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TECHNIQUES FOR STUDYING SEA ICE DRIFT AND DEFORMATION AT SITES --ETC(U)  
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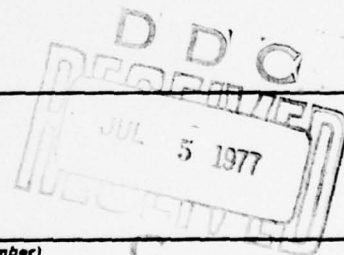
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20. Abstract (cont'd)

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→ system of a given floe is used as the common coordinate system, is discussed. Errors in coordinate system rotation, center location, distortions and non-linearities in the images caused errors in vorticity of the order of 0.5% and in strain of the order of 0.1% per day. Both these errors are less than typical sea ice vorticity and strain rate.



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TECHNIQUES FOR STUDYING SEA ICE DRIFT AND  
DEFORMATION AT SITES FAR FROM  
LAND USING LANDSAT IMAGERY

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ABSTRACT

In order to estimate sea ice drift and deformation using LANDSAT imagery far from shore, it is necessary to transfer the location coordinates of a common set of ice features from one image to another. The accuracy of the drift rate and the vorticity determination is dependent upon the accuracy of the coordinate transfer. The strain rate, on the other hand, is, with certain important exceptions, dependent only upon the accuracy with which common ice features on a pair of images can be brought into coincidence. The key exception here is that errors in the rotation of the coordinate system that are incurred during the transfer process will induce spurious strain rates if velocities are estimated by merely subtracting location coordinates in a rectangular coordinate system. This important effect will also occur if a given rotating floe is used to establish a common coordinate system. Such spurious effects can be avoided by using an appropriate least-squares strain program in polar coordinates.

A semi-automatic procedure for rapidly and accurately transferring ice coordinates from one LANDSAT image to another and for simultaneously estimating all linear measures of the ice deformation is described. The procedure takes into account the non-parallel nature of the longitude lines and the finite curvature of the latitude lines, factors which are particularly critical in the polar regions. Necessary inputs are the location coordinates (latitude and longitude) of the center of each image and the location of two arbitrary points on a line of longitude on the image. This input information is then utilized in a set of equations which give the effective rotation and translation needed to transfer the north-south, east-west axes centered on one image onto similar axes on another image. These equations, which are valid over distances of several hundred kilometers, bypass the complex and time-consuming procedure of projecting points on the spheroid. After the transfer of common ice feature locations (on successive days) is completed, a least-squares program yields the average strain rate and vorticity with the strain rate being independent of errors in the transfer of the coordinate system.

The accuracies of the various elements of the procedure are examined using imagery over land and are found to be dominated by deviations (as large as 8 km) of the actual position of the center of the image from its stated position. The average errors produced by these deviations are found to be less than typical one day ice drift distances observed in the Beaufort Sea. Similarly errors in the rotation of the coordinate system due to inaccuracies in longitude lines and in the center locations were found to induce vorticity errors of the order of 0.5%. Although these errors were also significant, they were less than typical sea ice vorticities observed in the same area. With regard to strain, distortions and non-linearities in images were found to cause spurious strains of the order of 0.1% per day. This is somewhat smaller than typical spatial variations in strain that are produced by the non-linear nature of the ice velocity field when it is observed on the scale of a LANDSAT image.

## 1. INTRODUCTION

One use of satellite imagery of considerable importance in sea ice modeling is the estimation of sea ice drift and deformation in order to provide observational checks on model results. This can be done by determining the absolute (on the earth's surface) and the relative positions of the ice by examining satellite images that are sequential in time. Although long time series are rare because of the inability of most sensors to penetrate clouds current imagery is, nevertheless, well suited for examining spatial variations in ice deformation and drift. In particular, the use of LANDSAT imagery for estimating ice drift rates near land has been examined by Hibler et al. (1974a) and by Shapiro and Burns (1975).

In these papers the basic procedure has been to identify common features on sequential images so that relative distance changes can then be measured. If some points on the image of interest are on land then drift rates in addition to deformation rates may be estimated since the land points do not move. It appears that for greatest accuracy the identification of common ice points is best accomplished by overlaying two transparencies and shifting these until a common ice floe or land feature comes into coincidence; a technique first used by Crowder et al. (1974) and Nye and Thomas (1974).

However, when LANDSAT images far from shore are used for analysis, the problem of obtaining both drift and strain rates becomes considerably more difficult. The principal difficulty with drift rate estimation is that no immovable land points are available for "calibration" and consequently one has to use the coordinates of the centers of the images as well as available longitude marks to fix the image locations. In principal the problem of drift then becomes one of first projecting common ice features (on sequential images) onto the spheroid and then back onto a common tangent plane. A somewhat more useful procedure, which we will discuss in this paper, is to transfer the coordinates of both images (considered to be tangent planes) onto a common intermediately located, tangent plane. In practice, such a procedure simply consists of appropriately translating and rotating the polar coordinates of all points in sequential images to the coordinate system of the first image.

The estimation of the strain rate, on the other hand, is dependent in a more subtle way on the transfer of coordinates from one image to another. The key problem here is caused by spurious rotations of the coordinate system that can be induced by errors in longitude lines. Such spurious rotations can also be induced by using a common rotating floe to establish a common coordinate system. The basic problem is that if velocities are estimated by merely subtracting location coordinates in a rectangular coordinate system then such spurious rotations will induce concomitant errors in the diagonal components of the strain rate tensor. Such undesirable effects can be avoided by using an appropriate least-squares strain program in polar coordinates, an example of which will be discussed later.

The final deformation parameter, vorticity, is dependent on the transferral accuracy in much the same manner as the drift rates. This is because the vorticity is closely related to the rotation [see for example Nye (1974)] of the pack ice (for a solid body rotation the vorticity is proportional to the rotation) which in turn is a function of the drift rate as well as the azimuth orientation.

In the following section of this paper the various elements of the coordinate transferral technique and the least squares strain estimation procedure mentioned above are discussed in more detail. In addition the accuracies and limitations of the various components of the technique are examined using imagery over land. Finally several sequences of sea ice LANDSAT images are examined in order to illustrate typical drift rates, deformation rates and spatial variations in the strain rates.

## 2. COORDINATE TRANSFER AND STRAIN ESTIMATION TECHNIQUE

### 2.1 COORDINATE TRANSFER PROCEDURE

In order to estimate drift rates from LANDSAT imagery far from shore it is necessary to transfer the location of a common set of ice features from one image to another. For this purpose it is convenient to view each LANDSAT image as a plane tangent to the Earth's surface at the center of the image. The coordinates of the center of this plane, as estimated from the satellite orbit, are given on

the image. For orienting an x-y axis on this plane we consider a right handed coordinate system with the x axis pointing North in alignment with the longitude line passing through the center of the image.

Let us now consider two images whose centers are tangent to the earth's surface at latitudes ( $\theta$ ) and longitudes ( $\phi$ ) of ( $\theta, \phi$ ) and ( $\theta', \phi'$ ) respectively. One way of transferring the coordinates from the plane of image 2 to the plane of image 1 would be to project the coordinates on the spheroid from plane 2 and then from the spheroid onto plane 1. Such a procedure is relatively complex, especially if a nonspherical surface, such as the International Spheroid is used for projection. A somewhat simpler procedure would be to project both images onto a common plane tangent at some intermediate location. Moreover, as a first approximation we can assume that all three surfaces are essentially co-planar, i.e., all distances between two ice features projected on any of the three planes are taken to be the same. The errors induced by such an assumption are of the order of  $d^3/R^2$  where  $d$  is the separation between photo centers and  $R$  is the earth's radius. Once such an assumption is made then the transfer of coordinates becomes one of simple translation and rotation.

In order to set up a geometry for this transfer we assume that both images are in the plane whose perpendicular is given by the cross product of the two north vectors tangent to the earth at latitude  $\theta$ , and parallel to the longitude lines  $\phi$  and  $\phi'$ . The angle between these two vectors defines the angle between the x-axes of the two image coordinate systems for image centers on the same latitude line. For simplicity we also take this angle to be the same if the second image is on both a different longitude line and a different latitude than the first, again an assumption inducing errors of order  $d^3/R^2$ . By taking an inner product, the angle between these two vectors  $\gamma$ , is

$$\gamma = \cos^{-1} \left\{ \cos^2 \theta + \sin^2 \theta \cos [\phi' - \phi] \right\}$$

For small values of  $(\phi' - \phi)$  this expression reduces to

$$\begin{aligned} \gamma &\approx \cos^{-1} \left\{ 1 - \frac{[\phi' - \phi]^2}{2} \sin^2 \theta \right\} \\ &\approx \cos^{-1} \left\{ \cos [(\phi' - \phi) \sin \theta] \right\} = [\phi' - \phi] \sin \theta \end{aligned}$$

With the above assumptions the geometry for the coordinate transfer is given in Figure 1, where points 1 and 2 are the centers of images 1 and 2 respectively. The angle of rotation between the coordinate systems is given by  $\gamma$  and the translation distance is defined by the chord distances  $D_1$  and  $D_2$  along the latitude and longitude lines. The values of  $D_1$  and  $D_2$  can be calculated from the expressions:

$$D_1 = \frac{2 \sin \left[ \frac{\phi' - \phi}{2} \right]}{[\phi' - \phi]} D_{LAT}$$

$$D_2 = \frac{2 \sin \left[ \frac{\theta' - \theta}{2} \right]}{[\theta' - \theta]} D_{LONG}$$

where  $D_{LAT}$  is the distance on the International Spheroid between longitude  $\phi'$  and  $\phi$  measured along latitude line  $\theta$  and  $D_{LONG}$  is the International Spheroid distance between latitudes  $\theta'$  and  $\theta$  measured along longitude line  $\phi'$ . Approximate expressions for these distances are given on page 1187 of Bowditch (1958). Another alternate approximation for these distances may be obtained by using a sphere with a radius equal to the average radius of the International Spheroid as a model for the earth's surface. If this is done, however, it is fairly important to convert the geographical coordinates of the image centers to parametric values.

In order to give an idea of the errors induced by the approximations in the coordinate transfer procedure, we have compared the distance and angles between a number of points obtained using our LANDSAT projection procedure for both a sphere (referred to as the LANDSAT SPHERE) and the International Spheroid (referred

to as the LANDSAT SPHEROID) with the results obtained by using a planar projection onto a fixed plane tangent to point 1. For the spherical cases, we considered point 1 to be at lat 80°, long 0° and have also considered 9 other points, 3 located along lat 80° at increasing distances toward the west, 3 located along long 0° at increasing distances toward the north and 3 located at increasing distances toward the northwest. The results are summarized in Table I with positive angles being measured in a clockwise direction from the x (north) axis. Point 1 is taken to be the origin. In the spherical calculations a radius equal to the average radius of the International Spheroid was used. The great circle distance was computed using the spherical surface.

Table I shows that there is very little difference between the various projections with the maximum difference being about 1 km at 600 km. In actual practice the maximum transfer distances are rarely greater than 250 km so that differences in the various projections are hardly significant. Other characteristics worth noting are that the LANDSAT SPHERE projection gives slightly greater distances than the planar projection and are consequently closer to great circle distances. In the case of point 2 located in a northwest direction from point 1, the LANDSAT SPHERE transfer technique gives slightly greater distances than the great circle distances. This is a consequence of assuming, as shown in Figure 1, that the longitude lines are straight. For a more graphic illustration of the deviation between the Planar and the LANDSAT SPHERE projections we have in Figure 2 plotted the difference in angle and distance between these two projections for the three northwest points listed in Table I.

TABLE I. COMPARISON OF PLANAR AND LANDSAT PROJECTIONS.

Location of initial pt. 80°N, 0°W.  
Radius of Reference Sphere was 6371.299 km

Location of Final Pt		Great Circle Distance	LANDSAT SPHERE Projection	Planar Projection using Sphere	LANDSAT SPHEROID Projection using International Spheroid*			
Long	Lat	Distance	Angle	Distance	Angle	Distance	Angle	Distance
0°W	81.2°N	133.439 km	0°	133.436 km	0°	133.429 km	0°	133.572 km
0°W	82.6°N	289.117	0°	289.092	0°	289.018	0°	289.414
0°W	85 °N	555.995	0°	555.819	0°	555.290	0°	556.415
6°W	80 °N	115.806	-87.046	115.804	-87.045	115.799	-87.044	115.932
14°W	80 °N	269.681	-83.106	269.661	-83.105	269.600	-83.105	269.959
25°W	80 °N	479.030	-77.690	478.917	-77.684	478.579	-77.688	479.446
6°W	81.2°N	172.110	-36.308	172.113	-36.302	172.089	-36.310	172.294
14°W	82.6°N	370.874	-32.413	370.915	-32.390	370.693	-32.411	371.323
25°W	85 °N	651.477	-21.262	651.759	-21.152	650.342	-21.261	652.459

\*The parametric latitude values of the points used for the spherical calculations were converted to geographical latitude values for use with the Spheroid program.

## 2.2 LEAST-SQUARES STRAIN RATE AND VORTICITY ESTIMATION

In practice the estimation of strain rates in most geophysical media, such as ice sheets or rock masses, is made by measuring the relative positions of a number of points versus time. Such information is then used to estimate least-squares strain rates in one of two ways. The first of these, which has normally been used in studies of glacier deformation, has been to calculate linear strains along a variety of strain lines as a function of time, and then note that such one dimensional strain rates along given lines, say  $pp'$ , which we shall denote by  $\dot{\epsilon}(pp')$ , are related to the strain-rate tensor by

$$\dot{\epsilon}(pp') = \dot{\epsilon}_{11} \cos^2 \theta + \dot{\epsilon}_{22} \sin^2 \theta + 2\dot{\epsilon}_{12} \sin \theta \cos \theta$$

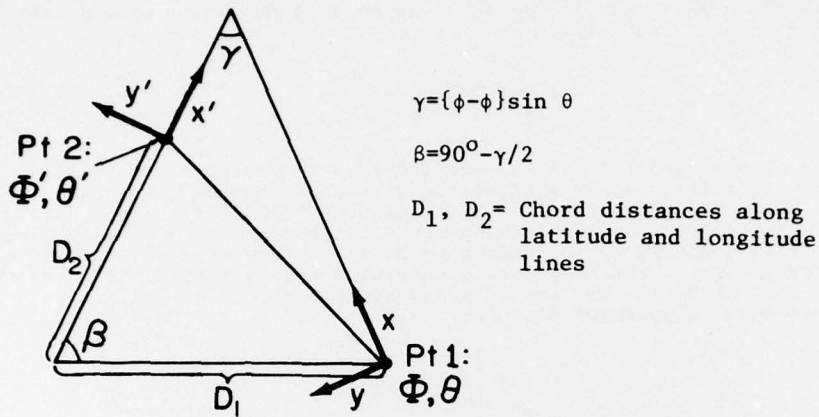


Figure 1. GEOMETRY FOR "LANDSAT" COORDINATE TRANSFERRAL

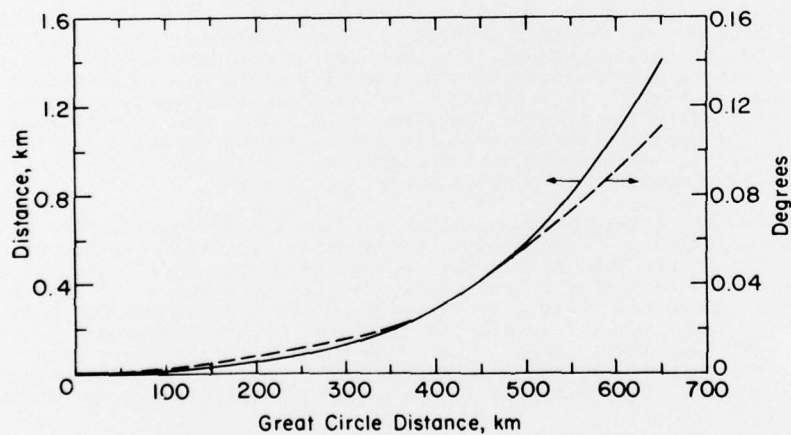


Figure 2. COMPARISON OF PLANAR AND "LANDSAT SPHERE" PROJECTIONS

Difference between the polar co-ordinates of a point on the Earth's surface (versus great circle distance) as obtained by conventional planar projection and by the "LANDSAT SPHERE" technique used in this paper.

where  $\theta$  is the angle between the line  $pp'$  and the  $x$  axis of a chosen coordinate system. This result is easily obtained by transforming  $\epsilon_{ij}$  into a coordinate system with the  $x$  axis parallel to  $pp'$ . In this equation the strain rate tensor is denoted by  $\dot{\epsilon}_{ij}$  and is defined by

$$\dot{\epsilon}_{ij} \equiv \frac{1}{2} \left[ \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right]$$

This procedure was used for sea ice measurements by Hibler et al. (1973a) and a derivation of a least-squares application of such an approach is given in Hibler et al. (1973b). One drawback of this technique is that all lines are equally weighted regardless of their length, whereas in fact it is physically reasonable that longer lines should be weighted more heavily since they will presumably be more representative of the broad scale continuum motion that is of principle interest. They also will be less affected by measurement errors. A second drawback is that such an approach does not yield a vorticity,  $w$ , defined by

$$w \equiv \frac{1}{2} \left[ \frac{\partial u_2}{\partial x_1} - \frac{\partial u_1}{\partial x_2} \right]$$

The second way of estimating least-squares strain has been to note that the  $x$  and  $y$  ice velocities at position  $i$ ,  $(u_{x_i}, u_{y_i})$  are related to the strain rate and vorticity by (assuming a linear velocity field)

$$\begin{aligned} u_{x_i} &= x_i \dot{\epsilon}_{11} + y_i [\dot{\epsilon}_{12} - w] + A_1 \\ u_{y_i} &= x_i [\dot{\epsilon}_{12} + w] + y_i \dot{\epsilon}_{22} + A_2 \end{aligned} \quad (1)$$

A complete discussion of the application of least-squares equations of this type to sea ice deformation is given in Hibler et al. (1974b). This reference also discusses the estimation through least squares of the experimental measurement error for the strain rates and the inhomogeneity error due to nonlinearities in the ice velocity field. In standard analysis these two types of errors, which come from very different origins, are not usually separated. This second approach also has the advantage that the velocity points further apart automatically affect the strain-rate estimates more than do velocities of points close together. Also the vorticity is computed in a normal least-squares way.

A drawback to this approach is that if velocities are estimated by merely subtracting location coordinates in a rectangular coordinate system errors will be induced in the strain rate by spurious rotations of the coordinate system. The basic problem can be easily visualized by examining Figure 3. In it we consider three points, which are stationary relative to one another and consequently undergo zero strain. Using the three relative distances between these points we would in fact find a strain rate of zero for all components of the strain rate tensor. However, using rectangular coordinates and equations (1) we would obtain for a clockwise rotation of  $\theta$  radians per unit time the strain rates and vorticity:

$$\begin{aligned} \dot{\epsilon}_{12} &= 0 \\ \dot{\epsilon}_{11} &= \dot{\epsilon}_{22} = \cos\theta - 1 \\ w &= \sin\theta \end{aligned}$$

or for a  $3^\circ$  rotation  $\dot{\epsilon}_{11} = \dot{\epsilon}_{22} = -1.37 \cdot 10^{-3}$  per unit time. A consistent way around this problem is to calculate the coordinates and velocities of the points in polar coordinates using as the direction of the unit  $r$  and  $\theta$  vectors the initial polar coordinates of the points. In particular if the initial and final positions of a point are given by the polar coordinates  $(r, \theta)$  and  $(r', \theta')$  with  $\theta$  measured clockwise from the  $y$  axis, then the  $x$  and  $y$  velocities are given by

$$u_x = (r'-r) \sin\theta + r[\theta'-\theta] \cos\theta$$

$$u_y = (r'-r) \cos\theta - r[\theta'-\theta] \sin\theta$$

Using these velocity estimates in equation 1, we would obtain (using Figure 3) for a clockwise rotation of  $\theta$  per unit time:

$$\epsilon_{11} = \epsilon_{22} = \epsilon_{12} = 0$$

$$w = [\theta'-\theta]$$

For the general case of many points utilizing least-squares equations based on Equations 1, it can be shown (Hibler et al., 1974b) that adding a constant rotation to all angles only changes the vorticity.

Besides the special example considered here, this rotation effect can be illustrated by actual data. Such an illustration is given in Figure 4, where various rotational errors in the coordinate transfer of five common ice features between two LANDSAT images was purposely added. As can be seen, in agreement with the simple example of Figure 3, rotations of  $3^\circ$  can cause significant undesirable effects.

In using this "polar" technique it should be noted that as long as one chooses one of the points as the origin, where the center of rotation is located has no effect on the results. This is because all lines connecting points will rotate by the same amount regardless of where the center of rotation is located. This will not, however, be true if one chooses for the origin some arbitrary point which may not actually represent a point on the ice.

### 3. EXAMINATION OF ERRORS IN THE TRANSFERRAL AND STRAIN TECHNIQUES

In order to test the proposed technique in actual practice we have utilized a number of overlapping LANDSAT images of Alaska. From pairs of such images common land features were identified and their coordinates were transferred from the second image to the coordinate system of the first image. Because the points were "immovable", apparent motions could be identified as errors.

#### 3.1 TRANSFERRAL ERRORS

After examination of a number of such image pairs it became apparent that the dominant source of error in the coordinate transfer was the result of inadequacies in either the information given about the images or in the images themselves. In fact, the greatest problem was caused by deviations between the actual location of the center of the image and its stated location. On the average these positional inadequacies induced errors of the order of one kilometer. Such errors, although much greater than the errors inherent in the transferral equations, are still generally acceptable for purposes of estimating ice drift rates.

The actual errors obtained from a series of 6 photo pairs are given in Table II and Table III. Table II gives the average distance error, with a standard deviation, of several common points on pairs of overlapping images. Note that the standard deviation is quite small ( $\approx .1$  km) indicating that the translation error varies little with the location of a given point on the image. In Table III rms errors before and after correcting the location of the center position are given. The center positions were corrected by overlaying the images on accurate survey maps with a zoom transfer scope. As can be seen from Table III, using the actual center position improves the translation errors by about half a kilometer on the average. The actual errors, however, between the true and stated center location can be as large as 8 km. This is illustrated in Table IV, which summarizes the center position errors of 17 photographs. The maximum deviation observed was 8.12 km (in the east-west direction). These numbers are in general agreement with the study by Colvocoresses (1974) who has found 1 to 8 km errors in the latitude and longitude center indicators on LANDSAT photos.

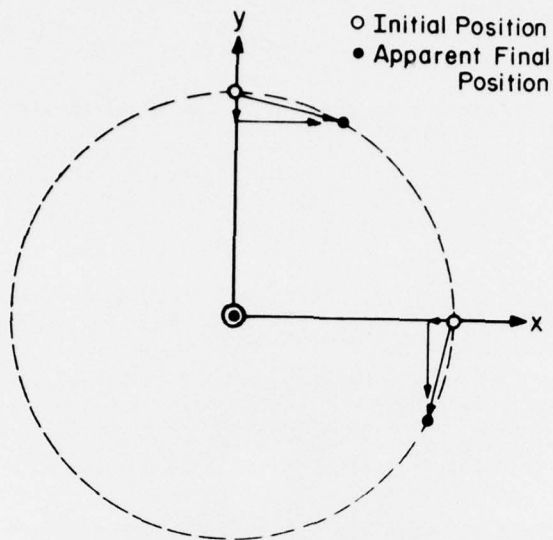


Figure 3. EFFECT OF SPURIOUS ROTATIONS ON ESTIMATED VELOCITIES

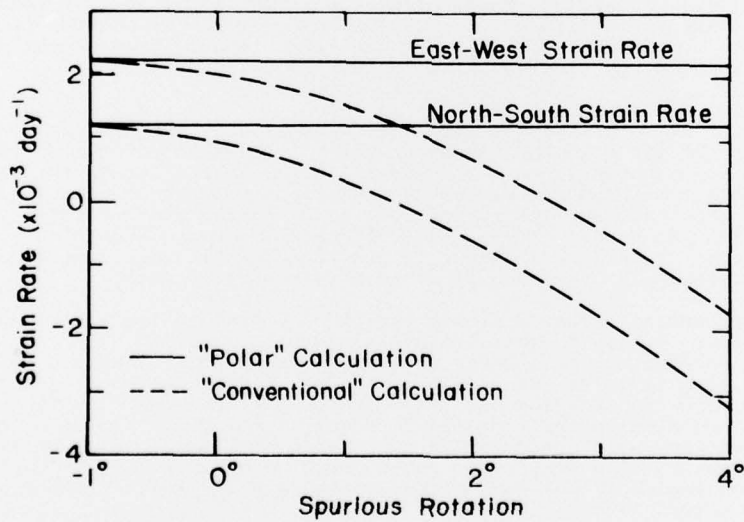


Figure 4. EFFECT OF SPURIOUS ROTATIONS ON ESTIMATED STRAINS

TABLE II. MEAN TRANSLATION ERRORS FROM SEQUENTIAL IMAGERY OVER ALASKA.

[Actual - Translated Distances] in km

Photo Pair	$\Delta X(\text{North})$	$\Delta Y(\text{East})$
1	-1.66 $\pm$ .055	1.14 $\pm$ .152
2	-2.13 $\pm$ .231	- .41 $\pm$ .329
3	-1.63 $\pm$ .171	- .475 $\pm$ .126
4	-2.43 $\pm$ .208	- .190 $\pm$ .182
5	- .53 $\pm$ .163	.10 $\pm$ .051
6	+ .17 $\pm$ .058	.21 $\pm$ .162

TABLE III. RMS TRANSLATION ERRORS.

Photo Pair	Before Center $\Delta X(\text{North})$	Position Correction $\Delta Y(\text{East})$	After Center $\Delta X(\text{North})$	Position Correction $\Delta Y(\text{East})$
1	1.66 km	1.14 km	.37 km	.11 km
2	2.14	.49	1.22	.49
3	1.63	.49	.72	.15
4	2.43	.24	.53	.13
5	.55	.11	.55	.58
6	.17	.25	.20	.24
Average	1.43 $\pm$ .89	.45 $\pm$ .37	.60 $\pm$ .35	.28 $\pm$ .20

TABLE IV. SUMMARY OF CENTER POSITION ERRORS FOR 17 IMAGES OVER ALASKA.

	X(North)	Y(East)
Max. Deviation	1.73 km	8.12 km
Min. Deviation	0.20	0.19
RMS Deviation	0.84	3.50

The reason that such center errors do not induce greater errors in the transfer appears to be due to the consistency of the offset errors; that is, pairs of photos will have similar errors in both the magnitude and the direction of their center positions. This appears to be especially true for the longitude errors which although large (see Table IV) do not seem to particularly effect east-west transferral accuracy (see Table III). Latitude errors on the other hand seem to be smaller but perhaps more random (see Table III and IV). However, there is no assurance that the updating of orbital parameters will not be made in the middle of a sequence. Consequently, for this and other reasons related to strain, discussed later, it would seem advisable to examine at least three photos in a sequence when estimating drift and deformation.

### 3.2 STRAIN AND VORTICITY ERRORS

As discussed earlier, if appropriate least-squares programs are used, strain estimates will be independent of coordinate transferral errors. These estimates will, however, depend upon the registration error of common features and upon nonlinearities in the photographs. Besides being dependent on such factors, vorticity estimates additionally depend on errors in the orientation estimates of the coordinate systems on the two images. To make a direct estimate of these errors we calculated least-squares strain and vorticity for the photo pairs used in the above transferral study. The results are shown in Table V where we list

the average strain rates and vorticities (which should be zero since there has been no motion) together with the inhomogeneity errors. As can be seen, on the average the strains are of the order of 0.1 to 0.2% per day which would represent a 0.2 km variation over 100 km, the approximate size of the arrays we were using.

TABLE V. APPARENT STRAINS ON LAND MASSES.

Photo Pair	Units: Percent per day			
	$\epsilon_{xx}$	$\epsilon_{yy}$	Shear	Vorticity
1	-0.069 ± 0.106	-0.276 ± 0.087	0.044 ± 0.069	0.063 ± 0.069
2	-0.178 ± 0.041	-0.033 ± 0.108	0.247 ± 0.058	-0.386 ± 0.058
3	0.024 ± 0.049	0.098 ± 0.053	-0.246 ± 0.036	0.143 ± 0.036
4	-0.241 ± 0.104	0.118 ± 0.098	0.108 ± 0.072	-0.051 ± 0.072
5	-0.142 ± 0.046	0.159 ± 0.053	0.109 ± 0.035	0.294 ± 0.035
6	-0.020 ± 0.157	0.402 ± 0.089	-0.078 ± 0.090	0.125 ± 0.090
RMS	0.139	0.219	0.159	0.215

However, not all the error can be attributed to random errors such as the registration errors. If this were the case then the average strain rates in Table V would cluster around zero with the error being somewhat larger than the average strain. A more consistent way to interpret the results in Table V would be to say that in addition to registration errors there are certain distortions in the photos. Assuming such distortions are approximately linear, the estimated strain would be equal to the average distortion with the residual error indicative of the registration error. For the six photos analyzed the average residual error is 86 m which is almost exactly the stated resolution (80 m) of the LANDSAT multi-spectral scanning system. The rest of the strain can be accounted for by saying that distortions induce about 100 m errors per 100 km. Consequently as a practical manner for estimating experimental errors in strain we can assign equal errors of about 100 m to the registration of points (which is in agreement with Colvocoresses (1974) estimates of geometric fidelity) and to image nonlinearities over distances of 100 km.

The rms vorticity was only a little larger than the rms strain rate indicating that rotation effects were relatively minor. However, comparisons of the angles of the line of longitude passing through the center of a LANDSAT image, as obtained from different lines of longitude marked on the image, indicate that the estimated angles often differ by 0.01 radian. This effect is somewhat ameliorated by taking the average estimated angle from two longitude lines located as near to the image center as the image tic marks allow. Still, because of such errors, it would seem safer to assume the experimental vorticity error to be of the order of 0.5% per day.

#### 4. EXAMPLES OF DRIFT AND DEFORMATION RATES FAR FROM LAND

To illustrate the application of these techniques to actual sea ice drift and deformation measurements, we examined three sequences of images from March 1973 in the region 78°N', 170°W. The starting position and day for each sequence is shown in Figure 5. Seq. 11 consisted of 7 images; Seq. 12, 6 images; and Seq. 13, 4 images. Four to six common ice features were digitized on each sequential image pair and the average drift and deformation calculated. The drift rates are shown in Figure 6. The westward drift rates generally decrease as one moves to the north, a fact which is in general agreement with the clockwise motion of the Pacific Gyre. Also the drift rates are on the average several kilometers a day which is larger than the observed transferral error of about 1 km per day. In general these drift rates also agree with average drift rates of ice islands which are found to typically drift about 3.6 km/day in this region (Dunbar and Wittmann, 1962).

In presenting the deformation rate results, it is convenient to consider two day running means, which are illustrated in Figure 7. In this figure we show the

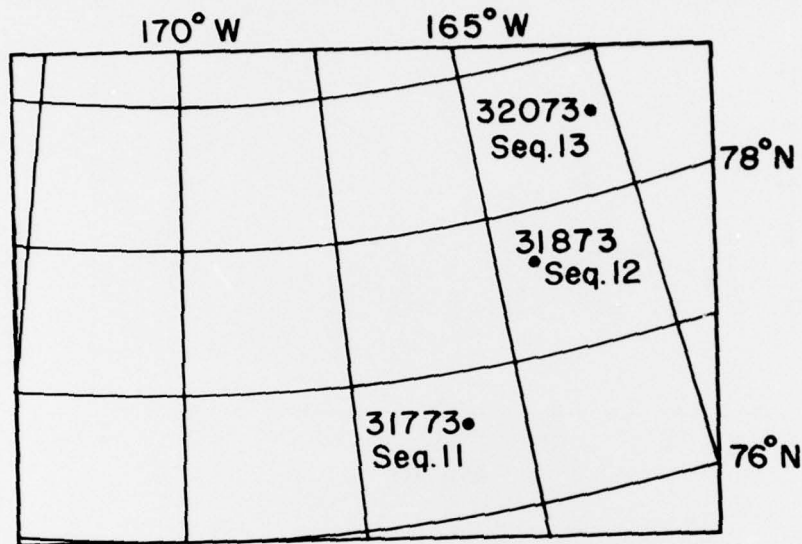


Figure 5. LOCATION OF INITIAL IMAGES IN SEA ICE IMAGE SEQUENCES

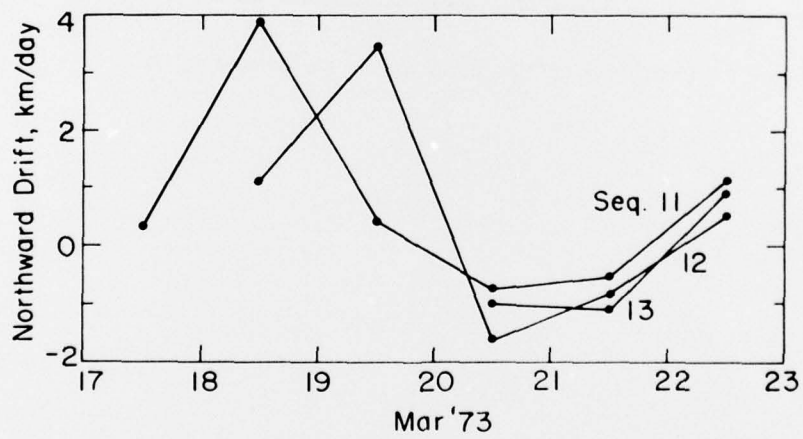
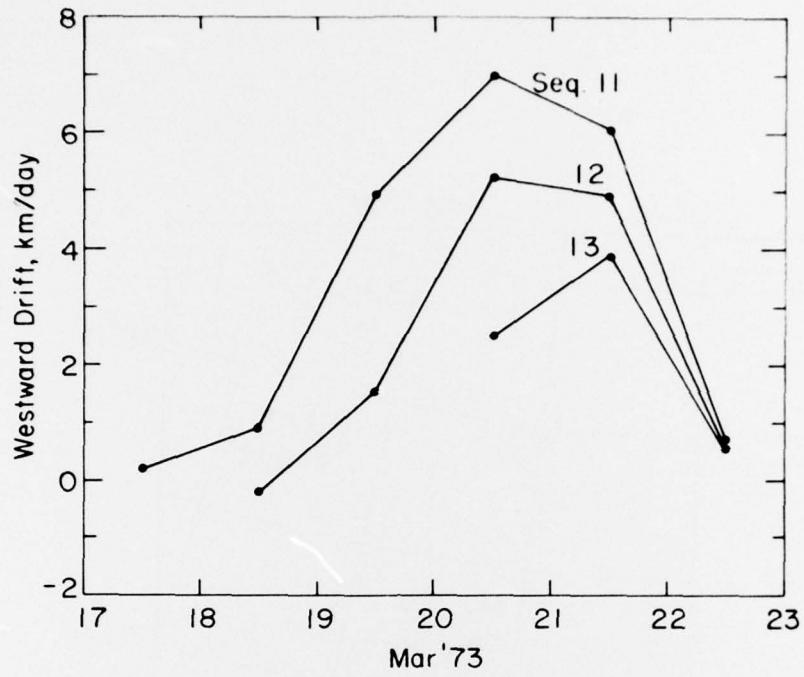


Figure 6. DRIFT RATES FROM SEA ICE IMAGE SEQUENCES

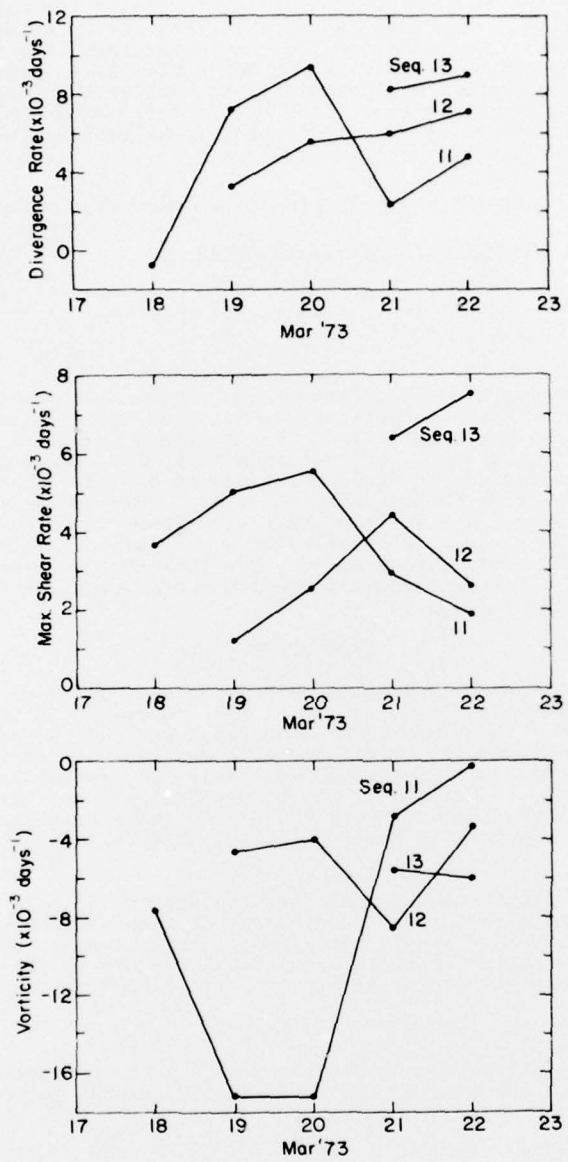


Figure 7. STRAIN RATE INVARIANTS AND VORTICITY FROM SEA ICE IMAGE SEQUENCES

two invariants of the strain rate tensor (in the form of the divergence rate and the maximum shear) and the vorticity. One reason for using two day running means is that the inhomogeneity errors (and the experimental errors as measured in the last section) are rather large. A summary of the divergence rate inhomogeneity errors, for these three sequences is given in Table VI. By averaging the strain rates over two days these errors will generally be smaller than the divergence rates. Moreover, the deformation rates from the various sequences seem to be much closer to one another when averaged, a fact which seems reasonable since the sequences are only several hundred kilometers apart.

TABLE VI. AVERAGE LEAST SQUARES DIVERGENCE RATE ERROR

Sequence	Error ( $10^{-3}$ days $^{-1}$ )
11	3.29 $\pm$ 1.43
12	1.68 $\pm$ 0.69
13	2.26 $\pm$ 0.39

It is also informative to compare the strain rates, as a function of time, both one with another and with the vorticity. Detailed deformation time series over a 30 day interval in March-April 1972 were analyzed by Hibler et al. (1974b) who found that the vorticity and divergence rate generally showed a significant negative correlation, a fact which could be predicted by the large viscosity limit of an infinite boundary drift theory (Hibler, 1974). This indeed seems to be the case in Figure 7. Moreover, the maximum shear rate seems to correlate well with the divergence rate. This was also observed by Hibler et al. (1974b) albeit with a sign change for negative divergence rates. The fact that such behaviour is observed in Figure 7 reinforces our confidence in the deformation and drift results obtained from these LANDSAT Images.

#### 5. CONCLUSIONS

We believe that this study has shown that, when analyzed with appropriate techniques, LANDSAT imagery is adequate for estimating sea ice drift rates at locations far from land. The transferral geometry developed in this paper gives a simple and rapid way to transfer coordinates from one image to another with the errors in the transferral equations being negligible over the transferral distances used. In general, the errors in the transferral process are dominated by the deviation of the actual image centers from their stated centers. The effect of such uncertainties, however, is not enough to mask effects caused by typical sea ice drift.

As regards deformation rates, typical results show that it is important to use a least squares procedure that is independent of rotation effects. Also, the typical magnitude of errors in strain rates, due to experimental errors and non-linearities in the ice velocity field, suggest that two day averages of strain rates should be used when estimating deformation rates from LANDSAT images.

#### 6. ACKNOWLEDGEMENTS

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#### APPENDIX: Digitization Process

In order to implement the above transferral and strain estimation procedures it is first necessary to digitize the coordinates of a number of common ice features as well as of points defining the location of the image. We have found the following procedures useful in accomplishing this. First identification of common ice features is accomplished by overlaying the two adjacent positive transparencies and adjusting until a given feature comes into coincidence. A fine needle is then used to make a small hole through both images, thus marking the common feature, which is subsequently circled and labeled. Approximately six common points are identified between any two overlapping images. These points, in addition to the corner tick marks and the tick marks defining two lines of longitude on each image are then given relative x-y coordinates on a manual digitizing

table capable of measuring to twenty-five ten thousandths of a millimeter. A computer program then checks for certain errors such as the orientation of the calculated center line of longitude and the location of the center position. Once past the error routines, the point locations are transferred to the coordinate system of the first image in the sequence and stored as such for subsequent drift and deformation calculations.

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