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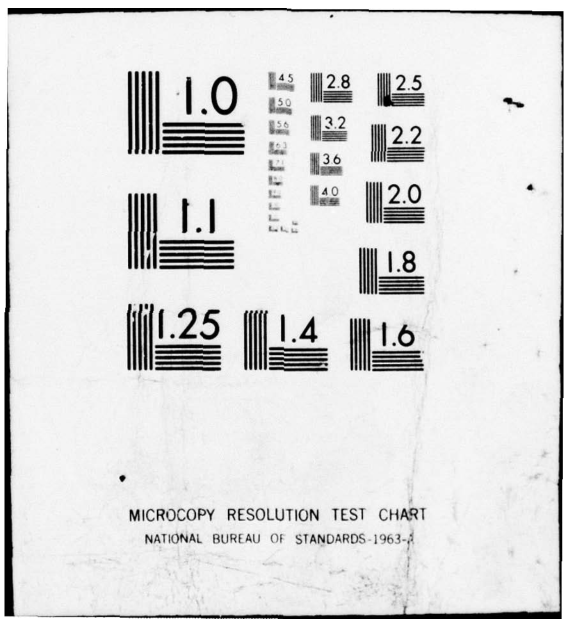
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NRL Report 8123



# Statistical Correlation Distance for Platform-to-Platform Radar Integration

B. H. Cantrell

*Radar Analysis Staff*
*Radar Division*

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The concept of using a statistical distance for associating target detections and tracks and for track maneuver detection has been used for sometime in tracking with search radars. The statistic normally used with collocated radars was derived using fundamental hypothesis-testing techniques. A parallel development using predicted positions as well as measured positions was performed, and a new statistical distance measure was obtained which would probably be of more use in integrating data from noncollocated sites. In general the form is quite complicated, but if the radar measurements are unbiased, if no bias exists between different radars, and if the predicted position is not corrupted

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20 Abstract (Continued)

by target maneuvers or previously misassigned detections, the statistical distance becomes easy to implement.

# STATISTICAL CORRELATION DISTANCE FOR PLATFORM-TO-PLATFORM RADAR INTEGRATION

## INTRODUCTION

In search-radar tracking systems the detections must be associated or correlated with the tracks. One way of correlating the detections with tracks is to compute a statistical distance between each track location and measurement and choose the detection to go with the track which has the minimum distance function. Usually those detection and track combinations are limited to a small region of space such that the distance would be below some threshold, thus reducing the number of calculations required dramatically. This type of correlation has been used with most search-radar tracking systems to date.

This report will first rederive the basic statistical distance used in most collocated radar tracking systems. The derivation will be based on the standard hypothesis-test procedure. Next the test will be complicated somewhat, and a new statistical measure will be obtained which would probably be of more use in integrating radar data from noncollocated sites.

## STANDARD STATISTICAL DISTANCE

We begin formulating the standard hypothesis test. Given a detection and measured position $X_m$, find which hypothesis $H_i$, $i = 1, ..., N$, has the highest probability of being true. This is given by

$$P(H_j/X_m) > P(H_i/X_m), \tag{1}$$

where $H_i$ is the hypothesis that the detection belongs to $i$th track. This should be read that the hypothesis $H_j$ is chosen if the probability of $H_j$ being true given the measurement is greater than the probability of $H_i$ being true given the measurement for all values of $i$. By use of Bayes rule the hypothesis test can be rewritten

$$\frac{P(X_m/H_j)P(H_j)}{P(X_m)} \geqslant \frac{P(X_m/H_i)P(H_i)}{P(X_m)}. \tag{2}$$

The $P(X_m)$ is irrelevant and can be removed. The measured position can be written as

$$X_m = X_T + N_m, \tag{3}$$

where $X_T$ is the target true position and $N_m$ is the measurement noise. By change of variable using (3), the hypothesis test (2) can be rewritten as

$$P_{N_m}(X_m - X_{T_j}/H_j)P(H_j) \geqslant P_{N_m}(X_m - X_{T_i}/H_i)P(H_i). \tag{4}$$

since the noise and the hypothesis are independent, (4) becomes

$$P_{N_m}(X_m - X_{T_j}) \geqslant P_{N_m}(X_m - X_{T_i})\frac{P(H_i)}{P(H_j)}. \tag{5}$$

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If the noise is Gaussian or near Gaussian, the hypothesis test (5) is equivalent to

$$[(X_m - X_T) - (\overline{X_m - X_T})]' R_{N_m}^{-1} [(X_m - X_T) - (\overline{X_m - X_T})]$$
$$< [(X_m - X_{T_i}) - (\overline{X_m - X_{T_i}})]' R_{N_m}^{-1} [X_m - X_{T_i}) - (\overline{X_m - X_{T_i}})] + C, \quad (6)$$

where $R_{N_m}^{-1}$ is the covariance matrix of noise, $C = ln\, P(H_i/P(H_j)$, and the prime denotes the transpose. If the measurement is unbiased ($\overline{X}_m = \overline{X}_{T_i}$) and the hypotheses have equal probability of occurrences $(P(H_i) = P(H_j))$, then (6) reduces to

$$[(X_m - X_{T_j})]' R_{N_m}^{-1} [(X_m - X_{T_j})] < [(X_m - X_{T_i})]' R_{N_m}^{-1} [(X_m - X_{T_i})]. \quad (7)$$

For "good" collocated radars the measurements can be made unbiased in most cases. (This is not true in general unless considerable effort at the radar level is made and bias-removal algorithms are used between different collocated radars.) With the assumption of no bias, for collocated radars the quantities are defined

$$X_m = \begin{bmatrix} r_m \\ az_m \end{bmatrix}, X_{T_i} = \begin{bmatrix} r_{T_i} \\ az_{T_i} \end{bmatrix}, \text{and } R_{n_m}^{-1} = \begin{bmatrix} 1/\sigma_R^2 & 0 \\ 0 & 1/\sigma_{az}^2 \end{bmatrix}. \quad (8)$$

The noise is uncorrelated in the range and azimuth measurement, and the hypothesis test becomes

$$\frac{(r_m - r_{T_j})^2}{\sigma_R^2} + \frac{(az_m - az_{T_j})^2}{\sigma_{az}^2} < \frac{(r_m - r_{T_i})^2}{\sigma_R^2} + \frac{(az_m - az_{T_i})^2}{\sigma_{az}^2}. \quad (9)$$

It is assumed that the $\sigma_R$ and $\sigma_{az}$ are estimated when the measurement is taken, and $r_m$ and $az_m$ are the measured quantities. However, the true position $r_{T_j}$ and $az_{T_j}$ are not known. To complete the test, $r_{T_j}$ and $az_{T_j}$ must also be estimated. The best estimate of the target's position comes from the tracking filter, which may be the Kalman filter or some variation of it. In any case the best estimate of the target's true position is the predicted position of the filter

$$x_{p_i} = X_{T_i} + N_{p_i} = \begin{bmatrix} r_{p_i} \\ az_{p_i} \end{bmatrix}. \quad (10)$$

Using the best estimate of the target's position $X_{P_i}$, the best estimate of the measurement error $\sigma_R$ and $\sigma_{az}$, and the measured position $X_m$, the hypothesis test becomes

$$\frac{(r_m - r_{p_j})^2}{\sigma_R^2} + \frac{(az_m - az_{P_j})^2}{\sigma_{az}^2} \geqslant \frac{(r_m - r_{P_i})^2}{\sigma_R^2} + \frac{(az_m - az_{P_i})^2}{\sigma_{az}^2}. \quad (11)$$

The statistical distance $D_i$ is defined as

$$D_i = \frac{(r_m - r_{P_i})^2}{\sigma_R^2} + \frac{(az_m - az_{P_i})^2}{\sigma_{az}^2}. \quad (12)$$

This is the usual statistical distance used in minimum-distance correlators for tracking with collocated radars. The statistical distance is also sometimes used for determining if a target is turning or not. For example, if the predicted position is biased from the true position, the distance $D_i$ becomes large and the bandwidth of filter should be opened up if the association was correct. At this point we will consider a somewhat more complicated correlator based on knowing both the measured and predicted position.

## NEW STATISTICAL DISTANCE

The formulation of the new statistical distance parallels the previous development. The hypothesis is

$$P(H_j/X_m, X_p) > P(H_i/X_m, X_p), \tag{13}$$

in which $H_i$ is the hypothesis that the measurement $X_m$ belongs to the $i$th track,

$$X_m = X_t + N_m, \tag{14}$$

and

$$X_p = X_t + N_p, \tag{15}$$

where $X_m$ is the measured position, $X_t$ is the target's true position, $N_m$ is the measurement noise, and $N_p$ is the predicted position noise. By use of Bayes rule the hypothesis test (13) can be written as

$$\frac{P(X_m, X_p/H_j)P(H_j)}{P(X_m, X_p)} > \frac{P(X_m, X_p/H_i)P(H_i)}{P(X_m, X_p)}. \tag{16}$$

The $P(X_m, X_p)$ is irrelevant to the test and can be ignored. Equations (14) and (15) can be used to change the variable in (16), yielding

$$P_{N_m,N_p}[(X_m - X_{t_j}), (X_{p_j} - X_{t_j})/H_j)P(H_j)]$$
$$\geqslant P_{N_m,N_p}[(X_m - X_{t_i}), (X_{p_i} - X_{t_i})/H_i)P(H_i)]. \tag{17}$$

The noise is independent of the hypothesis, and the a-priori probabilities are assumed equal $(P(H_i) = P(H_j))$ ; hence (17) can be written as

$$P_{N_m,N_p}[(X_m - X_{t_j})(X_{p_j} - X_{t_j})] \geqslant P_{N_m,N_p}[(X_m - X_{t_i}), (X_{p_i} - X_{t_i})]. \tag{18}$$

The probability distributions are assumed to be Gaussian. Even if they are not, this assumption often yields good tests. The test can then be written equivalently as

$$(V_j - \overline{V}_j)' R^{-1}_{N_m N_{p_j}} (V_j - \overline{V}_j) < (V_i - \overline{V}_i)' R^{-1}_{N_m N_{p_i}} (V_i - \overline{V}_i), \tag{19}$$

where

$$V_i = \begin{bmatrix} X_m - X_{t_i} \\ X_{p_i} - X_{t_i} \end{bmatrix} \text{ and } R_{N_m N_{P_j}} = \begin{bmatrix} R_{N_m N_m} & R_{N_m N_p} \\ R_{N_m N_p} & R_{N_p N_p} \end{bmatrix}.$$

Rather than rewrite the hypothesis test, the statistical distance is defined as

$$D_i = (V_i - \overline{V}_i) R^{-1}_{N_m N_{p_i}} (V_i - \overline{V}_i). \tag{20}$$

The inverse of the covariance matrix can be rewritten using a vector identity as

$$Q = R^{-1}_{N_m N_{p_i}} = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix}, \tag{21}$$

where

$$Q_{11} = (R_{n_m n_m} - R_{n_m n_p} R^{-1}_{n_p n_p} R_{n_p n_m})^{-1},$$
$$Q_{12} = -Q_{11} R_{n_m n_p} R^{-1}_{n_p n_p},$$
$$Q_{21} = -R^{-1}_{n_p n_p} R_{n_p n_m} Q_{11},$$

and

$$Q_{22} = R^{-1}_{n_p n_p} (I + R_{n_p n_m} Q_{11} R_{n_m n_p} R^{-1}_{n_p n_p})^{-1}.$$

The statistical distance defined in (20) is quite complex, and many of the parameters in the test are unknown. As before, the unknown parameters in the test must be estimated. The best least-square estimate $X_e$ of the target's true position $X_{t_i}$ given the predicted and measured position by standard calculations is

$$X_{t_i} = X_e = (Q_{11} + Q_{22} + Q_{12} + Q_{21})^{-1} [(Q_{11} - Q_{12}) X_{p_i} + (Q_{22} - Q_{21}) X_m]. \quad (22)$$

The other quantities to be estimated are as follows. The covariance matrix $R_{n_m n_m}$ is estimated at the radar, and the covariance matrix $R_{n_p n_p}$ is obtained from the tracking filter. For rather rapid update times the crosscorrelations $R_{n_m n_p}$ and the mean $\overline{V}_i$ can be estimated by averaging the results over the last few updates. The procedure just described is quite complicated, but the statistical distance can be simplified for a special case.

The first simplification is to set the mean value of $V_i$ to zero, which means $X_m$ and $X_{p_i}$ have mean values equal to the true target's position and are therefore unbiased estimators. The second simplification is to set the crosscorrelation between the measured and predicted positions $R_{n_m n_p}$ to zero. The statistical distance (20) then reduces to

$$D_i = (x_m - x_e)' R_{n_m n_m}^{-1} (x_m - x_e) + (x_p - x_e)' R_{n_p n_p}^{-1} (x_p - x_e), \quad (23)$$

where

$$X_e = (R_{n_m n_m}^{-1} + R_{n_p n_p}^{-1})^{-1} (R_{n_m n_m}^{-1} X_m + R_{n_p n_p}^{-1} X_p). \quad (24)$$

The quantity $D_i$ is the sum of two squared Mahalanobis distances.*

Equation (23) is a good statistical distance measure if the measured and predicted positions are unbiased estimates, which is usually the case when the targets are nonmaneuvering, the tracks have not previously been corrupted, and the measurements are unbiased because of the use of good radar and radar-alignment procedures. However, if this condition of nonbiased estimates does not hold, the mean value of the $\overline{V}_i$ and the crosscorrelations between the predicted and measured positions should be estimated, since they are now nonzero.

The question naturally arises as to how much better the test given by (20) is over the simplified test given by (23): Is it worthwhile performing all the extra computing of estimating the mean of $V_i$ and the crosscorrelation terms. This question is not addressed directly in this report. The answer requires a long-term simulation and analysis procedure with the use of many typical situations. However, a few comments are in order. First, given that the radar measurements are unbiased and no bias exists between different radars, the predicted position is unbiased typically when the target is not maneuvering or the track has not been previously corrupted most of the time. The simplified statistical distance is sufficient under these circumstances. If the predicted position is corrupted because of target maneuvers or previously misassigned detections, the estimates required for the hypothesis test will be rather poor anyway, and proper sorting of the data will probably require a longer time history of detections. Therefore it is felt that the simpler hypothesis test given by (23) is sufficient to use in all cases. The statistical distance measure (23) can also be used as a turn detector in that if the detection and tracks are properly associated and the statistical distance is large, the target is turning and hence the bandwidth of the filter should be opened.

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*G. V. Trunk and J. D. Wilson, "Tracking Filters for Multiple-Platform Radar Integration," NRL Report 8087, Dec. 14, 1976.

**SUMMARY**

The concept of correlating or associating detections with tracks uses a measure commonly called statistical distance. With the use of basic concepts in hypothesis testing, the statistical distance commonly used in search-radar tracking systems was derived. By use of more of the information available that is, by use of predicted positions as well as measured positions, a new hypothesis test was constructed and a new statistical distance was obtained which would probably be of more use in integrating radar data from noncollocated sites. In general the form is quite complicated, requiring some rather difficult estimation. Under a special case the statistical distance becomes easy to implement and is essentially the same as the Mahalanobis distance. All that is essentially required is that the measured and predicted positions be unbiased estimators, which is true in most cases. In the cases this is not true, such as maneuvering targets, corrupted tracks, and biased radar data, it is generally thought that the general form of the statistical distance is not neccessary to compute because other factors must enter into the correlation process as well. Finally the statistical distance can be used to determine if the target is maneuvering.