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A NUMERICAL INVESTIGATION OF THE THICKENING OF A SHOCK WAVE ON --ETC(U)

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A Numerical Investigation of the Thickening of a Shock Wave on Passing through Turbulence

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A Numerical Investigation of the Thickening of a Shock Wave on Passing through Turbulence

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Summary

The characteristic 'N' shaped pressure waves recorded at ground level below the flight path of supersonic aircraft have compressive regions very much less steep than predicted by theory, with a consequent reduction in the subjective annoyance. Attempts have been made to explain this phenomenon in terms of the effect of atmospheric turbulence. This Report is concerned with the particular proposal that since on passing through turbulence a shock will tend to become thicker for part of the time and to steepen for the remainder then if the thickening process operates faster than the steepening a net increase in thickness will result.

The problem is simulated in one-dimension by a variation of the 'piston problem' in which pressure waves representing turbulence are made to pass through a shock wave. The governing partial differential equations are solved using a finite difference technique.

The results of the calculations show that a shock wave tends to become thicker on passing through a disturbance wave having equal positive and negative parts. This is consistent with the proposed explanation of shock-thickening although the thickening calculated is small compared to the initial shock thickness, for the disturbance waves used. Distortion of the disturbance waves with time prevented a wider study of the effects of varying amplitude and wavelength being made so that precise quantitative conclusions could not be drawn but it is suggested that the process investigated is unlikely to be powerful enough to produce the shock thickening measured experimentally in sonic boom research.

* Replaces R.A.E. Technical Report 74131—A.R.C. 35 953

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Detachable Abstract Cards

1. Introduction

The leading and trailing shock waves generated by an aircraft flying at supersonic speed develop with increasing distance from the aircraft into a characteristic 'N' shaped pressure wave. An example of such a wave, generated by Concorde and reported by Holbeche, *et al.*[1], is given in Fig. 1. This example is typical of such waves when recorded at or near ground level below the flight path of a high-flying aircraft in that while the expansion part of the wave conforms with theoretical predictions, the compressive regions are very much thicker than expected. The factor by which the compressive regions are thickened can be as high as 10^3. The phenomenon of shock thickening is of considerable practical significance in that it is found to reduce the subjective annoyance resulting from the sonic boom.

Atmospheric turbulence is one possible cause of the shock thickening* and experiments by Bauer[3] have demonstrated some thickening with the introduction of artificially produced turbulence, although the scale of thickening measured in the experiments was at least an order of magnitude less than that commonly recorded under the flight path of high-flying supersonic aircraft. Several investigators, for example, Howe[4] and Plotkin and George[5] have sought an explanation of the thickening in the scattering of waves on passing through turbulence. However, in a review of theories based on wave-scattering mechanisms, Ffowcs Williams and Howe[6] have concluded that in practice a wave may be thickened in this way by a factor of 2 at the most.

An alternative explanation of shock thickening due to turbulence has been proposed by Broadbent[7]. He argues that as the shock enters atmospheric turbulence the equilibrium which may have existed between the nonlinear steepening and the thickening due to diffusion will be disrupted by large local disturbances that will be repeated in a random fashion while the turbulence persists. Thus each part of the shock front will be too thick for the ambient conditions during roughly half the time, when the nonlinear effect will cause it to steepen, and for the remaining time be too thin, when diffusion effects will tend to thicken it. The two opposing effects, however, operate at different rates, the variation due to nonlinear steepening depending on time while that due to diffusion depends on the square root of time. Thus, in small intervals of time, the diffusion process will make more progress than the steepening effect so that the shock will become progressively thicker. It should be possible to represent the essential features of this problem by considering a variation of the piston problem in which a shock wave travels down a tube and passes through a pressure wave or a series of waves travelling in the opposite direction. This Report describes a numerical study of such a representation in which the governing equations are solved, using a finite difference method, and the thickening of the shock wave by pressure waves of differing waveforms, amplitudes and wavelengths is calculated. Computational considerations imposed an upper limit on the amplitude and wavelength of the waves representing turbulence and restricted the number of interactions in any one calculation to one or two. However, it is thought that the scope of the calculations is sufficiently wide for the results to provide some insight into the validity of the suggested explanation of shock thickening.

The simulation of the problem is described in Section 2, the development and solution of the finite difference equations is explained in Section 3, the calculations performed are described in Section 4 and the results of the calculations are given in Section 5 and discussed in Section 6.

The results of the calculations show that some shock thickening is possible but the computational restraints prevent an estimation of the possible thickening due to this process in a real case of a shock wave moving through turbulence. However, it is inferred that on passing through a series of similar waves representing turbulence the shock wave would attain a time-average asymptotic thickness and that the total thickening is unlikely to be many times greater than that produced by the passage of a single wave. It is therefore concluded that the process considered is unlikely to be the principal cause of the observed shock thickening.

2. Simulation of the Problem

If the phenonemon of shock thickening on passing through turbulence is indeed due to the different rates at which steepening and thickening operate then the essential features of the problem should be present in one dimension in the variation of the 'piston problem' illustrated in Fig. 2. At one end of a long tube containing fluid there is a piston moving with a uniform velocity and sending a shock wave down the tube while at the other end a second piston is moving in an oscillatory manner and is sending a series of compression and expansion waves in the opposite direction to meet and pass through the oncoming shock wave. If the motion of the latter piston is such that the waves it produces have relatively large amplitudes and small gradients, compared to the shock wave, then the waves will resemble turbulence in their tendency to swamp the shock by

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* Real gas effects[2] have also been proposed.

an imposed velocity distribution that does not contain any sharp fronts such as the shock itself. Real turbulence will have other three-dimensional characteristics that may be important in the interaction with the shock (e.g. vorticity), but the simple waves used in the present paper are thought to be sufficient to test the theory outlined above, which depends only on disturbing the equilibrium of the shock.

Throughout this Report the shock will be regarded as having originated from the left and the waves representing turbulence from the right, as in Fig. 2.

The problem may be simplified if the pistons are regarded as being beyond the length of tube along which the calculation is to be carried out. The effect of the piston at the left may then be represented by a constant particle velocity at the left-hand boundary and the effect of the oscillating piston by a particle velocity varying in a suitably prescribed manner at the right-hand boundary. The boundaries referred to are the two ends of the length of tube over which the calculation is to be carried out. In this way difficulties in the numerical solution resulting from having a varying length of tube to consider are avoided.

Fig. 3 illustrates an example of the variation of particle velocity, denoted by v, with distance measured along the axis of the tube, denoted by x, before the two waves moving in opposite directions have met. The wave representing turbulence could be repeated to the right indefinitely although, as already noted, the computations reported here were limited to one or two cycles only.

A numerical method is required that will enable the wave representing a shock to be developed to its asymptotic form and allow the wave or waves representing turbulence to move to the left with little or no distortion. If the local particle velocities can then be calculated as the shock passes through and beyond the simulated turbulence it will be possible to calculate the effect of the turbulence upon the shock thickness.

3 Numerical Method

3.1. Basic Equations

Lighthill[8] has given the equations of plane sound waves of finite amplitude with linearized diffusion terms as

$$\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial x}+\frac{2}{\gamma-1}a\frac{\partial a}{\partial x}=\delta\frac{\partial^2 v}{\partial x^2} \tag{1}$$

$$\frac{\partial a}{\partial t}+v\frac{\partial a}{\partial x}+\frac{\gamma-1}{2}a\frac{\partial v}{\partial x}=0, \tag{2}$$

where t is time, a is the local speed of sound, γ is the ratio of specific heats and δ is the 'diffusivity of sound', that is the combination of different diffusivities which affect attenuation of sound waves. The left-hand sides of equations (1) and (2) are the exact forms of the equations of sound waves of finite amplitude under thermodynamically reversible conditions and the right-hand sides are a 'linearized' approximation to the effects of diffusion.

If the variables in equations (1) and (2) are made non-dimensional with respect to the speed of sound in undisturbed fluid, denoted by a_0, and to some time yet to be defined, denoted by t_0, so that

$$v=a_0V, \qquad a=a_0(1+A), \qquad t=t_0T,$$
$$x=a_0t_0X \quad \text{and} \quad \delta=a_0^2t_0D$$

then equations (1) and (2) may be written, respectively, as

$$\frac{\partial V}{\partial T}+V\frac{\partial V}{\partial X}+\frac{2}{\gamma-1}(1+A)\frac{\partial A}{\partial X}=D\frac{\partial^2 V}{\partial X^2} \tag{3}$$

and

$$\frac{\partial A}{\partial T}+V\frac{\partial A}{\partial X}+\frac{\gamma-1}{2}(1+A)\frac{\partial V}{\partial X}=0. \tag{4}$$

Now t_0 may be chosen to make $D=1$, that is $t_0=\delta/a_0^2$, so that equation (3) becomes

$$\frac{\partial V}{\partial T}+V\frac{\partial V}{\partial X}+\frac{2}{\gamma-1}(1+A)\frac{\partial A}{\partial X}=\frac{\partial^2 V}{\partial X^2}. \tag{5}$$

For a numerical solution it is convenient to transform the x co-ordinate further so that the shock remains nearly stationary in the new co-ordinate. Writing $\xi = X - \eta T$ and $\tau = T$ equations (4) and (5) become, respectively,

$$\frac{\partial A}{\partial \tau} - (\eta - V)\frac{\partial A}{\partial \xi} + \frac{\gamma - 1}{2}(1 + A)\frac{\partial V}{\partial \xi} = 0 \tag{6}$$

and

$$\frac{\partial V}{\partial \tau} - (\eta - V)\frac{\partial V}{\partial \xi} + \frac{2}{\gamma - 1}(1 + A)\frac{\partial A}{\partial \xi} = \frac{\partial^2 V}{\partial \xi^2}. \tag{7}$$

To hold the shock nearly stationary, in terms of ξ, η must have a value of approximately 1 but this value may be modified with experience.

3.2. Finite Difference Equations

To establish a finite difference scheme the variables τ and ξ are treated as rectangular co-ordinates and a mesh with equal elements $\Delta\tau$ and $\Delta\xi$ is adopted. Suffices m and n are used to denote position, thus $V_{m,n}$ is the value of V at $\tau = m\Delta\tau$ and $\xi = n\Delta\xi$. A difference scheme that is 'central' with respect to mesh intervals in both τ and ξ is adopted.

Considering equation (6) first, we write:

$$\left(\frac{\partial A}{\partial \tau}\right)_{m-\frac{1}{2},n-\frac{1}{2}} = \frac{A_{m,n} - A_{m-1,n} + A_{m,n-1} - A_{m-1,n-1}}{2\Delta\tau}, \tag{8}$$

$$V_{m-\frac{1}{2},n-\frac{1}{2}} = \frac{V_{m,n} + V_{m,n-1} + V_{m-1,n} + V_{m-1,n-1}}{4}, \tag{9}$$

$$\left(\frac{\partial A}{\partial \xi}\right)_{m-\frac{1}{2},n-\frac{1}{2}} = \frac{A_{m,n} - A_{m,n-1} + A_{m-1,n} - A_{m-1,n-1}}{2\Delta\xi}, \tag{10}$$

$$A_{m-\frac{1}{2},n-\frac{1}{2}} = \frac{A_{m,n} + A_{m,n-1} + A_{m-1,n} + A_{m-1,n-1}}{4} \tag{11}$$

and

$$\left(\frac{\partial V}{\partial \xi}\right)_{m-\frac{1}{2},n-\frac{1}{2}} = \frac{V_{m,n} - V_{m,n-1} + V_{m-1,n} - V_{m-1,n-1}}{2\Delta\xi}. \tag{12}$$

Substituting the above in equation (6), which is equivalent to evaluating equation (6) at $\tau = (m - \frac{1}{2})\Delta\tau$, $\xi = (n - \frac{1}{2})\Delta\xi$, gives:

$$P_n A_{m,n} + q_n A_{m,n-1} = r_n \tag{13}$$

where

$$P_n = w_n + y_n + z_n, \tag{14}$$

$$q_n = w_n - y_n + z_n \tag{15}$$

and

$$r_n = (w_n - y_n - z_n)A_{m-1,n} + (w_n + y_n - z_n)A_{m-1,n-1} - 4z_n, \tag{16}$$

with

$$w_n = \frac{1}{2\Delta\tau}, \tag{17}$$

$$y_n = -\frac{\eta - (V_{m,n} + V_{m,n-1} + V_{m-1,n} + V_{m-1,n-1})/4}{2\Delta\xi} \tag{18}$$

and

$$z_n = \frac{(\gamma - 1)(V_{m,n} - V_{m,n-1} + V_{m-1,n} - V_{m-1,n-1})}{16\Delta\xi}. \tag{19}$$

Now considering equation (7), the finite difference approximations used are:—

$$\left(\frac{\partial V}{\partial \tau}\right)_{m-\frac{1}{2},n} = \frac{V_{m,n} - V_{m-1,n}}{\Delta\tau}, \tag{20}$$

$$V_{m-\frac{1}{2},n} = \frac{V_{m,n} + V_{m-1,n}}{2}, \tag{21}$$

$$\left(\frac{\partial V}{\partial \xi}\right)_{m-\frac{1}{2},n} = \frac{V_{m,n+1} - V_{m,n-1} + V_{m-1,n+1} - V_{m-1,n-1}}{4\Delta\xi}, \tag{22}$$

$$A_{m-\frac{1}{2},n} = \frac{A_{m,n} + A_{m-1,n}}{2}, \tag{23}$$

$$\left(\frac{\partial A}{\partial \xi}\right)_{m-\frac{1}{2},n} = \frac{A_{m,n+1} - A_{m,n-1} + A_{m-1,n+1} - A_{m-1,n-1}}{4\Delta\xi} \tag{24}$$

and

$$\left(\frac{\partial^2 V}{\partial \xi^2}\right)_{m-\frac{1}{2},n} = \frac{V_{m,n+1} - 2V_{m,n} + V_{m,n-1} + V_{m-1,n+1} - 2V_{m-1,n} + V_{m-1,n-1}}{2(\Delta\xi)^2}. \tag{25}$$

Evaluating equation (7) at $\tau = (m - \frac{1}{2})\Delta\tau$, $\xi = n\Delta\xi$ gives

$$a_n V_{m,n-1} + b_n V_{m,n} + c_n V_{m,n+1} = d_n, \tag{26}$$

where

$$a_n = \frac{1}{8\Delta\xi}(2\eta - V_{m,n} - V_{m-1,n}) - \frac{1}{2(\Delta\xi)^2}, \tag{27}$$

$$b_n = \frac{1}{\Delta\tau} + \frac{1}{(\Delta\xi)^2}, \tag{28}$$

$$c_n = -a_n - \frac{1}{(\Delta\xi)^2} \tag{29}$$

and

$$d_n = \frac{-1}{4(\gamma - 1)\Delta\xi}(2 + A_{m,n} + A_{m-1,n})(A_{m,n+1} - A_{m,n-1} + A_{m-1,n+1} - A_{m-1,n-1})$$
$$- a_n V_{m-1,n-1} + \left(b_n - \frac{2}{(\Delta\xi)^2}\right) V_{m-1,n} - c_n V_{m-1,n+1}. \tag{30}$$

It may be noted that in the two equations to be solved ((13) and (26)) the coefficients are themselves functions of the variables. This is a result of the nonlinearity of the basic equations (6) and (7) and consequently a step-by-step solution will involve an iteration process. The coefficients of equations (13) and (26) are first evaluated after estimating the variables by some means and then the equations are solved in their linear form to obtain new values of the variables. These are then used to improve the values of the coefficients and the cycle repeated until convergence is achieved. The linear form adopted in equations (13) and (16) is to some extent a matter of choice, in that with a nonlinear product of two variables either variable may be retained explicitly while the other is relegated to the coefficients, but experience shows that stability is improved by keeping the derivatives explicit.

3.3. Solution of Finite Difference Equations

In order to solve equations (13) and (26) certain boundary conditions must be specified; these are given by the prescribed particle velocities at each end of the length of tube through which the equations are to be solved and, initially, by an assumed particle-velocity distribution along the tube at $\tau = 0$. That is, if the equations are to be solved from $\xi = 0$ to $\xi = N\Delta\xi$ then $V_{m,0}$, $V_{m,N}$ and $A_{m,N}$ must be specified for all m and $V_{0,n}$ and $A_{0,n}$ for all n. $A_{m,N}$ is found from $V_{m,N}$ using the equation for local acoustic speed (which holds in the absence of diffusion)

$$a = a_0 + \tfrac{1}{2}(\gamma - 1)v, \tag{31}$$

which may be written as

$$A_{m,n} = \tfrac{1}{2}(\gamma - 1)V_{m,n}. \tag{32}$$

Given $V_{0,n}$ it is assumed that $V_{1,n} = V_{0,n}$ for all n. $A_{1,n}$ is then calculated from equation (13), working down from $A_{1,N}$ and re-writing equation (13) as

$$A_{m,n-1} = (r_n - p_n A_{m,n})/q_n. \tag{33}$$

Now equation (26) is solved for $V_{1,n}$ by the resolution of a tri-diagonal matrix, using the values of $A_{1,n}$ just calculated and the assumed values of $V_{1,n}$ to evaluate the coefficients. The new values of $V_{1,n}$ are then used with equation (33) to find improved values of $A_{1,n}$ and thus an iterative procedure is established. When successive approximations to $V_{1,n}$ differ by less than a prescribed tolerance the calculation is advanced to compute $V_{2,n}$ and $A_{2,n}$ and so on to higher values of m. After the first step in m is completed, linear extrapolation is used for the first estimate of $V_{m,n}$ at the end of each subsequent step. The first and last rows of the tri-diagonal matrix are written as

$$b_1 V_{m,1} + c_1 V_{m,2} = d_1 - a_1 V_{m,0} \tag{34}$$

and

$$a_{N-1} V_{m,N-2} + b_{N-1} V_{m,N-1} = d_{N-1} - C_{N-1} Vm_{,N}, \tag{35}$$

respectively, thus introducing the boundary conditions $V_{m,0}$ and $V_{m,N}$.

The above procedure has been programmed for a digital computer in FORTRAN language; a standard ICL subroutine was used for the solution of the tri-diagonal matrix.

4. Calculations Performed

4.1. The Asymptotic Shock

In order to study the effect of the simulated turbulence upon the shock it was first necessary to establish the asymptotic shock in the absence of disturbances and to determine its thickness. Lighthill[8], has given the thickness of an asymptotic shock as

$$h = \frac{12\delta}{(\gamma + 1)v_0}, \tag{36}$$

where h is defined as the distance in which v changes from $0{\cdot}05v_0$ to $0{\cdot}95v_0$ and v_0 is the particle velocity on the compressive side of the shock. The non-dimensional shock thickness is therefore given by

$$H = \frac{12}{(\gamma+1)V_0} \tag{37}$$

where V_0 is the non-dimensional particle velocity corresponding to v_0. A value of $0{\cdot}1$ was assigned to V_0; this is large for a sonic boom but one of the problems of the numerical calculation was to obtain significant effects from a computer program without making extravagant demands on working storage space. Equation (37) shows that a shock thickness of approximately 50 units in ξ may be expected for a value of V_0 of $0{\cdot}1$. To start the calculation a linear approximation to the shock was used, with a thickness of 40 units in ξ. Thus $V_{0,n}$ was equal to $0{\cdot}1$ from $\xi = 0$ to some higher value of ξ, then decreased linearly with ξ to zero over 40 units in ξ and remained at zero to $\xi = N\Delta\xi$. $A_{0,n}$ was estimated from $V_{0,n}$ for all n, using equation (32). The calculation was then advanced in time as described above, keeping $V_{m,0} = 0{\cdot}1$ and $V_{m,N} = 0$, until the slope and thickness of the shock remained constant with time.

In order to check that the calculated asymptotic shock form is the same irrespective of whether the initial approximation was 'too-steep' or 'too-thick', a second computation was made similar to the above but with the initial linear approximation extending over 60 units in ξ. This did in fact confirm that the asymptotic shape was independent of the starting assumptions.

Two further computations were made, starting from shock forms that were obtained by scaling the asymptotic shock with respect to ξ; one calculation was started from a 'too-thick' and the other from a 'too-steep' shock. The asymptotic shock form was then developed in each case, thus allowing a comparison to be made between the rates at which the shock thickened and steepened.

4.2. Waves Representing Turbulence

Having established the asymptotic shock form, waves representing turbulence were introduced by varying, with increeasing τ, the boundary condition at $\xi = N\Delta\xi$, that is at the right-hand end of the region of computation. Calculations were carried out with waves of three different forms to be referred to as types (A), (B) and (C), the respective particle velocities at the boundary being given by

$$V_{m,N} = s\{\sin[2(1+\eta)\tau\pi/\lambda]\}, \tag{38}$$

$$V_{m,N} = s\{1 - \cos[2(1+\eta)\tau\pi/\lambda]\}/2 \tag{39}$$

and

$$V_{m,n} = s\{1-\cos[2(1+\eta)\tau\pi/\lambda]\}\sin[2(1+\eta)\tau\pi/\lambda]/\{[1-\cos(2\pi/3)]\sin(2\pi/3)\}. \tag{40}$$

Here s defines the amplitude (although its sign is significant) and λ the wavelength of the waves as they were introduced at the boundary. Values of $A_{m,N}$ corresponding to $V_{m,N}$ were computed from equation (32). The variation of the right-hand boundary condition, as given by equations (38), (39) and (40) is illustrated in Fig. 4. The waves produced travel to the left with a velocity, in the transformed co-ordinate ξ, of approximately 2 units per unit advance in τ.

Values of s from $-1{\cdot}0$ to $1{\cdot}0$ and of λ from 250 to 1000 were used; details are to be found in Section 5. Practical considerations imposed an upper limit on λ such that the values used, though large compared to the shock thickness, were small compared to the probable wavelengths of turbulence in the atmosphere.

Two additional calculations were made with wave forms of type B in which at the start of the computation the asymptotic shock form was replaced by one about 10 per cent thicker. The object of these two calculations was to investigate the effect of waves representing turbulence upon a shock initially having a strong tendency to revert to its asymptotic thickness.

4.3. Boundary Condition at $\xi = 0$

As the wave representing turbulence must pass through and clear of the shock before its effect on shock thickness can be calculated, a very long region of calculation is necessary to accommodate the wave unless some means can be found of 'absorbing' it, without 'reflections' occurring, at the left-hand boundary of the calculation. Computational considerations impose a practical limit on the length of the region considered and the difficulty is aggravated by the tendency of the waves to increase in wavelength with time and to be followed by 'tails' of waves of decreasing amplitude, both these effects being inherent in the numerical technique employed. It was therefore necessary to devise a means of modifying the boundary condition at $\xi = 0$ from the initial condition $V_{m,0} = V_0$ in order to absorb the turbulent wave. As the wave representing turbulence travels to the left with a velocity of approximately 2 units in ξ for each unit increase in τ it is possible to estimate a value of V at a given point from a value or values to the right at previous steps in τ. Thus, for example, a rough approximation is given for $V_{m,n}$ by

$$V_{m,n} \simeq V_{m-1,n+2}, \tag{41}$$

assuming that the step lengths $\Delta\tau$ and $\Delta\xi$ are equal. By this means it was possible to estimate values for the left-hand boundary conditions approximating to the values of V and A that would have occurred at the same point but with the region of calculation extending infinitely far to the left. In order to make a correction for the effects of wave distortion and variation in wave speed, second order approximations for the boundary conditions were adopted, thus

$$V_{m,0} = 2V_{m-1,2} - V_{m-1,4}, \tag{42}$$

where again it is assumed that $\Delta\tau = \Delta\xi$. Where $\Delta\tau$ and $\Delta\xi$ were not equal, equation (42) was modified as appropriate. For example with $\Delta\tau = 10$ and $\Delta\xi = 4$

$$V_{m,0} = 2V_{m-1,5} - V_{m-2,10} \tag{43}$$

was used.

As a check on the effectiveness of the above technique, calculations of the effect of turbulence upon shock thickness were made with the shock at two different initial positions. The results of the two calculations were substantially the same, demonstrating that the effect of any reflection from the left-hand boundary condition was negligible, since it would not be equal in the two calculations.

4.4. Calculations of Shock Thickness

In order to calculate the thickness of the shock, H, as defined in Section 4.1, the values of ξ corresponding to $0{\cdot}05\,V_0$ and $0{\cdot}95\,V_0$ were found by inverse interpolation from the values of V calculated at incremental points in ξ.

To calculate the increase in H due to the passage of the wave or waves representing turbulence, the value of H is required at a value of τ corresponding to the instant at which the last wave has just cleared the shock; any further advance in τ allows the shock to begin the process of reverting to its asymptotic thickness and the effect on H of the passage of the waves is apparently reduced. However, it was found that values of H computed for successive increments in τ just after the last wave representing turbulence had cleared the shock, fluctuated to such an extent that it was impossible to estimate H accurately directly; these fluctuations were traced to the presence of a 'tail' of minor waves, arising from the numerical method, following behind the wave or waves representing turbulence. The calculation was therefore advanced in τ until the fluctuations were damped out sufficiently for H to be estimated with confidence. The resulting value of H was then corrected to allow for the amount the shock would have changed in thickness in the time that had elapsed since the wave or waves representing turbulence had just cleared it, using the values of H obtained in the calculation of the asymptotic shock starting from 'too-steep' or 'too-thick' shapes.

4.5. Computation Parameters

After numerical experiments to establish a good compromise between computing time and accuracy the following values for certain parameters were used in the majority of the calculations: $N = 400$, $\Delta\tau = 10$ and $\Delta\xi = 4$. A value of $1{\cdot}06$ for the transformation constant η in equations (6) and (7) was found to hold the shock nearly stationary with respect to ξ while τ was advanced, in the absence of simulated turbulence. Waves of type B had the effect of moving the shock to the left if s was negative and to the right if s was positive. Values of η ranging from $0{\cdot}8$ to $1{\cdot}3$, as appropriate to a given case, were therefore used to hold the shock near the centre, with respect to ξ, of the range of computation.

5. Results of Calculations

5.1. The Asymptotic Shock

The calculated asymptotic shock is illustrated in Fig. 5. The thickness of the shock, as defined in Section 4·1, is 48·6 units in ξ. A similar shock shape was obtained with calculations started from both 'too-thin' and 'too-thick' linear approximations.

The result of calculations made starting from initial shock-shapes obtained by scaling the asymptotic shock with respect to ξ is shown in Fig. 6; δH is the change in shock thickness corresponding to an increment $\delta\tau$ in τ. It may be seen that there is a slight tendency for a too-thin shock to thicken at a faster rate than that at which a too-thick shock steepens.

5.2. Effect of Waves Representing Turbulence

5.2.1. Waves of Type A. Calculations of the thickness, H, of the shock after one wave of type A had passed through it showed that the value of H was fluctuating with successive increments in τ. This effect was traced to the presence of a succession of minor waves of progressively smaller amplitudes, following behind the wave representing turbulence. These minor waves were found to be present to some extent in the results of the calculations for all three types of wave considered, and a method of overcoming the resultant difficulty in estimating the effect on shock thickness of the passage of the wave representing turbulence has been described in Section 4. However, for waves of type A the fluctuations in the values of H were of such magnitude that it was impossible, by advancing τ, to obtain an accurate value for H before the shock had reverted to its asymptotic shape; thus no useful results could be obtained using waveforms of type A. The severity of the fluctuations in values of H for successive values of τ was probably due to the discontinuity in the variation of the particle velocity at the abrupt termination of the initial wave.

5.2.2. Waves of Type B. The results of calculations carried out with turbulence represented by waveforms of type B are given in Table 1; ΔH is the increment in shock thickness caused by the passage of the turbulence. It can be seen that the changes in H are small relative to the asymptotic value for H of 48·6 and that, in general

(i) a negative wave increases shock thickness
(ii) a positive wave reduces shock thickness
(iii) the influence of a wave increases with amplitude and wavelength and has a cumulative effect over a number of cycles.

However, it is difficult to draw any quantitative conclusions because of the lack of system in the variation of ΔH with the waveform parameters.

In order to find an explanation for the inconsistent behaviour of the computed values of ΔH the propagation of the waves representing turbulence across the region of computation has been examined. Calculations of the development of these waves have been made in the absence of a shock; examples of the results of these calculations are given in Figs. 7 and 8 where the variation of particle velocity with τ at several values of ξ for a wavelength of 500 is illustrated. The results of the calculations show that the waves undergo distortion, which becomes more severe with increasing amplitude and decreasing wavelength, and that the waves for positive values of s assume different shapes from their counterparts for negative values. Furthermore, because of the steepening of the region of decreasing velocity in some cases, for example $s = 1{\cdot}0$ in Fig. 7, the 'tail' of minor waves has developed to such an extent that some effect on shock thickness seems probable. However, in the majority of cases the distortion has not produced substantial changes in either the amplitudes or the lengths of the waves and it seems probable that if the analysis is confined to these cases then the results should have some significance. On this basis, the following results in Table 1 have been bracketed and should be disregarded:

$$\lambda = 250, \quad s = 0{\cdot}5 \quad \text{(1 and 2 cycles)}$$

$$\lambda = 500, \quad s = 1{\cdot}0.$$

The remaining values of ΔH have been plotted against s in Fig. 9; the scatter in the results is thought to be due to the presence of the minor waves, as described in Section 4.4, the effect of which may not have been completely eliminated. Despite the scatter it is apparent that there is a marked tendency for a negative wave to have a greater effect on shock thickness than a positive wave of the same amplitude.

The results of the two additional calculations that were made with the initial shock having a thickness about 10 per cent greater than that of the asymptotic shock are given below:

λ	s	ΔH
500	−0·5	−1·2
500	0·5	−4·4
—	—	−3·2

The third of the above three cases gives the steepening of the initial shock during the same advance in τ as occurred with the other two cases but in the absence of waves representing turbulence. It may be seen that in the first case, although the shock has steepened it is, after the passage of the wave, 2·0 units in ξ thicker than the third case at the corresponding value of τ. The value of 2·0 may be compared with a similar value in the corresponding case in Table 1. The difference between the second and third cases yields a value for ΔH of −1·2, which may be compared to one of −1·6 for the corresponding case in Table 1. Thus it seems that the thickening due to the tendency of the shock to revert to its asymptotic thickness may be added to the thickening or steepening due to the effect of the wave or waves representing turbulence to give the net thickening. If this is in fact the case then the calculated effects of the waves representing turbulence upon the asymptotic shock must also include the opposing effect of either diffusion or non-linear steepening, although this should be small in most cases since the rate of change of thickness with time is, of course, small for thicknesses close to the asymptotic, as shown in Fig. 6.

5.2.3. Waves of Type C. As with waves of type B, it was found that the waves of type C tended to become distorted as they moved to the left and that in some cases the wave shape at the position of the shock was so unlike the initial shape that the results could not be used for comparative purposes. However, in some cases where the velocity gradients in the waves were sufficiently small the waves retained their essential characteristics. Examples of such waves are shown in Fig. 10 which illustrates the variation of particle velocity with τ for $\lambda = 1000$, $s = 0{\cdot}5$ and $\lambda = 1000$, $s = -0{\cdot}5$ at a station corresponding approximately to the position of the centre of the shock when present, although Fig. 10 is derived from calculations made in the absence of a shock. To assist in comparing the shapes of the two waves the initial waves, that is the waves introduced at the right hand boundary, have been added to Fig. 10. It may be seen that the distortion is not severe and that, in particular, the positive and negative parts of the waves have remained similar.

Results of the calculations of the effect on shock thickness of waves of type C are given in Table 2; only cases where the waves representing turbulence were not severely distorted are retained. It is apparent that except for one case, which is of small amplitude, the effects of the negative and positive parts of the waves do not cancel out and that the net result is an increase in shock thickness; this is consistent with the results of Table 1. The difference between the values of H for positive and negative values of s for a wavelength of 1000 can be explained by the effect deduced above from the results obtained from the interaction of waves of type B with a shock initially too-thick. If the positive part of the wave passes through the shock first its steepening effect will be opposed throughout by the tendency of the shock to revert to its asymptotic shape but none the less a net steepening will be present when the negative part of the wave begins its passage through the shock. The thickening effect of the negative part of the wave will therefore be augmented by the tendency of the shock to revert to its asymptotic thickness until such time as the asymptotic thickness is achieved. Thereafter the two effects will be in opposition but it is evident that, no matter whether the positive part of the wave precedes or follows the negative part, the part that passes through the wave last will have its effect on the shock thickness increased. The results in Table 2 for the smaller waves conflict with this hypothesis, but the calculated values of H are very small and may be dominated by errors.

6. Discussion of Results

In the preceding Section numerical results are given that show that the thickening of the shock caused by an interaction with a negative wave is greater than the steepening produced by an interaction with a positive wave having the same amplitude and wavelength. This effect is confirmed by the results for a single wave having equal positive and negative parts, and it is consistent with the proposed explanation of shock thickening in which it is suggested that the two opposing effects, diffusion and steepening, operate at different rates. Some additional support to the suggestion is given by Fig. 6 which shows that a shock thinner than the asymptotic shock thickens at a faster rate than that at which a too-thick shock steepens. Other results are given that demonstrate that the influence of a wave increases with amplitude and wavelength and that there is a cumulative effect on thickening over a number of cycles, although calculations were made for one and two cycles only.

The results for calculations made to find the effect of waves passing through a shock initially too-thick showed that the steepening process goes on even during the passage through the shock of a wave that is tending to thicken it, the net effect being approximately equal to the sum of the two thickness changes calculated separately. If a shock were to pass through a series of similar waves representing turbulence then it seems likely that a time-average asymptotic thickness would be reached when the rate of thickening due to the 'turbulence' would balance that due to the tendency of the shock to revert to its undisturbed asymptotic thickness. For example, a succession of waves with $\lambda = 500$ and $s = -0{\cdot}5$, which individually would thicken the initial shock by 2·0 units in ξ in a time of approximately 250 units in τ, would produce an asymptotic thickness, deduced from Fig. 6, of about 52·5 units in ξ. Thus, in this case, an infinite number of waves would produce a thickening which is less than 10 per cent of the initial shock thickness and only twice that produced by a single wave. An increase in wavelength of the turbulence would increase the effect on thickness of the first wave but the effect of subsequent waves would soon be cancelled out by the rapidly increasing and continuous opposition of the steepening effects, which would have longer to operate during each interaction. An increase in amplitude would also increase the thickening due to the first wave but the second wave would have a greatly reduced thickening effect since the steepening tendency would be much increased at the new shock thickness, and equilibrium would be reached after the passage of fewer waves than for waves of smaller amplitude. It seems unlikely, therefore, that the mechanism investigated here could be the cause of the observed shock thickening.

7. Conclusions

The effect of turbulence upon the thickness of a shock wave has been investigated using a numerical method in which the turbulence was represented by pressure waves which, because of computational restraints, were not more than two in number. Distortion of the waves occurred as the calculations were advanced in time and resulted in waves of the same family becoming non-similar; consequently some of the results had to be discarded. Disturbance waves having initially small velocity gradients were less subject to distortion and the results from calculations carried out with such waves indicated a tendency for the shock to thicken after passing through waves having equal positive and negative parts. The shock thickening, for the limited range of amplitudes and wavelengths used in the calculations, was small compared to the initial shock thickness but the trend of the results is in accordance with the suggestion that on passing through turbulence a shock wave will tend to become thicker for part of the time and to steepen for the remainder, with the thickening process operating at the faster rate so that a net increase in thickness results, as shown by Fig. 9. Some additional support to the suggestion is given by Fig. 6 which shows that a too-thin shock thickens at a faster rate than that at which a too-thick shock steepens.

Further calculations of the effect of a single wave upon an initially too-thick shock showed that the tendency for the shock to steepen could be dominant over any thickening effect due to the disturbing waves. This suggested that on passing through a series of similar waves representing turbulence the shock would eventually attain a time-average asymptotic thickness. An estimate of this asymptotic thickness was made for a particular case and it gave a thickening only twice as great as that produced by a single wave. The amplitude and wavelength were again limited by computational restraints but it is argued that the effects of increasing either or both would be offset by the greatly increased tendency of the shock to steepen as each successive wave passed through it. It seems unlikely, therefore, that the process considered here could produce a thickening of the magnitude experienced by the shock wave associated with the sonic boom on passing through the atmosphere.

LIST OF SYMBOLS

A	Non-dimensional local speed of sound
a	Local speed of sound
a_0	Speed of sound in undisturbed fluid
a_n, b_n, c_n, d_n	Coefficients in equation (26)
D	Non-dimensional diffusivity of sound
H	Non-dimensional shock thickness
h	Shock thickness
m, n	Suffices to denote position
N	The maximum value of n
p_n, q_n, r_n	Coefficients in equation (13)
s	Amplitude of waves representing turbulence
T	Non-dimensional time
t	Time
t_0	A period of time used to make time non-dimensional, that is $T = t/t_0$
V	Non-dimensional particle velocity
V_0	Non-dimensional particle velocity on the compressive side of the shock in the absence of turbulence
v	Particle velocity
v_0	Particle velocity on the compressive side of the shock in the absence of turbulence
w_n, y_n, z_n	Coefficients in equations (14), (15) and (16)
X	Non-dimensional distance along the axis of the tube
x	Distance along the axis of the tube
γ	Ratio of the specific heat at constant pressure to the specific heat at constant volume
δ	Diffusivity of sound
η	Coefficient used in the transformation $\xi = X - \eta T$
λ	Wavelength of waves representing turbulence
ξ, τ	Transformed variables, treated as rectangular co-ordinates
ΔH	Increment in H, due to passage of turbulence
$\Delta\xi, \Delta\tau$	Mesh-lengths in ξ and τ

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TABLE 1

The Increment in Shock Thickness due to Turbulent Waves of Type B

(for significance of brackets, see text)

	ΔH				
	1 *wave-cycle*			2 *wave-cycles*	
s	$\lambda = 250$	$\lambda = 500$	$\lambda = 1000$	$\lambda = 250$	$\lambda = 500$
−1·0	—	+6·0	—	—	—
−0·5	+1·7	+2·0	+3·7	+2·3	+3·8
−0·2	+0·4	+0·9	+1·3	—	+1·4
+0·2	−0·2	−0·7	−1·1	—	—
+0·5	(−1·4)	−1·6	−1·9	(−1·4)	—
+1·0	—	(−7·1)	—	—	—

TABLE 2

The Increment in Shock Thickness due to Turbulent Waves of Type C; 1 Wave Cycle

ΔH

s	$\lambda = 500$	$\lambda = 1000$
−0·5	—	+0·4
−0·2	+0·2	—
+0·2	+0·0	—
+0·5	—	+1·0

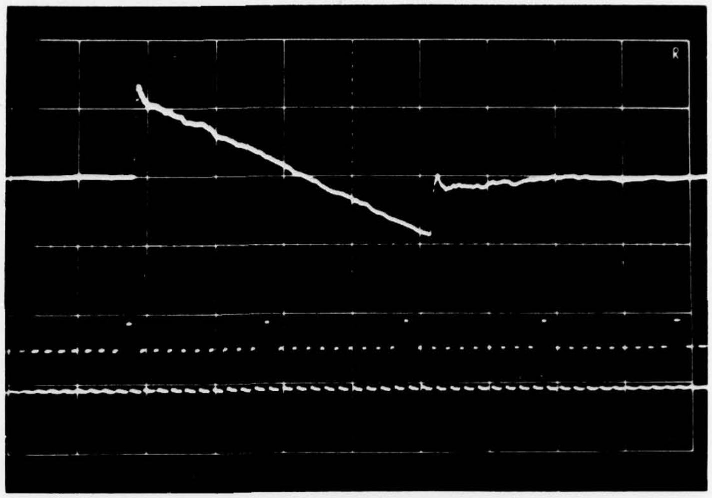

FIG. 1a. A recording of the pressure wave under the flight path of Concorde. Each division of the vertical scale corresponds to 100 N/m^2; the time marks nearest the foot of the figure are at 10 ms intervals.

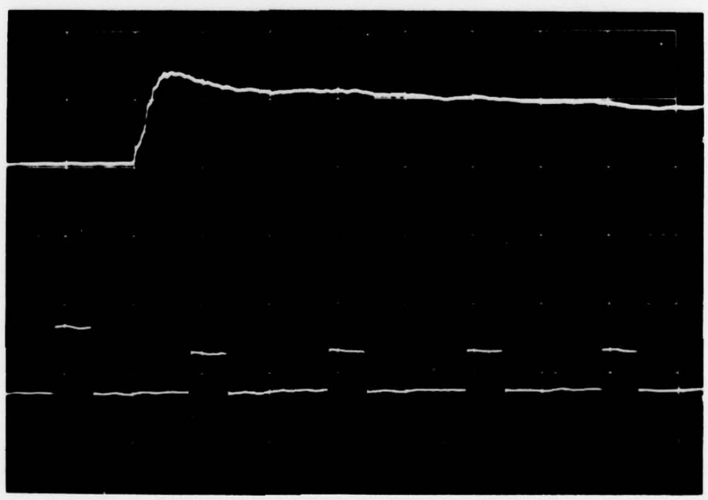

FIG. 1b. Expanded view of the early part of the wave of Fig. 1a.

FIG. 2. A variation of the 'piston problem'.

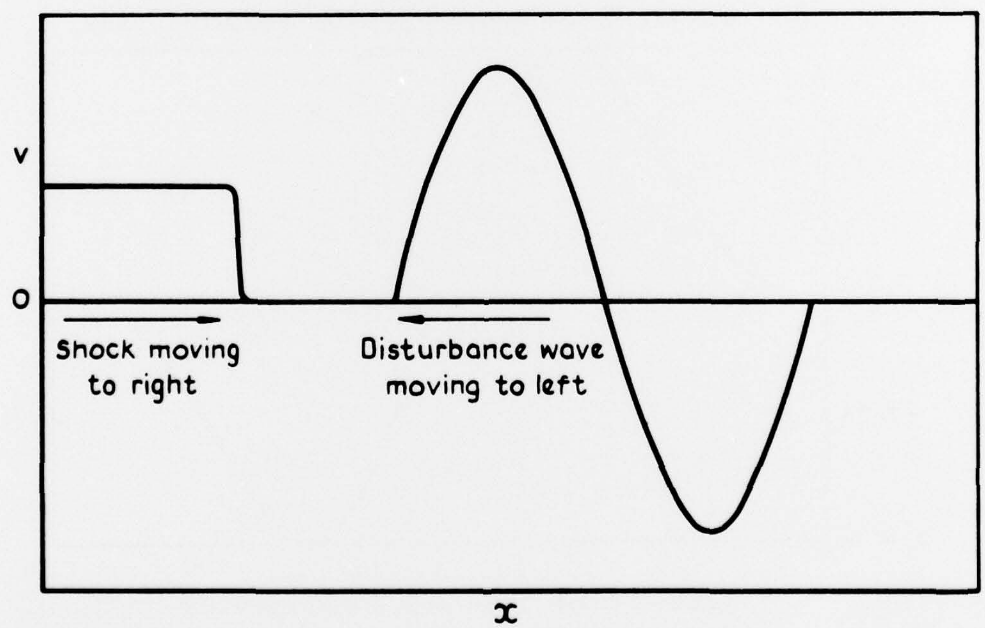

FIG. 3. An example of the variation of particle velocity along the tube.

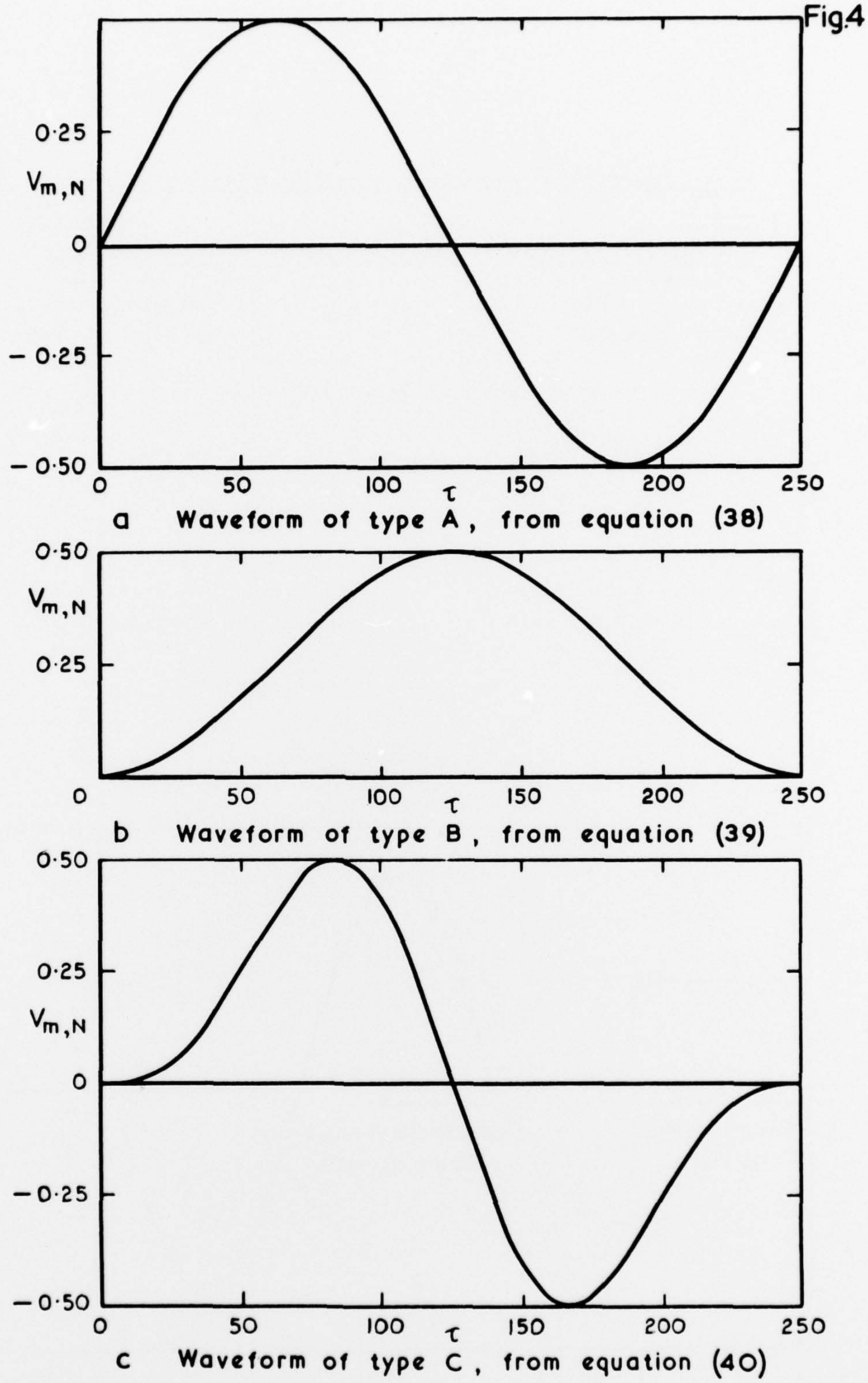

FIG. 4a–c. Variation of the right-hand boundary condition with τ; $s = 0{\cdot}5$, $\lambda = 500$, $\eta = 1{\cdot}0$.

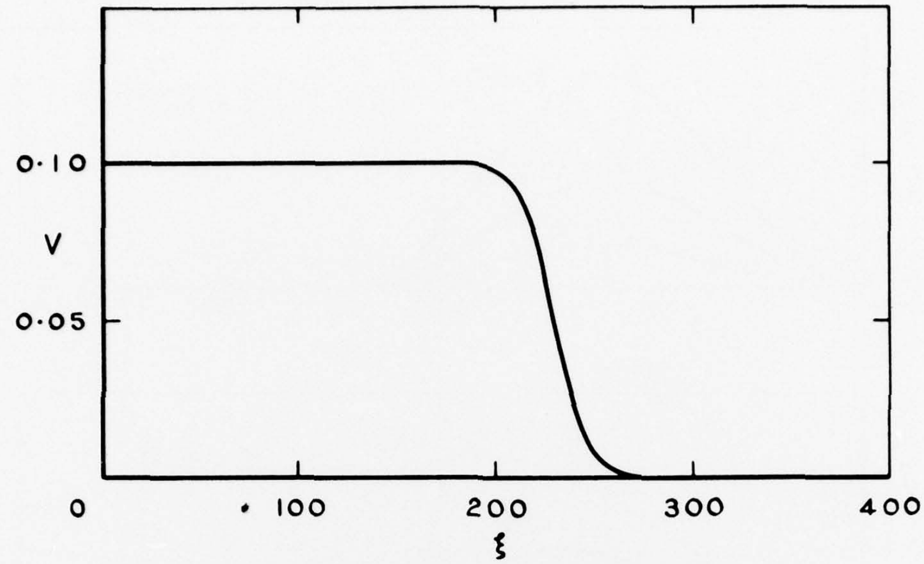

FIG. 5. The calculated asymptotic shock

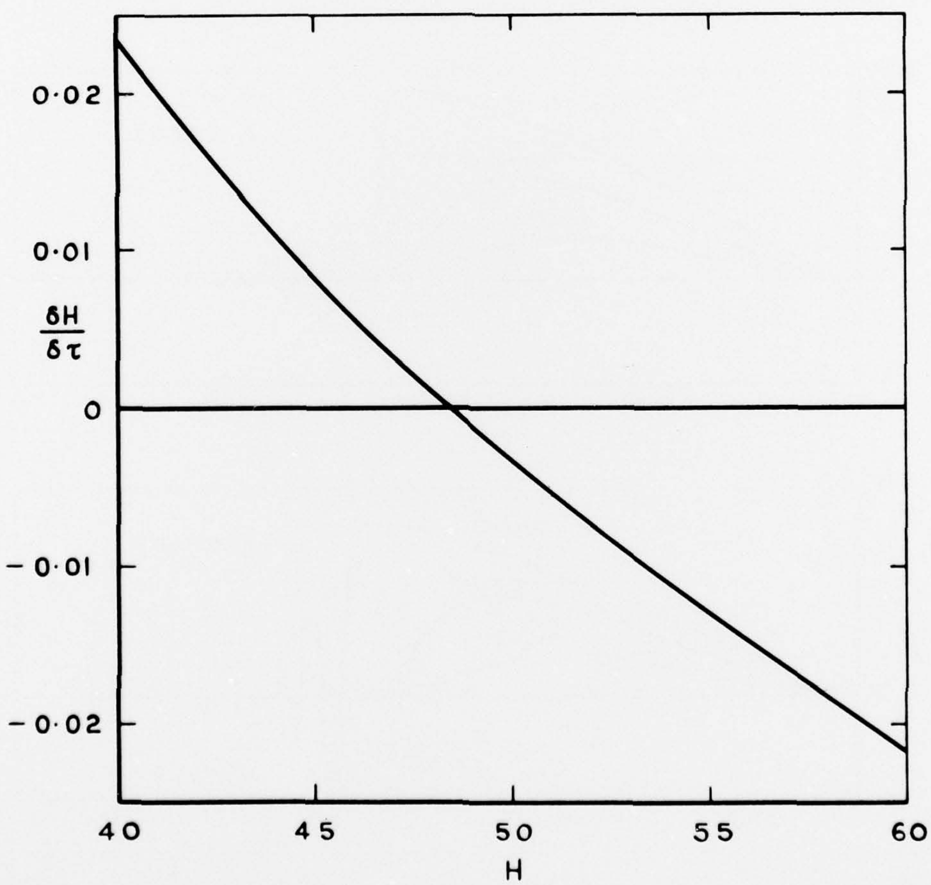

FIG. 6. The variation with shock thickness of the rate of change of shock thickness with τ.

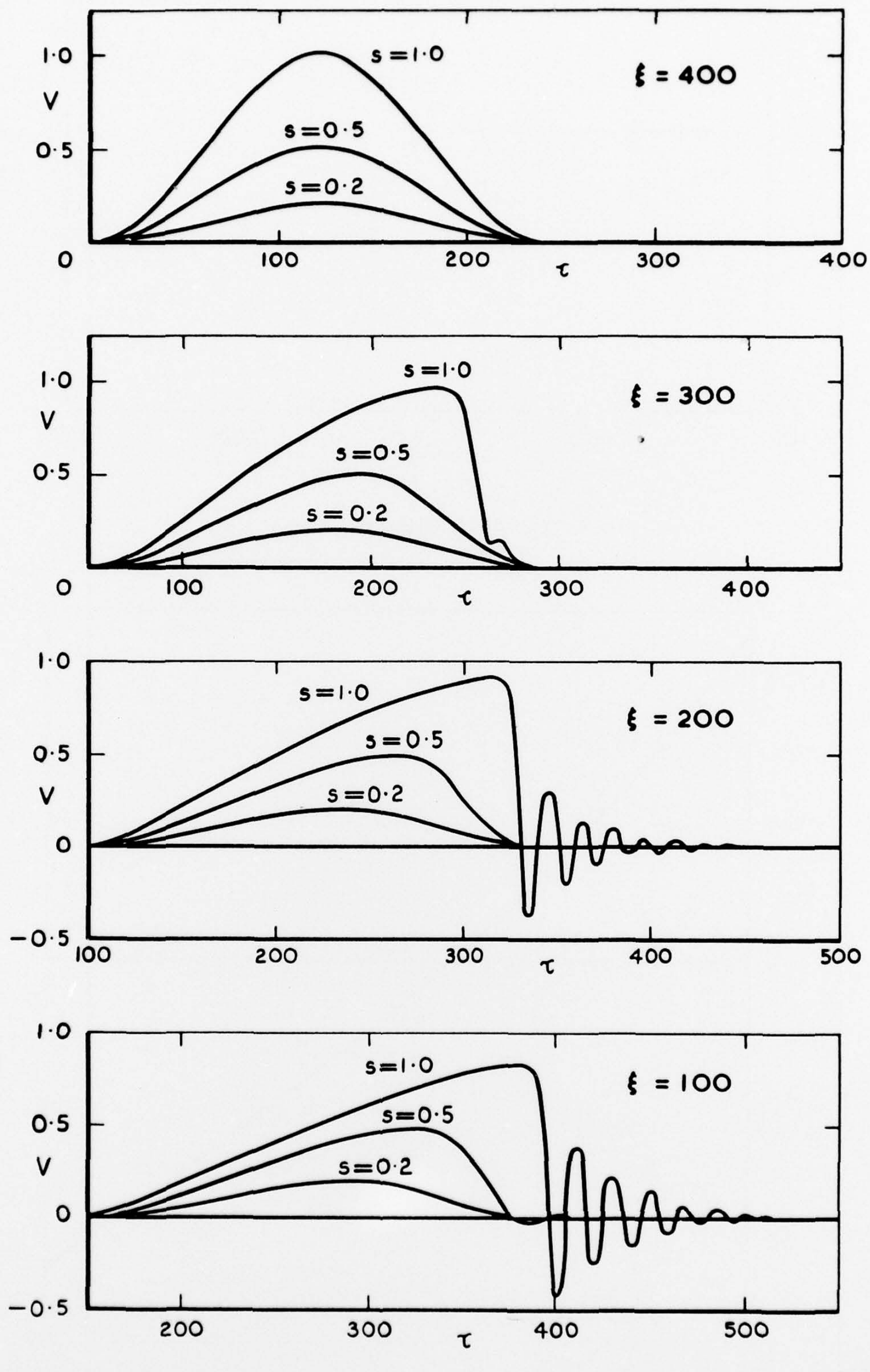

FIG. 7. The variation of particle velocity with τ for a positive wave of type B; no shock, $\lambda = 500$.

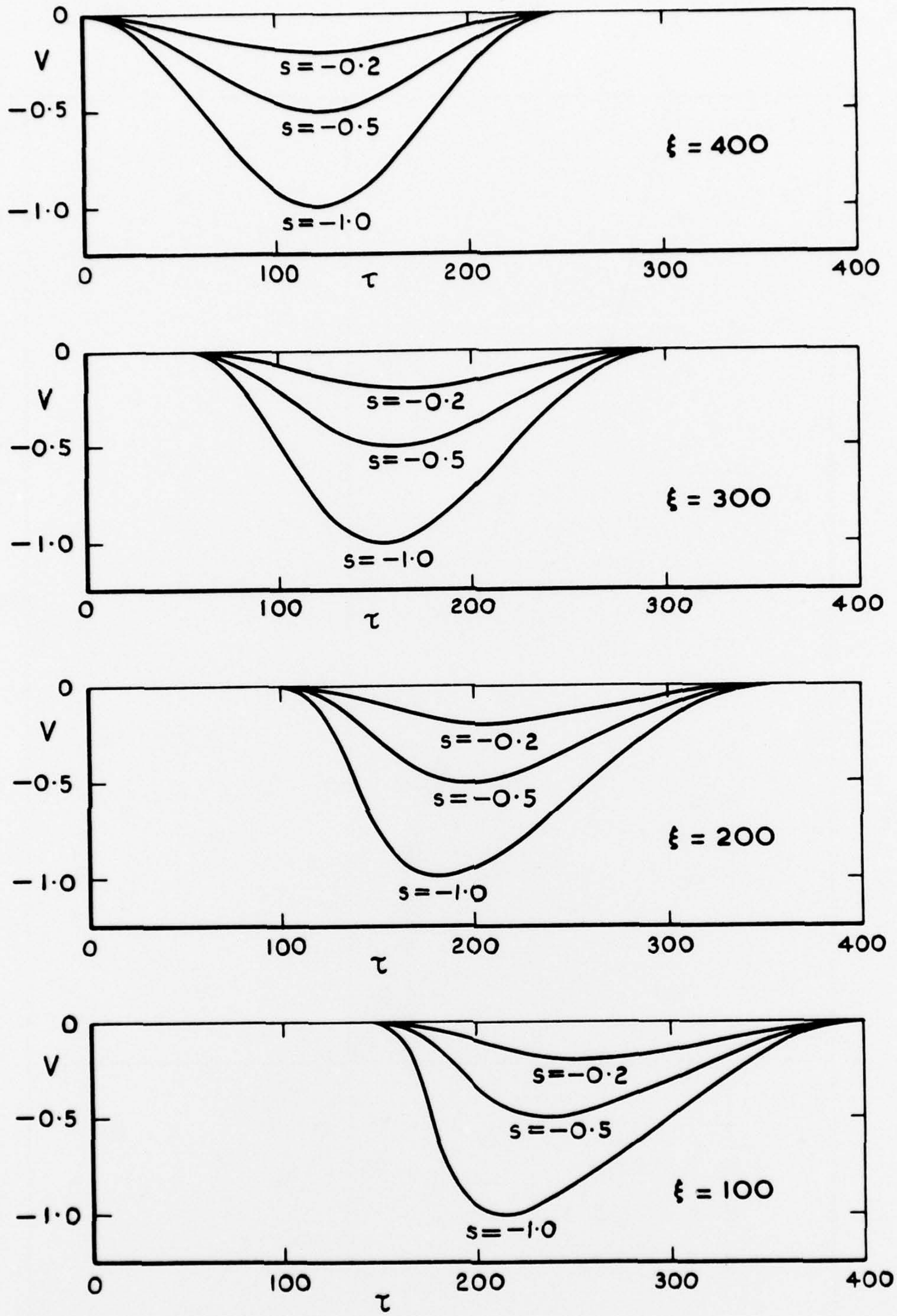

FIG. 8. The variation of particle velocity with τ for a negative wave of type B; no shock, $\lambda = 500$.

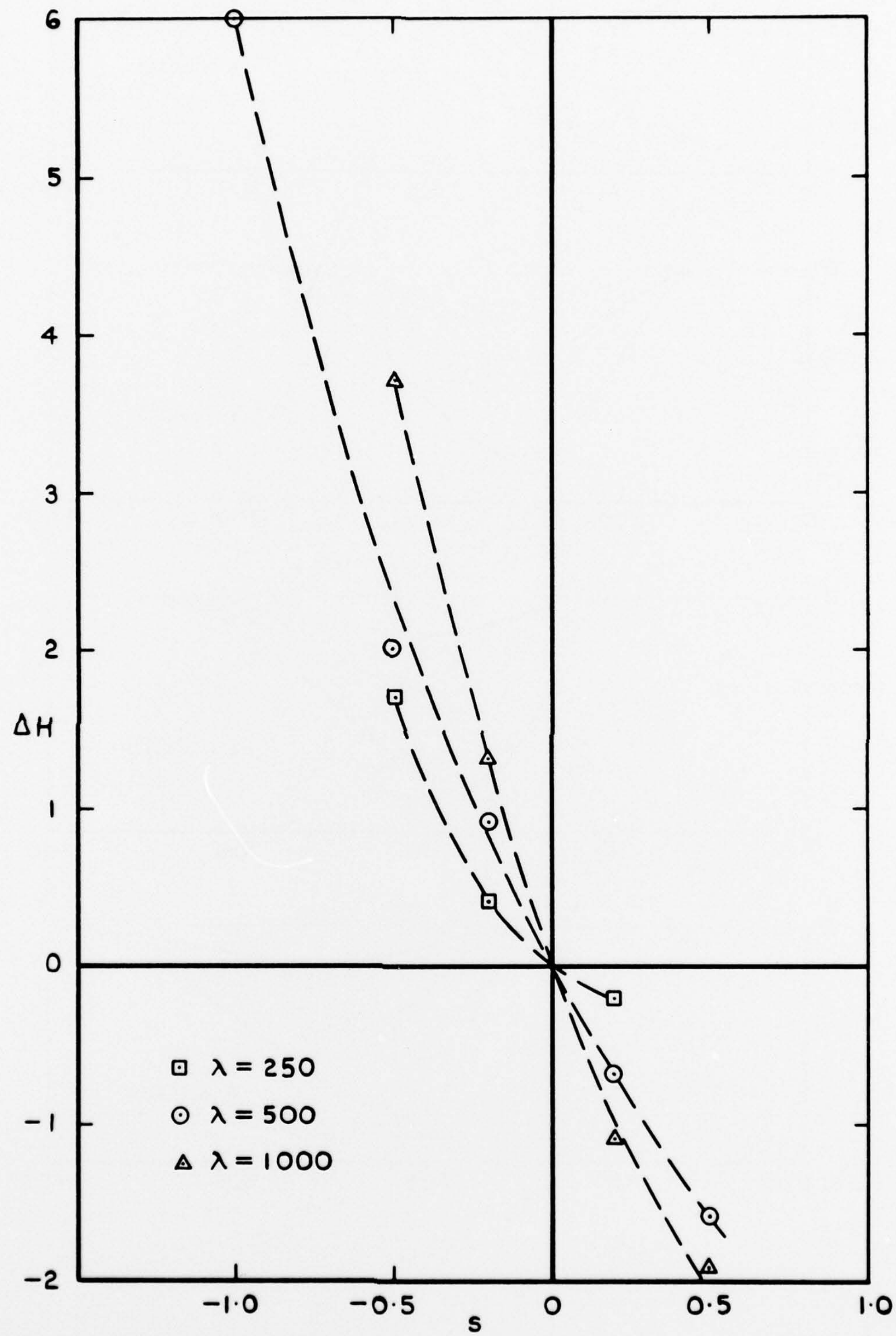

FIG. 9. The variation of ΔH with the amplitude of the disturbance wave for waves of type B.

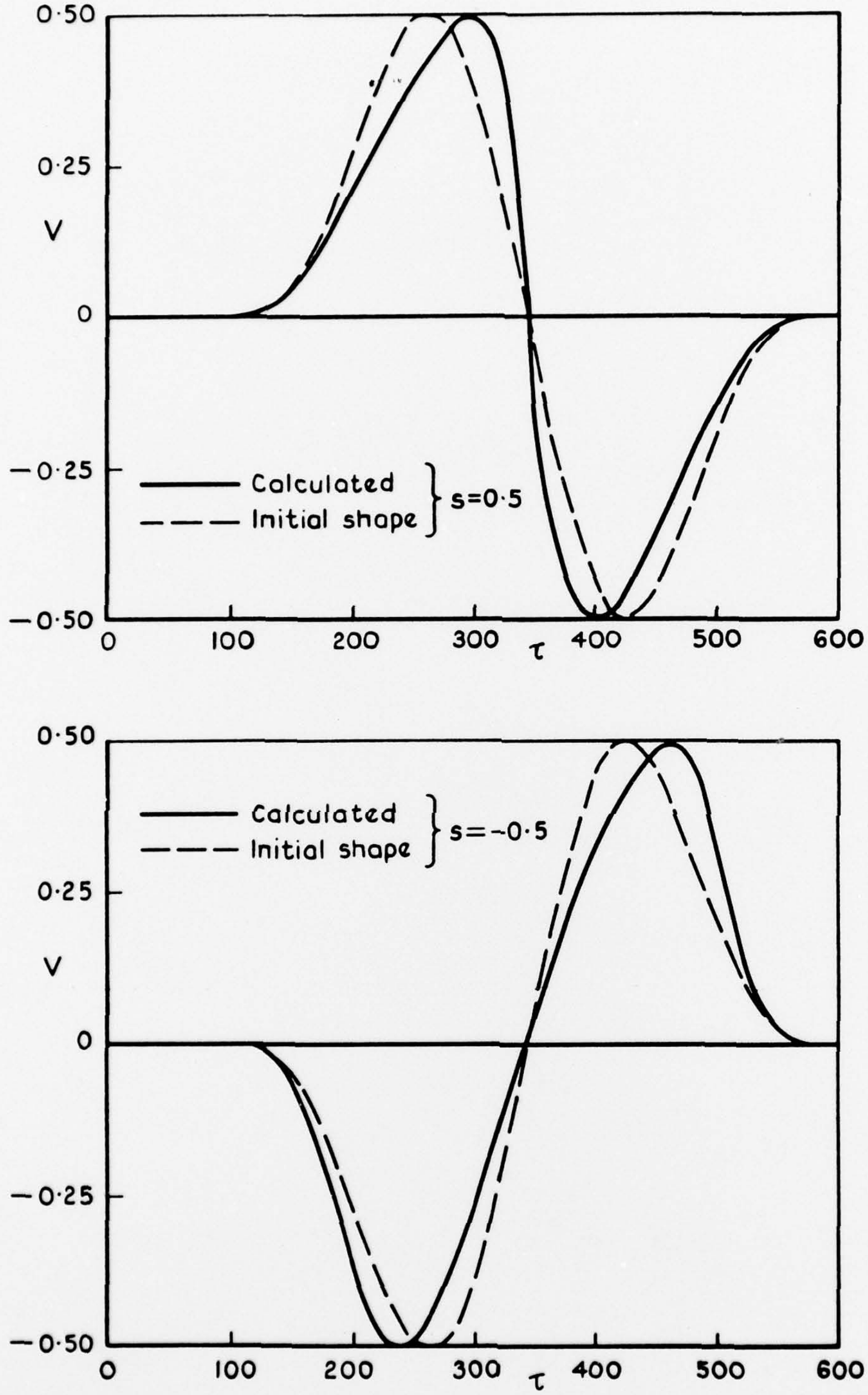

FIG. 10. The variation of particle velocity with τ for waves of type C at $\xi = 200$; no shock, $\lambda = 1000$.

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The problem is simulated in one-dimension by a variation of the 'piston problem' in which pressure waves representing turbulence are made to pass through a shock wave. The governing partial differential equations are solved using a finite difference technique.

The results of the calculations show that a shock wave tends to become thicker on passing through a disturbance wave having equal positive and negative parts. This is consistent with the proposed explanation of shock-thickening although the thickening calculated is small compared to the initial shock thickness, for the disturbance waves used. Distortion of the disturbance waves with time prevented a wider study of the effects of varying amplitude and wavelength being made so that precise quantitative conclusions could not be drawn but it is suggested that the process investigated is unlikely to be powerful enough to produce the shock thickening measured experimentally in sonic boom research.

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