AD-A042 348 BROWN UNIV PROVIDENCE R I DIV OF ENGINEERING F/G 20/4
ON THE LARGE-SCALE STRUCTURE IN TURBULENT FREE SHEAR FLOWS.(U)
1977 J T LIU, A ALPER N00014-76-C-0363

UNCLASSIFIED NL

1 OF 1
ADA042348

END
DATE
FILMED
8-77
DDC

On the large-scale structure in turbulent free shear fl

By J.T.C. Liu and A. Alper

Division of Engineering, Brown University
Providence, Rhode Island 02912 U.S.A.

ON THE LARGE-SCALE STRUCTURE IN TURBULENT FREE SHEAR FLOWS

J. T. C. LIU AND A. ALPER
DIVISION OF ENGINEERING
BROWN UNIVERSITY
PROVIDENCE, RHODE ISLAND 02912

ABSTRACT

The existence of organized structures in turbulent shear flow has been the subject of recent observational discoveries in both the laboratory and in the atmosphere and ocean. The recent work on modeling such structures in a temporally developing, horizontally homogeneous turbulent free shear layer (J. T. C. Liu and L. Merkine 1976 Proc. Roy. Soc. London, Ser. A, Vol. 352, pp. 213-247) has been extended to the spatially developing mixing layer, there being no available rational transformation between the two nonlinear problems. The basis for the consideration is the kinetic energy development of the mean flow, large-scale structure and fine-grained turbulence with a conditional average, supplementing the usual time average, to separate the nonrandom from the random part of the fluctuations. The integrated form of the energy equations and the accompanying shape assumptions, is used to derive "amplitude" equations for the mean flow, characterized by the shear layer thickness, the nonrandom and random components of flow which are characterized by their respective energy densities. In general, the large-scale structure augments the spreading of the shear layer and enhances the fine-grained turbulence by taking energy from the mean flow and transferring it to the turbulence as it amplifies and subsequently decays. The maximal amplitude of the large-scale structure is attained by the initially most amplified mode, however, the relative enhancement of the fine-grained turbulence is achieved by both the magnitude of the large-scale structure and its streamwise lifetime. Thus a greater enhancement of the turbulence is achievable by the lower frequency modes which have longer streamwise lifetimes. The large-scale structure can also be controlled by increasing the initial level of turbulence, which would render its decay more rapidly.

NOMENCLATURE

U_i, $\tilde{u}_i$, u_i'	dimensionless velocities of the mean, large-scale structure, fine-grained turbulence
$\mathcal{U}_i \equiv U_i + \tilde{u}_i$	
P, $\tilde{p}$, p'	dimensionless pressure of the mean, large-scale structure, fine-grained turbulence
$\mathcal{P} \equiv P + \tilde{p}$	
x_i	dimensionless coordinates (x, horizontal; y, vertical)
t	dimensionless time
$\overline{(\ )}$	average which produces a mean quantity, (the time average in the spatial problem, horizontal average in the temporal problem)
$\langle(\ )\rangle$	conditional average
$\overline{u_i'u_j'}$	mean turbulent stresses
$\langle u_i'u_j'\rangle$	conditionally averaged turbulent stresses
$\tilde{r}_{ij} \equiv \langle u_i'u_j'\rangle - \overline{u_i'u_j'}$	modulated turbulent stresses
Re	Reynolds number

Other quantities are defined as they appear.

INTRODUCTION

Recent reviews of laboratory observations have pointed towards the omnipresence of large-scale organized structures in turbulent shear flows, with each type of shear flow having its own peculiar organized structure (1), (2), (3), (4), (5). Several international conferences have also been held in which such organized structures in turbulent flows were discussed (6), (7). It is apparent that turbulence in shear flows are not as random as originally thought to be and, in his Dryden Research Lecture, Roshko (4) reminds us that Dryden (8) had pointed out that it is necessary to separate the random from the nonrandom processes in turbulent flow. This view was also emphasized by Liepmann (9). Observations strongly suggest that the large-scale organized structures are important to the mixing and development of the shear flow and it is beginning to be recognized that the study of their dynamics is fundamental to any study of turbulent shear flows. Such studies being technologically important to combustion and reactive flows (7), (10) and jet noise (11), (12) to mention a few. Such organized structures are also observed to occur in regions of strong local shear provided by frontal systems and internal waves of a much larger scale in the atmosphere and in the ocean and are now thought to play an important role in the spectral cascading process and in mixing (13), (14). The observational unmasking of the large-scale organized structures in turbulent flows has, at least in the authors' view, given turbulence research new and exciting impetus.

We shall focus our attention to *free* turbulent flows in this paper in which the mean flow velocity profiles are inflexional. The presence of the large-scale structure is more or less attributable to the dynamical instability inherent in such flows (9), (12), (15), (16). Although the large-scale organized structure may be deterministic, it is conceivable that it is subjected to random forcing. Take the mixing region, for instance; the large-scale structure as a consequence of upstream forcing at a given frequency will propagate downstream at a phase velocity characteristic of that frequency. Suppose now a slower large-scale structure is first set up and subsequently a faster one is generated, a "collision" of the latter with the first will take place downstream as the faster one catches up with the slower one. It is not entirely inconceivable that this provides one of the explanations of the "pairing" or "vortex coalescence" process that has been observed (17). This essentially emphasizes the importance of studying the large-scale organized structure under well-controlled rather than under natural conditions (9), much in the same spirit of Schubauer and Skramstad (18), for purposes of physically isolating and understanding the mechanisms involved.

While it is generally important to understand the coalescence of large-scale organized structures in a turbulent shear flow, it seems logical that theoretical understanding of the interactions between a single, monochomatic large-scale structure with the fine-grained turbulence be obtained without complicating the picture with the more complicated dynamical effects. On the other hand, "vortex coalescence" is known to occur even without the presence of fine-grained turbulence and contributions towards its theoretical explanation can be made from purely dynamical considerations (19). *We shall consider here the first major* problem discussed above, that of the interaction between the large-scale structure and the disparately fine-grained turbulence in a free turbulent shear flow, in particular, the mixing region.

Our aim is towards physical understanding of the interactions between the disparate scales of motion which lead to the growth and development of the shear layer, rather than emphasizing numerical prediction or accuracy. This necessitates approximate considerations in order to achieve our aim, one which may be helpful towards the eventual incorporation of the large-scale structure dynamics in predictive schemes or towards well-controlled experimental unmasking of those mechanisms which contribute to the understanding of the development of organized structures in turbulent shear flows.

To proceed, we essentially follow earlier suggestions (8), (9), (20) and theoretically sort out the monochromatic large-scale organized structure from the random fine-grained turbulence by a conditional averaging procedure which has been described and realized in the laboratory elsewhere (21), (22), (23). This forms the starting point for the subsequent approximate considerations for the study of simultaneous interactions between the mean flow, the large-scale structure and the fine-grained turbulence. We have reported on our previous study of an idealized mixing region (24), in which the mean flow consisted of horizontally homogeneous, oppositely directed streams. The shear layer and interactions develop in *time* rather than in the streamwise distance. Since all mean quantities there are horizontally homogeneous, there is no streamwise "diffusion" in the mean and this comprises one of the natural simplifications of the problem (there being no backwards diffusion in time because of causality, of course). Another simplification in Ref. (24) was that the only non-equilibrium behavior of the fine-grained turbulence is due to interactions with the large-scale structure. The large-scale structure there develops in time and is horizontally periodic. The problem considered there, then, was the temporal problem and there does not exist a physically rational one-to-one transformation to the streamwise developing shear layer or spatial problem because of the causality condition, among others.

In this paper we shall present new results pertaining to the *spatial* problem. The locally equilibrium (with the mean) behavior of the fine-grained turbulence is also relaxed so that in our integrated energy considerations its evolution is due to the balance or inbalance between production from both the mean and large-scale structure and viscous dissipation rather than attributing its nonequilibrium behavior to the production mechanism from large-scale structure alone. In order *to fix our ideas as to the physical picture which* might emerge from our analysis, it would be helpful to recapitulate the interesting results reported by Binder and Favre-Marinet (23). A well-controlled large-scale structure was imparted upon the shear layer of a round jet and measurements were taken on the jet axis. In the absence of the large-scale structure the turbulence level on the jet axis gradually increased downstream. With the presence of the large-scale organized structure, the fine-grained turbulence level increased at a faster rate and became more vigorous with the downstream distance. Simultaneously, the large-scale structure amplified and subsequently decayed, as if having extracted energy from the mean flow and subsequently transferring it to the fine-grained turbulence. The spreading of the jet was also enhanced by the presence of the large-scale structure. Although geometrical effects in the experiments certainly play an important role, the qualitative physical situation should hold for the plane mixing region that we are to consider here.

FORMULATION

The explicity study of the large-scale organized structure coexisting with random fine-grained turbulence necessitates the splitting of a flow quantity into three components consisting of the mean motion and the nonrandom and random parts of the fluctuations. The usual time average, over a period at least that of the organized structure, produces the equations of mean motion. A second, conditional average then enables one to sort out the nonrandom part of the fluctuations from the random part, in contrast to and different from filtering. Such a splitting procedure, suitable for the present study is, of course, not new and has been discussed and utilized in the literature (2), (8), (9), (15), (16), (21), (22), (23), (25). However, the first systematic derivation of the conservation equations for the three component splitting procedure is attributable to W. C. Reynolds, as much as the two component splitting procedure was to O. Reynolds.

We shall consider an incompressible fluid. The velocity components and coordinates are made dimensionless by the corresponding characteristic scales appropriate to the problem. For instance, for the mixing region, the characteristic velocity is the free stream velocity and the characteristic length the initial boundary layer thickness. The pressure is made dimensionless by the free stream dynamic pressure and the time by that formed from the characteristic velocity and length. The conditional average is denoted by $\langle\;\rangle$ and the usual time average by $\overline{(\;)}$. The mean flow quantities are denoted by capital symbols, the large-scale structure identified by the supercript tilde $\tilde{}$ and the random fine-grained turbulence by the supercript prime $'$. The conditional and time averages of the turbulence quantities are zero by definition. The conditional average of a large-scale organized quantity reproduces itself and its time average is zero. Because the two components of the fluctuations are not correlated, the time average of their products is zero. The conditional average of the products of two fine-grained turbulence quantities produces a mean Reynolds stress and a modulated stress, which are crucial to the interactions between the random fine-grained turbulence with the mean flow and the large-scale structure, respectively. The modulated turbulent stresses oscillate at the frequency of the large-scale organized structure.

Just as in the study of the two-component O. Reynolds turbulence problem, for all practical purposes, one is not interested in the details of the turbulence but in the stresses it sets up and the interaction with the mean flow in an averaged sense, in the explicit consideration of the large-scale structure one is also not interested in the details of the fine-grained turbulence but only in its modulated stresses and its mean stresses as it interacts with the large-scale and mean motions. Finally, the large-scale structure interacts with the mean flow through its own mean stresses. These can be best illustrated by writing down the momentum equations for the mean flow and the large-scale structure

$$\frac{\bar{D}U_i}{Dt} = -\frac{\partial P}{\partial x_i} + \frac{1}{Re}\frac{\partial^2 U_i}{\partial x_j \partial x_j} - \frac{\partial}{\partial x_j}\left(\overline{u_i'u_j'} + \overline{\tilde{u}_i\tilde{u}_j}\right)$$

$$\frac{\bar{D}\tilde{u}_i}{Dt} + \tilde{u}_j\frac{\partial U_i}{\partial x_j} = -\frac{\partial \tilde{p}}{\partial x_i} + \frac{1}{Re}\frac{\partial^2 \tilde{u}_i}{\partial x_j \partial x_j}$$

$$- \frac{\partial}{\partial x_j}\left(\tilde{u}_i\tilde{u}_j - \overline{\tilde{u}_i\tilde{u}_j} + \langle u_i'u_j'\rangle - \overline{u_i'u_j'}\right)$$

$$\frac{\partial U_i}{\partial x_i} = 0, \qquad \frac{\partial \tilde{u}_i}{\partial x_i} = 0$$

where $\bar{D}/Dt$ is the substantial derivative following the mean flow, Re is the Reynolds number. In the absence of the large-scale structure, or if it were not sorted out from the random part, the problem reduces to the usual two-component turbulent flow problem. In the absence of the fine-grained turbulence, the problem then provides the starting point of nonlinear hydrodynamic theory in a laminar flow (26). It is clear then, the present problem is a cooperative blend of the two. Just as conservation equations for the mean turbulent stresses,

$$\overline{u_i'u_j'}\,,$$

can be derived but with its accompanying closure problems (see, for instance, (27)), the conservation equations for the modulated turbulent stresses,

$$\tilde{r}_{ij} = \langle u_i'u_j'\rangle - \overline{u_i'u_j'},$$

can also be similarly obtained (see, for instance, (28)) and similar problems with respect to closure also follows. Eddy viscosity models for

$$\tilde{r}_{ij}$$

has been used to some advantage in describing certain features of the large-scale organized structure (12), (15), (16), (28), however, some progress towards removing this stigma by bringing in the active participation of the fine-grained turbulence has recently been made in a paper which we have discussed earlier (24), where the temporal mixing region problem was discussed. The ideas follow some of the earlier works on nonlinear hydrodynamic stability theory for laminar shear flows (26), (29), (30), (31) which have been found helpful in explaining the large-scale organized structure in developing laminar shear flows. In this paper, we shall present our work on the spatially developing mixing region problem.

ENERGY INTEGRAL DESCRIPTION

The considerations alluded to in our discussion of Ref. (24) utilizes techniques of nonlinear hydrodynamic stability theory while bringing in the active interactions with the fine-grained turbulence. Such a consideration enables one to provide a gross description of the interaction processes among the three components of motion and has as its aim the derivation of the respective coupled "amplitude" (or "envelope") equations, with the accompanying shape assumptions for the vertical distributions (24), (26). The amplitude equations are derived from the kinetic energy equations in the mean.

For the spatially developing mixing region, the boundary layer approximations are applied to the mean energy equations of the three components of flow. The application of such approximations to the mean flow quantities and to the time-averaged random turbulence quantities has been the usual practice. Such approximations to the time-averaged large-scale organized structure quantities require some care. In the previous works on laminar flows such approximations were found unnecessary in that the streamwise derivatives of the time-averaged correlations of the organized structure could be evaluated (29), (31). In Ref. (31) it was found that the purely dynamical contributions were unimportant and this is confirmed in the present problem also. Rather than retaining these terms, they are neglected at the outset for simplicity. The appropriate kinetic energy equations are then

$$\frac{\bar{D}}{Dt}\frac{1}{2}U^2 = -\frac{\partial}{\partial y}U(\overline{u'v'} + \overline{\tilde{u}\tilde{v}}) - (-\overline{u'v'} - \overline{\tilde{u}\tilde{v}})\frac{\partial U}{\partial y}$$

$$\frac{\bar{D}}{Dt}\bar{q} = -\frac{\partial}{\partial y}[\overline{v'(p'+q)} + \overline{\tilde{v}\langle q\rangle}] + (-\overline{u'v'})\frac{\partial U}{\partial y} + \left[-\overline{\tilde{r}_{xx}\frac{\partial \tilde{u}}{\partial x}} - \overline{\tilde{r}_{xy}\left(\frac{\partial \tilde{u}}{\partial y} + \frac{\partial \tilde{v}}{\partial x}\right)} - \overline{\tilde{r}_{yy}\frac{\partial \tilde{v}}{\partial y}}\right] - \bar{\varepsilon}$$

energy transfer from the large-scale structure

$$\frac{\bar{D}}{Dt}\bar{Q} = -\frac{\partial}{\partial y}[\overline{\tilde{v}(\tilde{p}+Q)} + \overline{\tilde{u}\tilde{r}_{xy}} + \overline{\tilde{v}\tilde{r}_{yy}}] + (-\overline{\tilde{u}\tilde{v}})\frac{\partial U}{\partial y} - \left[-\overline{\tilde{r}_{xx}\frac{\partial \tilde{u}}{\partial x}} - \overline{\tilde{r}_{xy}\left(\frac{\partial \tilde{u}}{\partial y} + \frac{\partial \tilde{v}}{\partial x}\right)} - \overline{\tilde{r}_{yy}\frac{\partial \tilde{v}}{\partial y}}\right]$$

energy transfer to turbulence

where

$$q = \frac{1}{2}(u_i'u_i'), \quad Q = \frac{1}{2}(\tilde{u}_i\tilde{u}_i)$$

and $\bar{\varepsilon}$ is the rate of viscous dissipation of the fine-grained turbulence. Viscous diffusion of the kinetic energies and dissipation of the mean flow and large-scale structure have been neglected. In the above, the respective diffusion and production from the mean flow terms are self-explanatory in the two fluctuation energy equations. The less familiar average energy transfer between the disparate scales are so indicated.

The integral kinetic energy equations are then

$$\frac{1}{2}\frac{d}{dx}\left[\int_{-\infty}^{0}U(U^2 - U_{-\infty}^2)dy + \int_{0}^{\infty}U(U^2 - U_{+\infty}^2)dy\right] = -\int_{-\infty}^{\infty}(-\overline{u'v'} - \overline{\tilde{u}\tilde{v}})\frac{\partial U}{\partial y}dy$$

with the upper stream $U_{+\infty}$ is normalized to unity and subsequently the lower shear stream $U_{-\infty}$ taken as zero,

$$\frac{d}{dx}\int_{-\infty}^{\infty}U\bar{q}dy = \int_{-\infty}^{\infty}(-\overline{u'v'})\frac{\partial U}{\partial y}dy + \int_{-\infty}^{\infty}\left[-\overline{\tilde{r}_{xx}\frac{\partial \tilde{u}}{\partial x}} - \overline{\tilde{r}_{xy}\left(\frac{\partial \tilde{u}}{\partial y} + \frac{\partial \tilde{v}}{\partial x}\right)} - \overline{\tilde{r}_{yy}\frac{\partial \tilde{v}}{\partial y}}\right]dy - \int_{-\infty}^{\infty}\bar{\varepsilon}dy$$

$$\frac{d}{dx}\int_{-\infty}^{\infty}U\bar{Q}dy = \int_{-\infty}^{\infty}(-\overline{\tilde{u}\tilde{v}})\frac{\partial U}{\partial y}dy - \int_{-\infty}^{\infty}\left[-\overline{\tilde{r}_{xx}\frac{\partial \tilde{u}}{\partial x}} - \overline{\tilde{r}_{xy}\left(\frac{\partial \tilde{u}}{\partial y} + \frac{\partial \tilde{v}}{\partial x}\right)} - \overline{\tilde{r}_{yy}\frac{\partial \tilde{v}}{\partial y}}\right]dy$$

Implicit in the above is that the mean shear flow is two dimensional, similarly with the large-scale organized structure which, in the subsequent shape assumption, will be taken as one whose vorticity axis is perpendicular to the streamwise direction. The recently observed spanwise periodicity (4) is not accounted here. The fine-grained turbulence, of course, is three dimensional.

The closure problem is obtained in the discussion of the shape assumptions. There, some of the detailed "microscopic" calculations of the modulated stresses and large-scale structure, prior to averaging, shall be done approximately.

Shape Assumptions

The mean flow is taken to be of the form

$$U = \frac{1}{2}(1 + \tanh\eta)$$

where $\eta = y/\delta(x)$ and $\delta(x)$ is the x-dependent shear layer thickness which will characterize the mean flow, to be determined jointly with the large-scale structure and the fine-grained turbulence problems.

The shape functions for the vertical distribution of the time-averaged fine-grained turbulence quantities are guided by measurements such as those of Wygnanski and Fiedler (32). The slight asymmetry of the profiles, though it could be accomodated here, is not taken into account. The Reynolds shear stress is taken to be of the form

$$-\overline{u'v'} = \frac{a_1}{\sqrt{\pi}}E(x)e^{-\eta^2}$$

and is related to the turbulent kinetic energy via the constant $a_1 = 0.3$, thus $\bar{q} = -\overline{u'v'}/a_1$. The factor $\sqrt{\pi}$ is a normalizing constant so that $\delta E(x)$ has the physical interpretation as being the total kinetic energy of the fine-grained turbulence across a slice of the entire shear layer. $E(x)$ is thus an energy density, the total energy per unit $\delta(x)$ and it characterizes the turbulence. Following Patankar and Spalding (33), and Bradshaw, Ferriss and Atwell (34), the dissipation of the fine-grained turbulence is taken to be

$$\varepsilon = a_2\bar{q}^{3/2}/2\delta$$

where the value $a_2 = 1.5$ will be used.

Following earlier works (24), (26), (30), the vertical shape of the large-scale structure is given by a *local* linearized theory but with an amplitude $A(x)$ which is to be jointly solved with $\delta(x)$ and $E(x)$,

$$\begin{pmatrix}\tilde{u}\\ \tilde{v}\end{pmatrix} = A(x)e^{-i\beta\tau}\begin{pmatrix}\phi'(\eta;\beta)\\ -i\alpha\phi(\eta;\beta)\end{pmatrix} + \begin{pmatrix}c.c.\\ c.c.\end{pmatrix}$$

where α is the dimensionless complex wave number scaled by $\delta(x)$, β is the dimensionless physical frequency scaled also according to the local shear layer thickness $\delta(x)$ and is real, ϕ is the streamfunction and ϕ' refers to its η-derivative, c.c. denotes the complex conjugate. Here τ is the dimensionless time also referred to $\delta(x)$. The local shape functions are coupled to and diffused by the modulated turbulent stresses. The structure of these stresses warrants their shape assumption to be of the form (24),

$$\tilde{r}_{ij} = A(x)E(x)e^{-i\beta\tau}r_{ij}(\eta;\beta) + c.c.$$

Hence the vertical shape functions ϕ and r_{ij} forms a coupled local linear problem and in obtaining their equations A is taken exponential in x and E constant locally. An approximate method of solving the local temporal problem is outlined and argued in (24) and can be carried out similarly for the spatial problem here. Namely, the "inviscid" form of ϕ is first solved and then used in the algebraic r_{ij} equations as known functions. Additional closure assumptions are required for use in the equations for the modulated stress functions r_{ij}. For the mean stresses, these are $\overline{v'^2} = \overline{w'^2}$ and $\overline{v'^2}/(-\overline{u'v'}) \triangleq 2$ from, say, (32). For the modulated turbulent quantities these are patterned after closure assumptions for the mean quantities, with the pressure velocity correlation taken simply as

$$-\left[\langle u_j' \frac{\partial p'}{\partial x_i}\rangle - \overline{u_j' \frac{\partial p'}{\partial x_i}} + \langle u_i' \frac{\partial p'}{\partial x_j}\rangle - \overline{u_i' \frac{\partial p'}{\partial x_j}}\right]$$

$$= \frac{1}{T}\left[\tilde{r}_{ij} - \frac{1}{3}\tilde{r}_{ii}\delta_{ij}\right]$$

where T is the time-scale for "return to isotropy" and is taken as $T^{-1} = c\partial U/\partial y$ with $c \triangleq 1.445$. The modulated dissipation is taken to be

$$\tilde{\varepsilon} \equiv \langle\varepsilon\rangle - \bar{\varepsilon} \triangleq \frac{3}{2}\frac{\hat{a}_2}{\delta}\sqrt{\bar{q}}\,\tilde{q}$$

where $\tilde{q} = \tilde{r}_{ii}/2$. Since the vertical shape function here is to be given by the local linear theory, the form for $\tilde{\varepsilon}$ as given above reflects a linearization with $\bar{q}$ there to be replaced by its local equilibrium value in the absence of the large-scale structure. Then

$$\tilde{\varepsilon} \triangleq \frac{\beta}{T}\tilde{q}$$

where the constant $\beta \triangleq 0.24$ (there is, of course, no confusion with the same symbol used for the frequency). This is similar to using the local homogeneous shear problem as a guide (see, for instance, (24)). These additional assumptions are used in the ϕ, r_{ij} problem which enters the main overall interaction problem for δ, E and A as quantities occurring under "interaction" integrals and are thus not considered to be sensitive. For completeness, we state the approximate ϕ, r_{ij} problem, with ϕ governed by the Rayleigh equation

$$(U - c)(\phi'' - \alpha^2\phi) - \phi U'' = 0$$

with appropriate boundary conditions, and r_{ij} by

$$i\alpha(U-c)\begin{bmatrix}r_{xx}\\ r_{yy}\\ r_{zz}\\ r_{xy}\end{bmatrix} + \begin{bmatrix}2r_{xy}\\ 0\\ 0\\ r_{yy}\end{bmatrix}U' + \frac{1}{T}\left\{\begin{bmatrix}r_{xx}\\ r_{yy}\\ r_{zz}\\ r_{xy}\end{bmatrix} - \frac{1}{3}\begin{bmatrix}r_{ii}\\ r_{ii}\\ r_{ii}\\ 0\end{bmatrix}\right\}$$

$$+ \frac{\beta}{3T}\begin{bmatrix}r_{ii}\\ r_{ii}\\ r_{ii}\\ 0\end{bmatrix} = i\alpha\phi\begin{bmatrix}R'_{xx}\\ R'_{yy}\\ R'_{zz}\\ R'_{xy}\end{bmatrix} + \begin{bmatrix}-2i\alpha R_{xx}\phi' - 2R_{xy}\phi''\\ 2i\alpha R_{yy}\phi' - 2\alpha^2 R_{xy}\phi\\ 0\\ -R_{yy}\phi'' - \alpha^2\phi R_{xx}\end{bmatrix}$$

where the vertical shape functions of the mean stresses are

$$\begin{bmatrix}R_{xx}\\ R_{yy}\\ R_{zz}\\ -R_{xy}\end{bmatrix} = \begin{bmatrix}0.45\\ 0.34\\ 0.34\\ 0.17\end{bmatrix}e^{-\eta^2}$$

and the prime on the R_{ij} and ϕ denotes, again, differentiation with respect to η. Calculations for ϕ and r_{ij} are ultimately to be used in the integral over the large-scale structure and turbulence energy transfer mechanism,

$$\overline{\tilde{r}_{ij}\partial\tilde{u}_i/\partial x_j}.$$

It is clear that, to the present order of approximations, such a term is proportional to $|A|^2E$ and the "constants" of proportionality are functions of $\delta(x)$. The eigenfunctions ϕ are normalized so as to render $|A(x)|^2$ to be the corresponding energy density of the large-scale organized structure. Both $|A|^2$ and E are thus similarly interpreted.

The Interaction Problem

After the substitution of the shape functions into the energy integral equations, and with some straightforward algebraic dexterity, we obtain

$$I_1\frac{d\delta}{dx} = -I'_{rs}E - \tilde{I}_{rs}(\delta)\cdot|A|^2$$

$$I_3\frac{d\delta E}{dx} = I'_{rs}E + I_{wt}(\delta)\cdot|A|^2E - I_\varepsilon E^{3/2}$$

$$I_2(\delta)\frac{d\delta|A|^2}{dx} = \tilde{I}_{rs}(\delta)\cdot|A|^2 - I_{wt}(\delta)\cdot|A|^2 E$$

with initial conditions $\delta(0) = 1$, $E(0) = E_o$, $|A(0)|^2 = |A|_o^2$. The physical interpretation of each term is obvious and can be reverted to the original conservation equations.

The integrals involving only the mean flow and time-averaged fine-grained turbulence quantities are constants,

$$I_1 = \frac{1}{16}\int_{-\infty}^{o}(1 + \tanh\eta)^3 d\eta$$

$$+ \frac{1}{4}\int_{o}^{\infty}(1 + \tanh\eta)\left[\frac{1}{4}(1 + \tanh\eta)^2 - 1\right]d\eta$$

$$= -0.202$$

$$I_3 = \frac{1}{2\sqrt{\pi}}\int_{-\infty}^{\infty}(1 + \tanh\eta)e^{-\eta^2}d\eta = 0.500$$

$$I'_{rs} = \frac{a_1}{2\sqrt{\pi}}\int_{-\infty}^{\infty}\mathrm{sech}^2\eta\; e^{-\eta^2}d\eta = 0.109$$

$$I_\epsilon = \frac{a_2}{2\pi^{3/4}}\int_{-\infty}^{\infty}e^{-3\eta^2/2}d\eta = 0.465$$

The integrals involving *the large-scale* organized quantities are dependent on the local shear layer thickness $\delta(x)$ through their dependence on the spectral properties via local frequency parameter $\beta(\delta)$. The large-scale structure production integral is

$$\tilde{I}_{rs}(\beta) = \tilde{I}_{rs}(\delta) = \int_{-\infty}^{\infty}R\ell(i\alpha\phi\bar{\phi}')\;\mathrm{sech}^2\eta d\eta$$

where $R\ell$ denotes the real part and $\bar{\phi}$ is the complex conjugate of ϕ, and the large-scale structure — fine-grained turbulence interaction integral is

$$I_{wt}(\beta) = I_{wt}(\delta) = \int_{-\infty}^{\infty}R\ell\Big[r_{xx}(-i\bar{\alpha}\bar{\phi}') + r_{xy}(\bar{\phi}'' + \bar{\alpha}^2\bar{\phi})$$

$$+ r_{yy}(i\bar{\alpha}\bar{\phi}')\Big]d\eta$$

with $I_{wt} > 0$. The details of the interpretation of the integrand are given in (24) for the temporal problem, and will not be repeated here. Finally,

$$I_2(\beta) = I_2(\delta) = \frac{1}{2}\left\{1 + \int_{-\infty}^{\infty}\tanh\eta\cdot\Big[|\phi'|^2 + |\alpha|^2|\phi|^2\Big]d\eta\right\}$$

and is very nearly constant.

The spectral properties of the interaction integrals associated with the large-scale structure are shown in Figure 1. They are numerically obtained as functions of $\beta(x) = \beta_o\delta(x)$, where β_o is the initial characteristic (dimensionless) frequency of the large-scale structure, $\beta(0) = \beta_o$.

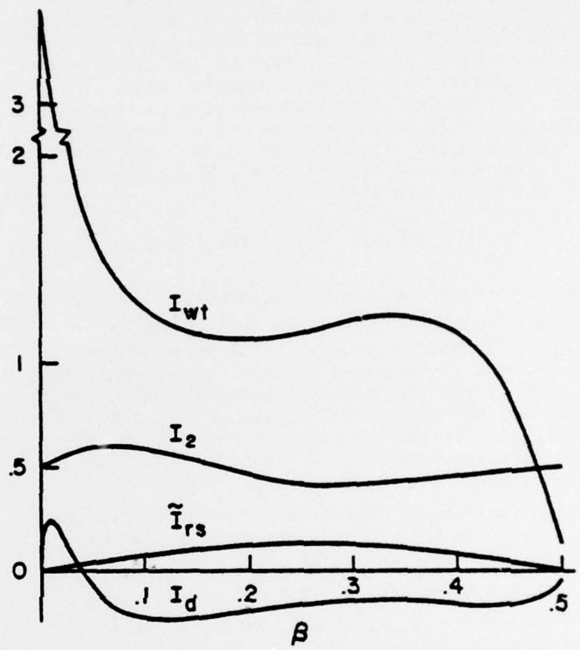

Figure 1. The spectral properties of the interaction integrals involving the large-scale structure.

The advection integral I_2 is very nearly constant and it will be approximated by 0.5 during the subsequent calculations since it was found that the problem is not sensitive to the variation of I_2. The Reynolds stress or production integral, $\tilde{I}_{rs}$, peaks at about $\beta \cong 0.21$, that value of β which corresponds to the maximal amplification in the local linear theory. The large-scale structure — fine-grained energy transfer integral I_{wt} shown here is different from that obtained for the temporal problem in Ref. (24) where the spectral properties were plotted as functions of the wave number. There the local turbulence was taken to be in equilibrium with the mean flow in order to isolate and illustrate the large-scale structure and fine-grained turbulence interaction and this rendered the modulated stresses to become singular as the local linear problem approached neutral. Here, because the fine-grained turbulence is not necessarily in equilibrium with the mean flow locally no such singularity occurred as $\beta \to 0.5$. Calculations of the properties beyond $\beta = 0.5$ were not obtained. The remaining integral, I_d, is shown only for illustrative purposes. It is associated with one of the streamwise large-scale structure diffusion terms,

$$\partial(\overline{u r}_{xx} + \overline{v r}_{xy})/\partial x.$$

Since $I_d \ll I_{wt}$ such streamwise diffusion terms contribute very little to the main features of the problem and they are neglected. There is no obvious rational transformation from a temporal to a spatial problem even in the present formulation.

RESULTS

With the large-scale structure interaction integrals interpreted as functions of $\beta = \beta_o \delta(x)$ and the single well controlled mode is specified to be β_o, the three first order nonlinear interaction equations are integrated subjected to the initial conditions $\delta(0) = 1$, $|A(0)|^2 = |A|^2_o$, $E(0) = E_o$. Energy densities corresponding to about 10^{-4} implies root mean square velocities of about a few percent of the free stream and this seems to be a reasonable value to be imposed upstream. The results are shown as functions of x, the streamwise distance normalized by the initial mixing layer thickness.

Physical Mechanisms for the Evolution of δ, $|A|^2$, and E

In Figure 2, for the case of $\beta_o = 0.1$, two sets of curves are shown. The dashed curve indicates the shear layer growth accomplished by the energy transfer from the mean flow to the turbulence energy density E alone. Thus the development of E is due to the imbalance between production from the mean and viscous dissipation alone. With the imposition of the large-scale structure the mean flow spreading becomes enhanced as is the turbulence energy density. It can be seen that the large-scale structure energy density first amplifies and subsequently decays. All this is quite reminiscent of the observations of Binder and Favre-Marinet (23). In order to obtain physical insight into the behavior shown in Figure 2, we use the solution in a diagnostic manner by showing the relevant interaction terms that bring about the evolution of δ, E and $|A|^2$. In Figure 3 we illustrate the following terms: the viscous dissipation of turbulence energy

$$I_\varepsilon E^{3/2}$$

shown as curve I, the turbulence production from the mean, $I'_{rs}E$ by curve II, the large-scale structure production from the mean $\tilde{I}_{rs}|A|^2$ by curve III, and the energy transfer between the large-scale and turbulence $I_{wt}|A|^2E$ by curve IV. Initially III > IV and the large-scale structure amplifies and starts to decay when III < IV after x ≈ 85. Of course, III itself decays because of the enhanced spreading caused by III itself. Initially when $|A|^2$ is still relatively small, the increase of E is due entirely to II > I. It continues to increase and becomes much enhanced due to IV and II in spite of the fact that I > II after x ≈ 55. The spread of the shear layer thickness δ is significantly enhanced when both $|A|^2$ and the enhanced E (because of $|A|^2$) becomes important, the growth being contributed by both II and III.

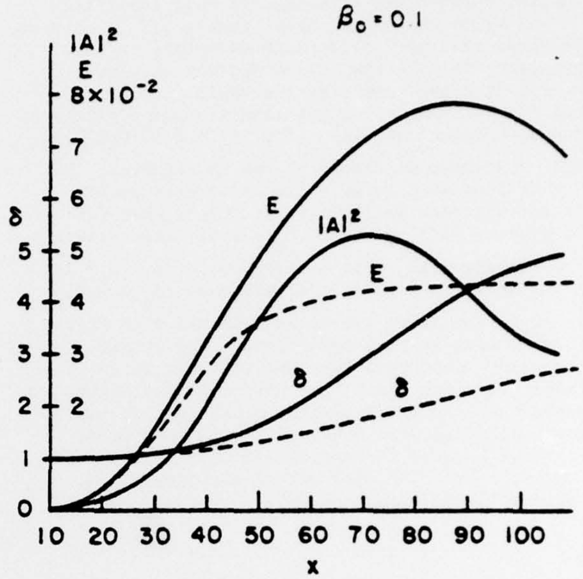

Figure 2. Streamwise development of δ, $|A|^2$ and E. Solid line: $\beta_o = 0.1$, $|A|^2_o = E_o = 10^{-4}$; Dotted line: $\beta_o = 0$, $|A|^2_o = 0$ and $E_o = 10^{-4}$.

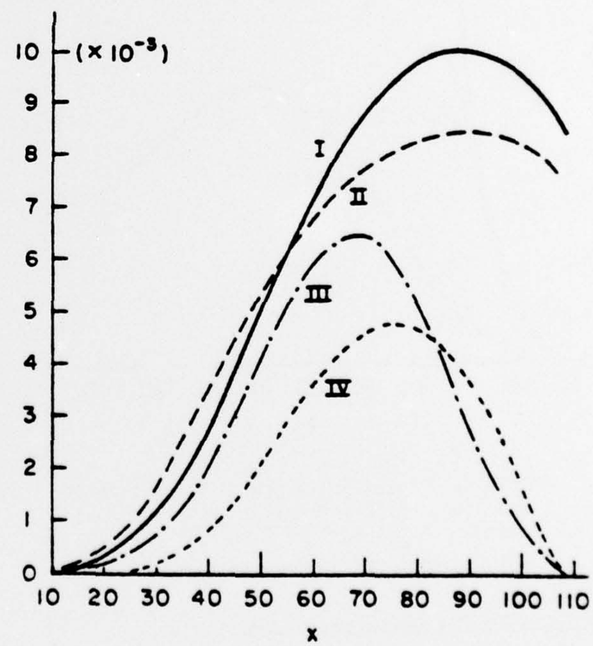

Figure 3. Energy transfer mechanisms, $\beta_o = 0.1$, $|A|^2_o = E_o = 10^{-4}$. I: $I_\varepsilon E^{3/2}$, II: $I'_{rs}E$, III: $\tilde{I}_{rs}|A|^2$, IV: $I_{wt}|A|^2E$.

The Spectral Dependence of the Interaction

Previously, the interaction was shown for $\beta_0 = 0.1$. The case $\beta_0 = 0.21$ is one for which the initial large-scale structure is most amplified according to the local linear theory and it is thus expected that the large-scale structure at this frequency will be the most efficient extractor of energy from the mean flow and would thus attain the highest peak. For the same initial conditions, the development of $|A|^2$ for $\beta_0 = 0.1$, 0.21 and 0.3 are shown in Figure 4. As is expected, the higher frequency large-scale structures peaked closer upstream while lower frequency ones further downstream (12), and the $\beta_0 = 0.21$ case attained the highest peak. The development of δ, $|A|^2$ and E is shown in Figure 5 for the case $\beta_0 = 0.21$ and 0.3, and these are to be compared with Figure 2 for the same initial conditions. The higher frequency components have shorter streamwise lifetimes; although $|A|^2$ may have attained higher peaks than the low frequency modes when it was most amplified, the enhanced E may not be as vigorous because it depends not only on the magnitude of $|A|^2$ but also on the lifetime of the large-scale structure as well.

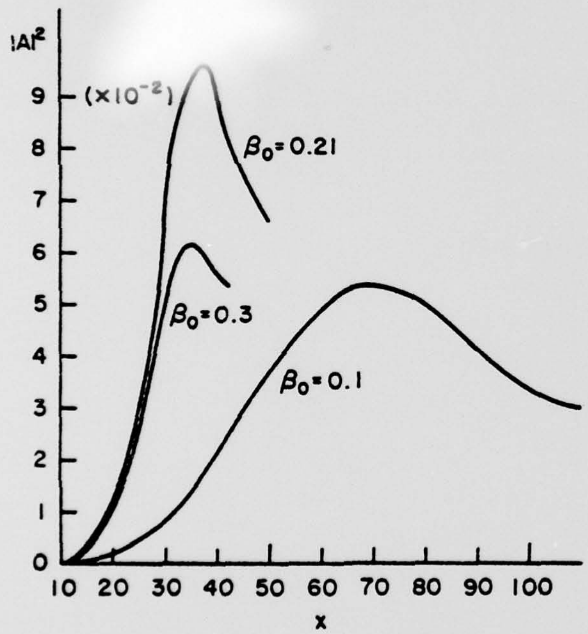

Figure 4. Spectral effects on the streamwise development of large-scale structure energy density, $|A|_0^2 = E_0 = 10^{-4}$. ($\beta_0 = 0.21$ is the initially most amplified case.)

Controlling the Large-Scale Structure

In addition to controlling the large-scale organized structure by direct imposition of definite spectral modes at a given strength, the use of fine-grained turbulence to control its development can also be achieved (12). In Figure 6 we show the $\beta_0 = 0.21$ mode with fixed $|A|_0^2$ but with different initial conditions for E_0. The initially enhanced fine-grained turbulence suppresses the subsequent development of the large-scale structure, curve II, and in so doing, it achieves a lower subsequent enhancement as indicated by E_{II}.

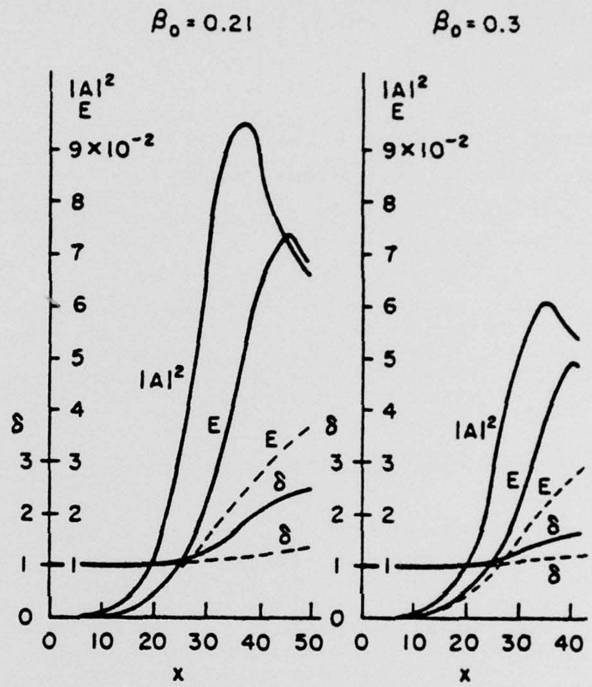

Figure 5. Spectral effects on the streamwise development of δ, $|A|^2$ and E. Solid line: $|A|_0^2 = E_0 = 10^{-4}$; Dotted line: $\varepsilon_0 = 0$, $|A|_0^4 = 0$ and $E_0 = 10^{-4}$. (Compare also with Figure 2.)

Discussion

The spatial nonlinear interaction problem has been obtained by consistantly interpreting the interaction integrals involving the large-scale structure, $\bar{I}_{rs}$ and I_{wt}, as functions of $\delta(x)$ through their dependence on $\beta(x) = \beta_0\delta(x)$ in the same spirit as (29), (30), and (31), as well as (24). In the temporal problem (24), the consequences of freezing these integrals as constants were further explored. The problem then became analytically tractable so as to enable us to obtain equilibrium states in the $|A|^2 - E$ plane and to study the nature of the singularity about such states. It was found that the $|A|^2 - E$ plane describes a stable spiral (24). The spiral behavior was sufficiently pronounced so as to resemble "bursts" or renewed large-scale structure activity after repeated cycles, which are not actually periodic, of amplification and decay. By a similar freezing of the interaction integrals in the spatial problem here, a spiral behavior about equilibrium points were also found but were not as pronounced as those in (24). The difference is not so much as between spatial and temporal problems, but that in (24) the local turbulence behavior was taken to be in equilibrium with the mean and this demanded a much larger I_{wt} for those modes of the large-scale structure approaching the neutral condition. In the present problem, the previous simplifying condition of local turbulence equilibrium with the mean is removed and this rendered I_{wt} to be finite as $\beta \to 0.5$. The results for the frozen interaction integrals are not presented here because of space limitations.

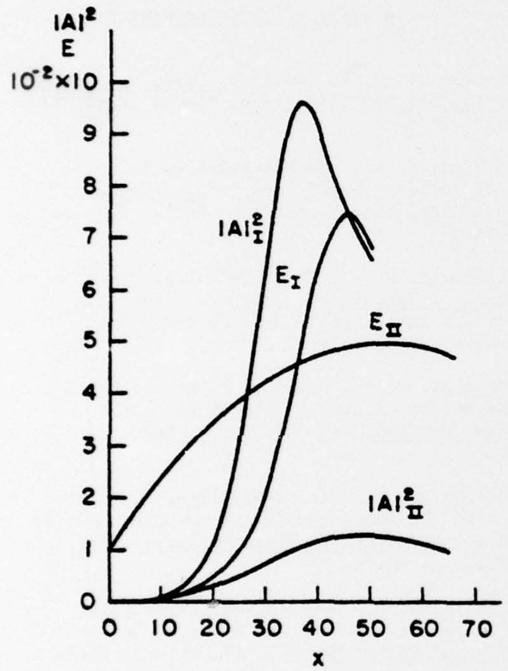

Figure 6. The effect of increasing the initial fine-grained turbulence energy level on the development of large-scale structure and turbulence energy densities. $\beta_o = 0.21$, I: $|A|_o^2 = E_o = 10^{-4}$; II: $|A|_o^2 = 10^{-4}$, $E_o = 10^{-2}$.

THE PROSPECTS FOR NUMERICAL SIMULATION

While much physical intuition could be gained about the interactions between large-scale structure and turbulence in a shear flow from the approximate energy integral considerations, much detail about the local structure is lacking. The analysis of the large-scale structure presented here resembles the analysis of nonlinear hydrodynamic stability theory in laminar flow (26), (29), (30), (31). Complementary to such analysis is the numerical simulation of finite-amplitude instabilities in laminar flow using the full nonsteady Navier-Stokes equations (see, for instance, (35), (36)). For the large-scale organized structure in a turbulent shear flow, a similar but more difficult numerical simulation could also be attempted. To this end, the senior author of the present paper recently posed the following problem.

To fix our ideas, let us revert to the temporal mixing layer problem of horizontally homogeneous and oppositely directed streams. The large-scale structure is horizontally periodic and develops in time. The "proper" dynamical dependent variable is the sum of the mean and the large-scale structure, denoted by the script notation

$$\mathcal{U}_i(x,y,t) = U(y,t) + \tilde{u}_i(x,y,t)$$

$$\mathcal{P}(x,y,t) = P(y,t) + \tilde{p}(x,y,t)$$

The full Navier-Stokes equations is then split into the mean and organized part and a turbulence part according to $\mathcal{U}_i + u_i'$ and $p = \mathcal{P} + p'$. Again denoting the conditional average by $<(\ )>$, we have for the dynamical variables

$$\frac{\partial \mathcal{U}_i}{\partial x_i} = 0$$

$$\frac{\partial \mathcal{U}_i}{\partial t} + \frac{\partial \mathcal{U}_i \mathcal{U}_j}{\partial x_j} = - \frac{\partial \mathcal{P}}{\partial x_i} - \frac{\partial <u_i'u_j'>}{\partial x_y} - \frac{1}{Re} \frac{\partial^2 \mathcal{U}_i}{\partial x_j \partial x_j}$$

and the coupled, yet unclosed, equation for the conditionally averaged stresses

$$\left(\frac{\partial}{\partial t} + \mathcal{U}_k \frac{\partial}{\partial x_k}\right)<u_i'u_j'> = - \left[<u_j'u_k'>\frac{\partial \mathcal{U}_i}{\partial x_k} + <u_i'u_k'>\frac{\partial \mathcal{U}_j}{\partial x_k}\right]$$

production

$$+ <p'\left(\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i}\right)>$$

redistribution

$$- \frac{\partial}{\partial x_k}\left[<u_i'u_j'u_k'> + <p'(u_i'\delta_{jk} + u_j'\delta_{ik})> - \frac{1}{Re}\frac{\partial <u_i'u_j'>}{\partial x_k}\right]$$

diffusion

$$- 2 \frac{1}{Re} <\frac{\partial u_i'}{\partial x_k} \frac{\partial u_j'}{\partial x_k}>$$

dissipation

The viscous diffusion effects will subsequently be neglected. A possible closure scheme is obtained by a direct extension of Launder et al.'s model (27) for the mean stresses to the present conditionally averaged stresses $<u_i'u_j'>$. Again, reverting to the temporal mixing layer problem with a two-dimensional large-scale structure, such a model then provides eight equations for the eight unknowns $\mathcal{U}$, $\mathcal{V}$, $\mathcal{P}$ and $<u'v'>$, $<u'^2>$, $<v'^2>$, $<w'^2>$ and $<\varepsilon>$. (Or, following Amsden and Harlow (35), the three dynamical variables are replaced by the vorticity and streamfunction.) U_i is horizontally periodic as are the conditionally averaged stresses $\langle u_i'u_j' \rangle$ and these decay far from the shear layer. Originally there existed the mean mixing layer with a set of consistent and compatible mean stresses and at $t = t_o$, a large-scale organized structure is imposed so that

$$U_i(x,y,t_o) \equiv U_i(y,t_o) + \tilde{u}_i(x,y,t_o),$$

$$\langle u_i'u_j' \rangle(x,y,t_o) \equiv \overline{u_i'u_j'}(y,t_o) + \tilde{r}_{ij}(x,y,t_o).$$

Hence, the "initialization" calls for a compatible coexistence of the mean problem $U_i(y,t_o)$ and $\overline{u_i'u_j'}(y,t_o)$. The imposed $\tilde{u}_i(x,y,t_o)$ is then made compatible with $U_i(y,t_o)$, being, say, the solution to the Rayleigh equation with $U(y,t_o)$ as the mean profile and $\tilde{r}_{ij}(x,y,t_o)$ taken as zero. The problem yields the total quantities, $U_i \equiv U_i + \tilde{u}_i$ and $\langle u_i'u_j' \rangle \equiv \overline{u_i'u_j'} + \tilde{r}_{ij}$, with the mean obtainable by horizontal averaging and the large-scale structure quantities by a subtraction of the mean from the total quantities. Of great interest, for instance, is the use of the numerical solution for diagnosing the energy transfer between the large-scale structure and the fine-grained turbulence, $\tilde{r}_{ij}\partial\tilde{u}_i/\partial x_j$ among others. The numerical problem here is certainly compounded compared to the relatively more straightforward laminar problem. Our colleague, T. B. Gatski, is courageously working on this problem and the results, when available, would be of great interest towards physically diagnosing the interaction problem. The extension of the temporally developing to the spatially developing mixing layer problem is certainly possible. However, much of the physics about the interaction could be gained from the simpler, temporal problem first before attempting the more difficult spatial problem.

ACKNOWLEDGMENTS

Various aspects of the research described in this paper are supported in part by the National Aeronautics and Space Administration, Langley Research Center, Grant NSG-1076, the National Science Foundation Grant ENG73-04104 and the Fluid Dynamics Program of the Office of Naval Research Contract N00014-76-C-0363. The authors express their thanks to Ruth Wattman for her assistance in the preparation of the manuscript.

REFERENCES

1 Laufer, J., "New Trends in Experimental Turbulence Research," Annual Review of Fluid Mechanics, Vol. 7, Annual Reviews, Palo Alto, 1975, pp. 307-326.

2 Mollo-Christensen, E., "Physics of Turbulent Flow," AIAA Journal, Vol. 9, No. 7, July 1971, pp. 1217-1228.

3 Mollo-Christensen, E., "Intermittency in Large-Scale Turbulent Flows," Annual Review of Fluid Mechanics, Vol. 5, Annual Reviews, Palo Alto, 1973, pp. 101-118.

4 Roshko, A., "Structure of Turbulent Shear Flows: A New Look," (Dryden Research Lecture), AIAA Journal, Vol. 14, No. 10, October 1976, pp. 1349-1357.

5 Willmarth, W. W., "Structure of Turbulence in Boundary Layers," Advances in Applied Mechanics, Vol. 15, Academic Press, New York, 1975, pp. 159-254.

6 Davis, P. O. A. L. and Yule, A. J., "Coherent Structures in Turbulence," Journal of Fluid Mechanics, Vol. 69, 1975, Part 3, pp. 517-537; and "Corrigendum," Journal of Fluid Mechanics, Vol. 74, Part 4, 1976, p. 797.

7 Murthy, S. N. B. (Editor), Turbulent Mixing in Nonreactive and Reactive Flows, Plenum Press, New York, 1975.

8 Dryden, H. L., "Recent Advances in the Mechanics of Boundary Layer Flow," Advances in Applied Mechanics, Vol. 1, Academic Press, New York, 1948, pp. 1-40.

9 Liepmann, H. W., "Free Turbulent Flows," The Mechanics of Turbulence, Int. Symp. Nat. Sci. Res. Center, Marseille 1961. Gordon and Breach, New York, 1964, pp. 211-227.

10 Carrier, G. F., Fendell, F. E. and Marble, F. E., "The Effect of Strain Rate on Diffusion Flames," SIAM Journal, Vol. 28, No. 2, March 1975, pp. 463-500.

11 Bishop, K. A., Ffowcs Williams, J. E. and Smith, W., "On the Noise Sources of the Unsuppressed High Speed Jet," Journal of Fluid Mechanics, Vol. 50, Part 1, 1971, pp. 21-31.

12 Liu, J. T. C., "Developing Large-Scale Wavelike Eddies and the Near Jet Noise Field," Journal of Fluid Mechanics, Vol. 62, Part 3, 1974, pp. 437-464.

13 Woods, J. D. and Wiley, R. L., "Billow Turbulence and Ocean Microstructure," Deep-Sea Research, Vol. 19, 1972, pp. 87-121.

14 Liu, J. T. C. and Merkine, L., "Energy Integral Description of the Development of Kelvin-Helmholtz Billows," Tellus, Vol. 28, No. 3, 1976, pp. 197-214.

15 Liu, J. T. C., "Nonlinear Development of an Instability Wave in a Turbulent Wake," Physics of Fluids, Vol. 14, No. 11, November 1971, pp. 2251-2257.

16 Reynolds, W. C., "Large-Scale Instabilities of Turbulent Wakes," Journal of Fluid Mechanics, Vol. 54, Part 3, 1972, pp. 481-488.

17 Winant, C. D. and Browand, F. K., "Vortex Pairing: The Mechanism of Turbulent Mixing-Layer Growth at Moderate Reynolds Number," Journal of Fluid Mechanics, Vol. 63, Part 2, 1974, pp. 237-255.

18 Schubauer, G. B. and Skramstad, H. K., "Laminar Boundary Layer Oscillations and Transition

on a Flat Plate," Rep. No. 909, 1948, National Advisory Committee on Aeronautics, Washington, D. C.

19 Kelly, R. E., "On the Stability of an Inviscid Shear Layer which is Periodic in Space and Time," Journal of Fluid Mechanics, Vol. 27, Part 4, pp. 657-689.

20 Townsend, A. A., The Structure of Turbulent Shear Flow, 1st ed., Cambridge University Press, 1956, p. 123.

21 Hussain, A. K. M. F. and Reynolds, W. C., "The Mechanics of an Organized Wave in Turbulent Shear Flow," Journal of Fluid Mechanics, Vol. 41, Part 2, 1970, pp. 241-258.

22 Kendall, J. M., "The Turbulent Boundary Layer Over a Wall with Progress Surface Waves," Journal of Fluid Mechanics, Vol. 41, Part 2, pp. 259-281.

23 Binder, G. and Favre-Marinet, M., "Mixing Improvement in Pulsating Turbulent Jets," Fluid Mechanics of Mixing, (E. M. Uram and V. W. Goldschmidt, eds.), American Society of Mechanical Engineers, New York, 1973, pp. 167-172.

24 Liu, J. T. C. and Merkine, L., "On the Interactions between Large-Scale Structure and Fine-Grained Turbulence in a Free Shear Flow," Proceedings of the Royal Society of London, Series A, Vol. 352, 29 December 1976, pp. 213-247.

25 Phillips, O. M., The Dynamics of the Upper Ocean, Cambridge University Press, 1966, p. 90.

26 Stuart, J. T., "On the Nonlinear Mechanics of Hydrodynamic Stability," Journal of Fluid Mechanics, Vol. 4, Part 1, 1958, pp. 1-21.

27 Launder, B. E., Reece, G. J. and Rodi, W., "Progress in the Development of a Reynolds Stress Turbulence Closure," Journal of Fluid Mechanics, Vol. 68, Part 3, 1975, pp. 537-566.

28 Reynolds, W. C. and Hussain, A. K. M. F., "The Mechanics of an Organized Wave in Turbulent Shear Flow. 3. Theoretical Models and Comparisons with Experiments," Journal of Fluid Mechanics, Vol. 54, Part 2, 1975, pp. 263-288.

29 Ko, D. R. S., Kubota, T., and Lees, L., "Finite Disturbance Effect in the Stability of a Laminar Incompressible Wake Behind a Flat Plate," Journal of Fluid Mechanics, Vol. 40, Part 2, 1970, pp. 315-341.

30 Liu, J. T. C. and Lees, L., "Finite-Amplitude Instability of the Compressible Laminar Wake: Strongly Amplified Disturbances," Physics of Fluids, Vol. 13, No. 12, December 1970, pp. 2932-2938.

31 Liu, J. T. C. and Gururaj, P. M., "Finite-Amplitude Instability of the Compressible Laminar Wake: Comparison with Experiments," Physics of Fluids, Vol. 17, No. 3, March 1974, pp. 532-543.

32 Wygnanski, I. and Fiedler, H. E., "The Two-Dimensional Mixing Region," Journal of Fluid Mechanics, Vol. 41, Part 2, pp. 327-362.

33 Patankar, S. V. and Spalding, D. B., "A Finite-Difference Procedure for Solving the Equations of the Two-Dimensional Boundary Layer," International Journal of Heat and Mass Transfer, Vol. 10, 1967, pp. 1389-1411.

34 Bradshaw, P., Ferriss, D. H. and Atwell, N. P., "Calculation of Boundary Layer Development Using the Turbulence Energy Equation," Journal of Fluid Mechanics, Vol. 28, Part 3, 1967, pp. 593-616.

35 Amsden, A. A. and Harlow, F. H., "Slip Instability," Physics of Fluids, Vol. 7, No. 3, March 1964, pp. 327-334.

36 Patnaik, P. C., Sherman, F. S., and Corcos, G. M., "A Numerical Simulation of Kelvin-Helmholtz Waves of Finite Amplitude," Journal of Fluid Mechanics, Vol. 73, Part 2, 1976, pp. 215-240.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) ON THE LARGE-SCALE STRUCTURE IN TURBULENT FREE SHEAR FLOWS.		5. TYPE OF REPORT & PERIOD COVERED Technical Report
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) J. T. C. Liu and A. Alper		8. CONTRACT OR GRANT NUMBER(s) N00014-76-C-0363, 9/1/75 - present NSF-ENG-73-04104
9. PERFORMING ORGANIZATION NAME AND ADDRESS Brown University Division of Engineering Providence, Rhode Island 02912		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Department of the Navy Office of Naval Research, Fluid Dynamics Program Arlington, Virginia 22217 (Code 438)		12. REPORT DATE 1977
		13. NUMBER OF PAGES 11
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

Reprinted from Proceedings of Symposium on Turbulent Shear Flows, pp. 11.1-11.11, April 18-20, 1977, The Pennsylvania State University

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Large-scale structure

Turbulence

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The existence of organized structures in turbulent shear flow has been the subject of recent observational discoveries in both the laboratory and in the atmosphere and ocean. The recent work on modeling such structures in a temporally developing, horizontally homogeneous turbulent free shear layer (J. T. C. Liu and L. Merkine 1976 Proc. Roy. Soc. London, Ser. A., Vol. 352, pp. 213-247) has been extended to the spatially developing mixing layer, there being no available rational transformation between the two nonlinear problems. (cont.)

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

20.

The basis for the consideration is the kinetic energy development of the mean flow, large-scale structure and fine-grained turbulence with a conditional average, supplementing the usual time average, to separate the nonrandom from the random part of the fluctuations. The integrated form of the energy equations and the accompanying shape assumptions, is used to derive "amplitude" equations for the mean flow, characterized by the shear layer thickness, the nonrandom and random components of flow which are characterized by their respective energy densities. In general, the large-scale structure augments the spreading of the shear layer and enhances the fine-grained turbulence by taking energy from the mean flow and transferring it to the turbulence as it amplifies and subsequently decays. The maximal amplitude of the large-scale structure is attained by the initially most amplified mode, however, the relative enhancement of the fine-grained turbulence is achieved by both the magnitude of the large-scale structure and its streamwise lifetime. Thus a greater enhancement of the turbulence is achievable by the lower frequency modes which have longer streamwise lifetimes. The large-scale structure can also be controlled by increasing the initial level of turbulence, which would render its decay more rapidly.