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GREEN'S FUNCTION CALCULATION OF A METAL-METAL INTERFACE. (U)  
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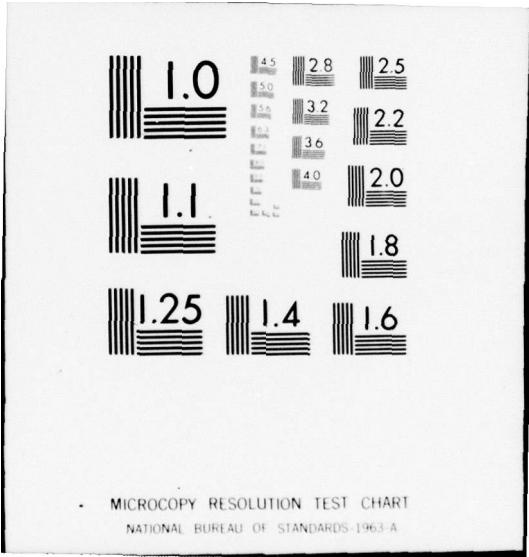
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Green's Function Calculation of a Metal-Metal Interface\*

A. Yaniv

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Abstract

The properties of metal-metal interfaces are of interest in many metallurgical applications. These include grain boundaries, crack growth, friction and bimetallic adhesion. The present work is a study of the electronic properties of a simple bimetallic junction. The method employed to investigate the interface are the Green's function technique and the phase shifts method. We calculate the Green's function of a simple junction within the tight-binding approximation. The conditions for the occurrence of bound states is deduced from the poles of the Green's function. It is shown that there are three types of wave functions associated with the interface. The first extends throughout the entire crystal, the second extends on one side of the interface only, whereas the third kind is localized near the interface. Using the phase shifts method we derive an expression for the change in density of states due to the creation of the interface. From this expression we derive the corresponding single-particle contribution to the interface energy and the interface specific heat.

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## I. Introduction

The properties of metal-metal interfaces, are of interest in many metallurgical applications. These include, for example, grain boundaries, crack growth, friction and bimetallic adhesion. The general metal-metal interface is a much more complicated problem than the corresponding metal-vacuum interface. Recently there has been, however, some success in applying certain methods developed for surface phenomena, to the investigation of bimetallic interfaces.

The electronic properties of 1-dimensional bimetallic junctions were discussed by several authors using various types of models. Aerts<sup>(1)</sup> studied the electronic structure of a 1-dimensional Kronig-Penney model, in the limit of a  $\delta$ -like potential. In this work he established the possibility of the existence of bound interface states. Very recently, a relativistic treatment of a similar model was given by Kandilarov and Detcheva<sup>(2)</sup>. A tight-binding approach, which was used successfully to several surface problems, was also applied to the 1-dimensional interface. Davison and Cheng<sup>(3)</sup> investigated the electronic properties of such a system using the molecular-orbital method. The model they use associates a single s-type orbital with each atom. A similar model was studied by Allan and Lannoo<sup>(4)</sup>, using an approximate Gaussian, density of states, having the correct second moment. Green's function formalism was used by Foo and Wong<sup>(5)</sup> to study the interface states of a 1-dimensional sp-hybrid junction.

As far as we know, the study of three-dimensional bimetallic interfaces was carried out by the density-functional formalism only. This formalism, developed by Hobenbery, Kohn, and Sham<sup>(6),(7)</sup>, was applied recently to the bimetallic interfaces by Bennett and Duke<sup>(8),(9)</sup>, Ferrante and Smith<sup>(10),(11)</sup>, Rouhani and Schuttler<sup>(12)</sup>, and by Mehrotra, Pant and Das<sup>(13)</sup>.

Whereas the density-functional formalism is applicable mainly to simple metals, the tight-binding approximation is more suitable for the description of transition metals. Since the purpose of the present work is to investigate

the electronic structure of a bimetallic junction, formed from two transition metals, we shall use the tight-binding approach. The model we consider is a highly simplified one. The two metals, on each side of the junction, are described by an s-type tight-binding Hamiltonians. The interface we consider is formed by bringing together two semi-infinite crystals, and creating bonds between the atoms on the two surfaces. We assume that the two semi-infinite crystals have the same, two-dimensional, translation symmetry parallel to the interface. The electronic properties of this model are investigated by using the Green's function method, described in detail by Kalkstein and Soven<sup>(14)</sup>.

The details of the tight-binding model are outlined in Section (2). Within this model we allow for a change in the self-consistent potential of the electrons near the interface. The diagonal matrix elements of the Green's function are calculated in Section (3) by considering the formation of the interface as a perturbation on the two semi-infinite crystals. This is accomplished by the use of Dyson's equation. Using the expression for the Green's function we discuss the electronic structure of the interface in Section (4). We show that there are three types of wave functions associated with the interface. The first one extends throughout the entire system, the second type extends on one side of the junction only, and the third, associated with bound states, is localized near the interface. The behavior of the bound states as a function of the coupling constant between the two metals is also discussed. The "phase shifts" method is applied in Section (5) to determine the change in the total density of states due to the creation of the interface. Applying this result we obtain an expression for the single-particle energy contribution to the interface energy and the corresponding contribution to the electronic specific heat.

## II The Model

Consider the formation of a metal-metal interface by bringing together two, semi-infinite, metallic crystals. As soon as a contact is formed, electrons will flow from the metal having the higher Fermi energy to the one having the lower energy. This flow of electrons stops when the potential energy difference between the two sides of the junction, which is created by the dipole layer produced at the interface, is equal in magnitude and opposite in sign to the difference between the two Fermi levels. Thus, the Fermi levels of the two metals are aligned, due to the interface dipole-layer, when the bimetallic junction is formed. If  $-\Delta v$  and  $+\Delta v$  are the electrostatic potential created by the dipole layer on the right and the left hand sides far away from the interface, then we have:

$$-e\Delta v = \frac{1}{2} (E_f^a - E_f^b) \quad (2.1)$$

where  $-e$  is the charge of the electron and  $E_f^a$  and  $E_f^b$  are the original Fermi levels of metals  $a$  and  $b$ , located to the right and to the left of the junction, respectively. The common Fermi level, after the junction is formed, is given by:

$$\begin{aligned} E_f &= E_f^a + e\Delta v \\ &= E_f^b - e\Delta v \\ &= \frac{1}{2} (E_f^a + E_f^b) \end{aligned} \quad (2.2)$$

In order to describe the electronic properties of the two bulk metals, the two metal-vacuum interfaces, and the metal-metal interface, we apply the tight-binding approximation for the various Hamiltonians. For simplicity we associate one  $s$ -type orbital with each lattice site. It is also assumed that each orbital has a  $g$ -fold degeneracy in order to partially account for the 10-fold degeneracy of the  $d$  orbitals in transition metals. Therefore, the Hamiltonians

of the two bulk metals,  $H_a$  and  $H_b$ , are given respectively by

$$H_a = \sum'_{ij} t_{ij}^a a_i^\dagger a_j \quad H_b = \sum'_{i,j} t_{ij}^b b_i^\dagger b_j \quad (2.3)$$

where  $a_i^\dagger, a_j$  and  $b_i^\dagger, b_j$  are the creation and destruction operators of the orbitals, associated with metals  $a$  and  $b$  localized near sites  $i$  and  $j$ . The prime on the summation sign denotes a summation over nearest neighbors only. The matrix elements  $t_{ij}^a$  and  $t_{ij}^b$  are given by

$$t_{ij}^a = \langle i | H_a | j \rangle = \begin{cases} E_{oa} & \text{if } i = j \\ E_{1a} \text{ or } E_{1a}^* & \text{if } i \text{ and } j \text{ are nearest neighbors} \\ 0 & \text{otherwise} \end{cases} \quad (2.4a)$$

$$t_{ij}^b = \langle i | H_b | j \rangle = \begin{cases} E_{ob} & \text{if } i = j \\ E_{1b} \text{ or } E_{1b}^* & \text{if } i \text{ and } j \text{ are nearest neighbors} \\ 0 & \text{otherwise} \end{cases} \quad (2.4b)$$

To cleave the crystal along a given low-order crystallographic plane, we have to break the bonds between two adjacent planes, parallel to the corresponding direction. We neglect any geometrical reconstruction of the crystal due to the formation of the surface and assume that the transfer integrals  $t_{ij}^a$  and  $t_{ij}^b$  with  $i \neq j$ , have the same value as in the bulk systems, provided that both sites  $i$  and  $j$  are occupied. In addition, we assume that the effect of the surface on the redistribution of the electronic charge near the surface can be described as a change in the self-consistent potential of the electrons near the first surface layer only. Therefore, the diagonal matrix elements of sites located on the surface plane will be different from the corresponding bulk value. Thus, for the metal-vacuum systems we have

$$\langle i | H'_a | i \rangle = E_{oa} + \bar{U}_a \delta_{i,0} \quad (2.5a)$$

$$\langle i | H'_b | i \rangle = E_{ob} + \bar{U}_b \delta_{i,-1} \quad (2.5b)$$

where  $\bar{U}_a$  and  $\bar{U}_b$  denote the change in the self-consistent potential of the electrons near the surface. In the above expressions we assumed that  $i = 0$  and  $i = -1$  denote surface sites of metal  $a$  and  $b$ , respectively.

In order to form the metal-metal interface we start from two semi-infinite metal vacuum interfaces, having the same translational symmetry parallel to the surface, and introduce a coupling between the two surface layers

$$\langle \underline{R}_{-1} | V | \underline{R}_0 \rangle = \langle \underline{R}_0 | V | \underline{R}_{-1} \rangle^* \quad (2.6)$$

where  $\underline{R}_{-1}$  and  $\underline{R}_0$  denote two nearest-neighbors sites located on the two sides of the interface. We also note that the dipole layer created at the interface will change the self-consistent potentials on the two sides of the junction. If we assume that charge redistribution is confined to the near vicinity of the interface we can write

$$\langle \underline{R}_i | H_a | \underline{R}_i \rangle = E_{0a} - e\Delta V + U_a \delta_{i,0} \quad (2.7a)$$

$$\langle \underline{R}_j | H_b | \underline{R}_j \rangle = E_{0b} + e\Delta V + U_b \delta_{j,-1} \quad (2.7b)$$

where  $i = 0$  and  $j = -1$  denote interface sites of metals  $a$  and  $b$ , respectively. In the above expressions  $U_a$  and  $U_b$  denote the change in the self-consistent potential of electrons near the interface relative to the bulk.

### III. The Bimetallic Interface Green's Function

As we already noted before, we assume that the two crystals forming the interface have the same translational symmetry parallel to the interface. As a result, the wave vector parallel to the interface  $\underline{k}_{||}$  is a good quantum number in such a system. In the following we assume that the interface is in the  $zy$  plane, and that metals  $a$  and  $b$  are to right and to the left respectively. The integers  $m, n, \dots$  will be used to denote the various planes parallel to the interface. Metal  $a$  is assumed to occupy the planes  $m \geq 0$  whereas  $m \leq -1$  planes are occupied by  $b$  atoms. Following Kalkstein and Soven<sup>(14)</sup>, we denote by  $\underline{\tau}_n$  the translation vector parallel to the interface which brings the atoms in the  $n$ -th plane to coincide with those on the plane  $n = 0$ . The general coordinate of an atom on the  $n$ -th plane is given by:

$$\underline{R}_n = \underline{R}_n^\perp + \underline{R}_{||} + \underline{\tau}_n \quad (3.1)$$

where  $\underline{R}_n^\perp$  is the distance between the planes  $0$  and  $n$ , and  $\underline{R}_{||}$  is a general translation vector parallel to the interface. Using the localized orbitals  $|\underline{R}_n^\perp + \underline{R}_{||} + \underline{\tau}_n\rangle$  we define the mixed Bloch-Wannier representation by the following two-dimensional Bloch sum<sup>(15)</sup>

$$|k_{||}, n\rangle = (N_{||})^{-\frac{1}{2}} \sum_{\underline{R}_{||}} |\underline{R}_n^\perp + \underline{R}_{||} + \underline{\tau}_n\rangle e^{i\underline{k}_{||} \cdot (\underline{R}_{||} + \underline{\tau}_n)} \quad (3.2)$$

where  $N_{||}$  is the number of atoms in the planes parallel to the interface. These functions are localized near the  $n$ -th plane. Since the translational symmetry parallel to the interface is preserved, the eigenfunctions of the interface Hamiltonian can be expanded in terms of the Bloch-Wannier functions (3.2) as follows

$$|E, j, \underline{k}_{||}\rangle = \sum_n C_{n\underline{k}_{||}}^j |k_{||}, n\rangle \quad (3.3)$$

where  $j$  stands for any quantum number needed to specify the eigenstates in addition to  $\underline{k}_{||}$ .

Using the expression (3.3) we can define various densities of states for the system with the interface. The local density of states, near the n-th plane, with a definite value of  $\underline{k}_n$ , is given in terms of the coefficients  $c_{n\underline{k}_n}^j$  by

$$\rho_n(E, \underline{k}_n) = \sum_j |c_{n\underline{k}_n}^j|^2 \delta(E - E_{j\underline{k}_n}) \quad (3.4)$$

The local density of states (per atom) near the n-th plane is

$$\rho_n(E) = \frac{1}{N_n} \sum_{\underline{k}_n} \rho_n(E, \underline{k}_n) = \frac{1}{N_n} \sum_{j, \underline{k}_n} |c_{n\underline{k}_n}^j|^2 \delta(E - E_{j\underline{k}_n}) \quad (3.5)$$

The sum over  $\underline{k}_n$  in this expression goes over all values of  $\underline{k}_n$  in the two-dimensional Brillouin zone defined by the crystal geometry parallel to the interface. Summing the expression (3.5) over the various crystal planes we obtain the total density of states. The density of states per atom is thus given by

$$\rho(E) = \frac{1}{N_{\perp}} \sum_n \rho_n(E) = \frac{1}{N} \sum_{j, \underline{k}_n} |c_{n\underline{k}_n}^j|^2 \delta(E - E_{j\underline{k}_n}) \quad (3.6)$$

where  $N_{\perp}$  is the number of planes parallel to the interface and  $N$  is the total number of atoms given by  $N = N_n N_{\perp}$ .

In order to derive the various densities of states of the bimetallic interface we shall use the Green's function technique. Let  $H$  and  $H_0$  be the Hamiltonians of the interfaced crystal and of the free metal-vacuum interfaces respectively. The metal-metal interface Green's operator is defined by

$$(E - i\delta - H) G = 1 \quad (3.7)$$

where  $E$  is the energy and  $\delta$  is a positive infinitesimal. The surface Green's operator  $G^0$  is related to the Hamiltonian  $H_0$  by a similar equation. The Green's operator of the interface is related to the surface Green's operator via Dyson's equation:

$$G = G^0 + G^0 V G \quad (3.8)$$

where  $V$  is the perturbation necessary to create the metal-metal interface from the free surfaces of metals  $a$  and  $b$ , i. e.

$$V = H - H_0 \quad (3.9)$$

We note that because of the translational symmetry parallel to the interface,  $G$ ,  $G^0$  and  $V$  will all be diagonal in the wave vector index  $\underline{k}_\parallel$ , in the Bloch-Wannier representation (3.2). Omitting the corresponding  $\delta$ -function,  $\delta(\underline{k}_\parallel - \underline{k}_\parallel')$ , we use the notations  $G(m, n; \underline{k}_\parallel)$ ,  $G^0(m, n; \underline{k}_\parallel)$  and  $V(m, n; \underline{k}_\parallel)$  for the matrix elements of  $G$ ,  $G^0$ , and  $V$  in the Bloch-Wannier representation. To further simplify the notation, we shall generally omit the explicit  $\underline{k}_\parallel$  dependence in these expressions.

The various densities of states defined earlier by Eqs (3.4) - (3.6), are simply related to the imaginary part of the diagonal matrix elements  $G(m, m; \underline{k}_\parallel)$  by

$$\rho(E) = \frac{1}{\pi N} \text{ImTr } G = \frac{1}{\pi N} \text{Im} \sum_{m, \underline{k}_\parallel} G(m, m; \underline{k}_\parallel) \quad (3.10)$$

$$\rho_n(E) = \frac{1}{\pi N_\parallel} \text{Im} \sum_{\underline{k}_\parallel} G(n, n; \underline{k}_\parallel) \quad (3.11)$$

$$\rho_n(E, \underline{k}_\parallel) = \frac{1}{\pi} \text{Im} G(n, n; \underline{k}_\parallel) \quad (3.12)$$

We turn now to the evaluation of the interface Green's function from Dyson's equation (3.8). This operator equation is reduced to the following algebraic equation in the Bloch-Wannier representation:

$$G(m, n) = G^0(m, n) + \sum_{\ell, r} G^0(m, \ell) V(\ell, r) G(r, n) \quad (3.13)$$

From the discussion of the model given in Section (2), it is obvious that the perturbation potential  $V$  has off-diagonal matrix elements, which couple the

interface planes  $n = 0$  and  $n = -1$ . In addition, on each side of the interface  $V$  has two types of diagonal matrix elements. The first one, due to the interface dipole layer, is given by  $\pm e\Delta v$ . This is just the perturbation necessary to align the Fermi levels on the two sides of the junction, Eq. (2.1). The second type of diagonal matrix elements is due to the change in the self-consistent potential of the electrons near the interface. This is given by  $U_a - \bar{U}_a$  and  $U_b - \bar{U}_b$  where  $U_a, \bar{U}_a$  and  $U_b, \bar{U}_b$  are the self-consistent potential relative to the bulk of interface and surface electrons of metals  $a$  and  $b$ , respectively.

As we shall see later on, it is possible to determine the surface Green's functions for an arbitrary value of the surface self-consistent potentials  $\bar{U}_a$  and  $\bar{U}_b$ . It is, therefore, simpler to start from a fictitious, intermediate surface problem where the surface self-consistent potentials  $\bar{U}_a$  and  $\bar{U}_b$ , have already the correct interface values,  $U_a$  and  $U_b$ , respectively. Using this system as our starting point, the perturbation potential necessary to create the metal-metal interface is simpler than in the original problem. It consists of diagonal terms  $\pm e\Delta v$ , which align the Fermi levels of the two metals and of off-diagonal terms, which couple the two semi-infinite crystals. Thus, the only non-vanishing matrix elements of the perturbation are given by

$$V(m,m) = \begin{cases} -e\Delta v & \text{for } m \geq 0 \\ +e\Delta v & \text{for } m \leq -1 \end{cases} \quad (3.14)$$

and

$$V(-1,0) = V(0,-1)^* = \beta(\underline{k}_{||}) e^{i\varphi(\underline{k}_{||})} \quad (3.15)$$

where  $\beta$  and  $\varphi$  are derived from the relation

$$V(-1,0) = \sum_{\underline{R}_0} \langle \underline{R}_{-1} | V | \underline{R}_0 \rangle e^{i\underline{k}_{||} \cdot (\underline{R}_0 + \underline{\tau})}$$

In this expression,  $\underline{\tau}$  is a translation vector parallel to the interface which brings the atomic sites of the two interface planes to coincide.

We start now from the intermediate surface problem discussed above, whose Green's function we denote by  $G^0$ , and apply the perturbation in two steps. First we align the Fermi levels of the two metals by applying a constant electrostatic potentials  $-\Delta v$  on metal a and  $+\Delta v$  on metal b. The only effect of applying these constant potentials on the surface green's functions is a shift in the corresponding energies. Explicitly we have

$$\tilde{G}_a^0(E) = G_a^0(E + e\Delta v) \quad (3.17a)$$

$$\tilde{G}_b^0(E) = G_b^0(E - e\Delta v) \quad (3.17b)$$

where  $\tilde{G}$  denotes the value of the Green's functions in the intermediate problem, after the application of the potentials  $\pm\Delta v$ . To simplify the notation we drop the wiggle from these Green's functions remembering that the energies have to be shifted according to (3.17).

At this stage we turn on the second part of the perturbation i.e. the off-diagonal coupling between the two metals. Due to the localized nature of this perturbation, the Dyson's equation (3.13) is greatly simplified and we obtain

$$G(m,n) = G^0(m,n) + G^0(m,-1) V(-1,0) G(0,n) + G^0(m,0) V(0,-1) G(-1,n) \quad (3.18)$$

Using the fact that the surface Green's function  $G^0(m,n)$  vanishes if  $m$  and  $n$  refer to planes on opposite sides of the interface, we can easily solve equation (3.18) for the perturbed Green's function. In this way we get

$$G(m,n) = G^0(m,n) + G^0(m,0) \left[ V(0,-1) G^0(-1,n) + |V(0,-1)|^2 G^0(-1,-1) \times \right. \\ \left. G^0(0,n) \right] \left[ 1 - |V(0,-1)|^2 G^0(0,0) G^0(-1,-1) \right]^{-1}$$

for  $m \geq 0$

and

(3.19a)

$$G(m,n) = G^0(m,n) + G^0(m,-1) \left[ V(-1,0) G^0(0,n) + |V(0,-1)|^2 G^0(0,0) \times \right. \\ \left. G^0(-1,n) \right] \left[ 1 - |V(0,-1)|^2 G^0(0,0) G^0(-1,-1) \right]^{-1}$$

for  $m \leq -1$ .

(3.19b)

In particular, the diagonal matrix elements of the interface Green's function are given by

$$G(m,m) = G^0(m,m) + \beta^2 G^0(-1,-1) G^0(m,0) G^0(0,m) \left[ 1 - \beta^2 G^0(0,0) G^0(-1,-1) \right]^{-1}$$

for  $m \geq 0$

(3.20a)

and

$$G(m,m) = G^0(m,m) + \beta^2 G^0(0,0) G^0(m,-1) G^0(-1,m) \left[ 1 - \beta^2 G^0(0,0) G^0(-1,-1) \right]^{-1}$$

for  $m \leq -1$

(3.20b)

In the above expressions  $\beta$  is related to the off-diagonal coupling between the two metals through Eq. (3.15), i.e.

$$\beta^2 = |V(0,-1)|^2$$

The diagonal matrix elements of the surface Green's function were calculated by Kalkstein and Soven<sup>(14)</sup>. Using the same method one can also derive the general matrix elements of this Green's function. In Appendix A we show that these matrix elements are given by

$$G^0(m,n) = i\mu_a^{-1} e^{-i(m-n)\theta_a} \left\{ \left( \frac{\omega + i\mu_a}{2T_a} \right)^{|m-n|} + \left( \frac{\omega + i\mu_a}{2T_a} \right)^{m+n} \frac{i\mu_a^+(\omega - 2\bar{U}_a)}{i\mu_a^-(\omega - 2\bar{U}_a)} \right\} \quad (3.21a)$$

for  $m, n \geq 0$

$$G^0(m,n) = i\mu_b^{-1} e^{-i(m-n)\theta_b} \left\{ \left( \frac{\omega + i\mu_b}{2T_b} \right)^{|m-n|} + \left( \frac{\omega + i\mu_b}{2T_b} \right)^{|m|+|n|-2} \times \right. \\ \left. \frac{i\mu_b^+(\omega - 2\bar{U}_b)}{i\mu_b^-(\omega - 2\bar{U}_b)} \right\} \quad (3.21b)$$

for  $m, n \leq -1$

and

$$G^0(m, n) = 0 \text{ otherwise.} \quad (3.21c)$$

In these expressions  $\mu$  and  $\omega$  are defined by

$$\mu = \begin{cases} (4T^2 - \omega^2)^{\frac{1}{2}} & \text{for } \omega^2 \leq 4T^2 \\ i \text{sign}(\omega) (\omega^2 - 4T^2)^{\frac{1}{2}} & \text{for } \omega^2 > 4T^2 \end{cases}$$

and

$$\omega = E - W(\underline{k}_{||})$$

where  $W$ ,  $T$ , and  $\theta$  are related to the matrix elements of the bulk Hamiltonians of the two metals in the Bloch-Wannier representation as follows

$$W_{a(b)}(\underline{k}_{||}) = \langle n\underline{k}_{||} | H_{a(b)}(\text{bulk}) | n\underline{k}_{||} \rangle$$

$$T_{a(b)}(\underline{k}_{||}) e^{i\theta_{a(b)}(\underline{k}_{||})} = \langle n\underline{k}_{||} | H_{a(b)}(\text{bulk}) | n+1, \underline{k}_{||} \rangle$$

In order to apply the expressions (3.21) to the solution of the interface problem, we replace first  $\bar{U}_a$  and  $\bar{U}_b$  by the corresponding interface values  $U_a$  and  $U_b$ , respectively. In addition we have to shift the energies according to Eq. (3.17) in order to align the Fermi levels of the two metals. This can be achieved by redefining the  $\omega$ -s in the following way

$$\omega_a = E - W_a(\underline{k}_{||}) + e\Delta v \quad (3.22a)$$

$$\omega_b = E - W_b(\underline{k}_{||}) - e\Delta v \quad (3.22b)$$

Substituting the explicit expression (3.21) for the surface Green's function into Eq. (3.20), we obtain that the diagonal matrix elements of the interface

Green's function are given by

$$G(m,m) = \frac{i}{\mu_a} + \frac{1}{\mu_a + i(\omega_a - 2U_a)} \left( \frac{\omega_a + i\mu_a}{2T_a} \right)^{2m} \left\{ \frac{i\mu_a + (\omega_a - 2U_a)}{\mu_a} - \frac{8\beta^2 i}{4\beta^2 + [\mu_a + i(\omega_a - 2U_a)][\mu_b + i(\omega_b - U_b)]} \right\} \quad (3.23a)$$

for  $m \geq 0$ , and

$$G(m,m) = \frac{i}{\mu_b} + \frac{1}{\mu_b + i(\omega_b - 2U_b)} \left( \frac{\omega_b + i\mu_b}{2T_b} \right)^{2|m|-2} \left\{ \frac{i\mu_b + (\omega_b - 2U_b)}{\mu_b} - \frac{8\beta^2 i}{4\beta^2 + [\mu_a + i(\omega_a - 2U_a)][\mu_b + i(\omega_b - 2U_b)]} \right\} \quad (3.23b)$$

for  $m \leq -1$ .

We recall that  $\omega_a$  and  $\omega_b$  in these equations are defined by the energy-shifted expressions (3.22a) - (3.22b), respectively.

#### IV. The Electronic Structure of the Interface

In this section we apply the Green's function derived in the preceding section in order to investigate the electronic structure of the interface. From the explicit expression (3.23) it follows that the diagonal matrix elements of the interface Green's function have a non-vanishing, continuous imaginary part for energies which are either inside the band of metal a or inside the band of metal b (shifted, of course, according to (3.17)). Therefore, as follows from Eq. (3.12) the band width of the crystal with the interface is the union of the shifted bands of the two separate crystals. However, the wave functions of the combined crystal can be classified into three distinct classes according to their different localization properties. To facilitate our further discussion we define the  $\underline{k}_{||}$  sub-bands of the two metals as the band structure obtained by the intersection of  $E_a(\underline{k})$  and  $E_b(\underline{k})$  with the plane  $\underline{k}_{||} = \text{constant}$ , where  $E_a(\underline{k})$  and  $E_b(\underline{k})$  are the single-particle energy spectra of the two bulk crystals.

The first type of states have wave functions which extend throughout the entire crystal. As can be seen from Eqs. (3.4) and (3.23), this kind of behavior is associated with states whose energy lies in the  $\underline{k}_{||}$  sub-band of the two metals, i.e. their energy and wave vector satisfy the relations

$$\begin{aligned} |E - W_a(\underline{k}_{||}) + e\Delta v| < 2T_a(\underline{k}_{||}) \\ |E - W_b(\underline{k}_{||}) - e\Delta v| < 2T_b(\underline{k}_{||}) \end{aligned} \tag{4.1}$$

The second type of states have wave functions which extend to  $\infty$  on only one side of the interface, and which decay exponentially with the distance from the interface on the other side. Using the expression (3.23) for the Green's function it is not hard to see that this behavior occurs when the energy lies in the  $\underline{k}_{||}$  sub-band of one metal but outside the corresponding sub-band of the other. Thus, for energies and wave vectors that satisfy

$$\begin{aligned} |E - W_a(\underline{k}_{||}) + e\Delta v| < 2T_a(\underline{k}_{||}) \\ |E - W_b(\underline{k}_{||}) - e\Delta v| > 2T_b(\underline{k}_{||}) \end{aligned} \tag{4.2}$$

the corresponding wave functions are Bloch-like inside metal a but decay exponentially from the interface inside metal b. In a similar way, the wave functions of states whose energies and wave vectors satisfy

$$\begin{aligned} |E - W_a(\underline{k}_n) + e\Delta v| > 2T_a(\underline{k}_n) \\ |E - W_b(\underline{k}_n) - e\Delta v| < 2T_a(\underline{k}_n) \end{aligned} \quad (4.3)$$

extend to infinity on the b side of the interface, but decay exponentially on the a side. The decay coefficient of the wave function is determined by the energy measured relative to the corresponding  $\underline{k}_n$  sub-band center. If we express this energy in units of the sub-band half-width, and write

$$\omega(\underline{k}_n) = \alpha 2T(\underline{k}_n), \quad |\alpha| > 1 \quad (4.4)$$

where  $\omega(\underline{k}_n)$  and  $T(\underline{k}_n)$  refer to the corresponding values on the side of the interface where the wave function decays exponentially, we obtain from Eqs. (3.4) and (3.23) that on the decaying side the wave function on plane m is proportional to  $e^{-\lambda m}$ . The decay coefficient  $\lambda > 0$  is given by

$$\lambda = -\ln \left[ \alpha - \text{sign}(\alpha) \sqrt{\alpha^2 - 1} \right] \quad (4.5)$$

Thus, the further the energy is from the center of the sub-band, the stronger is the exponential decay of the corresponding wave function.

We note that the behavior of the wave functions extending throughout the entire system and those extending only on one side of the interface is as expected. If we try to propagate a wave through the interfaced crystal this wave can propagate from one side to the other only if its frequency is in the common sub-bands. If, however, the frequency is in the sub-band of one of the crystals but outside the sub-band of the other, this wave cannot penetrate into the second crystal, and its amplitude will decay exponentially.

The third type of wave functions is associated with the existence of bound interface states. For a given  $\underline{k}_{||}$ , the energy of the possible bound states are determined by the poles of the Green's function, which lie outside the  $\underline{k}_{||}$  sub-bands of the two metals forming the junction. Using the explicit expression (3.23) for the interface Green's function, we see that the bound states energies are given by the roots of the equation:

$$4\beta^2 - \left[ \text{sign}(\omega_a) \bar{\mu}_a + \omega_a - 2U_a \right] \left[ \text{sign}(\omega_b) \bar{\mu}_b + \omega_b - 2U_b \right] = 0 \quad (4.6)$$

outside the  $\underline{k}_{||}$  sub-bands. In this equation  $\bar{\mu}$  is defined by

$$\bar{\mu} = (\omega^2 - 4T^2)^{\frac{1}{2}}$$

As can be seen from Eq. (3.23), the wave function of an interface bound state is localized near the interface and decays exponentially with distance on both sides of the interface. Thus, the bound electron is free to propagate parallel to the interface, but is confined to a finite stripe of width  $\lambda_a + \lambda_b$  (where  $\lambda_a$  and  $\lambda_b$  are given by Eq. (4.9)) in the direction perpendicular to the interface. The decay coefficients of the bound state wave function will be different, in general, on the two sides of the interface. If we express the bound state energy, relative to the center of the two  $\underline{k}_{||}$  sub-bands, in terms of the corresponding sub-bands half-widths, we can write

$$\omega_a^0 = \alpha_a 2T_a, \quad |\alpha_a| > 1; \quad \omega_b^0 = \alpha_b 2T_b, \quad |\alpha_b| > 1 \quad (4.7)$$

where

$$\omega_a^0 = E^0 - W_a + e\Delta v; \quad \omega_b^0 = E^0 - W_b - e\Delta v \quad (4.8)$$

and  $E^0$  is the energy of the bound state. It is easy to see from Eq. (3.23) that the exponential decay coefficients on side a and side b,  $\lambda_a$  and  $\lambda_b$  respectively, are given by

$$\lambda_a = - \ln \left[ \alpha_a - \text{sign}(\alpha_a) \sqrt{\alpha_a^2 - 1} \right] \quad (4.9a)$$

$$\lambda_n = - \ln \left[ \alpha_b - \text{sign}(\alpha_b) \sqrt{\alpha_b^2 - 1} \right] \quad (4.9b)$$

Thus, the greater the distance of the bound state energy from the center of the  $\underline{k}_n$  sub-bands, the more localized is the corresponding wave function.

The analytical solution of Eq. (4.6) for the bound states energies is not possible in general. However, by a suitable graphical analysis, one can determine the conditions for the occurrence of bound states, their number and their position with respect to the bands as a function of the strength of the coupling constant. It turns out to be very convenient to analyze the interface bound states in terms of the properties of the bound surface states of the intermediate surface problem considered in the preceding section (i.e. the surface problem having a surface self-consistent potential equal to the respective value of the interface problem). It is well known that the bound surface states are given by the roots of the equation<sup>(14)</sup>:

$$\text{sign}(\omega)\bar{\mu} + \omega - 2U = 0$$

The corresponding surface bound state energies are given by

$$E_a^0 = W_a(\underline{k}_{||}) - e\Delta v + U_a + \frac{T_a^2(\underline{k}_{||})}{U_a} \quad (4.10a)$$

$$E_b^0 = W_b(\underline{k}_{||}) + e\Delta v + U_b + \frac{T_b^2(\underline{k}_{||})}{U_b} \quad (4.10b)$$

provided  $|U_a| > T_a$  and  $|U_b| > T_b$ . For  $U < T$  there are no surface states.

In the following analysis we shall use the indices 1 and 2 to denote the metal having the higher sub-band top edge and the lower sub-band bottom edge, respectively.

The behavior of the interface bound states can be described as follows:

- a. In the intermediate surface problem there are no bound states (i.e.  $|U_a| < T_a$  and  $|U_b| < T_b$ ) there will be no bound interface states for small values of the coupling constant  $\beta$ . As the coupling constant is increased, a bound state will appear as soon as the critical value  $\beta_1$  is reached, where

$$\beta_1^2 = \min \left[ -\frac{1}{2}(T_2 + U_2) f_1(E_{\min}^2), \frac{1}{2}(T_1 - U_1) f_2(E_{\max}^1) \right] \quad (4.11)$$

and

$$f_i(E) = \text{sign}(\omega_i) \bar{\omega}_i + \omega_i - 2U_i \quad (4.12)$$

$E_{\max}^1$  and  $E_{\min}^2$  are the top edge and the bottom edge energies of the higher and the lower of the two  $k_{||}$  sub-bands respectively. If  $-(T_2 + U_2)f_1(E_{\min}^2) < (T_1 - U_1)f_2(E_{\max}^1)$  the bound state will appear below the sub-bands. If, however,  $-(T_2 + U_2)f_1(E_{\min}^2) > (T_1 - U_1)f_2(E_{\max}^1)$  the bound state will appear above the sub-bands. The bound state described above exists as long as  $\beta > \beta_1$ . When the coupling constant is further increased and the value  $\beta_2$  is reached, a second bound state appears where

$$\beta_2^2 = \max \left[ -\frac{1}{2}(T_2 + U_2)f_1(E_{\min}^2), \frac{1}{2}(T_1 - U_1)f_2(E_{\max}^1) \right] \quad (4.13)$$

for  $\beta > \beta_2$  there are always two bound states, one below and the other above the sub-bands.

b. Suppose that in the intermediate surface problem one of the following four situations holds:

1. There is one bound state above the sub-bands and the two sub-bands overlap.
2. There is one bound state above the sub-bands, which is associated with the metal having the higher sub-band and there is a gap between the two sub-bands.
3. There are two surface states, one above the sub-bands and the other lies in the sub-band of the second metal and there is an overlap between the sub-bands.
4. There are two bound states, a gap exists between the sub-bands and the bound state of the lower sub-band lies above the sub-bands, whereas, the one associated with the upper sub-band falls in the lower sub-band.

Then the behavior of the interface bound states is as follows: For small coupling constant there will be one bound state above the sub-bands. This state

develops in a continuous way from the corresponding surface state. If  $E_i^0$  is the energy of the surface state, which lies above the sub-bands, then for small  $\beta$  the energy of the corresponding interface state is given approximately by

$$E \simeq E_i^0 - \frac{4\beta^2}{f_j(E_i^0)} \left(1 - \frac{U_i}{T_i}\right) \quad (4.14)$$

where  $f_j(E)$  is the function defined by (4.12). As long as  $\beta$  is in the range  $0 \leq \beta < \beta_3$ , where  $\beta_3$  is defined by

$$\beta_3^2 = -\frac{1}{2} (T_2 + U_2) f_1(E_{\min}^2) \quad (4.15)$$

there is one bound state above the sub-bands. For  $\beta \geq \beta_3$  there are two bound interface states, one above and the other below the sub-bands.

c. Suppose that in the intermediate surface problem one of the following four situations exists:

1. There is an overlap between the sub-bands, a bound state exists below the sub-bands.
2. There is a gap between the sub-bands, there is one surface state below the sub-bands which is associated with the lower sub-band.
3. There is an overlap between the sub-bands and there are two surface states, one below the sub-bands and the second lies inside the sub-band of the other metal.
4. There is a gap between the sub-bands and there are two surface states, the one associated with the higher sub-band lies below the sub-bands, and the other falls inside the higher sub-band.

Then the behavior of the interface bound states is as follows: For small  $\beta$  there will be one bound state below the sub-bands, with an energy given approximately by (4.14). When the coupling constant is increased, a critical value  $\beta_4$ , is reached, where a second bound state appears above the sub-bands,  $\beta_4$  is defined by

$$\beta_4^2 = \frac{1}{2} (T_1 - U_1) f_2 (E_{\max}^1) \quad (4.16)$$

For  $\beta \geq \beta_4$  there are, thus, two bound states, one below and the other above the sub-bands.

d. Suppose that in the intermediate surface problem one of the following situations holds:

1. There is a gap between the sub-bands and there exists one surface state above the sub-bands, which is associated with the lower sub-band.
2. There is a gap between the sub-bands and there exist two surface states. The one associated with the higher sub-band is located above the sub-bands and the other lies inside the higher sub-band.

Then the behavior of the interface states is as follows: for a small coupling constant there is one interface state above the sub-bands. When  $\beta$  reaches the critical value  $\beta_5$  where

$$\beta_5^2 = -\frac{1}{2} (T_1 + U_1) f_2 (E_{\min}^1) \quad (4.17)$$

( $E_{\min}^1$  being the bottom edge energy of the higher sub-band) a second bound state appears inside the gap. This bound state exists as long as the coupling constant is in the range:  $\beta_5 \leq \beta \leq \beta_6$ , where  $\beta_6$  is given by

$$\beta_6^2 = \frac{1}{2} (T_2 - U_2) f_1 (E_{\max}^2) \quad (4.18)$$

$E_{\max}^2$  is the energy at the top of the lower sub-band. For  $\beta > \beta_6$  the bound state disappears from the gap. When  $\beta$  reaches the value  $\beta_3$ , given by Eq. (4.15), a new interface state appears below the sub-bands. For  $\beta > \beta_3$  there are two interface states, one below and the other above the sub-bands.

e. Suppose that in the intermediate surface problem one of the following situations holds:

1. There is a gap between the sub-bands, and there exists one surface state associated with the higher sub-band which lies below the sub-bands.
2. There is a gap between the sub-bands and there are two surface states. The one associated with the lower sub-band lies below the sub-bands and the other one lies in the lower sub-band.

Then the behavior of the interface states is as follows: For small values of the coupling constant there is one interface state below the sub-bands whose energy is given approximately by (4.14). When  $\beta$  is increased, and reaches the value  $\beta_6$  given by (4.18), a new interface state appears in the gap. This bound state exists in the gap for  $\beta_6 \leq \beta \leq \beta_5$  where  $\beta_5$  is given by (4.17). For  $\beta > \beta_5$  the bound state disappears from the gap and a new bound interface state appears above the sub-bands for  $\beta \geq \beta_4$ , where  $\beta_4$  is given by (4.16). For  $\beta \geq \beta_4$  there are two bound states, one below and the other above the sub-bands.

f. Suppose that one of the following situations holds in the intermediate surface problem:

1. There is a gap between the sub-bands and there is a single surface state which lies in the gap.
2. There is a gap between the sub-bands and there are two surface states, one in the gap and the other lies inside the sub-band of the other metal.

Then the behavior of the interface bound state is as follows: For small values of the coupling constant there is a single interface state which lies in the gap. When  $\beta$  is increased and reaches the value  $\beta_7$  where

$$\beta_7^2 = \max \left[ -\frac{1}{2}(T_1 + U_1) f_2(E_{\min}^1), \frac{1}{2}(T_2 - U_2) f_1(E_{\max}^2) \right] \quad (4.19)$$

this bound state disappears. For  $\beta \geq \beta_3$  there appears a new interface state below the sub-bands where  $\beta_3$  is given by (4.15). If  $\beta \geq \beta_4$  there is also a bound state above the sub-bands, where  $\beta_4$  is determined by (4.16).

g. If in the intermediate surface problem there are two surface states below the sub-bands, and a gap exists between those sub-bands, then for small values of  $\beta$  there will be two interface states below the sub-bands (with energies given approximately by (4.14)). When  $\beta$  reaches the value  $\beta_3$  given by (4.15), the interface state having the higher energy disappears. A new bound interface state appears in the gap when  $\beta$  reaches the value  $\beta_6$  where  $\beta_6$  is given by (4.18). This bound state exists as long as  $\beta_6 \leq \beta \leq \beta_5$  where  $\beta_5$  is determined by (4.17). For  $\beta > \beta_5$  the interface state disappears from the gap and a new bound state appears above the sub-bands when  $\beta \geq \beta_4$ , where  $\beta_4$  is given by (4.16).

h. If the situation in the intermediate surface problem is the same as described in (g.) except that there is an overlap between the sub-bands, there will be two interface states below the sub-bands for  $0 \leq \beta \leq \beta_3$ , where  $\beta_3$  is given by (4.15). A new interface state appears above the sub-bands for  $\beta \geq \beta_4$ , where  $\beta_4$  is given by (4.16).

i. If in the intermediate surface problem there are two surface states above the sub-bands, and a gap exists, then for small values of  $\beta$  there are two interface states above the sub-bands. When  $\beta$  reaches the value  $\beta_4$  given by (4.16), the interface state having the lower energy disappears. A new bound state appears inside the gap when  $\beta$  reaches the value  $\beta_5$  given by (4.17). This bound interface state exists as long as  $\beta_5 \leq \beta \leq \beta_6$  where  $\beta_6$  is given by (4.18). If  $\beta$  is further increased, the bound state disappears from the gap and a new interface state appears below the sub-bands when  $\beta \geq \beta_3$  where  $\beta_3$  is given by (4.15).

j. If the situation in the intermediate surface problem is the same as described in (i) except that there is no gap between the sub-bands, there will be two interface states above the sub-bands for  $0 \leq \beta \leq \beta_4$  where  $\beta_4$  is given by (4.16). For  $\beta > \beta_4$  the interface state having the lower energy disappears and a new one appears below the sub-bands when  $\beta > \beta_3$  where  $\beta_3$  is given by (4.15).

k. If in the intermediate surface problem there are two surface states inside the gap, then for small values of the coupling constant there will be two interface states inside the gap. Their energies are given approximately by (4.14). When  $\beta$  reaches the value  $\beta_8$  where

$$\beta_8^2 = \min \left[ \frac{1}{2} (T_2 - U_2) f_1 (E_{\max}^2), -\frac{1}{2} (T_1 + U_1) f_2 (E_{\min}^1) \right] \quad (4.20)$$

one of these bound states disappears. When  $\beta$  is further increased and reaches the value  $\beta_9$  given by

$$\beta_9^2 = \max \left[ \frac{1}{2} (T_2 - U_2) f_1 (E_{\max}^2), -\frac{1}{2} (T_1 + U_1) f_2 (E_{\min}^1) \right] \quad (4.21)$$

The other bound states also disappears from the gap. For  $\beta \geq \beta_3$ , where  $\beta_3$  is given by (4.15), an interface state appears below the sub-bands, whereas for  $\beta \geq \beta_4$ , where  $\beta_4$  is given by (4.16), there is a bound state above the sub-bands.

l. If in the intermediate surface problem there are two surface states, one below and the other above the sub-bands, then for any value of the coupling constant  $\beta$  there will be two interface states, one below and the other above the sub-bands.

m. If in the intermediate surface problem there are two surface states, one in the gap and the other below (above) the sub-bands, then for small values of  $\beta$  there will also be two interface states, one in the gap and the other below (above) the sub-bands. The bound state in the gap disappears for  $\beta > \beta_7$ , where  $\beta_7$  is given by (4.10). For  $\beta \geq \beta_4$  ( $\beta \geq \beta_3$ ) a new bound state appears above (below) the sub-bands, where  $\beta_4$  ( $\beta_3$ ) is given by (4.16) ((4.15)).

This analysis covers the various possible bound interface states.

We note that a general property of the bound interface states, derived from Eq. (4.6), is that whenever two interface states exist simultaneously, an increase

in the coupling constant will increase the energy of the interface state having the higher energy and will decrease the energy of the lower energy bound state.

**Another** remark that should be added here is that for very strong coupling between the two metals there are always two interface states, one below and the other above the sub-bands. The asymptotic energies of these bound states are given by

$$E_1 = U_a + W_a - e\Delta v + \beta$$

$$E_2 = U_b + W_b + e\Delta v - \beta$$

if  $|U_a + W_a - e\Delta v| > |U_b + W_b + e\Delta v|$  and by

$$E_1 = U_b + W_b + e\Delta v + \beta$$

$$E_2 = U_a + W_a - e\Delta v - \beta$$

if  $|U_b + W_b + e\Delta v| > |U_a + W_a - e\Delta v|$

## V. The Interface Energy

If many-body effects are neglected the total energy of the crystal with the interface is just the sum of the occupied single-particle energy levels. This, in turn, can be expressed as a corresponding integral over the system's density of states. The total density of states of the interfaced crystal can be obtained from the Green's function derived in Section (3), by summing the imaginary part over the various planes parallel to the interface. However, in order to determine the energy needed to break the metal-metal interface into two metal-vacuum interfaces we need to know the change in the total density of states due to this cleavage process. This can be done directly, without calculating the density of states of the two systems, by using the phase shifts method described by De Witt<sup>(16)</sup>, Callaway<sup>(17)</sup>, and Toulouse<sup>(18)</sup>. This method can be summarized as follows: Let  $H=H_0 + V$  be the perturbed Hamiltonian, and let  $E_j$  and  $\epsilon_j$  be the eigenvalues of  $H$  and  $H_0$  respectively. The change in density of states due to the perturbation  $V$  can be written as

$$\Delta\rho(E) = \rho(E) - \rho_0(E) = \sum_j \left[ \delta(E-E_j) - \delta(E-\epsilon_j) \right]$$

where  $\rho_0$  and  $\rho$  are the unperturbed and the perturbed densities of states. This can be also be expressed in the following way:

$$\begin{aligned} \Delta\rho(E) &= \frac{\text{Im}}{\pi} \frac{\partial}{\partial E} \sum_j \ln \left[ \frac{E-i\delta-E_j}{E-i\delta-\epsilon_j} \right] \\ &= \frac{1}{\pi} \text{Im} \frac{\partial}{\partial E} \text{Tr} \left[ \ln G^{-1} G^0 \right] \end{aligned}$$

where  $G$  and  $G^0$  are the Green's operators of the perturbed and the unperturbed Hamiltonians, respectively. Using Dyson's equation we can express  $G^{-1}$  in terms of  $G^0$ . In this way we get

$$\begin{aligned} \Delta_0(E) &= \frac{1}{\pi} \text{Im} \frac{\partial}{\partial E} \text{Tr} \left[ \ln (1-G^0 V) \right] \\ &= \frac{1}{\pi} \text{Im} \frac{\partial}{\partial E} \ln \left[ \det (1-G^0 V) \right] \end{aligned} \tag{5.1}$$

In the last step we used the identity  $\text{Tr} [\ln A] = \ln [\det A]$ , where  $A$  is a general operator. The application of Eq. (5.1) is especially useful for a localized perturbation, when  $\det (1-G^0V)$  can be expressed as a finite-order determinant.

Let us turn now to the specific problem of the interface. In this case we start from the two semi-infinite surface systems (the  $a$  and the  $b$  surfaces), and apply the perturbation necessary to create the interface. This perturbation was described in detail in Section (2). We introduce the perturbation in two steps. In the first one we apply a constant electrostatic potential  $-\Delta v$  on metal  $a$  and  $+\Delta v$  on metal  $b$ . As we have seen before the application of these potentials causes a shift in the densities of states of the two surface systems, which aligns the Fermi levels of the two metals. Since each metal is electrically neutral, there will be no net change in their energies due to the application of this perturbation. Our second and final step is to apply the remaining perturbation  $V$  needed to form the interface. As we have seen in Sections (2) and (3), the only non-vanishing matrix elements of this perturbation are given by

$$V(0,0) = U_a - \bar{U}_a \quad (5.2a)$$

$$V(-1,-1) = U_b - \bar{U}_b \quad (5.2b)$$

and

$$V(-1,0) = V(0,-1)^* = \beta e^{i\varphi} \quad (5.2c)$$

where, as before,  $U_a$ ,  $U_b$ , and  $\bar{U}_a$ ,  $\bar{U}_b$  are the self-consistent potentials near the interface and near the surface, respectively. Thus, in the present case we have

$$\det (1-G^0V) = \begin{vmatrix} 1 - G^0(-1,-1) V(-1,-1) & -G^0(-1,-1) V(-1,0) \\ -G^0(0,0) V(0,-1) & 1 - G^0(0,0) V(0,0) \end{vmatrix} \quad (5.3)$$

where  $G^0(0,0)$  and  $G^0(-1,-1)$  are the corresponding surface Green's functions of metals a and b respectively with the energies shifted according to (3.17). We note that the surface self-consistent potentials in  $G^0(0,0)$  and  $G^0(-1,-1)$  are  $\bar{U}_a$  and  $\bar{U}_b$  respectively and not  $U_a$  and  $U_b$ , as was the case in the preceding section. Since  $G^0$  and  $V$  are diagonal in the wave vector  $\underline{k}_{||}$ , the determinant (5.3) factorizes into similar terms with different  $\underline{k}_{||}$  values. Expanding (5.3) in terms of the indices m and n shows that each such factor is given by

$$\det_{\underline{k}_{||}} (1-G^0V) = \left[ 1-V(-1,-1) G^0(-1,-1) \right] \left[ 1-V(0,0) G^0(0,0) \right] - \beta^2 G^0(0,0) G^0(-1,-1) \quad (5.4)$$

In this expression we used the notation  $\det_{\underline{k}_{||}}$  for the partial determinant, with a specific  $\underline{k}_{||}$ .

Substituting the explicit expressions for the surface Green's functions  $G^0(0,0)$  and  $G^0(-1,-1)$ , from the general expression (3.21), into Eq. (5.4) gives

$$\det_{\underline{k}_{||}} (1-G^0V) = \frac{4\beta^2 + \left[ \mu_a + i(\omega - 2U_a) \right] \left[ \mu_b + i(\omega - 2U_b) \right]}{\left[ \mu_a + i(\omega - 2\bar{U}_a) \right] \left[ \mu_b + i(\omega - 2\bar{U}_b) \right]} \quad (5.5)$$

The bound state energies of the crystal with the interface are given by the roots of the equation (16-18):

$$\det(1-G^0V) = 0 \quad (5.6)$$

outside the shifted bands of the two metals. Using the explicit expression (5.5) we see that this is exactly the same condition derived earlier from the poles of the interface Green's function, Eq. (4.6).

In analogy with ordinary scattering theory, one defines the partial phase shifts by (17-18):

$$\begin{aligned} \eta(E, \underline{k}_{||}) &= \text{Im} \ln \left[ \det \underline{k}_{||} (1 - G^0 V) \right] \\ &= \text{Arg} \left\{ \frac{4\beta^2 + \left[ \mu_a + i(\omega_a - 2U_a) \right] \left[ \mu_b + i(\omega_b - 2U_b) \right]}{\left[ \mu_a + i(\omega_a - 2\bar{U}_a) \right] \left[ \mu_b + i(\omega_b - 2\bar{U}_b) \right]} \right\} \end{aligned} \quad (5.7)$$

In the second step, we applied the explicit expression (5.5) and used the identity  $\text{Im} \ln f = \text{Arg} f$ , where  $\text{Arg}$  denotes the argument of the complex function.

If we take the determinant of (5.5) with respect to  $\underline{k}_{||}$ , and use the relation (5.1) we see that the change in the density of states, due to the formation of the interface, is given by:

$$\begin{aligned} \Delta\rho(E) &= \frac{g}{\pi} \frac{A_{||}}{(2\pi)^2} \frac{\partial}{\partial E} \int d\underline{k}_{||} \eta(E, \underline{k}_{||}) \\ &= \frac{g}{\pi} \frac{A_{||}}{(2\pi)^2} \frac{\partial}{\partial E} \int d\underline{k}_{||} \text{Arg} \left\{ \frac{4\beta^2 + \left[ \mu_a + i(\omega_a - 2U_a) \right] \left[ \mu_b + i(\omega_b - 2U_b) \right]}{\left[ \mu_a + i(\omega_a - 2\bar{U}_a) \right] \left[ \mu_b + i(\omega_b - 2\bar{U}_b) \right]} \right\} \end{aligned} \quad (5.8)$$

In this expression we introduced explicitly the  $g$ -fold degeneracy of the bands under consideration. We also note that  $A_{||}$  is the area of the unit cell parallel to the interface and that the integration of  $\underline{k}_{||}$  goes over the two-dimensional Brillouin zone defined by the crystals structure parallel to the interface.

Equation (5.8) can be used to derive a relatively simple expression for the change in the integrated density of states. This is given by:

$$\begin{aligned} \Delta N(E) &= \int_{-\infty}^E \Delta\rho(E) dE \\ &= \frac{g}{\pi} \frac{A_{||}}{(2\pi)^2} \int d\underline{k}_{||} \text{Arg} \left\{ \frac{4\beta^2 + \left[ \mu_a + i(\omega_a - 2U_a) \right] \left[ \mu_b + i(\omega_b - 2U_b) \right]}{\left[ \mu_a + i(\omega_a - 2\bar{U}_a) \right] \left[ \mu_b + i(\omega_b - 2\bar{U}_b) \right]} \right\} \end{aligned} \quad (5.9)$$

Using Eq. (5.8), and integrating by parts, it is not hard to show that the single-particle contribution to the interface energy,  $\sigma_{ab}$  (i.e. the change in the total single-particle energy due to creation of the interface at  $T=0$ ), is

given by the expression

$$\begin{aligned} \sigma_{ab} &= \int_{-\infty}^{E_f} E \Delta\rho(E) dE \\ &= \frac{g}{\pi} \frac{A_{11}}{(2\pi)^2} \int_{-\infty}^{E_f} dE \int d\underline{k}_{11} \operatorname{Arg} \left\{ \frac{4\beta^2 + [\mu_a + i(\omega_a - 2U_a)] [\mu_b + i(\omega_b - 2U_b)]}{[\mu_a + i(\omega_a - 2\bar{U}_a)] [\mu_b + i(\omega_b - 2\bar{U}_b)]} \right\} \end{aligned} \quad (5.10)$$

where  $E_f$  is the common Fermi energy of the two metals, i.e.  $E_f = \frac{1}{2} (E_f^a + E_f^b)$  and  $\omega_a$  and  $\omega_b$  include the corresponding shifts in the energies, according to Eq. (3.22).

In a similar way, one can determine the single-particle contribution to the change in the electronic specific heat. This change is given by:

$$\Delta C_V = \int_{-\infty}^{\infty} E \Delta\rho(E) \frac{\partial f}{\partial T} dE$$

where  $f(E)$  is the Fermi-Dirac distribution function. For temperatures much lower than the Fermi temperature the change in the electronic specific heat is linear in the temperature and proportional to the change in the density of states at the Fermi level (assuming no Van Hove singularity occurring at the Fermi energy). Explicitly we have

$$\begin{aligned} \Delta C_V &= \frac{\pi^2}{3} K_B^2 \Delta\rho(E_f) T \\ &\equiv \gamma T \end{aligned}$$

where  $K_B$  is the Boltzmann constant. For the interface system the constant  $\gamma$  is given by

$$\gamma = \frac{gK_B^2}{12\pi} A_{11} \int d\underline{k}_{11} \frac{\partial}{\partial E} \operatorname{Arg} \left\{ \frac{4\beta^2 + [\mu_a + i(\omega_a - 2U_a)] [\mu_b + i(\omega_b - 2U_b)]}{[\mu_a + i(\omega_a - 2\bar{U}_a)] [\mu_b + i(\omega_b - 2\bar{U}_b)]} \right\} \quad (5.11)$$

where the integrand has to be evaluated at the common Fermi level.

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## APPENDIX A

In this appendix we derive the expressions for the general matrix elements of the surface Green's function. The technique we apply is the same as that used by Kalkstein and Soven<sup>(14)</sup> to derive the corresponding diagonal matrix elements.

To form the metal-vacuum interface we start from the bulk metal and break the bonds between the two adjacent planes  $n = 0$  and  $n = -1$ . In addition, we change the self-consistent potential of the electrons near the surface. The perturbation described above, which is necessary to create the surface, has the following non-vanishing matrix elements in the Bloch-Wannier representation:

$$V(0,0) = V(-1,-1) = \bar{U} \quad (\text{A.1})$$

$$V(-1,0) = V(0,-1)^* = -Te^{i\theta} \quad (\text{A.2})$$

With this perturbation potential, the Dyson's equation for the surface Green's function takes the following form:

$$G(m,n) = G^0(m-n) + G^0(m+1)V(-1,0)G(0,n) + G^0(m+1)V(-1,-1)G(-1,n) \\ + G^0(m)V(0,-1)G(-1,n) + G^0(m)V(0,0)G(0,n) \quad (\text{A.3})$$

In writing down the above equation we used the property that due to translational invariance, the bulk Green's function  $G^0$  depends on the indices  $m$  and  $n$  only through the difference  $m-n$ . Taking into account the fact that the surface Green's function  $G(m,n)$  vanishes if  $m$  and  $n$  refer to planes on opposite sides of the interface, we can solve Eq. (A.3) for the general matrix elements of the surface Green's function. This is given by:

$$G(m,n) = G^0(m-n) + \left[ G^0(m+1)V(-1,0) + G^0(m)V(0,0) \right] G^0(-n) \left[ 1 - G^0(1) \times \right. \\ \left. V(-1,0) - G^0(0)V(0,0) \right]^{-1} \quad (A.4a)$$

for  $m, n \geq 0$

$$G(m,n) = G^0(m-n) + \left[ G^0(m)V(0,-1) + G^0(m+1)V(-1,-1) \right] G^0(-1-n) \left[ 1 - G^0(-1) \times \right. \\ \left. V(0,-1) - G^0(-1)V(-1,-1) \right]^{-1} \quad (A.4b)$$

for  $m, n \leq -1$

$$G(m,n) = 0 \text{ otherwise.} \quad (A.4c)$$

The bulk Green's function,  $G^0(n)$ , was derived by Kalkstein and Soven<sup>(14)</sup> and is given by:

$$G^0(n) = \frac{i}{\mu} \left( \frac{\omega + i\mu}{2T} \right)^{|n|} e^{-in\theta}$$

Using this expression and the expressions (A.1) - (A.2) for the perturbation, we obtain from Eqs. (A.4) that the matrix elements of the surface Green's function are given by:

$$G(m,n) = \frac{i}{\mu} e^{-i(m-n)\theta} \left\{ \left( \frac{\omega + i\mu}{2T} \right)^{|m-n|} + \left( \frac{\omega + i\mu}{2T} \right)^{m+n} \frac{i\mu + (\omega - 2\bar{U})}{i\mu - (\omega - 2\bar{U})} \right\} \quad (A.5a)$$

for  $mn \geq 0$

$$G(m,n) = \frac{i}{\mu} e^{-i(m-n)\theta} \left\{ \left( \frac{\omega + i\mu}{2T} \right)^{|m-n|} + \left( \frac{\omega + i\mu}{2T} \right)^{|m|+|n|-2} \frac{i\mu + (\omega - 2\bar{U})}{i\mu - (\omega - 2\bar{U})} \right\} \quad (A.5b)$$

for  $m, n \leq -1$ , and

$$G(m,n) = 0 \quad (A.5c)$$

otherwise.