

AD-A042 732

WISCONSIN UNIV MADISON MATHEMATICS RESEARCH CENTER

F/G 12/1

SOME CRITICAL POINT THEOREMS AND APPLICATIONS TO SEMILINEAR ELL--ETC(U)

JUN 77 P H RABINOWITZ

DAAG29-75-C-0024

MRC-TSR-1758

NL

UNCLASSIFIED

| OF |

ADAD42 732



END

DATE

FILMED

9 - 77

DDC

12

2
ADA 042732

MRC Technical Summary Report #1758

SOME CRITICAL POINT THEOREMS AND
APPLICATIONS TO SEMILINEAR
ELLIPTIC PARTIAL DIFFERENTIAL
EQUATIONS

Paul H. Rabinowitz

**Mathematics Research Center
University of Wisconsin-Madison
610 Walnut Street
Madison, Wisconsin 53706**

June 1977

(Received May 2, 1977)

DDC
AUG 11 1977
C

**Approved for public release
Distribution unlimited**

DDC FILE COPY

Sponsored by

U.S. Army Research Office
P.O. Box 12211
Research Triangle Park
North Carolina 27709

and

Office of Naval Research
Arlington, Virginia 22217

UNIVERSITY OF WISCONSIN - MADISON
MATHEMATICS RESEARCH CENTER

SOME CRITICAL POINT THEOREMS AND APPLICATIONS TO
SEMILINEAR ELLIPTIC PARTIAL DIFFERENTIAL EQUATIONS

Paul H. Rabinowitz*

Technical Summary Report #1758

June 1977

ABSTRACT

Variational methods are used to obtain some new existence theorems for critical points of a real valued function on a Banach space. Applications are made to semilinear elliptic boundary value problems.

AMS (MOS) Subject Classification - 35J60, 58E05

Key Words - Critical point, critical value, minimax,
semilinear elliptic boundary value
problem

Work Unit Number 1 - Applied Analysis

ACCESSION for	
NTIS	White Section <input checked="" type="checkbox"/>
DDC	Buff Section <input type="checkbox"/>
UNANNOUNCED	<input type="checkbox"/>
JUSTIFICATION	
BY	
DISTRIBUTION/AVAILABILITY CODES	
Dist.	AVAIL. and/or SP:Cl
A	

* Mathematics Department and Mathematics Research Center, University of Wisconsin, Madison 53706. Reproduction in whole or in part is permitted for any purpose of the United States Government.

Sponsored by
the United States Army under Contract No. DAAG29-75-C-0024
and
the Office of Naval Research under Contract No. N00014-76-C-0300.

EXPLANATION

Using the calculus of variations, some abstract theorems are obtained which assert under certain hypotheses the existence of critical points of a real valued function defined on a Banach space. These results are then applied to some boundary value problems for semilinear-elliptic partial differential equations to conclude the solutions of these problems.

SOME CRITICAL POINT THEOREMS AND APPLICATIONS TO
SEMILINEAR ELLIPTIC PARTIAL DIFFERENTIAL EQUATIONS

Paul H. Rabinowitz*

Introduction

Let E be a real Banach space and I a continuously differentiable map from E to \mathbb{R} , i.e. $I \in C^1(E, \mathbb{R})$. We say I satisfies the Palais-Smale condition (PS) if any sequence (u_m) such that $I(u_m)$ is bounded and $I'(u_m) \rightarrow 0$ is precompact. In [1] and [2], by imposing various qualitative conditions on I near 0 and ∞ , Ambrosetti and the author obtained several results concerning the existence of critical points of I and applied them to semilinear elliptic boundary value problems to get existence theorems in that setting.

The purpose of this paper is to extend the theory of [1] and [2]. To be more precise, let $B_r = \{x \in E \mid \|x\| < r\}$. It was shown in [1, Theorem 2.1] that if $I(0) = 0$ and

1° There are constants $\rho, \alpha > 0$ such that $I > 0$ in $B_\rho \setminus \{0\}$ and $I \geq \alpha$ on ∂B_ρ ,

and 2° There is an $e \in E$, $e \neq 0$ such that $I(e) = 0$,

then I has a critical value $c \geq \alpha$. It was further shown in [1, Theorem 2.21], [2, Theorem 3.37] that if I is even and 2° is replaced by the requirement that I be negative at infinity in an appropriate sense (see condition (I_3) of §1), then 1° can be weakened to hold in $B_\rho \cap \hat{E}$ where \hat{E} is a subspace of

* Mathematics Department and Mathematics Research Center, University of Wisconsin, Madison 53706. Reproduction in whole or in part is permitted for any purpose of the United States Government.

E of finite codimension. Moreover for this case I has an unbounded sequence of positive critical values.

Our main results here show how one can still obtain critical points of I under weakened versions of 1° without requiring that I be even. The abstract critical point theorems will be given in §1 and some applications to semilinear elliptic partial differential equations will be carried out in §2.

§1. The critical point theorems

In this section we shall extend the results of [1] and [2] mentioned in the Introduction. Some additional variants will be presented for the special case of $E = \mathbb{R}^n$. Lastly a case where we only require l^0 for B_ρ intersected with a finite dimensional subspace of E will be treated. As in [1], [2], our arguments are based on minimax characterizations of critical values of I .

Our main result is

Theorem 1.1: Let E be a real Banach space and let $I \in C^1(E, \mathbb{R})$ and satisfy (PS). Suppose that $E = E_k \oplus \hat{E}$ where E_k is k dimensional and I satisfies

$$(I_1) \quad I|_{E_k} \leq 0,$$

(I₂) There are constants $\rho, \alpha > 0$ such that $I \geq 0$ in $B_\rho \cap \hat{E}$ and $I \geq \alpha$ on $\partial B_\rho \cap \hat{E}$,

(I₃) For each finite dimensional subspace \tilde{E} of E , there is a constant $R = R(\tilde{E})$ such that $I \leq 0$ on $\tilde{E} \setminus B_{R(\tilde{E})}$.

Then I has a positive critical value, c , characterized by

$$(1.2) \quad c = \inf_{h \in \Gamma} \max_{u \in \bar{B}_r \cap E_{k+1}} I(h(u))$$

where $E_{k+1} \equiv E_k \oplus \text{span}\{\varphi\}$ for some fixed $\varphi \in \tilde{E} \setminus \{0\}$, $r = R(E_{k+1})$, and

$$\Gamma = \{h \in C(\bar{B}_r \cap E_{k+1}, E) \mid h(u) = u \text{ if } I(u) \leq 0\}.$$

A finite dimensional version of Theorem 1.1 was given in [5] and it motivated this paper. Two preliminary results are required for the proof of Theorem 1.1. The first is a standard lemma from

the calculus of variations. Let $A_s = \{u \in E \mid I(u) \leq s\}$ and $K_s = \{u \in E \mid I(u) = s \text{ and } I'(u) = 0\}$.

Lemma 1.3: Let $I \in C^1(E, \mathbb{R})$ and satisfy (PS). Let $c \in \mathbb{R}$, \mathcal{O} be any neighborhood of K_c , and $\bar{\epsilon} > 0$. Then there is an $\epsilon \in (0, \bar{\epsilon})$ and an $\eta \in C([0, 1] \times E, E)$ such that

$$1^{\circ} \quad \eta(t, x) = x \text{ if } I(x) \notin [c - \bar{\epsilon}, c + \bar{\epsilon}],$$

$$2^{\circ} \quad \eta(1, A_{c+\epsilon} \setminus \mathcal{O}) \subset A_{c-\epsilon},$$

$$3^{\circ} \quad \text{If } K_c = \emptyset, \quad \eta(1, A_{c+\epsilon}) \subset A_{c-\epsilon}.$$

See e.g. [2] or [3] for a proof of this lemma. Next we need a topological lemma. For $j \leq m$, let $\mathbb{R}^j = \{x = (x_1, \dots, x_m) \mid x_i = 0, j+1 \leq i \leq m\}$ and $(\mathbb{R}^j)^\perp$ the orthogonal complement of \mathbb{R}^j in \mathbb{R}^m .

Lemma 1.4: Let $b_R = \mathbb{R}^{k+1} \cap \bar{B}_R$ and let $b_R^+ = \{x \in \bar{B}_R \mid x_{k+1} \geq 0\}$. If $g \in C(b_R^+, \mathbb{R}^m)$, $m > k$, $\rho < R$, and there is a homotopy $G \in C([0, 1] \times \partial b_R^+, \mathbb{R}^m \setminus (\partial B_\rho \cap (\mathbb{R}^k)^\perp))$ such that $G(0, x) = x$ and $G(1, x) = g(x)$, then $g(b_R^+) \cap \partial B_\rho \cap (\mathbb{R}^k)^\perp \neq \emptyset$.

Proof: A proof of Lemma 1.4 due to E. Fadell using intersection theory [4, p. 197] can be found in [5, Lemma A-2]. For the convenience of the reader, we include it here. The problem is normalized by taking $R = 1$ and $\rho < 1$. Let $D = \bar{B}_\rho \cap (\mathbb{R}^k)^\perp$, $\hat{D} = (\partial B_\rho) \cap (\mathbb{R}^k)^\perp$, $b^+ = b_1^+$, and $\partial b^+ = \{x \in b^+ \mid x_{k+1} = 0 \text{ or } |x| = 1\}$. The intersection pairing:

$$H_{m+k}(D, \hat{D}) \times H_k(\partial b^+) \rightarrow \mathbb{Z}$$

yields +1 as intersection number for appropriately chosen

generators in the homology groups above. Furthermore naturality yields the diagram

$$\begin{array}{ccc}
 H_{m+k}(D, \hat{D}) \times H_k(\partial b^+) & \rightarrow & \mathbb{Z} \\
 \downarrow \text{id} \times j_* & & \nearrow \\
 H_{m+k}(D, \hat{D}) \times H_k(\mathbb{R}^m \setminus \hat{D}) & &
 \end{array}$$

where $j: \partial b^+ \subset \mathbb{R}^m \setminus \hat{D}$ is inclusion. If $g \in C(b^+, \mathbb{R}^m \setminus \hat{D})$ and G is as in the statement of the lemma, then j_* would be the trivial homomorphism and the intersection number would be 0, a contradiction.

These preliminaries being completed, we can now give the

Proof of Theorem 1.1: Assume for the moment that $c \geq \alpha$. If c is not a critical value of I , we can invoke Lemma 1.3 with $\bar{\varepsilon} = \alpha/2$. Let $h \in \Gamma$ be such that

$$(1.5) \quad \max_{u \in \bar{B}_r \cap E_{k+1}} I(h(u)) \leq c + \varepsilon.$$

Since $\eta(1, h) \in C(\bar{B}_r \cap E_{k+1}, E)$ and if $I(u) \leq 0$, then $\eta(1, h(u)) = \eta(1, u) = u$ by 1° of Lemma 1.3 and our choice of $\bar{\varepsilon}$, it follows that $\eta(1, h) \in \Gamma$. But then by (1.5) and 3° of Lemma 1.3,

$$(1.6) \quad \max_{u \in \bar{B}_r \cap E_{k+1}} I(\eta(1, h(u))) \leq c - \varepsilon$$

contrary to the definition of c .

To complete the proof, we must show $c \geq \alpha$. Using (I_2) , it suffices to show $h(\bar{B}_r \cap E_{k+1}) \cap \partial B_\rho \cap \hat{E} \neq \emptyset$ for all $h \in \Gamma$. Let $K = h(\bar{B}_r \cap E_{k+1})$. Since K is compact, by an approximation

lemma of Leray-Schauder [9], for all $\varepsilon > 0$, there exists a finite dimensional subspace $F_\varepsilon \subset E$ and a mapping $i_\varepsilon \in C(K, F_\varepsilon)$ such that $\|i_\varepsilon(u) - u\| \leq \varepsilon$ for all $u \in K$. We can assume $F_\varepsilon \supset E_{k+1}$. Let $h_\varepsilon = i_\varepsilon \circ h$. Then $h_\varepsilon \in C(\bar{B}_r \cap E_{k+1}, F_\varepsilon)$ and $h_\varepsilon(u) \rightarrow h(u)$ as $\varepsilon \rightarrow 0$ uniformly for $u \in \bar{B}_r \cap E_{k+1}$. Observe that if $u \in (\partial B_r \cap E_{k+1}) \cup E_k$, by (I_1) and (I_3) we have

$$(1.7) \quad \|h_\varepsilon(u) - h(u)\| = \|h_\varepsilon(u) - u\| \leq \varepsilon.$$

Since $E = E_k \oplus \hat{E}$, $u \in E$ implies $u = v + w$, where $v \in E_k$, $w \in \hat{E}$. We can assume $\|u\|_E = (\|v\|_{E_k}^2 + \|w\|_{\hat{E}}^2)^{1/2}$. Thus if $u \in (\partial B_r \cap E_{k+1}) \cup E_k$, it follows that

$$\|u - (\partial B_\rho \cap \hat{E})\| \geq \beta \equiv \min(r - \rho, \rho).$$

Choosing $\varepsilon < \beta$, identifying F_ε with \mathbb{R}^m , E_k, E_{k+1} with $\mathbb{R}^k, \mathbb{R}^{k+1}$, and defining $G(t, u) = th_\varepsilon(u) + (1-t)u$, it follows from (1.7)-(1.8) that G satisfies the hypotheses of Lemma 1.4. Hence $h_\varepsilon(\bar{B}_r \cap E_{k+1}) \cap \partial B_\rho \cap \hat{E} \neq \emptyset$. Consequently there exists $u_\varepsilon \in \bar{B}_r \cap E_{k+1}$ such that $h_\varepsilon(u_\varepsilon) \in \partial B_\rho \cap \hat{E}$. As $\varepsilon \rightarrow 0$, along a subsequence we have $u_\varepsilon \rightarrow u \in \bar{B}_r \cap E_{k+1}$ and

$$\|h_\varepsilon(u_\varepsilon) - h(u)\| \leq \|h_\varepsilon(u_\varepsilon) - h(u_\varepsilon)\| + \|h(u_\varepsilon) - h(u)\| \rightarrow 0$$

as $\varepsilon \rightarrow 0$. Therefore $h(u) \in \partial B_\rho \cap \hat{E}$ and the proof is complete.

Remark 1.9: If E is infinite dimensional, in general I will not be bounded from above in contrast to the finite dimensional case where this is a consequence of (I_3) . By (I_2) , I then

has a positive maximum \bar{c} in B_r . In general $c < \bar{c}$. However if $c = \bar{c}$, I possesses infinitely many critical points. Indeed (1.2) then shows I achieves its maximum on $h(B_r \cap E_{k+1})$ for each $h \in \Gamma$ and this implies I has infinitely many distinct critical points corresponding to $c = \bar{c}$. Our next result gives another characterization of critical values of I in the finite case and a more precise description of the degenerate situation.

Theorem 1.10: Let $I \in C^1(\mathbb{R}^m, \mathbb{R})$ and satisfy $(I_1)-(I_3)$.

Then I possesses at least two positive critical values characterized by

$$\bar{c} = \max_{u \in B_r} I(u), \quad r = r(\mathbb{R}^m)$$

and

$$(1.11) \quad b = \sup_{\substack{A \in M \\ \text{cat}_M A = 2}} \min_{u \in A} I(u),$$

where $M = \bar{B}_r \cap (\mathbb{R}^m \setminus \mathbb{R}^k)$. If $b = \bar{c}$, $\text{cat}_M K_b = 2$.

Before proving Theorem 1.10, a few remarks are in order. The notation $\text{cat}_M A$ refers to the Ljusternik-Schnirelman category of the subset A of M . In (1.11) as admissible A we only consider $A \subset M$ with A closed in \mathbb{R}^m . By definition $\text{cat}_M A = 1$ if A is contractible to a point in M and $\text{cat}_M A = j$ if j is the least integer such that A can be covered by j closed sets A_i with $\text{cat}_M A_i = 1$, $1 \leq i \leq j$. It is clear that any admissible $A \subset M$ is homotopic to a subset of the unit sphere S^{m-k} in $(\mathbb{R}^k)^\perp$. Moreover $\text{cat}_M S^{m-k} = 2$ since if $\text{cat}_M S^{m-k} = 1$, any homotopy of S^{m-k} to a point in M would

induce a homotopy of S^{m-k} to a point in S^{m-k} . However as is well known (see e.g. [6]), $\text{cat}_{S^{m-k}} S^{m-k} = 2$ so no such homotopy can exist.

Proof of Theorem 1.10: Define

$$(1.12) \quad c_i = \sup_{A \subset M} \min_{u \in A} I(u), \quad i = 1, 2.$$

$$\text{cat}_M^{A \geq i}$$

Then $c_1 = \bar{c}$ since we can take $A = \{x\}$ for any $x \in M$ and $c_2 = b \leq \bar{c}$. Note further that by (I_2) , $I > 0$ on ρS^{m-k} and therefore $b > 0$. (I_3) implies I satisfies (PS) on the set where $I > 0$. A slightly strengthened version of Lemma 1.3 and a standard argument (similar to the first paragraph of the proof of Theorem 1.1) shows that b is a critical value of I and if $\bar{c} = b$, $\text{cat}_M K_b = 2$. (See e.g. [6], [7], or [2]).

Remark 1.13: We believe that b as defined in (1.11) equals c defined in (1.2).

An examination of the proof of Theorem 1.10 shows that the argument in fact gives the following result which interchanges the finite dimensional and finite codimensional hypotheses (I_1) and (I_2) of Theorem 1.1.

Theorem 1.14: Let E be a real Banach space, $I \in C^1(E, \mathbb{R})$ and satisfy (PS). Suppose that $E = E_k \oplus \hat{E}$ where $k \geq 1$, E_k is k -dimensional, and I satisfies

$$(I_4) \quad I|_{\hat{E}} \leq 0,$$

$$(I_5) \quad \text{There are constants } \rho, \alpha > 0 \text{ such that } I \geq 0 \text{ in } B_\rho \cap E_k \text{ and } I \geq \alpha \text{ on } \partial B_\rho \cap E_k,$$

$$(I_6) \quad I \text{ is bounded from above.}$$

Then I possesses at least two positive critical values characterized by

$$(1.15) \quad b_i = \sup_{\substack{A \in M \\ \text{cat}_M A \geq i}} \min_{u \in A} I(u) \quad i = 1, 2$$

where A is compact in E and M = E \setminus \hat{E}. Moreover if

$$b_1 = b_2 = d, \quad \text{cat}_M K_d \geq 2.$$

Proof: Immediate from that of Theorem 1.10 and the remark preceding it.

§2. Applications to partial differential equations

In this section we shall show how Theorems 1.1 and 1.14 can be employed to obtain existence theorems for semilinear elliptic boundary value problems. Consider

$$(2.1) \quad \begin{cases} Lu \equiv - \sum_{i,j=1}^n (a_{ij}(x) u_{x_j})_{x_i} + c(x)u = a(x)u + p(x,u), & x \in \Omega \\ u = 0, & x \in \partial\Omega \end{cases}$$

where Ω is a bounded domain in \mathbb{R}^n with a smooth boundary, L is uniformly elliptic in $\bar{\Omega}$ with smooth coefficients, $c \geq 0$ in Ω , a is smooth and positive in $\bar{\Omega}$, and p is smooth and satisfies

$$(p_1) \quad |p(x,z)| \leq a_1 + a_2 |z|^s, \quad s < \frac{n+2}{n-2}, \quad n > 2,$$

$$(p_2) \quad p(x,z) = o(|z|) \quad \text{at } z = 0.$$

There are constants $M > 0$ and $\theta \in [0, \frac{1}{2})$ such that

$$(p_3) \quad 0 < P(x,z) = \int_0^z p(x,t) dt \leq \theta z p(x,z) \quad \text{for } |z| \geq M.$$

If $n \leq 2$, (p_1) can be considerably improved. See e.g. §3 of [1].

Set

$$I(u) = \int_{\Omega} \left[\frac{1}{2} \left(\sum_{i,j=1}^n a_{ij}(x) u_{x_i} u_{x_j} + c(x)u^2 - a(x)u^2 \right) - P(x,u) \right] dx.$$

Formally critical points of I in $E = W_0^{1,2}(\Omega)$ are weak solutions of (2.1). The smoothness of L , a , p , and (p_1) and standard regularity results then imply such weak solutions are smooth functions.

Thus we focus our attention on finding critical points of I in E .

Consider the linear eigenvalue problem

$$(2.3) \quad \begin{cases} Lv = \lambda av, & x \in \Omega \\ v = 0, & x \in \partial\Omega . \end{cases}$$

As is well known, (2.3) possesses an unbounded sequence of eigenvalues $0 < \lambda_1 < \lambda_2 \leq \dots \leq \lambda_m \rightarrow \infty$ as $m \rightarrow \infty$ with corresponding eigenfunctions v_1, \dots, v_m, \dots . It was shown in [1] that if p satisfies $(p_1) - (p_3)$ and $\lambda_1 > 1$, (2.1) possesses a positive solution (i.e. $u > 0$ in Ω) and a negative solution. If $\lambda_1 \leq 1$, the argument of [1] fails unless $p(x, z)$ is odd in z . We will show:

Theorem 2.4: If p satisfies $(p_1) - (p_3)$ and
 (p_4) $zp(x, z) \geq 0$ for $x \in \Omega, z \in \mathbb{R}$,

then (2.1) possesses a nontrivial solution.

Proof: Because of the result just mentioned, we can assume $\lambda_1 \leq 1$. In fact suppose $\lambda_k \leq 1 < \lambda_{k+1}$. Let $E_k = \text{span}\{v_1, \dots, v_k\}$ and $\hat{E} = E_k^\perp$, the orthogonal complement of E_k . By (p_4) , $I \leq 0$ on E_k . It was further shown in [1] that $(p_1) - (p_3)$ imply that $I \in C^1(E, \mathbb{R})$ and satisfies (I_2) , (I_3) and (PS). Hence Theorem 2.4 follows immediately from Theorem 1.1.

Remark 2.5: As was noted above for $\lambda_1 > 1$, (2.1) has a positive and a negative solution. In contrast we next show:

Corollary 2.6: Under the hypotheses of Theorem 2.4, if $\lambda_1 \leq 1$, (2.1) does not have a positive (or negative) solution $u(x)$ unless $\lambda_1 = 1$, $u(x)$ is a multiple of $v_1(x)$ and $p(x, u(x)) \equiv 0$.

Proof: Suppose u is a positive solution of (2.1) with $\lambda_1 \leq 1$. Then

$$(2.7) \quad \int_{\Omega} v_1 Lu \, dx = \int_{\Omega} v_1 (au + p(x,u)) \, dx = \int_{\Omega} uLv_1 \, dx \\ = \lambda_1 \int_{\Omega} auv_1 \, dx .$$

Since we can assume $v_1 > 0$ in Ω , we have

$$(2.8) \quad 0 \geq (\lambda_1 - 1) \int_{\Omega} auv_1 \, dx = \int_{\Omega} v_1 p(x,u) \, dx \geq 0$$

which is impossible unless $p(x,z) \equiv 0$ for $0 \leq z \leq \max u(x)$ and $\lambda_1 = 1$ in which case $u \equiv \beta v_1$.

Next we illustrate how Theorem 1.14 can be used in similar situations. Consider (2.1) again where p satisfies (p_2) and e.g.

$$(p_4) \quad 0 \geq z^{-1} p(x,z) \rightarrow -\infty \text{ as } |z| \rightarrow \infty .$$

No growth conditions are needed for this case since (p_4) implies that $az + p$ can be redefined to be independent of z for large $|z|$ so the modified nonlinearity is uniformly bounded. Moreover solutions of the modified equation are still solutions of (2.1). (See e.g. Theorem 3.4 of [2]). It is easily verified that if $\lambda_k < 1 \leq \lambda_{k+1}$, E_k, \hat{E} are as in Theorem 2.4 and

$$J(u) = -\int_{\Omega} \left[\frac{1}{2} \left(\sum_{i,j=1}^n a_{ij}(x) u_{x_i} u_{x_j} + c(x)u^2 \right) - \bar{P}(x,u) \right] dx$$

where \bar{P} is the primitive of the modification of $az + p$, then J satisfies the hypotheses of Theorem 1.14.

Since stronger results can be obtained for this problem using methods based on Leray-Schauder degree theory [8] we will not carry out the details here. However if L were replaced by a higher order divergence structure elliptic operator one could obtain results using Theorem 1.14 where degree theoretic methods would fail.

REFERENCES

- [1] Ambrosetti, A. and P. H. Rabinowitz, Dual variational methods in critical point theory and applications, *J. Functional Anal.*, 14, (1973), 349-381.
- [2] Rabinowitz, P. H., Variational methods for nonlinear eigenvalue problems, Eigenvalues of Nonlinear Problems, Edizioni Cremonese, Rome, (1974), 141-195.
- [3] Clark, D. C., A variant of the Lusternik-Schnirelman theory, *Ind. Univ. Math. J.*, 22, (1972), 65-74.
- [4] Dold, A., Lectures on Algebraic Topology, Springer-Verlag, Berlin, 1972.
- [5] Rabinowitz, P. H., Free vibrations for a semilinear wave equation, to appear *Comm. Pure Appl. Math.*
- [6] Schwartz, J. T., Nonlinear Functional Analysis, lecture notes, Courant Inst. of Math. Sc., New York Univ., 1965.
- [7] Palais, R. S., Critical point theory and the minimax principle, *Proc. Sym. Pure Math.* 15, A.M.S. Providence, R.I., 1970, 185-212.
- [8] Rabinowitz, P. H., Some aspects of nonlinear eigenvalue problems, *Rocky Mtn. Math. J.*, 3, (1973), 161-202.
- [9] Leray, J. and J. Schauder, Topologie et equations fonctionnelles, *Ann. Sci. Ecole Norm. Sup.* 3, 51, 1934, 45-78.

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM	
1. REPORT NUMBER 1758	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER	(9) <i>Technical</i>
4. TITLE (and Subtitle) SOME CRITICAL POINT THEOREMS AND APPLICATIONS TO SEMILINEAR ELLIPTIC PARTIAL DIFFERENTIAL EQUATIONS.		5. TYPE OF REPORT & PERIOD COVERED Summary Report, no specific reporting period	
6. PERFORMING ORG. REPORT NUMBER		7. CONTRACT OR GRANT NUMBER(s)	
8. AUTHOR(s) Paul H. Rabinowitz		(15) DAAG29-75-C-0024, N00014-76-C-0300	
9. PERFORMING ORGANIZATION NAME AND ADDRESS Mathematics Research Center, University of Wisconsin 610 Walnut Street Madison, Wisconsin 53706		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Work Unit No. 1 - Applied Analysis	
11. CONTROLLING OFFICE NAME AND ADDRESS See Item 18 below.		(11) 12. REPORT DATE June 1977	
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) (12) <i>16p.</i>		13. NUMBER OF PAGES 14	
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited. (14) <i>MRC-TSR-1758</i>		15. SECURITY CLASS. (of this report) UNCLASSIFIED	
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE	
18. SUPPLEMENTARY NOTES U.S. Army Research Office P.O. Box 12211 Research Triangle Park North Carolina 27709 and Office of Naval Research Arlington, Virginia 22217			
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Critical point, critical value, minimax, semilinear elliptic boundary value problem.			
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) Variational methods are used to obtain some new existence theorems for critical points of a real valued function on a Banach space. Applications are made to semilinear elliptic boundary value problems.			

221200

18