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A MULTI-LEVEL COHERENCE MODEL FOR EEO PLANNING.(U)

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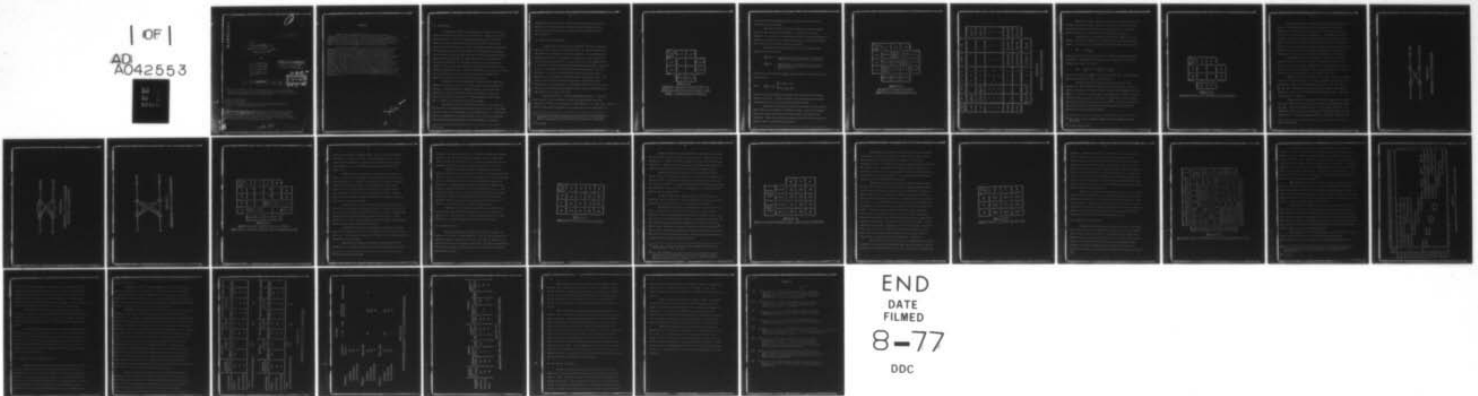
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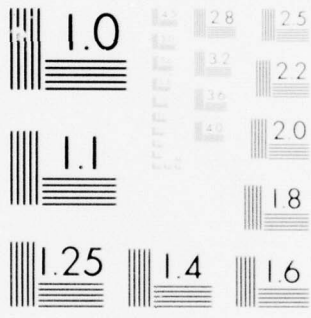
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A MULTI-LEVEL COHERENCE MODEL  
FOR EEO PLANNING

by

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ABSTRACT

2nd of 2

This is the second of two general classes of models for guiding EEO planning in the U.S. Navy's civilian workforce. The first, <sup>model -- FEEB</sup> called the FEEB (Flexible Equal Employment Opportunity) Model, provides overall guidance for such plans; The present one, <sup>Model --</sup> called the Coherence Model, is designed for more detailed application in ways which tie into FEEB. Together they <sup>use</sup> therefore provide a multi-level approach.

Both models utilize <sup>use</sup> a goal programming approach with embedded <sup>Markov</sup> Markoff processes. The Coherence Model embodies a new usage of piecewise linear goal functionals with "artifact goals" to approximate the transition relations of the Markoff processes. Other relations or constraints as well as objectives can be treated in this manner to thereby obtain a distribution format with a nonlinear functional (with linear equivalent) for which available current algorithms guarantee integer solutions. The mathematical development needed to achieve this reduction is provided along with a numerical (two-period) example which illustrates the dynamic (multi-period) aspects that need to be considered in such plans. This is followed by discussion and examples of the kinds of management reporting systems that might be utilized. The methods developed here are perfectly general, of course, and so other potential applications of this model to areas like personnel promotion planning, etc., are also briefly noted.

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## I. INTRODUCTION

An earlier paper [7] set forth an Equal Employment Opportunity (EEO) Model for Manpower Planning in the U.S. Navy. Consisting of Markoff process elements embedded in a goal programming framework, this might better be called a Flexible Equal Employment Opportunity (FEEO) Model because of the way in which the elements are altered to provide flexibility for achieving day-to-day manpower goals of the organization. The latter, i.e., the short range goals, are to be met "as closely as possible," in the sense of "goal programming."<sup>1/</sup> While this is being done, however, the transition probabilities of promotion, transfer, etc., are also to be altered so that, in time, the "correct" mixes of personnel--male, female, minority, non-minority, etc.-- will be achieved (also as closely as possible), but in ways which are consistent with accomplishing the first set of (day-to-day) working goals in the best possible manner.

The model we have just described is intended for comprehensive guidance at aggregate levels in the civilian manpower planning efforts of the Navy. Something further is required, however, when dealing with individual or "almost individual" assignments at the micro levels of local installations and operations -- such as shipyards, laboratories, etc. -- and this is the purpose of the present paper.

The model we shall now describe is directed to these ends, but in ways which are coherent with the earlier aggregative model. As we shall see, this involves the development of a new nonlinear model. The latter in turn is transformed in a way which provides access to the extraordinarily fast and efficient algorithms that are now available for capacitated

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<sup>1/</sup> See [2].

distribution-assignment models. This, we may observe, opens the way for developing interactive or conversational computer capabilities for use by management in guiding manpower planning efforts at both micro and macro levels.<sup>2/</sup>

## II. NOTATION AND MODEL DETAILS

Leaving aside, at least for the moment,<sup>3/</sup> the ways in which the macro and micro models are tied into each other, we turn to the details of the latter in the development that we now introduce. We are looking at a coherent transition from an aggregative dynamic model. Our requirements for personnel of each category  $\alpha$  (e.g., black male, white female) in job  $i$  in period  $t$  are here derived by proportionality factors  $p_i^\alpha$  for category  $\alpha$  in job  $i$  of the amount  $a_i(t)$  detailed by the aggregate model. The model structure we are about to set down applies to any determinate mode of derivation of these personnel requirements from the aggregative data. The further internal Markoff transitions from job to job will not appear explicitly in the model structure. Instead they will be accommodated within the format of a capacitated distribution model by means of "artifact goals" and nonlinear convex goal functionals.

Refer now to the  $2 \times 2$  table represented in Figure 1. The cells are intended to represent blocks of cells with distributions being made from typical  $i^{\text{th}}$  jobs in one period to typical  $j^{\text{th}}$  jobs in the next period. Where one has the "same" period represented in row and column, we are emphasizing the

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<sup>2/</sup> For further discussion of the uses of interactive-conversational computer capabilities in manpower planning and related activities, see Niehaus, Sholtz, and Thompson [8] and also Albanese, Niehaus, and Padalino [1].

<sup>3/</sup> See below.

TO FROM	t=1	t=2	
t=0			$p_i^\alpha a_i(0)$
t=1			$p_i^\alpha a_i(1)$
	$p_j^\alpha a_j(1)$	$p_j^\alpha a_j(2)$	

FIGURE 1  
PERSONNEL-JOB PROPORTIONS  
(PERIOD-TO-PERIOD DEVIATIONS  
FROM AGGREGATE MODEL)

so-called "transshipment" device which is used to go from network to distribution model format.<sup>4/</sup>

One further example may serve to clarify the general notation and structure we are about to introduce. In Figure 2 we indicate quantities on the rims of a 3-period model in which a typical distributional variable is entered into each unshaded cell block. The cell blocks which are shaded are excluded from receiving any entries.

To understand the entries in Figure 2 and to deal with the general case as well we let

$$\begin{aligned}x_{ij}^{\alpha}(t-1, t) &= \text{number of personnel of category } \alpha \text{ transferring} \\ &\quad \text{from job } i \text{ in period } t-1 \text{ to job } j \text{ in period } t. \\ x_{jj}^{\alpha}(t, t) &= \text{the "transshipment" amounts, e.g., the number of} \\ &\quad \text{personnel in category } \alpha \text{ not transiting into} \\ &\quad \text{job } j \text{ in period } t.\end{aligned}$$

We complete the definition of  $x_{ij}^{\alpha}(t_1, t_2)$  for arbitrary time period  $t_1$  and  $t_2$  by setting

$$(1.2) \quad x_{ij}^{\alpha}(t_1, t_2) = \begin{cases} 0 & \text{for } t_1 \neq t_2 - 1 \\ 0 & \text{for } t_1 = t_2, i \neq j. \end{cases}$$

In this way we have provided the notation for the general dyadic format that we now develop. In this development only the diagonal and immediate sub-diagonal blocks of cells can contain non-zero entries.

We are now in a position to present the structure for our general dyadic format over N periods as in Figure 3, below. Here we have omitted the shadings since the possible non-zero variables have already been explained. Observe that, as before, only the diagonal and immediately sub-diagonal cells can receive non-zero entries.

TO FROM	t=1	t=2	t=3	
t=0	$x_{ij}^{\alpha} (0,1)$			$p_i^{\alpha} a_i (0)$
t=1	$x_{jj}^{\alpha} (1,1)$	$x_{ij}^{\alpha} (1,2)$		$p_i^{\alpha} a_i (1)$
t=2		$x_{jj}^{\alpha} (2,2)$	$x_{ij}^{\alpha} (2,3)$	$p_i^{\alpha} a_i (2)$
	$p_j^{\alpha} a_j (1)$	$p_j^{\alpha} a_j (2)$	$p_j^{\alpha} a_j (3)$	

FIGURE 2  
ILLUSTRATIVE 3 PERIOD  
DISTRIBUTIONAL EXAMPLE

TO FROM	t=0	t=1	t=2		t=N-1	t=N
t=0	$\alpha$ $x_{ij} (0,1)$	$\alpha$ $x_{ij} (1,1)$	$\alpha$ $x_{ij} (1,2)$	$\alpha$ $x_{ij} (1,1)$	$\alpha$ $x_{ij} (1,1)$	$\alpha$ $x_{ij} (1,1)$
t=1	$\alpha$ $x_{jj} (1,1)$	$\alpha$ $x_{jj} (1,1)$	$\alpha$ $x_{jj} (1,2)$	$\alpha$ $x_{jj} (1,1)$	$\alpha$ $x_{jj} (1,1)$	$\alpha$ $x_{jj} (1,1)$
t=N-2	$\alpha$ $x_{ij} (N-2,1)$	$\alpha$ $x_{ij} (N-2,1)$	$\alpha$ $x_{ij} (N-2,2)$	$\alpha$ $x_{ij} (N-2,1)$	$\alpha$ $x_{ij} (N-2,1)$	$\alpha$ $x_{ij} (N-2,1)$
t=N-1	$\alpha$ $x_{jj} (N-1,1)$	$\alpha$ $x_{jj} (N-1,1)$	$\alpha$ $x_{jj} (N-1,2)$	$\alpha$ $x_{jj} (N-1,1)$	$\alpha$ $x_{jj} (N-1,1)$	$\alpha$ $x_{jj} (N-1,1)$
	$\alpha$ $p_i a_j (0)$	$\alpha$ $p_i a_j (1)$	$\alpha$ $p_i a_j (2)$	$\alpha$ $p_i a_j (N-1)$	$\alpha$ $p_i a_j (N)$	$\alpha$ $p_i a_j (N)$

FIGURE 3  
N-PERIOD DYADIC FORMAT

For clarity, Figure 3 omits rows and columns which will be needed (cf. Figure 12) to accommodate transfers to and from the "outside" as well as redundant rim data which will be needed to accommodate the nonlinear goal functions when we finally proceed to our distribution format.

In the model we are developing, the transfers  $x_{ij}^{\alpha}(t-1, t)$  are to conform "as closely as possible" to an organization's promotion transition experience (or policy) -- here represented by the Markoff transition matrix

$$(2) \quad M = (M_{ij}),$$

where  $M_{ij}$  equals the proportion in job  $i$  in period  $t-1$  transiting to job  $j$  in period  $t$ . To develop our "artifact goals," which must be in terms of numbers of personnel, we employ

$$(3) \quad g_{ij}^{\alpha}(t-1, t) = \langle r_i^{\alpha} a_i(t-1) M_{ij} \rangle$$

to guide our choice for  $x_{ij}^{\alpha}(t-1, t)$  where appropriate.<sup>5/</sup> Here the notation  $\langle u \rangle$  means the smallest integer not less than  $u$ .

We shall generally be concerned with convex artifact goal functionals. The structure and algorithmic solution procedures which we shall be utilizing must guarantee integer solutions for the  $x_{ij}^{\alpha}(t-1, t)$  and  $x_{jj}^{\alpha}(t, t)$ . Therefore we shall employ as our convex goal functions continuous piecewise linear functionals with changes of slope only at integer points. Thus using results from some of our past work,<sup>6/</sup> we can set up an equivalent (but larger) ordinary distribution model thereby preserving all of the advantages that such models now provide.

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<sup>5/</sup> For example, except for those variables not corresponding to Markoff transitions.

<sup>6/</sup> See [3], Chapter XVII.

TO	3	4	
FROM			
1			a
2			b
	$\frac{2(a+b)}{3}$	$\frac{(a+b)}{3}$	

FIGURE 4  
DYADIC FORMAT: ORIGINAL PROBLEM

It may be helpful here to consider the model format in Figure 12. Because of its small size, letters O, C, T, A rather than numbers are employed to mnemonically designate the "jobs" as follows: O = outside, C = clerical, T = technical, A = administrative. The cells corresponding to Markoff transitions have their "artifact goals" (as in (3)) designated within the double lines in the northeast corner of the cell. Transshipment cells appear in the southwest portion of the array and the northeast (shaded) portion exemplifies the first restriction of equation 1.2, namely, that no transitions are permitted from one period to a period beyond the next.

To simplify the present discussion, we shall omit the Markoff transition elements to O from C, T or A. We shall also employ only two pieces in our piecewise linear goal functional. We shall then exhibit our procedure for a 2 x 2 (two origin - two destination) distribution example. The extensions to the general  $m \times n$  case will then be clear.

Figure 4 below provides such a 2 x 2 example. It has the network representation shown in Figure 5. As shown in [3] <sup>7/</sup>, to accommodate a two-piece linear (convex) functional on the branches from (1) and (2) to (3) and (4), we double the number of branches and place an individual upper bound (indicated by  $\overline{\quad}$ ) on the flow in half of them. This moves us from Figure 5 to Figure 6.

We next reduce the network in Figure 6 to an equivalent transshipment form, as in Figure 7 where the transshipment modes are represented by (3') and (4'). The corresponding dyadic form is then shown in Figure 8. The individual bounds which were represented by  $\overline{\quad}$  in the network are now indicated by  $\backslash$  in the corresponding cell. The cells (3', 3') and (4', 4')

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<sup>7/</sup> Cf. Chapter XVIII.

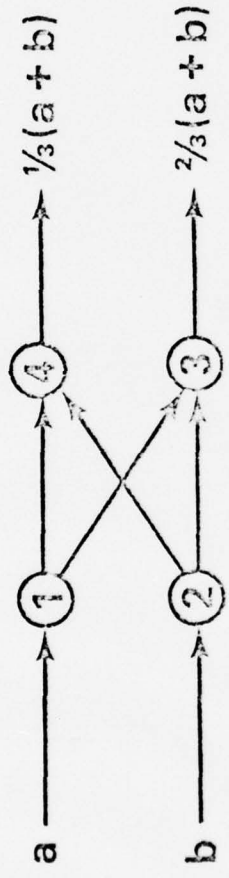


FIGURE 5  
NETWORK REPRESENTATION

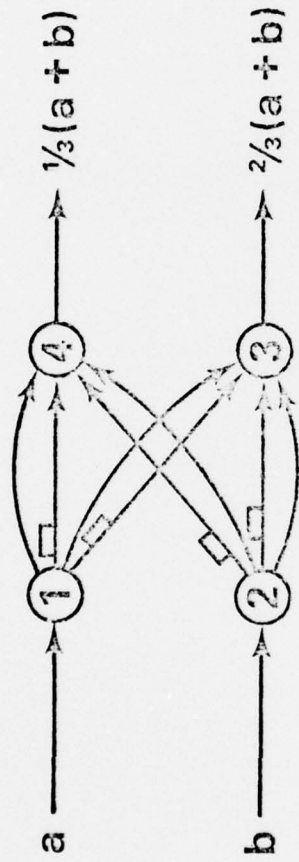


FIGURE 6

NETWORK WITH NON LINEAR  
FUNCTIONAL ELEMENTS

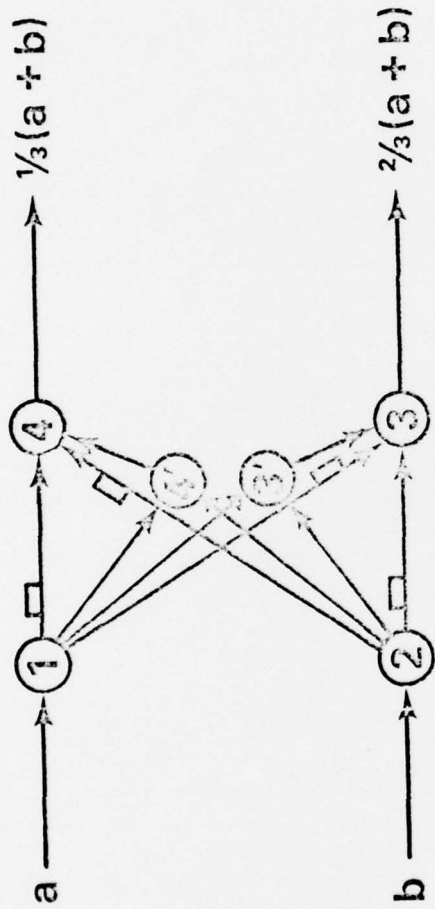


FIGURE 7  
 NETWORK WITH TRANSHIPMENTS

TO FROM	3	3'	4	4'	
1	/		/		a
2	/		/		b
3'			/	/	$\frac{2}{3}(a+b)$
4'	/	/			$\frac{1}{3}(a+b)$
	$\frac{2}{3}(a+b)$	$\frac{2}{3}(a+b)$	$\frac{1}{3}(a+b)$	$\frac{1}{3}(a+b)$	

**FIGURE 8**  
**EXPANDED DYADIC FORMAT WITH**  
**BOUNDS AND TRANSHIPMENT POSSIBILITIES**

are the corresponding transshipment cells. In this manner we have moved from Figure 4 to Figure 8 which is also in distribution format. The capacitated and transshipment cells in the latter <sup>§</sup> figure now accommodate the nonlinearities in a way which guarantees integer solutions from integer rims.

The general rule is now clear. We augment each destination which is to receive two-piece nonlinear (but piecewise linear) functional flows by introducing a "primed" destination and we augment the origins, one for each primed destination. The availabilities at the primed origins are those of the original destinations; the receipts at the primed destinations are those at the original destinations. We allow flow in only two cells of each primed origin row: the (primed, original) cell and the (primed, primed) cell.

If our dyadic form is  $m \times n$ , then the new dyadic form will be  $(m + h) \times (n + h)$ , where  $h$  is the number of primed origins or destinations. We observe that an "active basis" <sup>8/</sup> can always be taken to contain all the (primed, primed) cells and all the (primed, original) cells. In this designation of a cell, the first element of the pair denotes row (or origin) and the second denotes column (or destination).

To establish what we have just said, note first that an active basis for the original form has  $m + n - 1$  members and one for the new form has  $m + n - 1 + 2h$  members. This means that there are  $2h$  of (primed, primed) and (primed, original) cells.

Observe that if there are no (original, primed) cells in the active basis, it represents a solution for the original dyadic form, hence

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<sup>8/</sup> See [3], Chapters XV and XVI.

it contains an active basis of  $m + n - 1$  members of (original, original) cell type. The  $2h$  cells of (primed, original), (primed, primed) then connect properly via original columns to make up an active basis for the new dyadic form since the number of members is now  $m + n - 1 + 2h$ .

Observe further that since the functionals are convex, an (original, original) cell is always better than an (original, primed) cell. Thus, an (original, primed) cell need be employed only if its corresponding (original, original) cell is at its upper bound and hence not in the active basis. Therefore, the total number of (original, original) and (original, primed) cells in the active basis remains at  $m + n - 1$ . The connection needed for basis spanning is still at hand with the  $2h$  cells of (original, original), (primed, original) type but in the sequence (original, primed), (primed, primed), (primed, original) in place of (original, original), (primed, original), (primed, primed).

These last remarks generate a new modification of algorithmic procedure (e.g., special primal methods) which can speed computations substantially. Here, however, we direct our attention only to the FEOO Model which, as noted in section I, is the focus for this paper.

### III. NUMERICAL EXAMPLE

We shall consider two categories  $\alpha = 1, 2$ , (e.g., females and males) and  $t = 0, 1, 2$ . For job categories we let  $i, j = 0, 1, 2, 3$ , where the last three indices correspond to Clerical, Technical, and Administrative and 0 corresponds to Outside (the organization). For ease of comprehension in our simple example, we shall use C, T, A, and O, however, instead of the numerical indices.

TO FROM	N	C	T	A
C	.26	.7	.03	.01
T	.15	0	.8	.05
A	.13	0	.02	.85

FIGURE 9  
EXAMPLE OF MARKOV MATRIX

Figure 9 provides the pertinent numerical values for the elements of M, the Markoff matrix we shall use in this example. Figure 10 supplies the  $p_j^{\alpha}$  values. Figure 11 provides the  $a_i(t)$  for each of the indicated jobs and periods. Finally, Figure 12 designates the nonlinear goal programming model with constraints represented in distribution-assignment format.

The details in Figure 9 conform to the characteristic Markoff matrix conventions in which each row consists of non-negative elements that sum to unity.<sup>9/</sup> The letter "N" designates the column of transition rates for "natural attrition." We distinguish between this and "forced attrition," the latter being designated by "O." As noted above, the symbol "0" stands for "outside the organization."

In Figure 10 the actual  $p_j^{\alpha}$  proportions are obtained from the "on-board" starting population. The desired  $p_j^{\alpha}$  proportions represent policy statements concerning desired mixes of personnel for the future.

The  $a_i(t)$  in Figure 11 are obtained from the aggregate FEEO model, where, as already noted, i indexes the job type in C, T, or A, while t designates the period designated for the aggregated workforce goals -- i.e., the personnel goals established by reference to the tasks to be performed in each relevant category and period. Combining these  $a_i(t)$  values with the  $p_j^{\alpha}$  from Figure 10 produces the amounts shown on the lines of Figure 12.<sup>10/</sup> The location of nonlinear goal functional elements is indicated in the latter figure by the cells with numerical entries representing goal values enclosed in double lines in the upper right hand corner.

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<sup>9/</sup> In this way we depart from the conventions employed in some of our earlier reports. See, e.g., [4].

<sup>10/</sup> E.g.,  $525 = .75(700)$  is the goal for females in the Clerical category for period 1, as established by reference to the pertinent cells of Figures 10 and 11, while  $158 = .35(450)$  represents the goal targeted for females in the Technical category.

		C	T	A
ACTUAL PROPOR- TIONS	FEMALE	.89	.20	.40
	MALE	.11	.80	.60
DESIRED PROPOR- TIONS	FEMALE	.75	.35	.45
	MALE	.25	.65	.55

FIGURE 10  
EXAMPLE OF PERSONNEL—JOB PROPORTIONS

Recall that these nonlinear functional elements were introduced for the purpose of producing a change of format from one with a complex matrix structure to one with the simple structure of a dyadic or distribution model. We thereby replaced the original constraining relations by "artifact goals" in nonlinear goal functions to secure approximate equivalence between the more complex embedded Markoff constraint structure in our original goal programming FLEO model, and the simpler constraint structures of our new distribution type model.

To better understand the significance of these "artifact goals" we refer to the numbers that are represented as entries within the double lines in the upper right-hand corners of some of the cells in Figure 12. For specific reference we may use the value 420 which appears within the Clerical (C) cells going from  $t=0$  to  $t=1$ . To obtain this value we proceed as follows. Via the Markoff matrix of Figure 9 we first observe that 70% of all Clerical personnel will remain in that category from one period to another. Moreover, there is no inflow into Clerical from either the Technical or Administrative categories. Hence, we apply the pertinent transition probabilities in a very simple way to the period 0 total of 675 persons in this category from Figure 11 to obtain  $472.5 = .7(675)$ . Then to obtain the expected flow of female personnel in this category, we refer to Figure 10, where we observe that .89 is the relevant proportion to use. Hence, we secure the wanted  $420 = .89(472.5)$  as the pertinent value for entry within the double lines of this cell just as in Figure 12.

This 420 is the "artifact goal" and it is employed, as previously elaborated, in the form of an individual upper bound in accordance with our procedures for handling piecewise linear convex functionals in distribution

i \ t	0	1	2
C	675	700	650
T	875	450	400
A	225	200	200

FIGURE 11  
TARGETED WORKFORCE GOALS

problems. In this case the artifact goal corresponds to the normal flow value between the periods  $t=0$  and  $t=1$  for female employment in the Clerical category. Here the artifact goal represents the normal flow of female personnel into this category. This device may also be used to project management requirements explicitly for other purposes as well, of course, but at the expense of enlarging the dyadic format. On the other hand, recent developments in algorithmic combinations and computer codes make the computational costs of such elaborations relatively inexpensive, and so we may regard this kind of modeling strategy as suited also to these possible further uses and extensions.

The array of Figure 12 must be transformed, in a manner analogous to our example in going from Figure 4 to Figure 8, in order to obtain an equivalent distribution assignment format with linear functionals. This is done in Figure 13. Note the use of the "N columns" in the latter figure to distinguish "natural (or Markoff) attrition" from the "forced attrition" which is designated in the "0 columns." The designation of 0 for rows corresponds to "outside" sources for additional personnel.

#### IV. SOLUTION AND INTERPRETATION

As mentioned initially, one of the major reasons for developing this model in the form of a distribution-assignment type was to guarantee that solutions would be in integers. It follows that any extreme point method of solution will produce integer results. For this example we have employed the "stepping stone method" along with the row-column number procedures described in Chapter XIV in [3], which in turn have been incorporated as parts of the high speed computer codes developed in [6] and [7].

$\alpha = 1$		t = 1				t = 2				
		O	C	T	A	O	C	T	A	
t II 0	O	0								200
	C		420	18	6					300
	T		X	140	9					175
	A		X	2	75					90
t II 1	O	0				0				153
	C		0				300	15	9	525
	T			0			X	120	8	153
	A				0		X	2	75	90
		300	525	153	90	200	400	140	90	

FIGURE 12  
DYADIC FORMAT FOR NUMERICAL ILLUSTRATION

The numerical entries in the cells of Figure 13 correspond to a solution, as may be verified by summing to the rim totals. The row-column sum "numbers" are represented on the left and at the top of Figure 13 via their symbolic values of 0, H,  $-P+H-1$ , etc., as discussed above. Checking these values against the vacant cells and those indicated with bars above them -- to indicate that they are at their upper bounds -- will confirm that this program is optimal. That is, none of the vacant cells can be augmented to a positive program value and none of the barred values can be reduced without increasing the total of the deviations from the prescribed goals.<sup>11/</sup>

The availability of these row-column sum numbers provides the basis for the usual "dual evaluator analysis," and, as is well known,<sup>12/</sup> these dual evaluators provide a convenient way of studying potential program responses to changes in the goals or other policy conditions. Instead of following these well known paths, however, we turn rather to a consideration of the kinds of management reports that might be prepared from these results.

In this case, i.e., Figure 13, we have designated numerical values for the functional coefficients where they appear in the upper left-hand corner of the pertinent cells only for slack or transshipment cells as zero and as  $-1$  for the cells corresponding to being under (i.e., below or equal to) the goal values. Here these goal values are the individual upper bounds, as distinguished from the use of double lines for the bounding values in Figure 12 which were associated with the nonlinear nature of the goal functional.

The coefficients "P" correspond to these penalties which are associated with policy-directed changes in the Markoff transition rates that permit additional transfers or retentions of positions beyond those which

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<sup>11/</sup> I.e., apart from the cell associated with T-T' in going from period  $t=0$  to  $t=1$ , which has the entry "A" to indicate an alternate optimum is available there.

<sup>12/</sup> See [3].



would normally be produced by the organization. Note, for instance, that 420 in the cell C in going from  $t=0$  to  $t=1$  means that this (normal flow) value is at its upper bound so that this artifact goal is achieved. An addition of four more females, however, is planned at a penalty rate of "P" per unit via the cells C to C' in these same periods in order, thereby, to reduce the deviations from the real goal, which is not achieved.

The coefficients "R" in Figure 13 refer to high penalties for Reductions in Force (i.e., forced attrition) and the coefficients "H" refer to much smaller penalties incurred by hiring from the outside. For this example we specifically assume that R is larger than any combination of H and P.

In order to assure that the expected natural attrition is allowed for, the coefficients K are considered to be an order of magnitude larger than R. Since this determines the entries in the N columns completely, we have removed them from further consideration in the computations by means of the line drawn from the lower left to the upper right hand corner of these columnar arrays of cells. In this example the "Now" is not needed and hence is omitted entirely.

#### V. MANAGEMENT USES AND REPORTING SYSTEMS

One use of models like the above is in formulating plans at base, or other, levels in ways that also tie into overall plans such as those provided by the FEEO model discussed earlier (in section 1) in this paper. Another is in terms of helping to structure a system of reports which can be used as guides to down-the-line managers and (simultaneously) as a basis for decisions by the persons responsible for overall plans and guidance. Finally, they can be used to provide reports to others, including persons

outside the organization, who might be interested in EEO (and other) aspects of personnel planning.

Evidently the topic of report preparation and its relation to planning, etc., is a broad one in its own right. We do not propose to cover this topic comprehensively here. Instead we shall provide some suggestive illustrations which can serve one or more of the purposes indicated in the preceding paragraph. We shall do so, moreover, in a way that can also help to provide further illumination for the developments covered in the preceding section.

Figure<sup>s</sup> 14, 15 and 16 provide such illustrations drawn from the results portrayed in Figure 13. The first of these, i.e., Figure 14, represents the "optimal" plan. As drawn from Figure 13, these refer to the plans, including the projected transfers, etc., which "come closest to meeting the goals" set forth in the rims of Figure 13. Because these data refer to projections (via the Markoff transitions) as well as planned hires, etc., we have referred to these as projected personnel transfers in Figure 14. Observe, for instance, that the data for the Clerical, C, row at the top of Figure 13 are recapitulated in the first row of Figure 14. The data in the first Clerical, C, column of Figure 13 are also recapitulated in the column headed "Normal  $\pm$  Flexible" under the Clerical category in the table at the top of Figure 14. I.e., the planned value of 424 females under "Normal  $\pm$  Flexible" in the Clerical category is composed of 420 normal transfers plus 4 more females as part of a specific managerial plan to alter the present state of things. An addition of 98 persons from outside recruitment then produces a total of 522 persons -- which is shy of the 525 person goal noted at the bottom of the first C column in Figure 13.

Because two periods are involved, Figure 14 utilizes two separate tabulations that are drawn from the corresponding portions of Figure 13. The next table, Figure 15, provides a goal vs. plan comparison for each relevant period. Thus, in period 1 the recapitulation from Figure 13 now displays the discrepancy between plan and goal -- i.e., the projected vs. workload requirement amounts -- as drawn from the single "transshipment cell" outlined

JOB CATEGORY	NUMBER ABOARD AT PERIOD (0)	CLERICAL		TECHNICIANS		ADMINISTRATIVE		OUTSIDE LOSSES	
		NORMAL	NORMAL ± FLEXIBLE	NORMAL	NORMAL ± FLEXIBLE	NORMAL	NORMAL ± FLEXIBLE	NATURAL	FIRES
CLERICAL	600	420	424	18	16	6	4	155	
TECHNICIANS	175			140	140	9	10	25	
ADMINISTRATIVE	90			2	2	76	76	12	

HIRES 98

NUMBER ABOARD AT PERIOD (1) 522

158

90

JOB CATEGORY	NUMBER ABOARD AT PERIOD (1)	CLERICAL		TECHNICIANS		ADMINISTRATIVE		OUTSIDE LOSSES	
		NORMAL	NORMAL ± FLEXIBLE	NORMAL	NORMAL ± FLEXIBLE	NORMAL	NORMAL ± FLEXIBLE	NATURAL	FIRES
CLERICAL	522	368	368	15	12	5	5	137	
TECHNICIANS	158			125	125	8	8	24	
ADMINISTRATIVE	90			2	2	76	76	12	

HIRES 120

NUMBER ABOARD AT PERIOD (2) 488

140

1

89

FIGURE 14  
PROJECTED PERSONNEL TRANSFERS

	ON-BOARD ACTUAL	HIRES	FIRE	WORKLOAD REQUIREMENT	DISCREPANCY
FEMALE					
PERIOD 0:					
CLERICAL	600				
TECHNICIANS	175				
ADMINISTRATIVE	90				
PERIOD 1:	PROJECTED				
CLERICAL	522	98		525	-3
TECHNICIANS	153			158	
ADMINISTRATIVE	90			90	
PERIOD 2:	PROJECTED				
CLERICAL	433	120		438	
TECHNICIANS	140			140	
ADMINISTRATIVE	90	1		90	

FIGURE 15  
WORKFORCE REQUIREMENTS AND DISTRIBUTIONS

CATEGORY	NUMBER ABOARD AT PERIOD (0)	PERIOD 1			PERIOD 2			ABOARD END PERIOD (2)
		NORMAL TRANS.	FLEXIBLE TRANS.	HIRES FIRES	NORMAL TRANS.	FLEXIBLE TRANS.	HIRES FIRES	
CLERICAL	600	420	4	93	303		120	483
TECHNICAL	175	160	-2		143		-3	140
ADMIN.	90	91	-1		89		1	90

FIGURE 16  
SUMMARY OF PROJECTED PERSONNEL ACTIONS

There is more to be emphasized in the way of integrated reports for this kind of multiple-period planning. Hence, we provide an example of what else might be utilized for these purposes in Figure 16. The idea in any case is to provide adequate detail on the plans, the goals, etc., in each period but in a way that also emphasizes that an alteration in any one period (of plans or goals) also has consequence for the other periods.

These tabulations and figures are intended only to illustrate the kinds of considerations that need to enter into the design of any such system of reports. Adequate detail is evidently needed for management to understand fully what is planned for accomplishment and how these plans relate to the goals for each period. Progress toward stipulated goals over a sequence of periods also needs to be delineated. Here we have simply illustrated a two-period case for females only. The need for extending this to other aspects of EEO planning and for accommodating transitions (i.e., promotions) from one job level to another within each pertinent job category is also evident. The same principles will apply, of course, but there is also an additional need to ensure that the resulting reports are sufficiently simple so that they can be easily understood and used by persons who are interested in the details of such programs.

## VI. SUMMARY AND CONCLUSIONS

We have now covered the most recent in a series of researches dealing with modeling for EEO plans which can be succinctly summarized as follows. First, we provided a model, called FEEO, which can be used for overall EEO planning. Next, as discussed in this paper, we have provided a model for more detailed application, e.g., at base or other levels, which is consistent with -- or, more precisely, coherent with -- this FEEO model.

We have indicated how this model can be utilized as part of an integrated planning system extending over a sequence of periods. We have also indicated how it can be used as a guide for a suitable system of management reports.

This does not mean that the above modeling effort is intended as the one sole approach to this complex series of problems even in a single organization. It can be supplemental or replaced in a variety of applications. For instance, one would not want to use it in small installations or in situations where a great deal of turbulence indicates that the transition probabilities, etc., are likely to be unstable and hence not to be relied on for the projections embedded in these models.

On the other hand, these models can be used in a variety of additional ways. For instance, they might be used as guides for plans or for reports over a collection of such small units. They can also be readily adapted to problems like personnel promotion planning, etc., and other such functional parts of a total personnel planning system. How and in what form these additional applications or uses might be made is itself a subject of further continuing research which we shall report on at other occasions.

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