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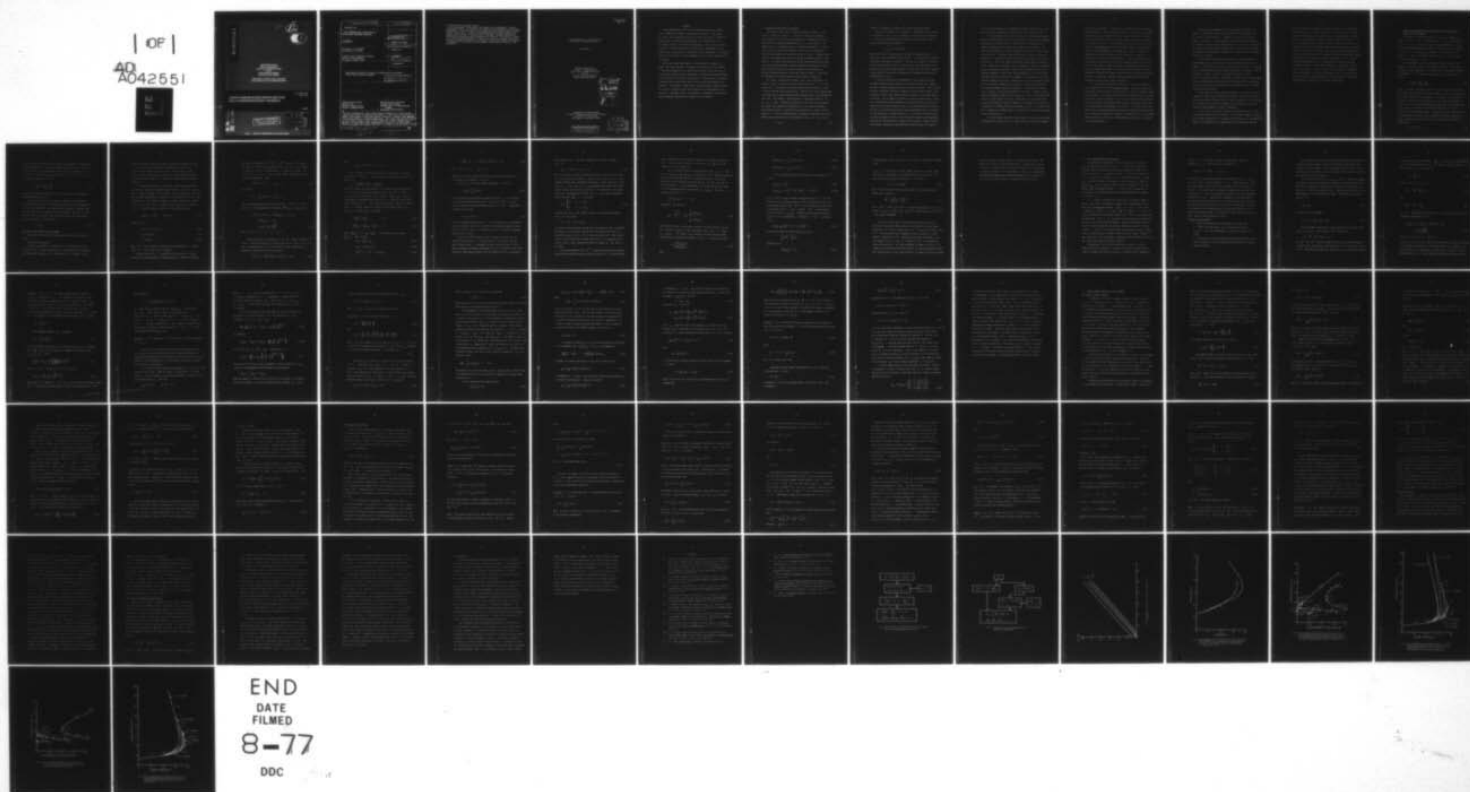
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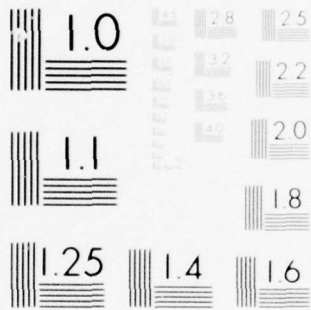
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GROUP RANDOM-ACCESS DISCIPLINES FOR MULTI-ACCESS BROADCAST CHANNELS

I. RUBIN

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Izhak Rubin

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ABSTRACT

A Group Random-Access (GRA) access-control discipline for a multi-access communication channel is presented and studied. A GRA scheme uses only certain channel time-periods to allow some network terminals to transmit their information-bearing packets on a random-access basis. The channel can thus be utilized at other times to grant access to other terminals, or other message types, by applying, as appropriate, group random-access, reservation or fixed access-control procedures. GRA schemes could also be utilized to provide channel access to various network protocol packets.

The average packet delay under a GRA discipline is evaluated by a Markov ratio limit theorem. To stabilize the channel, the GRA procedure is controlled dynamically by a control policy which rejects any newly arriving packets within certain time-periods. Studying the associated Markov decision problem, the optimal control policy is characterized as yielding a minimal average packet delay under a prescribed packet probability of rejection. This policy is shown to be a single-threshold scheme for which there exists a threshold value which attains the minimum probability of rejection. Performance curves are presented to demonstrate the excellent delay-throughput characteristics induced by GRA procedures.

1. Introduction and System Description

We consider a multi-access broadcast channel of capacity C bps serving a network of terminals. A satellite communication channel, a radio channel in a terrestrial radio network or a communication link in a broadcast line computer communication serve as a few examples. The channel utilizes a repeater (such as a satellite transponder or a radio relay station) to enable each terminal in the network to communicate (through the repeater) with any other terminal (see [1]). Messages transmitted by the terminals are directed, through the channel uplink, to the repeater. The latter then shifts the message uplink frequency-band into a disjoint downlink frequency-band and broadcasts the messages (so that each terminal can receive any signal reflected by the transmitter) through the downlink channel towards the network terminals. (Note that no schedulings for message downlink transmissions are required.)

We assume a synchronized structure. Thus, time (referenced w.r.t. repeater's time) is divided into fixed-length durations of τ sec. each, called slots. An appropriate network synchronization procedure is used to achieve network slot synchronization. Terminals will start transmissions of messages only at times coinciding with starting times of the synchronized time slots. The channel is characterized by a propagation delay of $R\tau$ sec., or R slots. Propagation delays are of the order of milliseconds for packet radio channels and around 0.25 sec. for satellite channels. Each terminal message is considered to be described as a packet of fixed-length of μ^{-1} bits (including protocol, information and parity-check bits). Packet transmission time across the channel is thus $(\mu C)^{-1}$ sec. We set

$$\tau = (\mu C)^{-1}, \quad (1.1)$$

so that a slot duration is equal to the packet transmission time.

We consider a network of M terminals. New messages arrive at the i -th terminal, $i=1,2,\dots,M$, according to a Poisson stream of intensity λ_i . The overall network message (or packet) arrival stream is thus a Poisson point process with intensity

$$\lambda = \sum_{i=1}^M \lambda_i \text{ mess./slot .}$$

Upon the arrival of a new message, a terminal will immediately try to gain access into the channel for this message. No terminal-buffer capacity or blocking constraints are imposed. One can also consider a terminal to possess buffer storage for only a single message, and subsequently to be blocked for new arrivals when occupied. However, when the network contains a large number of active bursty terminals, the blocking effects would be insignificant; see [1].

To utilize efficiently the bandwidth of such a channel and grant acceptable message response times to the terminals sharing this channel, one needs to apply an appropriate access-control discipline. Access-control procedures employing reservation schemes have been recently studied in [1]. Using these schemes, each terminal needs to transmit a reservation packet to reserve a slot (or number of slots) for a newly arrived message. Assuming a decentralized control mechanism, each terminal (while receiving the broadcasted reservation packets) stores in its own queueing table the present state of the reservation process, being subsequently able to determine its own allocation of transmission slots. In a centralized control mode, a central controller receives all the reservation packets and subsequently instructs the terminals when to transmit their messages. Dynamic reservation schemes, considering single and multi-packet messages, are shown in

[1] to yield excellent delay-throughput performance characteristics, over the whole range of moderate to high network traffic intensity values.

For low network traffic intensity values, when single-packet highly bursty terminal message processes are considered, a better delay-throughput performance, involving a much less sophisticated (distributed) access-control procedure, can be achieved by a random-access mechanism. The latter allows terminals to use the channel at any time to transmit a newly arrived packet. If, however, two or more packets collide, the involved messages are retransmitted following an appropriate random retransmission delay policy. Due to the simple distributed control mechanism involved with a random-access discipline, such a procedure can result in significant savings in hardware requirements (such as those involving various multiplexing mechanisms), protocol and system complexities. (See [3] and the references therein for the use of a random access technique in the ALOHA computer communication system.) Furthermore, very low message-delay values are attained when a random-access discipline is utilized, provided that low enough throughput values are acceptable. A simple slotted random-access procedure, called slotted ALOHA (see [3]-[8]), is noted to allow a maximal throughput (channel traffic capacity) of $1/e = 0.368$ packets/slot. Thus, an average number of at most 0.368 packets will be successfully transmitted through the channel (compared to a channel traffic capacity of 1 packet/slot for TDMA and reservation schemes, see [1]). We further note that a random-access channel needs to incorporate a flow-control mechanism to avoid instabilities.

In various actual situations, the designer is ready to accept channel throughput values lower than $1/e$, while requiring a simple distributed

access-control procedure. A random-access procedure, such as the slotted ALOHA technique, is then an attractive choice. In many cases, one desires to dedicate only certain portions of the channel time-frame to a family of network terminals wishing to share the channel (during these periods) on a random-access basis. For that purpose, we present and study in this paper the Group Random Access (GRA) discipline. Under a GRA discipline, a group of network terminals are provided with a periodic sequence of channel access periods, during which this group uses a random-access discipline to gain access into the channel. A packet experiencing collision during a certain period will be retransmitted during the next access period. Other groups of network terminal (distinguished by their priorities, performance requirements or by the statistics and nature of their information, emitting, for example, short-interactive or longer long-haul messages) can share the remaining time-frame duration using again GRA procedures or other access-control techniques.

It is many times of particular interest to use a GRA procedure to grant channel access to certain protocol packets. The latter packets are usually much shorter than the message packets so that low throughput values are acceptable. At the same time, the simple distributed-control structure of the GRA is highly desirable. This is the case when reservation access-control disciplines are considered and (shorter) reservation packets need to be transmitted by the terminals (see [1]-[2]). The latter reservation packets can be assigned periodically reservation periods during which they use a random-access procedure to compete for channel access. This procedure clearly results in a GRA access-control mechanism, utilized by the family of reservation packets.

The approximate throughput and delay-throughput performance of a regular (slotted ALOHA) random-access procedure have been studied (see [3]-[8] and references therein), assuming an approximating Markovian channel state process. Certain dynamic control schemes which stabilize the inherently unstable slotted ALOHA channel, have also been investigated (by proposing certain threshold control schemes, not necessarily optimal, and computing their performance through the associated dynamic-programming equation, see [6],[8]).

In this paper, we present a precise study of the performance of a Group Random-Access discipline and its optimal dynamic control. The channel is controlled so that the minimal average message delay is attained, under an appropriately prescribed packet probability of rejection. The GRA procedure is shown to yield delay-throughput performance characteristics comparable to those attained by a regular (slotted ALOHA) random-access procedure, while providing the network designer with a much higher degree of flexibility in granting access to different classes of information and protocol messages.

We note that our study of the GRA procedure as an access-control discipline for a multi-access communication channel, can be readily applied also to non-broadcast channels, where a central controller (or other means) is incorporated to provide the terminals with the relevant (positive or negative) acknowledgment and control information.

In Section II, we present the network performance measures and the slotted ALOHA random access discipline. For the latter scheme, we indicate the relevant characteristics associated with the evolution of the underlying Markov state sequence and the computation of the packet delay function.

The GRA procedure is presented in Section III, where we also derive the characteristics of its underlying Markov state sequence and the related formulas yielding the average packet delay. An optimal dynamic control policy for a GRA channel is characterized and studied in Section IV. An associated Markov decision problem is shown to induce an optimal control function which yields the minimal average packet delay under a prescribed value of probability of rejection. The resulting delay-throughput characteristics of the controlled GRA channel, under the optimal single-channel control scheme, are then indicated and demonstrated in a set of figures presenting performance curves. The appropriate preferable structure (threshold values) for a GRA channel controller is then noted. The controlled GRA channel is shown to exhibit excellent delay vs. throughput (or vs. probability of rejection) performance curves (even when not all the state variables are observable) over the whole range of acceptable network traffic intensities (or rejection probabilities).

II. Network Performance Measures and the Regular (Slotted ALOHA) Random-Access Discipline

We consider a synchronized multi-access broadcast communication channel of capacity C bps, a slot duration of τ sec. and propagation delay of R slots. The channel serves a large community of M terminals, generating new messages according to a Poisson stream of intensity λ mess./slot. Each message is considered to be a packet of fixed length of μ^{-1} bits. The packet transmission time is set to be equal to the slot duration, $\tau = (\mu C)^{-1}$.

The performance of an access-control discipline applied to this channel is assessed in terms of the following measures. A performance indicator of major importance is the steady-state average packet waiting-time function $\bar{W}$. The latter can be expressed as the limit (when it exists)

$$\bar{W} = \lim_{N \rightarrow \infty} N^{-1} E \left\{ \sum_{n=1}^N W_n \right\}, \quad (2.1)$$

where W_n denotes the waiting-time of the n -th message in the system. This waiting time, expressed in terms of number of slots, is measured from the instant the packet is transmitted by its terminal (which happens at the start of the next slot following its arrival, thus not including an average of $1/2$ -slot delay between actual arrival and first transmission) to the instant the packet is successfully transmitted. The overall steady-state average message delay D includes thus also single-slot and R -slot durations to account for the transmission time and propagation delay, respectively, associated with a successful packet transmission. We thus have,

$$D = \bar{W} + R + 1. \quad (2.2)$$

A successful packet transmission will occur if the packet is the only one being transmitted in its slot, while packet collisions occur if more than a single packet is being transmitted in the same slot. Denoting by S_i the number of successful transmissions in the i -th slot, $S_i = 0, 1, i \geq 1$, the channel throughput is given by

$$s = \lim_{N \rightarrow \infty} N^{-1} E \left\{ \sum_{i=1}^N S_i \right\}, \quad (2.3)$$

expressing the channel output rate (i.e., the limiting average number of successful packets per slot).

We will note, in Section III, that to stabilize the GRA channel, certain packets will have to be denied access and be (at least temporarily) rejected. The probability P_R indicating the probability of packet rejection will thus serve as another index of performance. Note that if access (eventual successful transmission) is then provided to all non-rejected (accepted) packets, we will then have

$$s = (1 - P_R) \quad . \quad (2.4)$$

The Slotted ALOHA Random-Access Procedure

The regular slotted ALOHA (SA) random-access discipline operates as follows.

Protocol (SA discipline):

A newly arrived packet is transmitted by its terminal at the start of the next slot. A packet transmission (or retransmission) which collides with other packet transmissions is retransmitted by its terminal. The re-

transmission slot is chosen according to a uniform distribution over the L slots following the reception of the broadcasted collision (i.e., R slots after the transmission of the latter colliding packet). Each packet is being retransmitted, governed by the latter random retransmission delay procedure, until it is successfully transmitted (avoiding any collisions). |

To indicate the evolution of the channel state process under an SA discipline, we define the following random variables. We let A_n , N_n , R_n and S_n denote the numbers of new arrivals, total transmissions, collisions and successful transmissions, respectively, associated with the n -th slot. We further set Z_n to denote the number of packets allocated for retransmission at the n -th slot. We then note that $\{A_n, n \geq 1\}$ is a sequence of i.i.d. random variables governed by a Poisson distribution,

$$P(A_n=k) = e^{-\lambda} \frac{\lambda^k}{k!}, \quad k=0,1,2,\dots \quad (2.5)$$

We have for $n \geq 1$,

$$N_n = Z_n + A_n = S_n + R_n, \quad (2.6)$$

$$R_n = N_n I(N_n \geq 1), \quad (2.7)$$

$$S_n = I(N_n = 1), \quad (2.8)$$

where $I(A)$ is the indicator function associated with event A , so that $I(A)=1$ if A occurs and $I(A)=0$, otherwise.

Consider now a set of K consecutive slots over which R transmissions (or retransmissions) are made according to a uniform distribution.

The joint distribution of $\{T^{(1)}, T^{(2)}, \dots, T^{(K)}\}$, where $T^{(i)}$ denotes the number of transmissions allocated to the i -th slot (out of these K slots and of the R transmissions), is computed as follows. Let $\{U^{(i)}, i=1, 2, \dots, R\}$ be R i.i.d. integer-valued random variables uniformly distributed over $[1, K]$,

$$P(U^{(i)}=k) = K^{-1}, \quad k=1, 2, \dots, K. \quad (2.9)$$

Then we have

$$T^{(j)} = \sum_{i=1}^R I(U^{(i)}=j), \quad j=1, 2, \dots, K. \quad (2.10)$$

Thus, we conclude that the joint distribution of $T^{(1)}, T^{(2)}, \dots, T^{(K)}$, given R , is the multinomial distribution $g_R^{(K)}(n_1, n_2, \dots, n_K)$ given by

$$\begin{aligned} P\{T^{(1)}=n_1, T^{(2)}=n_2, \dots, T^{(K)}=n_K \mid n_1+n_2+\dots+n_K = R\} \\ = g_R^{(K)}(n_1, n_2, \dots, n_K) \\ = \frac{R!}{n_1!n_2!\dots n_K!} \left(\frac{1}{K}\right)^R, \end{aligned} \quad (2.11)$$

where $0 \leq n_i \leq R, i=1, 2, \dots, K, \quad n_1+\dots+n_K = R$.

Considering now the SA procedure, we let $T_n^{(j)}$ denote the number of n -slot collisions allocated for retransmission in slot $n+R+j, j=1, 2, \dots, K$. Thus, we determine from (2.9)-(2.11) that the latter variables have a multinomial conditional distribution given by

$$P\{T_n^{(1)}=n_1, \dots, T_n^{(L)}=n_L \mid R_n=R\} = g_R^{(L)}(n_1, \dots, n_L), \quad (2.12)$$

where

$$\sum_{i=1}^L n_i = R, \quad 0 \leq n_i \leq R, \quad i=1, \dots, L.$$

The SA channel state process can then be represented as a vector Markov chain $\underline{Z} = \{Z_n, n \geq 1\}$, over the space of non-negative integers $\mathcal{J}$, where we set

$$\underline{Z}_n = \left\{ Z_{n+1}^{(n)}, Z_{n+2}^{(n)}, \dots, Z_{n+L+R}^{(n)} \right\},$$

and $Z_i^{(n)}$ denotes the overall number of retransmissions allocated to the i -th slot by any collisions occurring at the n -th slot, or earlier (i.e., at the j -th slot, with $j \leq n$). Clearly, $Z_{n+1}^{(n)} = Z_{n+1}$ since all allocations for retransmissions at the $(n+1)$ -st slot have already been made at the time slot n . The transition probability function for Markov chain $\underline{Z}$ is readily expressed in terms of the following expressions. Given $\underline{Z}_n$, we obtain $\underline{Z}_{n+1}$ by setting

$$Z_{n+1+j}^{(n+1)} = Z_{n+1+j}^{(n)}, \quad 1 \leq j \leq R, \quad (2.13a)$$

$$Z_{n+1+R+j}^{(n+1)} = Z_{n+1+R+j}^{(n)} + T_{n+1}^{(j)}, \quad 1 \leq j \leq L, \quad (2.13b)$$

where $Z_{n+1+R+L}^{(n)} = 0$, and $\{T_{n+1}^{(j)}\}$ are governed by distribution $g_{R_{n+1}}^{(L)}(\cdot)$. Thus, we have

$$N_{n+1} = Z_{n+1}^{(n)} + \Lambda_{n+1}, \quad (2.13c)$$

$$R_{n+1} = N_{n+1} I(N_{n+1} \geq 2), \quad (2.13d)$$

$$T_{n+1}^{(j)} = 0, \quad i \leq j \leq L, \quad \text{if } R_{n+1} = 0, \quad (2.13e)$$

$$P(T_{n+1}^{(j)} = n_j, \quad 1 \leq j \leq L \mid R_{n+1} = R) = g_R^{(L)}(n_1, \dots, n_L), \quad (2.13f)$$

where $0 \leq n_j \leq R$, $1 \leq j \leq L$, $\sum_{j=1}^L n_j = R > 0$.

Equations (2.13) thus specify the transition probability function for the channel state sequence $\underline{Z}$ (Fig. 2.1).

By Eqs. (2.3), (2.8), the channel throughput s satisfies

$$s = \lim_{N \rightarrow \infty} N^{-1} \sum_{n=1}^N P(N_n = 1). \quad (2.14)$$

If one assumes the retransmission counting variables Z_n to be governed by a Poisson distribution with mean $\bar{Z}$ (see [3]-[7]) then, by (2.13c), N_n will follow a Poisson distribution with mean $\bar{N} = \bar{Z} + \lambda$. Subsequently, we obtain by Eq. (2.14),

$$s = \bar{N} \exp(-\bar{N}) \leq e^{-1}. \quad (2.15)$$

It has in fact been observed that e^{-1} is the maximal possible throughput of an SA channel. We note in (2.15) that the channel throughput increases with $\bar{N}$ till $\bar{N}=1$ and, as $\bar{N}$ further increases, the channel throughput rapidly decays to zero.

Due to the bi-stable nature of the SA channel (see [5],[7]), the underlying Markov chain $\underline{Z}$ is readily noted to be transient, and the packet average delay D subsequently becomes arbitrarily high, $D = \bar{W} = \infty$. Delay-throughput performance measurements for the SA channel are many times obtained by admitting new packets into the channel only for an appropriate

finite duration N_1 . The latter duration can be chosen, assuming $R_1=0$, as

$$N_1(L_1) = \inf\{n: n \geq 1, R_{n+1} \geq L_1\} . \quad (2.16)$$

Thus, the channel is allowed to admit new packets as long as no more than L_1 slot collisions are observed. (Note by (2.15) that, for $L_1 > 1$, the resulting channel delay-throughput performance is quite insensitive to the value of L_1 since, when $R_n > 1$, the throughput and message-delay values rapidly decrease and increase, respectively.) To analyze the SA scheme with (2.16) incorporated, we replace A_{n+1} in (2.13c) with a controlled arrival variable $A_{n+1}^{(c)}$, $n \geq 1$, given by

$$A_n^{(c)} = \begin{cases} A_n , & \text{if } n \leq N_1(L_1) \\ 0 , & \text{if } n > N_1(L_1) . \end{cases} \quad (2.17)$$

Following the N_1 -th slot, packets continue to be retransmitted until the N_2 -th slot, where

$$N_2 = \inf\{n: n \geq N_1, R_n=0\} .$$

To obtain the average packet waiting time associated with such a controlled SA channel, we can further assume that the same channel operation is repeated following the N_2 -th slot of each operation period. (This readily follows by noting that the resulting state process is a regenerative stochastic process, with a regeneration period of length N_2 , and $E(N_2) < \infty$, see [10].)

The resulting Markov Chain $\underline{z}^{(c)}$ is then clearly positive-recurrent. The associated average message waiting-time function $\bar{W}$ is then calculated

using a Markov ratio limit theorem, following the procedure presented in [1]. Since the latter procedure is utilized here to analyze the GRA scheme, we briefly summarize it.

For an irreducible positive-recurrent Markov chain $\{Z_n\}$, or $\{Z_n^{(c)}\}$, we set $N(Z_n, Z_{n+1})$ and $W(Z_n, Z_{n+1})$ as the number of newly admitted packets and the sum of the waiting-times of all transmitted packets, respectively, during the n -th time period associated with Z_n (i.e., the n -th slot for the SA procedure). For our applications, the latter functions are noted to be time-homogeneous and to depend only on (Z_n, Z_{n+1}) , for each $n \geq 1$. Further note that, as $M \rightarrow \infty$,

$$\sum_{n=1}^M N(Z_n, Z_{n+1}) \rightarrow \infty, \quad \text{w.p.1.}$$

Subsequently, we can write

$$\lim_{M \rightarrow \infty} \frac{\sum_{n=1}^M W_n}{N} = \lim_{M \rightarrow \infty} \frac{\sum_{n=1}^M W(Z_n, Z_{n+1})}{\sum_{n=1}^M N(Z_n, Z_{n+1})}, \quad (2.18)$$

with probability one. We now apply a Markov ratio limit theorem (see [9], p.91, Theorem 1, and [1]) to the vector Markov chain $\{Y_n, n \geq 1\}$, where $Y_n = \{Z_n, Z_{n+1}\}$. The latter is an irreducible positive-recurrent Markov chain with the stationary distribution $\{\pi(i, j)\}$. We then conclude that

$$\bar{W} = \frac{E[W(Z_n, Z_{n+1})]}{E[N(Z_n, Z_{n+1})]}, \quad (2.19a)$$

where

$$E[W(\underline{Z}_n, \underline{Z}_{n+1})] = \sum_{\underline{i}, \underline{j}} W(\underline{i}, \underline{j}) \pi(\underline{i}, \underline{j}) , \quad (2.19b)$$

$$E[N(\underline{Z}_n, \underline{Z}_{n+1})] = \sum_{\underline{i}, \underline{j}} N(\underline{i}, \underline{j}) \pi(\underline{i}, \underline{j}) . \quad (2.19c)$$

For the SA channel described by the Markov state sequence $\underline{Z}^{(c)}$, we have

$$N(\underline{Z}_n, \underline{Z}_{n+1}) = A_n^{(c)} , \quad (2.20a)$$

$$W(\underline{Z}_n, \underline{Z}_{n+1}) = R R_n^{(c)} + [T_n^{(1)} + 2T_n^{(c)} + \dots + LT_n^{(L)}] . \quad (2.20b)$$

In Eq. (2.20b), we have included a propagation delay of R slots for each of the R_n colliding packets, as well as additional delay terms incorporating the positions $\{T_n^{(i)}\}$ of the retransmissions within the corresponding set of L slots. Variables $T_n^{(i)}$ are expressed by (2.13b) in terms of $(\underline{Z}_n, \underline{Z}_{n+1})$ and their conditional distribution is given by (2.12). Subsequently, $E(T_n^{(i)} | R_n^{(c)}) = L^{-1} R_n^{(c)}$, so that we obtain

$$E[W(\underline{Z}_n, \underline{Z}_{n+1}) | R_n^{(c)}] = [R + \frac{1}{2}(1+L)] R_n^{(c)} . \quad (2.21)$$

We note that since, for the present case, we have

$$\sum_{n=1}^{N_2} A_n^{(c)} = \sum_{n=1}^{N_2} S_n ,$$

we can also set

$$N(\underline{Z}_n, \underline{Z}_{n+1}) = S_n . \quad (2.22)$$

Using Equations (2.20a), (2.21)-(2.22) in (2.19), we obtain the following result.

Theorem 1. For the regular slotted (ALOHA) random-access scheme, input-controlled as represented by the underlying Markov chain process $\underline{Z}^{(c)}$, the limiting average packet waiting time $\bar{W}$ is given by

$$\bar{W} = [R + \frac{1}{2}(1+L)]\bar{R}_A^{(c)}, \quad (2.23)$$

where $\bar{R}_A^{(c)}$ denotes the limiting average number of retransmissions per packet, and is given by

$$\bar{R}_A^{(c)} = \frac{E(R^{(c)})}{E(A^{(c)})} = \frac{E(R^{(c)})}{s^{(c)}}, \quad (2.2)$$

where $E(R^{(c)})$, $E(A^{(c)})$ and $E(S^{(c)})$ are the limiting means (w.r.t. the measure $\pi(\underline{i}, \underline{j})$) of $R_n^{(c)}$, $A_n^{(c)}$ and $S_n^{(c)}$ respectively, and $s^{(c)} = E(S^{(c)})$ is the channel throughput. |

Theorem 1, which serves as the appropriate version of Little's Theorem for the present scheme, allows us to compute the average packet waiting-time $\bar{W}$ from the limiting mean $\bar{R}_A^{(c)}$ associated with Markov chain $\underline{Z}^{(c)}$. The latter mean can be computed by numerical methods or simply through a simulation of $\underline{Z}^{(c)}$ (using the recurrence relationship in the flow diagram of Fig. 2.1, with A_n replaced by $A_n^{(c)}$, and appropriate restarting conditions). The delay-throughput performance of such an SA scheme will be presented in the next Section. The delay-throughput performance characteristics of this input-controlled SA scheme are basically similar

to those presented in other studies of SA schemes (see [3]-[8]). The Markov-state process description and analysis presented above serve to present the essential points relevant to our analysis of a GRA scheme, as compared with those of an SA procedure, as well as to introduce and describe the underlying stochastic mechanisms to be utilized here. The GRA access-control scheme is presented in the next section.

III. The Group Random-Access Discipline

To present the Group Random-Access (GRA) procedure, we identify first the sequence of periods $\{B_n, n \geq 1\}$, during which the group of terminals under consideration are allowed to contend for channel access. The n -th period B_n is assumed to contain K successive channel slots $K > 1$, and thus be of duration $K\tau$ sec. The distance between B_n and B_{n+1} , measured in terms of the number of slots following B_n and preceding B_{n+1} , can be taken to be given by any fixed number of slots not smaller than the propagation delay R . With no loss in generality, we thus assume in the following analysis the latter distance to be equal to R , for any $n, n \geq 1$.

If N groups of terminals are set to utilize the whole channel capacity, each using a GRA procedure, channel time is decomposed into the union of N such periodic sequences $\{B_n^{(i)}, n \geq 1, 1 \leq i \leq N\}$, so that $B_n^{(i)}$ is followed by $B_n^{(i+1)}$, $i < N$, and by $B_n^{(N)}$ by $B_{n+1}^{(1)}$, $n \geq 1$. The i -th group of terminals with an overall traffic rate $\lambda^{(i)}$ uses the channel only during periods $\{B_n^{(i)}, n \geq 1\}$, and thus does not interfere with the transmissions of any other group of terminals. Since the performance characteristics are then the same for any group of terminals, except for varying values of λ , R and K , we need to study the performance of only a single group of terminals which we assume to have an overall traffic intensity λ and to emit newly arriving packets, following Poisson statistics (see (3.1)), only within the sequence of periods $\{B_n, n \geq 1\}$.

Alternatively, if we utilize only periods $\{B_n, n \geq 1\}$ to serve a family of network terminals on a GRA basis, we again assume the latter family to produce new packets according to a Poisson process supported on

$\{B_n, n \geq 1\}$; i.e., the number of newly arriving packets during B_n , denoted as A_n , is a Poisson variable with mean $K\lambda$,

$$P\{A_n=j\} = e^{-K\lambda} \frac{(K\lambda)^j}{j!} , \quad j=0,1,2,\dots, \quad (3.1)$$

with these A_n new arrivals uniformly distributed over the K slots of B_n , and $\{A_n, n \geq 1\}$ being a sequence of i.i.d. random-variables. Thus, new packet arrivals utilizing the GRA procedure and occurring outside $\{B_n, n \geq 1\}$, can be uniformly distributed for transmission over the following period of K slots, and subsequently included in the above-mentioned arrival model, with λ appropriately computed as the average number of new packet arrivals per utilized slot. The extra delay term, expressing the packet waiting-time from arrival until first transmission, is again not included in the following waiting time and delay functions, but is readily appropriately added. Considering the above-mentioned arrival patterns, the GRA scheme operates as follows.

Protocol (GRA discipline):

1. Newly arrived packets are transmitted in the next slot.
2. Packets colliding within B_n are retransmitted within $B_{n+1}, n \geq 1$, at a slot determined by a uniform distribution over $[1, K]$.
3. Each packet is being transmitted and retransmitted until successfully transmitted, or until rejected from the network by a network control procedure. |

We note that step 3 in the GRA protocol incorporates the possibility of packet rejection control to yield finite packet average delay-times, as noted for the SA scheme. The related optimal control analysis will be presented in the next section. We will present in this section a few basic characteristics of the GRA scheme.

As in Section II, we let A_n , N_n , R_n , S_n and Z_n denote the numbers of total new arrivals, transmissions, collisions, successful transmissions and packets allocated for retransmission, respectively, within the n -th period B_n . The arrival process $\{A_n, n \geq 1\}$ has been characterized by (3.1). Relationships (2.6)-(2.8) hold here as well. Furthermore, we can write here for $n \geq 1$,

$$Z_{n+1} = R_n, \quad (3.2)$$

so that Eq.(2.6) now becomes

$$N_{n+1} = R_n + A_{n+1} = S_{n+1} + R_{n+1}. \quad (3.3)$$

The GRA channel state process can be represented as a vector Markov chain $\underline{Z} = \{\underline{Z}_n, n \geq 1\}$, over the space $\mathcal{S}^{2K}$, where we set

$$\underline{Z}_n = \{T_n^{(i)}, A_n^{(i)}, i=1,2,\dots,K\},$$

and $A_n^{(i)}$ and $T_n^{(i)}$ denote the number of new arrivals and the number of retransmissions, respectively, allocated to the i -th slot within $B_n, n \geq 1, 1 \leq i \leq K$. The transition probability function of $\underline{Z}_n$ is obtained as follows:

We note that the arrival variables $\{A_{n+1}^{(i)}\}$ are statistically independent of $\underline{Z}_n$ and are characterized by (3.1). The variables $\{T_{n+1}^{(i)}\}$ depend on $\underline{Z}_n$ only through R_n , which is given by

$$R_n = \sum_{j=1}^K R_n^{(j)}, \quad (3.4a)$$

where for each $j, j=1,2,\dots,K$,

$$R_n^{(j)} = N_n^{(j)} I(N_n^{(j)} \geq 2), \quad (3.4b)$$

and

$$N_n^{(j)} = T_n^{(j)} + A_n^{(j)}, \quad (3.4c)$$

for $n \geq 1$. Conditional distribution (2.12) is then applied here to yield multinomial distribution (2.11),

$$\begin{aligned} P\{T_{n+1}^{(i)} = n_i, 1 \leq i \leq K | R_n = j\} &= g_j^{(K)}(n_1, \dots, n_K) \\ &= \frac{j!}{n_1! \dots n_K!} \left(\frac{1}{K}\right)^j, \end{aligned} \quad (3.5)$$

where $0 \leq n_i \leq j, i=1,2,\dots,K$, $\sum_{i=1}^K n_i = j$.

Equations (3.1), (3.4)-(3.5) thus yield the transition probability function for the Markov state chain $\underline{Z}$. One observes that $\underline{Z}_{n+1}$ depends on $\underline{Z}_n$ only through R_n . In particular, we note that $\underline{R} = \{R_n, n \geq 1\}$ is now a Markov chain over $\mathcal{R}$, with a transition probability function expressed

through Eqs.(3.4)-(3.5) . A flow diagram indicating the transition $R_n \rightarrow R_{n+1}$ is shown in Fig. 3.1 . Also included in the figure is a decision box involving the computation of the 0-1 control variable U_{n+1} , as determined by the values of R_n and U_n , $U_{n+1}=U(R_n, U_n)$. The control variable U_n is used to block any new arrivals, being set equal to 1 in a period during which all arrivals are rejected and equal to 0, otherwise , thus inducing the controlled arrival variable $\tilde{A}_n^{(j)} = (1-U_n)A_n^{(j)}$,

$$\tilde{A}_n = \sum_{j=1}^K \tilde{A}_n^{(j)} .$$

The throughput variable S_{n+1} is given by

$$S_{n+1} = \sum_{j=1}^K I(N_{n+1}^{(j)} = 1) . \quad (3.6)$$

By the multinomial distribution (3.5) and distribution (3.1), we conclude that $N_{n+1}^{(k)}$ (Eq.(3.4c)) is governed by a binomial distribution, given $N_{n+1} = A_{n+1} + R_n$, $1 \leq k \leq K$,

$$P\{N_{n+1}^{(k)} = i | N_{n+1} = j\} = \binom{j}{i} \left(\frac{1}{K}\right)^i \left(1 - \frac{1}{K}\right)^{j-i} , \quad (3.7)$$

where $0 \leq i \leq j$, $j \geq 0$. Hence, by (3.6)-(3.7), we obtain

$$E(S_{n+1}) = E\left\{N_{n+1} \left(1 - \frac{1}{K}\right)^{N_{n+1}-1}\right\} . \quad (3.8)$$

Noting that $xa^x \leq e^{-1} [\ln(a^{-1})]^{-1}$, for $x \geq 0, a \leq 1$, we obtain the GRA channel throughput s , expressing the average number of successful transmissions per slot.

upper bounded as

$$s \leq \frac{1}{e} \frac{1}{(K-1)\ln[(1+(K-1)^{-1})]} \stackrel{\Delta}{=} \frac{1}{e} G_K . \quad (3.9)$$

It is noted that the maximal value of $E\{S_{n+1} | N_{n+1}\}$ is attained at $N_{n+1} = N_{n+1}^*$, where N_{n+1}^* is given by the integer part of $\{\ln[1+(K-1)^{-1}]\}^{-1}$. Note also that $N_{n+1}^* \geq K-1$. Function G_K is very close to 1 for any $K > 2$, and $G_K \rightarrow 1$ as $K \rightarrow \infty$. Also, $K^{-1}N_{n+1}^* \rightarrow 1$ as $K \rightarrow \infty$. By actual simulation of a GRA channel, we further verify (see figures in Section 4) that we can attain $s = \frac{1}{e} G_K \approx \frac{1}{e}$. We have thus concluded the following result.

Theorem 2. A GRA scheme with K slot periods has a maximal throughput of $\frac{1}{e} G_K \approx \frac{1}{e}$. |

We note that, in deriving the above maximal throughput result in (3.0), we have only utilized the uniform distribution associated with allocating retransmissions and the conditional distribution of new arrivals. Otherwise, the distribution of A_n can be chosen arbitrarily, and need not be a Poisson distribution.

It is readily verified that the state sequences $\underline{Z}$ and $\underline{R}$ are transient, and thus $D=\bar{W}=\infty$, when the channel is uncontrolled; i.e., when $U_n=0, n \geq 1$. For that purpose, considering uncontrolled $\underline{R}$ with the transition probabilities P_{ij} , note that

$$\sum_{j > i+1} P_{ij} \geq p_i , \quad \sum_{j < i} P_{ij} \leq q_i ,$$

whereas $i \rightarrow \infty$, p_i increases monotonically to $1 - \exp(-\lambda K) > 0$, and q_i decreases exponentially to 0. Subsequently, a random walk with a transition probability function $\{\tilde{p}_{ij}\}$, where $\tilde{p}_{ii-K} = q_i$, $\tilde{p}_{ii+1} = p_i$, $\tilde{p}_{ii} = 1 - p_i - q_i$, is noted to be transient, thus implying that $\underline{R}$ itself is transient.

To note the evolution of the mean number of period collisions, we incorporate Eq.(3.8) into relation (3.3). We then obtain, for $n \geq 1$, noting that $N_{n+2} = A_{n+2} + N_{n+1} - S_{n+1}$,

$$E\{N_{n+2} | N_{n+1}\} - N_{n+1} = E(A_{n+2}) - N_{n+1} \left(1 - \frac{1}{K}\right)^{N_{n+1}-1}, \quad (3.10a)$$

and therefore,

$$E(N_{n+2}) - E(N_{n+1}) = E(A_{n+2}) - E\left\{N_{n+1} \left(1 - \frac{1}{K}\right)^{N_{n+1}-1}\right\}. \quad (3.10b)$$

Also, since $R_{n+1} = R_n + A_{n+1} - S_{n+1}$, we have for $n \geq 1$,

$$E(R_{n+1}) = E\left\{(R_n + A_{n+1}) \left[1 - \left(1 - \frac{1}{K}\right)^{R_n + A_{n+1} - 1}\right]\right\}. \quad (3.11)$$

Note in Eqs.(3.10) that, as the total number of period transmissions N_{n+1} increases, the throughput rapidly decreases and subsequently

$$E(N_{n+2}) - E(N_{n+1}) \approx E(A_{n+2}).$$

When the channel is controlled, Eqs.(3.10)-(3.11) still apply if we replace the arrival variable A_n by the controlled arrival variable $\tilde{A}_n$. In par-

ticular, we obtain from (3.11) the conditional means of R_{n+1} ,

$$\bar{R}_j(i) \triangleq E\{R_{n+1} | R_n = i, U_{n+1} = j\}, \quad (3.12)$$

where $i \geq 0, j=0,1$, given by the following Proposition.

Proposition 1. For a GRA scheme, we have for $i \geq 0$,

$$\bar{R}_1(i) = i \left[1 - \left(1 - \frac{1}{K} \right)^{i-1} \right], \quad (3.13a)$$

and

$$\bar{R}_0(i) = i \left[1 - e^{-\lambda} \left(1 - \frac{1}{K} \right)^{i-1} \right] + K\lambda \left[1 - e^{-\lambda} \left(1 - \frac{1}{K} \right)^i \right]. \quad (3.13b)$$

Proof. Eq.(3.13a) follows from (3.11) when we set $A_{n+1} = 0$. Eq.(3.13b) is obtained by evaluating the expectation in (3.11), with A_{n+1} governed by a Poisson distribution with mean λ and setting $R_n = i$.

Q.E.D.

Functions $\bar{R}_0(i)$ and $\bar{R}_1(i)$, given by Eqs.(3.13), are shown in Fig. 3.2. Note that $\bar{R}_1(i) = \bar{R}_0(i)$, $i \geq 0$, for $\lambda=0$. For any $\lambda \geq 0$, we observe $\bar{R}_0(i)$ to be a monotonically increasing function of i , with a slope monotonically increasing to a rather constant value (almost real line) within $0 \leq i \leq 2K$. For large i , $\bar{R}_0(i) \approx i$, as $\lim_{i \rightarrow \infty} i^{-1} \bar{R}_0(i) = 1$. We further observe that $\bar{R}_0(i) - \bar{R}_1(i) \approx K\lambda$ for $i > K$, and that

$$K\lambda(1 - e^{-\lambda}) = \bar{R}_0(0) \leq \bar{R}_0(i) \leq i + K\lambda, \quad (3.14)$$

and, by setting $\lambda=0$ in (3.14), we also note that

$$0 \leq \bar{R}_1(i) \leq i . \quad (3.15)$$

The lower bound in (3.14) clearly specifies the average number of collisions among newly arriving packets during any period.

The computation of the average packet waiting-time (or any other moment of its waiting-time) is now presented following the procedure used in Section II here and in [1]. For this purpose, we assume that an appropriate control function $U_n=U(\cdot)$ is chosen, so that the Markov state sequences $\underline{Z}$ and $\underline{R}$ are irreducible positive-recurrent. The transition probability functions and stationary distributions of $\underline{Z}$ and $\underline{R}$ are denoted by $\{P_Z(i,j)\}$, $\{\pi_Z(i)\}$, and $\{P_R(i,j)\}$, $\{\pi_R(i)\}$, respectively. As in Section II, we set $N(\underline{Z}_n, \underline{Z}_{n+1})$ and $W(\underline{Z}_n, \underline{Z}_{n+1})$ as the number of newly admitted packets in B_n and the sum of the waiting-times of all packets transmitted during B_n , respectively. Again, these functions are noted to be time-homogeneous and to depend only on $(\underline{Z}_n, \underline{Z}_{n+1})$, $n \geq 1$. Considering non-degenerate controls that yield a packet rejection probability (P_R) less than one, we have

$$\lim_{M \rightarrow \infty} \sum_{n=1}^M N(\underline{Z}_n, \underline{Z}_{n+1}) \rightarrow \infty , \text{ w.p.1.}$$

Subsequently, Eq.(2.18) holds here as well. Applying again a Markov ratio limit theorem, Eqs.(2.19) are obtained for computing the average packet waiting-time $\bar{W}$.

For the controlled GRA channel, we have

$$N(\underline{Z}_n, \underline{Z}_{n+1}) = \tilde{A}_n , \quad (3.16)$$

$$W(Z_n, Z_{n+1}) = RR_n + \left[T_{n+1}^{(1)} + 2T_{n+1}^{(2)} + \dots + KT_{n+1}^{(K)} \right] + \Delta W(Z_n) \quad (3.17)$$

where

$$\Delta W(Z_n) = \sum_{j=1}^K [(j-1)I(N_n^{(j)}=1)I(\tilde{A}_n^{(j)}=0)] \quad (3.18)$$

The first two terms in (3.17) are the same as those appearing in the SA channel expression (2.20b). The third term, unique to the GRA procedure and given by (3.18), accounts for the extra waiting-time experienced by a successful retransmission within its last period of transmission. As for the SA scheme, assuming the controlled channel to serve (eventually successfully transmit) all admitted packets, we can also set

$$N(Z_n, Z_{n+1}) = S_n \quad (3.19)$$

To evaluate the mean of (3.17), we use the multinomial distribution (3.5) to determine that $E(T_{n+1}^{(i)} | R_n) = K^{-1} R_n$, and subsequently

$$E \left\{ \left[T_{n+1}^{(1)} + 2T_{n+1}^{(2)} + \dots + KT_{n+1}^{(K)} \right] | R_n \right\} = \frac{1}{2}(K+1)R_n \quad (3.20)$$

To compute the steady-state mean of (3.18), we note that the limit

$$P_{SR} \stackrel{\Delta}{=} \lim_{n \rightarrow \infty} E \{ I(N_n^{(j)}=1) I(\tilde{A}_n^{(j)}=0) \} \quad (3.21)$$

is independent of j , $1 \leq j \leq K$, and yields the (steady-state) probability of a successful retransmission. Similarly, the limit

$$P_{SA} \stackrel{\Delta}{=} \lim_{n \rightarrow \infty} E \{ I(N_n^{(j)}=1) I(\tilde{A}_n^{(j)}=1) \} \quad (3.22)$$

is independent of j , $1 \leq j \leq k$, and yields the (steady-state) probability of successful first (newly arriving) packet transmission. Clearly, the throughput $s = \lim_{n \rightarrow \infty} E(S_n)$ satisfies

$$s = p_{SR} + p_{SA} \quad (3.23)$$

Probability p_{SA} is given by

$$\begin{aligned} p_{SA} &= \lim_{n \rightarrow \infty} P(\tilde{A}_n^{(j)}=1) E \left\{ \left(1 - \frac{1}{K}\right)^{R_{n-1}} \middle| \tilde{A}_n^{(j)}=1 \right\} \\ &= \lim_{n \rightarrow \infty} P(\tilde{A}_n^{(j)}=1) E \left\{ \left(1 - \frac{1}{K}\right)^{R_{n-1}} \middle| U_n=0 \right\}, \end{aligned} \quad (3.24)$$

since R_{n-1} depends on $\tilde{A}_n^{(j)}$ only through U_n and none of the R_{n-1} retransmissions are allowed to be allocated to the j -th slot. The probability of a single controlled new packet arrival in a slot is given by

$$\lim_{n \rightarrow \infty} p(\tilde{A}_n^{(j)}=1) = (1-p_R) \lambda e^{-\lambda} = s e^{-\lambda}, \quad (3.25)$$

where

$$p_R = \lim_{n \rightarrow \infty} p(U_n=1), \quad (3.26)$$

is the probability of packet rejection, and the (steady-state) throughput s is given by

$$s = K^{-1} \lim_{n \rightarrow \infty} E(S_n) = \lambda(1-p_R). \quad (3.27)$$

Using Eqs.(3.21)-(3.25) in Eq.(3.18), and incorporating Eq.(3.19), we conclude that

$$\Delta \bar{W} \triangleq \frac{E[\Delta W(Z_n)]}{E[N(Z_n, Z_{n+1})]} = \frac{1}{2}(K-1) \left[1 - e^{-\lambda} E \left\{ \left(1 - \frac{1}{K} \right)^{R_{n-1}} \middle| U_n = 0 \right\} \right], \quad (3.28)$$

where the expectation operation on the RHS of (3.28) is with respect to the conditional stationary distribution of R_{n-1} given $U_n = 0$. Using Eqs. (3.17), (3.19) and (3.28), we obtain by Eq.(3.19) the formula for computing the average packet waiting-time $\bar{W}$ in a GRA channel, in terms of the steady-state statistics of $\{R_n, n \geq 1\}$.

Theorem 3. For a controlled GRA channel, represented by an irreducible positive-recurrent state sequence Z , the limiting average packet waiting-time $\bar{W}$ is given by

$$\bar{W} = [R + \frac{1}{2} (1+K)] \bar{R}_A + \Delta \bar{W} \quad (3.29a)$$

where

$$\bar{R}_A = s^{-1} \bar{R} = s^{-1} \lim_{n \rightarrow \infty} E(R_n), \quad (3.29b)$$

and $\Delta \bar{W}$ is given by Eq.(3.28). |

Regarding the extra packet waiting-time term $\Delta \bar{W}$, we note the following points. Clearly,

$$\Delta \bar{W} \leq \frac{1}{2}(K-1). \quad (3.30)$$

Furthermore, in all practical applications, we will have $\bar{R} \ll K$ and, subsequently,

$$E \left\{ \left(1 - \frac{1}{K} \right)^{R_n-1} \middle| U_n=0 \right\} \approx 1 - K^{-1} \bar{R} . \quad (3.31)$$

Approximation (3.31) yields approximation $\Delta \bar{W}_1$ to $\Delta \bar{W}$, where

$$\Delta \bar{W} \approx \Delta \bar{W}_1 = \frac{1}{2}(K-1) [1 - e^{-\lambda} (1 - K^{-1} \bar{R})] , \quad (3.32)$$

and approximation $\bar{W}_1$ to $\bar{W}$, given by

$$\bar{W} \approx \bar{W}_1 = [\bar{R} + \frac{1}{2}(1+K)] s^{-1} \bar{R} + \Delta \bar{W}_1 . \quad (3.33)$$

In all our performance computations, we have found approximation $\bar{W}_1$ in (3.33) to be excellent, generally yielding average waiting-time results nearly non-distinguishable from those obtained by computing $\bar{W}$. We note that, for computing $\bar{W}$ in (3.29), only the stationary distribution of $\{(R_n, U_n), n \geq 1\}$ is involved, while the major term of (3.29) requires only the throughput s and the stationary mean $\bar{R}$ of $\{R_n, n \geq 1\}$. For computing $\bar{W}_1$ in (3.33), only means s and $\bar{R}$ are required. The latter mean (or related distribution) is readily obtained, for any control function, from the transition scheme of Fig. 3.1, using simple numerical computation techniques or a direct simulation procedure of $\{(R_n, U_n), n \geq 1\}$.

A control scheme similar to that presented in Section II, induced by thresholds $L_1, N_1(L_1)$ (see Eq.(2.16)) and N_2 , is simple to implement and can also be used here. Note that, under this scheme, the control function is given by

$$U_{n+1} = U(R_n, U_n) = \begin{cases} 0, & \text{if } R_n < L_1, U_n = 0 \\ 1, & \text{if } R_n \geq L_1, U_n = 0, 1 \\ 1, & \text{if } R_n > 0, U_n = 1 \\ 0, & \text{if } R_n = 0, U_n = 1 . \end{cases} \quad (3.34)$$

As indicated in Section 2, this scheme will also represent the waiting-time performance of a GRA channel which stops admitting new packet arrivals when the number of group retransmissions surpasses L_1 and is disconnected as soon as the latter number becomes zero. The delay-throughput performance curves for SA and GRA schemes, under input-control function (3.34), are shown in Fig. 3.3. We assume $K=1$, $L_1=2K$, $R=12$ and choose values $K=6$ and $K=12$. We note that, as the network traffic intensity λ is increased, the channel throughput s is increased until its corresponding maximal value is achieved (note that we attain $s_{\max} \approx e^{-1}$ for $K=12$). The delay-throughput performance characteristics under an SA or a GRA scheme are observed to be similar. Note that, under input control function (3.34), the operation of the channel is terminated after a random number of slots N_2 , with the mean function $E(N_2)^{<\infty}$ rapidly decreasing as $\lambda \rightarrow e^{-1}$. Alternatively, if the channel is restarted each time following its blocking period, the packet probability of rejection p_R will be noted to increase as λ is increased. A procedure for optimal control of a GRA channel, incorporating both packet average time-delay and probability of rejection as indices of performance, is developed and studied in the next section.

IV. Optimal Dynamic Control of a GRA Channel.

The Optimal Control Problem

The dynamics of the GRA channel, governed by control sequence $\{U_n, n \geq 1\}$, is described in Fig. 3.1 and Eqs.(3.3)-(3.5), in terms of the underlying state sequence $\underline{Z}$. Assuming Markov chain $\{(R_n, U_n), n \geq 1\}$ to be irreducible positive-recurrent, as is the case for all our applications of interest, the stationary probabilities of the latter chain have been noted to determine the major channel indices of performance. The average packet waiting-time $\bar{W}$ is the first measure of performance of interest. It is given by (3.29) and is well-approximated by $\bar{W}_1$ of (3.33), thus depending on the latter Markov chain only through $\bar{R}$ and throughput s . The second measure of performance of interest here is the probability of packet rejection P_R , given by (3.26) and directly related to the channel throughput s by Eq.(3.27). We wish to obtain the control sequence $\underline{U} = \{U_n, n \geq 1\}$ which will yield the minimum value of packet average waiting-time (or delay), while providing an appropriate prescribed value of packet rejection-probability P_R (or channel throughput s).

Note that rejected packets can be assumed to be lost, or to try again to access the channel following an appropriate random delay. The latter is then assumed to follow an exponential distribution so that the point process of new arrivals is still described as Poisson with intensity λ . Since the probability P_R for most applications will be very small (see latter curves), the precise rejection-reenter mechanism is not important for the present analysis.

Assuming causal observations of the controlled channel state sequence $\underline{Z}$ are available to the controller $\underline{U}$, only Markov sequence $\{(R_n, U_n), n \geq 1\}$

needs to be causally observed, since measures $\bar{W}$ and P_R are considered. The set of admissible control functions $\mathcal{U}$ is not constrained and includes all deterministic and randomized binary functions operating on all past observations of $\{(R_n, U_n)\}$. Thus, the control variable operating at the n -th slot U_n is expressed in terms of function $f_n(\cdot)$, where $U_n = f_n(R_{1,n}, U_{1,n-1})$, where $R_{1,n} = \{R_m, 1 \leq m \leq n\}$ and $U_{1,n} = \{U_m, 1 \leq m \leq n\}$, and $U_n \in \{0, 1\}$. Since only finite average packet delays are of interest, we need to consider only the subset of control disciplines $\mathcal{U}_E \subset \mathcal{U}$ which result in a positive-recurrent controlled Markov chain $\{R_n, n \geq 1\}$ with finite mean $\bar{R} < \infty$, where

$$\bar{R} = \lim_{N \rightarrow \infty} E(R_N) = \lim_{N \rightarrow \infty} N^{-1} E \left\{ \sum_{n=1}^N R_n \right\}, \quad (4.1)$$

and a packet probability of rejection P_R given by

$$P_R = \lim_{N \rightarrow \infty} N^{-1} E \left\{ \sum_{n=1}^N I(U_n = 1) \right\}. \quad (4.2)$$

The countable number of feasible control functions $\underline{u} \in \mathcal{U}_E$ induce the set $\mathcal{P}_R$ of values of attainable rejection probabilities. Thus,

$$\mathcal{P}_R = \{p: p = P_R(\underline{u}), \underline{u} \in \mathcal{U}_E\}, \quad (4.3)$$

where $P_R(\underline{u})$ denotes the rejection probability resulting under control function $\underline{u}$. The minimal attainable rejection probability is given by

$$P_R^0 = \inf\{p: p \in \mathcal{P}_R\}. \quad (4.4)$$

For $P_R > P_R^0$, we set

$$p(P_R) = \text{Max}\{p: p \in \mathcal{P}_R, p \leq P_R\}, \quad (4.5)$$

as the rejection probability closest (from below) to P_R . Given any value of rejection probability P_R , $P_R > P_R^0$, we wish to obtain the minimal attainable average packet waiting-time $\bar{W}^*(P_R)$, given as

$$\bar{W}^*(P_R) = \inf_{\underline{u} \in \mathcal{U}_E} \{\bar{W}(\underline{u}): P_R(\underline{u}) = p(P_R)\}, \quad (4.6)$$

where $\bar{W}(\underline{u})$ denotes the average packet waiting-time obtained when control function $\underline{u}$ is used. An optimal control function attaining waiting-time $\bar{W}^*(P_R)$ and yielding rejection probability P_R is denoted by $\underline{u}^*(P_R)$.

To generate the optimal delay-throughput curve of (4.6), as P_R varies between P_R^0 and 1, we can in turn derive function $\phi_W(\beta)$, as β varies in $[0, \infty)$, where

$$\phi_W(\beta) = \inf_{\underline{u} \in \mathcal{U}_E} \{\bar{W}(\underline{u}) + \beta P_R(\underline{u})\}. \quad (4.7)$$

Alternatively, since $\bar{W}$ is well-approximated by $\bar{W}_1$ of (3.33) or, as the contribution of $\Delta\bar{W}$ in (3.29) to $\bar{W}$ is generally small, we can derive function $\phi_R(\beta)$, given as

$$\phi_R(\beta) = \inf_{\underline{u} \in \mathcal{U}_E} \{\bar{R}(\underline{u}) + \beta P_R(\underline{u})\}, \quad (4.8)$$

where $\beta \geq 0$, and $\bar{R}(\underline{u})$ denotes the limiting average number of retransmissions

(4.1) under control function $\underline{u}$. Given β , let $\underline{u}_{-\beta}^*$ denote a control function attaining the minimum at (4.8). We have thus obtained the following result.

Proposition 2. For a GRA channel, curve $\phi_R(\beta)$ of Eq.(4.8), $\beta \geq 0$, induces the delay-throughput curve $\bar{W}^*(P_R)$ of Eq.(4.6), $P_R > P_R^0$. Thus, for each $\beta \geq 0$,

$$\bar{W}(\underline{u}_{-\beta}^*) = \bar{W}^*(P_R^*(\beta)), \quad (4.9)$$

where

$$P_R^*(\beta) = P_R(\underline{u}_{-\beta}^*), \quad (4.10)$$

so that

$$\underline{u}^*(P_R^*(\beta)) = \underline{u}_{-\beta}^*, \quad (4.11)$$

and $1 \geq P_R^*(\beta) > P_R^0$.

Proof. For $\beta \geq 0$, curve $\phi_W(\beta)$ of Eq. (4.7) induces curve $\bar{W}^*(P_R)$ of Eq.(4.6), since by (4.7) control function $\underline{u}_{-\beta}^*$ attains the minimal average waiting-time value $\bar{W}(\underline{u})$, considering all control functions $\underline{u}$ yielding $P_R(\underline{u}) = P_R(\underline{u}_{-\beta}^*)$. By Eq.(3.33) for $\bar{W}$, we note that $\bar{W}$ depends on $\bar{R}$ and P_R (through s , see Eq.(3.27)). Therefore, for any given P_R , $\bar{W}$ is minimized if and only if the corresponding $\bar{R}$ is minimized. Hence, curve $\phi_R(\beta)$ yields curve $\phi_W(\beta)$ and subsequently curve $\bar{W}^*(P_R)$. One readily verifies, as will be shown below, that varying β over $[0, \infty)$ covers the range $\{P_R : P_R > P_R^0\}$. Q.E.D.

Proposition 2 indicates that a solution to minimization problem (4.8) will yield the optimal delay-rejection probability curve $\bar{W}^*(P_R)$. As β increases, a solution to (3.8) will represent a scheme with a non-increasing rejection probability $P_R^*(\beta)$ and a non-decreasing average waiting-time function $\bar{W}^*(\beta) = \bar{W}(u_\beta^*)$. Subsequently, we obtain the desired curve $\bar{W}^*(P_R)$, and the optimal scheme $u^*(P_R) = u_\beta^*$ for some $\beta \geq 0$, $P_R > P_R^0$. Note also the role of β in (3.8) as a penalty cost for packet rejections.

Optimal control problem (4.6) has thus been represented in the form of a Markov Decision problem (4.8), described as follows. The stochastic process $\{R_n, n \geq 1\}$, with state-space $\mathcal{S} = \mathcal{I} = \{0, 1, 2, \dots\}$ is controlled by the binary control sequence $\{U_n, n \geq 1\}$, $U_n \in A = \{0, 1\}$. At time n , corresponding to the end of the n -th group B_n , state R_n is observed and action (control) $U_n = f(R_{1,n}, U_{1,n-1})$, is taken. Subsequently, a cost $C(R_n, U_n)$ is incurred, and the next state of the process is chosen according to transition probabilities $\{P_{ij}(U_n)\}$. Thus, the controlled process $\{R_n, n \geq 1\}$ transition probability function satisfies

$$P\{R_{n+1}=j | R_{1,n}, U_{1,n}, R_n=i, U_n=a\} = P_{ij}(a), \quad (4.12)$$

where $a \in A = \{0, 1\}$. Under an expected average cost criterion (see, for example, [10]-[12]), an admissible control function $u \in \mathcal{U}_E$ is chosen to minimize the long-run expected average cost per unit time. For control policy u , the associated cost function is then given by

$$\phi_u(i) = \lim_{N \rightarrow \infty} \sup N^{-1} E_u \left\{ \sum_{n=1}^N C(R_n, U_n) | R_1=i \right\}, \quad (4.13)$$

for $i \in \mathcal{I}$, where $E_{\underline{u}}$ indicates that the conditional expectation given $\underline{u}$ is used. A control function $\underline{u}^*$ is said to be average cost optimal if

$$\phi_{\underline{u}^*}(i) = \min_{\underline{u} \in \mathcal{U}_E} \phi_{\underline{u}}(i), \quad \text{all } i. \quad (4.14)$$

Incorporating Eqs. (4.1)-(4.2) in (4.8), we obtain

$$\phi_R(\beta) = \inf_{\underline{u} \in \mathcal{U}_E} \lim_{N \rightarrow \infty} N^{-1} E_{\underline{u}} \left\{ \sum_{n=1}^N [R_n + \beta U_n] \right\}, \quad (4.15)$$

since $I(U_n=1) = U_n, n \geq 1$. Comparing (4.15) with (4.13)-(4.14), we conclude the following result.

Proposition 3. The optimal control policy $\underline{u}_{\beta}^*$ yielding $\phi_R(\beta)$ for a GRA channel, for any $\beta \geq 0$, is an average cost optimal control policy for the Markov decision process $\{(R_n, U_n), n \geq 1\}$, under the long-run expected average cost per unit time measure (4.13), with an associated cost function $C(R_n, U_n)$ given by

$$C(R_n, U_n) = R_n + \beta U_n. \quad (4.16)$$

Results from Markov decision theory are incorporated in the following analysis to obtain the structure of optimal policy $\underline{u}_{\beta}^*$, under cost measure (4.13) and cost function (4.16). Note from (4.16) that following the n -th period B_n , when R_n is observed and policy $U_n \in \{0,1\}$ is chosen, the associated cost function is given by their linear combination weighted by

$$\beta, C(R_n, U_n) = R_n + \beta U_n.$$

A particularly important subclass of control disciplines is the class $\mathcal{U}_S \subset \mathcal{U}$ of stationary control policies. A control function u is said to be stationary if it is nonrandomized and it is described by a mapping $f(\cdot): \mathcal{P} \rightarrow A$, so that $U_n = f(R_n)$. Thus, under a stationary control function, the control U_n at time n depends only on the present observation R_n . Such a control procedure is clearly easily implementable. We note that, under a stationary control function $f(\cdot)$, the controlled process $\{R_n, n \geq 1\}$ becomes a Markov chain with the transition probability functions $\{P_{ij}(f(\cdot))\}$.

It will be useful in the following analysis to consider the above optimal control problem for $\{(R_n, U_n), n \geq 1\}$, also under an expected total discounted cost measure. The latter is given, for a control policy u , by

$$V_u(i) = \limsup_{N \rightarrow \infty} E_u \left\{ \sum_{n=1}^N \alpha^n C(R_n, U_n) \mid R_1 = i \right\}, \quad (4.17)$$

for $i \geq 0$ and a discount factor $\alpha \in (0, 1)$. We let

$$V_\alpha(i) = \inf_{u \in \mathcal{U}} V_u(i), \quad i \geq 0, \quad (4.18)$$

denote the minimal expected discounted cost function. A control policy u^* is said to be α -optimal if

$$V_{u^*}(i) = V_\alpha(i), \quad \text{for all } i \geq 0. \quad (4.19)$$

The Optimal Control Policy

We wish to derive and characterize the optimal control policy $u_{-\beta}^*$ for the Markov decision process $\{(R_n, U_n), n \geq 1\}$, under cost measure (4.15), or (4.13), (4.16). We note that the corresponding cost values become unbounded for control $u \notin \mathcal{U}_E$. At the same time, the simple control function $u_1 \in \mathcal{U}_E$ which rejects all arrivals clearly yields $P_R(u_1)=1$, $\bar{R}(u_1)=0$ and subsequently

$$\phi_{u_1}^*(i) = \phi_R(\beta) \leq \phi_{u_1}(i) = \beta. \quad (4.20)$$

Therefore, the search for an optimal control policy can be reduced to subclass $\mathcal{U}_E$. The same conclusion will be observed to hold under cost measure $V_\alpha(i)$, when α is taken to be close enough to 1.

We will first establish that an optimal policy $u_{-\beta}^*$ in fact exists, and that it is furthermore a stationary policy. The structure of the optimal policy will then be characterized. For that purpose, we first consider the α -discounted cost problem (4.18), and subsequently study the characteristics of the α -optimal policies. The policy obtained as a limit of the α -optimal policies is then shown to yield the desired optimal control function $u_{-\beta}^*$.

It is well known that a stationary α -optimal policy $u_{-\beta}^*(\alpha)$ for (4.18) exists whenever cost function $C(\cdot)$ is bounded (see [10]). However, for our problem R_n , and therefore $C(\cdot)$, are unbounded. It has however been shown in [12] that an optimal stationary deterministic policy for the α -discounted cost problem exists if the following conditions are met. The existence is required of an integer $m \geq 1$, a real-valued function $g(\cdot)$ on

$\mathcal{P}$ with $g(x) \geq 1$ for all $x \in \mathcal{P}$ and a real number $b \geq 0$ such that

$$L = \sup_{x \in \mathcal{P}} \{ \max_{a \in A} C(x, a) | g(x)^{-m} \} < \infty, \quad (4.21a)$$

and for all x , $n=1, 2, \dots, m$,

$$\max_{a \in A} \sum_{y \in \mathcal{P}} g(y)^n p_{xy}(a) \leq [g(x) + b]^n. \quad (4.21b)$$

We now verify that conditions (4.21) hold for our problem and subsequently deduce the following result.

Lemma 1. For a controlled GRA channel, an optimal stationary (deterministic) control policy $u_{\beta}^*(\alpha)$ exists for the α -discounted cost problem (4.18), $\alpha \in (0, 1)$. The minimal discounted cost $V_{\alpha}(i)$ is the unique solution to

$$\begin{aligned} V_{\alpha}(i) &= \min_{a \in A} \{ C(i, a) + \alpha \sum_{j \geq 0} V_{\alpha}(j) P_{ij}(a) \} \\ &= \min_{a \in A} \{ i + \beta a + \alpha \sum_{j \geq 0} V_{\alpha}(j) P_{ij}(a) \}, \end{aligned} \quad (4.22)$$

the functional equation of dynamic programming. Furthermore, $u_{\beta}^*(\alpha)$ is the policy which selects an action minimizing the right side of (4.22) for each $i, i \geq 0$.

Proof. The results follow by [12], once conditions (4.21) are verified. For that purpose, we choose $g(i) = \max(i, 1)$, $i \geq 0$, and $m=1$. Then we

obtain

$$L = \sup_{i \geq 0} \{ \max_{a \in A} [i + \beta a] g(i)^{-1} \} = \sup_{i \geq 1} [1 + \beta i^{-1}] = 1 + \beta < \infty ,$$

verifying (4.21a). For Eq.(4.21b), we obtain

$$\begin{aligned} \max_{a \in A} [\sum_{j \geq 0} g(j) P_{ij}(a)] &= \sum_{j \geq 0} g(j) P_{ij}(0) \\ &= \sum_{j \geq 0} j P_{ij}(0) + P_{i0}(0) \leq i + K\lambda + 1 \leq g(i) + K\lambda + 1 , \end{aligned}$$

for $i \geq 0$, using upper-bound (3.14).

Q.E.D.

It follows from Lemma 1 (see [12]) that the optimal discounted cost $V_\alpha(i)$ can be computed by the Policy Improvement Algorithm presented in Corollary 1. (See also [13] for discussions concerning policy computations using a policy improvement algorithm.)

Corollary 1. For a GRA channel, the α -discounted minimum cost function $V_\alpha(i)$, $i \geq 0$, is given by

$$V_\alpha(i) = \lim_{n \rightarrow \infty} V_\alpha(i, n) , \quad (4.23)$$

where $V_\alpha(i, 0) = 0$ for each $i, i \geq 0$, and $V_\alpha(i, n+1)$, $n \geq 0$, is computed by the recursive relationship

$$V_{\alpha}(i, n+1) = \min_{a \in \{0,1\}} \{i + \beta a + \alpha \sum_{j \geq 0} V_{\alpha}(j, n) P_{ij}(a)\} . \quad (4.24)$$

We can now proceed to study the average cost problem, using the results in Lemma 1 and Corollary 1.

Theorem 4. For a GRA channel, a stationary (deterministic) optimal control policy u_{β}^* exists, yielding a minimum cost $\phi_{u^*}(i) = \phi_R(\beta) = \phi_{u^*}(0)$ independent of $i, i \geq 0$, satisfying

$$\phi_R(\beta) = \phi_{u^*}(i) = \lim_{\alpha \uparrow 1} (1-\alpha)V_{\alpha}(i) = \lim_{\alpha \uparrow 1} (1-\alpha)V_{\alpha}(0) . \quad (4.25)$$

Proof. Incorporating Lemma 1 and Corollary 1, the proof proceeds following a similar procedure to that used in [14]. Considering the α -discounted cost problem, we readily verify (noting that u_{β}^* yields a positive-recurrent controlled chain) that

$$\overline{\lim}_{\alpha \uparrow 1} (1-\alpha)V_{\alpha}(i) = \overline{\lim}_{\alpha \uparrow 1} (1-\alpha)V_{\alpha}(0) . \quad (4.26)$$

Furthermore, using Eq.(4.24), we will observe that there exists a stationary policy $\hat{u}$ and an increasing sequence $\{\alpha_n\}$ with $\alpha_n \uparrow 1$ such that

$$\phi_{\hat{u}}(i) = \overline{\lim}_{n \rightarrow \infty} (1-\alpha_n)V_{\alpha_n}(i) , \quad (4.27)$$

for all $i, i \geq 0$. By an Abelian theorem (see [14],[15]) and conditions (4.21) for $i \geq 0$ and $u \in \mathcal{U}$, we obtain that

$$\phi_{\hat{u}}(i) \geq \overline{\lim}_{\alpha \uparrow 1} (1-\alpha)V_{\alpha}(i) . \quad (4.28)$$

Therefore, using relations (4.26)-(4.28) and sequence $\{\alpha_n\}$ used in (4.27), we conclude that for any $\underline{u} \in \mathcal{U}$, $i \geq 0$, we have

$$\phi_{\underline{u}}(i) \geq \phi_{\underline{u}}^{\wedge}(i) = \phi_{\underline{u}}^{\wedge}(0), \quad (4.29)$$

and therefore

$$\phi_R(\beta) = \phi_{\underline{u}^*}(i) = \phi_{\underline{u}}^{\wedge}(0), \quad (4.30)$$

and

$$\underline{u}_{\beta}^* = \hat{\underline{u}}. \quad (4.31)$$

More explicitly, we will note (see Lemma 2) by iterating Eq.(4.24) and observing the resulting policies that, for a large enough value of α , $\alpha > \alpha_{\beta}$, the stationary optimal control function $\underline{u}_{\beta}^*(\alpha) = \hat{\underline{u}}_{\alpha}$ is such that $\underline{u}_n(i)=1$ for any $i \geq M$, for some finite integer M . Therefore, we can choose a sequence $\{\alpha_n\}$ with $\alpha_n \uparrow 1$ yielding a stationary control policy $\underline{u}_{\beta}^*(\alpha_n) = \hat{\underline{u}}$, each n , where $\hat{\underline{u}}$ is the scheme derived from $\hat{\underline{u}}_{\alpha}$ as $\alpha \uparrow 1$. Subsequently, using (4.28), we obtain for $\underline{u} \in \mathcal{U}$, $i \geq 0$,

$$\phi_{\underline{u}}(i) \geq \lim_{n \rightarrow \infty} (1 - \alpha_n) V_{\hat{\underline{u}}}^{\wedge}(i, \alpha_n) = \phi_{\hat{\underline{u}}}^{\wedge}(i), \quad (4.32)$$

with the equality in (4.32) followed by an Abelian theorem ([15]) stating that

$$\lim_{\alpha \rightarrow 1} (1 - \alpha) \lim_{N \rightarrow \infty} \sum_{n=1}^N \alpha^n C_n = \lim_{N \rightarrow \infty} N^{-1} \sum_{n=1}^N C_n.$$

Therefore, $\underline{u}_{\beta}^* = \hat{\underline{u}}$.

Q.E.D.

Theorem 4 shows that an optimal control policy u_{β}^* exists, and can be chosen to be a stationary control function, denoted by $u_{\beta}(i)$, $i \geq 0$. Furthermore, the optimal control scheme can be obtained by solving for the α -discounted optimal scheme, using recurrence relationship (4.24), and then letting $\alpha \rightarrow 1$. We will use this procedure to establish the structure of the optimal scheme. Its performance function is then computed using the results presented in Section III.

We will not present here the tedious algebraic details related to performing iteration procedure (4.24), but will indicate the relevant associated results. To illustrate and explain the iteration results, we assume first that $\bar{R}_i(a)$, $a=0,1$, given by Eqs.(3.13) can be approximated by a straight line given as

$$\bar{R}_i(a) \cong ci + d_1 + d_2(1-a), \quad (4.33)$$

where $a=0,1$, $i \geq 0$, and $c > 0$, $d_2 > 0$ and d_1 are appropriate constants. We note by Eqs.(3.13) and Fig. 3.2 that, for $i \geq K$, Eq.(4.33) yields an excellent approximation to $\bar{R}_i(a)$. The slope function $\Delta_i(a) = \bar{R}_{i+1}(a) - \bar{R}_i(a)$ is also noted to increase monotonically, as i increases for $a=0,1$, to a value larger than 1 and, only at very high values of i , $\Delta_i(a)$ decreases monotonically to its asymptotic value of 1, $\lim_{i \rightarrow \infty} i^{-1} \Delta_i(a) = 1$. Therefore, slope c in approximation (4.33) actually varies with i , as indicated above. Applying Eq.(4.33) to compute $V_{\alpha}(i,n)$ following recurrence relationship (4.24), we note that $V_{\alpha}(i,n)$ is obtained to depend linearly on i . Subsequently, relation (4.33) can be reapplied in (4.24) to obtain $V_{\alpha}(i,n+1)$ from $\{V_{\alpha}(j,n), j \geq 0\}$. We then obtain $V_{\alpha}(i,n)$ to be given by, for $\alpha < 1$,

$$V_{\alpha}(i,n) = A(i,\alpha,n) + \min_{a \in A} a[\beta - D(\alpha,c,n)] , \quad (4.34a)$$

where

$$D(\alpha,c,n) = \alpha d_2 \frac{1 - (\alpha c)^{n-2}}{1 - \alpha c} . \quad (4.34b)$$

Consider now the set of states $\{i: i \geq 2K\}$. We have noted above that $L_1(a) > 1$ for $i > K$, $a=0,1$. Furthermore, we have

$$P_{ij}(a) = 0 , \quad \text{for } j \leq i-K , \ i > K , \ a=0,1 , \quad (4.35)$$

since, at most, $K-1$ successful transmissions can occur when more than K transmissions are attempted. Therefore, in carrying iteration (4.24) under assumption (4.33), we can incorporate the observation that

$$\sum_{j \geq 0} \bar{R}_j(a) P_{ij}(a) = \sum_{j \geq i-K+1} \bar{R}_j(a) P_{ij}(a) , \quad (4.36)$$

when $i \geq 2K$. Consequently, in evaluating $V_{\alpha}(i,n)$ by (4.24) for $i \geq 2K$, only functions $\{\bar{R}_j(a), j \geq K\}$ are involved. Since the latter functions have a slope larger than one, we conclude that $D(\alpha,c) = \lim_{n \rightarrow \infty} D(\alpha,c,n)$ becomes arbitrarily large for α close enough to 1; and subsequently $[\beta - D(\alpha,c)]$ is noted to become negative, resulting in an optimal control function satisfying the following property.

Lemma 2. For a GRA channel, for any $\beta \geq 0$, there exists an integer $M < \infty$, such that the α -discounted optimal stationary control, for any

α , $1 > \alpha > \alpha_\beta$, for some α_β , denoted by $u_\beta(\alpha, i)$, satisfies

$$u_\beta(\alpha, i) = 1, \quad \text{for } i \geq M, \quad \alpha > \alpha_\beta. \quad (4.37a)$$

The stationary optimal control policy $u_\beta(i)$ similarly satisfies

$$u_\beta(i) = 1, \quad \text{for } i \geq M. \quad (4.37b)$$

Furthermore, $M \leq 2K$.

For $0 \leq i < 2K$, the appropriate corresponding slope c increases monotonically from a small value, so that $D(\alpha, c, n)$ is replaced by a function $D(\alpha, i, n)$ which is increasing monotonically in i . Thus, letting $n \rightarrow \infty$, iteration (4.24) is observed to yield function $V_\alpha(i)$ described as

$$V_\alpha(i) = A(i, \alpha) + \min_{a \in A} a[\beta - D(\alpha, i)]. \quad (4.38)$$

Function $D(\alpha, i)$ is increasing monotonically in i , for $i < M$, where M is an appropriate finite integer. For $i \geq M$, any $\beta \geq 0$, we have

$$D(\alpha, i) > \beta, \quad \text{for } \alpha > \alpha_\beta, \quad i \geq M, \quad (4.39)$$

for some α_β close enough to 1 so that

$$\lim_{\alpha \rightarrow 1} D(\alpha, i) < \infty \quad \text{if and only if } i < M. \quad (4.40)$$

Properties (4.39)-(4.40) are incorporated in Lemma 2. Using expressions

(4.38)-(4.40) , we can thus deduce the character of the optimal control policy as summarized in Theorem 5.

Theorem 5. For $\beta \geq 0$, the optimal control function u_{β}^* attaining $\phi_R(\beta)$ is characterized as a single-threshold control function $u_{\beta}^*(i)$ given by

$$u_{\beta}^*(i) = u_S(i - K_1(\beta)) = \begin{cases} 1, & \text{if } i \geq K_1(\beta) \\ 0, & \text{if } i < K_1(\beta) \end{cases}, \quad (4.41)$$

where $u_S(\cdot)$ is the unit-step function. Threshold $K_1(\beta)$ satisfies

$$\begin{aligned} 0 < K_1(\beta) < M, & \quad \text{for } \beta_0 < \beta < \beta_{max} \\ K_1(\beta) = M, & \quad \text{for } \beta \geq \beta_{max} \\ K_1(\beta) = 0, & \quad \text{for } \beta \leq \beta_0, \end{aligned} \quad (4.42)$$

where

$$\beta_{max} = \lim_{\alpha \rightarrow 1} D(\alpha, M-1), \quad (4.43a)$$

$$\beta_0 = K\lambda [1 - \exp(-\lambda)], \quad (4.43b)$$

and $M \leq 2K$ is the integer appearing in Lemma 2.

Proof. Eq.(4.41) follows, by (4.38), the monotonicity of $D(\alpha, i)$ for $i < M$ and relation (4.39) or (4.40) , for $i \geq M$. These relationships also yield (4.42) for $\beta > \beta_0$. To prove that $K_1(\beta) = 0$ for $\beta \leq \beta_0$, so that

$u_{\beta}^*(i)=1$, each i , we note that if $c_n(a)$ denotes the average cost in the n -th period when $u_n=a$, we clearly have

$$c_n(0) - c_n(1) \geq \beta_0 - \beta \geq 0,$$

since β_0 expresses the average number of collisions, during a period, due to new arrivals. Therefore, we attain $\phi_R(\beta)=\beta$ by rejecting all transmissions, and $K_1(\beta)=0$. Q.E.D.

Theorem 5 characterizes the stationary optimal control policy $u_{\beta}^*(i)$ as a simple single-threshold scheme. For $\beta \leq \beta_0$, the optimal scheme rejects all arrivals, yielding a packet rejection probability equal to one, $P_R=1$, and thus an average waiting-time $\bar{W}=0$. As β is increased, the threshold $K_1(\beta)$ of the associated optimal single threshold scheme is increased, to yield a lower P_R value and the corresponding minimal $\bar{R}$ value, and subsequently minimal average packet waiting-time $\bar{W}$ value. However, for any value of β not smaller than β_{max} , $\beta \geq \beta_{max}$, the same optimal single-threshold scheme, with threshold $K_1(\beta)=M$, is obtained. The latter scheme yields a packet rejection probability $P_R=P_R^0$ which is clearly equal to the minimal attainable P_R value. Using Proposition 2, the optimal control scheme has thus been shown to be governed by the following characteristics.

Corollary 2. For a GRA channel, there exists a stationary optimal control scheme $\hat{u}_p(i)$ which yields the minimal $\bar{R}$ and $\bar{W}_1$ values, under a prescribed packet rejection-probability $P_R=p$, where $1 \geq p \geq P_R^0$. Such a

scheme assumes a single threshold structure, given by

$$\hat{u}_p(i) = \begin{cases} 1, & \text{if } i \geq \hat{K}(p) \\ 0, & \text{if } i < \hat{K}(p) \end{cases} \quad (4.44)$$

Threshold $\hat{K}(p)$ increases monotonically from $\hat{K}(1)=0$ to $\hat{K}(P_R^0)=M$, as p is decreased from 1 to P_R^0 . Rejection probability P_R^0 , attained by the single threshold scheme with threshold M , is the minimum attainable such probability for optimization problem (4.8). |

It is interesting to observe that, as the threshold K of single-threshold scheme (4.44) is increased from 0 to M , the packet rejection probability and average waiting-time are increased and decreased, respectively. However, a further increase of the threshold above M in such a scheme only increases P_R while also increasing $\bar{W}$, and is therefore avoided. This phenomenon is explained by noting that, for such a scheme with a threshold higher than M , the time gained to threshold upcrossing is more than offset by the extra time required for threshold downcrossing, since a larger number of successful transmissions is required now.

Performance Results

The performance curve $\bar{W}^*(P_R)$ of the optimal single-threshold scheme can now be computed using Eqs.(3.29) or (3.33), with control function (4.44) incorporated in Fig. 3.1 to yield the necessary statistics of $\{R_n\}$ (being just s and $\bar{R}$ if (3.33) is used). The latter have been computed by a straightforward simulation of Markov chain $\{R_n, n \geq 1\}$ under control function (4.44). The resulting performance curves are shown in Figs. 4.1-4.2

for $K=12$ and in Figs. 4.3-4.4 for $K=9$. In both cases, we set $R=12$. Average packet delay (D) vs. packet probability of rejection (P_R) curves are shown in Figs. 4.1, 4.3. For a fixed value of λ , $\lambda=0.2, 0.3, 0.4$, we note the variation of D vs. P_R as the threshold K_1 of a single-threshold scheme is increased from $K_1=3$ to $K_1=30$. The characteristics of the optimal schemes as stated in Theorem 4 are well demonstrated in these figures. Note that the minimal probability of rejection for the GRA channel with $K=12$ is equal to a very small number (measured 0) for $\lambda=0.1, 0.2$, and is $P_R^0=0.004$ for $\lambda=0.3$ and $P_R^0=0.114$ for $\lambda=0.4$. For $K=9$, $P_R^0=0.002$ for $\lambda=0.3$, $P_R^0=0.1$ for $\lambda=0.4$, and P_R^0 is very small for $\lambda \leq 0.2$. (Note that, as the Markov chain simulation is run for a large but finite number of slots, no threshold crossings would occur for low λ values and high K_1 values, thus accounting for the form of the curves shown for low λ values.) For $K=12$, the minimal P_R values (P_R^0) are attained at thresholds $K_1 \approx 16$ and $K_1=12$ for $\lambda=0.3$ and $\lambda=0.4$, respectively. For $K=9$, the thresholds attaining P_R^0 are $K_1 \approx 18$ and $K_1 \approx 9$ for $\lambda=0.3$ and $\lambda=0.4$, respectively. It can be noted that a scheme with a threshold $K_1=K$ yields an excellent D vs. P_R performance over the whole range of network traffic intensities (including generally any λ , $0 \leq \lambda \leq 0.4$) yielding a rejection probability not higher than $P_R \approx 0.1$. We further note that a threshold value $K_1=M$ yielding a minimal probability of rejection would not cause an average packet delay much higher than a threshold value yielding a much higher P_R value. Therefore, it is generally preferable to assign a threshold value of $K_1 \approx M$ to the GRA channel controller. For example, for $K=12$, $\lambda=0.3$, for threshold values $K_1=3, 6, 12$, we obtain P_R values of $P_R=0.162, 0.062, 0.01$ and delay (D)

values of $D=22, 25, 29$ (slots), respectively.

The associated delay (D) vs. throughput curves are shown in Figs. 4.2, 4.4. Note that, for $K=12$, the maximal throughput value of $s=e^{-1}$ is attained at $\lambda=0.8$ by a scheme with threshold $K_1=6$, yielding therefore (at this traffic value) a rejection probability value of $P_R=1-(\lambda e)^{-1} \approx 0.54$. For $K=9$, $s=e^{-1}$ is attained at $\lambda=0.6$ by a scheme with $K_1=7$, yielding $P_R=1-(\lambda e)^{-1} \approx 0.39$. It is noted that over the (practical) throughput range $0 \leq s \leq 0.3$, the average packet delay varies slowly, and nearly linearly, from $D=13(=R+1)$ at $s=0$, to $D \approx 20$ at $s=0.2$, to only $D \approx 25$ at $s=0.3$, at both $K=12$ and $K=9$ GRA schemes and any threshold value K_1 , with $3 \leq K_1 \leq K$.

Other Control Schemes for a GRA Channel

When the GRA discipline is governed by a distributed control procedure, applied over a broadcast channel, the process $\{R_n, n \geq 1\}$ many times cannot be observed by the individual terminals. Then terminals generally can observe, in each slot, only whether a successful transmission or collisions have occurred. In the latter case, the terminal obtains no information as to the number of collisions involved. Thus, the process observed by each terminal is given as $\{(S_n, C_n), n \geq 1\}$, where S_n denotes the number of successful transmissions within the n -th group B_n , and C_n gives the total number of slots in B_n experiencing collisions, and is therefore given by

$$C_n = \sum_{j=1}^K C_n^{(j)} = \sum_{j=1}^K I(N_n^{(j)} \geq 2), \quad (4.45)$$

for $n \geq 1$, where $\{N_n^{(j)}\}$ is given by Eq.(3.4c). Note that $\{(S_n, C_n), n \geq 1\}$

is not a Markov chain. An optimal control procedure incorporating the latter observation chain can be developed in a manner similar to that presented above. However, due to the special character of the underlying state sequence, we can readily make the following observations.

We note that $R_n \geq 2C_n$. Within the range of acceptable packet delay values, we further expect each collision to involve an average number of transmissions which is very close to 2, and is lower than 3. Thus, we should have $R_n \approx rC_n$ within this range, with $r \approx 2$. When $R_n > rC_n$, the number of group collisions rapidly increases and subsequently higher, generally unacceptable, packet delay values are obtained. Therefore, estimating $\{R_n, n \geq 1\}$ by $\{\hat{R}_n = rC_n, n \geq 1\}$, we can employ the optimal single-threshold scheme developed above. The latter scheme, denoted by $\hat{C}$, now uses observations of $\{C_n, n \geq 1\}$ and a threshold K_2 . We thus expect this scheme to exhibit a delay-throughput curve very close to that obtained by the optimal scheme which uses $\{R_n\}$ observations and threshold K_1 , and serves as a lower-bound to the performance curve of $\hat{C}$, with $K_2 = r^{-1}K_1$.

Performance points for a single-threshold scheme $\hat{C}$ are shown in Figs. 4.1-4.2, for $K=R=12$. The results completely verify the above-mentioned observations. The optimal performance points for $\hat{C}$ are all noted to lie on the lower-bound performance curves for the scheme using $\{R_n\}$ observations. Furthermore, scheme $\hat{C}$ with threshold $K_2 = K_1/2$ is noted to attain performance curves very close to those obtained by the optimal scheme using $\{R_n\}$ observations and threshold K_1 , $K_1 \leq M$. Only when the latter scheme has utilized a threshold $K_1 > M$, we note that scheme $\hat{C}$ would utilize threshold $K_2 > 2K_1$. The latter situation, however,

represents a range of undesirable threshold values, since schemes using lower threshold values yield lower packet delays under the same P_R values (see Fig. 4.1). We thus conclude that a single-threshold scheme using $\{C_n\}$ observations operates as well as the optimal single-threshold scheme using $\{R_n\}$ observations within the acceptable range of $D-P_R$ values.

The methods presented and used in this paper can be applied to study a variety of other related GRA access-control disciplines. For example, in certain applications we might wish to reject certain colliding packets rather than new transmissions. We can thus study a GRA scheme with a 0-1 control function $\{U_n\}$ which uses $\{R_n\}$ or $\{C_n\}$ observations and rejects, at appropriate times, all collisions within the corresponding period. The performance analysis for such a scheme follows that presented in Sections III-IV. In particular, we can note that the associated Markov decision problem involves now the cost function $C(R_n, U_n) = R_n [1 + \beta U_n]$. The resulting optimal single-threshold scheme, denoted by C_2 , is readily shown to have similar performance characteristics to those indicated in Theorem 5 and Corollary 2. Performance points for a C_2 scheme for $\lambda=0.3$, $K=R=12$ are shown in Fig. 4.1. The average packet delay value here incorporates both successful and rejected transmissions. We note that this scheme yields lower packet delay values at higher rejection probabilities ($P_R \geq 0.05$) when compared with the previous scheme. For rejection probabilities $0.014 \leq P_R \leq 0.05$, comparable packet delays are attained by both schemes. Scheme C_2 , however, yields a minimal probability of rejection $P_R^0 = 0.014$, while the previous scheme yields $P_R^0 = 0.004$. The latter is thus preferable at lower P_R values.

V. Conclusions

We have presented and studied Group Random-Access access-control disciplines for a multi-access communication channel. A GRA scheme uses only certain channel time-periods, during which some network terminals attempt to transmit their information-bearing packets, on a random-access basis. The channel can thus be utilized at other times to grant access to other terminals, or other message types.

To stabilize the GRA channel, an appropriate dynamic control procedure is applied. The state of the underlying channel state sequence is observed by each terminal and subsequently, within certain periods, no transmissions are allowed. During these periods, newly arriving packets are thus rejected. The performance of a dynamically controlled GRA channel is characterized in terms of the average packet delay (D) and the packet probability of rejection (P_R), or network throughput(s). The optimal control policy, yielding the minimal average packet delay under a prescribed rejection probability, is derived and characterized by studying the associated Markov decision process. A Markov ratio limit theorem is used to evaluate the packet average waiting-time function.

The performance results presented here demonstrate the excellent delay-throughput performance of a GRA channel, over the acceptable range of traffic intensities. We further note that a threshold value $K_1=M$, yielding the minimal probability of rejection, is many times a good choice. Furthermore, if only the sequence $\{C_n, n \geq 1\}$, indicating the number of colliding slots within a group, can be observed, as is generally the case for distributed control broadcast channels, we have shown that a corresponding single-threshold scheme $\hat{C}$ with threshold $K_2 \approx K_1/2$ yields a nearly

optimal delay-throughput performance. Other control schemes are noted to be governed by similar characteristics, and analyzed using similar methods. As for a slotted-ALOHA (SA) random-access procedure, the GRA scheme is shown to allow a maximal throughput of e^{-1} . We also note the delay-throughput characteristics of a controlled GRA channel to be similar to those of an appropriately controlled SA channel. A GRA access-control procedure, however, allows for a much higher degree of dynamic and efficient utilization of a multi-access channel which utilizes integrated random-access, reservation and fixed access-control procedures, or utilizes the GRA scheme only to provide channel access to certain protocol packets.

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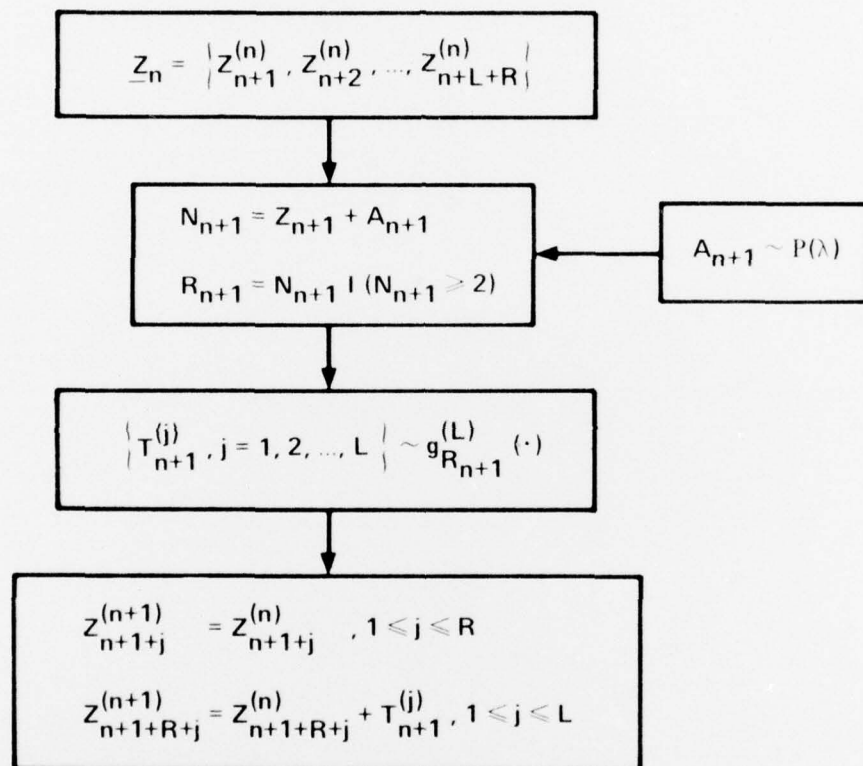


Fig. 2.1 The Transition $Z_n \rightarrow Z_{n+1}$ for the Channel State Process Z Under the Regular (Slotted ALOHA) Random Access Procedure

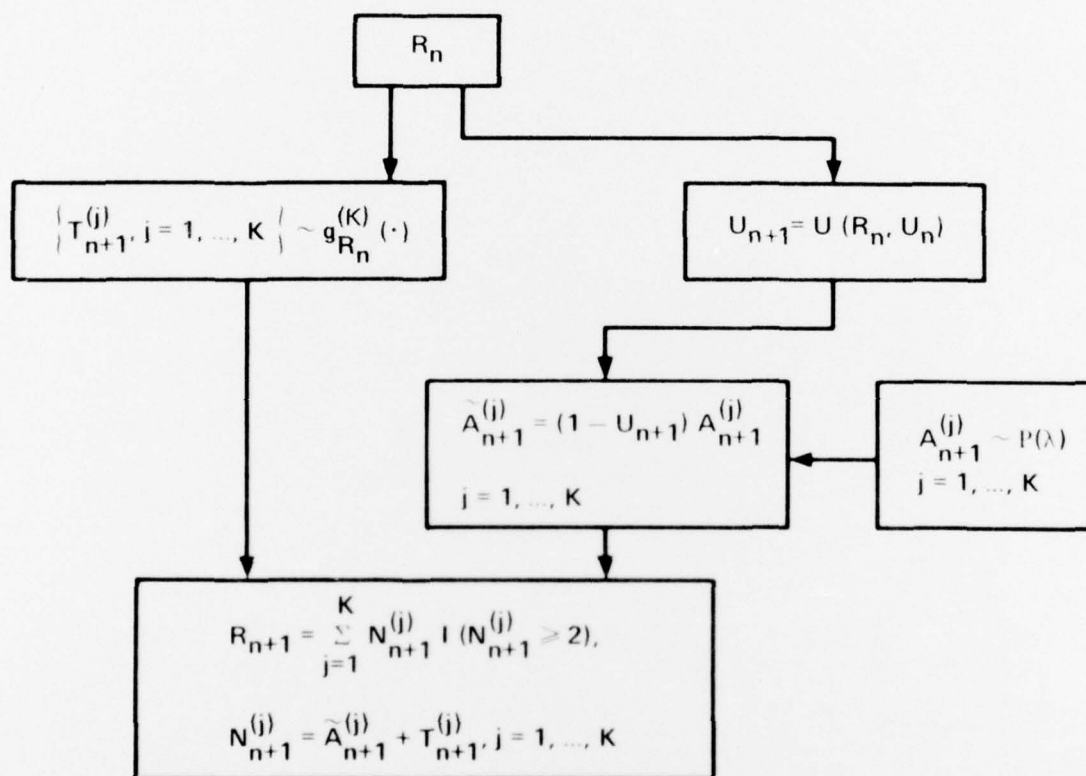


Fig. 3.1. Transition $R_n \rightarrow R_{n+1}$ for the Markov State Chain $\underline{R}$ Associated with a GRA Discipline

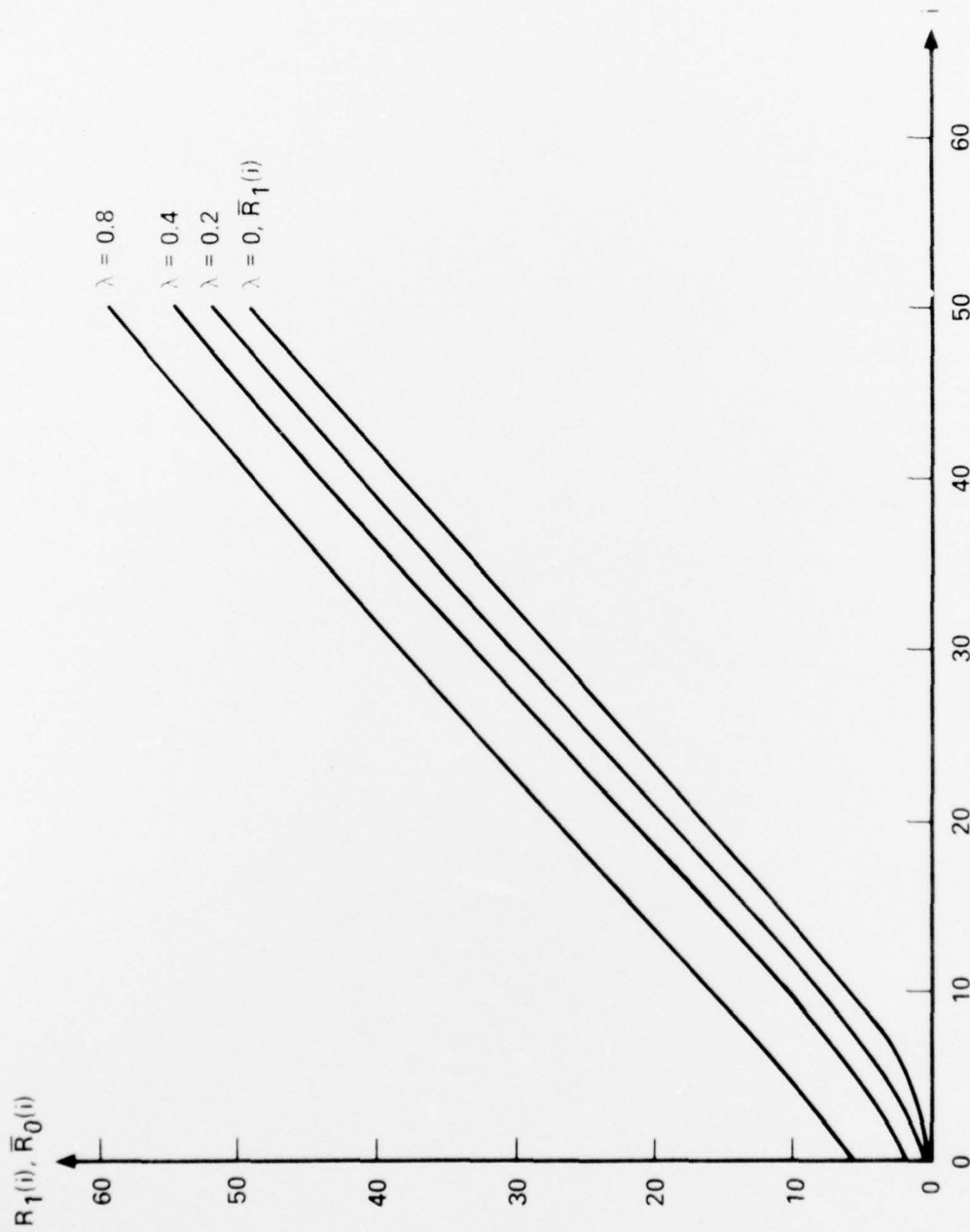


Fig. 3.2. Curves $\bar{R}_0(i)$ vs i , with Parameter λ , where $\bar{R}_0(i) = \bar{R}_1(i)$ for $\lambda = 0$, for $K = 12$

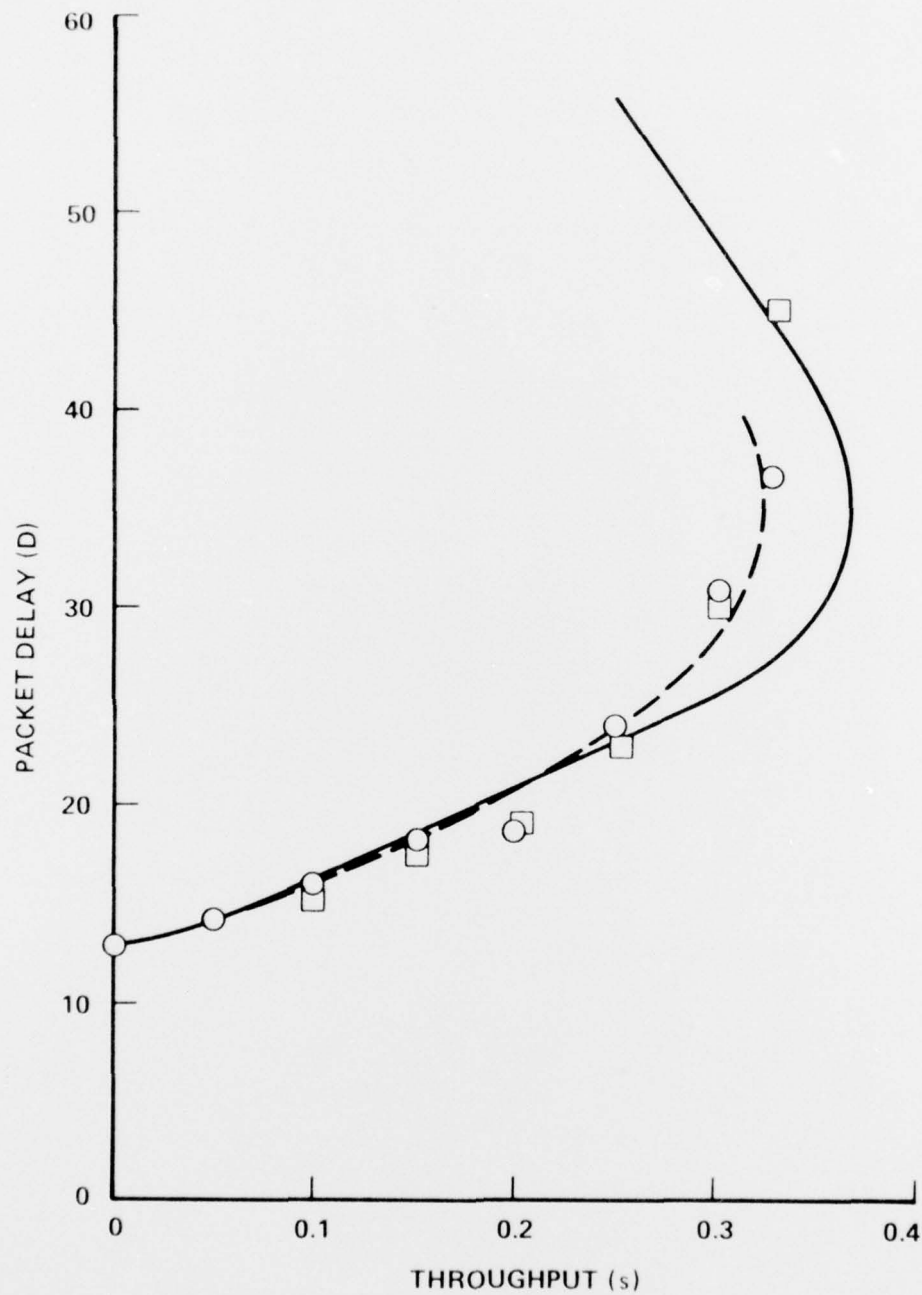


Fig. 3.3. Delay-Throughput Curves for GRA and SA Schemes Under an Input-Control with Threshold $2K$. Curves — — and — are for a GRA Scheme with $K = 6$ and $K = 12$, Resp. Points $\square$ and $\circ$ are for a SA Scheme with $L = K = 12$ and $L = K = 6$, Resp.

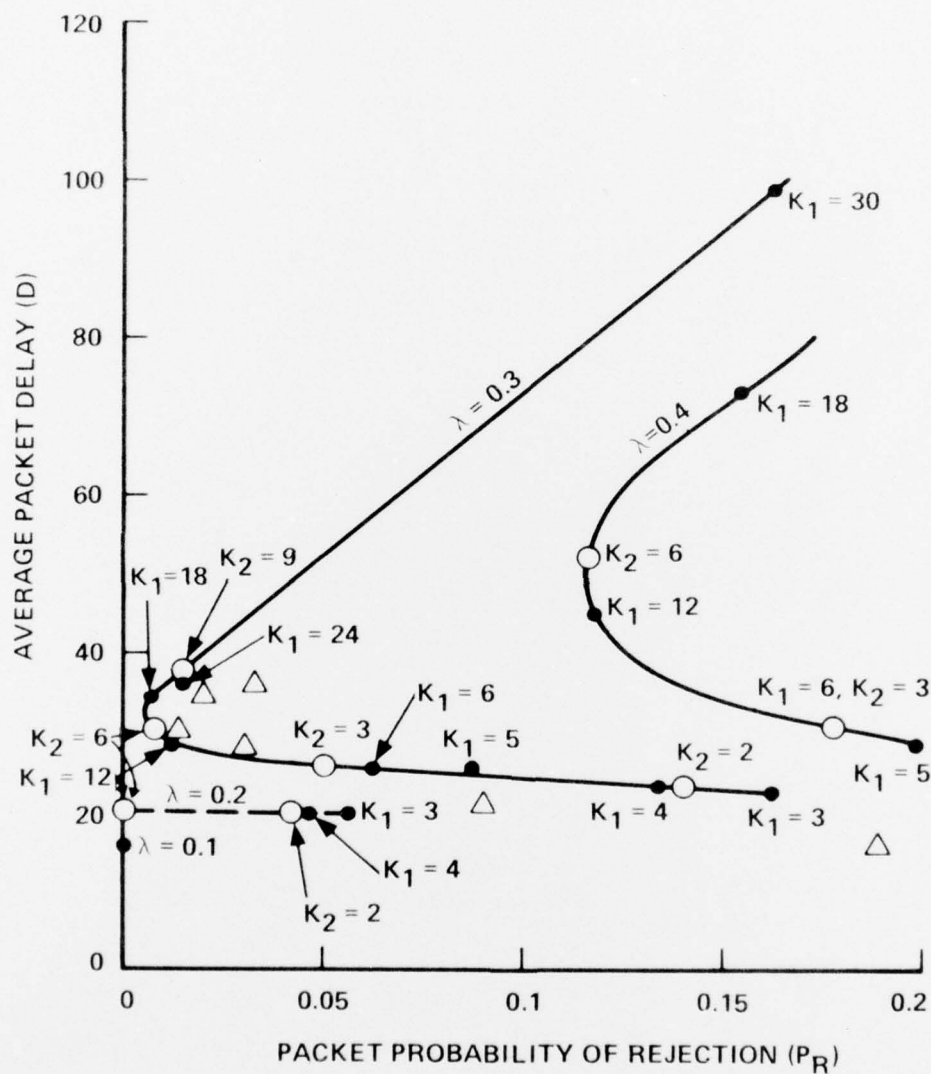


Fig. 4.1. Delay vs Probability of Rejection Curves for a GRA Channel with $K = R = 12$ Under a Single Threshold Control Scheme with Threshold K_1 . Constant λ Curves with Parameter K_1 are Shown. Also Shown are Performance Points ○ for Scheme $\hat{C}$ with Threshold K_2 , and Points △ for Scheme C_2 for $\lambda = 0.3$

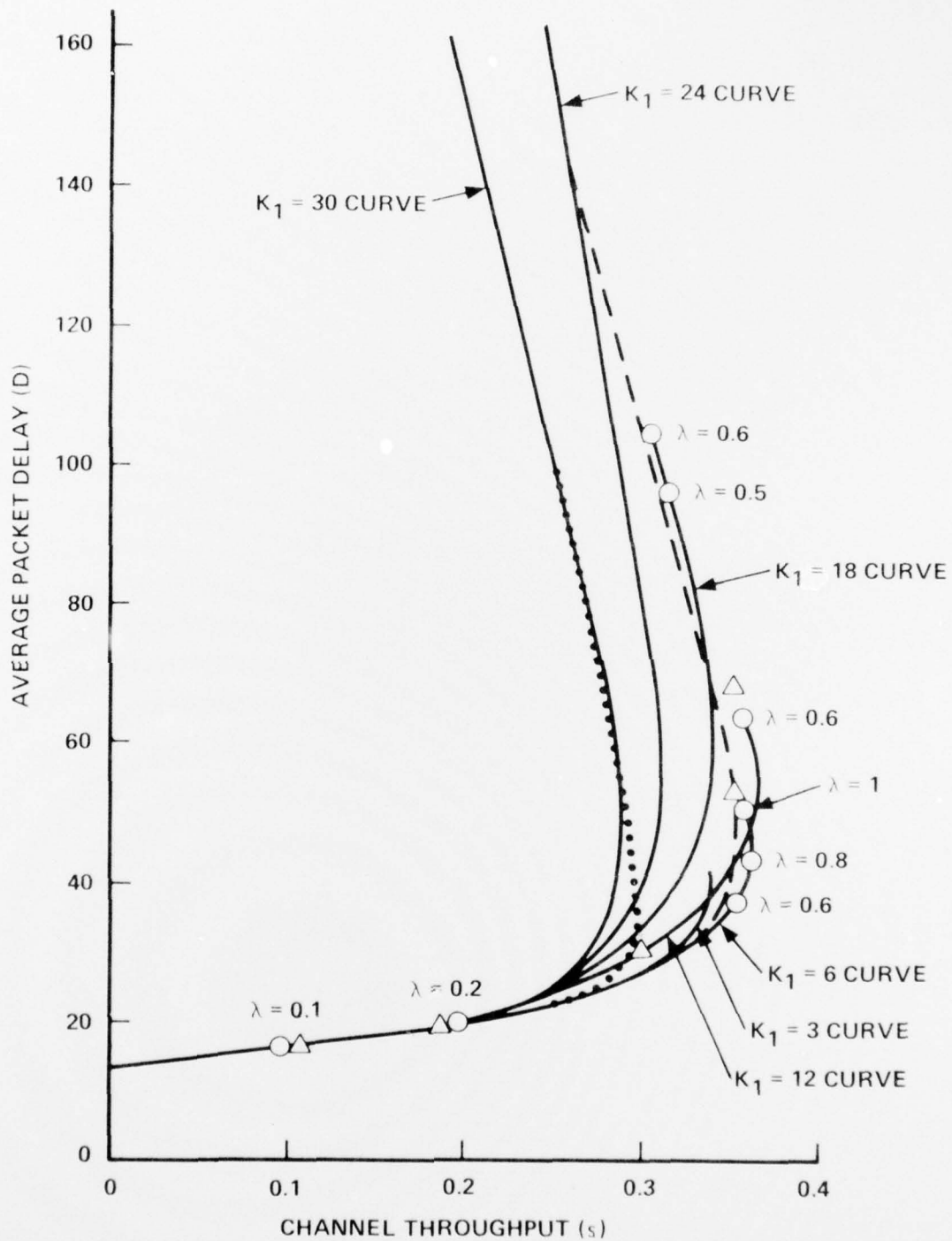


Fig. 4.2. Delay vs Throughput Curves for a GRA Channel with $K = R = 12$ Under Single-Threshold Schemes with Threshold K_1 and Parameter λ . Also Shown constant $-\lambda$ Curves for $\lambda = 0.3$ (.....) and $\lambda = 0.4$ (---), and Performance Points $\triangle$ for Scheme $\hat{C}$ with Threshold K_2

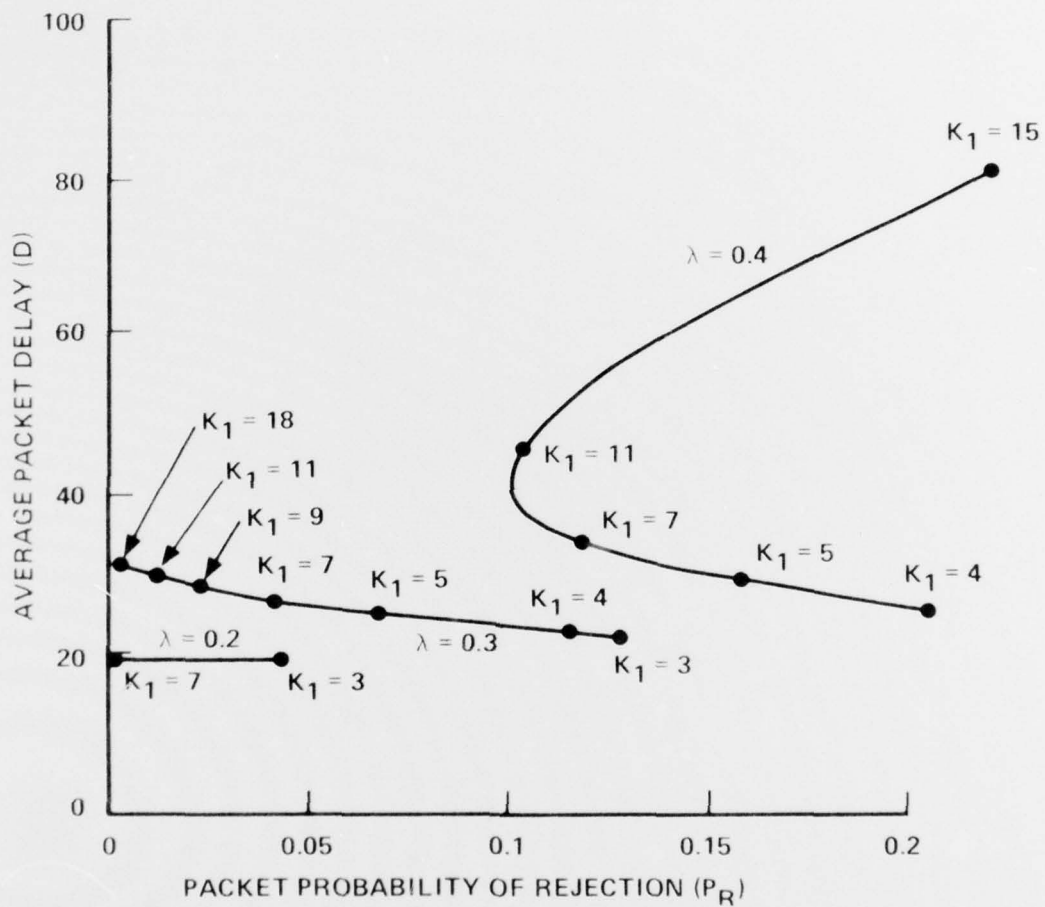


Fig. 4.3. Delay vs Probability of Rejection Curves for a GRA Channel with $K = 9$, $R = 12$, Under a Single Threshold Scheme with Threshold K_1 . Constant λ Curves with Parameter K_1 are Shown

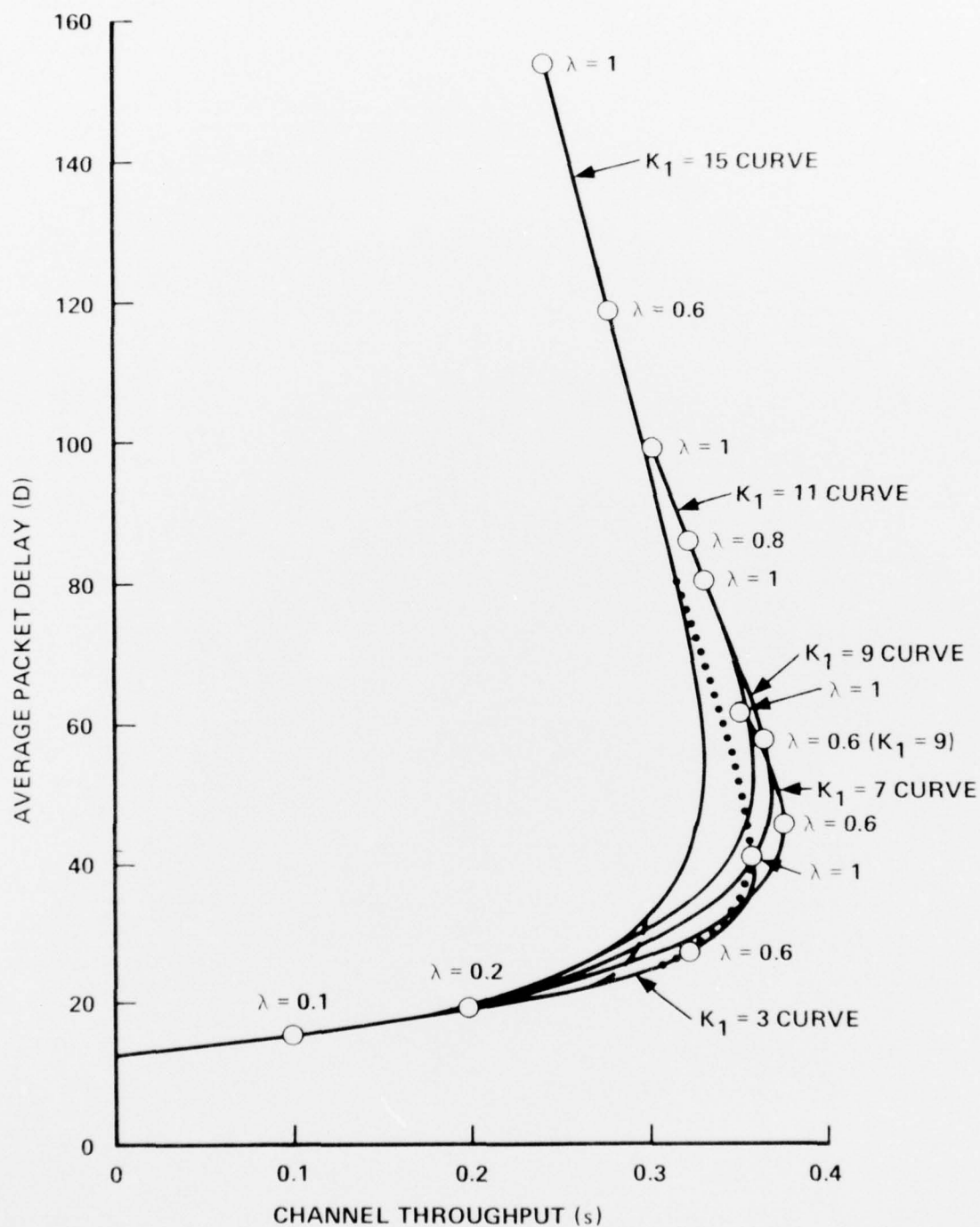


Fig. 4.4. Delay vs Throughput Curves for a GRA Channel with $K = 9$, $R = 12$, Under Single-Threshold Schemes with Threshold K_1 and Parameter λ . Also Shown Constant λ Curves for $\lambda = 0.3$ (---) and $\lambda = 0.4$ (.....)