	0.38906E-01	0.11123E 01	0.28510E-03
0.16	0.14397E 01	0.46097E-01	0.11216E 01	0.33861E-03
0.18	0.14838E 01	0.53657E-01	0.11302E 01	0.39587E-03
0.20	0.15269E 01	0.61605E-01	0.11382E 01	0.45751E-03
0.22	0.15690E 01	0.69919E-01	0.11456E 01	0.52372E-03
0.24	0.16104E 01	0.78608E-01	0.11526E 01	0.59521E-03
0.26	0.16509E 01	0.87665E-01	0.11592E 01	0.67257E-03
0.28	0.16908E 01	0.97088E-01	0.11654E 01	0.75667E-03
0.30	0.17300E 01	0.10688E 00	0.11713E 01	0.84885E-03
0.32	0.17685E 01	0.11703E 00	0.11769E 01	0.95031E-03
0.34	0.18064E 01	0.12756E 00	0.11821E 01	0.10636E-02
0.36	0.18438E 01	0.13844E 00	0.11872E 01	0.11906E-02
0.38	0.18806E 01	0.14970E 00	0.11920E 01	0.13350E-02
0.40	0.19168E 01	0.16131E 00	0.11965E 01	0.14996E-02
0.42	0.19525E 01	0.17329E 00	0.12009E 01	0.16889E-02
0.44	0.19877E 01	0.18563E 00	0.12051E 01	0.19073E-02
0.46	0.20224E 01	0.19833E 00	0.12091E 01	0.21591E-02
0.48	0.20567E 01	0.21140E 00	0.12130E 01	0.24504E-02
0.50	0.20904E 01	0.22482E 00	0.12167E 01	0.27844E-02
0.52	0.21237E 01	0.23860E 00	0.12202E 01	0.31695E-02
0.54	0.21565E 01	0.25273E 00	0.12237E 01	0.36073E-02
0.56	0.21889E 01	0.26721E 00	0.12270E 01	0.41056E-02
0.58	0.22208E 01	0.28202E 00	0.12301E 01	0.46674E-02
0.60	0.22523E 01	0.29717E 00	0.12332E 01	0.52979E-02
0.62	0.22834E 01	0.31265E 00	0.12362E 01	0.60019E-02
0.64	0.23141E 01	0.32845E 00	0.12390E 01	0.67811E-02
0.66	0.23443E 01	0.34457E 00	0.12418E 01	0.76426E-02
0.68	0.23742E 01	0.36097E 00	0.12445E 01	0.85846E-02
0.70	0.24036E 01	0.37768E 00	0.12470E 01	0.96150E-02
0.72	0.24326E 01	0.39467E 00	0.12496E 01	0.10731E-01
0.74	0.24612E 01	0.41193E 00	0.12520E 01	0.11938E-01
0.76	0.24895E 01	0.42945E 00	0.12543E 01	0.13235E-01
0.78	0.25174E 01	0.44721E 00	0.12566E 01	0.14624E-01
0.80	0.25449E 01	0.46522E 00	0.12589E 01	0.16106E-01
0.82	0.25720E 01	0.48344E 00	0.12610E 01	0.17679E-01
0.84	0.25987E 01	0.50189E 00	0.12631E 01	0.19347E-01
0.86	0.26251E 01	0.52053E 00	0.12651E 01	0.21104E-01
0.88	0.26511E 01	0.53937E 00	0.12671E 01	0.22955E-01
0.90	0.26768E 01	0.55838E 00	0.12690E 01	0.24894E-01
0.92	0.27021E 01	0.57756E 00	0.12709E 01	0.26924E-01
0.94	0.27271E 01	0.59689E 00	0.12727E 01	0.29041E-01
0.96	0.27518E 01	0.61637E 00	0.12745E 01	0.31243E-01
0.98	0.27761E 01	0.63598E 00	0.12763E 01	0.33532E-01
1.00	0.28001E 01	0.65571E 00	0.12779E 01	0.35901E-01

Table 1 (cont.) (τ = 8.0)

μ	X_1	Y_1	X_2	Y_2
0.00	0.10000E 01	0.	0.10000E 01	0.
0.02	0.10355E 01	0.44564E-05	0.10232E 01	0.18826E-04
0.04	0.10614E 01	0.92976E-05	0.10413E 01	0.38866E-04
0.06	0.10836E 01	0.14518E-04	0.10575E 01	0.60129E-04
0.08	0.11035E 01	0.20101E-04	0.10723E 01	0.82571E-04
0.10	0.11216E 01	0.26058E-04	0.10862E 01	0.10622E-03
0.12	0.11384E 01	0.32397E-04	0.10993E 01	0.13110E-03
0.14	0.11541E 01	0.39123E-04	0.11117E 01	0.15722E-03
0.16	0.11687E 01	0.46268E-04	0.11235E 01	0.18467E-03
0.18	0.11826E 01	0.53828E-04	0.11347E 01	0.21341E-03
0.20	0.11957E 01	0.61851E-04	0.11455E 01	0.24358E-03
0.22	0.12082E 01	0.70335E-04	0.11559E 01	0.27515E-03
0.24	0.12200E 01	0.79326E-04	0.11659E 01	0.30824E-03
0.26	0.12314E 01	0.88846E-04	0.11755E 01	0.34288E-03
0.28	0.12422E 01	0.98927E-04	0.11849E 01	0.37915E-03
0.30	0.12526E 01	0.10962E-03	0.11938E 01	0.41716E-03
0.32	0.12625E 01	0.12095E-03	0.12025E 01	0.45692E-03
0.34	0.12721E 01	0.13299E-03	0.12110E 01	0.49864E-03
0.36	0.12813E 01	0.14576E-03	0.12191E 01	0.54230E-03
0.38	0.12902E 01	0.15937E-03	0.12271E 01	0.58812E-03
0.40	0.12987E 01	0.17384E-03	0.12348E 01	0.63613E-03
0.42	0.13070E 01	0.18928E-03	0.12423E 01	0.68653E-03
0.44	0.13150E 01	0.20578E-03	0.12495E 01	0.73945E-03
0.46	0.13227E 01	0.22341E-03	0.12566E 01	0.79503E-03
0.48	0.13301E 01	0.24235E-03	0.12635E 01	0.85352E-03
0.50	0.13373E 01	0.26266E-03	0.12702E 01	0.91499E-03
0.52	0.13443E 01	0.28461E-03	0.12768E 01	0.97988E-03
0.54	0.13511E 01	0.30828E-03	0.12831E 01	0.10482E-02
0.56	0.13577E 01	0.33397E-03	0.12893E 01	0.11205E-02
0.58	0.13641E 01	0.36188E-03	0.12954E 01	0.11968E-02
0.60	0.13703E 01	0.39230E-03	0.13013E 01	0.12776E-02
0.62	0.13763E 01	0.42560E-03	0.13071E 01	0.13633E-02
0.64	0.13822E 01	0.46205E-03	0.13128E 01	0.14543E-02
0.66	0.13879E 01	0.50222E-03	0.13183E 01	0.15511E-02
0.68	0.13935E 01	0.54635E-03	0.13237E 01	0.16540E-02
0.70	0.13989E 01	0.59520E-03	0.13290E 01	0.17639E-02
0.72	0.14041E 01	0.64907E-03	0.13341E 01	0.18810E-02
0.74	0.14093E 01	0.70874E-03	0.13392E 01	0.20061E-02
0.76	0.14143E 01	0.77474E-03	0.13441E 01	0.21399E-02
0.78	0.14192E 01	0.84774E-03	0.13490E 01	0.22830E-02
0.80	0.14239E 01	0.92861E-03	0.13537E 01	0.24363E-02
0.82	0.14286E 01	0.10178E-02	0.13584E 01	0.26002E-02
0.84	0.14331E 01	0.11166E-02	0.13629E 01	0.27760E-02
0.86	0.14375E 01	0.12251E-02	0.13674E 01	0.29638E-02
0.88	0.14419E 01	0.13450E-02	0.13717E 01	0.31653E-02
0.90	0.14461E 01	0.14764E-02	0.13760E 01	0.33805E-02
0.92	0.14502E 01	0.16207E-02	0.13802E 01	0.36108E-02
0.94	0.14543E 01	0.17786E-02	0.13844E 01	0.38568E-02
0.96	0.14582E 01	0.19508E-02	0.13884E 01	0.41193E-02
0.98	0.14621E 01	0.21387E-02	0.13924E 01	0.43996E-02
1.00	0.14659E 01	0.23424E-02	0.13963E 01	0.46976E-02

Table 1 (cont.) (τ = 8.0)

μ	X_3	Y_3	X_4	Y_4
0.00	0.10000E 01	0.	0.10000E 01	0.
0.02	0.10810E 01	0.23702E-02	0.10292E 01	0.30816E-06
0.04	0.11450E 01	0.50222E-02	0.10489E 01	0.64208E-06
0.06	0.12033E 01	0.79348E-02	0.10649E 01	0.10017E-05
0.08	0.12583E 01	0.11085E-01	0.10788E 01	0.13864E-05
0.10	0.13110E 01	0.14467E-01	0.10911E 01	0.17974E-05
0.12	0.13619E 01	0.18074E-01	0.11022E 01	0.22360E-05
0.14	0.14114E 01	0.21899E-01	0.11123E 01	0.27030E-05
0.16	0.14598E 01	0.25945E-01	0.11216E 01	0.32016E-05
0.18	0.15072E 01	0.30199E-01	0.11302E 01	0.37322E-05
0.20	0.15537E 01	0.34671E-01	0.11382E 01	0.42995E-05
0.22	0.15995E 01	0.39348E-01	0.11456E 01	0.49043E-05
0.24	0.16447E 01	0.44236E-01	0.11526E 01	0.55514E-05
0.26	0.16892E 01	0.49329E-01	0.11592E 01	0.62439E-05
0.28	0.17331E 01	0.54628E-01	0.11654E 01	0.69861E-05
0.30	0.17766E 01	0.60134E-01	0.11713E 01	0.77842E-05
0.32	0.18196E 01	0.65839E-01	0.11769E 01	0.86420E-05
0.34	0.18621E 01	0.71754E-01	0.11821E 01	0.95699E-05
0.36	0.19042E 01	0.77865E-01	0.11872E 01	0.10572E-04
0.38	0.19458E 01	0.84184E-01	0.11920E 01	0.11663E-04
0.40	0.19871E 01	0.90700E-01	0.11965E 01	0.12850E-04
0.42	0.20281E 01	0.97419E-01	0.12009E 01	0.14152E-04
0.44	0.20686E 01	0.10434E 00	0.12051E 01	0.15587E-04
0.46	0.21089E 01	0.11146E 00	0.12091E 01	0.17175E-04
0.48	0.21488E 01	0.11878E 00	0.12130E 01	0.18963E-04
0.50	0.21884E 01	0.12629E 00	0.12167E 01	0.20971E-04
0.52	0.22276E 01	0.13402E 00	0.12202E 01	0.23275E-04
0.54	0.22666E 01	0.14193E 00	0.12237E 01	0.25923E-04
0.56	0.23053E 01	0.15005E 00	0.12270E 01	0.29018E-04
0.58	0.23437E 01	0.15837E 00	0.12301E 01	0.32655E-04
0.60	0.23818E 01	0.16688E 00	0.12332E 01	0.36970E-04
0.62	0.24196E 01	0.17559E 00	0.12362E 01	0.42130E-04
0.64	0.24572E 01	0.18450E 00	0.12390E 01	0.48297E-04
0.66	0.24945E 01	0.19361E 00	0.12418E 01	0.55746E-04
0.68	0.25315E 01	0.20292E 00	0.12445E 01	0.64654E-04
0.70	0.25683E 01	0.21242E 00	0.12471E 01	0.75418E-04
0.72	0.26048E 01	0.22212E 00	0.12496E 01	0.88263E-04
0.74	0.26411E 01	0.23202E 00	0.12520E 01	0.10364E-03
0.76	0.26771E 01	0.24211E 00	0.12544E 01	0.12190E-03
0.78	0.27129E 01	0.25240E 00	0.12566E 01	0.14349E-03
0.80	0.27484E 01	0.26289E 00	0.12589E 01	0.16896E-03
0.82	0.27837E 01	0.27357E 00	0.12610E 01	0.19866E-03
0.84	0.28188E 01	0.28445E 00	0.12631E 01	0.23339E-03
0.86	0.28536E 01	0.29551E 00	0.12652E 01	0.27342E-03
0.88	0.28882E 01	0.30678E 00	0.12671E 01	0.31965E-03
0.90	0.29225E 01	0.31823E 00	0.12691E 01	0.37246E-03
0.92	0.29567E 01	0.32988E 00	0.12709E 01	0.43263E-03
0.94	0.29906E 01	0.34172E 00	0.12728E 01	0.50086E-03
0.96	0.30242E 01	0.35374E 00	0.12745E 01	0.57753E-03
0.98	0.30577E 01	0.36595E 00	0.12763E 01	0.66378E-03
1.00	0.30909E 01	0.37835E 00	0.12780E 01	0.75976E-03

Table 1 (cont.) (τ = 16.0)

μ	X_1	Y_1	X_2	Y_2
0.00	0.10000E 01	0.	0.10000E 01	0.
0.02	0.10355E 01	0.28695E-08	0.10232E 01	0.49984E-07
0.04	0.10614E 01	0.59859E-08	0.10413E 01	0.10319E-06
0.06	0.10836E 01	0.93453E-08	0.10575E 01	0.15964E-06
0.08	0.11035E 01	0.12938E-07	0.10723E 01	0.21921E-06
0.10	0.11216E 01	0.16768E-07	0.10862E 01	0.28198E-06
0.12	0.11384E 01	0.20844E-07	0.10993E 01	0.34803E-06
0.14	0.11541E 01	0.25167E-07	0.11117E 01	0.41734E-06
0.16	0.11687E 01	0.29757E-07	0.11235E 01	0.49017E-06
0.18	0.11826E 01	0.34612E-07	0.11347E 01	0.56644E-06
0.20	0.11957E 01	0.39761E-07	0.11455E 01	0.64648E-06
0.22	0.12082E 01	0.45205E-07	0.11559E 01	0.73024E-06
0.24	0.12200E 01	0.50970E-07	0.11659E 01	0.81802E-06
0.26	0.12314E 01	0.57071E-07	0.11755E 01	0.90991E-06
0.28	0.12422E 01	0.63528E-07	0.11849E 01	0.10061E-05
0.30	0.12526E 01	0.70372E-07	0.11938E 01	0.11069E-05
0.32	0.12625E 01	0.77616E-07	0.12025E 01	0.12123E-05
0.34	0.12721E 01	0.85312E-07	0.12110E 01	0.13229E-05
0.36	0.12813E 01	0.93466E-07	0.12191E 01	0.14386E-05
0.38	0.12902E 01	0.10214E-06	0.12271E 01	0.15600E-05
0.40	0.12987E 01	0.11136E-06	0.12348E 01	0.16872E-05
0.42	0.13070E 01	0.12118E-06	0.12423E 01	0.18206E-05
0.44	0.13150E 01	0.13165E-06	0.12495E 01	0.19607E-05
0.46	0.13227E 01	0.14283E-06	0.12566E 01	0.21077E-05
0.48	0.13301E 01	0.15479E-06	0.12635E 01	0.22624E-05
0.50	0.13373E 01	0.16759E-06	0.12702E 01	0.24248E-05
0.52	0.13443E 01	0.18137E-06	0.12768E 01	0.25961E-05
0.54	0.13511E 01	0.19616E-06	0.12831E 01	0.27762E-05
0.56	0.13577E 01	0.21214E-06	0.12893E 01	0.29664E-05
0.58	0.13641E 01	0.22940E-06	0.12954E 01	0.31670E-05
0.60	0.13703E 01	0.24812E-06	0.13013E 01	0.33791E-05
0.62	0.13763E 01	0.26850E-06	0.13071E 01	0.36035E-05
0.64	0.13822E 01	0.29071E-06	0.13128E 01	0.38410E-05
0.66	0.13879E 01	0.31510E-06	0.13183E 01	0.40934E-05
0.68	0.13935E 01	0.34186E-06	0.13237E 01	0.43610E-05
0.70	0.13989E 01	0.37154E-06	0.13290E 01	0.46464E-05
0.72	0.14041E 01	0.40443E-06	0.13341E 01	0.49502E-05
0.74	0.14093E 01	0.44123E-06	0.13392E 01	0.52749E-05
0.76	0.14143E 01	0.48256E-06	0.13441E 01	0.56222E-05
0.78	0.14192E 01	0.52927E-06	0.13490E 01	0.59946E-05
0.80	0.14239E 01	0.58252E-06	0.13537E 01	0.63953E-05
0.82	0.14286E 01	0.64333E-06	0.13584E 01	0.68262E-05
0.84	0.14331E 01	0.71375E-06	0.13629E 01	0.72927E-05
0.86	0.14375E 01	0.79510E-06	0.13674E 01	0.77969E-05
0.88	0.14419E 01	0.89038E-06	0.13717E 01	0.83455E-05
0.90	0.14461E 01	0.10018E-05	0.13760E 01	0.89420E-05
0.92	0.14502E 01	0.11333E-05	0.13802E 01	0.95939E-05
0.94	0.14543E 01	0.12887E-05	0.13844E 01	0.10308E-04
0.96	0.14582E 01	0.14726E-05	0.13884E 01	0.11091E-04
0.98	0.14621E 01	0.16917E-05	0.13924E 01	0.11955E-04
1.00	0.14659E 01	0.19508E-05	0.13963E 01	0.12905E-04

Table 1 (cont.) (τ = 16.0)

	X_3	Y_3	X_4	Y_4
0.00	0.10000E 01	0.	0.10000E 01	0.
0.02	0.10821E 01	0.12655E-02	0.10292E 01	0.
0.04	0.11474E 01	0.26813E-02	0.10489E 01	0.
0.06	0.12070E 01	0.42364E-02	0.10649E 01	0.14946E-09
0.08	0.12635E 01	0.59183E-02	0.10788E 01	0.20662E-09
0.10	0.13177E 01	0.77237E-02	0.10911E 01	0.26755E-09
0.12	0.13703E 01	0.96499E-02	0.11022E 01	0.33240E-09
0.14	0.14216E 01	0.11692E-01	0.11123E 01	0.40127E-09
0.16	0.14719E 01	0.13852E-01	0.11216E 01	0.47460E-09
0.18	0.15213E 01	0.16123E-01	0.11302E 01	0.55239E-09
0.20	0.15699E 01	0.18511E-01	0.11382E 01	0.63528E-09
0.22	0.16179E 01	0.21008E-01	0.11456E 01	0.72335E-09
0.24	0.16653E 01	0.23617E-01	0.11526E 01	0.81721E-09
0.26	0.17122E 01	0.26337E-01	0.11592E 01	0.91724E-09
0.28	0.17586E 01	0.29166E-01	0.11654E 01	0.10240E-08
0.30	0.18046E 01	0.32105E-01	0.11713E 01	0.11382E-08
0.32	0.18502E 01	0.35151E-01	0.11769E 01	0.12602E-08
0.34	0.18955E 01	0.38309E-01	0.11821E 01	0.13914E-08
0.36	0.19404E 01	0.41572E-01	0.11872E 01	0.15321E-08
0.38	0.19851E 01	0.44945E-01	0.11920E 01	0.16840E-08
0.40	0.20294E 01	0.48424E-01	0.11965E 01	0.18478E-08
0.42	0.20735E 01	0.52012E-01	0.12009E 01	0.20252E-08
0.44	0.21173E 01	0.55706E-01	0.12051E 01	0.22180E-08
0.46	0.21608E 01	0.59506E-01	0.12091E 01	0.24278E-08
0.48	0.22041E 01	0.63415E-01	0.12130E 01	0.26578E-08
0.50	0.22472E 01	0.67428E-01	0.12167E 01	0.29096E-08
0.52	0.22901E 01	0.71550E-01	0.12202E 01	0.31887E-08
0.54	0.23327E 01	0.75776E-01	0.12237E 01	0.34974E-08
0.56	0.23752E 01	0.80111E-01	0.12270E 01	0.38429E-08
0.58	0.24175E 01	0.84550E-01	0.12301E 01	0.42313E-08
0.60	0.24596E 01	0.89095E-01	0.12332E 01	0.46724E-08
0.62	0.25014E 01	0.93746E-01	0.12362E 01	0.51795E-08
0.64	0.25432E 01	0.98502E-01	0.12390E 01	0.57679E-08
0.66	0.25847E 01	0.10337E 00	0.12418E 01	0.64668E-08
0.68	0.26261E 01	0.10833E 00	0.12445E 01	0.73039E-08
0.70	0.26673E 01	0.11341E 00	0.12471E 01	0.83393E-08
0.72	0.27083E 01	0.11859E 00	0.12496E 01	0.96326E-08
0.74	0.27492E 01	0.12387E 00	0.12520E 01	0.11294E-07
0.76	0.27899E 01	0.12926E 00	0.12544E 01	0.13460E-07
0.78	0.28305E 01	0.13476E 00	0.12566E 01	0.16324E-07
0.80	0.28709E 01	0.14036E 00	0.12589E 01	0.20179E-07
0.82	0.29112E 01	0.14606E 00	0.12610E 01	0.25342E-07
0.84	0.29513E 01	0.15187E 00	0.12631E 01	0.32404E-07
0.86	0.29913E 01	0.15779E 00	0.12652E 01	0.41886E-07
0.88	0.30312E 01	0.16381E 00	0.12671E 01	0.54803E-07
0.90	0.30709E 01	0.16994E 00	0.12691E 01	0.72101E-07
0.92	0.31104E 01	0.17617E 00	0.12709E 01	0.95275E-07
0.94	0.31499E 01	0.18250E 00	0.12728E 01	0.12608E-06
0.96	0.31892E 01	0.18894E 00	0.12745E 01	0.16656E-06
0.98	0.32283E 01	0.19549E 00	0.12763E 01	0.21976E-06
1.00	0.32673E 01	0.20214E 00	0.12780E 01	0.28820E-06

Table 1 (cont.) (τ = 100.0)

	X_1	Y_1	X_2	Y_2
0.00	0.10000E 01	0.	0.10000E 01	0.
0.02	0.10355E 01	0.	0.10232E 01	0.
0.04	0.10614E 01	0.	0.10413E 01	0.
0.06	0.10836E 01	0.	0.10575E 01	0.
0.08	0.11035E 01	0.	0.10723E 01	0.
0.10	0.11216E 01	0.	0.10862E 01	0.
0.12	0.11384E 01	0.	0.10993E 01	0.
0.14	0.11541E 01	0.	0.11117E 01	0.
0.16	0.11687E 01	0.	0.11235E 01	0.
0.18	0.11826E 01	0.	0.11347E 01	0.
0.20	0.11957E 01	0.	0.11455E 01	0.
0.22	0.12082E 01	0.	0.11559E 01	0.
0.24	0.12200E 01	0.	0.11659E 01	0.
0.26	0.12314E 01	0.	0.11755E 01	0.
0.28	0.12422E 01	0.	0.11849E 01	0.
0.30	0.12526E 01	0.	0.11938E 01	0.
0.32	0.12625E 01	0.	0.12025E 01	0.
0.34	0.12721E 01	0.	0.12110E 01	0.
0.36	0.12813E 01	0.	0.12191E 01	0.
0.38	0.12902E 01	0.	0.12271E 01	0.
0.40	0.12987E 01	0.	0.12348E 01	0.
0.42	0.13070E 01	0.	0.12423E 01	0.
0.44	0.13150E 01	0.	0.12495E 01	0.
0.46	0.13227E 01	0.	0.12566E 01	0.
0.48	0.13301E 01	0.	0.12635E 01	0.
0.50	0.13373E 01	0.	0.12702E 01	0.
0.52	0.13443E 01	0.	0.12768E 01	0.
0.54	0.13511E 01	0.	0.12831E 01	0.
0.56	0.13577E 01	0.	0.12893E 01	0.
0.58	0.13641E 01	0.	0.12954E 01	0.
0.60	0.13703E 01	0.	0.13013E 01	0.
0.62	0.13763E 01	0.	0.13071E 01	0.
0.64	0.13822E 01	0.	0.13128E 01	0.
0.66	0.13879E 01	0.	0.13183E 01	0.
0.68	0.13935E 01	0.	0.13237E 01	0.
0.70	0.13989E 01	0.	0.13290E 01	0.
0.72	0.14041E 01	0.	0.13341E 01	0.
0.74	0.14093E 01	0.	0.13392E 01	0.
0.76	0.14143E 01	0.	0.13441E 01	0.
0.78	0.14192E 01	0.	0.13490E 01	0.
0.80	0.14239E 01	0.	0.13537E 01	0.
0.82	0.14286E 01	0.	0.13584E 01	0.
0.84	0.14331E 01	0.	0.13629E 01	0.
0.86	0.14375E 01	0.	0.13674E 01	0.
0.88	0.14419E 01	0.	0.13717E 01	0.
0.90	0.14461E 01	0.	0.13760E 01	0.
0.92	0.14502E 01	0.	0.13802E 01	0.
0.94	0.14543E 01	0.	0.13844E 01	0.
0.96	0.14582E 01	0.	0.13884E 01	0.
0.98	0.14621E 01	0.	0.13924E 01	0.
1.00	0.14659E 01	0.	0.13963E 01	0.

Table 1 (cont.)   (τ = 100.0)

μ	X_3	Y_3	X_4	Y_4
0.00	0.10000E 01	0.	0.10000E 01	0.
0.02	0.10831E 01	0.21471E-03	0.10292E 01	0.
0.04	0.11496E 01	0.45493E-03	0.10489E 01	0.
0.06	0.12105E 01	0.71877E-03	0.10649E 01	0.
0.08	0.12684E 01	0.10041E-02	0.10788E 01	0.
0.10	0.13241E 01	0.13104E-02	0.10911E 01	0.
0.12	0.13783E 01	0.16373E-02	0.11022E 01	0.
0.14	0.14313E 01	0.19837E-02	0.11123E 01	0.
0.16	0.14834E 01	0.23502E-02	0.11216E 01	0.
0.18	0.15347E 01	0.27355E-02	0.11302E 01	0.
0.20	0.15853E 01	0.31406E-02	0.11382E 01	0.
0.22	0.16353E 01	0.35643E-02	0.11456E 01	0.
0.24	0.16849E 01	0.40070E-02	0.11526E 01	0.
0.26	0.17340E 01	0.44684E-02	0.11592E 01	0.
0.28	0.17828E 01	0.49484E-02	0.11654E 01	0.
0.30	0.18313E 01	0.54471E-02	0.11713E 01	0.
0.32	0.18794E 01	0.59639E-02	0.11769E 01	0.
0.34	0.19273E 01	0.64997E-02	0.11821E 01	0.
0.36	0.19750E 01	0.70533E-02	0.11872E 01	0.
0.38	0.20224E 01	0.76256E-02	0.11920E 01	0.
0.40	0.20696E 01	0.82159E-02	0.11965E 01	0.
0.42	0.21166E 01	0.88245E-02	0.12009E 01	0.
0.44	0.21635E 01	0.94513E-02	0.12051E 01	0.
0.46	0.22102E 01	0.10096E-01	0.12091E 01	0.
0.48	0.22568E 01	0.10759E-01	0.12130E 01	0.
0.50	0.23032E 01	0.11440E-01	0.12167E 01	0.
0.52	0.23495E 01	0.12140E-01	0.12202E 01	0.
0.54	0.23957E 01	0.12857E-01	0.12237E 01	0.
0.56	0.24417E 01	0.13592E-01	0.12270E 01	0.
0.58	0.24877E 01	0.14345E-01	0.12301E 01	0.
0.60	0.25335E 01	0.15116E-01	0.12332E 01	0.
0.62	0.25793E 01	0.15905E-01	0.12362E 01	0.
0.64	0.26250E 01	0.16712E-01	0.12390E 01	0.
0.66	0.26705E 01	0.17538E-01	0.12418E 01	0.
0.68	0.27160E 01	0.18380E-01	0.12445E 01	0.
0.70	0.27615E 01	0.19241E-01	0.12471E 01	0.
0.72	0.28068E 01	0.20120E-01	0.12496E 01	0.
0.74	0.28521E 01	0.21017E-01	0.12520E 01	0.
0.76	0.28973E 01	0.21931E-01	0.12544E 01	0.
0.78	0.29424E 01	0.22863E-01	0.12566E 01	0.
0.80	0.29875E 01	0.23814E-01	0.12589E 01	0.
0.82	0.30325E 01	0.24781E-01	0.12610E 01	0.
0.84	0.30775E 01	0.25767E-01	0.12631E 01	0.
0.86	0.31224E 01	0.26771E-01	0.12652E 01	0.
0.88	0.31672E 01	0.27793E-01	0.12671E 01	0.
0.90	0.32120E 01	0.28832E-01	0.12691E 01	0.
0.92	0.32567E 01	0.29889E-01	0.12709E 01	0.
0.94	0.33014E 01	0.30964E-01	0.12728E 01	0.
0.96	0.33460E 01	0.32057E-01	0.12745E 01	0.
0.98	0.33906E 01	0.33168E-01	0.12763E 01	0.
1.00	0.34352E 01	0.34296E-01	0.12780E 01	0.

Table 2

ORDINARY MOMENTS OF X- AND Y-FUNCTIONS

$\tau = 0.15$

Order	X_1	Y_1	X_2	Y_2
0	0.11298E 01	0.76138E 00	0.10896E 01	0.72553E 00
1	0.57117E 00	0.45016E 00	0.54927E 00	0.42979E 00
2	0.38183E 00	0.31445E 00	0.36695E 00	0.30042E 00
3	0.28670E 00	0.24057E 00	0.27544E 00	0.22990E 00
4	0.22950E 00	0.19453E 00	0.22046E 00	0.18593E 00

Order	X_3	Y_3	X_4	Y_4
0	0.12461E 01	0.86845E 00	0.11045E 01	0.73655E 00
1	0.63485E 00	0.51064E 00	0.55713E 00	0.43628E 00
2	0.42521E 00	0.35603E 00	0.37223E 00	0.30494E 00
3	0.31951E 00	0.27216E 00	0.27942E 00	0.23336E 00
4	0.25587E 00	0.21997E 00	0.22364E 00	0.18872E 00

$\tau = 0.25$

Order	X_1	Y_1	X_2	Y_2
0	0.11718E 01	0.66872E 00	0.11218E 01	0.62802E 00
1	0.59655E 00	0.41315E 00	0.56880E 00	0.38901E 00
2	0.39973E 00	0.29426E 00	0.38073E 00	0.27735E 00
3	0.30046E 00	0.22737E 00	0.28604E 00	0.21441E 00
4	0.24066E 00	0.18493E 00	0.22905E 00	0.17443E 00

Order	X_3	Y_3	X_4	Y_4
0	0.13368E 01	0.81353E 00	0.11328E 01	0.63129E 00
1	0.68929E 00	0.49804E 00	0.57425E 00	0.39147E 00
2	0.46351E 00	0.35349E 00	0.38432E 00	0.27920E 00
3	0.34892E 00	0.27268E 00	0.28872E 00	0.21588E 00
4	0.27970E 00	0.22157E 00	0.23118E 00	0.17565E 00

$\tau = 0.50$

Order	X_1	Y_1	X_2	Y_2
0	0.12329E 01	0.50425E 00	0.11718E 01	0.46593E 00
1	0.63552E 00	0.33219E 00	0.60079E 00	0.30772E 00
2	0.42794E 00	0.24458E 00	0.40392E 00	0.22683E 00
3	0.32245E 00	0.19257E 00	0.30414E 00	0.17872E 00
4	0.25864E 00	0.15846E 00	0.24386E 00	0.14713E 00

Order	X_3	Y_3	X_4	Y_4
0	0.14936E 01	0.71089E 00	0.11682E 01	0.44660E 00
1	0.78819E 00	0.46045E 00	0.59695E 00	0.29685E 00
2	0.53476E 00	0.33644E 00	0.40077E 00	0.21939E 00
3	0.40435E 00	0.26383E 00	0.30156E 00	0.17309E 00
4	0.32497E 00	0.21657E 00	0.24169E 00	0.14260E 00

Table 2 (cont.)

$\tau = 0.70$

Order	X_1	Y_1	X_2	Y_2
0	0.12597E 01	0.41037E 00	0.11955E 01	0.37894E 00
1	0.65342E 00	0.27887E 00	0.61667E 00	0.25786E 00
2	0.44124E 00	0.20907E 00	0.41573E 00	0.19348E 00
3	0.33298E 00	0.16647E 00	0.31349E 00	0.15413E 00
4	0.26734E 00	0.13800E 00	0.25159E 00	0.12781E 00

Order	X_3	Y_3	X_4	Y_4
0	0.15819E 01	0.64842E 00	0.11810E 01	0.34514E 00
1	0.84610E 00	0.43090E 00	0.60561E 00	0.23763E 00
2	0.57740E 00	0.31958E 00	0.40724E 00	0.17723E 00
3	0.43794E 00	0.25293E 00	0.30669E 00	0.14318E 00
4	0.35263E 00	0.20888E 00	0.24593E 00	0.11894E 00

$\tau = 1.0$

Order	X_1	Y_1	X_2	Y_2
0	0.12825E 01	0.30582E 00	0.12173E 01	0.28571E 00
1	0.66917E 00	0.21445E 00	0.63171E 00	0.20007E 00
2	0.45315E 00	0.16398E 00	0.42712E 00	0.15293E 00
3	0.34252E 00	0.13226E 00	0.32262E 00	0.12334E 00
4	0.27528E 00	0.11062E 00	0.25919E 00	0.10316E 00

Order	X_3	Y_3	X_4	Y_4
0	0.16809E 01	0.57378E 00	0.11901E 01	0.23822E 00
1	0.91270E 00	0.39062E 00	0.61202E 00	0.17026E 00
2	0.62718E 00	0.29413E 00	0.41212E 00	0.13142E 00
3	0.47754E 00	0.23511E 00	0.31062E 00	0.10658E 00
4	0.38544E 00	0.19549E 00	0.24921E 00	0.89453E-01

$\tau = 2.0$

Order	X_1	Y_1	X_2	Y_2
0	0.13054E 01	0.12024E 00	0.12432E 01	0.12360E 00
1	0.68561E 00	0.88911E-01	0.65022E 00	0.90261E-01
2	0.46592E 00	0.70543E-01	0.44146E 00	0.71152E-01
3	0.35293E 00	0.58416E-01	0.33431E 00	0.58693E-01
4	0.28405E 00	0.49810E-01	0.26903E 00	0.49920E-01

Order	X_3	Y_3	X_4	Y_4
0	0.18733E 01	0.41306E 00	0.11968E 01	0.73695E-01
1	0.10461E 01	0.29080E 00	0.61688E 00	0.56484E-01
2	0.72892E 00	0.22412E 00	0.41594E 00	0.45721E-01
3	0.55958E 00	0.18213E 00	0.31375E 00	0.38340E-01
4	0.45409E 00	0.15328E 00	0.25186E 00	0.32973E-01

Table 2 (cont.)

$\tau = 4.0$

Order	X_1	Y_1	X_2	Y_2
0	0.13092E 01	0.19418E-01	0.12494E 01	0.26534E-01
1	0.68842E 00	0.14939E-01	0.65483E 00	0.19798E-01
2	0.46817E 00	0.12222E-01	0.44512E 00	0.15899E-01
3	0.35481E 00	0.10366E-01	0.33735E 00	0.13318E-01
4	0.28566E 00	0.90087E-02	0.27163E 00	0.11472E-01

Order	X_3	Y_3	X_4	Y_4
0	0.20366E 01	0.26000E 00	0.11973E 01	0.78645E-02
1	0.11621E 01	0.18554E 00	0.61732E 00	0.64199E-02
2	0.81680E 00	0.14451E 00	0.41630E 00	0.54470E-02
3	0.63297E 00	0.11841E 00	0.31406E 00	0.47359E-02
4	0.51609E 00	0.10031E 00	0.25213E 00	0.41903E-02

$\tau = 8.0$

Order	X_1	Y_1	X_2	Y_2
0	0.13093E 01	0.51107E-03	0.12497E 01	0.13404E-02
1	0.68849E 00	0.40355E-03	0.65507E 00	0.10070E-02
2	0.46823E 00	0.33731E-03	0.44531E 00	0.81431E-03
3	0.35486E 00	0.29129E-03	0.33751E 00	0.68651E-03
4	0.28571E 00	0.25705E-03	0.27177E 00	0.59478E-03

Order	X_3	Y_3	X_4	Y_4
0	0.21505E 01	0.14716E 00	0.11974E 01	0.10392E-03
1	0.12434E 01	0.10522E 00	0.61733E 00	0.89570E-04
2	0.88224E 00	0.82087E-01	0.41631E 00	0.79348E-04
3	0.68500E 00	0.67353E-01	0.31406E 00	0.71467E-04
4	0.56020E 00	0.57127E-01	0.25213E 00	0.65122E-04

$\tau = 16.0$

Order	X_1	Y_1	X_2	Y_2
0	0.13093E 01	0.34694E-06	0.12497E 01	0.35566E-05
1	0.68849E 00	0.27785E-06	0.65507E 00	0.26751E-05
2	0.46823E 00	0.23507E-06	0.44531E 00	0.21658E-05
3	0.35486E 00	0.20516E-06	0.33751E 00	0.18282E-05
4	0.28571E 00	0.18276E-06	0.27177E 00	0.15858E-05

Order	X_3	Y_3	X_4	Y_4
0	0.22191E 01	0.78575E-01	0.11974E 01	0.22440E-07
1	0.12925E 01	0.56184E-01	0.61733E 00	0.20246E-07
2	0.92051E 00	0.43834E-01	0.41631E 00	0.18617E-07
3	0.71639E 00	0.35967E-01	0.31406E 00	0.17308E-07
4	0.58683E 00	0.30507E-01	0.25213E 00	0.16211E-07

Table 2 (cont.)

τ = 100.0

Order	X_1	Y_1	X_2	Y_2
0	0.13093E 01	0.	0.12497E 01	0.
1	0.68849E 00	0.	0.65507E 00	0.
2	0.46823E 00	0.	0.44531E 00	0.
3	0.35486E 00	0.	0.33751E 00	0.
4	0.28571E 00	0.	0.27177E 00	0.

Order	X_3	Y_3	X_4	Y_4
0	0.22843E 01	0.13331E-01	0.11974E 01	0.
1	0.13391E 01	0.95325E-02	0.61733E 00	0.
2	0.95691E 00	0.74371E-02	0.41631E 00	0.
3	0.74626E 00	0.61024E-02	0.31406E 00	0.
4	0.61216E 00	0.51759E-02	0.25213E 00	0.

Table 3   K- AND L-FUNCTIONS   (τ = 0.15)

μ	K_1	K_2	K_3	K_4
0.00	0.	0.10000E 01	0.10000E 01	0.
0.02	0.57996E-02	0.10546E 01	0.10345E 01	0.26177E-02
0.04	0.11937E-01	0.10909E 01	0.10595E 01	0.49744E-02
0.06	0.18116E-01	0.11170E 01	0.10789E 01	0.69836E-02
0.08	0.24328E-01	0.11334E 01	0.10928E 01	0.84873E-02
0.10	0.30719E-01	0.11451E 01	0.11040E 01	0.97208E-02
0.12	0.37571E-01	0.11515E 01	0.11124E 01	0.10652E-01
0.14	0.44859E-01	0.11553E 01	0.11194E 01	0.11438E-01
0.16	0.52891E-01	0.11556E 01	0.11249E 01	0.12057E-01
0.18	0.61606E-01	0.11538E 01	0.11295E 01	0.12580E-01
0.20	0.70951E-01	0.11503E 01	0.11335E 01	0.13033E-01
0.22	0.81224E-01	0.11446E 01	0.11368E 01	0.13409E-01
0.24	0.92150E-01	0.11377E 01	0.11397E 01	0.13743E-01
0.26	0.10406E 00	0.11289E 01	0.11422E 01	0.14028E-01
0.28	0.11665E 00	0.11190E 01	0.11445E 01	0.14285E-01
0.30	0.13026E 00	0.11074E 01	0.11464E 01	0.14508E-01
0.32	0.14457E 00	0.10949E 01	0.11482E 01	0.14712E-01
0.34	0.15991E 00	0.10808E 01	0.11497E 01	0.14891E-01
0.36	0.17610E 00	0.10656E 01	0.11512E 01	0.15053E-01
0.38	0.19304E 00	0.10494E 01	0.11525E 01	0.15203E-01
0.40	0.21104E 00	0.10318E 01	0.11536E 01	0.15337E-01
0.42	0.22975E 00	0.10133E 01	0.11547E 01	0.15462E-01
0.44	0.24952E 00	0.99352E 00	0.11557E 01	0.15575E-01
0.46	0.27002E 00	0.97281E 00	0.11566E 01	0.15680E-01
0.48	0.29159E 00	0.95081E 00	0.11574E 01	0.15777E-01
0.50	0.31388E 00	0.92793E 00	0.11582E 01	0.15867E-01
0.52	0.33725E 00	0.90378E 00	0.11590E 01	0.15950E-01
0.54	0.36149E 00	0.87860E 00	0.11596E 01	0.16028E-01
0.56	0.38649E 00	0.85254E 00	0.11603E 01	0.16102E-01
0.58	0.41256E 00	0.82522E 00	0.11608E 01	0.16169E-01
0.60	0.43937E 00	0.79706E 00	0.11614E 01	0.16234E-01
0.62	0.46725E 00	0.76767E 00	0.11619E 01	0.16294E-01
0.64	0.49587E 00	0.73744E 00	0.11624E 01	0.16351E-01
0.66	0.52557E 00	0.70597E 00	0.11629E 01	0.16404E-01
0.68	0.55614E 00	0.67352E 00	0.11633E 01	0.16455E-01
0.70	0.58748E 00	0.64020E 00	0.11637E 01	0.16503E-01
0.72	0.61990E 00	0.60566E 00	0.11641E 01	0.16548E-01
0.74	0.65306E 00	0.57029E 00	0.11645E 01	0.16592E-01
0.76	0.68730E 00	0.53371E 00	0.11649E 01	0.16633E-01
0.78	0.72227E 00	0.49631E 00	0.11652E 01	0.16672E-01
0.80	0.75833E 00	0.45770E 00	0.11655E 01	0.16709E-01
0.82	0.79512E 00	0.41826E 00	0.11658E 01	0.16745E-01
0.84	0.83300E 00	0.37763E 00	0.11661E 01	0.16779E-01
0.86	0.87175E 00	0.33601E 00	0.11664E 01	0.16811E-01
0.88	0.91128E 00	0.29354E 00	0.11667E 01	0.16842E-01
0.90	0.95189E 00	0.24987E 00	0.11669E 01	0.16872E-01
0.92	0.99323E 00	0.20539E 00	0.11672E 01	0.16900E-01
0.94	0.10357E 01	0.15971E 00	0.11674E 01	0.16928E-01
0.96	0.10788E 01	0.11321E 00	0.11676E 01	0.16954E-01
0.98	0.11231E 01	0.65512E-01	0.11679E 01	0.16979E-01
1.00	0.11681E 01	0.17004E-01	0.11681E 01	0.17004E-01

Table 3 (cont.) (τ = 0.15)

μ	L_1	L_2	L_3	L_4
0.00	0.	-0.	0.	0.
0.02	0.44906E-02	0.24246E-01	0.18991E-01	0.22047E-02
0.04	0.90789E-02	0.78492E-01	0.66035E-01	0.43942E-02
0.06	0.13456E-01	0.15757E 00	0.13774E 00	0.63601E-02
0.08	0.17530E-01	0.24896E 00	0.22333E 00	0.78888E-02
0.10	0.21610E-01	0.33560E 00	0.30567E 00	0.91589E-02
0.12	0.25961E-01	0.41033E 00	0.37826E 00	0.10131E-01
0.14	0.30656E-01	0.47634E 00	0.44351E 00	0.10954E-01
0.16	0.35990E-01	0.53056E 00	0.49884E 00	0.11606E-01
0.18	0.41947E-01	0.57668E 00	0.54743E 00	0.12160E-01
0.20	0.48491E-01	0.61642E 00	0.59081E 00	0.12640E-01
0.22	0.55916E-01	0.64848E 00	0.62803E 00	0.13040E-01
0.24	0.63968E-01	0.67605E 00	0.66178E 00	0.13397E-01
0.26	0.72974E-01	0.69780E 00	0.69113E 00	0.13701E-01
0.28	0.82644E-01	0.71610E 00	0.71797E 00	0.13976E-01
0.30	0.93309E-01	0.72986E 00	0.74160E 00	0.14214E-01
0.32	0.10466E 00	0.74086E 00	0.76340E 00	0.14432E-01
0.34	0.11703E 00	0.74818E 00	0.78280E 00	0.14624E-01
0.36	0.13025E 00	0.75281E 00	0.80057E 00	0.14798E-01
0.38	0.14420E 00	0.75524E 00	0.81705E 00	0.14959E-01
0.40	0.15919E 00	0.75482E 00	0.83193E 00	0.15103E-01
0.42	0.17490E 00	0.75260E 00	0.84587E 00	0.15237E-01
0.44	0.19166E 00	0.74787E 00	0.85855E 00	0.15358E-01
0.46	0.20915E 00	0.74156E 00	0.87047E 00	0.15472E-01
0.48	0.22769E 00	0.73301E 00	0.88140E 00	0.15576E-01
0.50	0.24696E 00	0.72302E 00	0.89172E 00	0.15673E-01
0.52	0.26730E 00	0.71101E 00	0.90123E 00	0.15763E-01
0.54	0.28850E 00	0.69741E 00	0.91016E 00	0.15846E-01
0.56	0.31047E 00	0.68248E 00	0.91861E 00	0.15925E-01
0.58	0.33351E 00	0.66574E 00	0.92647E 00	0.15998E-01
0.60	0.35727E 00	0.64780E 00	0.93394E 00	0.16068E-01
0.62	0.38211E 00	0.62815E 00	0.94091E 00	0.16133E-01
0.64	0.40768E 00	0.60736E 00	0.94757E 00	0.16194E-01
0.66	0.43433E 00	0.58495E 00	0.95381E 00	0.16252E-01
0.68	0.46186E 00	0.56125E 00	0.95973E 00	0.16306E-01
0.70	0.49015E 00	0.53644E 00	0.96539E 00	0.16358E-01
0.72	0.51952E 00	0.51010E 00	0.97071E 00	0.16407E-01
0.74	0.54962E 00	0.48273E 00	0.97583E 00	0.16454E-01
0.76	0.58080E 00	0.45387E 00	0.98065E 00	0.16498E-01
0.78	0.61272E 00	0.42402E 00	0.98530E 00	0.16540E-01
0.80	0.64572E 00	0.39273E 00	0.98969E 00	0.16581E-01
0.82	0.67945E 00	0.36046E 00	0.99393E 00	0.16619E-01
0.84	0.71427E 00	0.32679E 00	0.99794E 00	0.16656E-01
0.86	0.74996E 00	0.29198E 00	0.10018E 01	0.16691E-01
0.88	0.78643E 00	0.25619E 00	0.10055E 01	0.16724E-01
0.90	0.82397E 00	0.21902E 00	0.10090E 01	0.16756E-01
0.92	0.86226E 00	0.18094E 00	0.10125E 01	0.16787E-01
0.94	0.90167E 00	0.14150E 00	0.10157E 01	0.16817E-01
0.96	0.94172E 00	0.10115E 00	0.10189E 01	0.16845E-01
0.98	0.98291E 00	0.59475E-01	0.10219E 01	0.16872E-01
1.00	0.10248E 01	0.16899E-01	0.10248E 01	0.16899E-01

Table 3 (cont.) (τ = 0.25)

μ	K_1	K_2	K_3	K_4
0.00	0.	0.10000E 01	0.10000E 01	0.
0.02	0.65367E-02	0.10564E 01	0.10357E 01	0.29729E-02
0.04	0.13745E-01	0.10957E 01	0.10629E 01	0.58304E-02
0.06	0.21372E-01	0.11264E 01	0.10858E 01	0.85137E-02
0.08	0.29376E-01	0.11488E 01	0.11043E 01	0.10849E-01
0.10	0.37600E-01	0.11668E 01	0.11204E 01	0.12926E-01
0.12	0.46219E-01	0.11791E 01	0.11335E 01	0.14654E-01
0.14	0.55186E-01	0.11882E 01	0.11450E 01	0.16181E-01
0.16	0.64762E-01	0.11933E 01	0.11544E 01	0.17458E-01
0.18	0.74922E-01	0.11957E 01	0.11627E 01	0.18574E-01
0.20	0.85636E-01	0.11959E 01	0.11699E 01	0.19567E-01
0.22	0.97191E-01	0.11934E 01	0.11762E 01	0.20418E-01
0.24	0.10935E 00	0.11892E 01	0.11818E 01	0.21189E-01
0.26	0.12244E 00	0.11827E 01	0.11866E 01	0.21859E-01
0.28	0.13619E 00	0.11748E 01	0.11911E 01	0.22473E-01
0.30	0.15092E 00	0.11650E 01	0.11950E 01	0.23013E-01
0.32	0.16635E 00	0.11538E 01	0.11986E 01	0.23512E-01
0.34	0.18279E 00	0.11409E 01	0.12018E 01	0.23957E-01
0.36	0.20009E 00	0.11265E 01	0.12047E 01	0.24364E-01
0.38	0.21814E 00	0.11110E 01	0.12074E 01	0.24742E-01
0.40	0.23725E 00	0.10938E 01	0.12098E 01	0.25084E-01
0.42	0.25709E 00	0.10756E 01	0.12121E 01	0.25404E-01
0.44	0.27802E 00	0.10558E 01	0.12142E 01	0.25696E-01
0.46	0.29968E 00	0.10350E 01	0.12162E 01	0.25970E-01
0.48	0.32244E 00	0.10127E 01	0.12180E 01	0.26222E-01
0.50	0.34594E 00	0.98944E 00	0.12197E 01	0.26459E-01
0.52	0.37054E 00	0.96471E 00	0.12212E 01	0.26679E-01
0.54	0.39605E 00	0.93880E 00	0.12227E 01	0.26885E-01
0.56	0.42234E 00	0.91190E 00	0.12241E 01	0.27080E-01
0.58	0.44975E 00	0.88359E 00	0.12254E 01	0.27261E-01
0.60	0.47791E 00	0.85433E 00	0.12266E 01	0.27434E-01
0.62	0.50719E 00	0.82369E 00	0.12278E 01	0.27595E-01
0.64	0.53722E 00	0.79212E 00	0.12289E 01	0.27749E-01
0.66	0.56838E 00	0.75918E 00	0.12299E 01	0.27894E-01
0.68	0.60045E 00	0.72514E 00	0.12309E 01	0.28031E-01
0.70	0.63332E 00	0.69014E 00	0.12318E 01	0.28162E-01
0.72	0.66732E 00	0.65379E 00	0.12327E 01	0.28285E-01
0.74	0.70208E 00	0.61653E 00	0.12335E 01	0.28404E-01
0.76	0.73797E 00	0.57793E 00	0.12343E 01	0.28516E-01
0.78	0.77462E 00	0.53842E 00	0.12351E 01	0.28624E-01
0.80	0.81240E 00	0.49759E 00	0.12358E 01	0.28726E-01
0.82	0.85096E 00	0.45586E 00	0.12365E 01	0.28824E-01
0.84	0.89064E 00	0.41281E 00	0.12372E 01	0.28917E-01
0.86	0.93124E 00	0.36868E 00	0.12378E 01	0.29007E-01
0.88	0.97264E 00	0.32363E 00	0.12384E 01	0.29093E-01
0.90	0.10152E 01	0.27725E 00	0.12390E 01	0.29175E-01
0.92	0.10585E 01	0.22999E 00	0.12396E 01	0.29255E-01
0.94	0.11029E 01	0.18142E 00	0.12401E 01	0.29331E-01
0.96	0.11481E 01	0.13196E 00	0.12406E 01	0.29404E-01
0.98	0.11945E 01	0.81193E-01	0.12411E 01	0.29474E-01
1.00	0.12416E 01	0.29543E-01	0.12416E 01	0.29542E-01

Table 3 (cont.) (τ = 0.25)

μ	L_1	L_2	L_3	L_4
0.00	0.	-0.	0.	0.
0.02	0.45562E-02	0.16796E-01	0.14277E-01	0.22299E-02
0.04	0.95241E-02	0.40218E-01	0.33745E-01	0.46146E-02
0.06	0.14654E-01	0.75015E-01	0.63576E-01	0.70238E-02
0.08	0.19756E-01	0.12658E 00	0.10973E 00	0.92713E-02
0.10	0.24803E-01	0.18440E 00	0.16256E 00	0.11327E-01
0.12	0.29886E-01	0.24478E 00	0.21909E 00	0.13089E-01
0.14	0.35116E-01	0.30341E 00	0.27489E 00	0.14665E-01
0.16	0.40706E-01	0.35745E 00	0.32766E 00	0.16001E-01
0.18	0.46722E-01	0.40677E 00	0.37701E 00	0.17179E-01
0.20	0.53173E-01	0.45184E 00	0.42326E 00	0.18233E-01
0.22	0.60326E-01	0.49101E 00	0.46519E 00	0.19143E-01
0.24	0.68008E-01	0.52637E 00	0.50436E 00	0.19970E-01
0.26	0.76522E-01	0.55644E 00	0.53976E 00	0.20692E-01
0.28	0.85637E-01	0.58318E 00	0.57285E 00	0.21355E-01
0.30	0.95670E-01	0.60535E 00	0.60283E 00	0.21941E-01
0.32	0.10635E 00	0.62464E 00	0.63093E 00	0.22483E-01
0.34	0.11800E 00	0.63993E 00	0.65651E 00	0.22967E-01
0.36	0.13047E 00	0.65237E 00	0.68028E 00	0.23411E-01
0.38	0.14366E 00	0.66231E 00	0.70259E 00	0.23824E-01
0.40	0.15788E 00	0.66902E 00	0.72306E 00	0.24198E-01
0.42	0.17281E 00	0.67367E 00	0.74240E 00	0.24549E-01
0.44	0.18879E 00	0.67544E 00	0.76023E 00	0.24868E-01
0.46	0.20550E 00	0.67536E 00	0.77713E 00	0.25170E-01
0.48	0.22328E 00	0.67268E 00	0.79278E 00	0.25446E-01
0.50	0.24179E 00	0.66832E 00	0.80765E 00	0.25708E-01
0.52	0.26135E 00	0.66158E 00	0.82148E 00	0.25949E-01
0.54	0.28187E 00	0.65299E 00	0.83455E 00	0.26176E-01
0.56	0.30313E 00	0.64284E 00	0.84699E 00	0.26392E-01
0.58	0.32549E 00	0.63056E 00	0.85863E 00	0.26592E-01
0.60	0.34860E 00	0.61668E 00	0.86977E 00	0.26782E-01
0.62	0.37281E 00	0.60120E 00	0.88021E 00	0.26960E-01
0.64	0.39778E 00	0.58421E 00	0.89023E 00	0.27131E-01
0.66	0.42386E 00	0.56531E 00	0.89965E 00	0.27290E-01
0.68	0.45084E 00	0.54492E 00	0.90864E 00	0.27442E-01
0.70	0.47862E 00	0.52324E 00	0.91727E 00	0.27587E-01
0.72	0.50751E 00	0.49978E 00	0.92542E 00	0.27724E-01
0.74	0.53717E 00	0.47514E 00	0.93328E 00	0.27855E-01
0.76	0.56794E 00	0.44879E 00	0.94072E 00	0.27979E-01
0.78	0.59948E 00	0.42129E 00	0.94790E 00	0.28098E-01
0.80	0.63214E 00	0.39214E 00	0.95471E 00	0.28211E-01
0.82	0.66557E 00	0.36188E 00	0.96130E 00	0.28320E-01
0.84	0.70012E 00	0.33001E 00	0.96756E 00	0.28424E-01
0.86	0.73559E 00	0.29684E 00	0.97359E 00	0.28523E-01
0.88	0.77186E 00	0.26256E 00	0.97941E 00	0.28619E-01
0.90	0.80926E 00	0.22672E 00	0.98497E 00	0.28710E-01
0.92	0.84743E 00	0.18984E 00	0.99036E 00	0.28799E-01
0.94	0.88672E 00	0.15143E 00	0.99551E 00	0.28883E-01
0.96	0.92678E 00	0.11200E 00	0.10005E 01	0.28965E-01
0.98	0.96798E 00	0.71063E-01	0.10053E 01	0.29043E-01
1.00	0.10099E 01	0.29118E-01	0.10099E 01	0.29118E-01

Table 3 (cont.) (τ = 0.50)

	K_1	K_2	K_3	K_4
0.00	0.	0.10000E 01	0.10000E 01	0.
0.02	0.78556E-02	0.10584E 01	0.10375E 01	0.36109E-02
0.04	0.16598E-01	0.11002E 01	0.10669E 01	0.71839E-02
0.06	0.26034E-01	0.11341E 01	0.10924E 01	0.10695E-01
0.08	0.36349E-01	0.11611E 01	0.11148E 01	0.14074E-01
0.10	0.47194E-01	0.11845E 01	0.11356E 01	0.17331E-01
0.12	0.58755E-01	0.12031E 01	0.11542E 01	0.20374E-01
0.14	0.70774E-01	0.12187E 01	0.11715E 01	0.23257E-01
0.16	0.83466E-01	0.12304E 01	0.11870E 01	0.25897E-01
0.18	0.96722E-01	0.12392E 01	0.12011E 01	0.28347E-01
0.20	0.11049E 00	0.12455E 01	0.12142E 01	0.30637E-01
0.22	0.12503E 00	0.12487E 01	0.12260E 01	0.32717E-01
0.24	0.14011E 00	0.12498E 01	0.12370E 01	0.34667E-01
0.26	0.15603E 00	0.12482E 01	0.12469E 01	0.36440E-01
0.28	0.17255E 00	0.12448E 01	0.12561E 01	0.38106E-01
0.30	0.18998E 00	0.12389E 01	0.12645E 01	0.39626E-01
0.32	0.20805E 00	0.12314E 01	0.12724E 01	0.41058E-01
0.34	0.22710E 00	0.12216E 01	0.12796E 01	0.42370E-01
0.36	0.24696E 00	0.12100E 01	0.12864E 01	0.43597E-01
0.38	0.26755E 00	0.11970E 01	0.12927E 01	0.44753E-01
0.40	0.28918E 00	0.11819E 01	0.12985E 01	0.45821E-01
0.42	0.31152E 00	0.11654E 01	0.13040E 01	0.46834E-01
0.44	0.33494E 00	0.11470E 01	0.13091E 01	0.47773E-01
0.46	0.35910E 00	0.11273E 01	0.13140E 01	0.48666E-01
0.48	0.38436E 00	0.11058E 01	0.13185E 01	0.49497E-01
0.50	0.41038E 00	0.10831E 01	0.13227E 01	0.50290E-01
0.52	0.43751E 00	0.10585E 01	0.13267E 01	0.51030E-01
0.54	0.46558E 00	0.10325E 01	0.13305E 01	0.51732E-01
0.56	0.49444E 00	0.10053E 01	0.13341E 01	0.52402E-01
0.58	0.52446E 00	0.97644E 00	0.13375E 01	0.53031E-01
0.60	0.55526E 00	0.94639E 00	0.13408E 01	0.53634E-01
0.62	0.58722E 00	0.91468E 00	0.13438E 01	0.54202E-01
0.64	0.61997E 00	0.88184E 00	0.13467E 01	0.54748E-01
0.66	0.65389E 00	0.84737E 00	0.13495E 01	0.55264E-01
0.68	0.68876E 00	0.81158E 00	0.13521E 01	0.55756E-01
0.70	0.72448E 00	0.77465E 00	0.13547E 01	0.56230E-01
0.72	0.76137E 00	0.73612E 00	0.13571E 01	0.56679E-01
0.74	0.79908E 00	0.69651E 00	0.13594E 01	0.57112E-01
0.76	0.83797E 00	0.65533E 00	0.13616E 01	0.57523E-01
0.78	0.87768E 00	0.61307E 00	0.13637E 01	0.57920E-01
0.80	0.91857E 00	0.56926E 00	0.13657E 01	0.58298E-01
0.82	0.96029E 00	0.52439E 00	0.13677E 01	0.58664E-01
0.84	0.10032E 01	0.47797E 00	0.13696E 01	0.59013E-01
0.86	0.10471E 01	0.43030E 00	0.13713E 01	0.59349E-01
0.88	0.10918E 01	0.38154E 00	0.13731E 01	0.59675E-01
0.90	0.11378E 01	0.33125E 00	0.13747E 01	0.59986E-01
0.92	0.11846E 01	0.27992E 00	0.13763E 01	0.60287E-01
0.94	0.12325E 01	0.22707E 00	0.13779E 01	0.60576E-01
0.96	0.12814E 01	0.17319E 00	0.13794E 01	0.60857E-01
0.98	0.13314E 01	0.11780E 00	0.13808E 01	0.61126E-01
1.00	0.13822E 01	0.61388E-01	0.13822E 01	0.61388E-01

Table 3 (cont.) (τ = 0.50)

	L_1	L_2	L_3	L_4
0.00	0.	-0.	0.	0.
0.02	0.45218E-02	0.10266E-01	0.10496E-01	0.22045E-02
0.04	0.95355E-02	0.22069E-01	0.22025E-01	0.45928E-02
0.06	0.14924E-01	0.35482E-01	0.34701E-01	0.71219E-02
0.08	0.20743E-01	0.52537E-01	0.50335E-01	0.98002E-02
0.10	0.26772E-01	0.73129E-01	0.69112E-01	0.12525E-01
0.12	0.33025E-01	0.99118E-01	0.92952E-01	0.15241E-01
0.14	0.39398E-01	0.12818E 00	0.11987E 00	0.17900E-01
0.16	0.45960E-01	0.16048E 00	0.15032E 00	0.20434E-01
0.18	0.52720E-01	0.19400E 00	0.18248E 00	0.22841E-01
0.20	0.59693E-01	0.22792E 00	0.21559E 00	0.25131E-01
0.22	0.67040E-01	0.26139E 00	0.24919E 00	0.27254E-01
0.24	0.74695E-01	0.29402E 00	0.28265E 00	0.29266E-01
0.26	0.82872E-01	0.32489E 00	0.31553E 00	0.31123E-01
0.28	0.91455E-01	0.35430E 00	0.34776E 00	0.32881E-01
0.30	0.10069E 00	0.38131E 00	0.37888E 00	0.34502E-01
0.32	0.11041E 00	0.40656E 00	0.40913E 00	0.36038E-01
0.34	0.12090E 00	0.42914E 00	0.43807E 00	0.37457E-01
0.36	0.13205E 00	0.44950E 00	0.46590E 00	0.38791E-01
0.38	0.14380E 00	0.46791E 00	0.49278E 00	0.40054E-01
0.40	0.15643E 00	0.48358E 00	0.51833E 00	0.41226E-01
0.42	0.16969E 00	0.49740E 00	0.54300E 00	0.42342E-01
0.44	0.18390E 00	0.50855E 00	0.56641E 00	0.43380E-01
0.46	0.19876E 00	0.51791E 00	0.58902E 00	0.44371E-01
0.48	0.21462E 00	0.52468E 00	0.61047E 00	0.45297E-01
0.50	0.23117E 00	0.52972E 00	0.63118E 00	0.46181E-01
0.52	0.24875E 00	0.53228E 00	0.65084E 00	0.47010E-01
0.54	0.26718E 00	0.53285E 00	0.66971E 00	0.47798E-01
0.56	0.28637E 00	0.53173E 00	0.68791E 00	0.48551E-01
0.58	0.30664E 00	0.52827E 00	0.70522E 00	0.49260E-01
0.60	0.32764E 00	0.52325E 00	0.72195E 00	0.49941E-01
0.62	0.34975E 00	0.51598E 00	0.73789E 00	0.50584E-01
0.64	0.37261E 00	0.50720E 00	0.75331E 00	0.51202E-01
0.66	0.39658E 00	0.49626E 00	0.76802E 00	0.51786E-01
0.68	0.42147E 00	0.48359E 00	0.78218E 00	0.52346E-01
0.70	0.44717E 00	0.46943E 00	0.79588E 00	0.52885E-01
0.72	0.47401E 00	0.45320E 00	0.80897E 00	0.53396E-01
0.74	0.50163E 00	0.43560E 00	0.82166E 00	0.53890E-01
0.76	0.53040E 00	0.41599E 00	0.83381E 00	0.54360E-01
0.78	0.55996E 00	0.39504E 00	0.84560E 00	0.54814E-01
0.80	0.59068E 00	0.37214E 00	0.85690E 00	0.55247E-01
0.82	0.62219E 00	0.34793E 00	0.86787E 00	0.55666E-01
0.84	0.65488E 00	0.32183E 00	0.87840E 00	0.56066E-01
0.86	0.68852E 00	0.29420E 00	0.88859E 00	0.56452E-01
0.88	0.72300E 00	0.26525E 00	0.89848E 00	0.56825E-01
0.90	0.75866E 00	0.23447E 00	0.90799E 00	0.57183E-01
0.92	0.79512E 00	0.20245E 00	0.91725E 00	0.57530E-01
0.94	0.83277E 00	0.16863E 00	0.92616E 00	0.57863E-01
0.96	0.87123E 00	0.13360E 00	0.93484E 00	0.58186E-01
0.98	0.91089E 00	0.96794E-01	0.94320E 00	0.58496E-01
1.00	0.95135E 00	0.58799E-01	0.95135E 00	0.58798E-01

Table 3 (cont.) (τ = 0.70)

	K_1	K_2	K_3	K_4
0.00	0.	0.10000E 01	0.10000E 01	0.
0.02	0.86240E-02	0.10592E 01	0.10385E 01	0.39831E-02
0.04	0.18230E-01	0.11020E 01	0.10669E 01	0.79611E-02
0.06	0.28614E-01	0.11368E 01	0.10956E 01	0.11906E-01
0.08	0.40012E-01	0.11651E 01	0.11193E 01	0.15769E-01
0.10	0.52058E-01	0.11899E 01	0.11416E 01	0.19556E-01
0.12	0.65006E-01	0.12102E 01	0.11620E 01	0.23207E-01
0.14	0.78535E-01	0.12278E 01	0.11813E 01	0.26747E-01
0.16	0.92889E-01	0.12417E 01	0.11991E 01	0.30108E-01
0.18	0.10790E 00	0.12528E 01	0.12158E 01	0.33313E-01
0.20	0.12348E 00	0.12616E 01	0.12315E 01	0.36379E-01
0.22	0.13988E 00	0.12672E 01	0.12460E 01	0.39249E-01
0.24	0.15684E 00	0.12709E 01	0.12597E 01	0.41990E-01
0.26	0.17465E 00	0.12717E 01	0.12724E 01	0.44546E-01
0.28	0.19306E 00	0.12706E 01	0.12845E 01	0.46987E-01
0.30	0.21237E 00	0.12670E 01	0.12956E 01	0.49261E-01
0.32	0.23232E 00	0.12616E 01	0.13061E 01	0.51430E-01
0.34	0.25321E 00	0.12539E 01	0.13159E 01	0.53454E-01
0.36	0.27490E 00	0.12442E 01	0.13252E 01	0.55369E-01
0.38	0.29729E 00	0.12329E 01	0.13340E 01	0.57195E-01
0.40	0.32071E 00	0.12194E 01	0.13421E 01	0.58904E-01
0.42	0.34483E 00	0.12044E 01	0.13500E 01	0.60538E-01
0.44	0.37001E 00	0.11873E 01	0.13573E 01	0.62071E-01
0.46	0.39592E 00	0.11688E 01	0.13642E 01	0.63540E-01
0.48	0.42291E 00	0.11483E 01	0.13708E 01	0.64720E-01
0.50	0.45066E 00	0.11264E 01	0.13771E 01	0.66245E-01
0.52	0.47952E 00	0.11026E 01	0.13830E 01	0.67494E-01
0.54	0.50930E 00	0.10772E 01	0.13886E 01	0.68685E-01
0.56	0.53988E 00	0.10505E 01	0.13940E 01	0.69828E-01
0.58	0.57162E 00	0.10219E 01	0.13990E 01	0.70910E-01
0.60	0.60414E 00	0.99204E 00	0.14039E 01	0.71952E-01
0.62	0.63782E 00	0.96037E 00	0.14086E 01	0.72939E-01
0.64	0.67231E 00	0.92748E 00	0.14130E 01	0.73892E-01
0.66	0.70797E 00	0.89280E 00	0.14173E 01	0.74796E-01
0.68	0.74460E 00	0.85669E 00	0.14213E 01	0.75664E-01
0.70	0.78208E 00	0.81933E 00	0.14252E 01	0.76502E-01
0.72	0.82076E 00	0.78022E 00	0.14290E 01	0.77300E-01
0.74	0.86026E 00	0.73994E 00	0.14326E 01	0.78073E-01
0.76	0.90097E 00	0.69795E 00	0.14360E 01	0.78809E-01
0.78	0.94250E 00	0.65479E 00	0.14393E 01	0.79523E-01
0.80	0.98526E 00	0.60994E 00	0.14425E 01	0.80205E-01
0.82	0.10288E 01	0.56394E 00	0.14456E 01	0.80867E-01
0.84	0.10737E 01	0.51626E 00	0.14485E 01	0.81500E-01
0.86	0.11195E 01	0.46723E 00	0.14514E 01	0.82112E-01
0.88	0.11661E 01	0.41701E 00	0.14541E 01	0.82705E-01
0.90	0.12141E 01	0.36513E 00	0.14567E 01	0.83275E-01
0.92	0.12628E 01	0.31214E 00	0.14593E 01	0.83828E-01
0.94	0.13128E 01	0.25750E 00	0.14618E 01	0.84360E-01
0.96	0.13637E 01	0.20174E 00	0.14642E 01	0.84877E-01
0.98	0.14158E 01	0.14435E 00	0.14665E 01	0.85375E-01
1.00	0.14687E 01	0.85860E-01	0.14687E 01	0.85859E-01

Table 3 (cont.) (τ = 0.70)

r	L_1	L_2	L_3	L_4
0.00	0.	-0.	0.	0.
0.02	0.44227E-02	0.77042E-02	0.89070E-02	0.21526E-02
0.04	0.93374E-02	0.16449E-01	0.18600E-01	0.44847E-02
0.06	0.14637E-01	0.26091E-01	0.28971E-01	0.69592E-02
0.08	0.20427E-01	0.37218E-01	0.40488E-01	0.96140E-02
0.10	0.26510E-01	0.49745E-01	0.53153E-01	0.12371E-01
0.12	0.32967E-01	0.64901E-01	0.68103E-01	0.15236E-01
0.14	0.39634E-01	0.82093E-01	0.84925E-01	0.18138E-01
0.16	0.46576E-01	0.10217E 00	0.10453E 00	0.21049E-01
0.18	0.53727E-01	0.12418E 00	0.12616E 00	0.23923E-01
0.20	0.61070E-01	0.14754E 00	0.14932E 00	0.26748E-01
0.22	0.68709E-01	0.17216E 00	0.17416E 00	0.29478E-01
0.24	0.76578E-01	0.19717E 00	0.19981E 00	0.32133E-01
0.26	0.84834E-01	0.22227E 00	0.22624E 00	0.34668E-01
0.28	0.93391E-01	0.24707E 00	0.25295E 00	0.37118E-01
0.30	0.10244E 00	0.27107E 00	0.27979E 00	0.39441E-01
0.32	0.11187E 00	0.29428E 00	0.30652E 00	0.41679E-01
0.34	0.12189E 00	0.31610E 00	0.33295E 00	0.43796E-01
0.36	0.13245E 00	0.33659E 00	0.35898E 00	0.45817E-01
0.38	0.14351E 00	0.35581E 00	0.38461E 00	0.47758E-01
0.40	0.15530E 00	0.37315E 00	0.40958E 00	0.49590E-01
0.42	0.16763E 00	0.38913E 00	0.43406E 00	0.51354E-01
0.44	0.18077E 00	0.40305E 00	0.45779E 00	0.53020E-01
0.46	0.19449E 00	0.41554E 00	0.48099E 00	0.54624E-01
0.48	0.20909E 00	0.42588E 00	0.50341E 00	0.56141E-01
0.50	0.22432E 00	0.43475E 00	0.52529E 00	0.57603E-01
0.52	0.24048E 00	0.44144E 00	0.54639E 00	0.58988E-01
0.54	0.25742E 00	0.44636E 00	0.56687E 00	0.60314E-01
0.56	0.27507E 00	0.44973E 00	0.58680E 00	0.61592E-01
0.58	0.29372E 00	0.45092E 00	0.60600E 00	0.62805E-01
0.60	0.31306E 00	0.45065E 00	0.62471E 00	0.63976E-01
0.62	0.33344E 00	0.44822E 00	0.64272E 00	0.65091E-01
0.64	0.35455E 00	0.44434E 00	0.66027E 00	0.66168E-01
0.66	0.37672E 00	0.43833E 00	0.67716E 00	0.67195E-01
0.68	0.39977E 00	0.43063E 00	0.69355E 00	0.68182E-01
0.70	0.42361E 00	0.42143E 00	0.70950E 00	0.69136E-01
0.72	0.44855E 00	0.41016E 00	0.72487E 00	0.70049E-01
0.74	0.47426E 00	0.39750E 00	0.73985E 00	0.70933E-01
0.76	0.50109E 00	0.38280E 00	0.75429E 00	0.71779E-01
0.78	0.52869E 00	0.36673E 00	0.76837E 00	0.72599E-01
0.80	0.55745E 00	0.34865E 00	0.78195E 00	0.73385E-01
0.82	0.58698E 00	0.32923E 00	0.79521E 00	0.74148E-01
0.84	0.61768E 00	0.30784E 00	0.80799E 00	0.74879E-01
0.86	0.64933E 00	0.28486E 00	0.82042E 00	0.75587E-01
0.88	0.68181E 00	0.26051E 00	0.83255E 00	0.76274E-01
0.90	0.71548E 00	0.23423E 00	0.84426E 00	0.76935E-01
0.92	0.74995E 00	0.20666E 00	0.85570E 00	0.77578E-01
0.94	0.78561E 00	0.17719E 00	0.86676E 00	0.78196E-01
0.96	0.82209E 00	0.14644E 00	0.87757E 00	0.78798E-01
0.98	0.85976E 00	0.11382E 00	0.88803E 00	0.79378E-01
1.00	0.89826E 00	0.79944E-01	0.89825E 00	0.79944E-01

Table 3 (cont.) (τ = 1.0)

μ	K_1	K_2	K_3	K_4
0.00	0.	0.10000E 01	0.10000E 01	0.
0.02	0.95012E-02	0.10599E 01	0.10395E 01	0.44080E-02
0.04	0.20090E-01	0.11035E 01	0.10711E 01	0.88468E-02
0.06	0.31542E-01	0.11392E 01	0.10990E 01	0.13282E-01
0.08	0.44127E-01	0.11684E 01	0.11240E 01	0.17677E-01
0.10	0.57448E-01	0.11943E 01	0.11477E 01	0.22027E-01
0.12	0.71816E-01	0.12158E 01	0.11697E 01	0.26295E-01
0.14	0.86873E-01	0.12348E 01	0.11908E 01	0.30495E-01
0.16	0.10292E 00	0.12502E 01	0.12105E 01	0.34580E-01
0.18	0.11974E 00	0.12630E 01	0.12293E 01	0.38558E-01
0.20	0.13724E 00	0.12736E 01	0.12474E 01	0.42436E-01
0.22	0.15567E 00	0.12812E 01	0.12644E 01	0.46165E-01
0.24	0.17474E 00	0.12869E 01	0.12807E 01	0.49789E-01
0.26	0.19473E 00	0.12898E 01	0.12961E 01	0.53256E-01
0.28	0.21536E 00	0.12909E 01	0.13109E 01	0.56616E-01
0.30	0.23693E 00	0.12893E 01	0.13249E 01	0.59820E-01
0.32	0.25916E 00	0.12861E 01	0.13383E 01	0.62919E-01
0.34	0.28234E 00	0.12803E 01	0.13509E 01	0.65868E-01
0.36	0.30633E 00	0.12726E 01	0.13631E 01	0.68701E-01
0.38	0.33103E 00	0.12632E 01	0.13747E 01	0.71434E-01
0.40	0.35674E 00	0.12515E 01	0.13857E 01	0.74034E-01
0.42	0.38316E 00	0.12383E 01	0.13963E 01	0.76546E-01
0.44	0.41061E 00	0.12228E 01	0.14063E 01	0.78936E-01
0.46	0.43879E 00	0.12059E 01	0.14159E 01	0.81247E-01
0.48	0.46804E 00	0.11868E 01	0.14251E 01	0.83446E-01
0.50	0.49804E 00	0.11662E 01	0.14339E 01	0.85574E-01
0.52	0.52913E 00	0.11436E 01	0.14423E 01	0.87601E-01
0.54	0.56114E 00	0.11192E 01	0.14504E 01	0.89551E-01
0.56	0.59395E 00	0.10935E 01	0.14582E 01	0.91437E-01
0.58	0.62789E 00	0.10657E 01	0.14656E 01	0.93237E-01
0.60	0.66262E 00	0.10366E 01	0.14728E 01	0.94982E-01
0.62	0.69850E 00	0.10055E 01	0.14796E 01	0.96649E-01
0.64	0.73519E 00	0.97306E 00	0.14862E 01	0.98266E-01
0.66	0.77305E 00	0.93870E 00	0.14926E 01	0.99813E-01
0.68	0.81187E 00	0.90280E 00	0.14986E 01	0.10131E 00
0.70	0.85154E 00	0.86554E 00	0.15046E 01	0.10276E 00
0.72	0.89242E 00	0.82640E 00	0.15102E 01	0.10414E 00
0.74	0.93413E 00	0.78599E 00	0.15157E 01	0.10549E 00
0.76	0.97704E 00	0.74372E 00	0.15210E 01	0.10679E 00
0.78	0.10208E 01	0.70019E 00	0.15261E 01	0.10805E 00
0.80	0.10658E 01	0.65482E 00	0.15310E 01	0.10926E 00
0.82	0.11116E 01	0.60822E 00	0.15357E 01	0.11044E 00
0.84	0.11586E 01	0.55980E 00	0.15403E 01	0.11157E 00
0.86	0.12067E 01	0.50991E 00	0.15448E 01	0.11267E 00
0.88	0.12557E 01	0.45874E 00	0.15491E 01	0.11374E 00
0.90	0.13059E 01	0.40578E 00	0.15532E 01	0.11477E 00
0.92	0.13570E 01	0.35161E 00	0.15573E 01	0.11577E 00
0.94	0.14093E 01	0.29566E 00	0.15612E 01	0.11674E 00
0.96	0.14625E 01	0.23851E 00	0.15650E 01	0.11768E 00
0.98	0.15169E 01	0.17960E 00	0.15687E 01	0.11860E 00
1.00	0.15723E 01	0.11949E 00	0.15723E 01	0.11949E 00

Table 3 (cont.) (τ = 1.0)

μ	L_1	L_2	L_3	L_4
0.00	0.	-0.	0.	0.
0.02	0.42266E-02	0.54236E-02	0.73755E-02	0.20539E-02
0.04	0.89304E-02	0.11533E-01	0.15365E-01	0.42776E-02
0.06	0.14011E-01	0.18207E-01	0.23860E-01	0.66376E-02
0.08	0.19582E-01	0.25676E-01	0.33061E-01	0.91765E-02
0.10	0.25466E-01	0.33728E-01	0.42775E-01	0.11829E-01
0.12	0.31787E-01	0.42791E-01	0.53378E-01	0.14635E-01
0.14	0.38382E-01	0.52700E-01	0.64737E-01	0.17531E-01
0.16	0.45350E-01	0.64036E-01	0.77409E-01	0.20535E-01
0.18	0.52593E-01	0.76629E-01	0.91288E-01	0.23598E-01
0.20	0.60067E-01	0.90309E-01	0.10625E 00	0.26695E-01
0.22	0.67854E-01	0.10539E 00	0.12271E 00	0.29811E-01
0.24	0.75854E-01	0.12129E 00	0.14010E 00	0.32922E-01
0.26	0.84181E-01	0.13813E 00	0.15872E 00	0.36003E-01
0.28	0.92743E-01	0.15541E 00	0.17805E 00	0.39051E-01
0.30	0.10168E 00	0.17307E 00	0.19825E 00	0.42038E-01
0.32	0.11089E 00	0.19078E 00	0.21890E 00	0.44975E-01
0.34	0.12055E 00	0.20833E 00	0.24006E 00	0.47830E-01
0.36	0.13060E 00	0.22551E 00	0.26148E 00	0.50615E-01
0.38	0.14102E 00	0.24226E 00	0.28305E 00	0.53334E-01
0.40	0.15200E 00	0.25819E 00	0.30470E 00	0.55961E-01
0.42	0.16339E 00	0.27346E 00	0.32634E 00	0.58525E-01
0.44	0.17542E 00	0.28760E 00	0.34787E 00	0.60994E-01
0.46	0.18790E 00	0.30089E 00	0.36927E 00	0.63401E-01
0.48	0.20110E 00	0.31281E 00	0.39042E 00	0.65717E-01
0.50	0.21480E 00	0.32374E 00	0.41137E 00	0.67972E-01
0.52	0.22927E 00	0.33315E 00	0.43198E 00	0.70140E-01
0.54	0.24441E 00	0.34123E 00	0.45228E 00	0.72240E-01
0.56	0.26014E 00	0.34814E 00	0.47229E 00	0.74282E-01
0.58	0.27673E 00	0.35335E 00	0.49188E 00	0.76244E-01
0.60	0.29392E 00	0.35736E 00	0.51118E 00	0.78155E-01
0.62	0.31203E 00	0.35960E 00	0.53003E 00	0.79993E-01
0.64	0.33077E 00	0.36060E 00	0.54857E 00	0.81782E-01
0.66	0.35047E 00	0.35978E 00	0.56665E 00	0.83503E-01
0.68	0.37096E 00	0.35746E 00	0.58438E 00	0.85172E-01
0.70	0.39216E 00	0.35380E 00	0.60178E 00	0.86796E-01
0.72	0.41437E 00	0.34827E 00	0.61872E 00	0.88359E-01
0.74	0.43729E 00	0.34146E 00	0.63536E 00	0.89883E-01
0.76	0.46124E 00	0.33277E 00	0.65156E 00	0.91352E-01
0.78	0.48592E 00	0.32278E 00	0.66747E 00	0.92784E-01
0.80	0.51167E 00	0.31089E 00	0.68295E 00	0.94165E-01
0.82	0.53817E 00	0.29772E 00	0.69814E 00	0.95512E-01
0.84	0.56576E 00	0.28264E 00	0.71293E 00	0.96812E-01
0.86	0.59425E 00	0.26602E 00	0.72739E 00	0.98076E-01
0.88	0.62354E 00	0.24805E 00	0.74158E 00	0.99309E-01
0.90	0.65396E 00	0.22819E 00	0.75538E 00	0.10050E 00
0.92	0.68516E 00	0.20704E 00	0.76893E 00	0.10166E 00
0.94	0.71751E 00	0.18400E 00	0.78212E 00	0.10279E 00
0.96	0.75065E 00	0.15967E 00	0.79507E 00	0.10389E 00
0.98	0.78495E 00	0.13347E 00	0.80767E 00	0.10495E 00
1.00	0.82005E 00	0.10599E 00	0.82005E 00	0.10599E 00

Table 3 (cont.) (τ = 2.0)

μ	K_1	K_2	K_3	K_4
0.00	0.	0.10000E 01	0.10000E 01	0.
0.02	0.11223E-01	0.10609E 01	0.10415E 01	0.52417E-02
0.04	0.23734E-01	0.11055E 01	0.10752E 01	0.10582E-01
0.06	0.37269E-01	0.11424E 01	0.11054E 01	0.15973E-01
0.08	0.52152E-01	0.11729E 01	0.11330E 01	0.21399E-01
0.10	0.67910E-01	0.12001E 01	0.11593E 01	0.26829E-01
0.12	0.84925E-01	0.12231E 01	0.11841E 01	0.32249E-01
0.14	0.10277E 00	0.12437E 01	0.12080E 01	0.37653E-01
0.16	0.12183E 00	0.12608E 01	0.12309E 01	0.43021E-01
0.18	0.14186E 00	0.12755E 01	0.12530E 01	0.48350E-01
0.20	0.16272E 00	0.12881E 01	0.12746E 01	0.53641E-01
0.22	0.18475E 00	0.12979E 01	0.12953E 01	0.58870E-01
0.24	0.20757E 00	0.13058E 01	0.13155E 01	0.64052E-01
0.26	0.23153E 00	0.13111E 01	0.13351E 01	0.69160E-01
0.28	0.25627E 00	0.13147E 01	0.13542E 01	0.74213E-01
0.30	0.28215E 00	0.13157E 01	0.13728E 01	0.79182E-01
0.32	0.30879E 00	0.13152E 01	0.13909E 01	0.84090E-01
0.34	0.33656E 00	0.13122E 01	0.14084E 01	0.88905E-01
0.36	0.36524E 00	0.13073E 01	0.14255E 01	0.93641E-01
0.38	0.39472E 00	0.13008E 01	0.14422E 01	0.98308E-01
0.40	0.42531E 00	0.12919E 01	0.14584E 01	0.10287E 00
0.42	0.45667E 00	0.12815E 01	0.14743E 01	0.10737E 00
0.44	0.48915E 00	0.12689E 01	0.14897E 01	0.11176E 00
0.46	0.52239E 00	0.12547E 01	0.15047E 01	0.11609E 00
0.48	0.55675E 00	0.12384E 01	0.15193E 01	0.12031E 00
0.50	0.59189E 00	0.12205E 01	0.15336E 01	0.12446E 00
0.52	0.62816E 00	0.12005E 01	0.15475E 01	0.12850E 00
0.54	0.66536E 00	0.11787E 01	0.15610E 01	0.13246E 00
0.56	0.70337E 00	0.11555E 01	0.15742E 01	0.13635E 00
0.58	0.74252E 00	0.11300E 01	0.15870E 01	0.14015E 00
0.60	0.78247E 00	0.11032E 01	0.15996E 01	0.14387E 00
0.62	0.82357E 00	0.10742E 01	0.16118E 01	0.14750E 00
0.64	0.86547E 00	0.10439E 01	0.16237E 01	0.15106E 00
0.66	0.90854E 00	0.10114E 01	0.16353E 01	0.15453E 00
0.68	0.95257E 00	0.97730E 00	0.16466E 01	0.15792E 00
0.70	0.99744E 00	0.94171E 00	0.16577E 01	0.16126E 00
0.72	0.10435E 01	0.90408E 00	0.16684E 01	0.16450E 00
0.74	0.10904E 01	0.86507E 00	0.16790E 01	0.16768E 00
0.76	0.11385E 01	0.82404E 00	0.16892E 01	0.17079E 00
0.78	0.11874E 01	0.78163E 00	0.16992E 01	0.17383E 00
0.80	0.12375E 01	0.73721E 00	0.17090E 01	0.17680E 00
0.82	0.12885E 01	0.69144E 00	0.17186E 01	0.17971E 00
0.84	0.13407E 01	0.64367E 00	0.17278E 01	0.18255E 00
0.86	0.13938E 01	0.59429E 00	0.17369E 01	0.18533E 00
0.88	0.14479E 01	0.54350E 00	0.17458E 01	0.18806E 00
0.90	0.15032E 01	0.49074E 00	0.17545E 01	0.19072E 00
0.92	0.15594E 01	0.43664E 00	0.17630E 01	0.19333E 00
0.94	0.16168E 01	0.38058E 00	0.17713E 01	0.19587E 00
0.96	0.16751E 01	0.32320E 00	0.17794E 01	0.19837E 00
0.98	0.17347E 01	0.26387E 00	0.17873E 01	0.20081E 00
1.00	0.17951E 01	0.20321E 00	0.17951E 01	0.20321E 00

Table 3 (cont.) $(\tau = 2.0)$

μ	L_1	L_2	L_3	L_4
0.00	0.	-0.	0.	0.
0.02	0.34946E-02	0.23846E-02	0.48633E-02	0.16942E-02
0.04	0.73893E-02	0.50506E-02	0.10113E-01	0.35250E-02
0.06	0.11601E-01	0.79418E-02	0.15676E-01	0.54662E-02
0.08	0.16230E-01	0.11134E-01	0.21660E-01	0.75547E-02
0.10	0.21130E-01	0.14523E-01	0.27921E-01	0.97393E-02
0.12	0.26418E-01	0.18202E-01	0.34586E-01	0.12062E-01
0.14	0.31962E-01	0.22077E-01	0.41522E-01	0.14475E-01
0.16	0.37878E-01	0.26245E-01	0.48859E-01	0.17020E-01
0.18	0.44087E-01	0.30661E-01	0.56528E-01	0.19668E-01
0.20	0.50551E-01	0.35305E-01	0.64496E-01	0.22404E-01
0.22	0.57365E-01	0.40290E-01	0.72903E-01	0.25260E-01
0.24	0.64413E-01	0.45523E-01	0.81624E-01	0.28192E-01
0.26	0.71798E-01	0.51141E-01	0.90828E-01	0.31230E-01
0.28	0.79407E-01	0.57030E-01	0.10038E 00	0.34334E-01
0.30	0.87342E-01	0.63323E-01	0.11045E 00	0.37527E-01
0.32	0.95495E-01	0.69885E-01	0.12088E 00	0.40773E-01
0.34	0.10397E 00	0.76828E-01	0.13185E 00	0.44088E-01
0.36	0.11270E 00	0.84049E-01	0.14325E 00	0.47449E-01
0.38	0.12167E 00	0.91489E-01	0.15501E 00	0.50844E-01
0.40	0.13097E 00	0.99167E-01	0.16728E 00	0.54278E-01
0.42	0.14050E 00	0.10702E 00	0.17988E 00	0.57730E-01
0.44	0.15038E 00	0.11498E 00	0.19293E 00	0.61203E-01
0.46	0.16051E 00	0.12298E 00	0.20626E 00	0.64682E-01
0.48	0.17101E 00	0.13097E 00	0.21998E 00	0.68166E-01
0.50	0.18178E 00	0.13890E 00	0.23394E 00	0.71646E-01
0.52	0.19297E 00	0.14667E 00	0.24823E 00	0.75116E-01
0.54	0.20451E 00	0.15422E 00	0.26274E 00	0.78571E-01
0.56	0.21637E 00	0.16155E 00	0.27743E 00	0.82012E-01
0.58	0.22871E 00	0.16849E 00	0.29235E 00	0.85426E-01
0.60	0.24138E 00	0.17511E 00	0.30738E 00	0.88819E-01
0.62	0.25457E 00	0.18119E 00	0.32258E 00	0.92179E-01
0.64	0.26812E 00	0.18685E 00	0.33785E 00	0.95512E-01
0.66	0.28224E 00	0.19184E 00	0.35323E 00	0.98806E-01
0.68	0.29683E 00	0.19623E 00	0.36866E 00	0.10206E 00
0.70	0.31186E 00	0.20005E 00	0.38413E 00	0.10529E 00
0.72	0.32751E 00	0.20302E 00	0.39962E 00	0.10847E 00
0.74	0.34361E 00	0.20537E 00	0.41512E 00	0.11162E 00
0.76	0.36038E 00	0.20676E 00	0.43060E 00	0.11472E 00
0.78	0.37762E 00	0.20746E 00	0.44606E 00	0.11779E 00
0.80	0.39557E 00	0.20711E 00	0.46147E 00	0.12080E 00
0.82	0.41403E 00	0.20601E 00	0.47684E 00	0.12379E 00
0.84	0.43324E 00	0.20378E 00	0.49213E 00	0.12672E 00
0.86	0.45308E 00	0.20058E 00	0.50735E 00	0.12960E 00
0.88	0.47348E 00	0.19652E 00	0.52249E 00	0.13246E 00
0.90	0.49470E 00	0.19121E 00	0.53753E 00	0.13526E 00
0.92	0.51648E 00	0.18503E 00	0.55248E 00	0.13803E 00
0.94	0.53911E 00	0.17754E 00	0.56731E 00	0.14074E 00
0.96	0.56233E 00	0.16914E 00	0.58205E 00	0.14342E 00
0.98	0.58643E 00	0.15937E 00	0.59664E 00	0.14606E 00
1.00	0.61114E 00	0.14865E 00	0.61114E 00	0.14865E 00

Table 3 (cont.) (τ = 4.0)

μ	K_1	K_2	K_3	K_4
0.00	0.	0.10000E 01	0.10000E 01	0.
0.02	0.12681E-01	0.10615E 01	0.10433E 01	0.59477E-02
0.04	0.26819E-01	0.11068E 01	0.10789E 01	0.12050E-01
0.06	0.42113E-01	0.11444E 01	0.11110E 01	0.18249E-01
0.08	0.58932E-01	0.11757E 01	0.11407E 01	0.24542E-01
0.10	0.76740E-01	0.12038E 01	0.11693E 01	0.30880E-01
0.12	0.95971E-01	0.12278E 01	0.11964E 01	0.37266E-01
0.14	0.11614E 00	0.12493E 01	0.12229E 01	0.43673E-01
0.16	0.13769E 00	0.12675E 01	0.12483E 01	0.50101E-01
0.18	0.16032E 00	0.12833E 01	0.12732E 01	0.56535E-01
0.20	0.18391E 00	0.12970E 01	0.12975E 01	0.62970E-01
0.22	0.20882E 00	0.13080E 01	0.13212E 01	0.69399E-01
0.24	0.23462E 00	0.13172E 01	0.13445E 01	0.75819E-01
0.26	0.26173E 00	0.13238E 01	0.13673E 01	0.82220E-01
0.28	0.28972E 00	0.13287E 01	0.13897E 01	0.88606E-01
0.30	0.31901E 00	0.13313E 01	0.14117E 01	0.94963E-01
0.32	0.34917E 00	0.13322E 01	0.14333E 01	0.10130E 00
0.34	0.38061E 00	0.13308E 01	0.14545E 01	0.10760E 00
0.36	0.41309E 00	0.13275E 01	0.14754E 01	0.11387E 00
0.38	0.44648E 00	0.13226E 01	0.14960E 01	0.12011E 00
0.40	0.48114E 00	0.13155E 01	0.15163E 01	0.12630E 00
0.42	0.51666E 00	0.13069E 01	0.15363E 01	0.13246E 00
0.44	0.55344E 00	0.12961E 01	0.15559E 01	0.13857E 00
0.46	0.59109E 00	0.12838E 01	0.15754E 01	0.14465E 00
0.48	0.62999E 00	0.12693E 01	0.15944E 01	0.15067E 00
0.50	0.66975E 00	0.12534E 01	0.16133E 01	0.15666E 00
0.52	0.71077E 00	0.12354E 01	0.16319E 01	0.16259E 00
0.54	0.75281E 00	0.12157E 01	0.16502E 01	0.16847E 00
0.56	0.79575E 00	0.11945E 01	0.16653E 01	0.17431E 00
0.58	0.83993E 00	0.11711E 01	0.16861E 01	0.18009E 00
0.60	0.88497E 00	0.11464E 01	0.17037E 01	0.18582E 00
0.62	0.93126E 00	0.11196E 01	0.17210E 01	0.19149E 00
0.64	0.97840E 00	0.10914E 01	0.17381E 01	0.19712E 00
0.66	0.10268E 01	0.10611E 01	0.17550E 01	0.20269E 00
0.68	0.10762E 01	0.10292E 01	0.17716E 01	0.20820E 00
0.70	0.11265E 01	0.99579E 00	0.17881E 01	0.21367E 00
0.72	0.11781E 01	0.96036E 00	0.18043E 01	0.21907E 00
0.74	0.12305E 01	0.92354E 00	0.18203E 01	0.22442E 00
0.76	0.12841E 01	0.88470E 00	0.18360E 01	0.22971E 00
0.78	0.13387E 01	0.84447E 00	0.18516E 01	0.23495E 00
0.80	0.13944E 01	0.80223E 00	0.18669E 01	0.24012E 00
0.82	0.14510E 01	0.75862E 00	0.18820E 01	0.24525E 00
0.84	0.15089E 01	0.71299E 00	0.18970E 01	0.25031E 00
0.86	0.15678E 01	0.66574E 00	0.19117E 01	0.25532E 00
0.88	0.16276E 01	0.61706E 00	0.19262E 01	0.26028E 00
0.90	0.16887E 01	0.56638E 00	0.19405E 01	0.26518E 00
0.92	0.17506E 01	0.51434E 00	0.19547E 01	0.27002E 00
0.94	0.18138E 01	0.46031E 00	0.19686E 01	0.27481E 00
0.96	0.18779E 01	0.40493E 00	0.19824E 01	0.27954E 00
0.98	0.19432E 01	0.34756E 00	0.19960E 01	0.28421E 00
1.00	0.20094E 01	0.28884E 00	0.20094E 01	0.28884E 00

Table 3 (cont.) (τ = 4.0)

μ	L_1	L_2	L_3	L_4
0.00	0.	-0.	0.	0.
0.02	0.23969E-02	0.10436E-02	0.29911E-02	0.11606E-02
0.04	0.50690E-02	0.22074E-02	0.62184E-02	0.24132E-02
0.06	0.79597E-02	0.34667E-02	0.96366E-02	0.37402E-02
0.08	0.11138E-01	0.48521E-02	0.13310E-01	0.51667E-02
0.10	0.14504E-01	0.63194E-02	0.17152E-01	0.66584E-02
0.12	0.18138E-01	0.79045E-02	0.21235E-01	0.82442E-02
0.14	0.21950E-01	0.95679E-02	0.25480E-01	0.98927E-02
0.16	0.26021E-01	0.11345E-01	0.29960E-01	0.11633E-01
0.18	0.30298E-01	0.13213E-01	0.34628E-01	0.13445E-01
0.20	0.34754E-01	0.15161E-01	0.39461E-01	0.15322E-01
0.22	0.39461E-01	0.17219E-01	0.44525E-01	0.17287E-01
0.24	0.44336E-01	0.19353E-01	0.49743E-01	0.19312E-01
0.26	0.49457E-01	0.21597E-01	0.55190E-01	0.21424E-01
0.28	0.54744E-01	0.23915E-01	0.60791E-01	0.23595E-01
0.30	0.60275E-01	0.26345E-01	0.66620E-01	0.25853E-01
0.32	0.65970E-01	0.28850E-01	0.72604E-01	0.28168E-01
0.34	0.71907E-01	0.31467E-01	0.78815E-01	0.30568E-01
0.36	0.78038E-01	0.34176E-01	0.85214E-01	0.33037E-01
0.38	0.84338E-01	0.36967E-01	0.91775E-01	0.35564E-01
0.40	0.90877E-01	0.39873E-01	0.98569E-01	0.38174E-01
0.42	0.97577E-01	0.42859E-01	0.10552E 00	0.40838E-01
0.44	0.10451E 00	0.45960E-01	0.11271E 00	0.43581E-01
0.46	0.11161E 00	0.49140E-01	0.12005E 00	0.46377E-01
0.48	0.11894E 00	0.52435E-01	0.12764E 00	0.49249E-01
0.50	0.12643E 00	0.55805E-01	0.13538E 00	0.52170E-01
0.52	0.13416E 00	0.59283E-01	0.14337E 00	0.55163E-01
0.54	0.14208E 00	0.62843E-01	0.15154E 00	0.58211E-01
0.56	0.15016E 00	0.66470E-01	0.15989E 00	0.61306E-01
0.58	0.15849E 00	0.70184E-01	0.16847E 00	0.64465E-01
0.60	0.16698E 00	0.73948E-01	0.17721E 00	0.67664E-01
0.62	0.17572E 00	0.77777E-01	0.18619E 00	0.70922E-01
0.64	0.18463E 00	0.81638E-01	0.19532E 00	0.74216E-01
0.66	0.19379E 00	0.85535E-01	0.20467E 00	0.77561E-01
0.68	0.20315E 00	0.89443E-01	0.21420E 00	0.80945E-01
0.70	0.21271E 00	0.93350E-01	0.22389E 00	0.84361E-01
0.72	0.22253E 00	0.97237E-01	0.23378E 00	0.87820E-01
0.74	0.23254E 00	0.10109E 00	0.24382E 00	0.91304E-01
0.76	0.24284E 00	0.10489E 00	0.25405E 00	0.94825E-01
0.78	0.25333E 00	0.10862E 00	0.26442E 00	0.98367E-01
0.80	0.26412E 00	0.11224E 00	0.27497E 00	0.10194E 00
0.82	0.27512E 00	0.11577E 00	0.28563E 00	0.10553E 00
0.84	0.28644E 00	0.11913E 00	0.29647E 00	0.10914E 00
0.86	0.29803E 00	0.12234E 00	0.30744E 00	0.11277E 00
0.88	0.30986E 00	0.12540E 00	0.31852E 00	0.11642E 00
0.90	0.32203E 00	0.12821E 00	0.32975E 00	0.12008E 00
0.92	0.33445E 00	0.13083E 00	0.34107E 00	0.12375E 00
0.94	0.34724E 00	0.13316E 00	0.35253E 00	0.12743E 00
0.96	0.36029E 00	0.13526E 00	0.36407E 00	0.13111E 00
0.98	0.37373E 00	0.13701E 00	0.37572E 00	0.13480E 00
1.00	0.38745E 00	0.13850E 00	0.38745E 00	0.13850E 00

Table 3 (cont.) (τ = 8.0)

μ	K_1	K_2	K_3	K_4
0.00	0.	0.10000E 01	0.10000E 01	0.
0.02	0.13716E-01	0.10619E 01	0.10445E 01	0.64490E-02
0.04	0.29008E-01	0.11077E 01	0.10815E 01	0.13092E-01
0.06	0.45552E-01	0.11458E 01	0.11151E 01	0.19864E-01
0.08	0.63743E-01	0.11776E 01	0.11464E 01	0.26772E-01
0.10	0.83006E-01	0.12063E 01	0.11765E 01	0.33754E-01
0.12	0.10381E 00	0.12309E 01	0.12054E 01	0.40824E-01
0.14	0.12563E 00	0.12530E 01	0.12336E 01	0.47943E-01
0.16	0.14893E 00	0.12720E 01	0.12610E 01	0.55121E-01
0.18	0.17341E 00	0.12885E 01	0.12878E 01	0.62337E-01
0.20	0.19892E 00	0.13030E 01	0.13142E 01	0.69582E-01
0.22	0.22587E 00	0.13147E 01	0.13401E 01	0.76859E-01
0.24	0.25378E 00	0.13248E 01	0.13656E 01	0.84153E-01
0.26	0.28310E 00	0.13323E 01	0.13906E 01	0.91467E-01
0.28	0.31333E 00	0.13381E 01	0.14154E 01	0.98790E-01
0.30	0.34506E 00	0.13416E 01	0.14398E 01	0.10612E 00
0.32	0.37768E 00	0.13435E 01	0.14640E 01	0.11346E 00
0.34	0.41169E 00	0.13431E 01	0.14878E 01	0.12080E 00
0.36	0.44683E 00	0.13409E 01	0.15114E 01	0.12814E 00
0.38	0.48294E 00	0.13371E 01	0.15348E 01	0.13547E 00
0.40	0.52043E 00	0.13311E 01	0.15579E 01	0.14280E 00
0.42	0.55885E 00	0.13236E 01	0.15808E 01	0.15012E 00
0.44	0.59865E 00	0.13140E 01	0.16035E 01	0.15742E 00
0.46	0.63937E 00	0.13029E 01	0.16260E 01	0.16472E 00
0.48	0.68146E 00	0.12897E 01	0.16483E 01	0.17200E 00
0.50	0.72447E 00	0.12751E 01	0.16704E 01	0.17927E 00
0.52	0.76884E 00	0.12585E 01	0.16923E 01	0.18652E 00
0.54	0.81432E 00	0.12401E 01	0.17140E 01	0.19376E 00
0.56	0.86077E 00	0.12203E 01	0.17356E 01	0.20097E 00
0.58	0.90856E 00	0.11984E 01	0.17569E 01	0.20816E 00
0.60	0.95728E 00	0.11751E 01	0.17782E 01	0.21534E 00
0.62	0.10074E 01	0.11498E 01	0.17992E 01	0.22249E 00
0.64	0.10583E 01	0.11231E 01	0.18201E 01	0.22962E 00
0.66	0.11107E 01	0.10944E 01	0.18408E 01	0.23672E 00
0.68	0.11641E 01	0.10641E 01	0.18614E 01	0.24380E 00
0.70	0.12185E 01	0.10323E 01	0.18819E 01	0.25086E 00
0.72	0.12743E 01	0.99852E 00	0.19022E 01	0.25788E 00
0.74	0.13309E 01	0.96339E 00	0.19223E 01	0.26489E 00
0.76	0.13890E 01	0.92627E 00	0.19423E 01	0.27186E 00
0.78	0.14479E 01	0.88780E 00	0.19622E 01	0.27881E 00
0.80	0.15082E 01	0.84734E 00	0.19819E 01	0.28572E 00
0.82	0.15693E 01	0.80553E 00	0.20015E 01	0.29261E 00
0.84	0.16319E 01	0.76175E 00	0.20210E 01	0.29947E 00
0.86	0.16955E 01	0.71636E 00	0.20403E 01	0.30629E 00
0.88	0.17601E 01	0.66957E 00	0.20595E 01	0.31309E 00
0.90	0.18260E 01	0.62081E 00	0.20785E 01	0.31986E 00
0.92	0.18928E 01	0.57071E 00	0.20975E 01	0.32660E 00
0.94	0.19610E 01	0.51865E 00	0.21162E 01	0.33330E 00
0.96	0.20301E 01	0.46526E 00	0.21349E 01	0.33997E 00
0.98	0.21005E 01	0.40991E 00	0.21534E 01	0.34661E 00
1.00	0.21719E 01	0.35323E 00	0.21719E 01	0.35322E 00

Table 3 (cont.) (τ = 8.0)

μ	L_1	L_2	L_3	L_4
0.00	0.	-0.	0.	0.
0.02	0.13997E-02	0.55435E-03	0.17156E-02	0.67764E-03
0.04	0.29603E-02	0.11724E-02	0.35666E-02	0.14088E-02
0.06	0.46485E-02	0.18410E-02	0.55271E-02	0.21832E-02
0.08	0.65049E-02	0.25762E-02	0.76341E-02	0.30155E-02
0.10	0.84707E-02	0.33547E-02	0.98371E-02	0.38857E-02
0.12	0.10593E-01	0.41954E-02	0.12179E-01	0.48107E-02
0.14	0.12820E-01	0.50773E-02	0.14613E-01	0.57722E-02
0.16	0.15198E-01	0.60190E-02	0.17182E-01	0.67870E-02
0.18	0.17696E-01	0.70085E-02	0.19859E-01	0.78443E-02
0.20	0.20300E-01	0.80396E-02	0.22630E-01	0.89389E-02
0.22	0.23050E-01	0.91286E-02	0.25532E-01	0.10085E-01
0.24	0.25898E-01	0.10257E-01	0.28523E-01	0.11267E-01
0.26	0.28890E-01	0.11442E-01	0.31644E-01	0.12500E-01
0.28	0.31980E-01	0.12666E-01	0.34853E-01	0.13767E-01
0.30	0.35213E-01	0.13946E-01	0.38191E-01	0.15086E-01
0.32	0.38542E-01	0.15264E-01	0.41617E-01	0.16439E-01
0.34	0.42013E-01	0.16639E-01	0.45170E-01	0.17843E-01
0.36	0.45598E-01	0.18059E-01	0.48828E-01	0.19287E-01
0.38	0.49284E-01	0.19519E-01	0.52576E-01	0.20768E-01
0.40	0.53110E-01	0.21034E-01	0.56452E-01	0.22299E-01
0.42	0.57031E-01	0.22587E-01	0.60414E-01	0.23864E-01
0.44	0.61091E-01	0.24196E-01	0.64504E-01	0.25479E-01
0.46	0.65247E-01	0.25842E-01	0.68679E-01	0.27128E-01
0.48	0.69542E-01	0.27543E-01	0.72982E-01	0.28827E-01
0.50	0.73931E-01	0.29281E-01	0.77370E-01	0.30561E-01
0.52	0.78459E-01	0.31075E-01	0.81885E-01	0.32344E-01
0.54	0.83101E-01	0.32914E-01	0.86503E-01	0.34167E-01
0.56	0.87840E-01	0.34792E-01	0.91211E-01	0.36026E-01
0.58	0.92718E-01	0.36724E-01	0.96045E-01	0.37935E-01
0.60	0.97689E-01	0.38694E-01	0.10096E 00	0.39877E-01
0.62	0.10280E 00	0.40719E-01	0.10601E 00	0.41869E-01
0.64	0.10800E 00	0.42781E-01	0.11114E 00	0.43894E-01
0.66	0.11334E 00	0.44897E-01	0.11640E 00	0.45969E-01
0.68	0.11880E 00	0.47058E-01	0.12176E 00	0.48083E-01
0.70	0.12435E 00	0.49257E-01	0.12721E 00	0.50232E-01
0.72	0.13004E 00	0.51510E-01	0.13279E 00	0.52431E-01
0.74	0.13582E 00	0.53800E-01	0.13845E 00	0.54662E-01
0.76	0.14174E 00	0.56142E-01	0.14424E 00	0.56942E-01
0.78	0.14775E 00	0.58520E-01	0.15011E 00	0.59254E-01
0.80	0.15391E 00	0.60950E-01	0.15611E 00	0.61615E-01
0.82	0.16015E 00	0.63414E-01	0.16220E 00	0.64008E-01
0.84	0.16654E 00	0.65928E-01	0.16841E 00	0.66448E-01
0.86	0.17304E 00	0.68480E-01	0.17472E 00	0.68726E-01
0.88	0.17964E 00	0.71066E-01	0.18112E 00	0.71438E-01
0.90	0.18637E 00	0.73697E-01	0.18765E 00	0.73995E-01
0.92	0.19321E 00	0.76357E-01	0.19426E 00	0.76584E-01
0.94	0.20018E 00	0.79058E-01	0.20100E 00	0.79218E-01
0.96	0.20725E 00	0.81784E-01	0.20782E 00	0.81883E-01
0.98	0.21447E 00	0.84546E-01	0.21476E 00	0.84572E-01
1.00	0.22178E 00	0.87330E-01	0.22178E 00	0.87330E-01

Table 3 (cont.) (τ = 16.0)

	K_1	K_2	K_3	K_4
0.00	0.	0.10000E 01	0.10000E 01	0.
0.02	0.14359E-01	0.10671E 01	0.10453E 01	0.67602E-02
0.04	0.30368E-01	0.11082E 01	0.10831E 01	0.13739E-01
0.06	0.47687E-01	0.11466E 01	0.11177E 01	0.20866E-01
0.08	0.66731E-01	0.11788E 01	0.11499E 01	0.28157E-01
0.10	0.86897E-01	0.12078E 01	0.11810E 01	0.35539E-01
0.12	0.10867E 00	0.12328E 01	0.12110E 01	0.43034E-01
0.14	0.13152E 00	0.12554E 01	0.12403E 01	0.50594E-01
0.16	0.15591E 00	0.12747E 01	0.12689E 01	0.58238E-01
0.18	0.18154E 00	0.12917E 01	0.12970E 01	0.65940E-01
0.20	0.20825E 00	0.13066E 01	0.13246E 01	0.73688E-01
0.22	0.23646E 00	0.13189E 01	0.13518E 01	0.81491E-01
0.24	0.26567E 00	0.13295E 01	0.13787E 01	0.89328E-01
0.26	0.29637E 00	0.13375E 01	0.14052E 01	0.97208E-01
0.28	0.32807E 00	0.13439E 01	0.14314E 01	0.10511E 00
0.30	0.36123E 00	0.13480E 01	0.14573E 01	0.11305E 00
0.32	0.39538E 00	0.13505E 01	0.14831E 01	0.12101E 00
0.34	0.43099E 00	0.13507E 01	0.15086E 01	0.12899E 00
0.36	0.46777E 00	0.13491E 01	0.15338E 01	0.13699E 00
0.38	0.50558E 00	0.13460E 01	0.15589E 01	0.14501E 00
0.40	0.54482E 00	0.13407E 01	0.15838E 01	0.15304E 00
0.42	0.58505E 00	0.13340E 01	0.16085E 01	0.16108E 00
0.44	0.62671E 00	0.13251E 01	0.16331E 01	0.16913E 00
0.46	0.66934E 00	0.13148E 01	0.16575E 01	0.17718E 00
0.48	0.71340E 00	0.13024E 01	0.16818E 01	0.18524E 00
0.50	0.75842E 00	0.12886E 01	0.17059E 01	0.19331E 00
0.52	0.80488E 00	0.12727E 01	0.17298E 01	0.20138E 00
0.54	0.85249E 00	0.12552E 01	0.17537E 01	0.20945E 00
0.56	0.90111E 00	0.12362E 01	0.17774E 01	0.21752E 00
0.58	0.95115E 00	0.12152E 01	0.18010E 01	0.22559E 00
0.60	0.10022E 01	0.11928E 01	0.18245E 01	0.23366E 00
0.62	0.10546E 01	0.11684E 01	0.18479E 01	0.24172E 00
0.64	0.11080E 01	0.11427E 01	0.18711E 01	0.24978E 00
0.66	0.11628E 01	0.11150E 01	0.18943E 01	0.25784E 00
0.68	0.12187E 01	0.10856E 01	0.19173E 01	0.26589E 00
0.70	0.12756E 01	0.10549E 01	0.19403E 01	0.27393E 00
0.72	0.13340E 01	0.10221E 01	0.19631E 01	0.28197E 00
0.74	0.13933E 01	0.98805E 00	0.19859E 01	0.29000E 00
0.76	0.14541E 01	0.95200E 00	0.20085E 01	0.29802E 00
0.78	0.15157E 01	0.91461E 00	0.20311E 01	0.30603E 00
0.80	0.15788E 01	0.87527E 00	0.20536E 01	0.31404E 00
0.82	0.16429E 01	0.83460E 00	0.20760E 01	0.32203E 00
0.84	0.17084E 01	0.79197E 00	0.20983E 01	0.33001E 00
0.86	0.17750E 01	0.74776E 00	0.21205E 01	0.33798E 00
0.88	0.18426E 01	0.70217E 00	0.21426E 01	0.34594E 00
0.90	0.19116E 01	0.65463E 00	0.21646E 01	0.35389E 00
0.92	0.19815E 01	0.60577E 00	0.21866E 01	0.36183E 00
0.94	0.20529E 01	0.55497E 00	0.22085E 01	0.36975E 00
0.96	0.21252E 01	0.50286E 00	0.22303E 01	0.37766E 00
0.98	0.21990E 01	0.44881E 00	0.22520E 01	0.38556E 00
1.00	0.22737E 01	0.39345E 00	0.22737E 01	0.39344E 00

Table 3 (cont.) (τ = 16.0)

μ	L_1	L_2	L_3	L_4
0.00	0.	-0.	0.	0.
0.02	0.75730E-03	0.29930E-03	0.92784E-03	0.36662E-03
0.04	0.16016E-02	0.63298E-03	0.19289E-02	0.76219E-03
0.06	0.25150E-02	0.99396E-03	0.29893E-02	0.11812E-02
0.08	0.35194E-02	0.13909E-02	0.41288E-02	0.16314E-02
0.10	0.45829E-02	0.18112E-02	0.53203E-02	0.21022E-02
0.12	0.57313E-02	0.22651E-02	0.65868E-02	0.26027E-02
0.14	0.69361E-02	0.27412E-02	0.79033E-02	0.31229E-02
0.16	0.82226E-02	0.32497E-02	0.92927E-02	0.36719E-02
0.18	0.95743E-02	0.37838E-02	0.10740E-01	0.42439E-02
0.20	0.10983E-01	0.43405E-02	0.12239E-01	0.48361E-02
0.22	0.12471E-01	0.49265E-02	0.13809E-01	0.54563E-02
0.24	0.14012E-01	0.55374E-02	0.15426E-01	0.60956E-02
0.26	0.15631E-01	0.61773E-02	0.17114E-01	0.67625E-02
0.28	0.17302E-01	0.68377E-02	0.18850E-01	0.74483E-02
0.30	0.19051E-01	0.75291E-02	0.20655E-01	0.81617E-02
0.32	0.20853E-01	0.82409E-02	0.22508E-01	0.88937E-02
0.34	0.22730E-01	0.89830E-02	0.24430E-01	0.96531E-02
0.36	0.24670E-01	0.97496E-02	0.26408E-01	0.10435E-01
0.38	0.26664E-01	0.10538E-01	0.28435E-01	0.11236E-01
0.40	0.28734E-01	0.11356E-01	0.30531E-01	0.12064E-01
0.42	0.30855E-01	0.12194E-01	0.32674E-01	0.12911E-01
0.44	0.33053E-01	0.13062E-01	0.34886E-01	0.13785E-01
0.46	0.35301E-01	0.13951E-01	0.37144E-01	0.14677E-01
0.48	0.37625E-01	0.14869E-01	0.39471E-01	0.15596E-01
0.50	0.40000E-01	0.15807E-01	0.41844E-01	0.16534E-01
0.52	0.42450E-01	0.16775E-01	0.44286E-01	0.17499E-01
0.54	0.44961E-01	0.17768E-01	0.46783E-01	0.18486E-01
0.56	0.47525E-01	0.18781E-01	0.49329E-01	0.19492E-01
0.58	0.50165E-01	0.19824E-01	0.51943E-01	0.20525E-01
0.60	0.52855E-01	0.20887E-01	0.54603E-01	0.21576E-01
0.62	0.55619E-01	0.21979E-01	0.57332E-01	0.22654E-01
0.64	0.58435E-01	0.23092E-01	0.60106E-01	0.23751E-01
0.66	0.61325E-01	0.24234E-01	0.62949E-01	0.24874E-01
0.68	0.64275E-01	0.25400E-01	0.65847E-01	0.26019E-01
0.70	0.67279E-01	0.26586E-01	0.68794E-01	0.27183E-01
0.72	0.70357E-01	0.27803E-01	0.71808E-01	0.28374E-01
0.74	0.73486E-01	0.29039E-01	0.74869E-01	0.29584E-01
0.76	0.76689E-01	0.30304E-01	0.77997E-01	0.30820E-01
0.78	0.79942E-01	0.31590E-01	0.81172E-01	0.32074E-01
0.80	0.83270E-01	0.32905E-01	0.84414E-01	0.33356E-01
0.82	0.86649E-01	0.34240E-01	0.87703E-01	0.34655E-01
0.84	0.90101E-01	0.35604E-01	0.91059E-01	0.35981E-01
0.86	0.93614E-01	0.36992E-01	0.94471E-01	0.37329E-01
0.88	0.97180E-01	0.38401E-01	0.97930E-01	0.38697E-01
0.90	0.10082E 00	0.39839E-01	0.10146E 00	0.40090E-01
0.92	0.10451E 00	0.41297E-01	0.10503E 00	0.41503E-01
0.94	0.10827E 00	0.42784E-01	0.10867E 00	0.42941E-01
0.96	0.11209E 00	0.44291E-01	0.11236E 00	0.44398E-01
0.98	0.11598E 00	0.45828E-01	0.11612E 00	0.45882E-01
1.00	0.11992E 00	0.47384E-01	0.11992E 00	0.47384E-01

Table 3 (cont.) (τ = 100.0)

μ	K_1	K_2	K_3	K_4
0.00	0.	0.10000E 01	0.10000E 01	0.
0.02	0.14986E-01	0.10624E 01	0.10461E 01	0.70639E-02
0.04	0.31694E-01	0.11087E 01	0.10847E 01	0.14370E-01
0.06	0.49770E-01	0.11475E 01	0.11201E 01	0.21845E-01
0.08	0.69646E-01	0.11800E 01	0.11533E 01	0.29509E-01
0.10	0.90692E-01	0.12093E 01	0.11854E 01	0.37280E-01
0.12	0.11342E 00	0.12347E 01	0.12164E 01	0.45187E-01
0.14	0.13726E 00	0.12576E 01	0.12469E 01	0.53180E-01
0.16	0.16272E 00	0.12774E 01	0.12766E 01	0.61279E-01
0.18	0.18947E 00	0.12948E 01	0.13058E 01	0.69455E-01
0.20	0.21734E 00	0.13102E 01	0.13347E 01	0.77693E-01
0.22	0.24678E 00	0.13230E 01	0.13632E 01	0.86010E-01
0.24	0.27728E 00	0.13340E 01	0.13914E 01	0.94377E-01
0.26	0.30932E 00	0.13426E 01	0.14193E 01	0.10281E 00
0.28	0.34240E 00	0.13496E 01	0.14470E 01	0.11128E 00
0.30	0.37701E 00	0.13542E 01	0.14745E 01	0.11981E 00
0.32	0.41265E 00	0.13573E 01	0.15017E 01	0.12838E 00
0.34	0.44981E 00	0.13581E 01	0.15288E 01	0.13699E 00
0.36	0.48820E 00	0.13572E 01	0.15557E 01	0.14564E 00
0.38	0.52766E 00	0.13547E 01	0.15825E 01	0.15431E 00
0.40	0.56862E 00	0.13501E 01	0.16091E 01	0.16303E 00
0.42	0.61060E 00	0.13440E 01	0.16356E 01	0.17177E 00
0.44	0.65408E 00	0.13359E 01	0.16620E 01	0.18054E 00
0.46	0.69857E 00	0.13263E 01	0.16883E 01	0.18934E 00
0.48	0.74456E 00	0.13147E 01	0.17144E 01	0.19816E 00
0.50	0.79155E 00	0.13017E 01	0.17405E 01	0.20700E 00
0.52	0.84004E 00	0.12866E 01	0.17665E 01	0.21587E 00
0.54	0.88973E 00	0.12699E 01	0.17924E 01	0.22476E 00
0.56	0.94047E 00	0.12518E 01	0.18183E 01	0.23366E 00
0.58	0.99270E 00	0.12316E 01	0.18440E 01	0.24259E 00
0.60	0.10459E 01	0.12101E 01	0.18697E 01	0.25153E 00
0.62	0.11006E 01	0.11867E 01	0.18954E 01	0.26048E 00
0.64	0.11564E 01	0.11618E 01	0.19209E 01	0.26945E 00
0.66	0.12135E 01	0.11350E 01	0.19464E 01	0.27844E 00
0.68	0.12719E 01	0.11067E 01	0.19719E 01	0.28744E 00
0.70	0.13314E 01	0.10769E 01	0.19973E 01	0.29645E 00
0.72	0.13923E 01	0.10452E 01	0.20226E 01	0.30547E 00
0.74	0.14542E 01	0.10121E 01	0.20479E 01	0.31450E 00
0.76	0.15176E 01	0.97710E 00	0.20731E 01	0.32355E 00
0.78	0.15820E 01	0.94078E 00	0.20983E 01	0.33260E 00
0.80	0.16478E 01	0.90252E 00	0.21235E 01	0.34166E 00
0.82	0.17147E 01	0.86296E 00	0.21486E 01	0.35073E 00
0.84	0.17830E 01	0.82146E 00	0.21737E 01	0.35981E 00
0.86	0.18525E 01	0.77840E 00	0.21987E 01	0.36890E 00
0.88	0.19231E 01	0.73397E 00	0.22237E 01	0.37799E 00
0.90	0.19951E 01	0.68762E 00	0.22487E 01	0.38710E 00
0.92	0.20681E 01	0.63997E 00	0.22736E 01	0.39620E 00
0.94	0.21426E 01	0.59041E 00	0.22985E 01	0.40532E 00
0.96	0.22181E 01	0.53954E 00	0.23233E 01	0.41444E 00
0.98	0.22950E 01	0.48677E 00	0.23482E 01	0.42356E 00
1.00	0.23730E 01	0.43269E 00	0.23730E 01	0.43269E 00

Table 3 (cont.) (τ = 100.0)

μ	L_1	L_2	L_3	L_4
0.00	0.	-0.	0.	0.
0.02	0.13008E-03	0.51449E-04	0.15941E-03	0.62972E-04
0.04	0.27510E-03	0.10881E-03	0.33140E-03	0.13092E-03
0.06	0.43198E-03	0.17086E-03	0.51357E-03	0.20288E-03
0.08	0.60450E-03	0.23909E-03	0.70935E-03	0.28022E-03
0.10	0.78719E-03	0.31133E-03	0.91405E-03	0.36109E-03
0.12	0.98445E-03	0.38934E-03	0.11316E-02	0.44705E-03
0.14	0.11914E-02	0.47118E-03	0.13578E-02	0.53641E-03
0.16	0.14124E-02	0.55856E-03	0.15965E-02	0.63071E-03
0.18	0.16446E-02	0.65037E-03	0.18452E-02	0.72897E-03
0.20	0.18865E-02	0.74604E-03	0.21027E-02	0.83069E-03
0.22	0.21421E-02	0.84708E-03	0.23724E-02	0.93723E-03
0.24	0.24068E-02	0.95174E-03	0.26503E-02	0.10470E-02
0.26	0.26849E-02	0.10617E-02	0.29403E-02	0.11616E-02
0.28	0.29721E-02	0.11752E-02	0.32385E-02	0.12794E-02
0.30	0.32725E-02	0.12940E-02	0.35486E-02	0.14019E-02
0.32	0.35820E-02	0.14163E-02	0.38669E-02	0.15277E-02
0.34	0.39045E-02	0.15438E-02	0.41971E-02	0.16581E-02
0.36	0.42378E-02	0.16755E-02	0.45369E-02	0.17924E-02
0.38	0.45803E-02	0.18109E-02	0.48852E-02	0.19300E-02
0.40	0.49359E-02	0.19515E-02	0.52453E-02	0.20723E-02
0.42	0.53003E-02	0.20955E-02	0.56135E-02	0.22177E-02
0.44	0.56778E-02	0.22447E-02	0.59934E-02	0.23678E-02
0.46	0.60640E-02	0.23973E-02	0.63814E-02	0.25211E-02
0.48	0.64632E-02	0.25551E-02	0.67811E-02	0.26791E-02
0.50	0.68712E-02	0.27163E-02	0.71889E-02	0.28402E-02
0.52	0.72921E-02	0.28827E-02	0.76084E-02	0.30059E-02
0.54	0.77235E-02	0.30531E-02	0.80374E-02	0.31754E-02
0.56	0.81641E-02	0.32272E-02	0.84748E-02	0.33482E-02
0.58	0.86175E-02	0.34064E-02	0.89239E-02	0.35257E-02
0.60	0.90796E-02	0.35890E-02	0.93809E-02	0.37062E-02
0.62	0.95546E-02	0.37766E-02	0.98497E-02	0.38914E-02
0.64	0.10038E-01	0.39677E-02	0.10326E-01	0.40798E-02
0.66	0.10535E-01	0.41639E-02	0.10815E-01	0.42728E-02
0.68	0.11042E-01	0.43641E-02	0.11313E-01	0.44695E-02
0.70	0.11558E-01	0.45680E-02	0.11819E-01	0.46695E-02
0.72	0.12087E-01	0.47769E-02	0.12337E-01	0.48741E-02
0.74	0.12624E-01	0.49892E-02	0.12862E-01	0.50818E-02
0.76	0.13174E-01	0.52066E-02	0.13400E-01	0.52942E-02
0.78	0.13733E-01	0.54274E-02	0.13945E-01	0.55097E-02
0.80	0.14305E-01	0.56532E-02	0.14502E-01	0.57298E-02
0.82	0.14886E-01	0.58825E-02	0.15067E-01	0.59530E-02
0.84	0.15479E-01	0.61168E-02	0.15644E-01	0.61808E-02
0.86	0.16082E-01	0.63552E-02	0.16230E-01	0.64124E-02
0.88	0.16695E-01	0.65971E-02	0.16825E-01	0.66473E-02
0.90	0.17320E-01	0.68441E-02	0.17431E-01	0.68867E-02
0.92	0.17954E-01	0.70944E-02	0.18044E-01	0.71293E-02
0.94	0.18601E-01	0.73498E-02	0.18670E-01	0.73765E-02
0.96	0.19257E-01	0.76086E-02	0.19304E-01	0.76268E-02
0.98	0.19925E-01	0.78725E-02	0.19949E-01	0.78817E-02
1.00	0.20602E-01	0.81397E-02	0.20602E-01	0.81397E-02

Table 4

ORDINARY MOMENTS OF K- AND L-FUNCTIONS

$\tau = 0.15$

Order	K_1	K_2	K_3	K_4
0	0.40608E 00	0.80640E 00	0.11454E 01	0.14450E-01
1	0.29909E 00	0.31087E 00	0.57984E 00	0.80201E-02
2	0.23749E 00	0.16790E 00	0.38776E 00	0.54844E-02
3	0.19712E 00	0.10592E 00	0.29119E 00	0.41560E-02
4	0.16855E 00	0.73215E-01	0.23311E 00	0.33430E-02

Order	L_1	L_2	L_3	L_4
0	0.33828E 00	0.49828E 00	0.77690E 00	0.14199E-01
1	0.25285E 00	0.23409E 00	0.45881E 00	0.79323E-02
2	0.20247E 00	0.13465E 00	0.32037E 00	0.54352E-02
3	0.16896E 00	0.87453E-01	0.24506E 00	0.41223E-02
4	0.14500E 00	0.61470E-01	0.19814E 00	0.33174E-02

$\tau = 0.25$

Order	K_1	K_2	K_3	K_4
0	0.43991E 00	0.84946E 00	0.11985E 01	0.23659E-01
1	0.32179E 00	0.33262E 00	0.61190E 00	0.13529E-01
2	0.25465E 00	0.18118E 00	0.41036E 00	0.93555E-02
3	0.21095E 00	0.11499E 00	0.30856E 00	0.71272E-02
4	0.18015E 00	0.79903E-01	0.24720E 00	0.57500E-02

Order	L_1	L_2	L_3	L_4
0	0.33305E 00	0.43740E 00	0.69517E 00	0.22814E-01
1	0.24826E 00	0.21790E 00	0.42838E 00	0.13197E-01
2	0.19870E 00	0.12938E 00	0.30482E 00	0.91630E-02
3	0.16583E 00	0.85708E-01	0.23543E 00	0.69935E-02
4	0.14236E 00	0.61093E-01	0.19143E 00	0.56481E-02

$\tau = 0.50$

Order	K_1	K_2	K_3	K_4
0	0.50555E 00	0.91385E 00	0.12880E 01	0.44399E-01
1	0.36616E 00	0.36802E 00	0.66867E 00	0.26548E-01
2	0.28817E 00	0.20397E 00	0.45138E 00	0.18720E-01
3	0.23791E 00	0.13118E 00	0.34051E 00	0.14406E-01
4	0.20271E 00	0.92172E-01	0.27331E 00	0.11692E-01

Order	L_1	L_2	L_3	L_4
0	0.31587E 00	0.33458E 00	0.55729E 00	0.40500E-01
1	0.23382E 00	0.18226E 00	0.36442E 00	0.24765E-01
2	0.18667E 00	0.11454E 00	0.26745E 00	0.17625E-01
3	0.15565E 00	0.78920E-01	0.21023E 00	0.13627E-01
4	0.13358E 00	0.57947E-01	0.17282E 00	0.11090E-01

Table 4 (cont.)

$\tau = 0.70$

Order	K_1	K_2	K_3	K_4
0	0.54596E 00	0.94473E 00	0.13368E 01	0.58672E-01
1	0.39375E 00	0.38637E 00	0.70093E 00	0.35856E-01
2	0.30906E 00	0.21643E 00	0.47522E 00	0.25555E-01
3	0.25471E 00	0.14038E 00	0.35934E 00	0.19784E-01
4	0.21676E 00	0.99350E-01	0.28883E 00	0.16117E-01
Order	L_1	L_2	L_3	L_4
0	0.30133E 00	0.27895E 00	0.48262E 00	0.50981E-01
1	0.22192E 00	0.15919E 00	0.32400E 00	0.32088E-01
2	0.17671E 00	0.10343E 00	0.24154E 00	0.23169E-01
3	0.14715E 00	0.73063E-01	0.19174E 00	0.18061E-01
4	0.12619E 00	0.54716E-01	0.15864E 00	0.14774E-01

$\tau = 1.0$

Order	K_1	K_2	K_3	K_4
0	0.59381E 00	0.97521E 00	0.13904E 01	0.76691E-01
1	0.42669E 00	0.40549E 00	0.73726E 00	0.47905E-01
2	0.33410E 00	0.22993E 00	0.50247E 00	0.34537E-01
3	0.27489E 00	0.15063E 00	0.38106E 00	0.26918E-01
4	0.23365E 00	0.10753E 00	0.30685E 00	0.22024E-01
Order	L_1	L_2	L_3	L_4
0	0.28021E 00	0.21967E 00	0.40224E 00	0.61752E-01
1	0.20501E 00	0.13190E 00	0.27649E 00	0.40064E-01
2	0.16263E 00	0.89097E-01	0.20933E 00	0.29407E-01
3	0.13511E 00	0.64889E-01	0.16788E 00	0.23150E-01
4	0.11570E 00	0.49813E-01	0.13991E 00	0.19059E-01

$\tau = 2.0$

Order	K_1	K_2	K_3	K_4
0	0.69265E 00	0.10246E 01	0.14933E 01	0.11624E 00
1	0.49574E 00	0.43867E 00	0.80872E 00	0.75178E-01
2	0.38706E 00	0.25459E 00	0.55704E 00	0.55280E-01
3	0.31779E 00	0.17012E 00	0.42511E 00	0.43623E-01
4	0.26968E 00	0.12358E 00	0.34375E 00	0.35992E-01
Order	L_1	L_2	L_3	L_4
0	0.22114E 00	0.12049E 00	0.25960E 00	0.71969E-01
1	0.15963E 00	0.79756E-01	0.18334E 00	0.49166E-01
2	0.12549E 00	0.58312E-01	0.14170E 00	0.37219E-01
3	0.10359E 00	0.45308E-01	0.11541E 00	0.29894E-01
4	0.88300E-01	0.36700E-01	0.97291E-01	0.24954E-01

Table 4 (cont.)

$\tau = 4.0$

Order	K_1	K_2	K_3	K_4
0	0.76063E 00	0.10611E 01	0.15826E 01	0.15215E 00
1	0.55821E 00	0.46446E 00	0.87179E 00	0.10049E 00
2	0.43553E 00	0.27452E 00	0.60584E 00	0.74851E-01
3	0.35738E 00	0.18635E 00	0.46492E 00	0.59575E-01
4	0.30313E 00	0.13726E 00	0.37736E 00	0.49452E-01

Order	L_1	L_2	L_3	L_4
0	0.14814E 00	0.62038E-01	0.15529E 00	0.58036E-01
1	0.10609E 00	0.43789E-01	0.11020E 00	0.40866E-01
2	0.82875E-01	0.33784E-01	0.85627E-01	0.31575E-01
3	0.68078E-01	0.27456E-01	0.70073E-01	0.25732E-01
4	0.57798E-01	0.23095E-01	0.59321E-01	0.21714E-01

$\tau = 8.0$

Order	K_1	K_2	K_3	K_4
0	0.84424E 00	0.10863E 01	0.16478E 01	0.17798E 00
1	0.60357E 00	0.48248E 00	0.91799E 00	0.11851E 00
2	0.47097E 00	0.28858E 00	0.64172E 00	0.89071E-01
3	0.38645E 00	0.19787E 00	0.49427E 00	0.71208E-01
4	0.32778E 00	0.14703E 00	0.40220E 00	0.59299E-01

Order	L_1	L_2	L_3	L_4
0	0.86162E-01	0.34094E-01	0.88497E-01	0.34925E-01
1	0.61612E-01	0.24372E-01	0.62769E-01	0.24765E-01
2	0.48070E-01	0.19010E-01	0.48763E-01	0.19235E-01
3	0.39444E-01	0.15595E-01	0.39906E-01	0.15739E-01
4	0.33457E-01	0.13225E-01	0.33788E-01	0.13324E-01

$\tau = 16.0$

Order	K_1	K_2	K_3	K_4
0	0.88381E 00	0.11020E 01	0.16884E 01	0.19403E 00
1	0.63196E 00	0.49366E 00	0.94680E 00	0.13019E 00
2	0.49304E 00	0.29730E 00	0.66410E 00	0.97915E-01
3	0.40456E 00	0.20503E 00	0.51258E 00	0.78446E-01
4	0.34314E 00	0.15310E 00	0.41771E 00	0.65427E-01

Order	L_1	L_2	L_3	L_4
0	0.46613E-01	0.18420E-01	0.47855E-01	0.18910E-01
1	0.33331E-01	0.13171E-01	0.33941E-01	0.13412E-01
2	0.26004E-01	0.10276E-01	0.26367E-01	0.10419E-01
3	0.21337E-01	0.84314E-02	0.21578E-01	0.85263E-02
4	0.18098E-01	0.71514E-02	0.18269E-01	0.72189E-02

Table 4 (cont.)

$\tau = 100.0$

Order	K_1	K_2	K_3	K_4
0	0.92241E 00	0.11172E 01	0.17280E 01	0.20970E 00
1	0.65957E 00	0.50457E 00	0.97491E 00	0.14130E 00
2	0.51458E 00	0.30581E 00	0.68594E 00	0.10654E 00
3	0.42223E 00	0.21201E 00	0.53045E 00	0.85507E-01
4	0.35813E 00	0.15902E 00	0.43284E 00	0.71406E-01

Order	L_1	L_2	L_3	L_4
0	0.80076E-02	0.31648E-02	0.82216E-02	0.32482E-02
1	0.57259E-02	0.22629E-02	0.58311E-02	0.23038E-02
2	0.44673E-02	0.17654E-02	0.45299E-02	0.17897E-02
3	0.36656E-02	0.14485E-02	0.37071E-02	0.14646E-02
4	0.31091E-02	0.12286E-02	0.31386E-02	0.12401E-02