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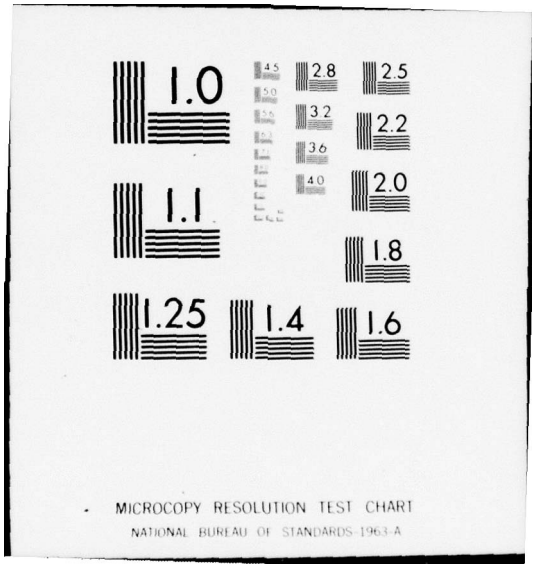
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DIRECT DETERMINATION OF FLEXURAL STRAINS IN PLATES USING PROJECTED GRATINGS

BY

C. Y. LIANG, Y. Y. HUNG, A. J. DURELLI AND J. D. HOVANESIAN

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ON

CONTRACT No. N00014-76-C-0487
O.U. PROJECT No. 30339-49
REPORT No. 45

SCHOOL OF ENGINEERING
OAKLAND UNIVERSITY
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29 p.

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Abstract

It is proposed in this paper to use projected gratings to determine principal strains in plates subjected to bending. The method requires the rotation of one photograph of the deformed grating over a copy of itself, the rotation taking place about the point at which the bending strains are desired. The idea previously suggested by Stetson to double differentiate holograms of bent plates has been extended and verified with an application to a plate subjected to twisting. The proposed method is easy to use and permits the determination of principal strains in plates subjected to much larger deflections than the ones that can be studied using holograms. Suggestions to improve the precision of the determination are also made.

Previous Technical Reports to the Office of Naval Research

1. A. J. Durelli, "Development of Experimental Stress Analysis Methods to Determine Stresses and Strains in Solid Propellant Grains"--June 1962.
Developments in the manufacturing of grain-propellant models are reported. Two methods are given: a) cementing routed layers and b) casting.

2. A. J. Durelli and V. J. Parks, "New Method to Determine Restrained Shrinkage Stresses in Propellant Grain Models"--October 1962.
The birefringence exhibited in the curing process of a partially restrained polyurethane rubber is used to determine the stress associated with restrained shrinkage in models of solid propellant grains partially bonded to the case.

3. A. J. Durelli, "Recent Advances in the Application of Photoelasticity in the Missile Industry"--October 1962.
Two- and three-dimensional photoelastic analysis of grains loaded by pressure and by temperature are presented. Some applications to the optimization of fillet contours and to the redesign of case joints are also included.

4. A. J. Durelli and V. J. Parks, "Experimental Solution of Some Mixed Boundary Value Problems"--April 1964.
Means of applying known displacements and known stresses to the boundaries of models used in experimental stress analysis are given. The application of some of these methods to the analysis of stresses in the field of solid propellant grains is illustrated. The presence of the "pinching effect" is discussed.

5. A. J. Durelli, "Brief Review of the State of the Art and Expected Advance in Experimental Stress and Strain Analysis of Solid Propellant Grains"--April 1964.
A brief review is made of the state of the experimental stress and strain analysis of solid propellant grains. A discussion of the prospects for the next fifteen years is added.

6. A. J. Durelli, "Experimental Strain and Stress Analysis of Solid Propellant Rocket Motors"--March 1965.
A review is made of the experimental methods used to strain-analyze solid propellant rocket motor shells and grains when subjected to different loading conditions. Methods directed at the determination of strains in actual rockets are included.

7. L. Ferrer, V. J. Parks and A. J. Durelli, "An Experimental Method to Analyze Gravitational Stresses in Two-Dimensional Problems"--October 1965.
Photoelasticity and moiré methods are used to solve two-dimensional problems in which gravity-stresses are present.

8. A. J. Durelli, V. J. Parks and C. J. del Rio, "Stresses in a Square Slab Bonded on One Face to a Rigid Plate and Shrunk"--November 1965.
A square epoxy slab was bonded to a rigid plate on one of its faces in the process of curing. In the same process the photoelastic effects associated with a state of restrained shrinkage were "frozen-in." Three-dimensional photoelasticity was used in the analysis.

9. A. J. Durelli, V. J. Parks and C. J. del Rio, "Experimental Determination of Stresses and Displacements in Thick-Wall Cylinders of Complicated Shape"--April 1966.
Photoelasticity and moiré are used to analyze a three-dimensional rocket shape with a star shaped core subjected to internal pressure.

10. V. J. Parks, A. J. Durelli and L. Ferrer, "Gravitational Stresses Determined Using Immersion Techniques"--July 1966.
The methods presented in Technical Report No. 7 above are extended to three-dimensions. Immersion is used to increase response.

11. A. J. Durelli and V. J. Parks. "Experimental Stress Analysis of Loaded Boundaries in Two-Dimensional Second Boundary Value Problems"--February 1967.
The pinching effect that occurs in two-dimensional bonding problems, noted in Reports 2 and 4 above, is analyzed in some detail.

12. A. J. Durelli, V. J. Parks, H. C. Feng and F. Chiang, "Strains and Stresses in Matrices with Inserts,"-- May 1967.
Stresses and strains along the interfaces, and near the fiber ends, for different fiber end configurations, are studied in detail.

13. A. J. Durelli, V. J. Parks and S. Uribe, "Optimization of a Slot End Configuration in a Finite Plate Subjected to Uniformly Distributed Load,"--June 1967.
Two-dimensional photoelasticity was used to study various elliptical ends to a slot, and determine which would give the lowest stress concentration for a load normal to the slot length.

14. A. J. Durelli, V. J. Parks and Han-Chow Lee, "Stresses in a Split Cylinder Bonded to a Case and Subjected to Restrained Shrinkage,"--January 1968.
A three-dimensional photoelastic study that describes a method and shows results for the stresses on the free boundaries and at the bonded interface of a solid propellant rocket.

15. A. J. Durelli, "Experimental Stress Analysis Activities in Selected European Laboratories"--August 1968.
This report has been written following a trip conducted by the author through several European countries. A list is given of many of the laboratories doing important experimental stress analysis work and of the people interested in this kind of work. An attempt has been made to abstract the main characteristics of the methods used in some of the countries visited.

16. V. J. Parks, A. J. Durelli and L. Ferrer, "Constant Acceleration Stresses in a Composite Body"--October 1968.
Use of the immersion analogy to determine gravitational stresses in two-dimensional bodies made of materials with different properties.

17. A. J. Durelli, J. A. Clark and A. Kochev, "Experimental Analysis of High Frequency Stress Waves in a Ring"--October 1968.
A method for the complete experimental determination of dynamic stress distributions in a ring is demonstrated. Photoelastic data is supplemented by measurements with a capacitance gage used as a dynamic lateral extensometer.

18. J. A. Clark and A. J. Durelli, "A Modified Method of Holographic Interferometry for Static and Dynamic Photoelasticity"--April 1968.
A simplified absolute retardation approach to photoelastic analysis is described. Dynamic isopachics are presented.

19. J. A. Clark and A. J. Durelli, "Photoelastic Analysis of Flexural Waves in a Bar"--May 1969.
A complete direct, full-field optical determination of dynamic stress distribution is illustrated. The method is applied to the study of flexural waves propagating in a urethane rubber bar. Results are compared with approximate theories of flexural waves.

20. J. A. Clark and A. J. Durelli, "Optical Analysis of Vibrations in Continuous Media"--June 1969.
Optical methods of vibration analysis are described which are independent of assumptions associated with theories of wave propagation. Methods are illustrated with studies of transverse waves in prestressed bars, snap loading of bars and motion of a fluid surrounding a vibrating bar.

21. V. J. Parks, A. J. Durelli, K. Chandrashekhara and T. L. Chen, "Stress Distribution Around a Circular Bar, with Flat and Spherical Ends, Embedded in a Matrix in a Triaxial Stress Field"--July 1969.
A Three-dimensional photoelastic method to determine stresses in composite materials is applied to this basic shape. The analyses of models with different loads are combined to obtain stresses for the triaxial cases.

22. A. J. Durelli, V. J. Parks and L. Ferrer, "Stresses in Solid and Hollow Spheres Subjected to Gravity or to Normal Surface Tractions"--October 1969.
The method described in Report No. 10 above is applied to two specific problems. An approach is suggested to extend the solutions to a class of surface traction problems.

23. J. A. Clark and A. J. Durelli, "Separation of Additive and Subtractive Moiré Patterns"--December 1969.
A spatial filtering technique for adding and subtracting images of several gratings is described and employed to determine the whole field of Cartesian shears and rigid rotations.

24. R. J. Sanford and A. J. Durelli, "Interpretation of Fringes in Stress-Holo-Interferometry"--July 1970.
Errors associated with interpreting stress-holo-interferometry patterns as the superposition of isopachics (with half order fringe shifts) and isochromatics are analyzed theoretically and illustrated with computer generated holographic interference patterns.

25. J. A. Clark, A. J. Durelli and P. A. Laura, "On the Effect of Initial Stress on the Propagation of Flexural Waves in Elastic Rectangular Bars"--December 1970.
Experimental analysis of the propagation of flexural waves in prismatic, elastic bars with and without prestressing. The effects of prestressing by axial tension, axial compression and pure bending are illustrated.

26. A. J. Durelli and J. A. Clark, "Experimental Analysis of Stresses in a Buoy-Cable System Using a Birefringent Fluid"--February 1971.
An extension of the method of photoviscous analysis is presented which permits quantitative studies of strains associated with steady state vibrations of immersed structures. The method is applied in an investigation of one form of behavior of buoy-cable systems loaded by the action of surface waves.

27. A. J. Durelli and T. L. Chen, "Displacements and Finite-Strain Fields in a Sphere Subjected to Large Deformations"--February 1972.
Displacements and strains (ranging from 0.001 to 0.50) are determined in a polyurethane sphere subjected to several levels of diametral compression. A 500 lines-per-inch grating was embedded in a meridian plane of the sphere and moiré effect produced with a non-deformed master. The maximum applied vertical displacement reduced the diameter of the sphere by 27 per cent.

28. A. J. Durelli and S. Machida, "Stresses and Strain in a Disk with Variable Modulus of Elasticity"--March 1972.
A transparent material with variable modulus of elasticity has been manufactured that exhibits good photoelastic properties and can also be strain analyzed by moiré. The results obtained suggests that the stress distribution in the homogeneous disk. It also indicates that the strain fields in both cases are very different, but that it is possible, approximately, to obtain the stress field from the strain field using the value of E at every point, and Hooke's law.

29. A. J. Durelli and J. Buitrago, "State of Stress and Strain in A Rectangular Belt Pulled Over a Cylindrical Pulley"--June 1972.
Two- and three-dimensional photoelasticity as well as electrical strain gages, dial gages and micrometers are used to determine the stress distribution in a belt-pulley system. Contact and tangential stress for various contact angles and friction coefficients are given.

30. T. L. Chen and A. J. Durelli, "Stress Field in a Sphere Subjected to Large Deformations"--June 1972.
Strain fields obtained in a sphere subjected to large diametral compressions from a previous paper were converted into stress fields using two approaches. First, the concept of strain-energy function for an isotropic elastic body was used. Then the stress field was determined with the Hookean type natural stress-natural strain relation. The results so obtained were also compared.

31. A. J. Durelli, V. J. Parks and H. M. Hasseem, "Helices Under Load"--July 1973.
Previous solutions for the case of close coiled helical springs and for helices made of thin bars are extended. The complete solution is presented in graphs for the use of designers. The theoretical development is correlated with experiments.

32. T. L. Chen and A. J. Durelli, "Displacements and Finite Strain Fields in a Hollow Sphere Subjected to Large Elastic Deformations"--September 1973.
The same methods described in No. 27, were applied to a hollow sphere with an inner diameter one half the outer diameter. The hollow sphere was loaded up to a strain of 30 per cent on the meridian plane and a reduction of the diameter by 20 per cent.

33. A. J. Durelli, H. H. Hasseem and V. J. Parks, "New Experimental Method in Three-Dimensional Elastostatics"--December 1973.
A new material is reported which is unique among three-dimensional stress-freezing materials, in that, in its heated (or rubbery) state it has a Poisson's ratio which is appreciably lower than 0.5. For a loaded model, made of this material, the unique property allows the direct determination of stresses from strain measurements taken at interior points in the model.

34. J. Wolak and V. J. Parks, "Evaluation of Large Strains in Industrial Applications"--April 1974.
It was shown that Mohr's circle permits the transformation of strain from one axis of reference to another, irrespective of the magnitude of the strain, and leads to the evaluation of the principal strain components from the measurement of direct strain in three directions.

35. A. J. Durelli, "Experimental Stress Analysis Activities in Selected European Laboratories"--April 1975.
Continuation of Report No. 15 after a visit to Belgium, Holland, Germany, France, Turkey, England and Scotland.

36. A. J. Durelli, V. J. Parks and J. O. Bühler-Vidal, "Linear and Non-linear Elastic and Plastic Strains in a Plate with a Big Hole Loaded Axially in its Plane"--July 1975.
Strain analysis of the ligament of a plate with a big hole indicates that both geometric and material non-linearity may take place. The strain concentration factor was found to vary from 1 to 2 depending on the level of deformation.

37. A. J. Durelli, V. Pavlin, J. O. Bühler-Vidal and G. Ome, "Elastostatics of a Cubic Box Subjected to Concentrated Loads"--August 1975.
Analysis of experimental strain, stress and deflection of a cubic box subjected to concentrated loads applied at the center of two opposite faces. The ratio between the inside span and the wall thickness was varied between approximately 5 and 121.

38. A. J. Durelli, V. J. Parks and J. O. Bühler-Vidal, "Elastostatics of Cubic Boxes Subjected to Pressure"--March 1976.
Experimental analysis of strain, stress and deflections in a cubic box subjected to either internal or external pressure. Inside span-to-wall thickness ratio varied from 5 to 14.

39. Y. Y. Hung, J. D. Hovanesian and A. J. Durelli, "New Optical Method to Determine Vibration-Induced Strains with Variable Sensitivity After Recording"--November 1976.
A steady state vibrating object is illuminated with coherent light and its image slightly misfocused. The resulting specklegram is "time-integrated" as when Fourier filtered gives derivatives of the vibrational amplitude.

40. Y. Y. Hung, C. Y. Liang, J. D. Hovanesian and A. J. Durelli, "Cyclic Stress Studies by Time-Averaged Photoelasticity"--November 1976.
"Time-averaged isochromatics" are formed when the photographic film is exposed for more than one period. Fringes represent amplitudes of the oscillating stress according to the zeroth order Bessel function.

41. Y. Y. Hung, C. Y. Liang, J. D. Hovanesian and A. J. Durelli, "Time-Averaged Shadow Moiré Method for Studying Vibrations"--November 1976.
Time-averaged shadow moiré permits the determination of the amplitude distribution of the deflection of a steady vibrating plate.

42. J. Buitrago and A. J. Durelli, "On the Interpretation of Shadow-Moiré Fringes"--April 1977.
Possible rotations and translations of the grating are considered in a general expression to interpret shadow-moiré fringes and on the sensitivity of the method. Application to an inverted perforted tube.

43. J. der Hovanesian, "18th Polish Solid Mechanics Conference." Published in European Scientific Notes of the Office of Naval Research, in London, England, Dec. 31, 1976.
Comments on the planning and organization of, and scientific content of paper presented at the 18th Polish Solid Mechanics Conference held in Wisla-Jawornik from September 7-14, 1976.

44. A. J. Durelli, "The Difficult Choice," -- May 1977.
The advantages and limitations of methods available for the analyses of displacements, strain, and stresses are considered. Comments are made on several theoretical approaches, in particular approximate methods, and attention is concentrated on experimental methods: photo-elasticity, moiré, brittle and photoelastic coatings, gages, grids, holography and speckle to solve two- and three-dimensional problems in elasticity, plasticity, dynamics and anisotropy.

Nomenclature

a_n	minimum diameter of ellipse or hyperbola associated with fringe order n
b_n	maximum diameter of ellipse or hyperbola associated with fringe order n
E	modulus of elasticity
h	distance of plate surface from neutral axis
I(x,y)	light intensity distribution of undeformed plate
I'(x,y)	light intensity distribution of deformed plate
n	fringe order
p	pitch of grating
t	thickness of test plate
T(x,y)	intensity transmittance of photographic plate recording
T'(x,y)	intensity transmittance of contact copy
w(x,y)	deformation function corrected
x,y,z	co-ordinates
ε_1, ε_2	maximum and minimum principal strain, respectively.
θ	angle between normal to the test plate and viewing direction of camera.
ν	Poisson's ratio
σ_1,σ_2	maximum and minimum principal stress, respectively
ϕ	angle denoting the orientation of principal direction.

DIRECT DETERMINATION OF FLEXURAL STRAINS IN PLATES USING PROJECTED GRATINGS

Introduction

Plates are frequently used in engineering to resist bending loads. Depending on geometry and loading conditions, plates may be difficult to analyze mathematically. Experimental techniques are then useful. Mechanical and electrical strain gages give strains at the two faces of the plate, and from these strains moments can be computed directly. However, this information is obtained only point-by-point. Optical methods are more interesting because they give whole field information, and because no contact with the plates is required[1]. Of particular importance for this application are the projected grating,[2,3] Ligtenberg's[4] and Salet-Ikeda techniques,[5] holographic[6,7] and speckle-shearing interferometry[8]. All these optical methods measure either deflections or slope of deflections; thus it is necessary to differentiate the information once, or twice, to obtain the bending strains. These operations besides being very laborious are a source of error. The results obtained using these methods are also subjected to the limitations of the theory of plates. One of the authors used gratings directly printed on the surface of the plate and a superposed master grating to produce moiré.[9] Whole-field in-plane displacements are obtained this way and only one differentiation is required to obtain strains, but the method has not been applied yet to vibrations and is limited to relatively small deflections.

An innovative idea was introduced by Stetson[10] whereby bending moments can be determined directly using holographic interferometry. The method consists in superposing and aligning two identical transparencies of the fringe pattern of deflections obtained by holographic interferometry.

By rotating one transparency through an angle of 180° (in the plane of the film) with respect to the other about a point of interest, a moiré in the form of either an ellipse or a hyperbola is produced. Stetson showed that the directions and the length of the principal axes of these figures indicate the directions and the magnitude of the principal strians, respectively. Also, the relative sign of the two principal strains can be obtained from the form of the moiré pattern. An ellipse indicates that the principal strains are of the same sign. Opposite signs correspond to the hyberbola. The method permits only point-by-point analysis, but it is the only one so far developed for double differentiation.

The present paper extends the Stetson's idea to the projected grating method. While the hologram interferometry is mainly suitable for measuring very small deflections, the projected grating method can be used to determine relatively large ones. The paper includes considerations on the error inherent in the double differentiation operation and suggests how the results can be improved. A verification is also given.

Description of the Method

A grating is projected onto the plate (Fig. 1), in a direction making an angle θ with the normal to the plate. The projection may be achieved by using either a projector[2] or by interference of two coherent beams[3]. When the plate is deformed, the shadow of the grating on the surface of the plate is perturbated because of the deflection. The perturbated grating is photographed by a camera viewing normally and is recorded on a photographic plate. A contact copy of the recorded grating is made on a second plate. The two plates are superposed and carefully aligned so that no moiré is seen. One of the plates is then rotated 180° about a point of interest. A moiré will appear in the shape of an ellipse or a hyberbola.

Analysis

The light intensity distribution of a sinusoidal grating projected onto a flat plate can be represented by:

$$I(x,y) = 1 + \cos \frac{2\pi}{p} x \tag{1}$$

where $I(x,y)$ is the intensity distribution; p is the grating pitch observed on the plate. The grating is running parallel to the y-axis. When the plate is deformed, the grating will be distorted accordingly and its intensity distribution becomes:

$$I'(x,y) = 1 + \cos \frac{2\pi}{p} [x - w(x,y)\cdot\tan\theta] \tag{2}$$

where $I'(x,y)$ is the distorted grating intensity; θ is the angle between the direction of projection and the normal to the plate, and $w(x,y)$ is the deflection of the plate.

Assuming negative and linear recording, the intensity transmittance $T(x,y)$ of the photographic plate recording the intensint of Eq. (2) is:

$$T(x,y) = 1 - \cos \frac{2\pi}{p} [x - w(x,y)\cdot\tan\theta] \tag{3}$$

and the intensity transmittance $T'(s,y)$ of the contact copy is:

$$T'(x,y) = 1 + \cos \frac{2\pi}{p} [x - w(x,y)\cdot\tan\theta] \tag{4}$$

The intensity transmittance of the two transparencies superposed and aligned is represented by the product of the individual transmittances. With one plate being rotated 180°, and choosing the point of rotation as the origin, the resulting intensity transmittance $T_R(x,y)$ is given by:

$$T_R(x,y) = T(x,y) \cdot T'(-x,-y) \tag{5}$$

The above equation can be expanded to yield:

$$T_R(x,y) = 1 + \frac{1}{2}\cos\{\frac{2\pi}{p}\tan\theta[w(x,y) + w(-x,-y)]\}$$
$$- \cos\frac{2\pi}{p}[x - w(x,y)\tan\theta] + \cos\frac{2\pi}{p}[-x - w(-x,-y)\tan\theta]$$
$$- \frac{1}{2}\cos\frac{2\pi}{p}[2x - w(x,y)\tan\theta + w(-x,-y)\tan\theta] \quad (6)$$

Except for the first two terms, all the terms in the above equation are of high spatial frequency.

If the pair of plates is illuminated by a uniform intensity quasi plane wave of unity magnitude, and if one considers the average illuminence over a resolution cell ΔA, the resulting transmitted intensity field I_o may be represented by:

$$I_o(x,y) = 1 + \frac{1}{2}\cos[\frac{2\pi}{p}\tan\theta[w(x,y) + w(-x,-y)] \quad (7)$$

In Eq. (7), the linear dimensions of ΔA are considered to be large compared to the grating pitch p. Dark fringes form when

$$\frac{2\pi}{p}\tan\theta[w(x,y) + w(-x,-y)] = n\pi \quad (8)$$

where $n = 1, 3, 5, 7, 9, \ldots\ldots\ldots$, is the fringe order.

By Taylor's series expansion about the origin and terminating at the fourth term, w(x,y) can be expressed as

$$w(x,y) = w(0,0) + [x\frac{\partial w(0,0)}{\partial x} + y\frac{w(0,0)}{\partial y}]$$
$$+ \frac{1}{2!}[x^2\frac{\partial^2 w(0,0)}{\partial x^2} + 2xy\frac{\partial^2 w(0,0)}{\partial x\partial y} + y^2\frac{\partial^2 w(0,0)}{\partial^2 y}$$
$$+ \frac{1}{3!}[x^3\frac{\partial^3 w(0,0)}{\partial x^3} + 3x^2y\frac{\partial^3 w(0,0)}{\partial x^2\partial y} + 3xy^2\frac{\partial^3 w(0,0)}{\partial x\partial y^2}$$
$$+ y^3\frac{\partial^3 w(0,0)}{\partial y^3}] \quad (9)$$

and

$$w(-x,-y) = w(0,0) + \left[-x \frac{\partial w(0,0)}{\partial x} - y \frac{\partial w(0,0)}{\partial y}\right]$$

$$+ \frac{1}{2!}\left[x^2 \frac{\partial^2 w(0,0)}{\partial x^2} + 2xy \frac{\partial^2 w(0,0)}{\partial x \partial y} + y^2 \frac{\partial^2 w(0,0)}{\partial y^2}\right]$$

$$+ \frac{1}{3!}\left[-x^3 \frac{\partial^3 w(0,0)}{\partial x^3} - 3x^2 y \frac{\partial^3 w(0,0)}{\partial x^2 \partial y} - 3xy^2 \frac{\partial^3 w(0,0)}{\partial x \partial y^2}\right.$$

$$\left. - y^3 \frac{\partial^3 w(0,0)}{\partial y^3}\right] \qquad (10)$$

Define the origin such that $w(0,0) = 0$. If the orientation of the coordinate axes is chosen such that one of the partial derivatives of $w(0,0)$ vanishes, then the crossed partial derivative $\partial^2 w(0,0)/\partial x \partial y$ would vanish as well. Under these conditions,

$$w(x,y) - w(-x,-y) = x^2 \frac{\partial^2 w(0,0)}{\partial x^2} + y^2 \frac{\partial^2 w(0,0)}{\partial y^2} \qquad (11)$$

and Eq. (8) can be written as

$$x^2 \frac{\partial^2 w(0,0)}{\partial x^2} + y^2 \frac{\partial^2 w(0,0)}{\partial y^2} = \frac{np}{2\tan\theta} \qquad (12)$$

The above equation represents an ellipse or a hyperbola depending on the relative signs of $\partial^2 w(0,0)/\partial x^2$ and $\partial^2 w(0,0)/\partial y^2$. Since the cross derivative $\partial w(0,0)/\partial x \partial y$ is zero, $\partial^2 w(0,0)/\partial x^2$ and $\partial^2 w(0,0)/\partial y^2$ measure the principal bending strains at the origin, the point about which the photographic plates is rotated. They are related to the principal diameters of the ellipse or hyperbola by:

$$\frac{\partial^2 w(0,0)}{\partial x^2} = \frac{np}{2\tan\theta} \cdot \frac{1}{a_n^2} \qquad (13)$$

$$\frac{\partial^2 w(0,0)}{\partial y^2} = \frac{np}{2\tan\theta} \cdot \frac{1}{b_n^2} \qquad (14)$$

where $2a_n$ and $2b_n$ are the principal diameters associated with finge order n.

By measuring the principal diameters, the principal bending strain can be calculated by the following equations

$$\varepsilon_1 = h \frac{\partial^2 w(0,0)}{\partial x^2} = h \frac{np}{2\tan\theta} \cdot \frac{1}{a_n^2} \tag{15}$$

and

$$\varepsilon_2 = h \frac{\partial^2 w(0,0)}{\partial y^2} = h \frac{np}{2\tan\theta} \cdot \frac{1}{b_n^2} \tag{16}$$

where h is the distance from the neutral axis to the plate surface, and ε_1 and ε_2 are the maximum and minimum principal strains, respectively. The directions of the principal strains are indicated by the orientation of the principal diameters. To use Eqs. (15) and (16), it is necessary to obtain values of $\frac{n}{a_n^2}$ and $\frac{n}{b_n^2}$ as near to the point of interest as possible. Precisely at the origin, both n and a_n (or b_n) approach zero, therefore the ratios become mathematically indeterminate.

To increase the precision of the procedure it is advisable to determine the limit of the above ratios. As one moves along a major diameter, away from the origin, values of $\frac{1}{a_1}$, $\frac{3}{a_2}$ . . . . $\frac{n}{a_n}$ can be computed. A graphical extrapolation of these ratios to the origin is herein proposed as a means to estimate $\frac{n}{a_n^2}$ as n approaches zero. This procedure was used in the next section for the experimental verification.

The above analysis is based on a sinusoidal grating. Should a square wave grating be used, the same result with better fringe visibility will be obtained.[10]

Experimental Verification

A thin flat titanium alloy plate clamped along one side was chosen for demonstration. The plate measures 12.7 cm x 12.7 cm x 0.394 mm. Two point loads, acting in parallel but opposite direction, were applied to the test plate at locations (1) and (2) shown in Fig. 2. A grating of 16.5 lines/cm was projected onto the plate with $\theta = 45°$. Kodak contrast process films were used for recording and copying. Figure 2 shows the test plate under loading with the grating projected on it.

In order to compare the results obtained by the proposed moiré method with the well-established electrical resistance strain gage techniques, three rectangular strain rosettes were mounted at three selected points on the plate. The exact locations of these points are shown on Fig. 2.

The pitch of the grating was first determined from a photograph of the projected grating on the undeformed test plate. The loading was then applied, and the grating on the deformed plate was photographed. Strain measurements were made before and after the photographic recording.

The experimental data obtained by the moiré method were processed in the manner as described earlier in the text. Equations (15) and (16) were employed to calculate the maximum principal strains, and h was taken to be half the thickness of the test plate.

Results and Discussion

A summary of the experimental results is presented in Table 1. Figure 3(a) shows the elliptical moiré fringe pattern at point A on the test plate while Fig. 3(b) reveals the case of hyperbolic bending at point B. As can be observed in Table 1, the maximum principal strain and orientation of the principal direction obtained by the moiré method check well with those obtained by strain gage measurements, but the discrepancy in the case of the minimum principal strain measurements is larger. The value of the minimum strain is small and larger difference with strain gage results could be expected. The sources of error will be discussed below.

One of the major sources of error comes from the fact that the governing Eqs. (13) and (14) are based on a four term Taylor expansion of the deformation function $w(x,y)$ and $w(-x,-y)$ in Eqs. (9) and (10). The curvature at the center of the ellipse (or hyperbola) is determined from measurements at the extremities of the ellipse (or hyperbola). Therefore, it can be expected that the errors will grow with increase in the principal diameters. This kind of error can be reduced by increasing the density of the projected grating. Another way to improve the accuracy of the measurement is to estimate the principal strains at the center of rotation by means of the extrapolation explained earlier. Figure 4 presents a graph of principal strains for point B determined from fringe orders taken at different diameters. By means of extrapolation it is estimated that $|\varepsilon_1| = 617 \times 10^{-6}$ and $|\varepsilon_2| = 230 \times 10^{-6}$. These results check quite well with those obtained from strain gages (see table 1).

It is assumed in the analysis of the proposed moiré method that the plate is initially flat. In fact, Eqs. (13) and (14) predict the principal

curvatures of the plate at the point of interest due to bending only. Any initial curvatures in the test plate would cause errors in the final determination.

When this proposed moiré method is used, care should be exercised in the initial alignment and subsequent rotation of the photographs. In the present experiment, this important step of data reduction was carried out on a piece of ground glass sitting on a uniform light source. The two photographic plates were aligned such that the recorded grating patterns coincided with each other. This condition was ensured by the absence of moiré fringes. When one of the photographic plates was rotated, the angle of rotation would be 180° when the light intensity at and near the point of rotation reached a maximum value. This condition can be ensured, for example, by employing a light meter to continuously monitor the light intensity at the point of rotation.

One major advantage of this method over the conventional strain gage technique is that the strain at any point of the plate can be determined from a single photographic record. The experimental technique is easy to apply, and unlike some other optical methods which require expensive optical equipment, this experiment can be performed with just a projector, a master grating and a camera. Although only the absolute values and the relative signs of the principal strains can be determined using this method, the actual signs of the strains at any point of the plate can be determined if the data reduction is started at a point at which the strain sign is independently known.

While holographic interferomety[10] can be used to study bending strains due to very small deformation, the proposed moiré method using a projected grating is suitable for the determination of bending strains of relatively

large magnitude. Since the quality of the photographic recording of the projected grating is inherently superior to that of the holographic fringes, the moiré pattern obtained by the projected grating method is of much better visibility, as evidenced by the results presented in Fig. 3.

Conclusion

A technique has been developed to determine bending strains directly using projected gratings. The method is particularly useful to study strains associated with relatively large deflections. The sensitivity of the method is proportional to the density of the projected grating and depends on the resolving capability of the imaging system.

Acknowledgments

The research reported in this paper has been conducted using funds partially provided by the John F. Dodge Chair, at Oakland University, and partially by the Office of Naval Research (Contract N00014-76-C-0487). Professor Durelli likes to express his appreciation for the opportunity the awarding of the Chair offers him to continue developing experimental mechanics methods and to Dr. N. Perrone, monitor of ONR, for his continuous support.

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Table 1 Summary of Experimental Results

	$t = 0.394$ mm			$\tan\theta = 1.02$		
	$h = 0.197$ mm			$E = 1.01 \times 10^{11} N/m^2$		
	$p = 0.605$ mm			$\nu = 0.3$		
LOCATION	MEASUREMENT	OPTICAL*				STRAIN GAGE
		N = 0**	N = 1	N = 2	N = 3	
A***	a_n, cm	-	0.737	1.25	1.66	-
	b_n, cm	-	2.03	3.58	4.70	-
	$\varepsilon_1 \times 10^6$	1,090	1,100	1,120	1,080	1,020
	$\varepsilon_2 \times 10^6$	150	144	139	135	100
	ϕ, deg	25.3	25.3	25.3	25.3	23.6
	$\sigma_1 \times 10^{-6}, N/m^2$	126	127	131	124	117
	$\sigma_2 \times 10^{-6}, N/m^2$	52.9	52.6	53.4	50.9	45.1
B***	a_n, cm	-	0.996	1.75	2.39	-
	b_n, cm	-	1.53	2.50	3.18	-
	$\varepsilon_1 \times 10^6$	617	600	582	529	636
	$\varepsilon_2 \times 10^6$	230	255	286	295	-216
	ϕ, deg.	45	45	45	45	42.6
	$\sigma_1 \times 10^{-6}, N/m^2$	60.8	58.1	55.1	48.9	63.4
	$\sigma_2 \times 10^{-6}, N/m^2$	5.98	8.30	12.4	15.1	-2.80

* The proposed optical method only determines the absolute values of principal strain

** Values of principal strains at n = 0 are estimated by extrapolation

*** The principal strains and stresses at location A are of the same sign, and those at location B are of the opposite sign.

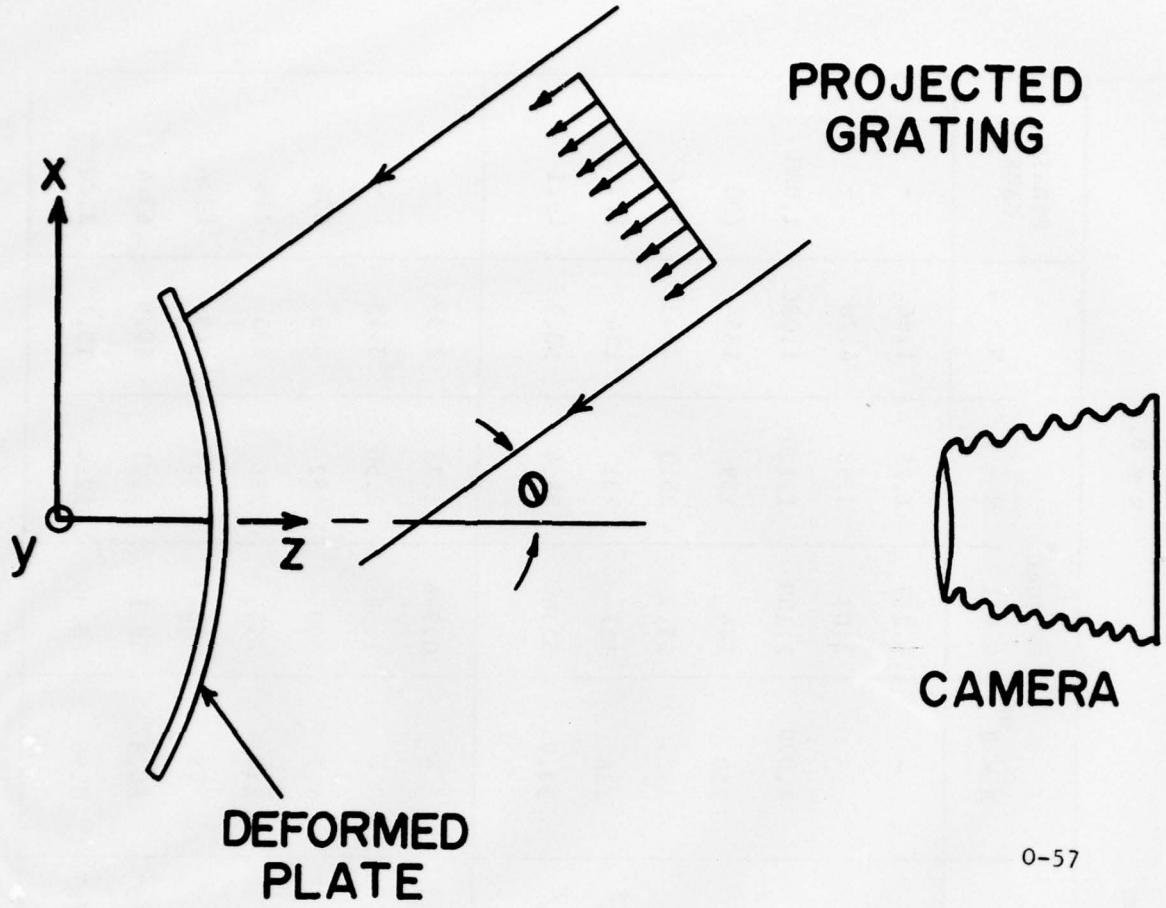
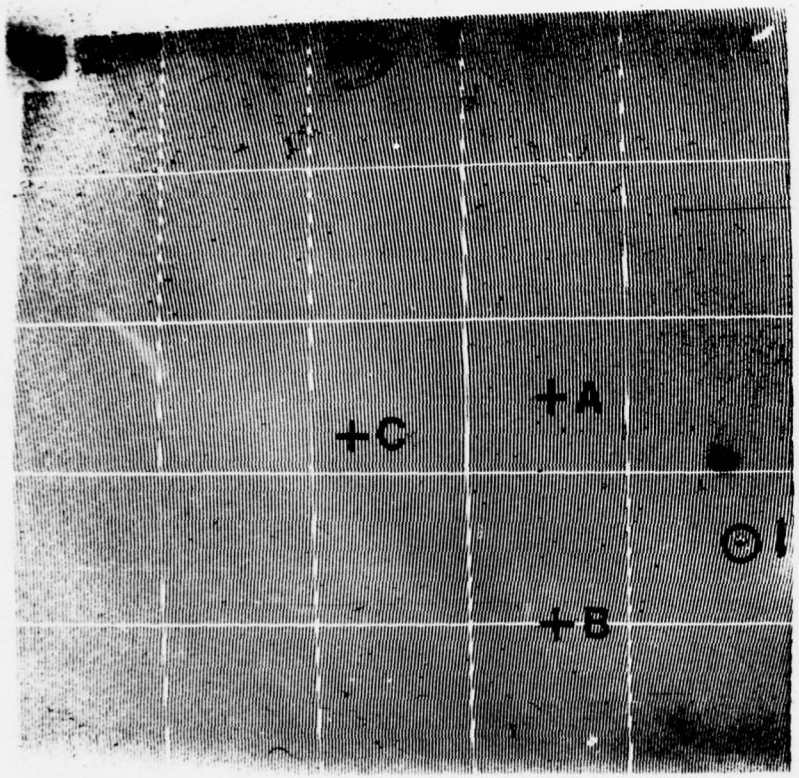
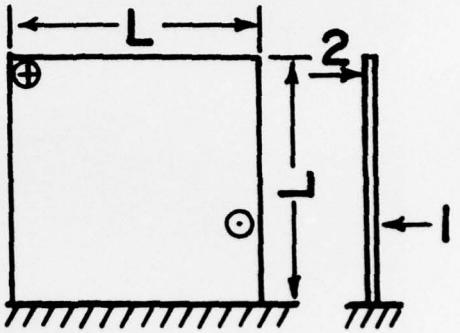


Figure 1 Experimental Set-Up of the Projected Grating Method



+ LOCATION OF STRAIN GAUGES

⊙, ⊕ LOCATION OF POINT LOAD



p=0.605 cm

L=12.7 cm

O-58

Figure 2 Grating Projected on Deformed Plate

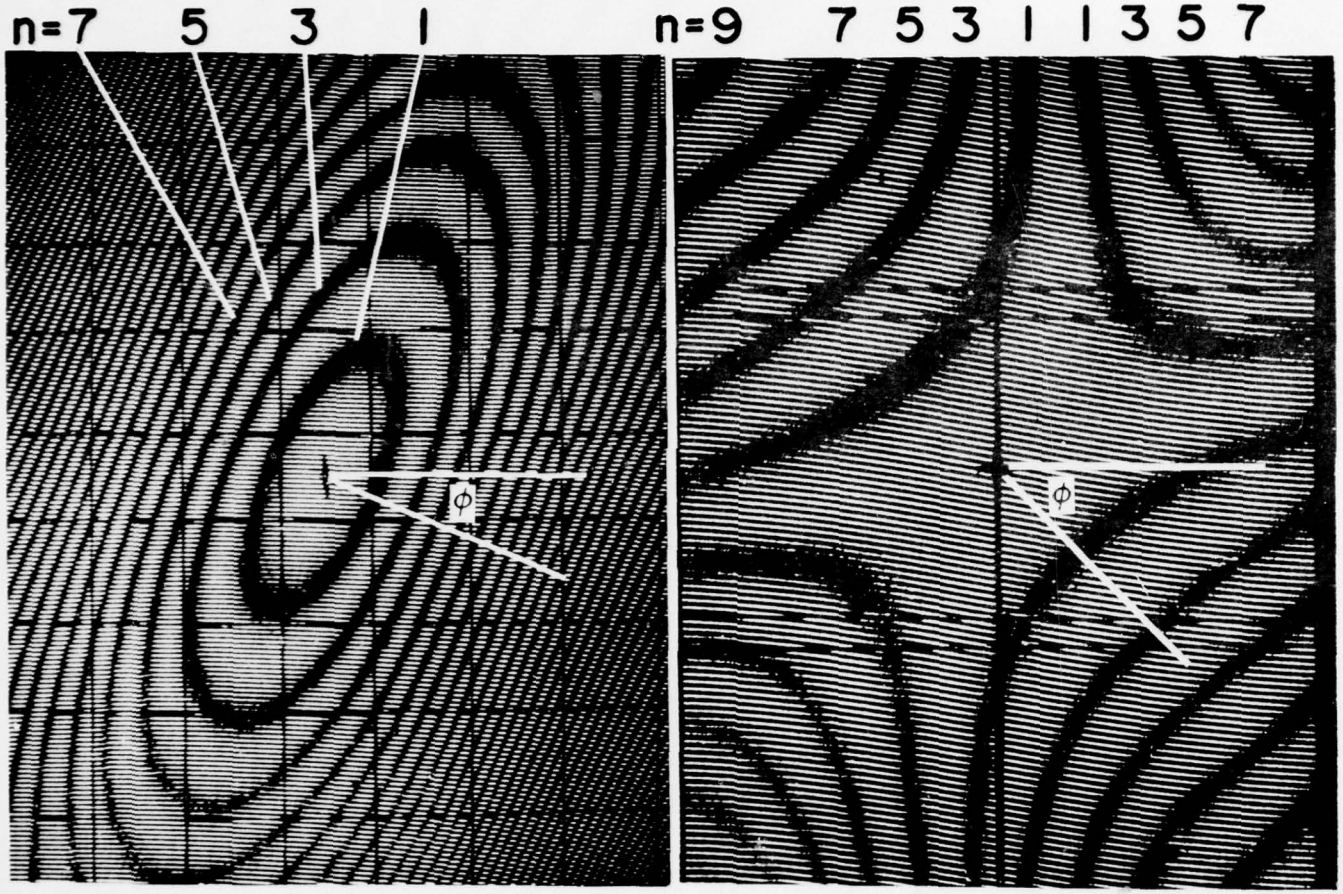


Figure 3 Moire Fring Patterns for Double Differentiation of Out-of-Plane Deflection at (a) Point A and (b) Point B on a Plate Subjected to Twisting

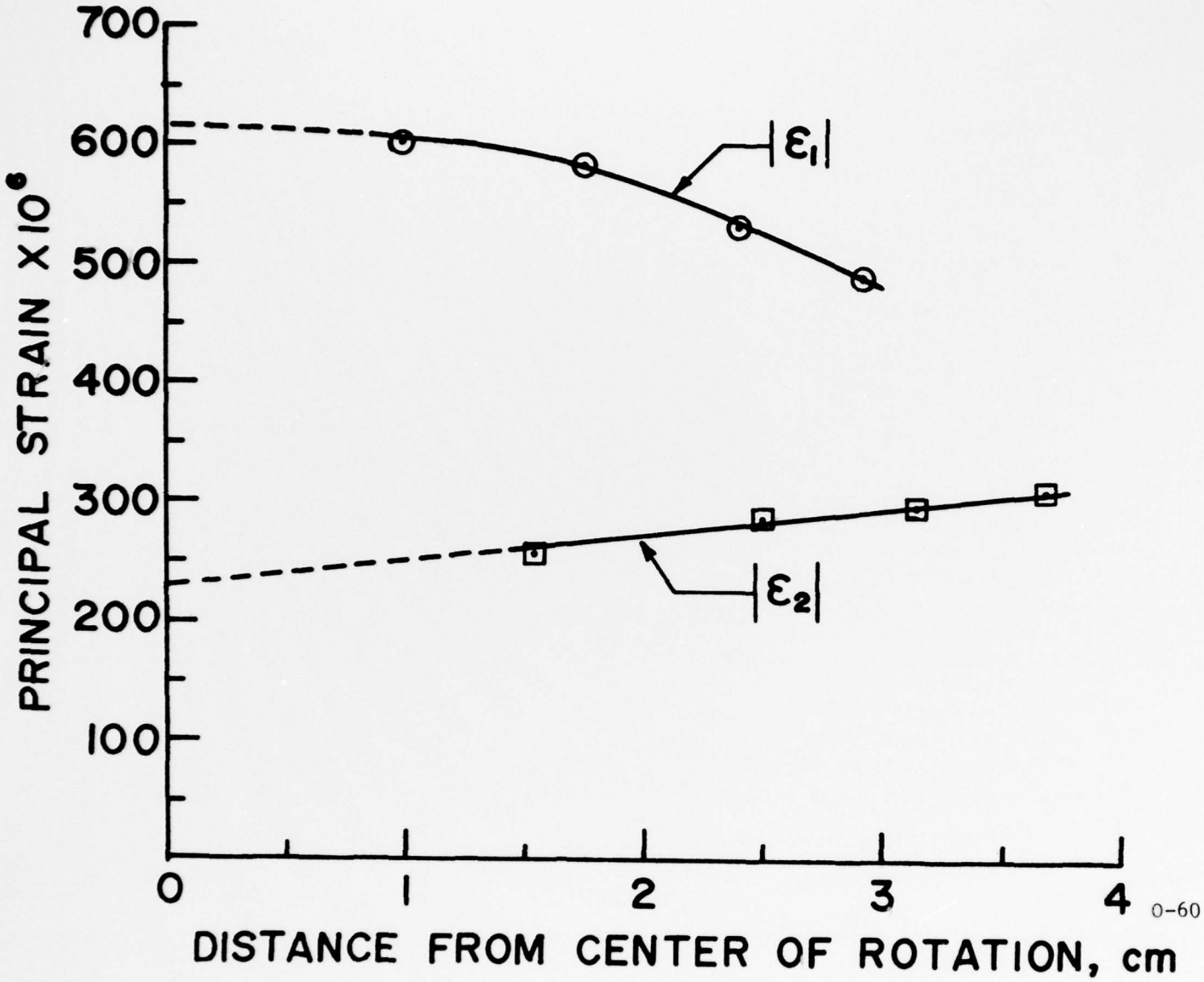


Figure 4 Principal Strain Measured at n = 1, 3, 5, 7 Versus Distance from Center of Rotation at Point B on Test Plate

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER 45	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) DIRECT DETERMINATION OF FLEXURAL STRAINS IN PLATES USING PROJECTED GRATINGS		5. TYPE OF REPORT & PERIOD COVERED 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) C. Y. Liang, Y. Y. Hung, A. J. Durelli and J. D. Hovanesian		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Oakland University Rochester, MI 48063		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Office of Naval Research Department of the Navy Washington, D.C. 20025		12. REPORT DATE June 1977 13. NUMBER OF PAGES 30
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Distribution of this report is unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Moiré
Projected grating
Flexural strain
Principal strains
Double differentiation

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

It is proposed in this paper to use projected gratings to determine principal strains in plates subjected to bending. The method requires the rotation of one photograph of the deformed grating over a copy of itself, the rotation taking place about the point at which the bending strains are desired. The idea previously suggested by Stetson to double differentiate holograms of bent plates has been extended and verified with an application to a plate subjected to twisting. The proposed method is easy to use and

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permits the determination of pricnipal strains in plates subjected to much larger deflections than the ones that can be studied using holograms. Suggestions to improve the precision of the determination are also made.