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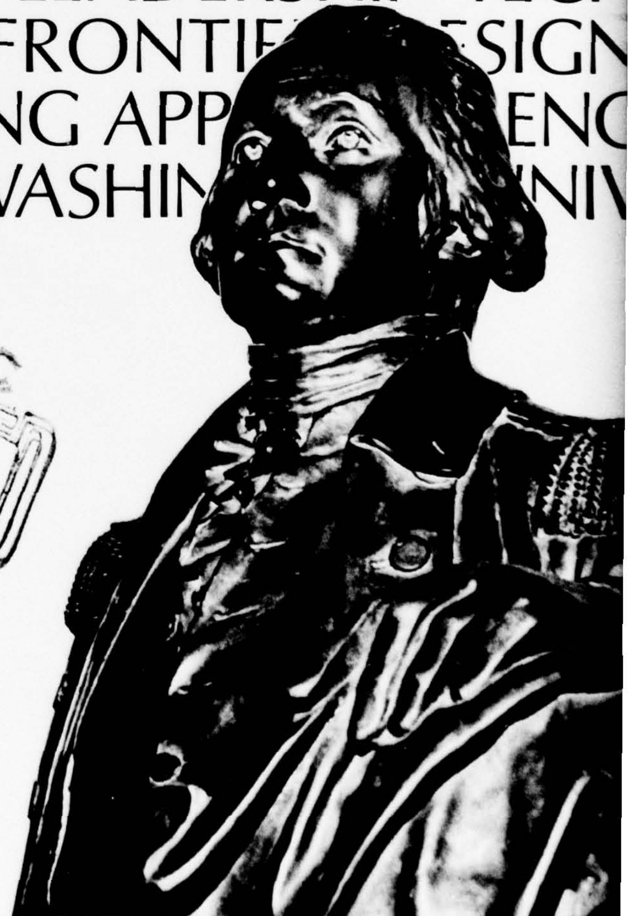

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ON PLATOON FORMATION ON TWO-LANE ROADS

by

Zeev Barzily*
Michael Rubinovitch†

Serial T-350
20 May 1977

The George Washington University
School of Engineering and Applied Science
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An approximate model for the study of platoon formation on two-lane highways is discussed in detail. The model assumes that the two-lane highway is divided in each traffic direction into alternating road sections of fixed lengths. The passing in one type of section is unrestricted and the passing in the other one is prohibited. It is assumed that there are slow and fast vehicles on the highway and that inputs follow independent Poisson processes. The results include the distribution of the number of vehicles in a platoon and the average speed of a typical fast vehicle.

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1. Introduction and Summary

This communication studies a simplified model for platoon (bunch) formation on two-lane two-way highways.

The behavior of vehicles on two-lane two-way highways is a very complex process and several models have been proposed by different authors under various simplified assumptions. Usually the objectives of such studies are to derive the distribution function of the number of vehicles in a platoon and to find the average speed of a fast test car moving in a stream of slow vehicles.

The models for traffic flow on roads may be divided, according to their method of study, into two groups, microscopic models and macroscopic models. In the micro approach a detailed description of the behavior of individual vehicles is the basis for the construction of the model. The macro approach, on the other hand, studies the behavior of sizable groups of vehicles without specifying the behavior of any single vehicle.

Several models have been proposed for traffic flow on two-lane roads. Tanner [9] assumes that traffic in one direction is moving at a constant speed v while traffic in the opposite direction moves at a constant speed V . Spacings between bunches are assumed to be exponentially distributed. The purpose of this study is to determine the average speed of a single vehicle having a free speed of u $(u > v)$ and traveling in the v stream. Miller [7] discussed a macro model for the determination of the average passing rate from bunches. He assumed exponential spacings between platoons and that the platoons are in a state of equilibrium in the sense that the average rate at which vehicles join a bunch equals the rate at which they leave it. Taylor, Miller, and Ogden [10] compared numerical results of several bunching models using both experimental and simulated bunch size data. Galin and Epstein [5] studied the steady-state situation on a road in which passing is possible only in passing points located at equal distances along the road. Models for traffic flow in a no-passing zone are proposed by Cowan [2], Hodgson [6], and Epstein, Galin, and Shlifer [4].

The present study is an extension of the models proposed by Galin and Epstein [5], Cowan [2], Hodgson [6], and Epstein, Galin, and Shlifer [4]. We assume that the road consists of alternating free-passing zones and no-passing zones in each traffic direction. The free-passing zones are named Type I sections and the no-passing zones are named Type II sections. It is also assumed that all road sections of the same type have the same length. What we in fact have is a sequence of no-passing zones of fixed length separated by a sequence of free-passing zones of fixed length.

In reality the lengths of the Type I and Type II sections are random variables dependent on traffic in the opposite direction. However, we assume that the Type I and Type II sections have constant lengths because it enables us to analyze traffic in one direction independently of traffic in the opposite direction. Even under this simplified assumption, the analysis is quite complex. We believe that the present model may give some insight into the mechanism of platoon formation. Moreover, there are situations in which this model may in fact provide an approximation to the behavior of vehicles in a road. This will be the case when traffic intensity is low and passing is frequently prohibited due to sight and road conditions.

A summary of the paper now follows. In Section 2 we give a detailed description of our model and its underlying assumptions. We also derive several properties which will provide the basis for the study of platoon formation in Section 3. Section 4 contains a numerical example and Section 5 is a discussion of the model.

2. The Model and Some Preliminary Results

Consider a two-lane two-way highway and assume that vehicles enter and leave it only at its end points. The highway consists in each traffic direction of alternating Type I and Type II sections. As traffic in one direction is (assumed) independent of traffic in the opposite direction, we will focus our attention on traffic moving in a traffic direction that will be named "our" direction. Let ℓ_1 denote the length of a Type I section in our direction, while ℓ_2 denotes the length of a Type II section.

Assume that there are two types of zero size vehicles moving in the highway; slow vehicles have a free speed v_1 and fast vehicles have a free speed v_2 $(v_2 > v_1)$. Input processes of slow and fast vehicles are independent Poisson processes with arrival rate λ_1 for the slow vehicles and λ_2 for the fast vehicles. Regarding the movement of vehicles, we shall assume the following. A slow vehicle always maintains its free speed v_1. A fast vehicle moves at its free speed v_2 except when it comes up against a slow vehicle in a Type II section. When this happens the fast vehicle slows down immediately and follows the slow one at a zero distance up to the end of the section. At the end of the Type II section it immediately passes the slow vehicle and resumes its free speed v_2.

We assume that fast vehicles do not disturb one another. Hence, the analysis of the movement of fast vehicles along the highway can be carried out through analyzing the movement of a typical fast vehicle -- a "test vehicle." Let $t=0$ denote the time at which the test car arrives at the highway, and let $\{\tau_n : n=1,2,\ldots\}$ be the interarrival times of the slow vehicles which precede it. Thus, the slow vehicle closest to the entrance arrives τ_1 time units prior to $t=0$, the slow vehicles in front of it arrives $\tau_1 + \tau_2$ time units prior to $t=0$, etc. By assumption and by the well-known properties of Poisson processes, it is clear that $\tau_1, \tau_2, \ldots$ are i.i.d. exponential random variables with mean λ_1^{-1} .

The fast test vehicle now starts its trip on the highway. The following are some fundamental observations regarding its movement and interactions with other vehicles:

(a) Let $D_m(u)$, $(m=1,2,\ldots ;\ u \leq \ell_1)$ denote the distance between the test car and the closest slow vehicle ahead of it when the test car is at a distance u from the beginning of the mth Type I road section. Then $D_m(u)$ are i.i.d. exponential random variables, all with mean v_1/λ_1 .

(b) Consider two fast vehicles, say No. 1 and No. 2, and suppose No. 1 is the one that arrives first at our highway. Suppose also that No. 1 is impeded by a slow vehicle (henceforth Vehicle A) at the ith Type II section. Then if No. 2 is impeded by A at the (i+j)th Type II section $(j \geq 1)$, then No. 1 and No. 2 will never be in the same platoon.

We first establish (a). Let

$$D_n = \tau_n v_1 \quad , \quad n=1,2,\ldots$$

$$S_n = \sum_{i=1}^{n} D_i \quad , \quad n=1,2,\ldots ,$$

and denote by $f_n(\cdot)$ the probability density function of S_n . By definition, $D_1(0) = D_1$, and since D_n are independent and exponentially distributed (parameter λ_1/v_1) it follows that $f_n(\cdot)$ is a gamma density with parameters $(n, \lambda_1/v_1)$. The distance between the test car and the nth preceding slow vehicle at $t=0$ is S_n , and when the test car is at a distance u $(u \leq \ell_1)$ from the entrance, this distance reduces to

$$S'_n = S_n - u(1 - v_1/v_2) .$$

If the test car has already passed the nth slow vehicle, S'_n is negative. Let $F_{m,u}(\cdot)$ denote the distribution function (d.f.) of $D_m(u)$; then

$$F_{1,u}(x) = P\left[0 \leq D_1 - u(1 - v_1/v_2) \leq x\right]$$

$$+ \sum_{n=2}^{\infty} P\left[S_{n-1} - u(1 - v_1/v_2) < 0, \quad 0 \leq S_n - u(1 - v_1/v_2) \leq x\right]$$

$$= e^{-(\lambda_1/v_1)u(1 - v_1/v_2)} - e^{-(\lambda_1/v_1)\left[u(1 - v_1/v_2) + x\right]}$$

$$+ \sum_{n=1}^{\infty} \int_{y=0}^{u(1 - v_1/v_2)} f_n(y)P\left[u(1 - v_1/v_2) - y < D_n \leq u(1 - v_1/v_2) - y + x\right]dy$$

$$= 1 - e^{-(\lambda_1/v_1)y} .$$

Now set

$$k = \min_n \left\{n : S_n - \ell_1(1- v_1/v_2) \geq 0\right\} ,$$

and obtain

$$D_2(0) = \begin{cases} \tau_{k+1}v_1 , & \text{if } D_1(\ell_1) \leq \ell_2(1 - v_1/v_2) \\ D_1(\ell_1) - \ell_2(1 - v_1/v_2) , & \text{if } D_1(\ell_1) > \ell_2(1 - v_1/v_2) . \end{cases}$$

From this we conclude that $D_2(0)$ is an exponential random variable (r.v.) due to the exponentiality of τ_{k+1} and $D_1(\ell_1)$, and the lack of memory property of this distribution. Thus starting with $D_1(0)$ as exponential (with parameter λ_1/v_1) we find that $D_1(u)$ has the same distribution for $u \leq \ell_1$, which in turn leads to $D_2(0)$ following also the same exponential distribution. By induction, it follows then that $D_m(u)$, $(0 \leq u \leq \ell_1)$ is exponential with parameter λ_1/v_1 for any $m = 1,2,3,\ldots$.

Now we establish (b). Let X_{mn}^{k} denote the length of time Vehicle No. k, k=1,2, maintains a speed v_n, n=1,2, for the mth time after passing Vehicle A. The values of X_{mn}^{k} are determined by v_1, v_2, ℓ_1, ℓ_2, and the distances between A and the slow vehicles preceding it. None of the values of these parameters changes, hence,

$$X_{mn}^{1} = X_{mn}^{2}, \quad n=1,2 \; ; \; m=1,2,3,\ldots .$$

Now we assume that No. 1 passes A at time t, and therefore it passes the ℓth impeding vehicle at

$$t_{\ell}^{1} = t + \sum_{m=1}^{\ell} X_{m1}^{1} + \sum_{m=1}^{\ell} X_{m2}^{1} .$$

No. 2 passes the ℓth impeding vehicle at

$$t_{\ell}^{2} = t+j \frac{\ell_1+\ell_2}{v_1} + \sum_{m=1}^{\ell} X_{m1}^{2} + \sum_{m=1}^{\ell} X_{m2}^{2} = t+j \frac{\ell_1+\ell_2}{v_1} + \sum_{m=1}^{\ell} X_{m1}^{1} + \sum_{m=1}^{\ell} X_{m1}^{1} > t_{\ell}^{1} \; ;$$

consequently, No. 1 and No. 2 will never simultaneously pass any slow vehicle, which means that they will never move in the same platoon. This proves Statement (b).

From here we can make two further conclusions essential to the analysis of the next section:

(c) Vehicle No. 2 will never move in a platoon with No. 1 if it passes Vehicle A later than the end of the (i+1)st Type II section.

(d) If the distance between No. 1 and No. 2 exceeds $(\ell_1+\ell_2) \times (1/v_1 - 1/v_2)$, then No. 1 and No. 2 will never move in the same platoon.

To see why (c) is true, imagine that there is a (fictitious) vehicle in a platoon which is moving after A in the (i+1)st Type II section. Then by (a) the fictitious vehicle will never come up against No. 1. Hence, since fast vehicles never pass one another, then fast vehicles moving behind the fictitious one will never come up against No. 1. Assertion (d) is now obvious since No. 2 will be unable to pass a slow vehicle, which impedes No. 1 at a Type II section before the beginning of the next consecutive Type II section.

3. The Platoon Formation

We will now determine the distribution function of the number of fast vehicles in platoons which arrive at the ends of Type II sections. A platoon is formed when several fast vehicles together are impeded by a slow vehicle. When such a platoon arrives at the end of the Type II section in which it is formed, the fast vehicles pass the slow leader, continue moving as a fast platoon, and the zero relative distances among the constituent fast vehicles never change. The platoon may join (or be joined by) other platoons later. In the discussion here we do not differentiate between platoons moving at their free speed (v_2) and platoons moving at a speed v_1 (although this separation may be added).

We begin with the determination of the distribution of the time spent in a Type II section. From (a) of Section 2 we deduce that the times spent by fast vehicles in Type II sections are i.i.d. random variables. Let T denote the length of a time period spent by a fast vehicle in the mth Type II section, and let $F_T(\cdot)$ denote the d.f. of T. Clearly,

$$T = \max\left\{\ell_2/v_2 \;;\; \left(\ell_2 - D_m(\ell_2)\right)/v_1\right\} ,$$

hence

$$F_T(t) = \begin{cases} 0 & , \quad t < \ell_2/v_2 \\ e^{-\lambda_1(\ell_2/v_1 - t)} & , \quad \ell_2/v_2 \le t \le \ell_2/v_1 \\ 1 & , \quad \ell_2/v_1 < t \end{cases}$$

and

$$F_T(dt) = \begin{cases} e^{-\lambda_1 \ell_2(1/v_1 - 1/v_2)} & , \quad t = \ell_2/v_2 \\ \lambda_1 e^{-\lambda_1(\ell_2/v_1 - t)}\, dt & , \quad \ell_2/v_2 < t \le \ell_2/v_1 \\ 0 & , \quad \text{otherwise.} \end{cases} \tag{3.1}$$

Using a different approach, $F_T(t)$ was calculated previously in [4].

Now we define:

<u>a Type A interval of order m</u> as a time interval $(t, t+x]$ satisfying (i) all fast vehicles which arrive at the highway in this interval are unimpeded in the first m Type II sections, and (ii) the fast vehicles which arrive at the highway at $t-$ (an instant before t) or at $(t+x)+$ (an instant after t) are impeded in at least one of the first m Type II sections; and

<u>a Type B interval of order m</u> as a time interval $(t, t+x]$ satisfying (i) the fast vehicles which arrive at the highway in $(t, t+x]$ form a platoon while arriving at the end of the mth Type II section, and (ii) vehicles that do not arrive at the highway in this interval do not move in this platoon at that point.

Since the arrivals of fast vehicles at the highway are assumed independent and Poisson, one can calculate the distribution function of the number of fast vehicles in a platoon if the distribution function of the Type B intervals is known. We therefore begin with the determination of this distribution.

We notice that the Type A and Type B intervals of order m (formed) in any time interval $(t_1, t_2]$ constitute a partition of this interval. Furthermore, this partition is a <u>fixed</u> function of ℓ_1, ℓ_2, v_1, v_2 and of the arrivals at the highway of slow vehicles in $\left(t_1 - m(\ell_1+\ell_2)(1/v_1 - 1/v_2),\ t_2\right]$. (The subtraction of $m(\ell_1+\ell_2)(1/v_1 - 1/v_2)$ results from the fact that while moving in a stretch of road of length $m(\ell_1+\ell_2)$, a fast vehicle may pass slow vehicles which arrived at the beginning of this section no earlier than $m(\ell_1+\ell_2)(1/v_1 - 1/v_2)$ ahead of it.) Since the slow vehicles arrive according to a Poisson process and since ℓ_1, ℓ_2, v_1, and v_2 are constants, then the partition is a stationary process. Moreover, the Type A and Type B intervals in $(t_1, t_2]$ are not independent due to the cyclical effect caused by the entrance of any given slow vehicle to consecutive Type II sections every fixed time period -- $(\ell_1+\ell_2)/v_1$. We will determine therefore the marginal density of the Type B intervals of order m .

To determine the marginal density of a Type B interval, we calculate $S(u,m)$, the probability that two fast vehicles, arriving at the highway u unit of time apart, move in one platoon at the end of the mth Type II section. Let T_j denote the time spent by the leading fast vehicle in the jth Type II section. Denote by U_j the interarrival time of these fast vehicles at the end of the jth Type II section, and define $U_0 = u$. Let $F_{U_1T_1}(\cdot,\cdot)$ and $G_{U_jT_j}(\cdot,\cdot)$ be the joint distribution functions of (U_1,T_1) and (U_j,T_j) , $j > 1$, respectively. While calculating $S(u,m)$ the following [(3.2) - (3.5)] must be taken into account:

$$S(u,m) = P[U_m=0 \mid U_0=u] \tag{3.2}$$

$$U_k = 0 \text{ implies that } U_i = 0 \text{ for all } i \geq k . \tag{3.3}$$

Let DS_j denote the distance between the second fast vehicle and the first slow vehicle preceding it when the fast vehicle is at the entrance of the jth Type II section, and let

$$H = 1/v_1 - 1/v_2 .$$

Then

$$\begin{gathered}(U_m=0 \text{ and } T_j = \ell_2/v_2 \text{ for } j \leq m) \text{ implies that} \\ \left(U_j < (\ell_1+\ell_2)H \text{ and } DS_{j+1} \geq v_1[U_j - \ell_1H]^+\right) ,\end{gathered} \tag{3.4}$$

where $[a]^+$ is the positive part of a . The condition on U_j results from (d) of Section 2 and the information on DS_{j+1} is due to the leading fast vehicle having been unimpeded in the jth Type II section. Finally,

$$(U_m=0 \text{ and } T_j > \ell_2/v_2 \text{ for } j \leq m) \text{ implies that } U_j < \ell_1H . \tag{3.5}$$

Unless (3.5) holds, No. 2 is not able to pass the slow vehicle that impeded No. 1 at the jth Type II section before the beginning of the (j+1)st Type II section. From (a) of Section 2 and Equations (3.4) and (3.5), we deduce that $G_{U_jT_j}(\cdot,\cdot)$, j=2,3,... are identical. Furthermore, we realize that the reason for the difference between $F_{U_1T_1}(\cdot,\cdot)$ and $G_{U_jT_j}(\cdot,\cdot)$ is that when $T_j = \ell_2/v_2$, $j \geq 1$, we have prior information on DS_{j+1} .

The probability $S(u,1)$ has a simple expression (see Equation (A.1) in the appendix). The determination of $S(u,m)$, $m \geq 2$, is carried out as follows. Define

$$Q_{m-1}(u_{m-2},t_{m-2}) = \int_{t=t_1}^{t_2} \int_{u=0}^{t_0} dG_{U_{m-1},T_{m-1}}(u,t \mid U_{m-2} = u_{m-2}, T_{m-2} = t_{m-2})$$

$$\cdot\ P[U_m=0 \mid U_{m-1}=u,\ T_{m-1}=t] \ , \quad m=2,3,\ldots \tag{3.6}$$

and

$$Q_j(u_{j-1},t_{j-1}) = \int_{t=t_1}^{t_2} \int_{u_j=0}^{t_0} dG_{U_jT_j}(u_j,t \mid U_{j-1}=u_{j-1},\ T_{j-1}=t_{j-1})Q_{j+1}(u_j,t) \ ,$$

$$j=2,3,\ldots,m-2 \ . \tag{3.7}$$

Hence

$$Q_1(u,m) = S(u,m) = \int_{t=t_1}^{t_2} \int_{u_1=0}^{t_0} dF_{U_1T_1}(u_1,t \mid U_0=u)Q_2(u_1,t) \ , \tag{3.8}$$

where

$$t_0 = (\ell_1+\ell_2)(1/v_1 - 1/v_2) \ , \quad t_1 = \ell_2/v_2 \ , \quad \text{and} \quad t_2 = \ell_2/v_1 \ . \tag{3.9}$$

The conditional distributions of G and F and the probability $P[U_{m=0} \mid U_{m-1} = u \ , \ T_{m-1} = t]$ are derived in the appendix.

Now we determine the distribution function of a Type B interval. Let $A(m)$ denote the event that the first of the two fast vehicles is impeded in at least one of the first m Type II sections, and let $B(u,m)$ denote the event that the time from a random arrival to the end of the first Type B interval of order m is longer than u. Clearly,

$$S(u,m) = P[A(m) \cap B(u,m)] = P[A(m)]P[B(u,m) \mid A(m)] , \tag{3.10}$$

$$1 - P[A(m)] = P[T_1 = \ell_2/v_2, T_2 = \ell_1/v_2, \ldots, T_m = \ell_2/v_2] ,$$

and due to the independence of T_i , $i=1,\ldots,m$, one obtains

$$1 - P[A(m)] = \Big(P[T = \ell_2/v_2]\Big)^m = e^{-m\lambda_1 \ell_2 H} . \tag{3.11}$$

Now let $X(m)$ denote a Type B interval of order m , let $X'(m)$ denote a Type B interval of order m containing a random arrival, and let $F_{X(m)}(\cdot)$ and $F_{X'(m)}(\cdot)$ denote the respective distributions. It is known that

$$dF_{X'(m)}(x) = \begin{cases} \dfrac{x dF_{X(m)}(x)}{E[X(m)]} , & 0 \le x \le (\ell_1+\ell_2)H \\ 0 , & \text{otherwise;} \end{cases} \tag{3.12}$$

hence, because the allocation of a random point is uniformly distributed in $X'(m)$, we obtain

$$P[B(u,m) \mid A(m)] = \int_{x=u}^{t_0} \frac{x-u}{x} \frac{x dF_{X(m)}(x)}{E[X(m)]} . \tag{3.13}$$

We insert (3.13) and (3.11) into (3.10) and get

$$S(u,m) = \frac{1 - e^{-m\lambda_1 \ell_2 H}}{E[X(m)]} \left[\int_{x=u}^{t_0} x dF_{X(m)}(x) - u\Big(1-F_{X(m)}(u)\Big) \right] . \tag{3.14}$$

Differentiating (3.14) with respect to u and denoting

$$S'(u,m) = \frac{d}{du} S(u,m)$$

yields

$$S'(u,m) = \frac{1 - e^{-m\lambda_1 \ell_2 H}}{E[X(m)]} [1-F_{X(m)}(u)] . \tag{3.15}$$

Using the fact that $F_{X(m)}(0) = 0$, we get from (3.15) that

$$E[X(m)] = \frac{1 - e^{-m\lambda_1 \ell_2 H}}{S'(0,m)} \tag{3.16}$$

and

$$F_{X(m)}(u) = \frac{S'(u,m)}{S'(0,m)} . \tag{3.17}$$

The expression $S'(u,m)$ is simple for $m=1$ and becomes more messy as m increases.

We are now able to calculate the probability function of the number of fast vehicles in a platoon. Let $N(m)$ denote the number of fast vehicles in a platoon arriving at the end of the mth Type II section, let $p_{N(m)}(\cdot)$ denote its probability function, and let $C(m)$ denote the event that the platoon has been impeded in at least one of the first m Type II sections. Clearly,

$$p_{N(m)}(n) = P[C(m)]P[N(m)=n \mid C(m)] + P[\bar{C}(m)]P[N(m)=n \mid \bar{C}(m)] ,$$

where $\bar{C}(m)$ is the complement of $C(m)$. The arrival of fast vehicles at the highway is Poisson, hence

$$P[N(m)=n \mid \bar{C}(m)] = \begin{cases} 1 , & n = 1 \\ 0 , & \text{otherwise.} \end{cases} \tag{3.18}$$

Denote by $q_{N(m)}$ the probability that n fast vehicles arrive at a Type B interval.

$$q_{N(m)}(n) = \int_0^{\ell_0} \frac{(\lambda_2 x)^n}{n!} e^{-\lambda_2 x} dF_{X(m)}(x) , \tag{3.19}$$

and therefore

$$P[N(m)=n \mid C(m)] = \frac{q_{N(m)}(n)}{1 - q_{N(m)}(0)} , \quad n=1,2,3,\ldots . \tag{3.20}$$

The platoons that are impeded in at least one of the m first Type II sections arrive at the end of the mth Type II section at a rate of $\lambda_2\left(1 - e^{-m\lambda_1 \ell_2 H}\right) / E[N(m) \mid C(m)]$; the platoons that are not impeded arrive at the same point with rate $\lambda_2 e^{-m\lambda_1 \ell_2 H}$; hence

$$P[C(m)] = \frac{\dfrac{\left(1 - e^{-m\lambda_1 \ell_2 H}\right)}{E[N(m) \mid C(m)]}}{\dfrac{\left(1 - e^{-m\lambda_1 \ell_2 H}\right)}{E[N(m) \mid C(m)]} + e^{-m\lambda_1 \ell_2 H}} . \tag{3.21}$$

We calculate the conditional expectation of $N(m)$ using (3.19) and (3.16), and the result we insert into (3.21) to obtain

$$P[C(m)] = \frac{-\left(1 - q_{N(m)}(0)\right) S'(0,m)}{-\left(1 - q_{N(m)}(0)\right) S'(0,m) + \lambda_2 e^{-m\lambda_1 \ell_2 H}} . \tag{3.22}$$

Combining (3.22), (3.20), (3.19), (3.18), (3.17), (3.6), (3.7), and (3.8) yields the desired probability function, from which we obtain

$$E[N(m)] = \frac{\lambda_2 E[X(m)]}{\left(1 - e^{-m\lambda_1 \ell_2 H}\right)\left(1 - \int_0^{t_0} e^{-\lambda_2 x} dF_{X(m)}(x)\right) + \lambda_2 E[X(m)] e^{-m\lambda_1 \ell_2 H}} . \tag{3.23}$$

The expectation of $N(m)$ can be bounded without calculating $dF_{X(m)}(\cdot)$. For this we use Jensen's inequality to get

$$\int_0^{t_0} e^{-\lambda_2 x} dF_{X(m)}(x) > e^{-\lambda_2 E[X(m)]} , \tag{3.24}$$

and as

$$e^{-\lambda_2 x} \leq 1 - \frac{1 - e^{-\lambda_2 t_0}}{t_0} x , \quad \text{for any } 0 \leq x \leq t_0 ,$$

we obtain

$$\int_0^{t_0} e^{-\lambda_2 x} dF_{X(m)}(x) \leq 1 - \frac{1 - e^{-\lambda_2 t_0}}{t_0} E[X(m)] . \tag{3.25}$$

Using (3.24) and (3.25) we get

$$E_1[N(m)] = \frac{\lambda_2 E[X(m)]}{\left(1 - e^{-m\lambda_1 \ell_2 H}\right)\left(1 - e^{-m\lambda_2 E[X(m)]}\right) + \lambda_2 E[X(m)] e^{-m\lambda_1 \ell_2 H}} \leq E[N(m)]$$

$$\leq \frac{\lambda_2}{\left(1 - e^{-m\lambda_1 \ell_2 H}\right) \dfrac{1 - e^{-\lambda_2 t_0}}{t_0} + \lambda_2 e^{-m\lambda_1 \ell_2 H}} = E_2[N(m)] .$$

It is obvious that $E[N(m)]$ is nondecreasing, hence its upper and lower bounds, $EU[N(m)]$ and $EL[N(m)]$, respectively, can be established from $E_1[N(m)]$ and $E_2[N(m)]$ as follows:

$$EU[N(m)] = \max\{E_2[N(m)], EU[N(m-1)]\} ,$$

and

$$EL[N(m)] = \max\{E_1[N(m)], EL[N(m-1)]\} .$$

The expected number of fast vehicles in a platoon behind a slow vehicle can be calculated without applying the procedure outlined above. Let $J(\ell)$ denote the number of fast vehicles in a platoon moving behind a slow vehicle at a distance ℓ from the entrance, and let $r_1(\ell)$ be the probability that a fast vehicle is traveling at a speed v_1 at that point. We now show that

$$E[J(\ell)] = \frac{\lambda_2}{\lambda_1} r_1(\ell) . \tag{3.26}$$

To prove (3.26), suppose we observe the arrival process of vehicles at the point located at a distance ℓ from the entrance. Let t denote the length of the observation period and let $J_1(t,\ell)$ and $J_2(t,\ell)$ denote the number of slow and fast vehicles, respectively, which arrive at that point in the time interval considered. Denote by Y_i , $i=1,2,\ldots,J_1(t,\ell)$, the number of fast vehicles in the platoon behind the ith slow vehicle. Using the strong law of large numbers, one may obtain

$$\lim_{t\to\infty} \frac{\sum_{i=1}^{J_1(t,\ell)} Y_i}{J_1(t,\ell)} = E[J(\ell)] \ . \tag{3.27}$$

The left-hand side of (3.27) can be rewritten as

$$\lim_{t\to\infty} \frac{\sum_{i=1}^{J_1(t,\ell)} Y_i}{J_1(t,\ell)} = \lim_{t\to\infty} \frac{\sum_{i=1}^{J_1(t,\ell)} Y_i}{J_2(t,\ell)} \frac{J_2(t,\ell)}{t} \frac{t}{J_1(t,\ell)}$$

$$= \lim_{t\to\infty} \frac{\sum_{i=1}^{J_1(t,\ell)} Y_i}{J_2(t,\ell)} \lim_{t\to\infty} \frac{J_2(t,\ell)}{t} \lim_{t\to\infty} \frac{t}{J_1(t,\ell)} \ . \tag{3.28}$$

Using the strong law of large numbers for the three series in (3.28) yields the desired result. The probability $r_1(\ell)$ is obtained from (3.1) and is given by

$$r_1(\ell) = \begin{cases} 0 & , \text{ if } \ell - \left[\frac{\ell}{\ell_1+\ell_2}\right](\ell_1+\ell_2) \leq \ell_1 \\ 1 - e^{-\lambda_1 x(1/v_1 - 1/v_2)} & , \text{ if } 0 < x = \ell - \left[\frac{\ell}{\ell_1+\ell_2}\right](\ell_1+\ell_2) - \ell_1 < \ell_2 \ , \end{cases} \tag{3.29}$$

where [a] is the biggest integer which is equal to or smaller than a .

4. Numerical Examples

We calculate here $E[X(1)]$, $E[X(2)]$, $E[N(1)]$, and the upper and lower bounds of $E[N(1)]$ and $E[N(2)]$. Using the procedure outlined in the previous section, we obtain

$$E[X(1)] = \frac{1 - e^{-\lambda_1 \ell_2 H}}{\lambda_1} \ ,$$

$$E[X(2)] = \frac{1 - e^{-2\lambda_1 \ell_2 H}}{\frac{\lambda_1}{2} + 2\lambda_1 e^{-\lambda_1 \ell_2 H} - \frac{1}{2}\lambda_1 e^{-2\lambda_1 \ell_2 H}} \ ,$$

and

$$E[N(1)] = \frac{1}{\frac{\lambda_1}{\lambda_1+\lambda_2}\left(1 - e^{-(\lambda_1+\lambda_2)\ell_2 H}\right) + e^{-\lambda_1 \ell_2 H}} ,$$

(note, $E[X(1)] \leq E[X(2)]$). The following are the numerical values assumed: $v_1 = 60_{Km/h}$, $v_2 = 80_{Km/h}$, $\lambda_2 = 225$ vehicles per hour, and $\ell_1 = \ell_2 = 1.0$ Km (see graphs).

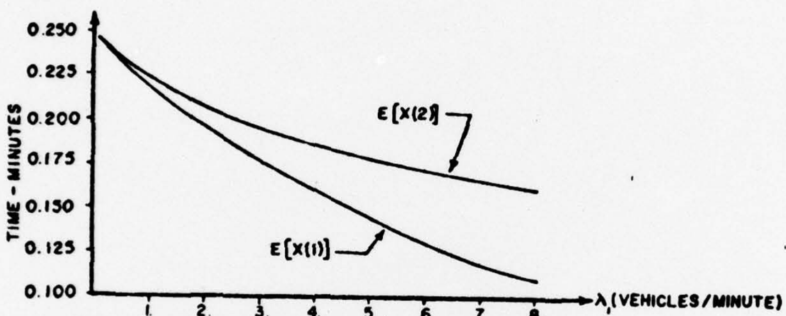

FIG. 1: EXPECTED LENGTH OF TYPE B SECTION AS A FUNCTION OF ARRIVAL RATE OF SLOW VEHICLES.

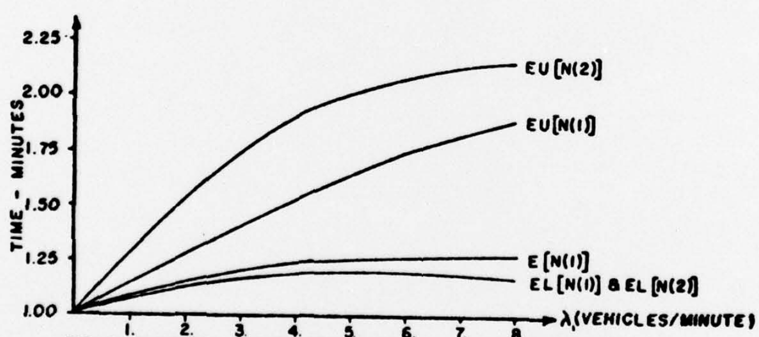

FIG. 2: EXPECTED LENGTH OF A PLATOON AS A FUNCTION OF THE ARRIVAL RATE OF THE SLOW VEHICLES.

FIG. 3: THE AVERAGE SPEED OF A FAST VEHICLE AS A FUNCTION OF THE ARRIVAL RATE OF THE SLOW VEHICLE.

5. Discussion

This study discusses the platoon formation in a two-lane two-way highway under low to moderate traffic intensity, i.e., up to 4000 vehicles per day in each traffic direction. Under this traffic load, it is reasonable to assume Poisson arrivals of vehicles at the highway (see Taylor et al. [10] and Breiman [1]). As for the assumptions on the sizes of the vehicles, distances between vehicles in platoons, and the passing mechanism, none of them seems to be too restrictive under this traffic intensity. We believe that the most restrictive assumption here is that the highway is divided into Type I and Type II sections having constant lengths. Actually, it would have been more realistic to assume that the lengths of these sections are random variables, since they depend on sight and road conditions and on traffic in the opposing direction. Nevertheless, we preferred this assumption because it enabled us to analyze the movements of vehicles in one lane independently of the traffic in the other lane, and consequently to study the process of platoon formation, a difficult and complex process under any set of reasonable assumptions.

We note that the lengths of Type I and Type II road sections may be determined as functions of the traffic in the opposite lane according to the following procedure. Let Y denote the interarrival times of consecutive vehicles at the entrance of the opposite lane, let $F_Y(\cdot)$ be the distribution of Y, and let d denote the minimal interarrival time which enables safe passing. Hence,

$$\ell_1 = (E[Y>d] - d)v_2 ,$$

and ℓ_2 satisfies

$$\ell_2/v_2 = \int_{x=0}^{d} \left(y + \ell_2/v_2\right)dF_Y(y) ,$$

which yields

$$\ell_2 = \frac{\int_0^d ydF_Y(y)}{1 - F_Y(d)} v_2 .$$

Since platoon formation increases the gaps between consecutive platoons, we may actually assign to ℓ_1 a larger value and to ℓ_2 a smaller value than the ones calculated from the expression above.

We would also like to point out that even though the result that the length of a Type B section is bounded from above by $(\ell_1+\ell_2)H$, and consequently the expected length of the platoon is bounded by $\lambda_2(\ell_1+\ell_2)H$ is derived under the assumption that ℓ_1 and ℓ_2 are constants, we expect that under more general assumptions it can be shown that the expected length of a platoon is bounded by an equivalent expression.

Our final remark is in regard to three-lane highways, which are not too common. For such highways the present model may provide a very good fit if the center lane is assigned alternately to one of the traffic directions as a passing lane.

REFERENCES

[1] BREIMAN, L. (1963). The Poisson tendency in traffic distribution. Ann. Math. Stat. **34** 308-311.

[2] COWAN, J. R. (1971). A road with no overtaking. Australian J. Stat. **13** (2) 94-115.

[3] COX, D. R. and P. A. W. LEWIS (1966). The Statistical Analysis of Series of Events. Methuen and Co., Ltd., London; John Wiley and Sons, Inc., New York.

[4] EPSTEIN, B., D. GALIN and E. SHLIFER (1974). Behavior of vehicles along roads for which passing is not permitted. Transportation Res. **8** 517-522.

[5] GALIN, D. and B. EPSTEIN (1974). Speeds and delays on two-lane roads, where passing is possible at given points of the road. Transportation Res. **8** 29-37.

[6] HODGSON, V. (1968). The time to drive through a no-passing zone. Transportation Sci. **2** 252-264.

[7] MILLER, A. J. (1963). An analysis of bunching in rural two-lane traffic. Operations Res. **11** 236-247.

[8] RUBINOVITCH, M. (1970). A survey of some recent models for traffic on two-lane roads. Technical Report No. 87, Department of Operations Research, Cornell University, Ithaca, New York.

[9] TANNER, J. C. (1961). Delays on a two-lane road. J. Roy. Statist. Soc. Ser. B **23** 38-63.

[10] TAYLOR, M. A. P., A. J. MILLER and K. W. OGDEN (1974). A comparison of some bunching models for rural traffic flow. Transportation Res. **8** 1-9.

APPENDIX

We calculate here some distributions needed for the evaluation of $S(u,m)$. Let R_j denote the time spent by No. 2 at the jth Type II section, and let $M_j(t)$ be the number of slow vehicles which arrive at the jth Type II section in the time interval which begins when No. 1 enters the Type II section and ends t- units of time later. To simplify the notations define $T_0 = 0$. Clearly,

$$\begin{aligned}
P[U_m=0 \mid U_{m-1}=u,\ T_{m-1}=t] &= P[M_m(u)=0,\ T_m \geq \ell_2/v_2 + u] \\
&= P[M_m(u)=0]\ P[T_m \geq \ell_2/v_2 + u] \\
&= \begin{cases} 1 & , \text{ if } u=0 \\ e^{-\lambda_1 u}\left(1 - e^{-\lambda_1(\ell_2 H - u)}\right) & , \text{ if } m=1 \text{ and } u < \ell_2 H \\ & \quad \text{or } u \leq \ell_1 H \text{ and } t = \ell_2/v_2 \\ & \quad \text{or } u < H\min\{\ell_1,\ell_2\} \text{ and } t > \ell_2/v_2 \\ e^{-\lambda_1 \ell_1 H}\left(1 - e^{-\lambda_1(\ell_2 H - u)}\right) & , \text{ if } \ell_1 H < u < \ell_2 H \text{ and } t = \ell_2/v_2 \\ 0 & , \text{ otherwise.} \end{cases}
\end{aligned} \tag{A.1}$$

For the evaluation of F we use the relation

$$U_j = U_{j-1} + (R_j - T_j) .$$

Hence,

$$\begin{aligned}
F_{U_1 T_1}(u_1, t \mid U_0=0) &= P[U_1 \leq u_1,\ T_1 \leq t \mid U_0=u] = P[R_1 \leq u_1 - u + T_1,\ T_1 \leq t \mid U_0=u] \\
&= \int_{x=\ell_2/v_2}^{t} \int_{r=\max\{\ell_2/v_2,\ t-u\}}^{u_1-u+t} dP_{r,x}(R_1 \leq r,\ T_1 \leq x \mid U_0=u)
\end{aligned} \tag{A.2}$$

When $u > \ell_2 H$, then T_1 and R_1 are i.i.d. distributed according to (3.1). If $u \leq \ell_2 H$, then the distribution of T_1 is as before, but here T_1 and R_1 are not independent and the calculation of F is based on the fact that $M_1(u)$ is a Poisson random variable with parameter $\lambda_1 u$. After evaluation of F one can derive dF , which yields, in the case $u \leq \ell_2 H$:

$$dF_{U,T_1}(u_1, t \mid U_0=u)$$

$$= \begin{cases} e^{-\lambda_1 u}\, e^{-\lambda_1 \ell_2 H} & , \quad u_1=u,\ t = \ell_2/v_2 \\ e^{-2\lambda_1 \ell_2 H}\, \lambda_1 e^{\lambda_1(u_1-u)}\, du_1 & , \quad \ell_2 H \leq u_1 \leq u + \ell_2 H,\ t = \ell_2/v_2 \\ \lambda_1^2\, e^{-2\lambda_1(\ell_2/v_1)}\, e^{\lambda_1(2t_1+u_1-u)}\, du_1 dt & , \quad \ell_2/v_1 - t < u_1 < \min\{u + \ell_2/v_1 - t,\ \ell_1 H\}, \\ & \quad\ u_1 \neq u + \ell_2/v_2 - t,\ \ell_2/v_2 < t \leq \ell_2/v_1 \\ \lambda_1^2\, e^{-2\lambda_1(\ell_2/v_1)}\, e^{\lambda_1(2t+u_1-u)}\, du_1 dt + \lambda_1 e^{-\lambda_1(\ell_2/v_1 + u_1 - u)}\, dt & , \quad \ell_2/v_2 < t < \ell_2/v_2 + u, \\ & \quad\ u_1 = u + \ell_2/v_2 - t < \ell_1 H \\ \lambda_1\, e^{-\lambda_1 u}\, e^{-\lambda_1(\ell_2/v_1 - t)}\, dt & , \quad u_1=0,\ \ell_2/v_2 + u \leq t \leq \ell_2/v_1 \\ 0 & , \quad \text{otherwise.} \end{cases} \tag{A.3}$$

In the case $u > \ell_2 H$, then

$$dF_{U,T_1}(u_1, t \mid U_0 = u) =$$

$$= \begin{cases} e^{-2\lambda_1 \ell_2 H} & , \quad u_1 = u,\ t = \ell_2/v_2 \\ e^{-2\lambda_1 \ell_2 H} \lambda_1 e^{\lambda_1(u_1 - u)} du_1 & , \quad u < u_1 \le u + \ell_2 H,\ t = \ell_2/v_2, \\ & \quad\ \ \ell_2/v_2 < t \le \ell_2/v_1 \\ \lambda_1^2 e^{-2\lambda_1(\ell_2/v_1)} e^{\lambda_1(2t + u_1 - u)} du_1 dt & , \quad u - \ell_2 H \le u_1 < \min\{u + \ell_2 H, \ell_1 H\}, \\ & \quad\ \ u_1 \neq u + \ell_2/v_2 - t \\ \lambda_1^2 e^{-2\lambda_1(\ell_2/v_1)} e^{\lambda_1(2t + u_1 - u)} du_1 dt + \lambda_1 e^{-2\lambda_1 \ell_2 H} e^{\lambda_1(u - u_1)} dt & , \quad \ell_2/v_2 < t \le \ell_2/v_1, \\ & \quad\ \ u_1 = u + \ell_2/v_2 - t \\ 0 & , \quad \text{otherwise.} \end{cases} \tag{A.4}$$

As for the evaluation of G, from (3.5) and (3.6) we deduce that for any $j \geq 2$:

(i) When $T_{j-1} > \ell_2/v_2$ and $U_{j-1} < \ell_1 H$, or when $T_j = \ell_2/v_2$ and $U_{j-1} < \ell_1 H$, then $G = F$.

(ii) When $T_{j-1} > \ell_2/v_2$ and $U_{j-1} \geq \ell_1 H$, then No. 2 will never move with No. 1 in the same platoon. Here we define $dG = 0$.

(iii) When $T_{j-1} = \ell_2/v_2$ and $U_{j-1} > \ell_1 H$, then $M_j(\ell_1 H)$ is a Poisson random variable with parameter $\lambda_1 \ell_1 H$ and

$$P[M_j(U_{j-1}) - M_j(\ell_1 H) = 0] = 1 .$$

Here we have

$$G_{U_j T_j}(u_j, t_j \mid U_{j-1} = u_{j-1},\ T_{j-1} = \ell_2/v_2)$$

$$= \int_{x = \ell_2/v_2}^{t} \int_{r - \max\{\ell_2/v_2,\ t - u_{j-1}\}}^{\ell_2/v_1 - (u_{j-1} - \ell_1 H)} dP_{r,x}(R_j \le r,\ T_j \le x \mid U_{j-1} = u_{j-1},\ T_{j-1} = \ell_2/v_2) . \tag{A.5}$$

If $\ell_1 H < U_{j-1} < \ell_2 H$, then R_j and T_j are not independent and (A.5) yields

$$dG_{U_jT_j}(u_j,t_j \mid U_{j-1}=u_{j-1},\ T_{j-1}=\ell_2/v_2)$$

$$= \left\{ \begin{array}{ll} e^{-\lambda_1(\ell_1+\ell_2)H} & ,\quad u_j=u_{j-1},\ t_j = \ell_2/v_2 \\ e^{-\lambda_1\ell_2H}\ \lambda_1\ e^{-\lambda_1[(\ell_1+\ell_2)H - u_j]} & ,\quad \ell_2H < u_j < (\ell_1+\ell_2)H,\ t_j = \ell_2/v_2 \\ \lambda_j^2\ e^{-\lambda_1(\ell_2/v_1 - t_j)}\ e^{-\lambda_1(\ell_2/v_1 - u_j - t_j + \ell_1H)}\, du_j dt_j & ,\quad \ell_2/v_2 < t_j \leq \ell_2/v_1, \\ & \quad\ \ \ell_2/v_1 - t_j < u_j < \ell_1H + \ell_2/v_1 - t_j, \\ & \quad\ \ u_j \neq u_{j-1} + \ell_2/v_2 - t \\ \lambda_1\ e^{-\lambda_1\ell_1H}\ e^{-\lambda_1(\ell_2/v_1 - t_j)}\, dt_j & \\ + \lambda_1^2\ e^{-\lambda_1(\ell_2/v_1 - t_j)}\ e^{-\lambda_1(\ell_2/v_1 - u_j - t_j + \ell_1H)}\, du_j dt_j & ,\quad \ell_2/v_2 < t_j < \ell_2/v_2 + u_{j-1}, \\ & \quad\ \ u_j = u_{j-1} + \ell_2/v_2 - t \\ \lambda_1\ e^{-\lambda_1\ell_1H}\ e^{-\lambda_1(\ell_2/v_1 - t_j)}\, dt_j & ,\quad u_{j-1} + (\ell_2/v_2) < t_j < \ell_2/0,\ u_j=0 \\ 0 & ,\quad \text{otherwise.} \end{array} \right. \tag{A.6}$$

If $H\max\{\ell_2,\ell_1\} < U_{j-1} < (\ell_1+\ell_2)H$, then R_j and T_j are independent, and (A.5) yields

$$dG_{U_j,T_j}(u_j,t_j \mid U_{j-1}=u_{j-1},\ T_{j-1} = \ell_2/v_2)$$

$$= \left\{ \begin{array}{ll} e^{-2\lambda_1\ell_2H}\ e^{-\lambda_1(u_{j-1} - \ell_1H)} & ,\quad u_j=u_{j-1},\ t_j = \ell_2/v_2 \\ \lambda_1\ e^{-\lambda_1\ell_2H}\ e^{-\lambda_1[(\ell_1+\ell_2)H - u_j]}\, du_j & ,\quad u_{j-1} < u_j < (\ell_1+\ell_2)H,\ \ t_j = \ell_2/v_2 \\ \lambda_1^2\ e^{-\lambda_1(\ell_2/v_1 - t_j)}\ e^{-\lambda_1(\ell_2/v_1 - u_j - t_j + \ell_1H)}\, du_j dt_j & ,\quad \ell_2/v_2 < t_j \leq \ell_2/v_1, \\ & \quad\ \ u_{j-1} + \ell_2/v_2 - t_j < u_j < \ell_1H + \ell_2/v_1 - t, \\ & \quad\ \ u_j \neq u_{j-1} + \ell_2/v_2 - t_j \\ \lambda_1\, e^{-\lambda_1[(\ell_1+\ell_2)H - u_{j-1}]}\ e^{-\lambda_1(\ell_2/v_1 - t_j)}\, dt_j\ + & \\ \lambda_1^2\ e^{-\lambda_1(\ell_2/v_1 - t_j)}\ e^{-\lambda_1(\ell_2/v_1 - u_j - t_j + \ell_2H)}\, du_j dt_j & ,\quad \ell_2/v_2 < t_j \leq \ell_2/v_1,\ u_j = u_{j-1} + \ell_2/v_2 - t_j \\ 0 & ,\quad \text{otherwise.} \end{array} \right. \tag{A.7}$$

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