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SOLVING INVENTORY PROBLEMS USING THE
FACTORABLE NONLINEAR PROGRAMMING LANGUAGE

by

Garth P. McCormick

Serial T-357
1 June 1977

The George Washington University
School of Engineering and Applied Science
Institute for Management Science and Engineering

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This memorandum is intended to aid the logistics practitioner in setting up his algebraic statement of an optimization model in such a way that it can be automatically processed and solved by a canned nonlinear programming algorithm. The example used is the Schradley-Choe optimal inventory model. The model algorithm interface is accomplished by a new version of the factorable nonlinear programming language which allows for symbolic input.

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1. Introduction

With the ability to solve general nonlinear optimization (programming) problems has come the development of more sophisticated and realistic models. One example of this in the logistics area is the presentation in [1] of an optimal inventory model. This model was solved by the Sequential Unconstrained Minimization Technique (SUMT) using as the interface between the algebraic statement of the problem and the canned algorithm, four FORTRAN subroutines. This traditional mode of input to the nonlinear programming algorithm was required since function value and derivative information is necessary to solve the problem efficiently. A first attempt at providing a simpler model/algorithm interface was done by Pugh, et al., [2] and was a very sophisticated computer code allowing for general forms of symbolic input. Unfortunately, this code was slow. A more sparse approach to problem representation was developed in [3]. Internally, the code was fast, but the user was restricted to numerical input. This code is in the process of undergoing revision [4] and in this memorandum the inventory model given in [1] is presented in the new format which allows for mnemonic symbolic input for variable, constant, and function names. This will make the existing nonlinear programming codes more accessible to practitioners. This memorandum is an adjunct to the users manual [4] and should be used in conjunction with it and the original paper [1].

2. The Inventory Model

Below is given a brief development of the optimization problem. For more details the reader is referred to [1] and [5].

Fixed Inputs and Constants

N = number of items in inventory ($n = 2N$ is the number of variables or unknowns in the nonlinear programming problem)

c_i = item unit cost (in dollars)

v_i = item unit volume

λ_i = mean demand per unit time

(μ_i, σ_i) = mean and standard deviation of lead time demand distribution for i th item.

Let

$\phi(x) = (1/\sqrt{2\pi}) e^{-x^2/2}$ the normal density function, and

$\Phi(z) = \int_z^{\infty} \phi(x) dx$ the right hand cumulative normal distribution function.

K_1 = investment limit in dollars

K_2 = reorder work load limit

K_3 = volume limit

Derived Quantities

$$\beta_i(z) = \frac{1}{2} [\sigma_i^2 + (z - \mu_i)^2] \phi\left(\frac{z - \mu_i}{\sigma_i}\right) - \frac{\sigma_i}{2} (z - \mu_i) \phi\left(\frac{z - \mu_i}{\sigma_i}\right)$$

$$B_i(Q_i, r_i) = \frac{1}{Q_i} [\beta_i(r_i) - \beta_i(Q_i + r_i)], \quad EOH_i = r_i + Q_i/2 - \mu_i + B_i(Q_i, r_i)$$

The optimization problem is

$$\text{minimize}_{\{Q_i, r_i\}} \sum_{i=1}^N B_i(Q_i, r_i) \quad (\text{expected time-weighted shortages})$$

subject to

$$\sum_{i=1}^N c_i \left(r_i + \frac{Q_i}{2} - \mu_i + B_i(Q_i, r_i) \right) \leq K_1 \quad (\text{investment constraint})$$

$$\sum_{i=1}^N \frac{\lambda_i}{Q_i} \leq K_2 \quad (\text{reorder workload constraint})$$

$$\sum_{i=1}^N v_i \left(r_i + \frac{Q_i}{2} - \mu_i + B_i(Q_i, r_i) \right) \leq K_3, \quad (\text{capacity constraint})$$

$$Q_i \geq 0, \quad i = 1, \dots, N.$$

3. The Model in Factorable Form

The factorable programming language which provides the interface between the algebraic statement of the optimization problem and canned optimization algorithms assumes that the problem can be written in the following canonical factorable form:

$$\min_{x \in E^n} X^N(x)$$

$$\text{Subject to: } L_i \leq X^i(x) \leq U_i, \quad \text{for } i=1, \dots, N-1$$

(possibly $L_i = -\infty$ and/or $U_i = +\infty$) where $X^i(x) \equiv x_i$ (for $i = 1, \dots, n$)

and the remainder are defined recursively as follows:

given $X^p(x)$, for $p = 1, \dots, i-1$, then for $i = n+1, \dots, N$,

$$X^i(x) = \sum_{p=1}^{i-1} T_p^i[X^p(x)] + \sum_{p=1}^{i-1} \sum_{q=1}^p V_{q,p}^i[X^p(x)] \cdot U_{p,q}^i[X^q(x)],$$

where the T's, U's, and V's are functions of a single variable.

In writing down his particular problem in canonical factorable form the user should obey two fundamental rules. The quantities $X^p(x)$ are called concomitant variable functions (cvfs) and on the right-hand side of the defining equation for any X^i there cannot appear any X^p where $p \geq i$. Furthermore, no more than two factors can appear in any product term.

The new format allowed for the factorable programming language allows for symbolic variable names instead of just numbers for the cvfs. The user may use any mnemonic devices to help in setting up the factorable canonical form. For the problem example above this could take the following form:

$$\begin{aligned} z_i &= (r_i - \mu_i) & , \quad i = 1, \dots, N & & (r_i - \mu_i) \\ y_i &= z_i + Q_i & , \quad i = 1, \dots, N & & (r_i + Q_i - \mu_i) \\ u_i &= z_i / \sigma_i & , \quad i = 1, \dots, N & & (r_i - \mu_i) / \sigma_i \\ v_i &= y_i / \sigma_i & , \quad i = 1, \dots, N & & (r_i + Q_i - \mu_i) / \sigma_i \\ w_i &= \sigma_i^2 + z_i^2 & , \quad i = 1, \dots, N & & (\sigma_i^2 + (r_i - \mu_i)^2) \\ t_i &= \sigma_i^2 + y_i^2 & , \quad i = 1, \dots, N & & (\sigma_i^2 + (r_i + Q_i - \mu_i)^2) \end{aligned}$$

$$p_i = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{u_i} e^{-t^2/2} dt$$

$$q_i = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{v_i} e^{-t^2/2} dt$$

$$\begin{aligned} D_i(r_i, Q_i) &= 1/2 w_i [1 - p_i] - \frac{\sigma_i}{2} z_i \phi(u_i) & & [\beta(r_i) - \\ & & & & \beta_i(r_i + Q_i)] \\ & & - \frac{1}{2} t_i [1 - q_i] + \frac{\sigma_i}{2} y_i \phi(v_i), \quad i = 1, \dots, N \end{aligned}$$

$$B_i(Q_i, r_i) = D_i(r_i, Q_i)/Q_i, \quad i = 1, \dots, N \quad B_i(Q_i, r_i)$$

$$EOH_i = r_i + Q_i/2 - \mu_i + B_i(Q_i, r_i), \quad i = 1, \dots, N \quad (EOH_i)$$

$$EXPINV = \sum_{i=1}^N c_i EOH_i \quad (\text{EXPECTED INVESTMENT})$$

$$EXPVOL = \sum_{i=1}^N v_i EOH_i \quad (\text{EXPECTED VOLUME})$$

$$REORD = \sum_{i=1}^N \lambda_i / Q_i \quad (\text{WORK LOAD})$$

$$SHORT = \sum_{i=1}^N B_i(Q_i, r_i) \quad (\text{EXPECTED TIME-WEIGHTED SHORTAGES})$$

Under the new system the user first declares the names of his variables and then the names of the data constants he uses. He also has the flexibility of placing bounds on ranges of variables using their symbolic names.

In Appendix A is given the listing of the card input for the inventory model developed in [1]. A slightly more elaborate model was presented in [5] based on material in [1]. The card listing for this model is given in Appendix B.

4. Conclusion

A brief description of the manner in which a particular logistics optimal inventory model can be described in the new factorable programming interface has been given. Hopefully the new flexible format which allows for symbolic names of variables, constants, and functions will make the use of optimization codes more widespread.

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APPENDIX A

SYMBOLIC FACTORABLE PROGRAMMING INPUT FOR
SIMPLE SCHRADY-CHOE MODEL DESCRIBED IN [1]

APPENDIX B

SYMBOLIC FACTORABLE PROGRAMMING INPUT FOR MODIFIED
SCHRADY-CHOE MODEL DESCRIBED IN [5]

UTILITY: VERSION 6. 3/8/75 MONDAY JAN 31, 1977 1614R

L1
L2
L3

-1:
-1:
0:
K1
K2
K3
1:
1:
1:

HEOR Q1 POL
REOR Q2 POL
S S S Q3 POL
U U U E INV
U U U E REOR
S S S E VOL
S S S R1 LIN
S S S R2 LIN
S S S R3 LIN

1
2
3
40
41
42

270.
2
0.10000D-02
1
DU

300.
1
200

270.
2
0.10000D-02
1
DU

600.
3
0.10000D-02
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270.
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