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NUMERICAL TREATMENT OF ARBITRARILY-SHAPED REGIONS  
IN FLUID DYNAMICS

Joanna W. Schot



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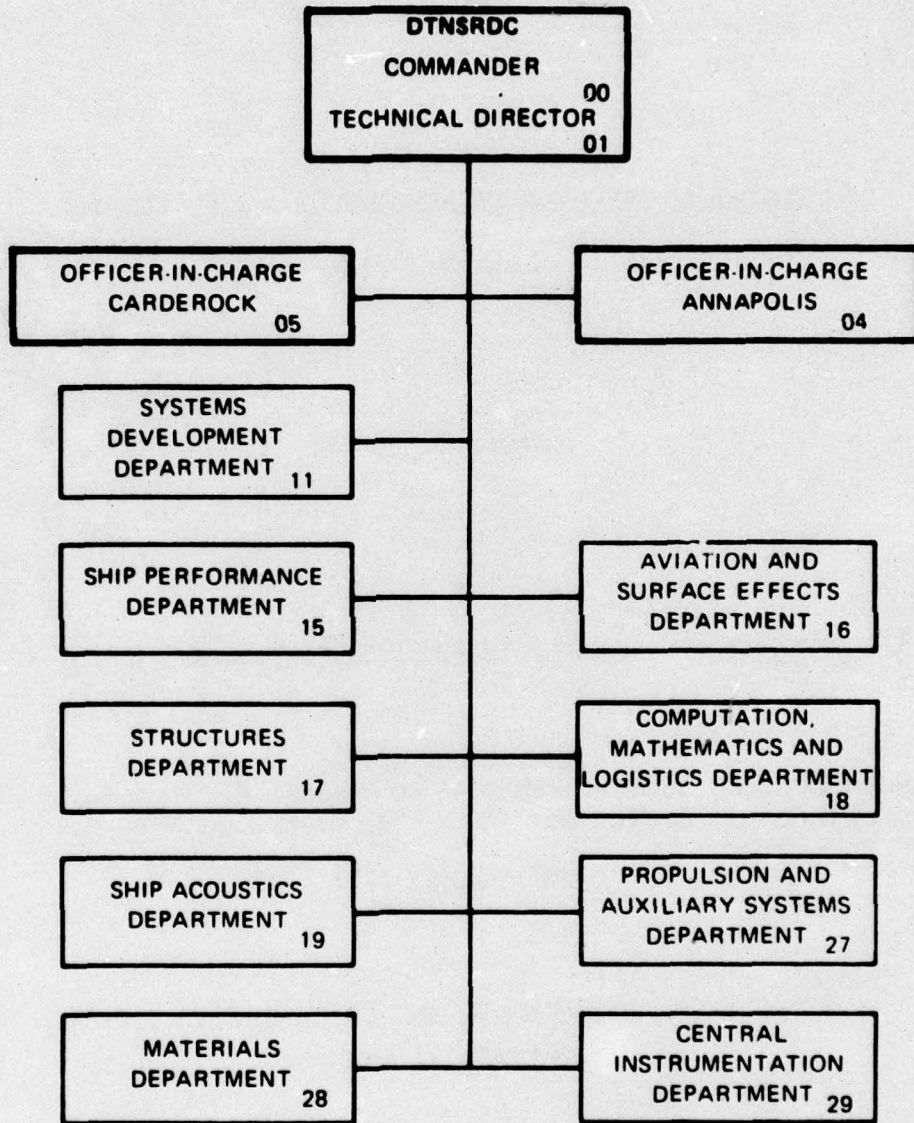
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NUMERICAL TREATMENT OF ARBITRARILY-SHAPED REGIONS  
IN FLUID DYNAMICS

by

Joanna W. Schot

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INTRODUCTION

In numerical aerodynamic and hydrodynamic flow calculations, both the shape of the body under investigation and the outer boundary of the surrounding fluid region determine the geometry of the computational flow field. Various procedures are in current use to translate a real flow problem into a discretized formulation for finite-difference or other numerical calculations on a computer. This paper addresses the need for improving and automating the numerical definition of arbitrarily-shaped flow domains and body surfaces and describes new techniques for using the computer to generate accurate geometric models for flow calculations.

CURRENT METHODS

Whenever a body and the flow region to be studied are of simple shape, the network of discrete points representing the body surface and the flow field can be adequately defined by using either standard Cartesian coordinates or a conformal transformation into another orthogonal coordinate system in which a coordinate line (or surface) may closely approximate the body contours. The use of polar, elliptical, spheroidal, and prolate-spheroidal coordinates are examples. It is no problem at all to prepare the geometric input data for such flow domains and body shapes -- only a few parameters have to be numerically specified. However, in the practical world of aircraft and ship design, the bodies about which potential flow and viscous flow calculations must be performed are often too complicated to fit easily into one of these "natural" coordinate systems. If, in addition, the outer boundary of the flow region is of irregular shape or is changing with time, as in free surface

flows involving ship- or wind-generated waves, then the problem of handling the geometry is even more challenging.

For such arbitrarily-shaped and time-dependent flow domains, the use of fixed rectangular networks or grids for finite-difference calculations presents computational difficulties which have been investigated by many authors. A major problem encountered is that the numerical solutions of the flow equations may be grossly inaccurate in critical locations close to the body surface, where rapid changes occur in the flow properties (velocity, pressure, vorticity, temperature, etc.) The seriousness of this inaccuracy depends of course on the type of flow problem as well as the geometric complexities.

For example, Dawson and Marcus [1] showed for two-dimensional viscous incompressible flow based on the Navier-Stokes equations that the accuracy of the vorticity calculation at points on and near the body is very sensitive to the grid point-density and the body curvature. The well-known MAC (Marker and Cell) method developed at the Los Alamos Scientific Laboratory has been used to compute both two- and three-dimensional flows with free surfaces, but most of the body or wall shapes have been composed of fixed rectangular segments of the grid. Improvements in MAC to handle curved bodies and moving surfaces, at least in two dimensions, have been developed by Viacelli [2] and others, including Hirt, Nichols, and Romero [3].

It is important to point out, on the other hand, that three-dimensional potential flow calculations based on the source-sink formulation are routinely performed for very complicated bodies, such as large ships and aircraft, by using the "panel" method in Cartesian coordinates to approximate the body geometry. In this approach, as developed originally by Hess and Smith [4] and refined by Dawson and Dean [5], among others, the body is specified by a set of points in Cartesian coordinates which are located at strategic positions on the body surface. The computer is programmed to connect these geometric data points with either straight lines or curves to form quadrilaterals or "panels". See Figure 1. Even with the use of electronic digitizing tablets, the preparation of the geometric input data for large and complex

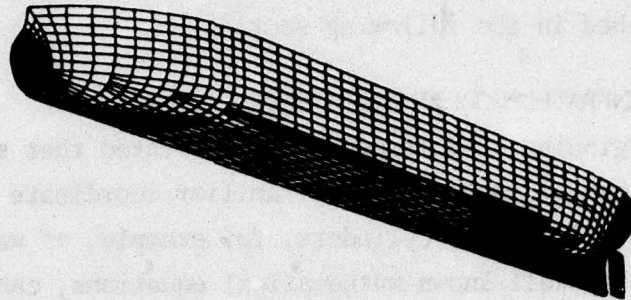


FIG. 1 SHIP HULL DEFINED BY PANEL METHOD FOR POTENTIAL FLOW CALCULATION

bodies is prone to error and time-consuming. Recently new methods for including free-surfaces in potential flow programs have been explored and are under development by Dawson [6] using source distribution formulations and by Ohring and Telste [7] using finite-difference operators and matrix imbedding techniques.

Among other approaches for dealing with free surface flow problems, Boris and Hain [8] have cited the geometrical advantages of using a "general connectivity triangular mesh" for two-dimensional free surface Lagrangian hydrodynamic calculations. This approach, which has some features of a finite-element method, does not appear suitable for calculations in three-dimensional space. Finite-element methods are also under development by Bai [9] and others (see next section) for handling free surface flows.

A very promising new approach for dealing with arbitrarily-shaped and time-dependent flow regions is to numerically generate a specific curvilinear coordinate system such that a coordinate line is coincident with each of the boundaries of the physical region. This technique has been developed by Thompson, Thames, and Mastin [10], and by others, see for example Chia, Chia, and Studerus [11] based on earlier work, to

numerically transform arbitrarily-shaped and multiply-connected flow domains in the physical plane  $(x,y)$  into convenient-to-use rectangular domains with uniform grid spacing in the computational plane  $(\xi,\eta)$ . This method of numerically generating boundary-fitted coordinate systems is briefly described in the following section.

#### NUMERICALLY GENERATED COORDINATE SYSTEMS

At the beginning of this paper it was stated that simple body shapes can be easily represented by familiar coordinate systems. Thus, all circular or elliptical cylinders, for example, as well as many other shapes defined by well-known mathematical equations, can be easily "discretized" for numerical calculations. For more complex shapes, explicit mathematical expressions are not so well known. However, by setting up a system of elliptic partial differential equations whose solutions are the desired coordinate curves, numerical methods can be used to solve these equations, thereby generating curvilinear coordinate systems. As described by Thompson and his students, see [12] for example, this method is summarized below, using their notation for convenience in referencing their articles.

Let  $x,y$  be the coordinates in the physical plane and  $\xi,\eta$  those in the transformed plane. Let the arbitrarily-shaped flow region contain a body with a closed boundary contour  $\Gamma_1$  and an irregular outer boundary  $\Gamma_2$ . Thus, a doubly-connected region is to be transformed into a rectangular domain such that the curve  $\Gamma_1$  corresponds to the constant coordinate line  $\eta = \eta_1$  and  $\Gamma_2$  to the line  $\eta = \eta_2$ , with  $\eta_1 < \eta_2$ . The desired curvilinear coordinate may be generated by an elliptic system of the form

$$\xi_{xx} + \xi_{yy} = P(\xi,\eta)$$

$$\eta_{xx} + \eta_{yy} = Q(\xi,\eta)$$

with Dirichlet boundary conditions which control the positioning of coordinate lines of prescribed values to coincide with the body contour  $\Gamma_1$  and the outer boundary  $\Gamma_2$ . In the above equations, subscripts denote partial differentiation and  $P$  and  $Q$  are functions which may be specified

to suit the problem at hand. Since it is most convenient to perform all calculations in the transformed (computational) plane, the dependent and independent variables must be interchanged to obtain the inverse mapping. This results in:

$$\begin{aligned} \alpha x_{\xi\xi} - 2\beta x_{\xi\eta} + \gamma x_{\eta\eta} &= -J^2[x_{\xi} P(\xi, \eta) + x_{\eta} Q(\xi, \eta)] \\ \alpha y_{\xi\xi} - 2\beta y_{\xi\eta} + \gamma y_{\eta\eta} &= -J^2[y_{\xi} P(\xi, \eta) + y_{\eta} Q(\xi, \eta)], \end{aligned}$$

where

$$\begin{aligned} \alpha &= x_{\eta}^2 + y_{\eta}^2 \\ \beta &= x_{\xi}x_{\eta} + y_{\xi}y_{\eta} \\ \gamma &= x_{\xi}^2 + y_{\xi}^2 \end{aligned}$$

and  $J = x_{\xi}y_{\eta} - x_{\eta}y_{\xi}$  is the Jacobian of the inverse transformation.

The flow equations to be solved must also be transformed from the original Cartesian system to the computational plane. For the Navier-Stokes equations of viscous flows this means the introduction of cross derivative terms which somewhat complicates the numerical solution. See [12], and the survey on the numerical treatment of Navier-Stokes equations by Lugt [13]. However, the computations are always carried out on the simple uniform grid of the  $\xi, \eta$  plane, which is a great advantage, especially for time-dependent flow regions. For transient free-surface potential flow calculations, for example, see the work of Haussling and Coleman [14] who have extended Thompson's method to permit greater variation in the point density of the generated grid systems. Examples of the types of curvilinear grid systems which they have generated are shown in Figures 2 and 3. Their numerical mesh generation program, known as NUMESH, see [15], has also been used by Zarda and Marcus [16] to obtain grids for finite-element calculations of free-surface flow problems using the NASTRAN (Nasa Structural Analysis) program.

EXAMPLES OF NUMERICALLY GENERATED COORDINATE SYSTEMS  
FOR FINITE DIFFERENCE CALCULATIONS

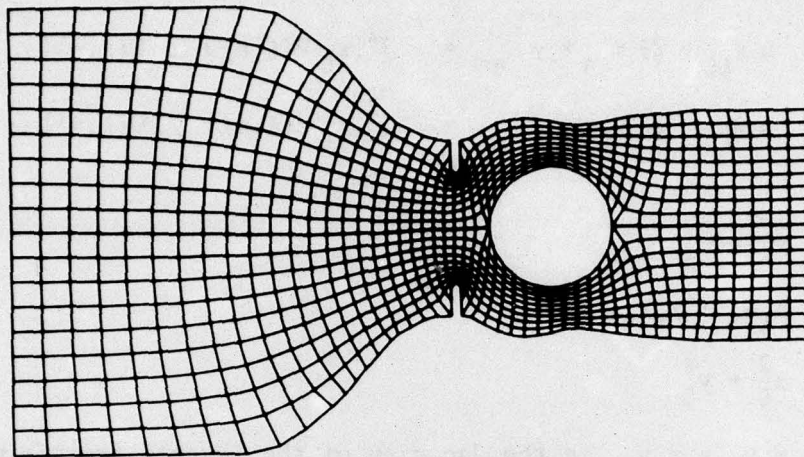


FIG. 2 TYPE 1 MESH FOR FLOW AROUND A BODY IN  
A CONSTRICTED CHANNEL

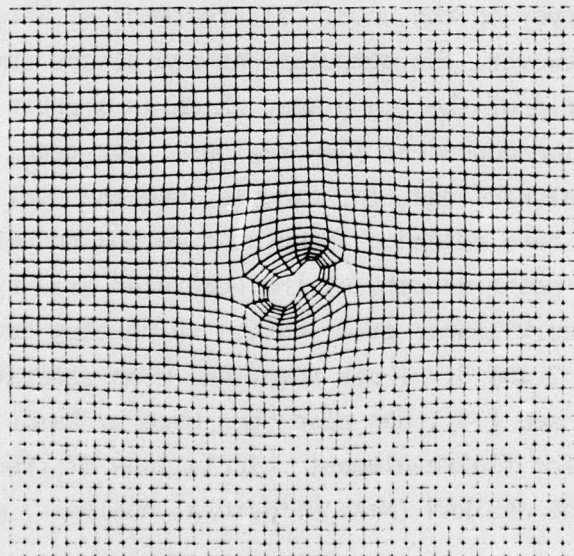


FIG. 3 TYPE 2 MESH FOR FLOW AROUND AN  
ARBITRARY BODY

## COMPUTER GRAPHICS DISPLAY

There are sophisticated computer methods also being developed for generating arbitrary three-dimensional body surfaces from a minimal amount of geometric input data. McKee and Kazden [17] have contributed to the development of an engineering aid which uses basic-spline functions to numerically fit complex three-dimensional surfaces. Figures 4, 5, and 6 illustrate the types of ship surfaces which they have generated from very few input data points with the aid of computer graphics display terminals.

An important requirement in any method for numerically generating body surfaces and grid systems is the availability of a fast-response computer graphics terminal with a "hard-copy" capability. By displaying the generated geometric patterns, the user can quickly evaluate the quality of the computed results and make required corrections. Such interactive techniques reduce errors and real time delays in the analysis of complex aerodynamic and hydrodynamic flow problems as well as structural analysis problems. Figure 7 illustrates a submarine configuration which was modeled by manually prepared input data for structural analysis computations. See Everstine, Schroeder, and Marcus [18] for a description of the analysis of such submerged bodies.

The field of structural analysis is well-advanced in the use of computers to analyze very large, complex structures, such as the destroyer illustrated in Figure 8. With the exploitation of faster numerical and computer graphics techniques developed in both the fields of fluid dynamics and structural mechanics, impressive breakthroughs can be achieved in the speed and accuracy of solving practical and important engineering design problems.

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EXAMPLES OF SHIP SURFACES DEFINED NUMERICALLY  
BY BASIC SPLINE FUNCTIONS

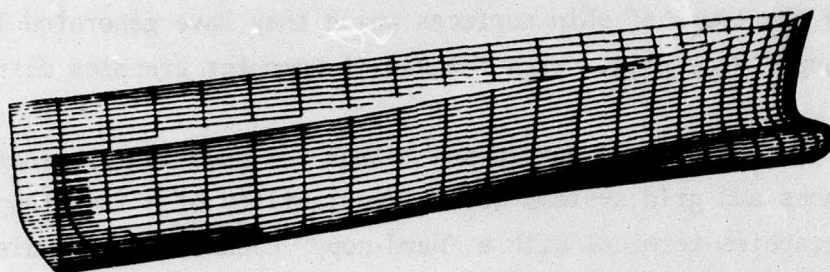


FIG. 4 PORTION OF HULL, SIDE VIEW

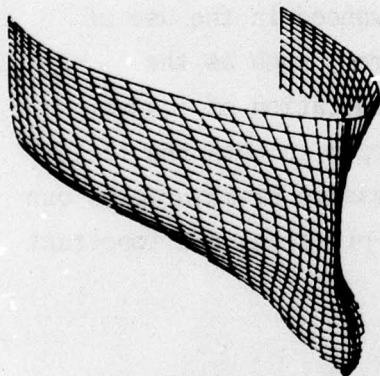


FIG. 5

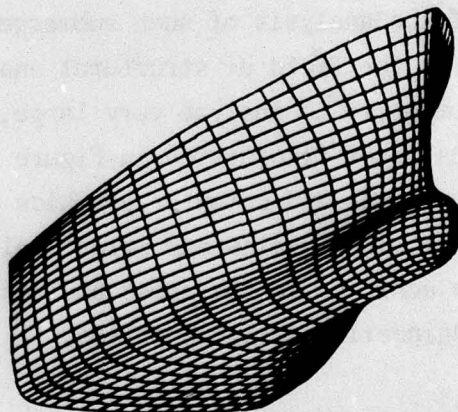


FIG. 6

PERSPECTIVE VIEWS OF BOW WITH SONAR DOME

EXAMPLES OF BODY GEOMETRY CONFIGURATIONS BASED ON  
MANUALLY DEFINED INPUT DATA

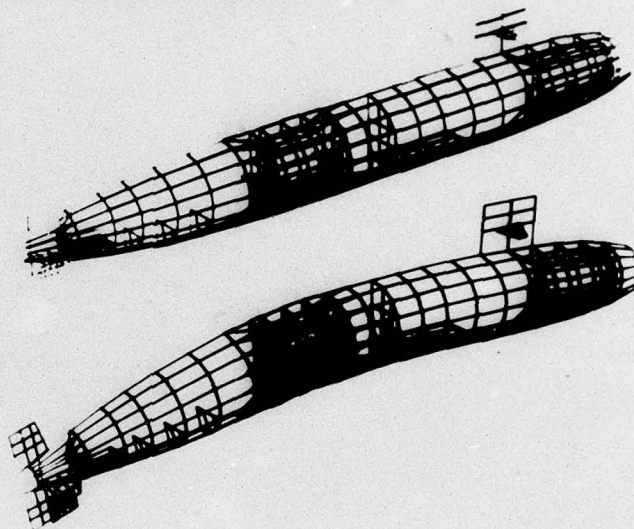
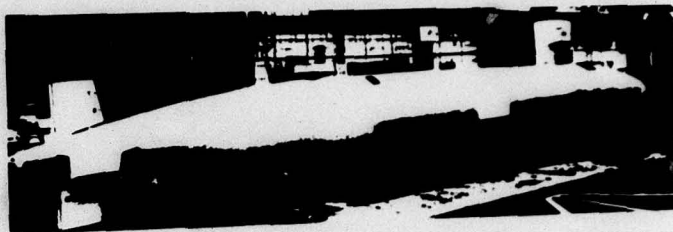


FIG. 7 TEST SUBMARINE AND NUMERICAL MODEL FOR  
FLUID-STRUCTURE INTERACTION ANALYSIS

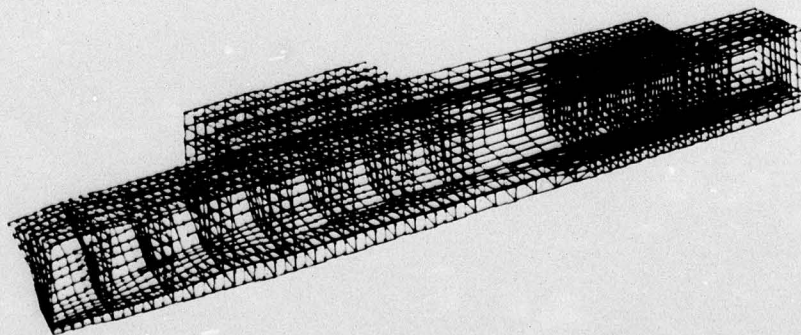


FIG. 8 FINITE-ELEMENT MODEL OF DESTROYER FOR STRUCTURAL ANALYSIS

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