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ON DECOMPOSITIONS OF A MULTI-GRAPH INTO SPANNING SUBGRAPHS, (U)

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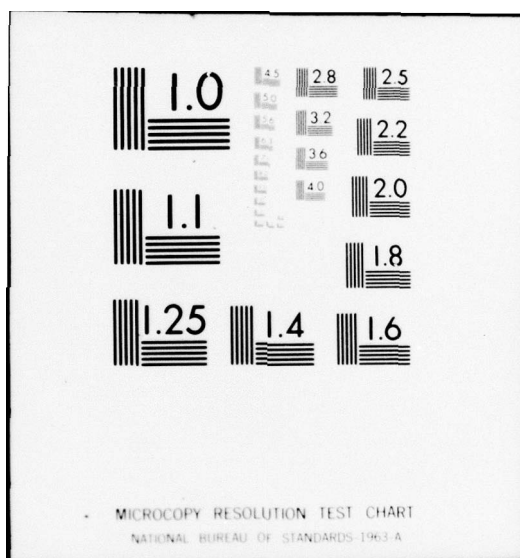
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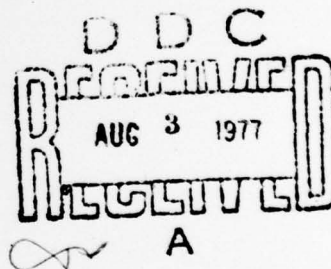
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6 ON DECOMPOSITIONS OF A MULTI-GRAPH
INTO SPANNING SUBGRAPHS,

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AMS Subject Classification 05C15.

Key Words and Phrases. Multigraph, bipartite graph, balanced hypergraph, spanning subgraph, cover, matching, cover index, chromatic index.

AUG. 1973

¹This research was supported in part by ONR contract N00014-67-A-0232-0016.

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1. Let G be a multi-graph, i.e., a finite graph with no loops.
 $V(G)$ and $E(G)$ denote the vertex-set and edge-set of G , respectively.
 For $x \in V(G)$, $d(x, G)$ denotes the degree (or valency) of x in G
 and $m(x, G)$ denotes the multiplicity of edges at x in G , i.e. the
 minimum number m such that x is joined to any other vertex in G
 by at most m edges.

A graph H is called a spanning subgraph of G if $V(H) = V(G)$
 and $E(H) \subseteq E(G)$. Let k be any positive integer. Let

$$(1.1) \quad \sigma: G = H_1 \cup H_2 \cup \dots \cup H_k$$

be a decomposition of G into k spanning subgraphs so that (1)
 $H_1, H_2, \dots, H_k$ are spanning subgraphs of G , (2) $H_1, H_2, \dots, H_k$ are
 pairwise edge-disjoint, and (3) $\bigcup_{1 \leq \alpha \leq k} E(H_\alpha) = E(G)$. For each

$x \in V(G)$, let $v(x, \sigma)$ denote the number of subgraphs H_α in σ
 such that $d(x, H_\alpha) \geq 1$. Evidently,

$$(1.2) \quad v(x, \sigma) \leq \min \{k, d(x, G)\} \text{ for all } x \in V(G).$$

2. Given a multi-graph G and any positive integer k , we
 consider the problem of determining a decomposition σ of G into k
 spanning subgraphs such that $v(x, \sigma)$ is as large as possible for each
 vertex $x \in V(G)$. In particular, we have proved the following two theorems.

Theorem 2.1: If G is a bipartite graph, then, for every positive integer k , there exists a decomposition σ of G into k spanning subgraphs such that

$$(2.1) \quad v(x, \sigma) = \min \{k, d(x, G)\} \text{ for all } x \in V(G).$$

Theorem 2.2: If G is a multi-graph, then for every positive integer k , there exists a decomposition σ of G into k spanning subgraphs such that

$$(2.2) \quad v(x, \sigma) \geq \begin{cases} \min\{k - m(x, G), d(x, G)\} & \text{if } d(x, G) \leq k \\ \min\{k, d(x, G) - m(x, G)\} & \text{if } d(x, G) \geq k \end{cases} \text{ for all } x \in V(G).$$

Moreover, if $W \subseteq V(G)$ is such that

$$W \cap \{x \in V(G) : k - m(x, G) < d(x, G) < k + m(x, G)\}$$

is independent, then σ can be so chosen that in addition to (2.2), we have

$$v(x, \sigma) = \min\{k, d(x, G)\} \text{ for all } x \in W.$$

3. The above theorems generalize some well-known theorems in graph theory.

Let G be a multi-graph; let H be a spanning subgraph of G . H is said to be a matching of G if for every vertex x , $d(x, H) \leq 1$; H is said to be a cover of G if for every vertex x , $d(x, H) \geq 1$.

The chromatic index of G , denoted by $\chi_1(G)$, is defined to be the minimum number k such that there exists a decomposition of G into k spanning subgraphs each of which is a matching of G . The cover index of G , denoted by $\kappa_1(G)$ is the maximum number k such that there exists a decomposition of G into k spanning subgraphs each of which is a cover of G .

Theorems 3.1 and 3.2 below are obtained from Theorem 2.1 by taking $k = \max_{x \in V(G)} d(x, G)$ and $k = \min_{x \in V(G)} d(x, G)$, respectively.

Theorem 3.1 [1]: If G is a bipartite graph, then,

$$\chi_1(G) = \max_{x \in V(G)} d(x, G).$$

Theorem 3.2 [2]: If G is a bipartite graph, then,

$$\kappa_1(G) = \min_{x \in V(G)} d(x, G).$$

Similarly, Theorems 3.3 and 3.4 are seen to be special cases of Theorem 2.2.

Theorem 3.3 [3,4]: If G is a multi-graph, then,

$$\chi_1(G) \leq \max_{x \in V(G)} (d(x, G) + m(x, G)).$$

Theorem 3.4 [5]: If G is a multi-graph, then,

$$\kappa_1(G) \geq \min_{x \in V(G)} (d(x, G) - m(x, G)).$$

Remark: We have also generalized Theorem 2.1 to a theorem for balanced hypergraphs which contains as special cases some theorems due to C. Berge [6].

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