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FINITE ELEMENT ANALYSIS OF A REINFORCED EARTH WALL

by

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Work Unit 004

FINITE ELEMENT ANALYSIS OF A REINFORCED EARTH WALL
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Reinforced earth consists of a soil mass whose engineering characteristics and performance have been improved by the introduction of small quantities of frictional material that possesses a relatively high tensile strength. The analysis of a reinforced earth system which incorporates metal strips placed at known intervals in the vertical and transverse directions within the soil mass is a complex problem. The complexity arises from the fact that reinforced earth is a three-dimensional problem and cannot be solved without imposing

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20. ABSTRACT (Continued).

simplification with regard to the mechanical behavior of the system as well as the constitutive relationship governing the interaction between various components of the reinforced earth mass.

The investigation reported herein describes a simplified approach for analyzing the instrumented reinforced earth wall which was previously constructed and loaded to failure at the U. S. Army Engineer Waterways Experiment Station. The three-dimensional problem was approximated by a structurally equivalent two-dimensional system and a two-dimensional finite element method was used in the analysis. The nonlinear behavior of the earth fill of the reinforced earth wall was simulated by hyperbolic formulation. The behavior of the metal skin element and reinforcing strips was assumed linear elastic until the yield stress was reached; thereafter, these metal components were assumed to fail plastically. Interface elements, to accommodate slippage between components of the reinforced earth wall, were also employed in the analysis.

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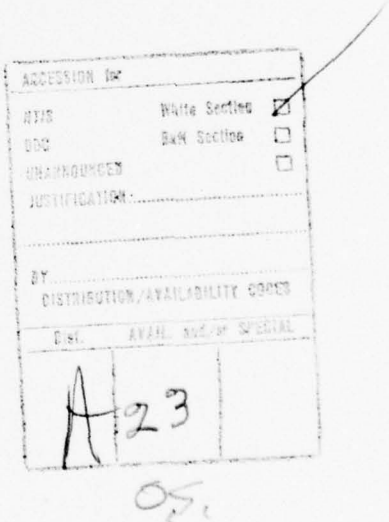

PREFACE

The study reported herein was conducted at the U. S. Army Engineer Waterways Experiment Station (WES) under the sponsorship of the Office, Chief of Engineers, Project 4A161102AT22, "Theory and Principles of Reinforced Earth," Task A2, Work Unit 004.

The finite element and parametric studies described were performed during the period July 1975 through February 1977 by Drs. M. M. Al-Hussaini and L. D. Johnson, assisted by Mr. Y. S. Jeng, all of the Soil Mechanics Division (SMD), Soils and Pavements Laboratory (S&PL). This report was written by Dr. Al-Hussaini, assisted by Dr. Johnson, under the general direction of Mr. C. L. McAnear, Chief, SMD, and Mr. J. P. Sale, Chief, S&PL.

Directors of WES during the investigation and the preparation of this report were COL G. H. Hilt, CE, and COL J. L. Cannon, CE. Technical Director was Mr. F. R. Brown.

CONTENTS

CONVERSION FACTORS, U. S. CUSTOMARY TO METRIC (SI)
UNITS OF MEASUREMENT

U. S. customary units of measurement used in this report can be converted to metric (SI) units as follows:

Multiply	By	To Obtain
inches	25.4	millimetres
feet	0.3048	metres
square inches	6.4516	square centimetres
square feet	0.09290304	square metres
pounds (mass)	0.4535924	kilograms
pounds (force) per square inch	6894.757	pascals
pounds (force) per square foot	47.88026	pascals
kips (force) per square inch	6894.757	kilopascals
pounds (mass) per cubic foot	16.01846	kilograms per cubic metre
inches per minute	25.4	millimetres per minute
degrees (angle)	0.01745329	radians

FINITE ELEMENT ANALYSIS OF A REINFORCED EARTH WALL

PART I: INTRODUCTION

Background

1. Reinforced earth consists of a soil mass whose engineering characteristics and performance have been improved by the introduction of small quantities of frictional material that possess a relatively high tensile strength. Many reinforced earth walls are designed as gravity structures, the stability of which depends on the frictional stresses between the reinforcing elements and the surrounding soil and the strength and elastic properties of the reinforcing material. The basic design concept of a reinforced earth wall assumes that the soil mass is in active failure governed by Rankine earth pressure theory. The developed lateral pressure is assumed to be counterbalanced by the friction force that develops between the reinforcement and the surrounding soil.

2. An instrumented reinforced earth wall was constructed at the U. S. Army Engineer Waterways Experiment Station (WES) to determine the performance of such a wall during construction and surcharge loading.[1] The wall, shown in Figure 1, was reinforced with galvanized steel strips, 4 in.* wide, 0.024 in. thick, and 10 ft long, spaced at intervals of 2 and 2.5 ft in the vertical and horizontal directions, respectively. Three galvanized strips along the center line of the wall, located at elevations 1, 5, and 9 ft above the bottom of the wall (Figure 2), were instrumented with full SR-4 strain gage bridges at points 1, 2.5, 5, and 7.5 ft from the face of the wall. At the elevation of each instrumented tie and 1 ft away from the face of the wall (i.e., skin element), two pressure cells were placed to measure the induced vertical and horizontal pressures within the fill material.

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* A table of factors for converting U. S. customary units of measurement to metric (SI) units is shown on page 4.

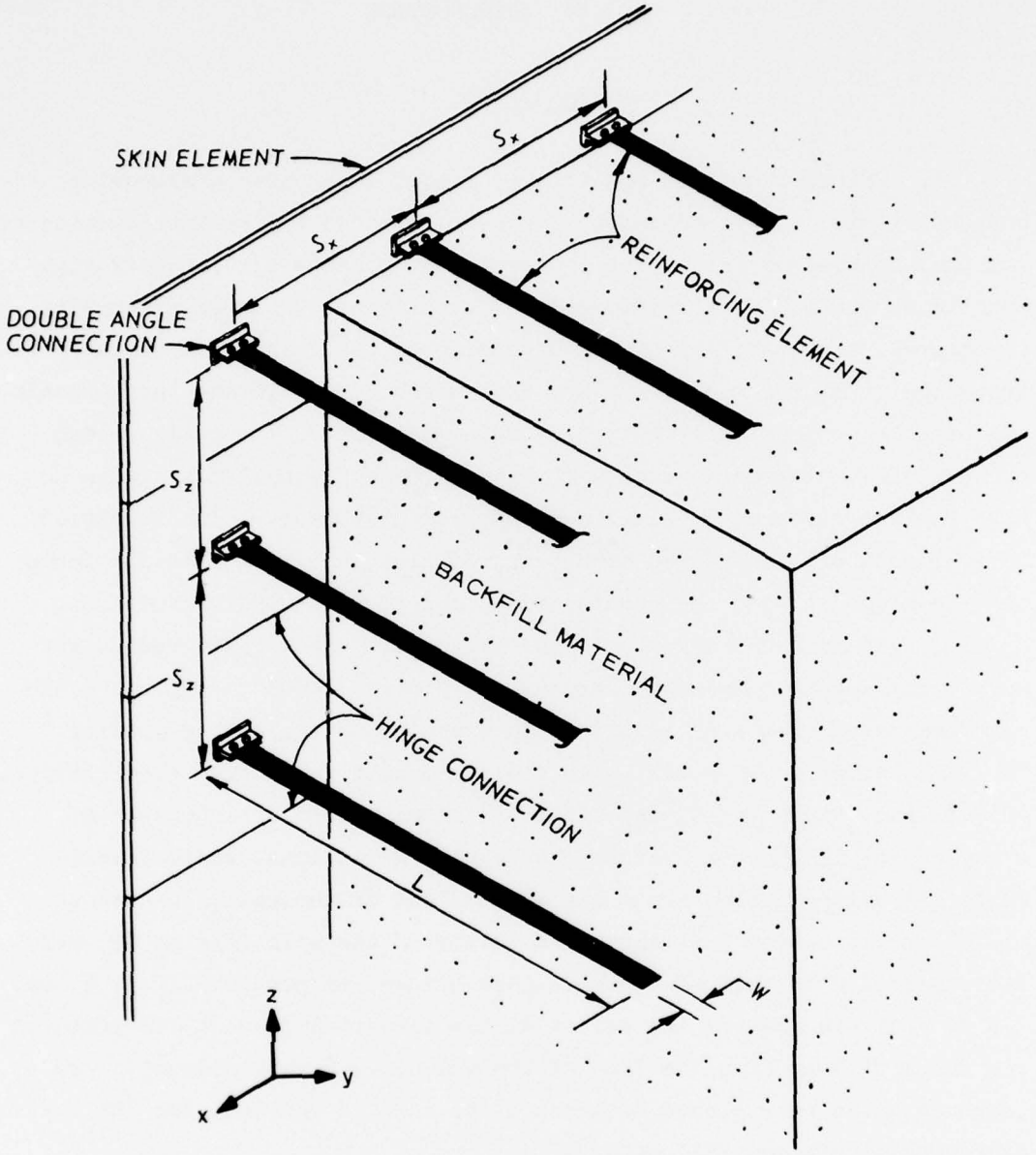

Figure 1. Schematic of major elements of reinforced earth wall

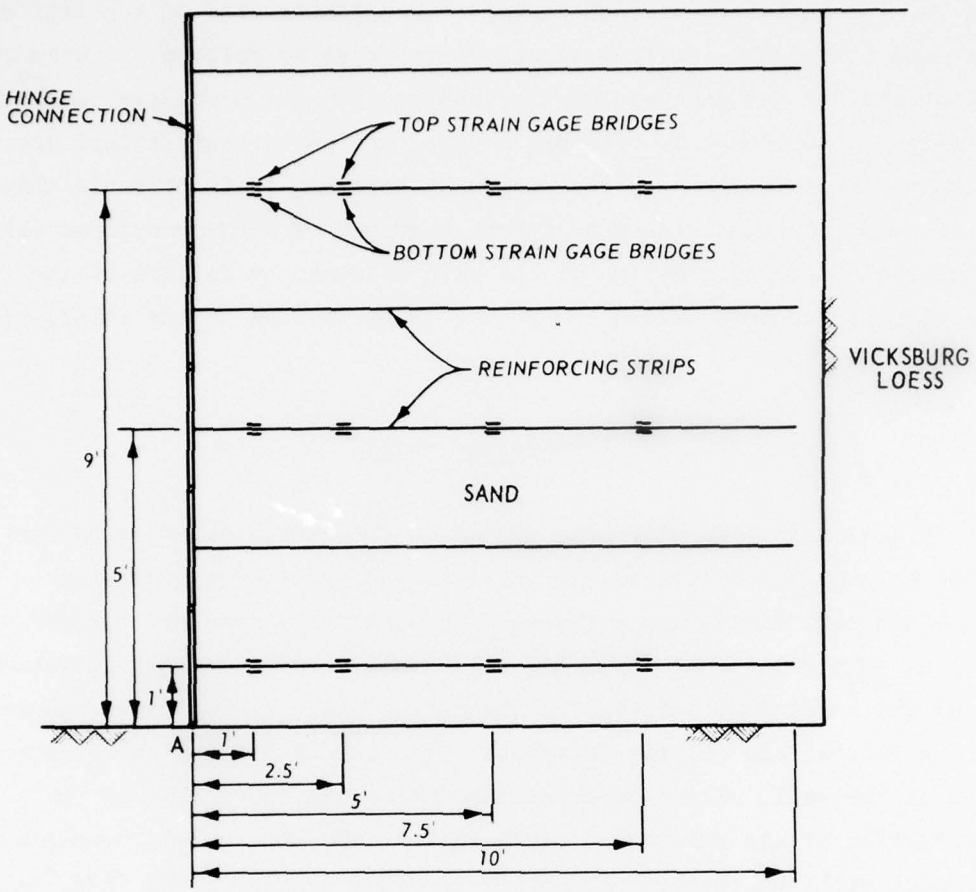

Figure 2. Schematic showing the location of the electric strain gage bridges along the center line of the retaining wall

3. The skin element, which comprised the exposed surface of the wall, was made of Alcoa T11 high-strength aluminum panels normally used in the expedient construction of airfields. Each panel was 2 ft wide, 12 ft long, and 1.6 in. thick and can be connected lengthwise to other panels by integral hinge-type connections, Figures 1 and 2. The reinforcing elements were fixed to the skin element by double angle connectors, Figure 1. Further details of these components are discussed later. A full description of the reinforced earth wall is presented by Al-Hussaini and Perry.[1]

4. The reinforced earth structure was constructed to a height of 12 ft and loaded by a uniform surcharge pressure to failure. Instrumentation readings and analyses during construction and surcharge loading are given in Reference 1. The exact cause and pattern of failure are not known since failure was catastrophic; however, it is probable that failure was initiated either by fracture of one of the connections which joined the reinforcing strips to the skin element, by failure of the skin element due to buckling and shear, or by failure of the reinforcing strips.

Purpose and Scope

5. Accurate analysis of a system incorporating reinforcing strips placed at known intervals in the vertical and transverse directions within the soil mass poses a three-dimensional (3D) problem. Proper modeling of a reinforced earth wall also involves reasonable simulation of (a) the mechanical behavior of the earth fill, skin element, and reinforcing strips, and (b) the interaction mechanisms between these components of the wall. Field observations during and subsequent to the construction of the reinforced earth wall at WES were not in complete agreement with the theoretical method proposed originally by Vidal[2] and later by Lee.[3]

6. The finite element (FE) method was consequently proposed to simulate as closely as possible various sequences of construction of the reinforced earth wall constructed at WES and to predict the distribution of stresses and deformations within the reinforced earth mass. Although 3D formulations and codes based on the FE method are possible, their use for reinforced earth walls would involve great amounts of human and computer effort. The 3D problem is therefore approximated by a structurally equivalent two-dimensional (2D) system.

7. The soil-structure plane strain 2D FE code originally developed by Clough and Duncan[4] for analyses of Port Allen and Old River Locks and subsequently modified by Radhakrishnan and Jones[5] was adopted for use in this study. This code (see Appendix A), which uses the

5-node isoparametric quadrilateral element, is capable of simulation of the incremental buildup of the wall and the concentrated loading construction sequences needed for this study. The nonlinear behavior of the earth fill of the reinforced wall is simulated by a hyperbolic formulation developed by Duncan and Chang.[6] The behavior of the metal skin element and reinforcing strips is assumed linear elastic until the yield stress is reached; thereafter, these metal components are assumed to fail plastically.

8. Interface elements available in the FE code were also used to accommodate slippage between and separation of the components of the reinforced wall. The interface elements are especially important in separating the ends of the reinforcing strips from the original in situ earth face which supports the back of the reinforced earth mass, thereby permitting the stresses in the reinforcing strips to approach zero at the ends.

9. FE analyses of the reinforced wall were performed for the construction of the wall and surcharge loading to failure. The numerical predictions of the FE analyses are compared with field observations, and conclusions regarding design analyses are presented.

PART II: MODELING THE MATERIAL BEHAVIOR OF THE REINFORCED EARTH WALL

10. Because of the composite nature of the reinforced earth wall, it is essential that the stress-strain relationships for the foundation soil, fill material, reinforcing strips, and skin element are accurately defined. It is also essential that the interaction between these composite materials within the reinforced mass be considered.

Stress-Strain Behavior of Foundation Material

11. The stress-strain relationship for the foundation material was approximated by the hyperbolic formulation, summarized in Appendix B, and stated by the following expressions:*

$$E_t = (1 - \lambda_1)^2 E_i \tag{1a}$$

$$E_i = KP_a \left(\frac{\sigma_3}{P_a}\right)^n \tag{1b}$$

$$\lambda_1 = \frac{R_f(\sigma_1 - \sigma_3)(1 - \sin \phi)}{2c \cos \phi + 2\sigma_3 \sin \phi} \tag{1c}$$

where

E_t = tangent modulus
E_i = initial modulus
K = hyperbolic loading parameter
P_a = atmospheric pressure
σ_3 = minor principal stress
n = pure number
R_f = failure ratio

---

* For convenience, symbols and abbreviations used herein are listed and defined in the Notation (Appendix C).

σ_1 = major principal stress

ϕ = angle of internal friction

c = cohesion

12. Laboratory tests were not conducted on the foundation material because these tests were not required in the conventional design analysis of the wall. However, several triaxial Q tests previously conducted on soil specimens with similar characteristics were used in the FE analysis. The stress-strain curves and Mohr-Coulomb criteria of the ML soil used in the FE analysis are presented in Figures 3 and 4, respectively. The stress-strain data were replotted in the linear form, Figure 5, which shows that most of the data points of each test fell on a straight line, indicating that the shape of the stress-strain curve of the ML material conforms with the hyperbolic representation. From Figure 5, the value of the initial modulus E_i, represented by the inverse of the intercept on the ordinate, and the value of $(\sigma_1 - \sigma_3)_{ult}$, represented by the inverse of the slope of the line, were obtained. The failure ratio R_f is equal to $(\sigma_1 - \sigma_3)_f / (\sigma_1 - \sigma_3)_{ult}$ where $(\sigma_1 - \sigma_3)_f$ is the principal stress difference at failure, as illustrated in Figure 3. A summary of the ML soil parameters needed for the hyperbolic stress-strain representation is tabulated below.

σ_3 psi	E_i psi	$(\sigma_1 - \sigma_3)_f$ psi	$(\sigma_1 - \sigma_3)_{ult}$ psi	R_f	$\left(\frac{\sigma_3}{P_a}\right)$
6.95	3,700	27.78	29.85	0.93	0.473
20.84	5,550	62.51	70.42	0.89	1.418
41.67	8,000	111.12	125.00	0.89	2.835
69.45	12,500	168.75	188.70	0.89	4.790
			Average	0.90	

13. The values of the loading stiffness K and the exponential constant n needed to incorporate the effect of the confining pressure σ_3 in the hyperbolic constitutive model are obtained by plotting E_i versus (σ_3/P_a) as shown in Figure 6. The best fit line was used for the

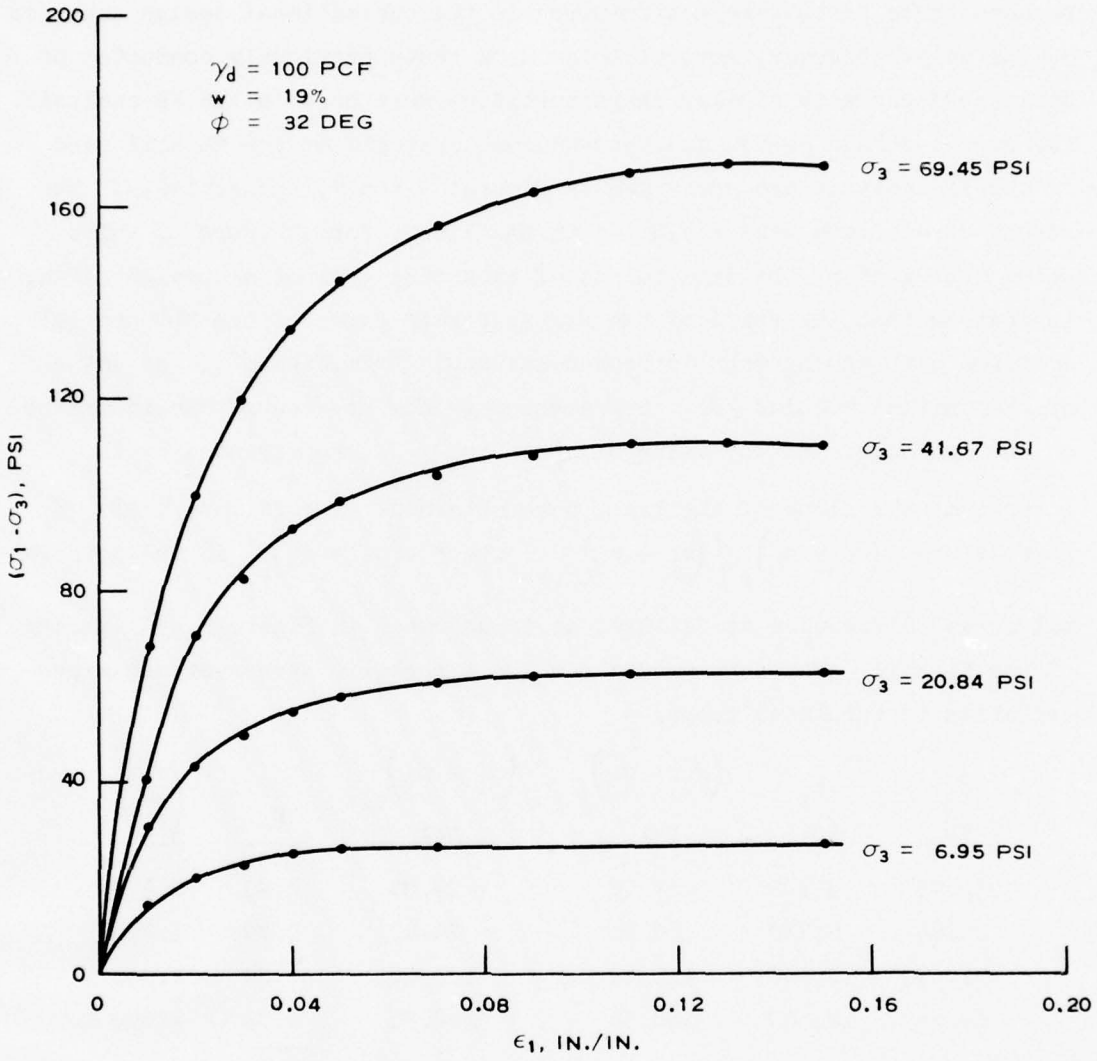

Figure 3. Stress-strain curves for the ML soil

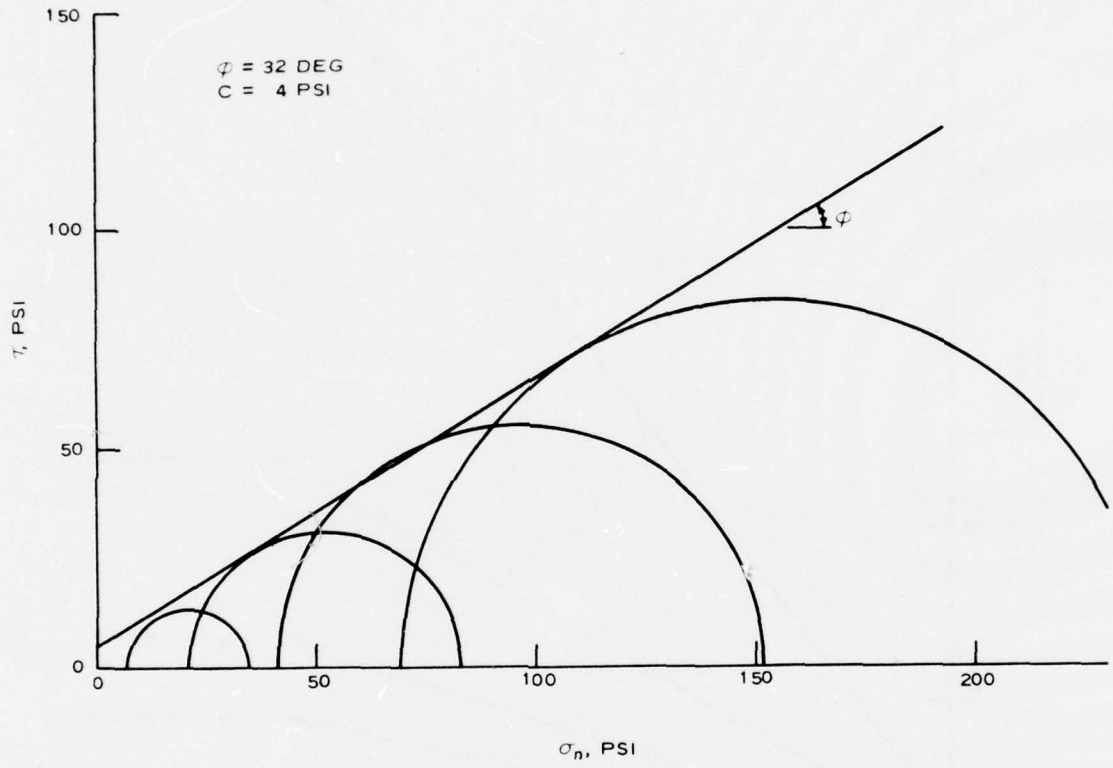

Figure 4. Mohr's failure envelope for the ML soil

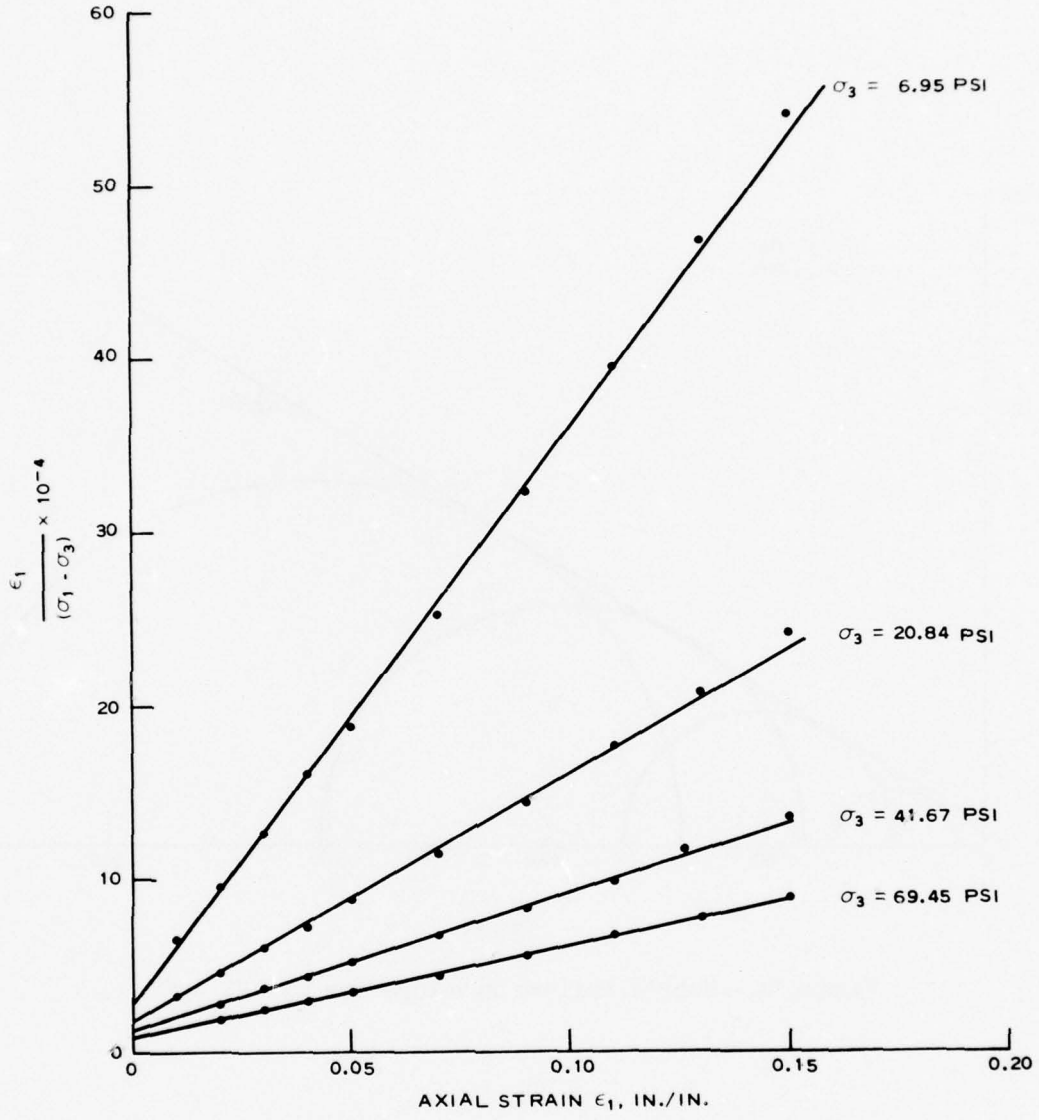

Figure 5. Linearized stress-strain curves for the ML soil

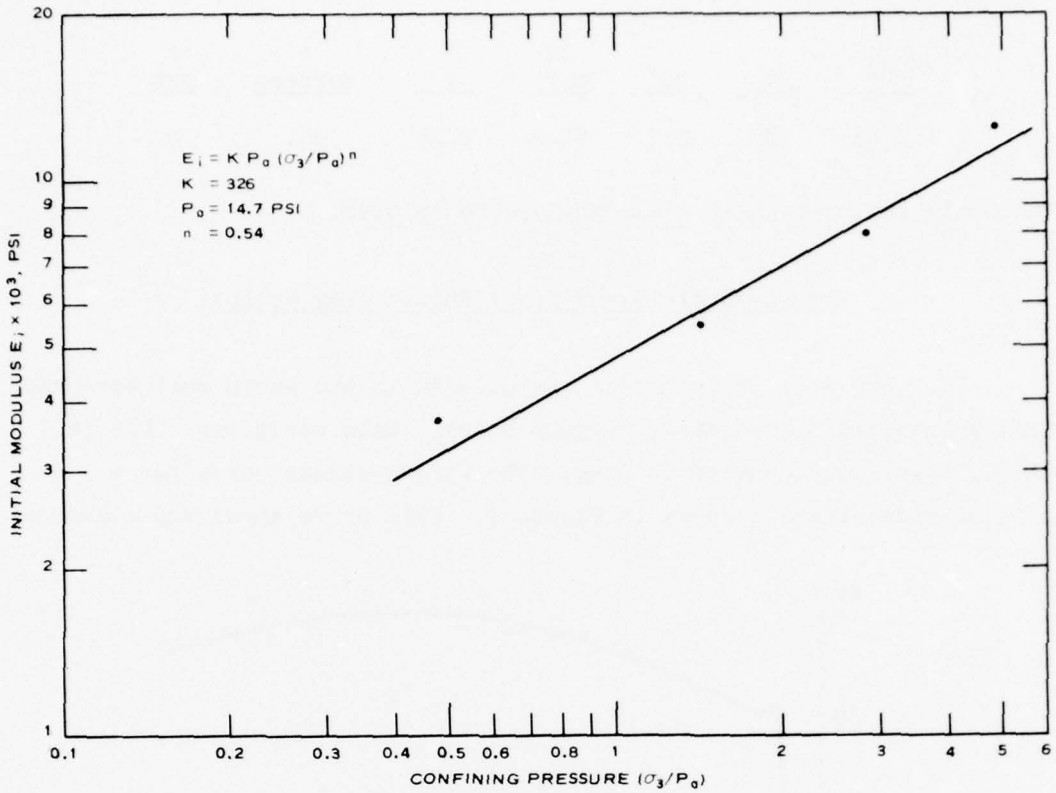

Figure 6. Variation of the initial modulus E_i versus (σ_3/P_a) for the ML soil

analysis. Since adequate information on the volumetric behavior of the ML soil was not available, Poisson's ratio ν was assumed constant at 0.33. Poisson's ratio at failure was assumed to be 0.48.

Stress-Strain Behavior of Sand Fill

14. The fill material used in the construction of the reinforced earth wall was clean concrete sand with particles ranging from subangular to angular. More detailed physical properties of the sand fill used are presented in an earlier report.[1] The same procedure of hyperbolic stress-strain analysis, presented previously in describing the behavior of the foundation soil, was used to obtain the constitutive equations for the sand fill. The hyperbolic parameters and other properties adopted for the FE analysis were as follows:

R_f	K	n	γ_d pcf	ν	ϕ degrees	c psf
0.85	580	0.5	97.2	0.30	36	0

Poisson's ratio at failure was assumed to be 0.48.

Stress-Strain Behavior of Reinforcing Strips

15. The steel reinforcing strips used in the earth wall were made from galvanized zinc-coated, 24-gage steel. Each strip was 0.025 in. thick, 4 in. wide, and 10 ft long. The stress-strain curve for a 1/2-in.-wide strip is shown in Figure 7. This curve shows the classical

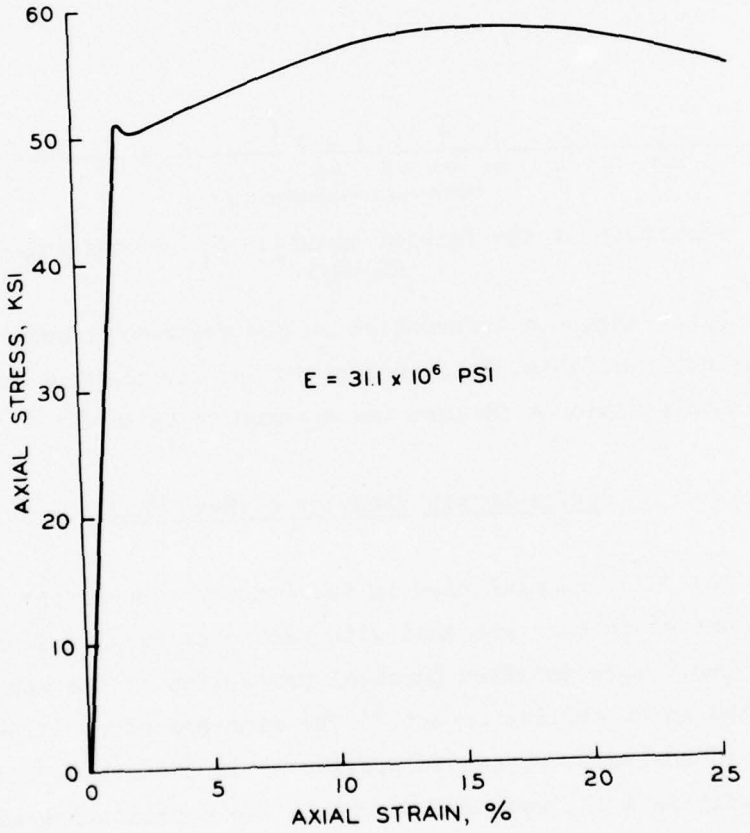

Figure 7. Average stress-strain relationship for the galvanized steel strips

stress-strain curve with modulus of elasticity $E = 31.1 \times 10^6$ psi and yield stress $\sigma_y = 51$ ksi. The Poisson's ratio ν needed for the FE analysis was assumed equal to 0.28.

Elastic Behavior of Skin Element

16. The skin element was made of Alcoa T11 high-strength aluminum panels, 2 ft wide, 12 ft long, and 1.6 in. thick, connected lengthwise to each other by a hinge connector. The physical and elastic properties of the panels are shown below.

Moment of inertia per foot of panel width	= 1.368 in.^{3}
Section modulus per foot of panel width	= 1.396 in.^{3}
Modulus of elasticity	= 10×10^6 psi
Weight per square foot	= 4.61 lb
Poisson's ratio	= 0.33
Thickness of the panel	= 1.6 in.
Thickness of the sheet	= 0.1 in.
Yield stress	= 35,000 psi

17. The reinforcing elements were fixed to the skin element through a connector as shown in Figure 8. The connector consisted of double angles, each of which was 1-1/2 by 3 by 1/4 in. and 12 in. long, used to grip the reinforcing strip and tie it to the skin element by two 1/4-in. bolts. This type of connection was simple and adequate as long as the skin element exhibited very small or no bending deformations.

Behavior of Interface Elements

18. The interface element formulation introduced by Goodman and his coworkers[7] was incorporated in the FE program used in this study. The constitutive behavior of the interface elements was based on the hyperbolic shear stress-displacement model originally formulated by Duncan and Chang[6] (Appendix A) and summarized by the following equations:

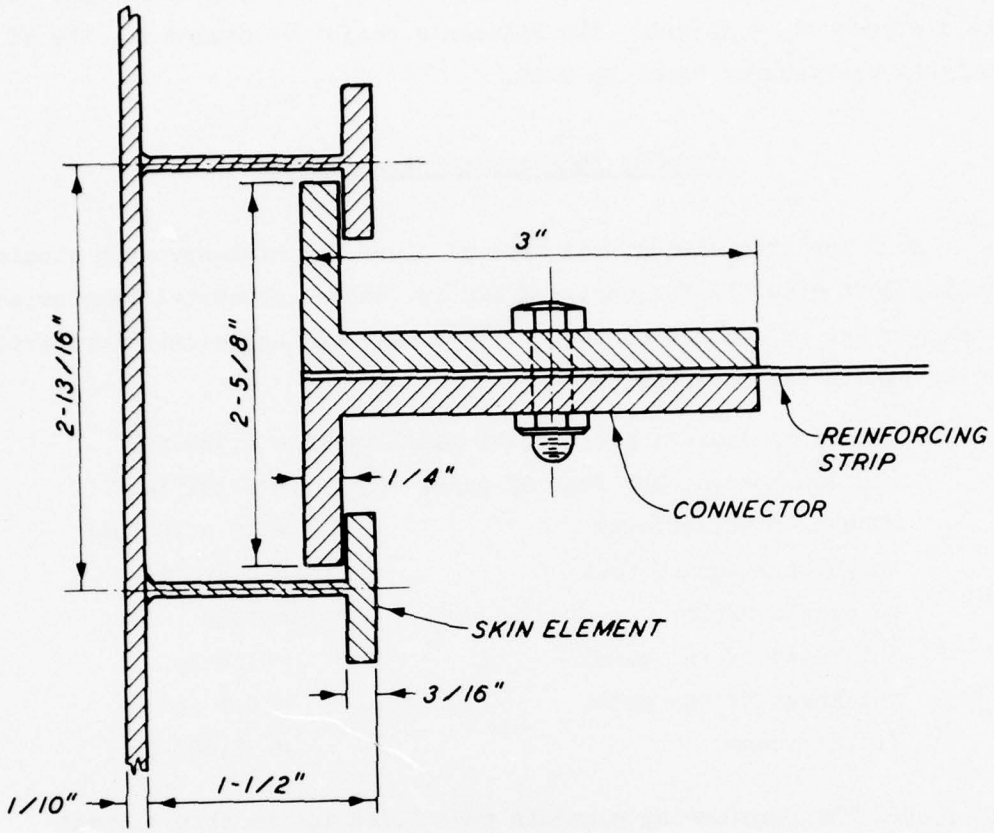

Figure 8. Details of the connector tying the reinforcing strip to the skin element

$$K_{st} = \left(1 - \lambda_2\right)^2 K_i \tag{2a}$$

$$K_i = K_j \gamma_w \left(\frac{\sigma_n}{P_a}\right)^m \tag{2b}$$

$$\lambda_2 = \frac{R_f \tau}{C_a + \sigma_n \tan \delta} \tag{2c}$$

where

K_{st} = tangent shear stiffness

K_i = initial shear stiffness

K_j = dimensionless number

γ_w = unit weight of water

σ_n = normal stress

m = exponential number

τ = shear stress

C_a = adhesion

δ = angle of friction

The normal stiffness of the interface element is assumed to behave elastically and, given a high value of 10^8 psf, to inhibit mutual penetration of the materials adjacent to the interface element. The tangent shear and normal stiffnesses were each set to a small residual value K_r of 10 psf if the normal stress of the interface element became tensile, permitting voids to develop in the interface. The shear stiffness is also reduced to the residual shear stiffness K_r if the shear stress at the interface equals or exceeds the Mohr-Coulomb strength.

Interface behavior
between reinforcing strip and sand

19. The nonlinear shear stress-displacement relationship between the sand and the reinforcing was determined, prior to the field test, using a specially designed shear box. The testing device, shown in Figure 9, is similar to the direct shear box except that the lower frame was replaced by a sheet of the galvanized steel glued to a wooden block.

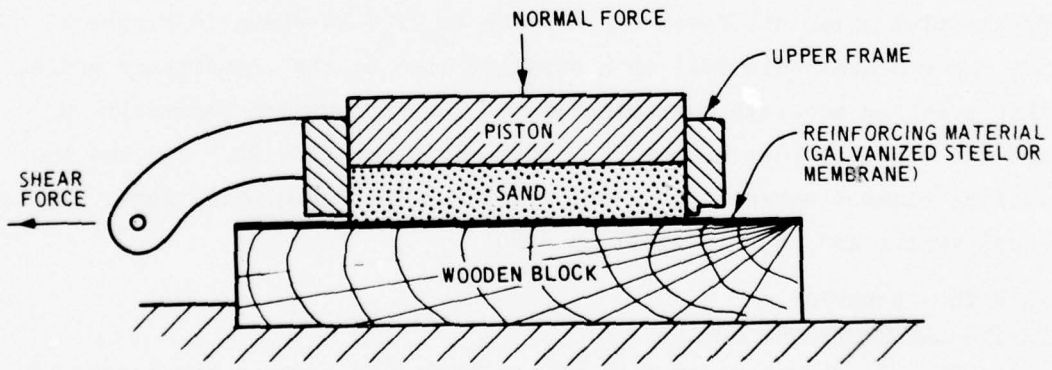

Figure 9. Modified shear box for determining skin friction angle between the sand and the reinforcing material

The sand was compacted to a dry density of 101.7 pcf. Three direct shear tests were conducted with normal stresses of 10, 50, and 100 psi, respectively, at a shear deformation rate of 0.5 in. per min.

20. The shear stress-deformation relationship between the sand and the galvanized steel is shown in Figure 10; these curves provide a reasonably accurate determination of the initial shear stiffness K_i. The ultimate shear stress τ_{ult} was obtained from the inverse of the slope of the linearized shear stress-deformation relationship as shown in Figure 11. The failure ratio R_f was obtained by dividing the peak shear stress τ_{max}, as measured from Figure 10, by the ultimate shear stress τ_{ult}; for these tests the value R_f was approximately equal to unity. The angle of friction δ and the adhesion C_a were obtained by plotting the shear stress at failure τ_{max} versus σ_n as shown in Figure 12. A summary of the parameters needed for the hyperbolic shear stress-displacement behavior of the interface elements between the sand fill and the galvanized steel is presented below.

σ_n psi	τ_{max} psi	K_i psi	C_a psi	δ degrees	R_f
10	3.06	81	0	18	1.0
50	11.95	444	0	18	1.0
100	32.09	1020	0	18	1.0

21. The influence of stress level was accounted for by plotting the initial shear stiffness K_i versus (σ_n/P_a) as shown in Figure 13. The experimental data fell on a straight line on the logarithmic scale, thus enabling accurate determination of the exponential parameter m and the dimensionless number K_j defined in Equation 2b. For the interface element between the steel strip and the surrounding sand, m is equal to 1.1 and K_j is equal to 3280.

Interface behavior
between skin element and sand

22. No direct shear test was conducted to examine the interface behavior between the skin element and the sand fill. However, the nonlinear parameter obtained previously for the sand on galvanized steel

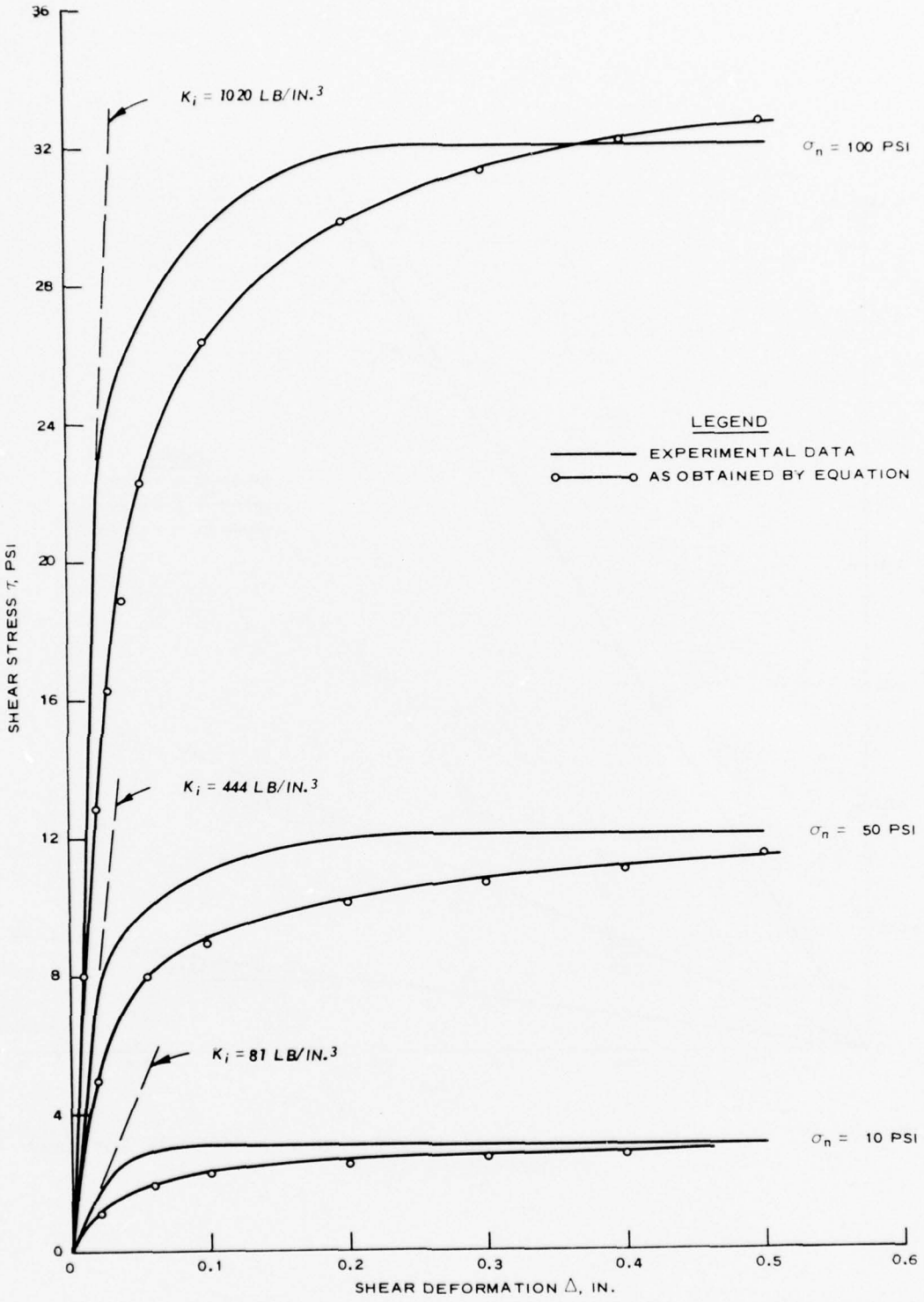

Figure 10. Shear stress-deformation relationship between the sand and the galvanized steel strips

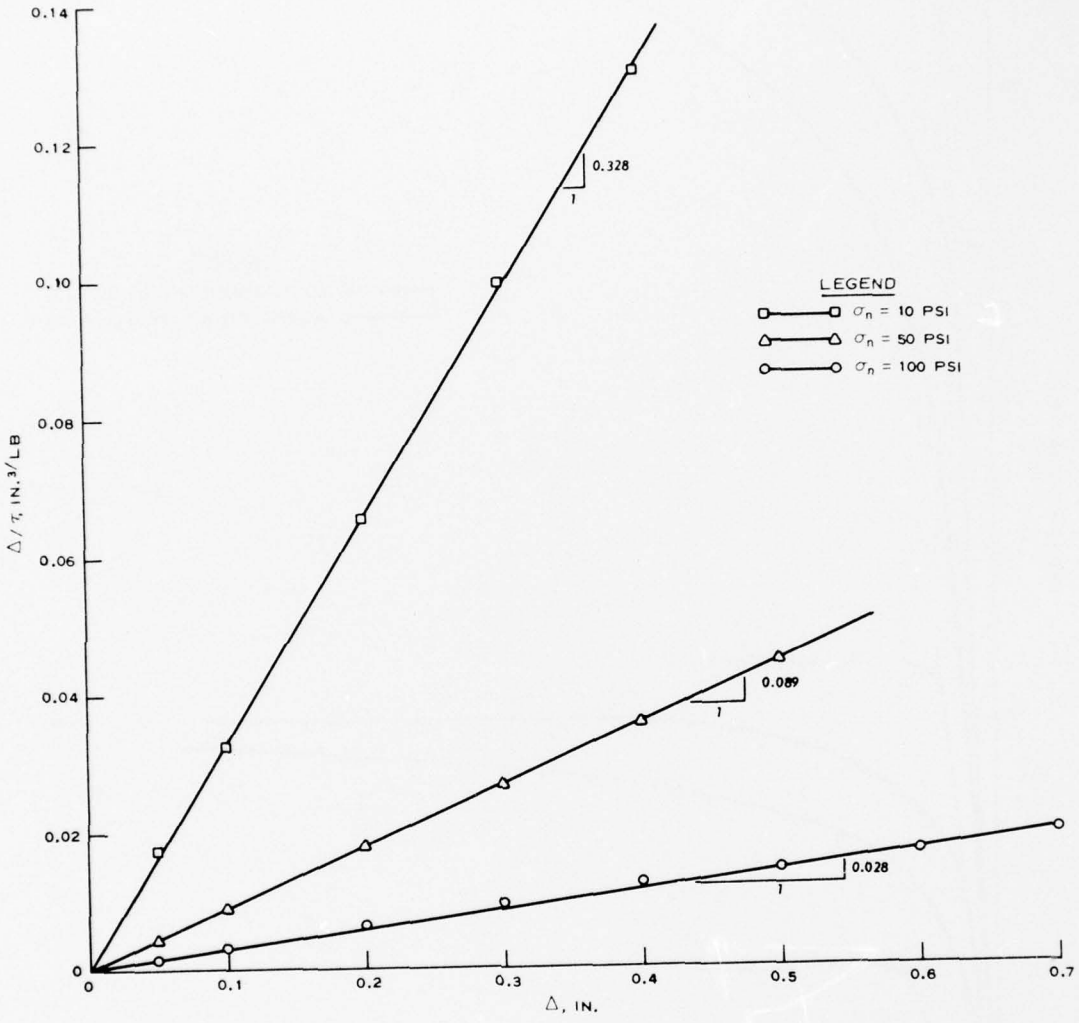

Figure 11. Linearized shear stress-deformation relationship for sand on galvanized steel

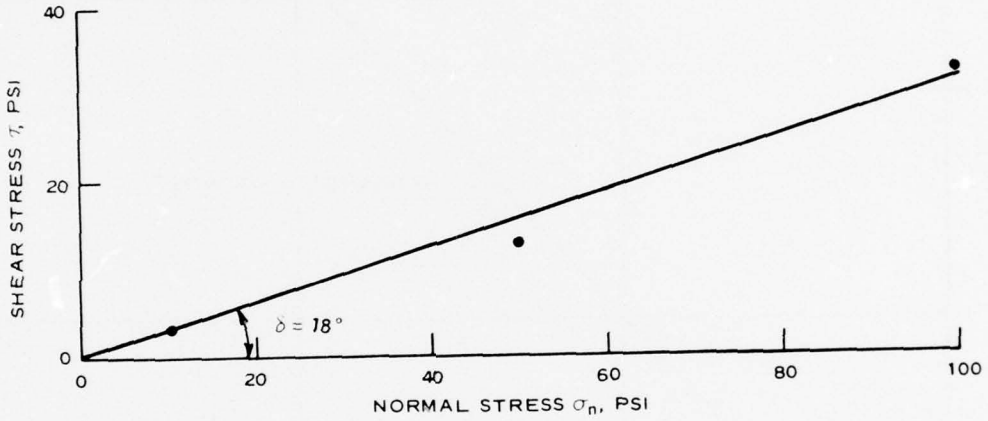

Figure 12. Mohr-Coulomb envelope for sand on galvanized steel

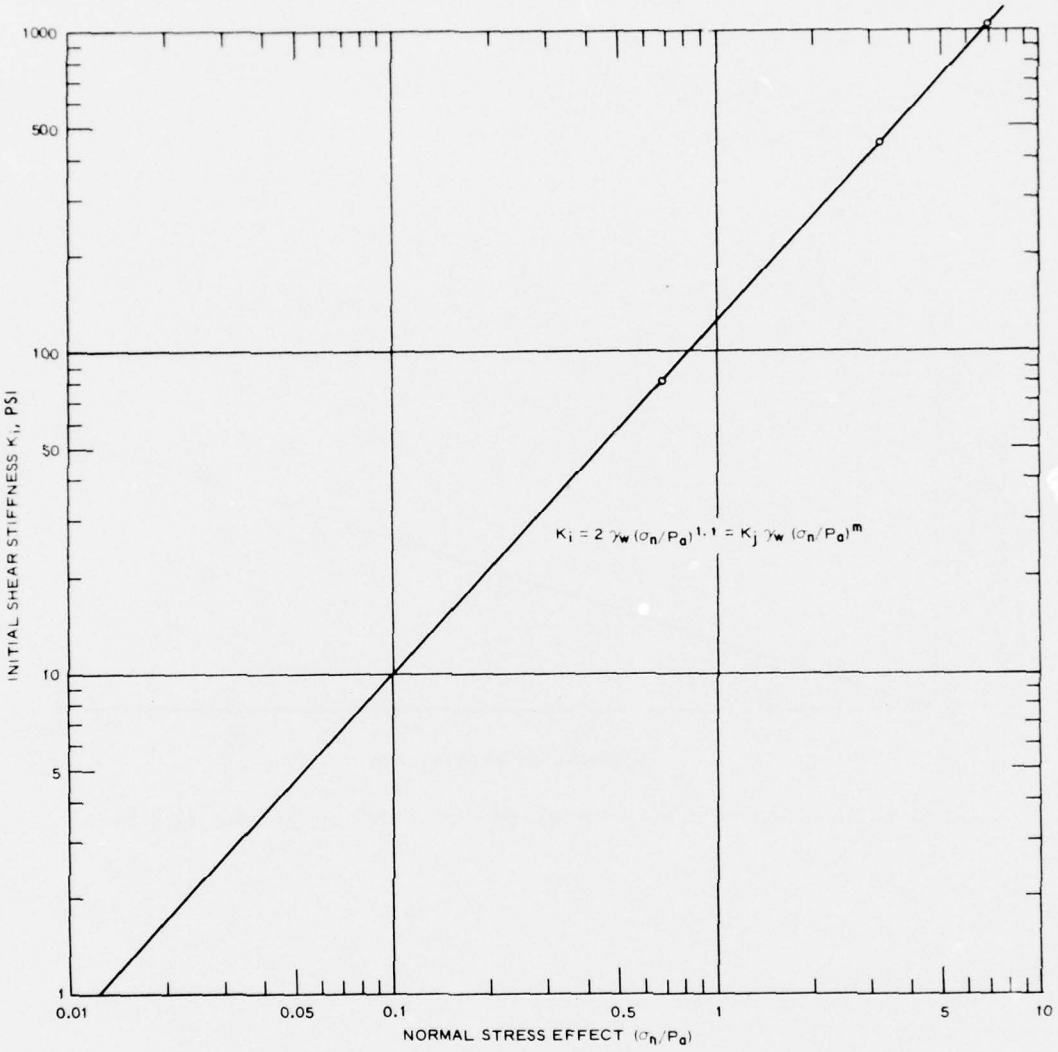

Figure 13. Variation of K_i versus (σ_n/P_a) for sand on galvanized steel

was used to describe the interface behavior between the skin element and the sand.

Interface behavior
between sand and natural soil

23. The nonlinear constitutive equation presented in Equation 2 was used to describe the interface behavior between the sand fill and the surrounding natural ground. Three direct shear tests in which the lower part of the shear box was filled with the ML soil and the upper part was filled with compacted sand were conducted, and the linearized shear stress-displacement relationship is presented in Figure 14. The figure shows that the experimental points deviate slightly from the straight line, which indicates that the hyperbolic representation is only an approximation for the behavior of this material. The Mohr-Coulomb failure envelope for sand on silt, presented in Figure 15, showed an angle of friction of 34 deg and a small adhesion of 0.05 psi.

24. The influence of the stress level on the behavior of the interface elements between the sand and the natural ML soil was accounted for by plotting the initial shear stiffness K_i versus (σ_n/P_a) as shown in Figure 16. The figure shows that the experimental points deviate significantly from the ideal straight line; however, the best fit line was used for evaluating m and K_j. A summary of the parameters needed for the nonlinear constitutive equation for the interface element between the sand and the natural ML soil is presented below.

C_a psi	δ degrees	R_f	K_j	m
0.05	34	0.86	14,523	0.68

25. Comparison between the shear stress-deformation curves obtained by experimental tests and those generated by the hyperbolic formulation, shown in Figure 17, indicates a reasonable agreement between the two respective curves.

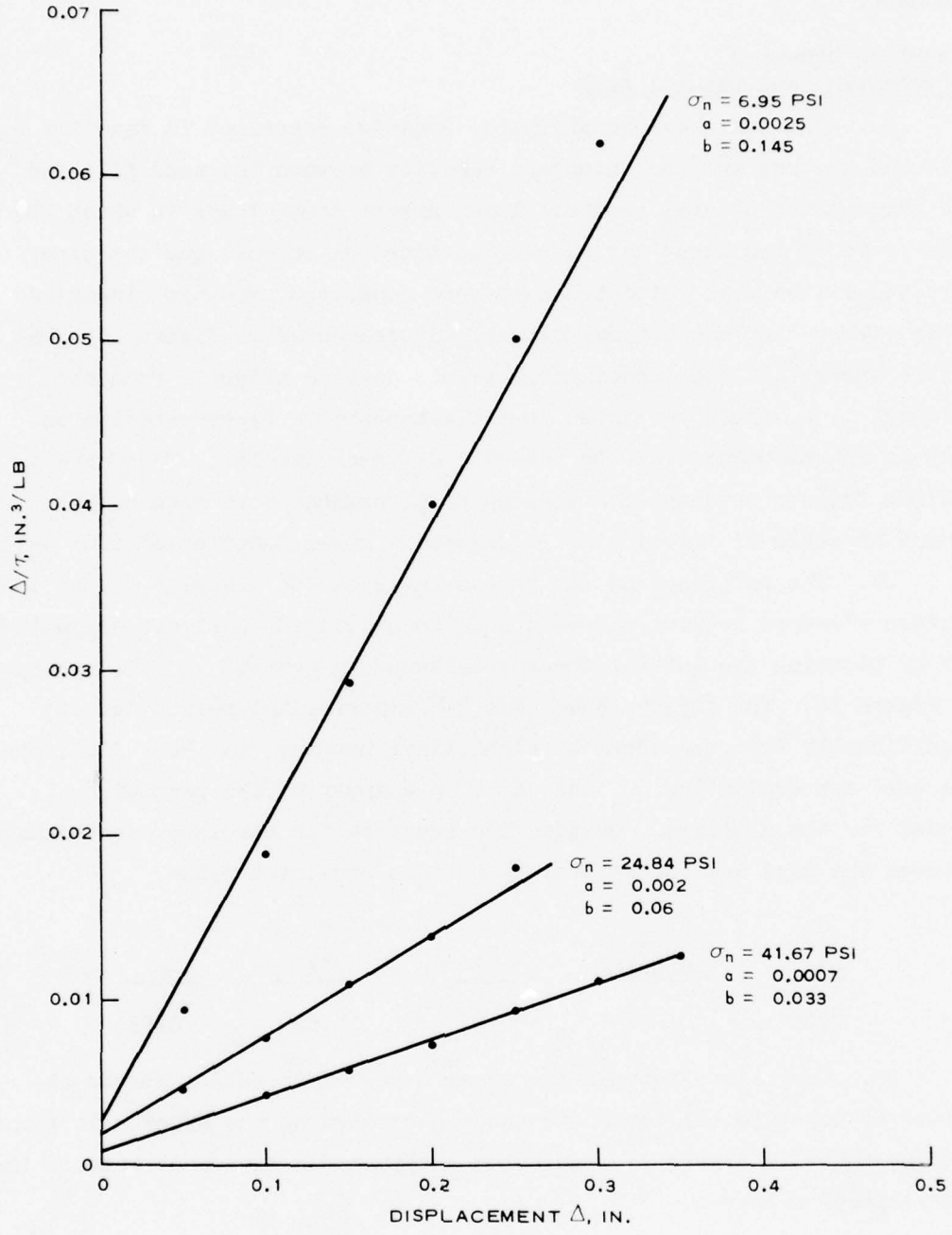

Figure 14. Linearized shear stress-displacement relationship for sand on silt

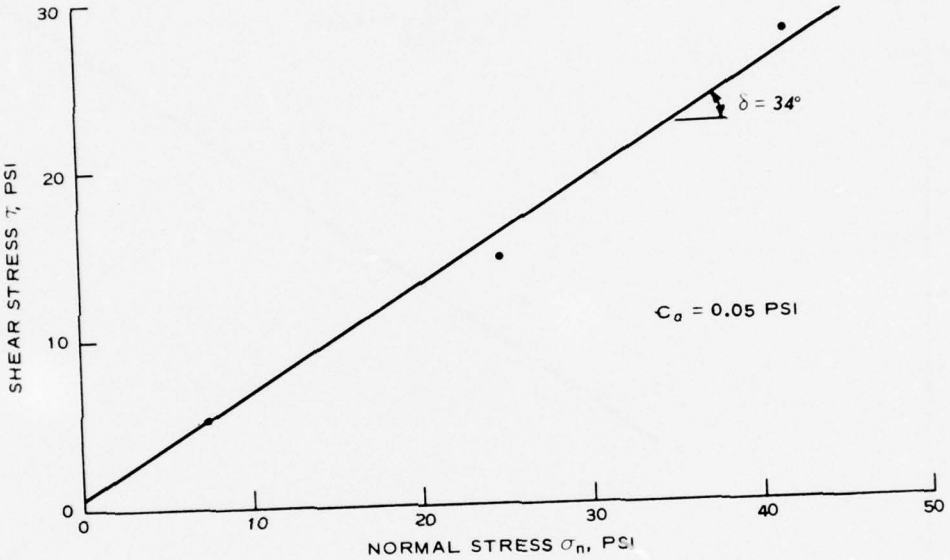

Figure 15. Shear stress versus normal stress at failure for sand on silt

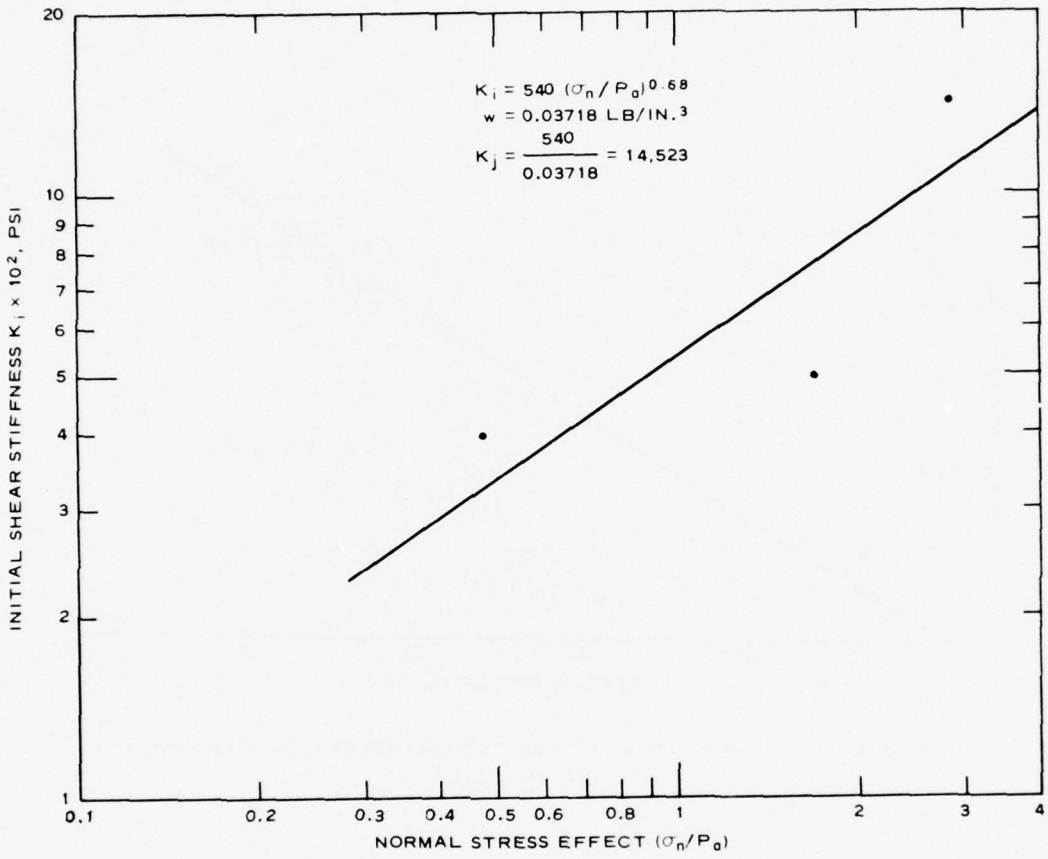

Figure 16. Variation of initial shear stiffness with respect to (σ_n/P_a) for interface element between the sand fill and the natural ML soil

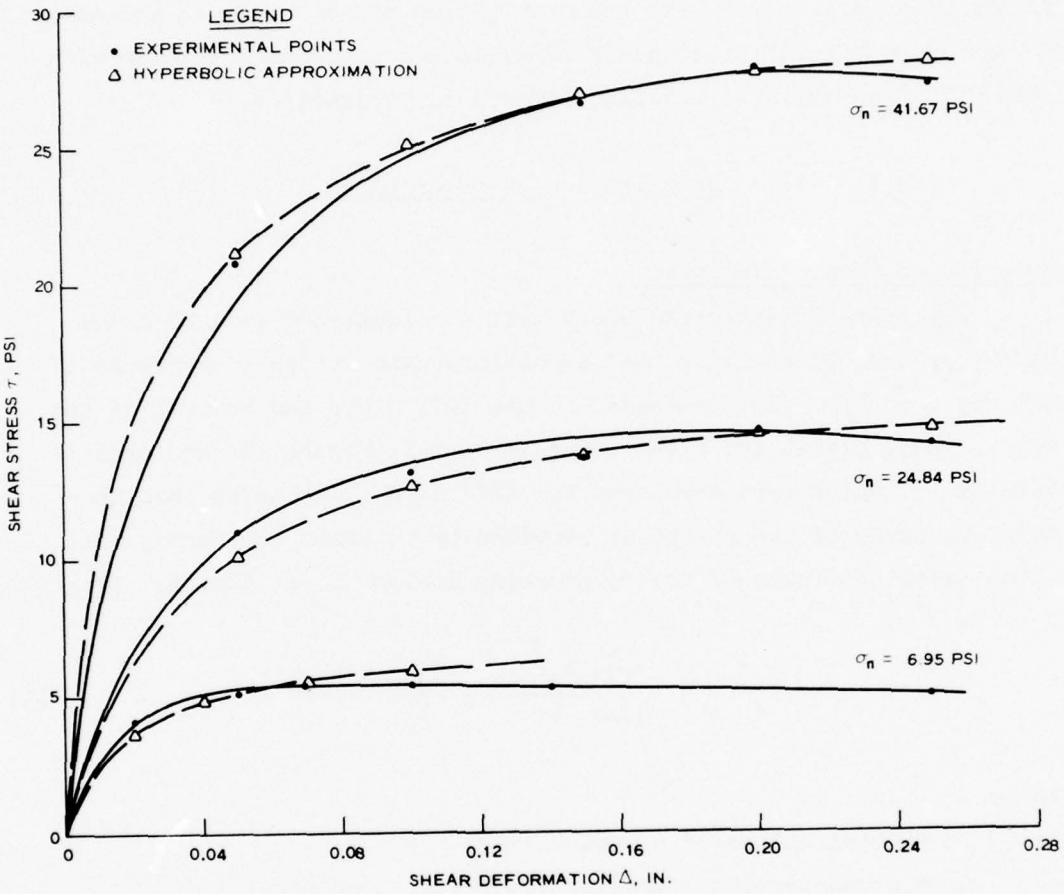

Figure 17. Shear stress-deformation relationship along plane between the sand fill and the ML soil

PART III: IDEALIZATION OF THE REINFORCED EARTH WALL

26. The reinforced earth wall is a complicated 3D problem, the complexity of which is compounded by the introduction of the steel strips as a reinforcing element. The solution of such a problem requires a substantial amount of human effort and computer time. In order to arrive at a solution with reasonable time and cost, it is necessary to introduce a certain amount of idealization and simplification with respect to geometry and modeling of material properties.

Idealization as 2D Problem

Modeling of reinforcing strip

27. The 3D reinforced earth wall was idealized as a 2D plane strain problem by assuming that the reinforcing strips (see Figure 1) are replaced by a plate extended to the full width and breadth of the wall. These plates are illustrated in mesh I (Figure 18) and mesh II (Figure 19) which were developed for this study. Assuming that the major response of the strips is provided by an axial stiffness, the total axial stiffness of the reinforcing strips S of the wall is:

$$S = \sum_{j=1}^{n} \frac{A_j E_j}{L_j} = n \frac{A_s E_s}{L_s} \tag{3}$$

where

n = total number of strips in each row

A_s = cross-sectional area of the reinforcing strip

E_s = modulus of elasticity of the galvanized steel

L_s = length of the reinforcing strip

28. The equivalent stiffness of the plate that substitutes for each row of steel strips S_e may be defined as

$$S_e = \frac{A_e E_e}{L_e} \tag{4}$$

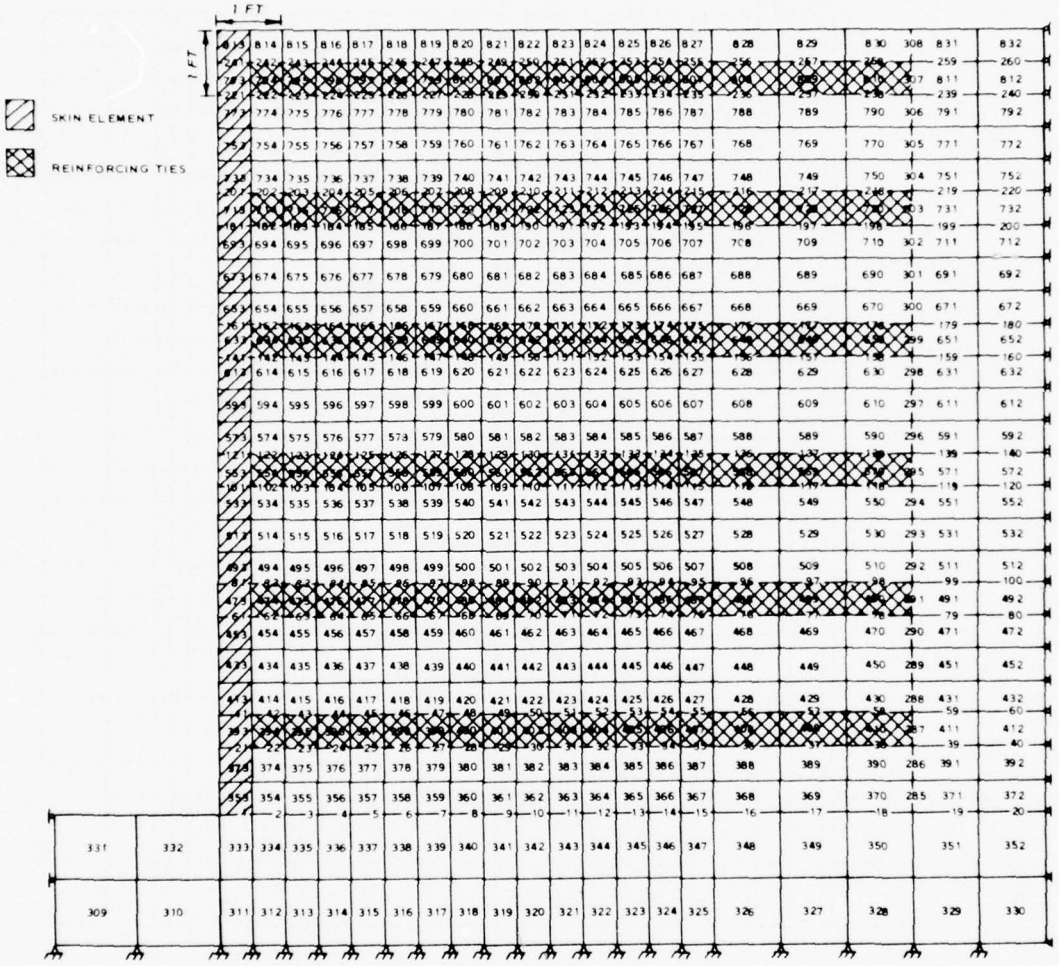

Figure 18. FE mesh I employed in the analysis of the reinforced earth wall

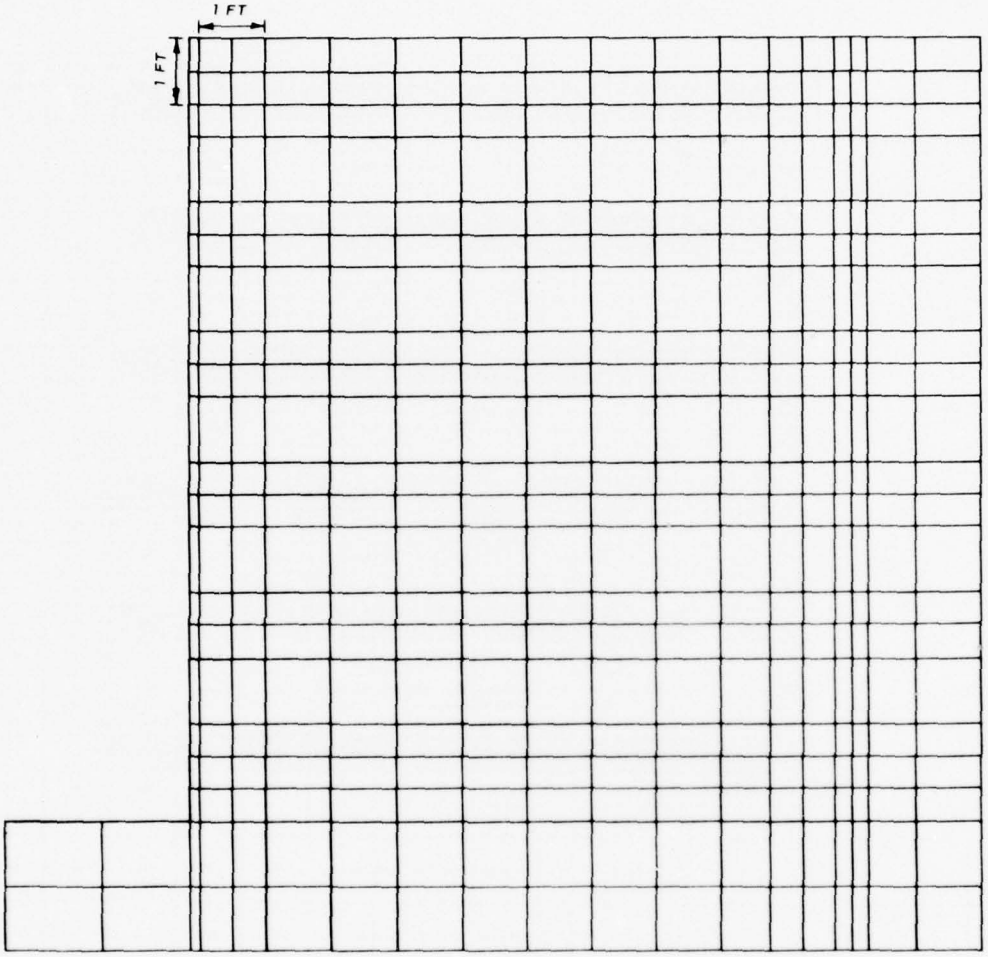

Figure 19. FE mesh II for the reinforced earth wall

where

A_e = equivalent cross-sectional area of the steel strips

E_e = equivalent modulus of the steel strips

L_e = equivalent length of the steel strips

28. Because the stiffness S and the equivalent stiffness S_e should be the same and also because the length L_s and the equivalent length L_e remain the same, thus by equating Equations 3 and 4, the equivalent stiffness E_e can be determined as

$$E_e = \frac{nA_s E_s}{A_e} \tag{5}$$

29. For the prototype reinforced earth wall the cross-sectional area of the reinforcing strip was 0.096 sq in., the modulus of elasticity of the galvanized steel was 31.1×10^6 psi, and the number of strips in each row was 6. The thickness of the equivalent reinforcing strip elements, as shown in mesh I (Figure 18), was 6 in. and as shown in mesh II (Figure 19), 0.024 in. The width of the equivalent plate for both meshes (i.e. length of the reinforced earth wall) was 16 ft. Substituting the appropriate values in Equation 5, the magnitude of E_e can be determined.

$$\text{Mesh I:} \quad E_e = \frac{6 \times 0.096 \times 31.1 \times 10^6}{0.5 \times 16} = 2.24 \times 10^6 \text{ psf}$$

$$\text{Mesh II:} \quad E_e = \frac{6 \times 0.096 \times 31.1 \times 10^6}{0.002 \times 16} = 5.598 \times 10^8 \text{ psf}$$

The unit weight of the equivalent reinforcing strips of mesh I was assumed identical with that of the sand backfill because of the very small cross-sectional area of the actual reinforcing strips. The unit weight of the equivalent reinforcing strips in mesh II was taken as 492 pcf, which is the unit weight of the actual strips; the very small area of these strips should cause little additional contribution to the stresses from gravity using mesh II.

Modeling of skin element

30. The behavior of the skin element, like that of the rest of the reinforced earth mass, was treated as a plane strain problem whose major response is provided by bending. The aluminum panels that represent the skin element, as shown in Figure 1, were replaced by an equivalent beam element of similar deflection response depending on the dimensions of the skin element in the FE meshes, Figures 18 and 19. The equation used to satisfy equal bending deformation response between the prototype and the model was:

$$\frac{E_a I_a}{L_a} = \frac{E_e I_e}{L_e} \tag{6}$$

where

E_a = modulus of elasticity of the aluminum panel

I_a = moment of inertia of the aluminum panel per unit width

L_a = length of the beam (S_z) between any two rows of reinforcing strips (see Figure 1)

E_e = equivalent modulus of elasticity

I_e = equivalent moment of inertia per unit width

L_e = equivalent length of the beam

31. Since the beam lengths of the skin element and its model are the same (i.e., $L_a = L_e$), the equivalent modulus of elasticity for the skin element can be expressed by:

$$E_e = E_a \frac{I_a}{I_e} \tag{7}$$

From paragraph 16, the modulus of elasticity and the moment of inertia are 10×10^6 psi and 1.368 in.^{4}, respectively. The depth of the equivalent beam, as shown in Figure 18 for mesh I, is 6 in.; therefore, the equivalent moment of inertia I_e is equal to 216 in.^{4} Substituting the values of E_a, I_a, and I_e in Equation 7, the magnitude of E_e for mesh I can be obtained as follows:

$$E_e = \frac{10 \times 10^6 \times 1.368}{216} = 6.33 \times 10^4 \text{ psi } (9.13 \times 10^6 \text{ psf})$$

The magnitude of E_e for mesh II (Figure 19) with skin element thickness of 1.6 in. was 3.34×10^6 psi (4.8×10^8 psf).

32. The aluminum panels of the skin element were hinged rather than bonded together into an elastically rigid unit; the actual modulus of the skin element may consequently have been substantially less than that assumed for the FE analyses using meshes I and II.

33. Two iterations were made for computation of stresses during each construction and loading sequence. The first iteration indicated the appropriate modulus (based on input material properties) to use for computation of the stresses during the second iteration.

FE Mesh Design

34. In addition to idealizing the reinforced earth wall as a 2D problem, there are a number of other factors which affect the design of the FE mesh. Some of these factors are:

a. Design of elements required to obtain a satisfactory result.

b. Boundary conditions.

c. Number of construction and loading sequences.

d. Location of interface elements.

Design of elements

35. It is always desirable to use a minimum number of elements that achieve a satisfactory degree of accuracy so that the computer time and hence computation cost are minimum. However, for a given number of elements, the degree of accuracy in simulating a particular behavior depends upon the correlation between the displacement pattern assumed and the displacement of the actual events. Because of the uncertainties regarding the assumed displacement and the preliminary nature of this study, simple meshes using relatively large numbers of rectangular elements, as shown in Figures 18 and 19, were developed to improve accuracy.

36. The elements in mesh I, Figure 18, were square to minimize distortion of results that might occur if the aspect ratio is different from unity. The elements in mesh II were reduced in total number

compared to mesh I to increase computation efficiency and dimensions were adjusted to closely simulate actual dimensions of the components of the wall.

Boundary conditions

37. The boundaries should be located so that they will have negligible effect on the main structure to be analyzed. In view of the preliminary nature of this analysis, the boundaries were kept at a distance of 2 ft from the reinforced earth mass. No calculation was used to determine the influence of the boundary conditions on the FE results.

38. The left and right vertical boundaries were confined laterally and the bottom horizontal boundary was confined both laterally and vertically. During each construction sequence of the wall, the lateral displacement was set to zero to simulate actual construction in which the lateral displacements of the skin element were restricted.

Construction and loading sequences

39. Twelve buildup increments 1 ft high each were used to construct the wall described by mesh I, while six buildup increments 2 ft high were used to construct the wall described by mesh II. Concentrated loads following construction were placed on each nodal point at the top of the wall mesh to cause surface pressures of 250 psf per increment. Six increments were applied to reach the failure condition of 1500 psf observed with the actual wall.

Location of interface elements

40. The interface elements within the FE meshes are presented in Figures 18 and 19. These interface elements increase flexibility in accounting with the interaction effects between the different components of the wall such as slippage and separation between the different components. Mesh I contains vertical interface elements between the skin element and backfill material and between the back face of the wall and the original in situ earth. Horizontal interface elements are located between the reinforcing strips and backfill material and between the backfill and original ground surface of the wall. Mesh II contains vertical interface elements between the back face of the wall and the

original in situ earth and horizontal interface elements between the reinforcing strips and backfill material.

Examination of the FE Mesh

41. Two FE analyses were performed using mesh I and mesh II in order to determine the influence of the shape of the FE mesh on the results. The results of the FE analyses obtained from both meshes were compared and correlated with field measurements at the end of construction as well as prior to failure. End of construction (EC) refers to completion of construction of the reinforced earth wall; prior to failure (PF) refers to placement of 1500 psf of surcharge load on the top surface of the wall.

Tensile stress distribution in reinforcing strips

42. The tensile stress obtained by the FE method, which is based on a reinforcing plate covering the entire area at a given reinforced elevation of the wall, was converted to tensile stress carried by the reinforcing strips as actually placed in the field by the following expression:

$$\sigma_t = \frac{t_e W_e}{NtW} \sigma_{te} \tag{8}$$

where

σ_t = tensile stress in the reinforcing strip

t_e = equivalent thickness of the reinforcing tie in the FE mesh

W_e = equivalent width of the reinforcing tie in the FE mesh

N = number of reinforcing ties in each reinforced elevation

t = actual thickness of the reinforcing tie

W = actual width of the reinforcing tie

σ_{te} = equivalent tensile stress computed by the FE analysis

The parameters expressed in Equation 8 for mesh I and mesh II are:

Parameter	Mesh I	Mesh II
t	0.024 in.	0.024 in.
W	4.00 in.	4.00 in.
N	6	6
t_e	6.00 in.	0.024 in.
W_e	192.0 in.	192.0 in.

43. Comparisons of tensile stresses obtained from the FE method using mesh I and mesh II and those obtained from actual measurement at the end of construction and prior to failure are shown in Figures 20 and 21, respectively. Figure 20 shows little difference in results between mesh I and mesh II at the end of construction. The tensile stress distribution from the FE analysis assumes a parabolic shape similar to that obtained from actual measurements for the tie at elevation 1 ft above the base of the wall. The locations of the maximum tensile stress as obtained from the FE analysis agreed well with those measured in the field.

44. The tensile stress distribution along the reinforcing ties prior to failure, Figure 21, indicates that the FE results of mesh II are slightly higher than those from mesh I. However, the shapes of the tensile stress distribution curves for mesh II are much closer to actual field data than those obtained for mesh I. These results indicate that the square elements as represented in mesh I reduce the maximum tensile stress as well as the curvature of the tensile stress distribution curve.

Lateral pressure distribution

45. The lateral pressure distributions along a vertical plane 1.25 ft from the skin element as obtained from the FE analyses of mesh I and mesh II are shown in Figure 22. These results indicate that while the lateral pressures at the end of construction are slightly higher for mesh I than for mesh II, the trend reverses itself prior to failure. The results of the FE analyses for both EC and PF cases are approximately similar but somewhat higher than those obtained from the Rankine active earth pressure theory. The PF lateral pressure near the top of

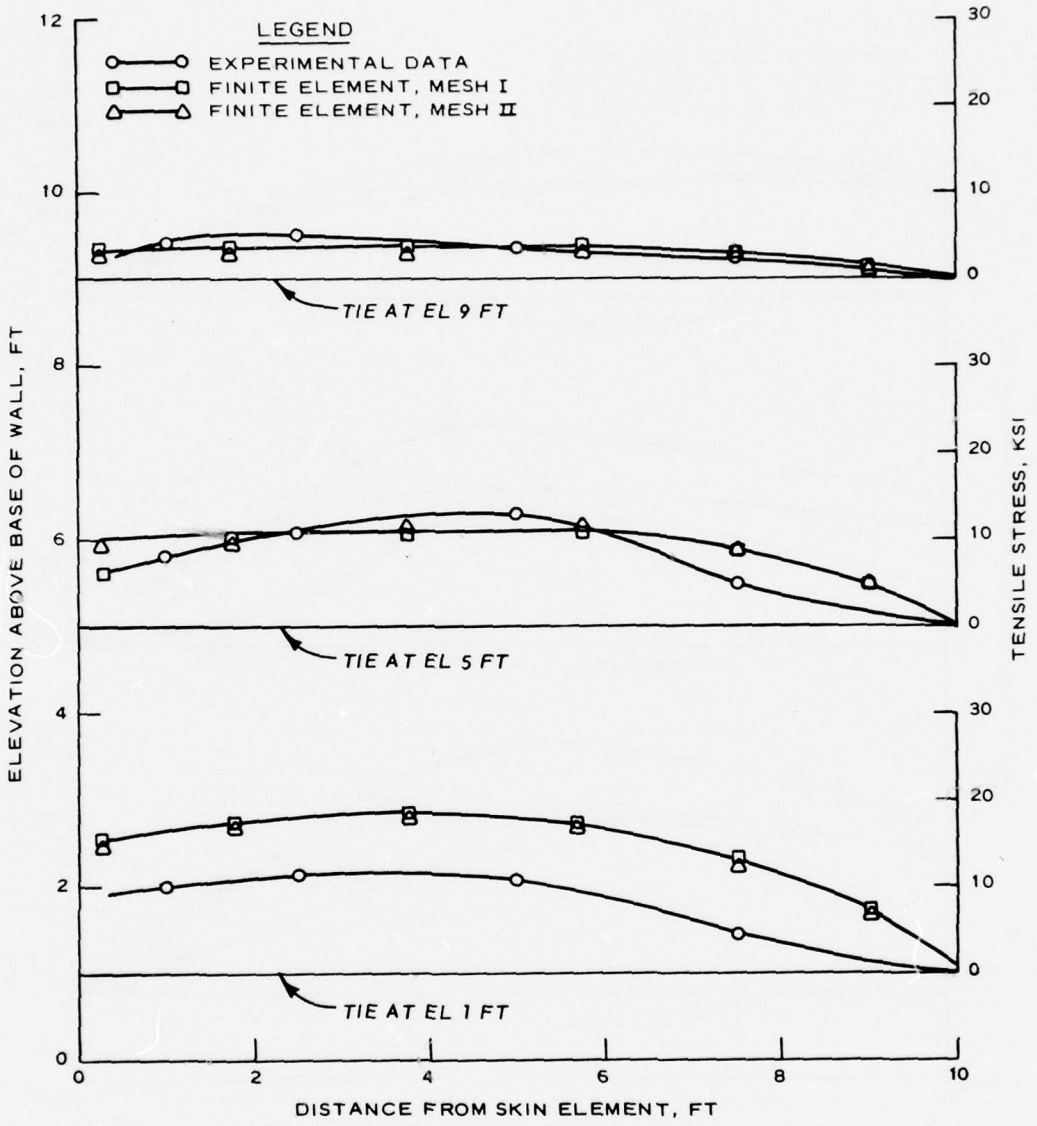

Figure 20. Influence of FE mesh on tensile stress distribution along the instrumented reinforcing strips at end of construction

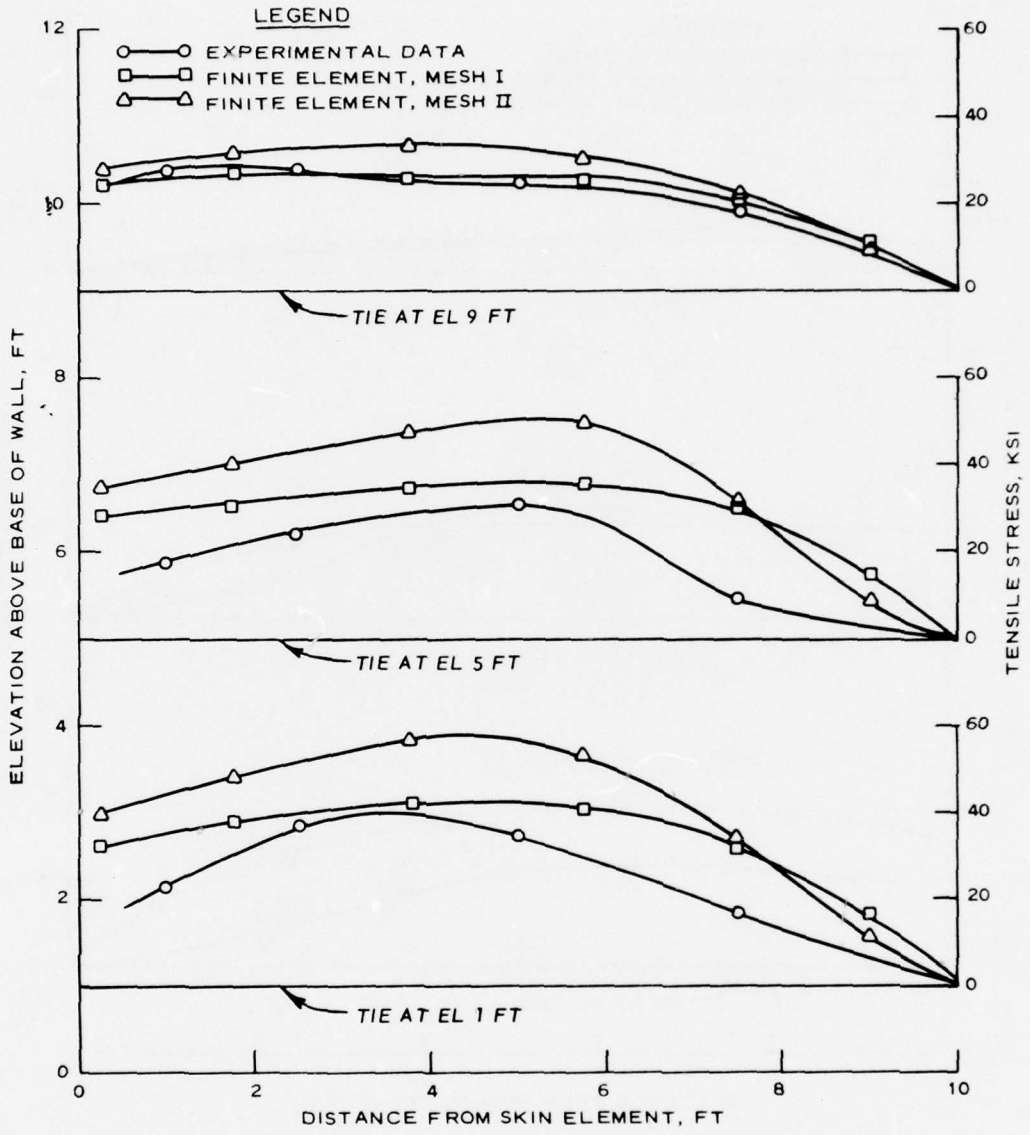

Figure 21. Influence of FE mesh on tensile stress distribution along the instrumented reinforcing strips prior to failure

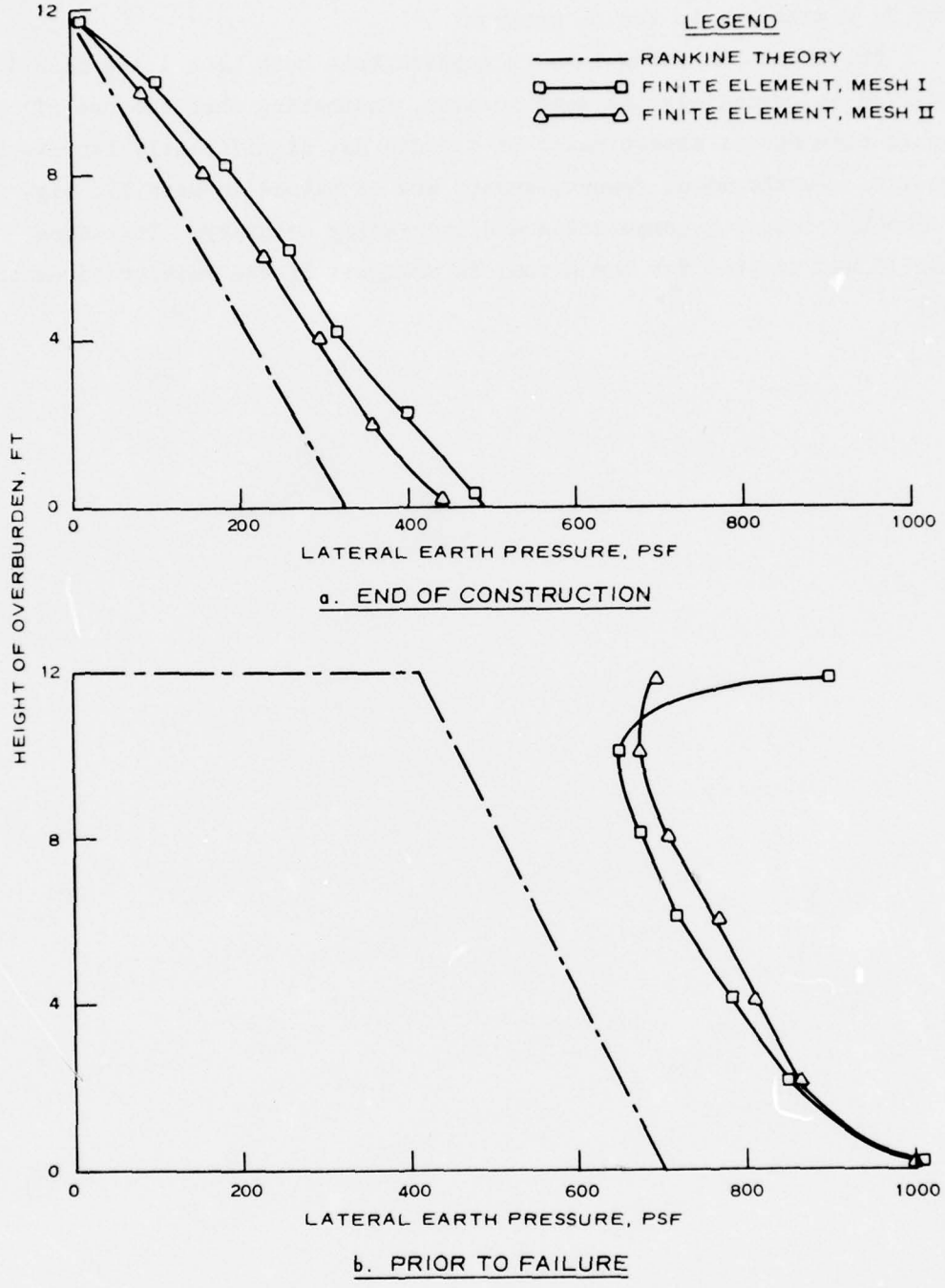

Figure 22. Influence of FE mesh on lateral pressure acting on vertical plane 1.25 ft from skin element

the wall as obtained from the FE analysis is greatly increased for mesh I, a weakness in the FE program.

46. From the foregoing, it appears that both mesh I and mesh II provided approximately the same results, indicating that the use of square elements of aspect ratio of 1.0 did not significantly improve the results. Furthermore, fewer elements are contained in mesh II, significantly reducing computation and increasing accuracy. Therefore, mesh II was adopted for the actual FE analysis of the reinforced earth wall.

PART IV: FE ANALYSIS OF THE REINFORCED EARTH WALL

47. Reliable FE analysis of the reinforced earth wall requires accurate information on material properties, proper modeling of deformation and failure mechanisms, and proper modeling of the interaction effects between the components of the wall. The previously described models of material behavior (Part II) and idealization of the wall by the computer code (Part III) provide only a rough approximation of the actual field situation. The results of this analysis and comparisons with field observation will consequently be used to help determine more appropriate parameters for material properties of the components of the wall and improved models that may better represent field behavior.

48. An important assumption needed to obtain the FE plane strain solution is that the reinforcing strips cover the entire length of the wall. Such an assumption increases the shear resistance between the reinforcing element and the surrounding sand by a magnification factor m_f, where m_f (see Figure 1) is the ratio of the horizontal cross-sectional area of the wall to the total area of the reinforcing strips at that elevation. Thus,

$$m_f = \frac{L \cdot L'}{N \cdot L \cdot W} = \frac{10 \times 16}{6 \times 10 \times 0.333} = 8 \tag{9}$$

Therefore, the increase in the shear stress at the interface element of mesh II due to the computer code is compensated for by reducing the shear stresses in Equation 2 by m_f. Thus, the constitutive equation for the interface element used in the actual FE analysis is:

$$K_{st} = (1 - \lambda_2)^2 K_i \tag{10a}$$

$$K_i = \frac{K_j}{m_f} \gamma_w \left(\frac{\sigma_n}{P_a}\right)^n \tag{10b}$$

$$\lambda_2 = \frac{R_f \tau}{\frac{C_a}{m_f} + \sigma_n \tan\left(\frac{\delta}{m_f}\right)} \qquad (10c)$$

Therefore, the material parameters input in the code are $K_j = 410$ and $\phi = 3$ deg.

49. It must be noted that Equation 10 can only provide a crude simplification of the real behavior between the reinforcing strips and the surrounding soil. More study is needed to provide a better constitutive model to describe the behavior of the interface elements. In Equation 10 the tangent shear stiffness K_{st} is set to a small residual value K_r of 10 psf whenever the shear stress at the interface element exceeds a Mohr-Coulomb strength envelope with cohesion equal to C_a/m_f and an angle of friction equal to δ/m_f.

50. The ribs of the skin element, Figure 8, are extended only along the length of the wall; thus, they contribute very little to the bending resistance of the skin element in the vertical direction. The moment of inertia used in the calculation is that for the aluminum sheet of 0.1-in. thickness rather than that of the integral aluminum panel.

FE Analysis

51. Two FE analyses were performed using mesh II. In the first analysis, case C_1, the toe of the wall represented by point A of Figure 2 was assumed fixed. In the second FE analysis, case C_2, point A was assumed free to move in any direction. It must be noted that under field conditions point A is neither fixed nor free and the true answer may be between the two extreme cases. The lateral pressures exerted by the backfill and surcharge loading following construction, lateral movement of the skin element, and the tensile stress and deformation behavior of the reinforcing strips are given below. Comparisons are made with field observations for the EC and PF cases.

Distribution of lateral pressure

52. The distributions of lateral pressure along a vertical plane

1 ft from the wall face determined from the FE analysis for the free- and fixed-end cases are compared with field observations. Figure 23 shows the variation of the lateral pressure as a function of the height of the wall at the end of construction. The differences in results between cases C_1 and C_2 at the end of construction are negligible except at the base of the wall where the fixed end, case C_1, showed higher lateral pressure than the free end, case C_2. The results of the FE analysis at the end of construction for C_1 and C_2 cases are somewhat higher than those determined by Rankine theory and those obtained by field measurements.

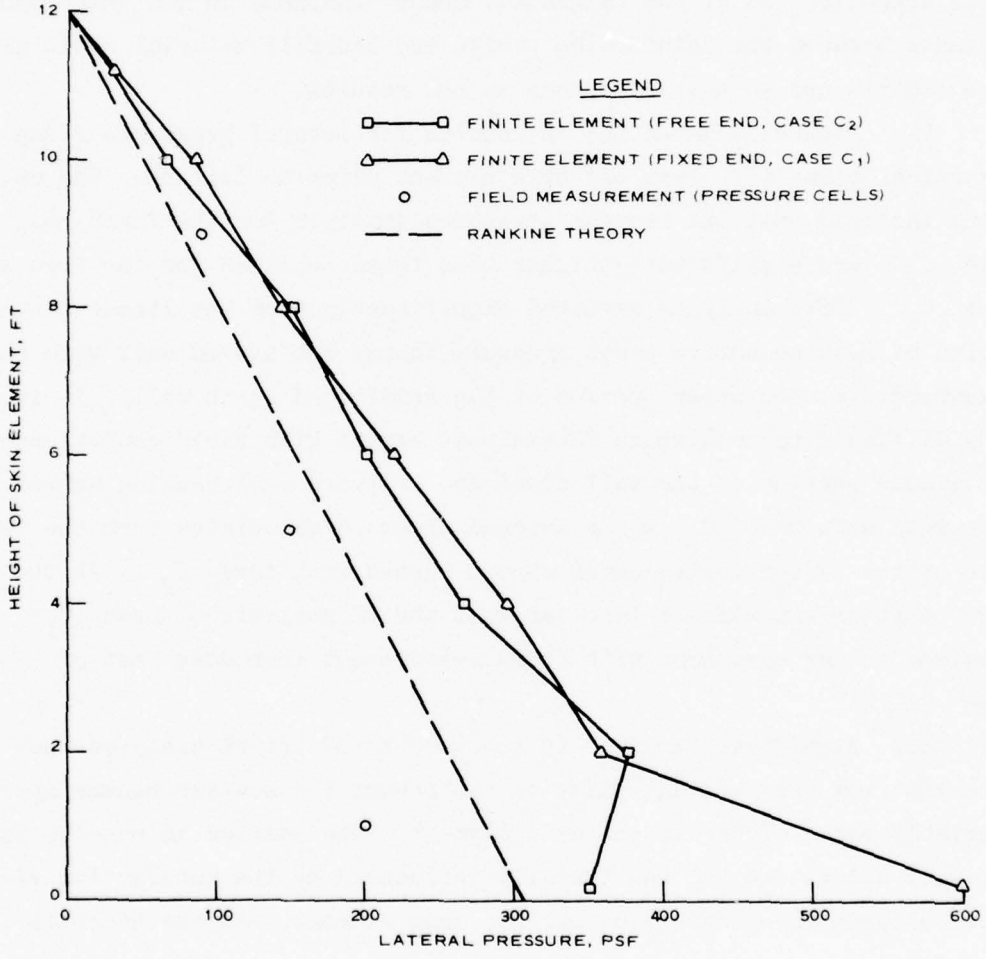

Figure 23. Pressure distribution at vertical plane 1 ft from skin element at end of construction

53. Field measurements for lateral pressures, Figure 23, were obtained directly from readings of earth pressure cells placed along the center line of the wall 1 ft from the skin element, and also from those stresses generated in the instrumented reinforcing strips within a segment of the wall bounded by $S_x S_z$, Figure 1, surrounding the strip. The excessive lateral pressures determined by the FE analysis may be attributed to deficiencies in the computer code and to the poor modeling of the skin element. It must be noted that the panels of the skin element were connected together by horizontal hinged joints, Figure 1, rather than continuous bonded joints as assumed in the FE analyses. The crude approximation of the tangential shear stiffness in the interface elements between the reinforcing strips and backfill material could also have contributed to the difference in the results.

54. Figure 24 shows the FE results for lateral pressures along a vertical plane 1 ft from the skin element prior to failure. The results indicate that the lateral pressures obtained for the fixed end, case C_1, are significantly higher than those obtained for the free end, case C_2. Both analyses deviated significantly from the linear distribution of Rankine active earth pressure theory but agreed well with field measurement at the upper portion of the reinforced earth wall. It is very difficult to tell which FE analysis agrees with field conditions at the middle portion of the wall since the pressure cell reading agreed very well with case C_1 while lateral pressure calculated from the tensile stress of the instrumented strips agreed with case C_2. At the lower part of the wall it is clear that the FE analysis of case C_1 provides better agreement with field measurement than does that of case C_2.

55. Significant scatter in the results of the FE analyses was observed from one vertical plane to the other; the scatter became appreciably worse closer to the skin element. The scatter in results is not well understood but was probably influenced by the interaction effects between the reinforcing strips, skin element, and the backfill material.

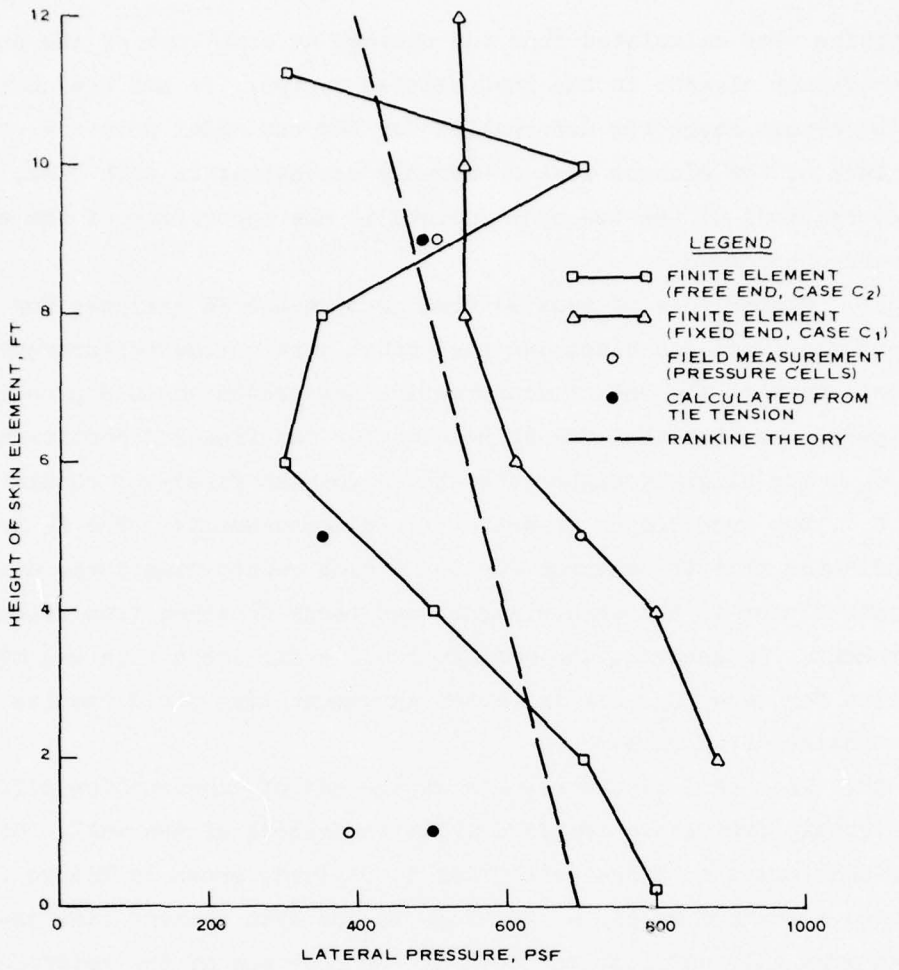

Figure 24. Pressure distribution at vertical plane 1 ft from skin element prior to failure

Distribution of tensile
strain in reinforcing strips

56. The reinforcing strips located at elevations of 1, 5, and 9 ft along the center line of the wall were instrumented with full SR-4 bridges on both surfaces at points 1, 2.5, 5, and 7.5 ft from the face of the skin element. The measured strains at the top and bottom of each instrumented point were averaged to eliminate the effect of bending strains; the average strain was considered equal to the tensile strain at that point in the reinforcing strip. Only the strains at the end of construction and prior to failure are presented. For the FE analysis

the strains were calculated from the average deformations of the nodal points of each element in the instrumented strips. It has been noticed that in certain cases the deformation for the two nodal points at the lower face of the element were unbearably excessive; in such case, only the deformations of the two nodal points at the upper face of the element were considered.

57. Comparisons of tensile strains from the FE analyses for the free-end and fixed-end cases and the actual strain gage measurements of the field test at the end of construction are presented in Figure 25. The figure indicates that the FE results for the free-end condition, case C_2, are slightly higher than those for the fixed-end condition, case C_1, but much closer to actual field measurements. The FE analysis indicated that the maximum strain in each reinforcing strip occurred at points closer to the skin element than those obtained from actual measurement. In general, the maximum tensile strains at the end of construction for case C_2 are in better agreement with field results than those obtained from case C_1.

58. The trend of the results at the end of construction differed significantly from those obtained prior to failure of the wall. Strains determined from the FE analysis prior to failure, shown in Figure 26, indicated a maximum strain value close to the skin element that decreased gradually until it vanished at the free end of the reinforcing strips. The results, unlike the end of construction condition, showed higher strains for case C_2 than for case C_1. For the strips at elevation 5 ft above the base, both C_1 and C_2 cases showed similar values and their maximum values were in good agreement with that obtained from field measurement.

Distribution of tensile
stress in reinforcing strips

59. The in situ tensile stress distribution along the reinforcing strips of the reinforced earth wall was calculated from the strain gage readings and the modulus of elasticity of the galvanized steel using modulus of elasticity $E = 31.1 \times 10^6$ psi. Therefore, the variation in the field tensile stress distribution is similar to the variation in

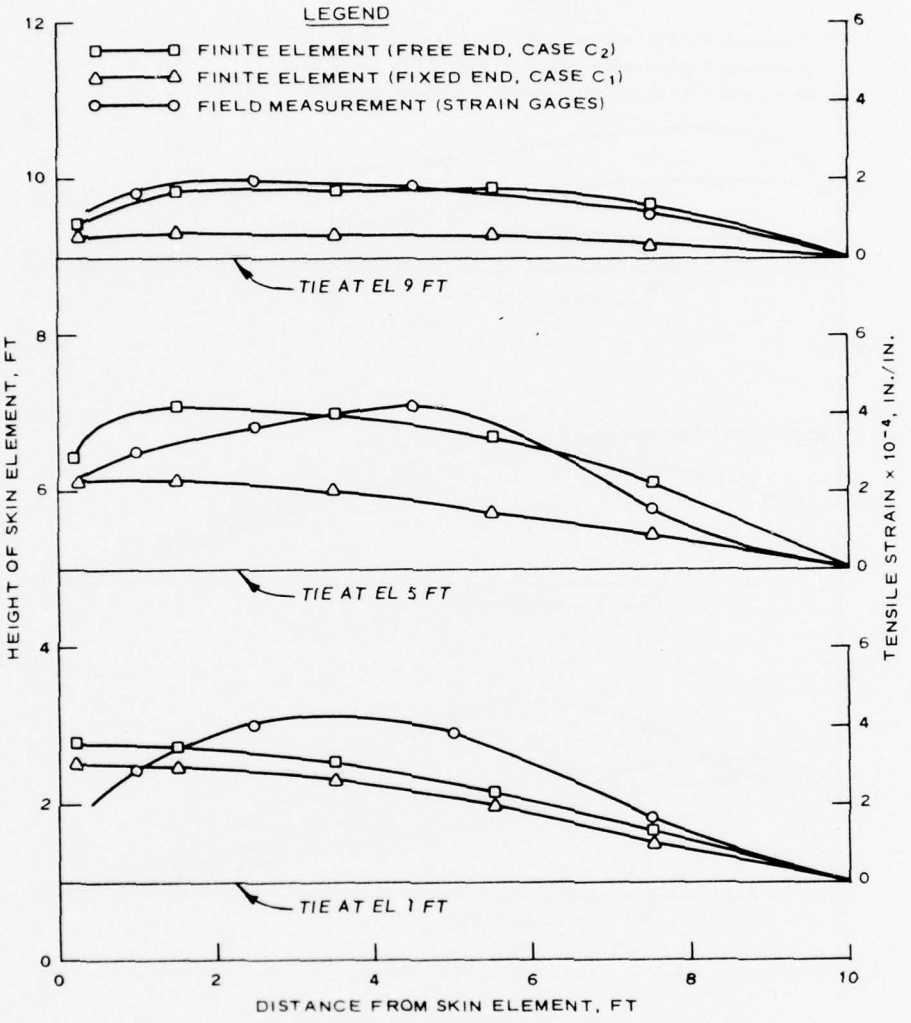

Figure 25. Tensile strain distribution along instrumented ties at end of construction

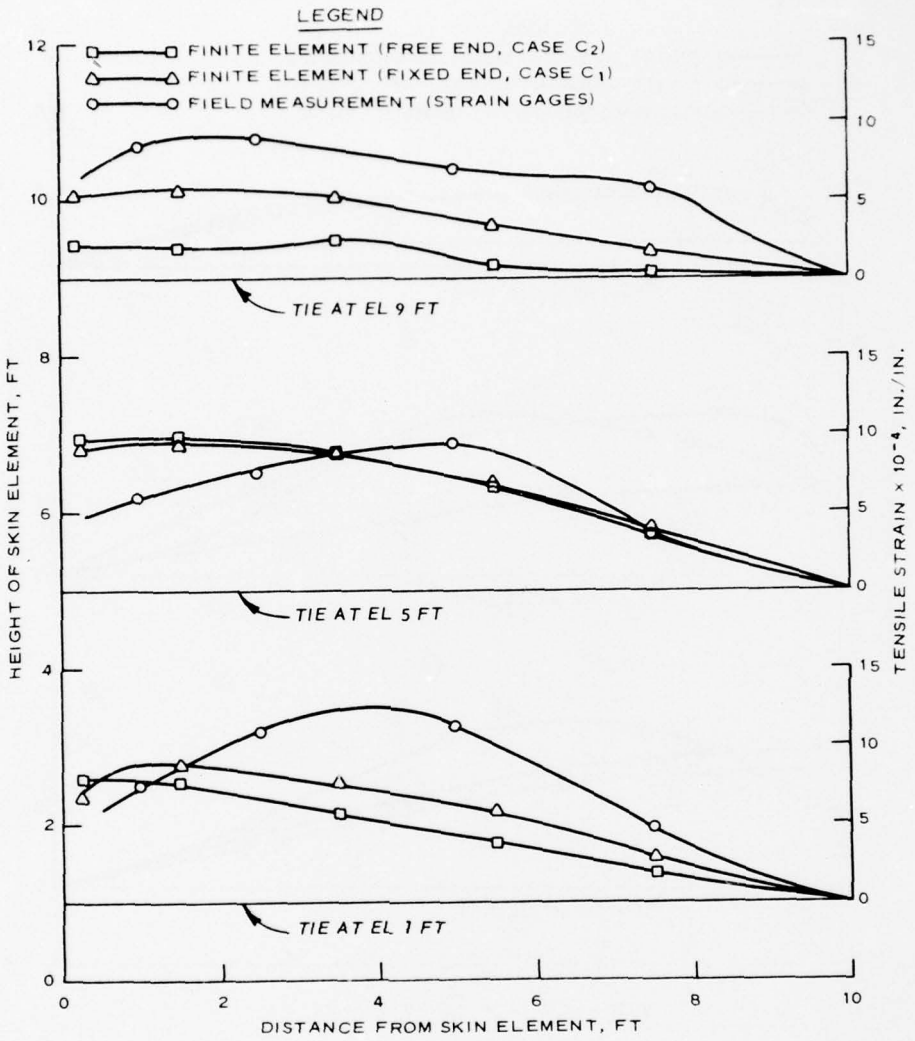

Figure 26. Tensile strain distribution along instrumented ties prior to failure

tensile strain distribution as long as the stresses in the reinforcing strips are within the elastic range. The tensile stress at any point for the FE analysis was obtained from the equivalent tensile stresses using Equation 8. Comparisons of the tensile stress distribution along the instrumented strips at the end of construction between the FE analysis and field test results are presented in Figure 27. The figure shows that the FE results for the free-end case and the fixed-end case are identical, but somewhat different from field measurements. The maximum

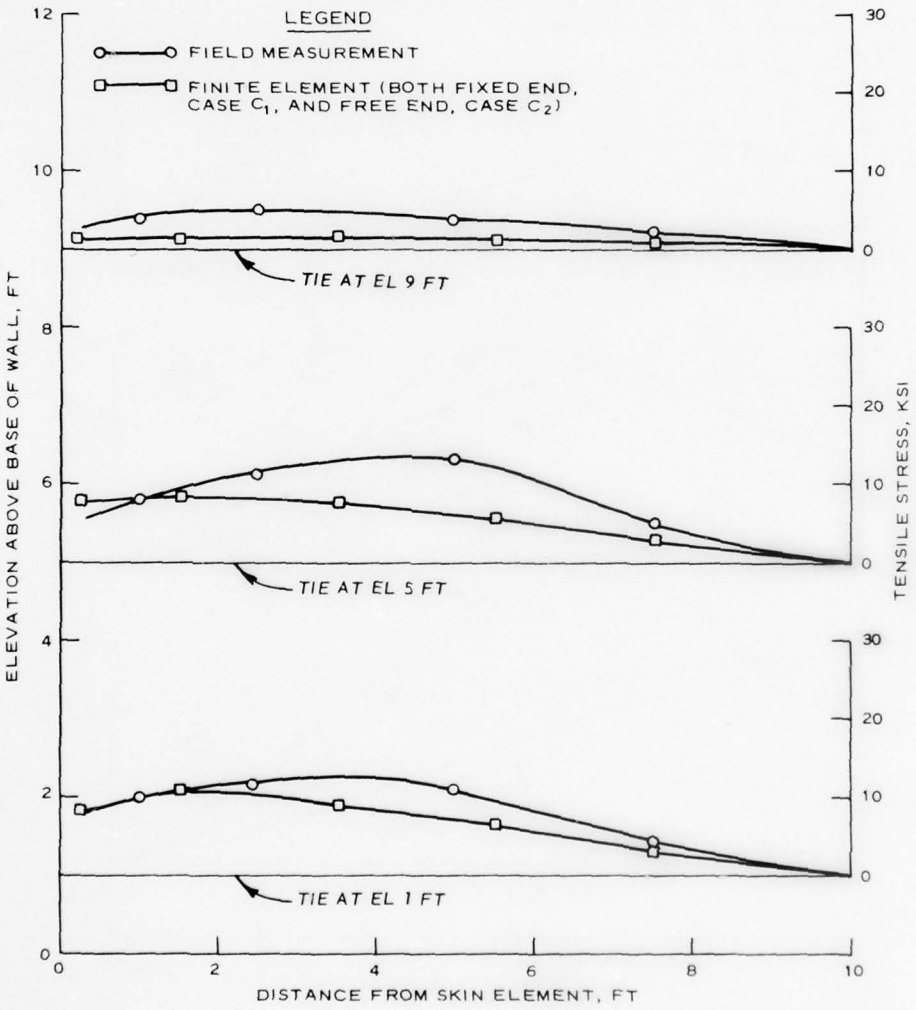

Figure 27. Tensile stress distribution along instrumented ties at end of construction

tensile stress for the FE analysis occurred at a point much closer to the skin element than that observed in the field. Both the FE results and the field data show a gradual decrease in the tensile stress from the maximum until it reached zero at the free end of the reinforcing strip.

60. Comparisons of results between the tensile stress prior to failure for the FE analysis and actual field observations are presented in Figure 28. The figure shows that the tensile stresses obtained from

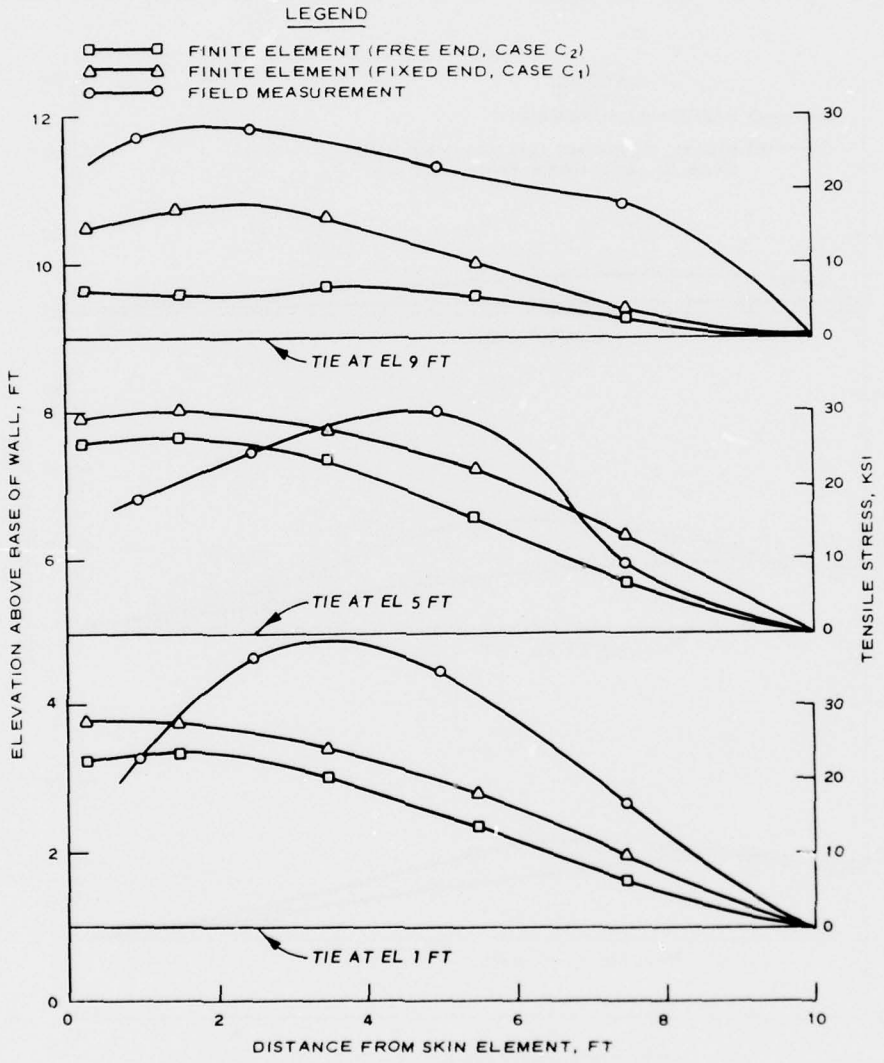

Figure 28. Tensile stress distribution along instrumented ties prior to failure

the FE analysis are generally lower than those obtained from field measurements and the results for the fixed-end condition, case C_1 , are higher than those for the free-end condition, case C_2 . FE analysis results also show that the maximum tensile stresses occurred at a point much closer to the skin element than those obtained from field measurement. Reasons for the differences between the FE results and field observations are not known with certainty; however, deficiencies in the computer code and modeling, difficulty in determining the actual stiffness of the skin element, and lack of precise constitutive equations for describing behavior for the materials are considered major contributing factors.

Lateral movement of skin element

61. The reinforced earth wall was constructed such that each section of aluminum paneling added to form the skin element was held in place to prevent lateral movement while the corresponding layer of backfill was added. Field observations indicated negligible deformations at the end of construction, but the skin element deformations became substantial prior to failure, as shown in Figure 29, with maximum deformation of 3.4 in. at the top of the wall. Comparison between the FE analyses and the field observations, based on the accumulative deformation from one panel to the next, shows good agreement between the field observations and FE results for the fixed-end condition, case C_1 . For the free-end condition, case C_2 , the FE analysis showed excellent agreement with field observations at the lower half of the wall; however, at the upper half of the wall the FE results deviated significantly from actual observations and indicated compression.

Concluding Remarks on the FE Analysis

62. Because of the preliminary nature of this study and the complex interaction between the various components of the reinforced earth wall, it is considered unlikely that total agreement between the FE analyses and field measurements could be obtained. However, it is shown that the boundary conditions of the wall play a significant role in the

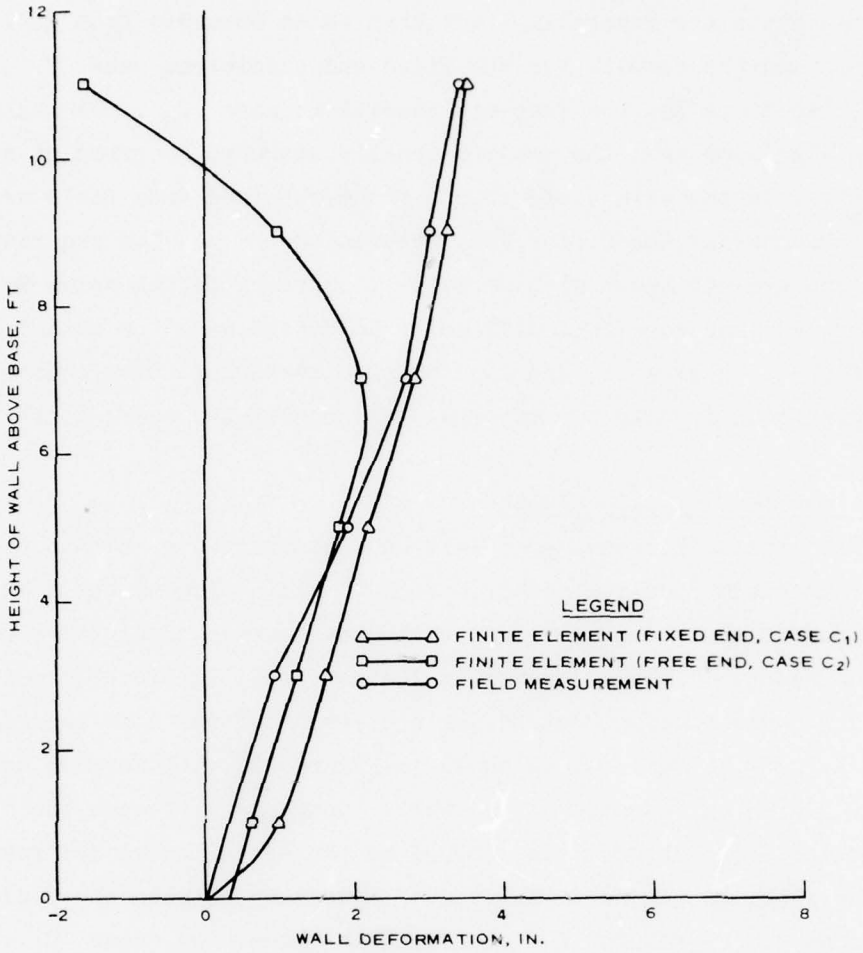

Figure 29. Accumulated lateral skin element deformation prior to failure

FE results. In certain aspects such as the deformation of the skin element, the fixed-end condition showed better agreement with field observations, while in other aspects such as the tensile strain in the reinforcing strips, the free-end condition showed better agreement. Other problems, such as improving the computer code, obtaining better constitutive equations for modelling material properties, and better representation of the skin element stiffness used in the calculation, need further development before a total agreement can be achieved.

PART V: PARAMETRIC STUDY

General Objectives

63. In the previous part of this report it was shown that there are some differences between results of the FE analysis and those obtained from field observation. The differences were attributed to many factors, some of which were discussed previously. Variables which may influence the performance of a reinforced earth wall are also numerous. The influence of the following variables on the tensile stress distribution along the instrumented reinforcing strips was evaluated using the FE analysis: (a) friction between the reinforcing strips and the fill material; (b) end conditions of the skin element; (c) stiffness of the skin element; and (d) length of the reinforcing strips.

Influence of friction between reinforcing strip and fill material

64. Equation 2 represents the constitutive equation which is used to describe the stress-deformation relationship between the galvanized steel and the concrete sand used. Analysis of the test results presented in Figures 12 and 13 indicated that $\delta = 18$ deg and $K_j = 3280$. These parameters were reduced by a magnification factor $m_f = 8$ when used in the FE code to obtain tensile stress distribution for cases C_1 and C_2, shown in Figures 27 and 28. In order to determine the influence of friction between the reinforcing strips and the fill material, FE runs were performed using the same parameters employed in cases C_1 and C_2 except that δ and K_j were not reduced. The tensile stress distributions for the fixed-end condition, case C_3, and the free-end condition, case C_4, are presented in Figures 30 and 31, respectively. Comparison between Figures 30 and 31 for the unreduced interface friction and Figures 27 and 28 for the reduced interface friction indicates that the increase in the interface friction increases the tensile stress in the reinforcing strips; the increase in stress is much higher for the free-end condition, case C_4, than for the fixed-end condition, case C_3. Based on this analysis it

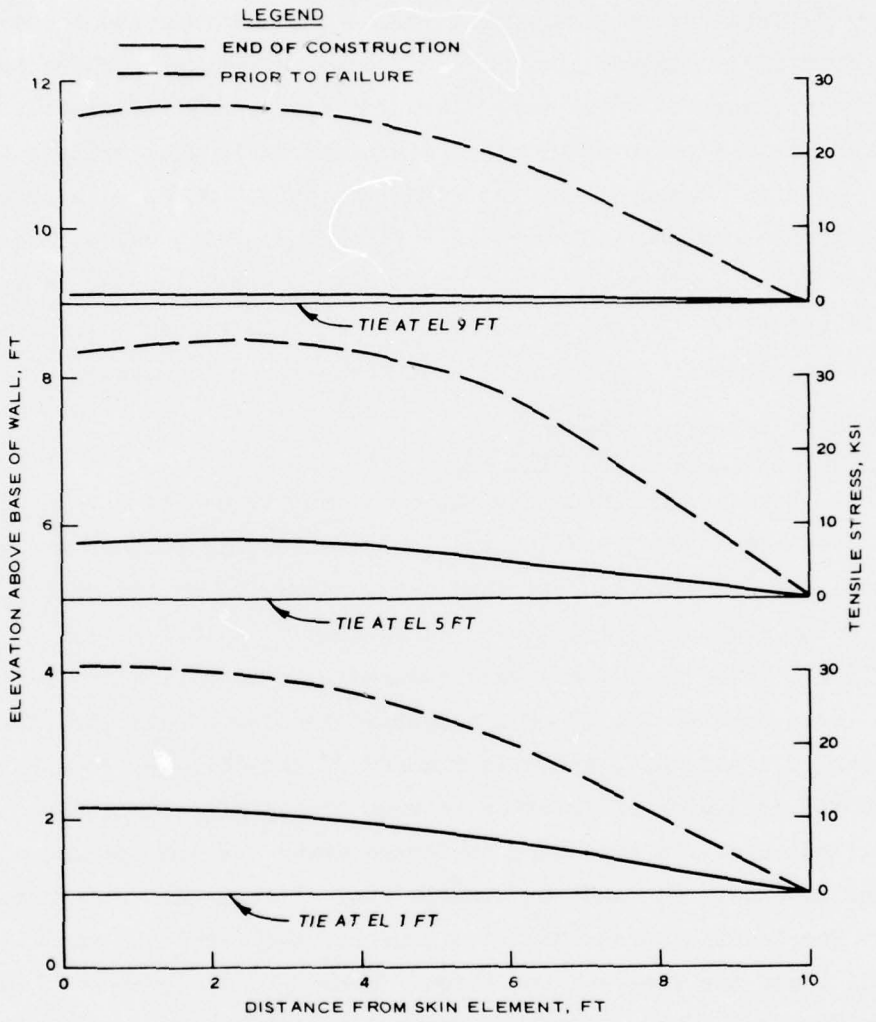

Figure 30. Tensile stress distribution along instrumented reinforcing strips at end of construction and prior to failure for case C_3 (fixed-end condition)

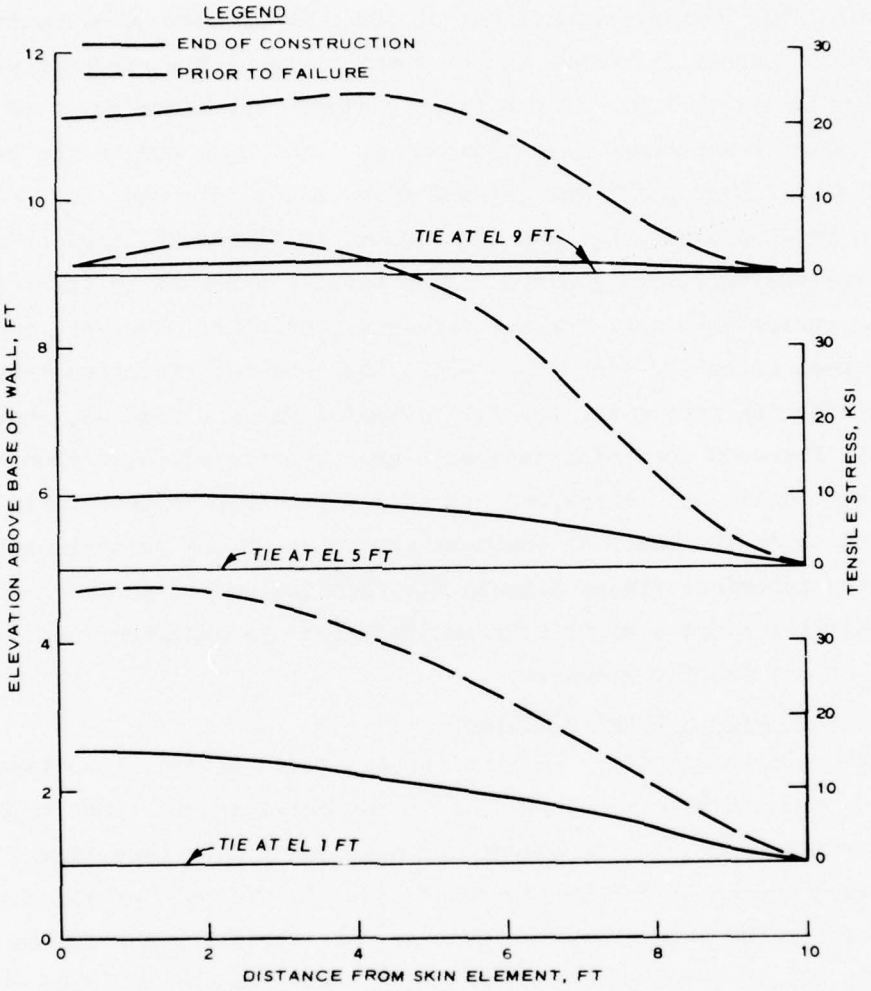

Figure 31. Tensile stress distribution along instrumented reinforcing strips at end of construction and prior to failure for case C_4 (free-end condition)

can be concluded that the increase in the interface friction between the reinforcing strips and the surrounding fill increases the maximum tensile stresses.

Influence of end conditions of skin element

65. Two boundary conditions of the skin element were investigated to determine their influence on the tensile stress distribution within the reinforcing strips. In the first analysis the lower point of the skin element was assumed fixed, cases C_1 and C_3, and in the second analysis the lower point was assumed free, cases C_2 and C_4. Comparison between cases C_1 and C_2, shown in Figure 28, indicates that the fixed-end condition yielded higher tensile stresses in the reinforcing strips than that for the free-end condition. However, comparison between cases C_3 and C_4, where the interface friction between the reinforcing strips and the fill material was not reduced, shows that the free-end condition induced higher tensile stresses than the fixed-end condition. Therefore, it is not possible to draw definite conclusions on the basis of boundary condition of the skin element alone since the interface stress between the reinforcing strips and the surrounding fill plays a major role in dictating the magnitude and distribution of the tensile stresses.

Influence of skin element stiffness

66. In the previous FE runs the stiffness of the skin element was based on the moment of inertia I_x of the metal sheet at the back of the aluminum panel, on the assumption that the T ribs (see Figure 2), which were placed in the horizontal direction, did not influence the stiffness of the skin element. Knowing that the thickness of the metal sheet is 0.1 in., the moment of inertia I_x per foot of the skin element is 0.0022 in.^{3} To evaluate the influence of skin element stiffness on the tensile stress along the reinforcing strips, another FE analysis, case C_5, was conducted. Case C_5 was similar to case C_4 in every respect except that the moment of inertia in case C_5 was 1.368 in.,^{3} which is the moment of inertia per foot of panel as shown in paragraph 16, rather than 0.0022 in.^{3}, which is the I_x for the aluminum

sheet only. The tensile stress distribution along the instrumented strips at the end of construction and prior to failure for case C_5 is shown in Figure 32. Comparison between results for case C_5 and those of case C_4, shown in Figure 31, indicates that the magnitudes of the maximum tensile stress are not significantly influenced by the stiffness of the skin element. However, maximum tensile stresses for low skin element stiffness, case C_4, occurred at points much closer

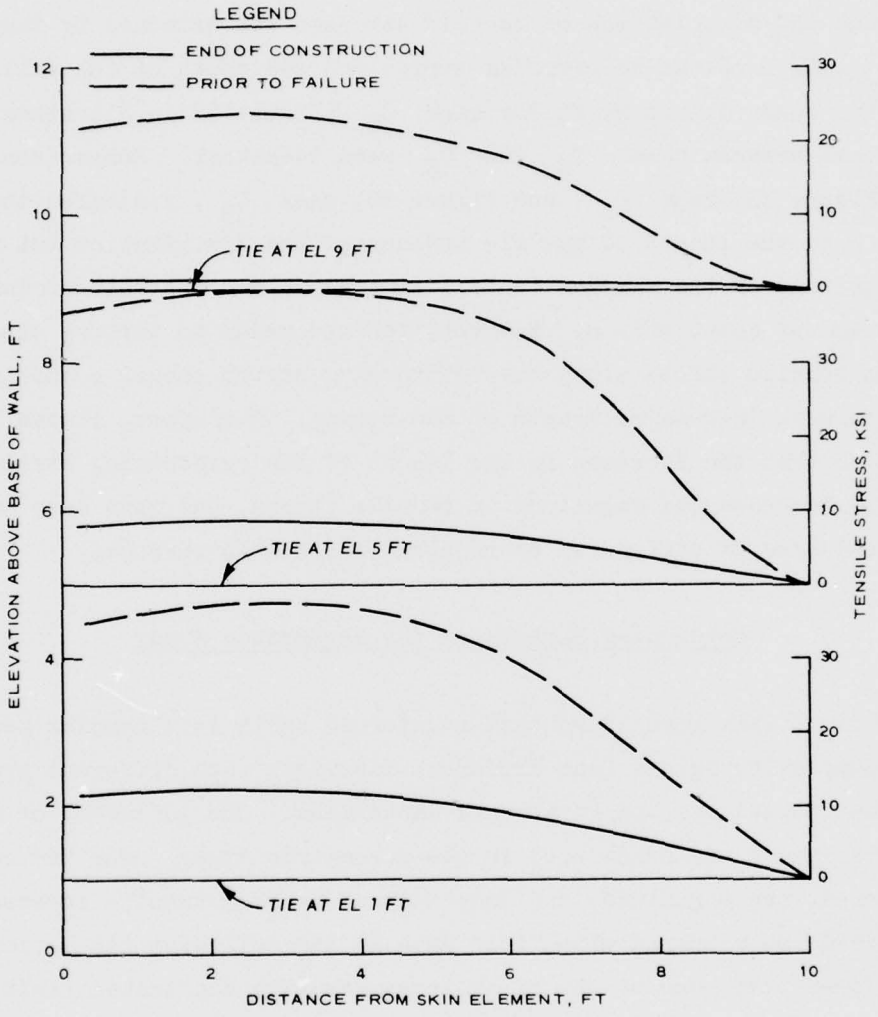

Figure 32. Tensile stress distribution along instrumented reinforcing strips at end of construction and prior to failure for case C_5

to the face of the wall than those for high skin element stiffness, case C_5. Therefore, it can be concluded that the stiffness of the skin element slightly changes the shape of the tensile stress distribution along the reinforcing strip but not the magnitude of the maximum stress.

Influence of length
of reinforcing strips

67. The influence of the length of the reinforcing strips on the magnitude and distribution of tensile stresses was examined by increasing the length of the reinforcing strips and the depth of the wall from 10 ft for case C_3 to 20 ft for case C_6 (Figure 33). All other parameters between cases C_3 and C_6 were identical. Comparison between Figure 33, case C_6, and Figure 30, case C_3, indicates that the increase in the length of the tie influenced the distribution but not the magnitude of the maximum tensile stresses along the reinforcing ties at the end of construction. However, for the prior to failure case, maximum tensile stress along the reinforcing strips showed a substantial decrease with increasing length of the strips. Therefore, it can be concluded that the increase in the length of the reinforcing strips tends to decrease the magnitude of tensile stress, and thus less cross-sectional area of strips may be required for design purposes.

Concluding Remarks on the Parametric Study

68. It has been shown that reinforced earth is a complex problem whose complexity arises from different materials with different properties that constitute the reinforced earth mass. The influence of only a few variables was considered in the parametric study. For the cases considered, the magnitudes and locations of maximum tensile stresses are summarized and compared with field data in the following tabulation. It can be seen that none of the FE analyses actually duplicated field conditions. While the magnitude of the tensile stress in case C_3 is closest to the field observation, the location of the maximum stresses in case C_5 correlates most favorably with the actual field data.

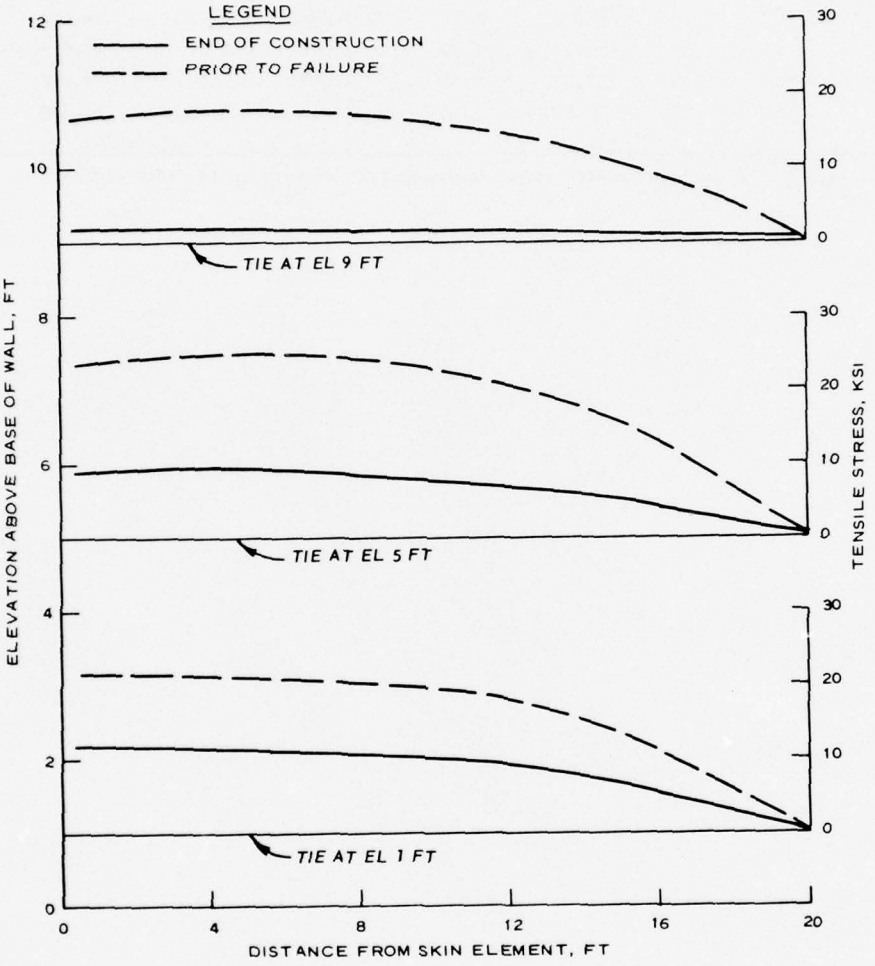

Figure 33. Tensile stress distribution along instrumented reinforcing strips at end of construction and prior to failure for case C_6

	Friction Angle δ, deg	Strip Length L, ft	Tie Elevation Above the Foundation: 1.0 ft — Maximum Tensile Stress psi	1.0 ft — Location* ft	5.0 ft — Maximum Tensile Stress psi	5.0 ft — Location* ft	9.0 ft — Maximum Tensile Stress psi	9.0 ft — Location* ft
Field test	--	10	39,000	2.00	30,000	4.50	29,000	3.50
Case C_1	3	10	28,700	0.75	29,410	1.50	17,220	2.50
Case C_2	3	10	25,140	0.75	26,460	1.50	6,770	0.25
Case C_3	18	10	31,040	0.75	34,980	2.50	26,260	2.50
Case C_4	18	10	37,020	1.50	44,280	2.50	23,650	4.50
Case C_5	18	10	37,600	2.50	39,760	3.50	23,670	2.50
Case C_6	18	20	22,260	1.50	24,510	5.00	17,650	5.00

* The location of the maximum tensile stress was measured relative to the skin element.

PART VI: CONCLUSIONS AND RECOMMENDATIONS

Conclusions

69. An FE study was performed on a reinforced earth wall to investigate the feasibility of using numerical methods in predicting stresses and deformations of the various elements of the wall. The wall used in the FE analyses was 16 ft long, 10 ft deep, and 12 ft high; the earth backfill was reinforced with galvanized steel strips 4 in. wide, 0.025 in. thick, and 10 ft long, spaced at 2- and 2.5-ft intervals in the vertical and horizontal directions, respectively. Based on the results of the FE analyses of this study and results collected from laboratory and field tests, the following conclusions are drawn:

a. The one-dimensional interface elements vertically inserted between the foundation soil and the back of the reinforced earth wall are necessary to permit separation between the reinforcing strips and foundation soil such that the tensile stresses developed in the reinforcing strips decrease to zero at the back of the wall as observed from field measurements. Interface elements horizontally inserted between the reinforcing strips and backfill of the wall are necessary to model the slippage and friction forces between the reinforcing strips and backfill.

b. The assumption of using the FE plane strain analysis to solve the 3D problem of a reinforced earth wall by increasing the area of the reinforcing strip, to cover the entire length of the wall, and decreasing the shear resistance between the fill and the reinforcement by the same proportion is a practical and satisfactory assumption.

c. The FE analyses of both fixed-end and free-end skin elements are necessary to obtain good correlation with field data. It is observed that the FE results of tensile stresses along the reinforcing strips and the lateral deformation of the skin element showed better agreement with field observation for the fixed-end condition than for the free-end condition. However, field measurements of strains along the reinforcing strips agreed better with the FE results of the free-end condition than with those obtained from the fixed-end condition. While lateral pressure cell measurements agreed better with FE results of the fixed-end rather than free-end conditions,

the opposite was observed when comparison was made with lateral earth pressure calculated from tensile stresses in the reinforcing strips.

d. The magnitude and distribution of tensile stresses along the reinforcing strips increase with increasing interface friction between the strip and surrounding fill material, decrease with increasing reinforcing strip length, and show no significant change with change in skin element stiffness.

Recommendations

70. Based on the findings of this study, it is recommended that the study be expanded as follows:

a. It is desirable to improve the computer code and make it more accurate and efficient by introducing elements that better simulate actual events. The variable node of bending element is recommended because fewer elements may be needed and oscillation observed in results using the isoparametric elements may be reduced or eliminated.

b. Better constitutive equations that not only simulate the behavior of each material in the reinforced earth mass but also describe the interaction between different materials should be developed.

c. More studies are needed to model and improve the behavior of interface elements between the reinforcing strips and surrounding soil.

d. Hinges between panels that form the skin element need to be considered in the FE code.

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APPENDIX A: COMPUTER CODE

```
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      PARAMETER MEL=690
      PARAMETER MND=761
      PARAMETER MBW=120
      PARAMETER MW=60
      PARAMETER NEE=2,NBE=100,N4=20,N5=2
C
C
C  CHANGES ADDED ARE LINES 19731-19739 AND 20841-20848
C   MND = MAXIMUM NUMBER OF NODES(IN PARAMETER STATEMENT)
C   MEL = MAXIMUM NUMBER OF ELEMENTS(IN PARAMETER STATEMENT)
C   MW = (MAXIMUM DIFFERENCE + 1) * 2    ( A(MBW,MW)   )
C   MBW = (MAXIMUM DIFFERENCE + 1 ) * 2 * 2  (SIZE OF B VECTOR)
C   NBE = MAXIMUM NUMBER OF PLACEMENT ELEMENTS PER LOAD CASE   LEL(NBE)
C   NEE = MAXIMUM NUMBER OF EXCAVATION ELEMENTS PER LOAD CASE LCL(NEE,8)
C   N4  = MAXIMUN NUMBER OF NODES TO BE SET TO ZERO AFTER A BUILD CASE
C   N5  = MAXIMUM NUMBER OF BOUNDARY WATER PRESSURE CONDITIONS PER CASE
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C    * A FINITE ELEMENT PROGRAM FOR SIMULATION OF ARBITRARY
C    * SEQUENCES OF INCREMENTAL EXCAVATION,SEEPAGE LOADING,
C    * EMBANKMENT OR STRUCTURAL LOADING,BOUNDARY PRESSURE
C    * LOADING,CONCENTRATED FORCE OR DISPLACEMENT LOADING,
C    * OR TEMPERATURE LOADING ON STRUCTURAL MATERIALS
C    * THE ORIGINAL CODE WAS WRITTEN BY DR. WAYNE CLOUGH, DUKE UNIVERSIT
C    * FOR THE IBM 360 COMPUTER
C    * CODE CONVERSION TO THE GE 600 SYSTEM WAS DONE BY MR. DANNY WALTER
C    * MODIFICATIONS TO THE CODE WERE MADE BY DR. R. RADHAKRISHNAN,
C    * MR. FRED TRACY ; AND MR. WAYNE JONES OF THE COMPUTER ANALYSIS
C    * BRANCH, ADP CENTER, WES
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      COMMON      HED(20),LL,NUMNP,NUMEL,NUMMAT,NUMSOL,ITRD,LORUL,NUMJT
     1,NUMAPP,INIT,MTYPE,NOY,NOX,NOXY,NCE,NLEL,NB1E,NONP,NTB,NCTYP,NB1TY
     2P,NB2TYP,NATYP,NEL,NTRIP,NLL,NF,NNPP,NDL,MB,NC,MQ,NB,NNP,KT,MBAND,
     3TEMP,NUMBLK,II,JJ,KK,VOL,UW,SL,XC,YC,RDF,RD,PATM,GAMW,RKS(20),COJ(
     420),PHJ(20),COEFJ(20),EXPJ(20),STFN(320),STFS(320),GAM(20),E(20),G
     5UI(20),GUF(20),AG(20),ALPHA(20),PHI(20),BOWE(20),HCOEF(20),AKP(20)
     6,ULCOEF(20),FR(20),Q(4),PSTR(5),SIG(MEL,6),EE(MEL),BMOD(MEL),GU(ME
     7L),X(MND),Y(MND),DISP(MND,2),FX(MND),FY(MND),PP(MND),DP(MND),IC(30
     80),IL(MEL,5),LCL(NEE,8),LEL(NBE),NP(N4),IBB(N5),JBC(N5),PR(N5),F(2
     9,4),XY(2,4),B(MBW),A(MBW,MW),S(10,10),R(10),C(4,4),LM(8),BA(3,2),A
     1BC(3,4,2),DD(3,10),ST(3,10)
      COMMON/PUNCH/IPUNCH
      COMMON/NXY/NNPX,NNPY
      COMMON/DRAIN/NDRAIN(20)
      REAL HED
      NFILE=0
      CALL FLGEOF(1,NFILE)
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C    *  READ AND PRINT CONTROL DATA  *
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
   10 READ(1,1000)(HED(I),I=1,20),NUMNP,NUMEL,NUMMAT,NUMSOL,NUMJT,NUMAPP
     1,INIT,NC,LORUL,ITRD,GAMW,PATM,IPUNCH
```

```
      IF(NPILE.EQ.1) STOP
      WRITE(3,2000)(MED(I),I=1,20),NUMNP,NUMEL,NUMMAT,NUMBOL,NUMJT,NC,NU
     1MAPP,INIT,LORUC,IAMB,IPATH,IPUNCH
      READ(1,1030) NATYP,NCTYP,NB1TYP,NB2TYP
      WRITE(3,2005) NATYP,NCTYP,NB1TYP,NB2TYP
      WRITE(3,2010)
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C     *  READ AND PRINT MATERIAL PROPERTIES                             *
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      DO 20 N = 1,NUMSOL
      READ(1,1010)GUI(N),GUF(N),GAM(N),FR(N),A0(N),PHI(N),AXP(N)
      READ(1,1013)HCOEF(N),ULCOEF(N),COME(N),E(N),ALPHA(N),NDRAIN(N)
   20 WRITE(3,2020)N,NDRAIN(N),GAM(N),GUI(N),GUF(N),A0(N),PHI(N),COME(N)
     1,AXP(N),HCOEF(N),ULCOEF(N),FR(N),E(N),ALPHA(N)
      IF(NUMSOL .EQ. NUMMAT) GO TO 30
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C     *  READ AND PRINT INTERFACE PROPERTIES                            *
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      NN = NUMBOL + 1
      WRITE(3,2030)
      READ(1,1012)(NDRAIN(N),EXPJ(N),PWJ(N),BKB(N),A0(N),COJ(N),
     1 COEFJ(N),GUI(N),FR(N),N=NN,NUMMAT)
      WRITE(3,2040) (N,BKB(N),COJ(N),PWJ(N),COEFJ(N),EXPJ(N),FR(N),
     1 N = NN,NUMMAT)
   30 CONTINUE
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C     *  READ AND PRINT OF NODAL POINT DATA -- INCLUDING COORDINATES    *
C      PORE PRESSURE AND CHANGE IN PORE PRESSURE OR TEMPERATURE         *
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      WRITE(3,2050)
      L = 0
   40 READ(1,1020) N,X(N),Y(N),PP(N),DP(N)
      NL = L + 1
      IF(N .EQ. 1) GO TO 50
      ZX = N-L
      DX = (X(N) - X(L))/ZX
      DY = (Y(N) - Y(L))/ZX
      DELP = (PP(N) - PP(L))/ZX
      DELPD = (DP(N) - DP(L))/ZX
   50 L = L + 1
      IF(N - L) 80,70,60
   60 X(L) = X(L - 1) + DX
      Y(L) = Y(L - 1) + DY
      PP(L) = PP(L-1) + DELP
      DP(L) = DP(L-1) + DELPD
      IF(L .LE. NUMNP) GO TO 50
      WRITE(3,2055)
      GO TO 60B
   70 WRITE(3,2060) N,X(N),Y(N),PP(N),DP(N),NONE,NL
      IF(NUMNP - N)80,90,40
   80 WRITE(3,2065)
      CALL EXIT
```

```
  98   CONTINUE
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C     * READ AND PRINT BOUNDARY CONDITIONS *
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      WRITE(3,2070)
      READ(1,1030) NOY,NOX,NOXY
      IP = NOY + 1
      IM = NOY + NOX
      IN = IM + 1
      IO = NOY +NOX + NOXY
      IF(NOY .EQ. 0) GO TO 93
      READ(1,1030) (IC(N),N=1,NOY)
      WRITE(3,2075) (IC(N),N=1,NOY)
  93  IF(NOX .EQ. 0) GO TO 97
      READ(1,1030) (IC(N),N=IP,IM)
      WRITE(3,2080) (IC(N),N=IP,IM)
  97  IF(NOXY .EQ. 0) GO TO 99
      READ(1,1030) (IC(N),N=IN,IO)
      WRITE(3,2090) (IC(N),N=IN,IO)
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C     * READ AND PRINT OF ELEMENT DATA *
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
  99  N = 0
      WRITE(3,2100)
 100  READ(1,1030) M,(IL(N,I),I=1,5)
 110  N = N + 1
      IF(M .EQ. N) GO TO 120
      IL(N,1) = IL(N-1,1) + 1
      IL(N,2) = IL(N-1,2) + 1
      IL(N,3) = IL(N-1,3) + 1
      IL(N,4) = IL(N-1,4) + 1
      IL(N,5) = IL(N-1,5)
 120  WRITE(3,2103) N,(IL(N,I),I=1,5)
      IF (M .EQ. N) GO TO 130
      IF(N .LE. NUMEL) GO TO 110
      WRITE(3,2107)
      GO TO 600
 130  IF(NUMEL .EQ. 8) GO TO 140
      GO TO 100
 140  CONTINUE
 150  CONTINUE
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C     * DETERMINE BAND WIDTH *
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      J=0
      DO 250 N=1,NUMEL
      DO 250 I=1,4
      DO 240 L=1,4
      KK=IABS(IL(N,I)-IL(M,L))
      IF(KK-J)240,240,230
 230  J=KK
      MW2=(MW-2)/2
```

```
      IF (J .LE. MW2) GO TO 240
      WRITE (3,2110) N
  240 CONTINUE
  250 CONTINUE
      MBAND=2*J+2
      IF(MBAND .LE. NW ) GO TO 300
      GO TO 600
  300 CONTINUE
      WRITE(3,2120) MBAND
      NNP = 0
      NLL = 0
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C     * ESTABLISH INITIAL STRESSES IN SUBROUTINE INITAL *
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      CALL INITAL
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C     * PERFORM INCREMENTAL LOADING *
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      IF(NC .EQ. 0) GO TO 600
      DO 550 MO = 1,NC
      WRITE(3,2130) MO
      READ (1,1030) NUL,NLL,MS,NNP,NT,NF
      NP(1) = 0
      DO 305 I = 1,NONNP
      FX(I) = 0.
  305 FY(I) = 0.
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C     * READ AND PRINT EXCAVATION DATA *
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      IF(NUL .EQ. 0) GO TO 310
      READ(1,1030) NEL
      READ(1,1030) ((LUL(N,I),I=1,8),N=1,NEL)
      WRITE(3,2195)
      WRITE(3,2160) ((LUL(N,I),I=1,8),N=1,NEL)
      CALL EXCAV
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C     * READ AND PRINT BUILD-UP DATA *
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
  310 IF(NLL .EQ. 0) GO TO 330
      READ(1,1050) NLEL,(LEL(N),N = 1,NLEL)
      READ(1,1050) NJ
      READ(1,1050) NONP,(NP(N),N = 1,NONP)
      READ(1,1040) NCE,NBLE,HTB
      IP1=10+1
      READ(1,1041)NNPX,NNPY
      IF(NNPX.EQ.0) GO TO 4011
 1041 FORMAT(2I5)
      IP2=IP1+NNPX-1
      READ(1,1042)(IC(I),I=IP1,IP2)
 1042 FORMAT(16I5)
 4001 FORMAT(/9X,50HNODES FIXED IN X-DIRECTION FOR THIS INCREMENT ONLY)
 4002 FORMAT(/9X,50HNODES FIXED IN Y-DIRECTION FOR THIS INCREMENT ONLY)
```

```
      WRITE(3,4001)
      WRITE(3,4003)(IC(I),I=IP1,IP2)
      DO 3051 I=IP1,IP2
      NN=IC(I)
 3051 FX(NN)=0.0
 4011 CONTINUE
      IF(NNPY.EQ.0) GO TO 4012
      IP3=IP2+NNPX
      IP4=IP1+NNPX+NNPY-1
      READ(1,1042)(IC(I),I=IP3,IP4)
      WRITE(3,4002)
 4003 FORMAT(10X,10I5)
      WRITE(3,4003)(IC(I),I=IP3,IP4)
      DO 3052 I=IP3,IP4
      NN=IC(I)
 3052 FY(NN)=0.0
 4012 CONTINUE
      WRITE(3,2190)
      WRITE(3,2030) NLBL,(LEL(N),N = 1,NLBL)
      CALL BUILD
      IF(NJ .EQ. (NLEL + 1)) GO TO 330
      DO 320 N = NJ,NLBL
      M = LEL(N)
      SIG(M,5) = 0.0
      STFS(M) = 100000000.
  320 STFN(M) = 100000000.
C . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .
C     . GENERATE SEEPAGE LOADING .
C . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .
  330 IF(MS .EQ. 0) GO TO 350
      WRITE(3,2200)
      CALL SEEP
C     .  READ AND PRINT BOUNDARY PRESSURE LOADING DATA  .
C . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .
  350 IF(NWP .EQ. 0) GO TO 360
      WRITE(3,2260)
      READ (1,1030) NBPF
      READ (1,1060) (IBC(L),JBC(L),PR(L),L=1,NBPF)
      WRITE(3,2300) (IBC(L),JBC(L),PR(L),L=1,NBPF)
C . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .
C     . ACKNOWLEDGE TEMPERATURE LOADING .
C . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .
  360 IF(NT .EQ. 0) GO TO 370
      WRITE(3,2320) MQ
C . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .
C     .  ZERO LOADS ON FIXED BOUNDARY POINTS  .
C . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .
  370 DO 400 M = 1,10
      N = IC(M)
      IF(M .GT. NOY .AND. M .LE. IM) GO TO 390
      FY(N) = 0.0
      IF(M .LE. NOY)GO TO 400
```

```
      390  FX(N) = 0.0
      400  CONTINUE
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C     * ACKNOWLEDGE CONCENTRATED FORCE OR DISPLACEMENT LOADING *
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      IF(NP .EQ. 0) GO TO 470
      WRITE(3,2340)
      READ(1,1030) NCL
      DO 450 I = 1,NCL
      READ(1,1070) M,X1,Y1,N,X2,Y2
      FX(M) = FX(M) + X1
      FY(M) = FY(M) + Y1
      FX(N) = FX(N) + X2
      FY(N) = FY(N) + Y2
  450 WRITE(3,2350) M,X1,Y1,N,X2,Y2
  470 CONTINUE
      DO 500 NQ = 1,NUMAPP
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C     * SET UP STRUCTURE STIFFNESS MATRIX AND LOAD VECTOR *
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      CALL SYRSYP
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C     * SOLVE SIMULTANEOUS EQUATIONS FOR DISPLACEMENTS *
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      CALL BANSOL
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C     * SOLVE FOR ELEMENT STRESSES AT CENTER NODE *
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      CALL STRESS
  500 CONTINUE
      NNPX=0
      NNPY=0
  550 CONTINUE
  600  CONTINUE
C     ***********************************************************
      GO TO 10
 1000 FORMAT(20A4/10I5/2F10.3,I5)
 1010 FORMAT(8F10.5)
 1012 FORMAT(I5,F9.2,7F10.5)
 1015 FORMAT(5E15.5,I5)
 1020 FORMAT(1I10,6F10.3)
 1030 FORMAT(8I10)
 1040 FORMAT(2I10,F16.2)
 1050 FORMAT(16I5)
 1060 FORMAT(3(2I8,F8.2))
 1070 FORMAT(2(I10,2F10.2))
 2000 FORMAT(1H1,20A4///30X,'NUMBER OF NODAL POINTS =',I5,30X,'NUMBER OF
     1ELEMENTS =',I5,30X,'TOTAL NUMBER OF DIFFERENT MATERIALS =',I5,30X,
     2'NUMBER OF DIFFERENT MATERIALS FOR TWO DIMENSIONAL ELEMENTS =',I5/
     330X,'NUMBER OF ONE-D ELEMENTS =',I5/30X,'NUMBER OF LOAD CASES =',
     419/30X,'NUMBER OF ITERATIONS PER LOAD CASE =',I5/30X,'INITIAL STRE
     5SS CODE =',I5/30X,'MODULUS CALCULATION CODE =',I5/30X,'UNIT WEIGHT
```

```
     6 OF WATER =',F10.2/30X,'ATMOSPHERIC PRESSURE =',F10.2/30X,'PUNCH
     7 = ',I5)
2005 FORMAT(///30X,'THE SPECIFIED INCREMENTAL MATERIAL TYPES ARE AS FO
    1LLOWS'//50X,'ATMO',I5/34X,'STRUCTURAL MATERIAL=',I5/35X,'BACKFILL
    2MATERIAL1=',I5/35X,'BACKFILL MATERIAL2=',I5)
2010 FORMAT(1H1,30X,'MATERIAL PARAMETERS'//1X,'MAT DRAIN GAMMA',4X,'MOI
    1',4X,'MUP',4X,'KO',4X,'PHI',4X,'COHESION',3X,'EXP',10X,'LCOEF.',9X
    2,'ULCOEF.',13X,'E',1X,'YOUNGS MODULUS',3X,'ALPHA'//)
2020 FORMAT(1X,I2,3X,I1,3X,F5.1,2X,F5.3,3X,F5.3,1X,F4.2,2X,F5.2,1X,F10
    1.0,4X,F5.3,2(3X,F10.1),5X,F4.2,2X,F14.1,8X,F8.6)
2030 FORMAT(///50X,'INTERFACE PROPERTIES'//15X,'MATERIAL',3X,'RESIDUAL
    [illegible]
2040 FORMAT(/[illegible],F5.2,7X,F5.2)
2050 FORMAT(1H1,     'NODAL POINT',4X,'X-ORDINATE',6X,'Y-ORDINATE',6X,'P
    1ORE PRESSURE',8X,'CHANGE IN PORE PRESSURE',4X,'NODAL POINT'//)
2055 FORMAT(1H1,30X,'ERROR IN NODAL POINT DATA')
2060 FORMAT( 3X,I5,9X,F7.3,11X,F7.3,12X,F8.3,19X,F8.3,13X,I5)
[illegible]
2070 FORMAT ([illegible],'NODES WITH BOUNDARY RESTRAINTS'///)
2075 FORMAT (5X,'NO Y MOVEMENT',20I5/20X,20I5)
2080 FORMAT(//5X,'NO X MOVEMENT',20I5/20X,20I5)
2090 FORMAT(//5X,'NO X OR Y MOVEMENT',20I5/20X,20I5/20X,20I5)
2100 FORMAT(1H1,30X,'ELEMENT NO.       I      J      K      L          MATERI
    1AL'/)
2103 FORMAT(30X,I7,4I7,A2,I12)
2105 FORMAT(1H1,30X,'ERROR IN ELEMENT DATA')
2110 FORMAT(1H1,'BAND WIDTH TOO LARGE AT ELEMENT = ',I5)
2120 FORMAT(////40X,'THE BANDWIDTH IS=',I5///)
2130 FORMAT(1H1,30X,'INCREMENTAL LOADING HAS BEEN SPECIFIED CASE LOADIN
    1G NUMBER',I5///)
2150 FORMAT(////40X,'EXCAVATION INPUT DATA'//15X,'CUT ELEM.',5X,'INTERP
    1OLATION ELEMENTS',5X,'LOADED NODAL POINTS - CODE = 3'//49X,
    2 'I',9X,'J',9X,'K',9X,'L'///)
2160 FORMAT(8(5X,I5))
2190 FORMAT(////40X,'BUILD-UP INPUT DATA'//15X,'NO. ELEM.',30X,
    1'ELEMENTS BUILT-UP')
2200 FORMAT(////30X,'INCREMENTAL SEEPAGE LOADING HAS BEEN SPECIFIED FOR
    1 THIS INCREMENT')
2260 FORMAT(////40X,'INCREMENTAL PRESSURE LOADING'///
    1 2X,'NODE I',5X,'NODE J',5X,'WATER PRESSURE',2X,'NODE I',5X,'NODE
    2J',5X,'WATER PRESSURE',2X,'NODE I',5X,'NODE J',5X,'WATER PRESSURE'
    3//)
2300 FORMAT(3(2I9,F17.2))
2320 FORMAT(//20X,'DIFFERENTIAL NODAL TEMPERATURES ARE BEING APPLIED F
    1OR LOADING CASE # ',I5)
2340 FORMAT(////30X,'INCREMENTAL FORCE OR DISPLACEMENT LOADING HAS BEEN
    1 SPECIFIED FOR THIS INCREMENT'//34X,'NODAL PT.   X-LOAD    Y-LOAD
    2   NODAL PT.   X-LOAD    Y-LOAD'//)
2350 FORMAT(31X,I10,2F10.2,2X,I10,3X,2F10.2)
3010 FORMAT(8F10.2)
3020 FORMAT(9F8.3)
3030 FORMAT(15I5)
```

```
      3040 FORMAT(8E10.4)
           END
*WARN*  7 MEMORY EXPANDED, USE $LIMITS OR CORE= OPTION FOR NEXT RUN
```

```
      SUBROUTINE INITAL
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      PARAMETER MEL=696
      PARAMETER MND=761
      PARAMETER MBW=120
      PARAMETER MW=60
      PARAMETER NEE=2,NBE=100,N4=20,N5=2
      COMMON        MEB(20),LL,NUMNP,NUMEL,NUMMAT,NUMSOL,IIRB,LORUL,NUMJT
     1,MAPP,INIT,MTYPE,NDY,NDX,NDXY,NCE,NLEL,NDIB,NDNP,HTB,NCTYP,NBITY
     2P,NB2,NYP,NATYP,NBL,NTRIP,NLL,NF,NWPF,NUL,MB,NC,MQ,NQ,NWP,NT,MBAND,
     3TEMP,NUMBLK,II,JJ,KK,VOL,UW,SL,XC,YC,RDF,RD,PATM,GAMW,RKS(20),COJ(
     420),PWJ(20),COEFJT(20),EXPJT(20),STFN(320),STFS(320),GAM(20),E(20),G
     5UI(20),GUF(20),AO(20),ALPHA(20),PHI(20),ONE(20),HCOEF(20),ANP(20)
     6,ULCOEF(20),FRC(20),G(4),PSTR(5),SIG(MEL,6),EE(MEL),BMOD(MEL),GU(ME
     7L),X(MND),Y(MND),DISP(MND,2),FX(MND),FZ(MND),PP(MND),DP(MND),IC(30
     80),IL(MEL,5),LOL(NEE,8),LEL(NBE),NP(N4),BC(N5),VBC(N5),PR(N5),F(2
     9,4),XY(2,4),B(MBW),A(MBW,MW),S(10,10),R(80),C(4,4),LM(8),BA(3,2),A
     1BC(3,4,2),DD(3,16),GY(3,10)
      COMMON/DRAIN/NORAIN(20)
      REAL MED
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
       NNPX=0
       NNPY=0
      WRITE(3,2000)
      LL = 0
      II = 0
      MQ = 1
      NQ = NUMAPP
      NTRIP = 1
      IF(INIT .EQ. 1) GO TO 100
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C     *  READ INITIAL STRESS,DISPLACEMENT,MATERIAL PROPERTY AND
C     INTERFACE STIFFNESS VALUES IF SPECIFIED *
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      READ(7,1000)((SIG(N,M),M=1,4),N=1,NUMEL)
      READ(7,1010)((DISP(N,M),M=1,2),PP(N),N=1,NUMNP)
      READ(7,1020)(IL(N,5),N=1,NUMEL)
      IF(NUMJT .EQ. 0) GO TO 170
      READ(7,1030)(STFS(N),STFN(N),N=1,NUMJT)
      GO TO 170
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C     * GENERAT INITIAL VERTICAL STRESSES BY GRAVITY TURN ON IF STRESSES
C     NOT READ IN *
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
  100 DO 150 N = 1,NUMEL
      DO 130 M = 1,4
  130 SIG(N,M) = 0.0
      IF(IL(N,5) .GT. NUMSOL) GO TO 140
      EE(N) = 500000.
      IF(IL(N,5) .EQ. NATYP) EE(N) = E(NATYP)
      MTYPE = IL(N,5)
      GUE = GUI(MTYPE)
```

```
          GU(N) = GU1(MTYPE)
          BMOD(N) = EE(N)/(2.*(1. + GUE)*(1.-2.*GUE))
          GO TO 150
      140 STFN(N) = 100000000.
          STFS(N) = 100000000.
      150 CONTINUE
          DO 160 N = 1,NONMP
          FY(N) = 0.0
      160 FX(N) = 0.0
          CALL STRSTF
          CALL BANSOL
          CALL STRESS
C
      170 DO 360 N = 1,NOMEL
          MTYPE = IL(N,5)
          IF(INIT .EQ. 0) GO TO 260
          IF(MTYPE .LE. NUMSOL) GO TO 240
          GO TO 260
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C     * GENERATE INITIAL HORIZONTAL STRESSES IF STRESSES NOT READ IN  *
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      240 SIG(N,1) = AO(MTYPE)*SIG(N,2)
          SIG(N,4) = ABS( SIG(N,1) - SIG(N,2))/2.
          SIG(N,3) = 0.0
          IF(NDRAIN(MTYPE).EQ.1) GO TO 260
C          NDRAIN=0,  UNDRAINED MATERIAL
          UU=0.0
          DO 250 I=1,4
          L=IL(N,I)
      250  UU=UU+PR(L)
          UW=UU/4.*GAMW
          SIG(N,1)=SIG(N,1)+UW
          SIG(N,2)=SIG(N,2)+UW
      260 IF(MTYPE .GT. NUMSOL) GO TO 360
          DO 265 I = 1,4
      265 Q(I) = SIG(N,I)
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C     * SET UP INITIAL MODULUS VALUES AND PRINT INITIAL STRESSES,
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C       MODULUS VALUES, AND PORE PRESSURES FOR 2-D ELEMENTS  *
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
          CALL MODCAL(N)
*****W   1457 DO LOOP INDEX N MAY NOT BE REDEFINED IN CALL OR ABNORMAL FUNCTION
          WRITE(6,2010) N,XC,YC,(SIG(N,I),I=1,4),PSTR(1),PSTR(2),EE(N),SL,UW
      360 CONTINUE
          IF(NUMJT .EQ. 0) GO TO 390
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C     * OUTPUT INITIAL INTERFACE PROPERTIES  *
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
          WRITE(3,2020)
          DO 380 J = 1,NOMJT
```

```
      K = IL(J,1)
      L = IL(J,2)
      XC = (X(K) + X(L))/2.
      YC = (Y(K) + Y(L))/2.
      IF(INIT .EQ. 0) GO TO 370
      IF (NDRAIN.NE.1) GO TO 366
C     NDRAIN = 0, UNDRAINED MATERIAL
      UU = 0.0
      DO 366 I = 1,4
      LL = IL(J,I)
  366 UU = UU + PP(LL)
      UW = UU/4.0 - SANW
      SIG(J,1) = SIG(J,1) + UW
  368 CONTINUE
      SIG(J,2) = 0.0
  370 WRITE(3,2030) J,XC,YC,(SIG(J,I),I=1,2),SIGN(J),SIGS(J),J
  380 CONTINUE
  390 CONTINUE

C     * ZERO INITIAL DISPLACEMENTS *
      IF( INIT .EQ. 0) GO TO 550
      DO 500 N = 1,NUMNP
      DO 500 M = 1,2
  500 DISP(N,M) = 0.0
  550 CONTINUE
      INIT = 0
      NTRIP = 0
      RETURN
 1000 FORMAT(8F10.2)
 1010 FORMAT(9F8.3)
 1020 FORMAT(15I5)
 1030 FORMAT(6E12.5)
 2000 FORMAT('1H1,40X,'INITIAL EFFECTIVE STRESSES AND MODULI',///  ,'EL NO
     1',5X,'X',5X,'Y',4X,'X-STRESS',2X,'Y-STRESS',2X,'TAU-XY',2X,'TAU MA
     2X',3X,'MAX STRES',1X,'MIN STRES',3X,'MODULUS',2X,'STRESS',
     35X,'ONE',/,40X,'PREVIOUS',3X,'LEVEL',6X,'PRESS')
 2010 FORMAT(I5,2F9.2,1P7E10.3,0PF6.2,F10.2)
 2020 FORMAT(///40X,'INITIAL INTERFACE STRESSES',///9X,'EL NO',4X,'X',
     16X,'Y',4X,'NORMAL STRESS',4X,'SHEAR STRESS',4X,'NORMAL STIFF',
     2 4X,'SHEAR STIFF',4X,'EL NO'//)
 2030 FORMAT(4X,I8,2E8.2,1P4E15.4,I8)
      END
```

```
R0588 01  11-18-76   17:350                                        LABEL EXCAV   PAGE     1

      1            SUBROUTINE EXCAV
      2            IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      3            PARAMETER MEL=690
      4            PARAMETER MND=761
      5            PARAMETER MBW=120
      6            PARAMETER MW=60
      7            PARAMETER NEE=2,NBE=100,N4=20,N5=2
      8            COMMON       HED(20),LL,NUMNP,NUMEL,NUMMAT,NUMSOL,IPRD,LORUL,NUMJT
      9           1,NUMAPP,INIT,MTYPE,NDY,NOX,NOXY,NCE,NLEL,NBIE,NONP,HTB,NCTYP,NB1TY
     10           2P,NB2TYP,NATYP,NEL,NTRIP,NLL,NF,NHPF,NULMS,NC,MQ,NQ,NWP,NT,MBAND,
     11           3TEMP,NUMBLK,II,JJ,KK,VOL,UH,SL,XC,YC,RDF,RD,PATM,GAMW,RKS(20),COJ(
     12           420),PHJ(20),COEFJ(20),EXPJ(20),STFN(320),STFS(320),SAM(20),E(20),G
     13           5UI(20),GUF(20),AO(20),ALPHA(20),PHI(20),COME(20),HCOEF(20),AXP(20)
     14           6,ULCOEF(20),PR(20),B(4),PSTR(5),SIG(MEL,4)TEB(MEL),BMOD(MEL),GU(ME
     15           7L),X(MND),Y(MND),DISP(MND,2),FX(MND),FY(MND),PP(MND),DP(MND),IC(30
     16           80),IL(MEL,5),LUL(NEE,8),LEL(NBE),NP(N4),IBE(N5),JBC(N5),PR(N5),F(2
     17           9,4),XY(2,4),B(MBW),A(MBW,MW),S(10,10),R(10),C(4,4),LM(8),BA(3,2),A
     18          1BC(3,4,2),DD(3,10),ST(3,10)
     19            DIMENSION Z(3,3),ZP(3,3)
     20            EQUIVALENCE (S,Z),(S(10,1),ZP)
     21            REAL HED
     22          C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
     23                III = 0
     24                ISTOP = 0
     25                S2=SQRT(2.)*0.5
     26                DO 300 M = 1,NEL
     27                NI = LUL(M,1)
     28                IF(M .EQ. 1) GO TO 50
     29                MK = LUL(M-1,1)
     30           90   IF(IL(NI,5) .LE. NUMSOL) GO TO 90
     31                STFS(NI) = 10.
     32                STFN(NI) = 10.
     33                GO TO 300
     34           90   IL(NI,5) = NATYP
     35                EE(NI) = E(NATYP)
     36                BMOD(NI)=E(NATYP)
     37                DO 100 I = 1,4
     38                IF(LUL(M,I+4) .NE. 0) GO TO 110
     39           100  CONTINUE
     40                GO TO 300
     41           110  CONTINUE
     42                III = III + 1
     43                DO 25 I = 1,4
     44                DO 25 J = 1,3
     45           25   DD(J,I) = C(J,I)
     46          C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
     47          C     * SET UP CENTER NODE COORDINATE MATRIX S( ) *
     48          C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
     49                DO 120 NN = 1,4
     50                MM = LUL(M,NN)
     51                I = IL(MM,1)
     52                J = IL(MM,2)
```

```
      K = IL(MM,3)
      L = IL(MM,4)
      P(NN) = (PP(I) + PP(J) + PP(K) + PP(L))*GAMMA/4.
      XX=(X(I)+X(J)+X(K)+X(L))*0.25
      YY=(Y(I)+Y(J)+Y(K)+Y(L))*0.25
      NN1 = NN+1
      IF(NN1) 112, 112, 114
  112 S(1,5) = XX
      S(2,5) = YY
      GO TO 120
  114 XX=XX-S(1,5)
      YY=YY-S(2,5)
      Z(NN1,1)=XX
      Z(NN1,2)=YY
      Z(NN1,3)=XX*YY
      ZP(NN1,1)=(XX+YY)*S2
      ZP(NN1,2)=(YY-XX)*S2
      ZP(NN1,3)=ZP(NN1,1)*ZP(NN1,2)
  120 CONTINUE
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C     * INVERT CENTER NODE COORDINATE MATRIX S( ) *
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      D1=0.
      D2=0.
      DO 122 I=1,3
      J1=MOD(I+1,3)+1
      J2=MOD(I,3)+1
      D1=Z(I,1)*Z(J2,2)*Z(J1,3)-Z(I,3)*Z(J2,2)*Z(J1,1)+D1
      D2=ZP(I,1)*ZP(J2,2)*ZP(J1,3)-ZP(I,3)*ZP(J2,2)*ZP(J1,1)+D2
  122 CONTINUE
      D3=ABS(D1)
      D4=ABS(D2)
C
      IF(D3+D4-1.E-4) 125, 125, 130
C
  125 ISTOP = 1
      WRITE(3,2000) NI
      GO TO 300
C
  130 D12=D3-D4
      CALL MIN03(D1,D2,D12)
C
      DO 135 J=1,3
      DO 135 I=1,3
      S(I+1,J+1)=Z(I,J)
  135 CONTINUE
      DO 142 I=2,4
C
      SUM = 0.
      S(1,I)=SUM
      DO 140 J=2,4
      SUM = S(I,J) + SUM
```

```
      240   CONTINUE
            S(I,1) = SUM
      142   CONTINUE
C
            S(1,1)=1.
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C     * SOLVE FOR INTERPOLATION COEFFICIENT MATRIX ABC( )  *
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
            DO 190 KK = 1,3
            DO 190 JJ = I,4
            ABC(KK,JJ,1) = 0.
            DO 190 II = 1,4
            IF(KK .EQ. 3) R(II) = 0.0
            MM = LUL(M,II)
      190   ABC(KK,JJ,1) = ABC(KK,JJ,1) + S(JJ,II)*(BIG(MM,KK) + P(II))
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C     * SOLVE FOR NODAL STRESS MATRIX  C( )  *
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
            DO 250 KK = 1,4
            MM = IL(NI,KK)
            XX=X(MM)-S(1,5)
            YY=Y(MM)-S(2,5)
            IF(D12) 202, 204, 204
      202   D1=XX
            XX=(D1*YY)*S8
            YY=(YY-D1)*S8
      204   DO 250 JJ = 1,3
            IF(III .EQ. 1) GO TO 220
            DO 210 I = 1,4
            IF(LUL(MM,I) .EQ. MM) GO TO 230
      210   CONTINUE
      220   C(JJ,KK) = ABC(JJ,1,1) + ABC(JJ,2,1)*XX    + ABC(JJ,3,1)*YY    +
           1 ABC(JJ,4,1)*XX    *YY
            GO TO 250
      230   DO 240 K = 1,3
      240   C(K,KK) = BDIM(I)
      250   CONTINUE
            CALL BONDFO(NI)
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C     * ADD EQUIVALENT NODAL FORCES TO CONCENTRATED NODAL FORCES *
C     * IF LUL(M,4-8) EQUALS ZERO - NODE NOT LOADED
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
            DO 260 I = 1,4
            IM = IL(NI,I)
            IF(LUL(M,I+4) .EQ. 0) GO TO 260
            FX(IM) = FX(IM) + F(1,I)
            FY(IM) = FY(IM) + F(2,I)
      260   CONTINUE
      300   CONTINUE
            IF(ISTOP .EQ. 1) CALL EXIT
            RETURN
      2000  FORMAT(///20X,'ELEMENT',I4,'HAS PRODUCED A SINGULAR INTERPOLATIO
```

```
157          IN MATRIX-REORDER (INTERPOLATION ELEMENT))
158           END
```

```
      SUBROUTINE MINV3(D1,D2,D12)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C
C
C     THIS SUBROUTINE INVERTS A 3 X 3 MATRIX.
C
C
C     DESCRIPTION OF MAJOR VARIABLES
C
C
C Z/ZP    THE MATRIX TO BE INVERTED,
C
C     B   THE TRANSPOSE OF THE ADJOINT MATRIX,
C
C    D1   THE DETERMINANT OF MATRIX Z,
C
C    D2   THE DETERMINANT OF MATRIX ZP.
C
C
      PARAMETER MEL=690
      PARAMETER MND=761
      PARAMETER MBW=120
      PARAMETER MW=60
      PARAMETER NEE=2,NBE=100,N4=20,ND=2
      COMMON        HED(20),LL,NUMNP,NUMEL,NUMMAT,NUMSOL,ITRD,LORUL,NUMJT
     1,NUMAPP,INIT,ATYRE,NOY,NOX,NOXY,NCE,NLEL,NB1E,NONP,HTB,NCTYP,NB1TY
     2P,NB2TYP,NATYP,NGL,NTRIP,NLL,NF,NHPF,NUL,MB,NC,MQ,NQ,NWP,NT,MBAND,
     3TEMP,NUMBLK,II,JJ,KK,VOL,UW,SL,XC,YC,RDF,RD,PATM,GAMW,RKS(20),COJ(
     420),PHJ(20),COEFJ(20),EXPJ(20),STFN(320),STFS(320),GAM(20),E(20),G
     5UI(20),GUF(20),AO(20),ALPHA(20),PHI(20),COME(20),HCOEF(20),AXP(20)
     6,ULCOEF(20),FR(20),Q(4),PSTR(5),SIG(MEL,4),EE(MEL),BMOD(MEL),GU(ME
     7L),X(MND),Y(MND),DISP(MND,2),FX(MND),FY(MND),PP(MND),DP(MND),IC(30
     80),IL(MEL,5),LUL(NEE,8),LEL(NBE),NP(N4),IBC(N5),JBC(N5),PR(N5),P(2
     9,4),XY(2,4),BX(MBW),A(MBW,MW),Z(3,3),ZP(3,3),B(3,3),ABUF(99),
     1LM(8),BA(3,2),ABC(3,4,2),DD(3,10),ST(3,10)
      REAL HED
C
      DO 20 I=1,3
      IA = MOD(I,3) +1
      IB = MOD(I+1,3) +1
      DO 10 J=1,3
      JA= MOD(J,3) +1
      JB= MOD(J+1,3) +1
      IF(D12) 2, 4, 4
    2 B(J,I) = ZP(IA,JA)*ZP(IB,JB) - ZP(IA,JB)*ZP(IB,JA)
      GO TO 10
    4 B(J,I)=Z(IA,JA)*Z(IB,JB)-Z(IA,JB)*Z(IB,JA)
   10 CONTINUE
   20 CONTINUE
C
      IF(D12) 22, 24, 24
   22 D=D2
```

```
      GO TO 26
   24 D=D1
   26 IF(D) 30,100,30
C
   30 DO 40 I=1,3
      DO 40 J=I,3
      Z(I,J)=B(I,J)/D
   40 CONTINUE
C
  100 RETURN
      END
```

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```
      SUBROUTINE EQNDFO(NI)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      PARAMETER MEL=690
      PARAMETER MND=761
      PARAMETER MBN=120
      PARAMETER MW=60
      PARAMETER NEE=2,NBE=100,N4=20,N5=2
      COMMON        HED(20),LL,NUMNP,NUMEL,NUMMAT,NUMSOL,ITRD,UORUL,NUMJT
     1,NUMAPP,INIT,MTYPE,NBY,NOX,NOXY,NCE,NLEL,NB1E,NONP,BTB,NCTYP,NB1TY
     2P,NB2TYP,NATYP,NBL,NTRIP,NLL,NF,NWPF,NULMB,NC,MQ,NQ,NWP,NT,MBAND,
     3TEMP,NUMBLK,II,JJ,KK,VOL,UW,SL,XC,YC,RDF,RD,PATM,GAMW,RKS(20),COJ(
     420),PHJ(20),COEFJ(20),EXPJ(20),STFN(320),STFS(320),GAM(20),E(20),G
     5OI(20),GUF(20),AO(20),ALPHA(20),PHI(20),COME(20),MCOEF(20),AXP(20)
     6,ULCOEF(20),FR(20),Q(4),PSTR(5),SIG(MEL,4),EE(MEL),BMOD(MEL),GU(ME
     7L),X(MND),Y(MND),DISP(MND,2),FX(MND),FY(MND),PP(MND),DP(MND),IC(30
     80),IL(MEL,5),LUL(NEE,8),LEL(NBE),NP(N4),IBB(N5),JBC(N5),PR(N5),F(2
     9,4),XY(2,4),B(MBW),A(MBW,MW),S(10,10),P(10),G(4,4),LM(8),BA(3,2),A
     1BC(3,4,2),DD(3,10),ST(3,10)
      REAL HED
C ****************************************
C     SET ELE GEOMETRY
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      DO 110 N = 1,4
      I = IL(NI,N)
      MM = N + 1
      IF(MM .EQ. 5) MM = 1
      J = IL(NI,MM)
      XY(1,N) = X(J) -X(I)
  110 XY(2,N) = Y(I) - Y(J)
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C     * SOLVE FOR EQUIVALENT NODAL FORCES *
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      DO 150 J = 1,2
      L = 4
      DO 150 I = 1,4
      K = I + 1
      IF(I .EQ. 4) K = 1
      J3=3-J
      F(J,I) = 0.1667*(XY(J3 ,L)*C(J,L) + 2.*C(J,I)*(XY(J3 ,L) +
     1 XY(J3 ,I)) + XY(J3 ,I)*C(J,K)) + 0.1667*(XY(J,L)*C(3,L)
     2 + 2.0*C(3,I)*(XY(J,L) + XY(J,I)) + XY(J,I)*C(3,K))
      L = I
  150 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE BUILD
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      PARAMETER MEL=690
      PARAMETER MND=761
      PARAMETER MBW=120
      PARAMETER MW=60
      PARAMETER NEE=2,NBE=100,N4=20,N5=2
      COMMON       HED(20),LL,NUMNP,NUMEL,NUMMAT,NUMSOL,ITRD,LORUL,BUMJT
     1,NUMAPP,INIT,MTYPE,NOY,NOX,NOXY,NCE,NLEL,NB1E,NONP,HTB,NCTYP,NB1TY
     2P,NB2TYP,NATYP,NBL,NTRIP,NLL,NF,NWPF,NUL,MS,MC,MQ,NQ,NWP,NT,MBAND,
     3TEMP,NUMBLK,II,JJ,KK,VOL,UW,SL,XC,YC,RDF,RD,PATM,GAMW,RHS(20),COJ(
     420),PWJ(20),COEFJ(20),EXPJ(20),STFN(320),STFS(320),GAM(20),E(40),G
     501(20),GOF(20),AO(20),ALPHA(20),PMI(20),COME(20),HCOEF(20),AKP(20)
     6,ULCOEF(20),Pa(20),Q(4),PSTR(5),gIG(MEL,4)YEB(MBL),BMOD(MEL),GU(ME
     7L),X(MND),Y(MND),DISP(MND,2),FX(MND),FY(MND),PP(MND),DP(MND),IC(30
     80),IL(MEL,5),LUL(NEE,8),LEL(NBE),NP(N4),IBB(N5),JBC(N5),PR(N5),F(2
     9,4),XY(2,4),B(MBW),A(MBW,MW),S(10,10),R(10),C(4,4),LM(8),BA(3,2),A
     1BC(3,4,2),DD(3,10),gY(3,10)
      REAL HED
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      LL = 0
      N1 = NCE + NB1E
 1001 FORMAT(////12X,7HELEMENT,10X,15HREDUCED MODULUS)
 1000 FORMAT(5X,I10,5X,E20.5)
      WRITE(3,1001)
      DO 300 L = 1,NLEL
       MM = LEL(L)
      DO 90 I = 1,4
   90 SIG(MM,I) = 0.0
      I = IL(MM,1)
      J = IL(MM,2)
      K = IL(MM,3)
      M = IL(MM,4)
      YAVG = (Y(I) + Y(J) + Y(K) + Y(M))/4.
      IF(IL(MM,5) .LE. NUMSOL) GO TO 180
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C     * ESTABLISH STRESSES AND MATERIAL PARAMETERS *
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      DX = X(J) - X(I)
      DY = Y(J) - Y(I)
      DL = SQRT(DX**2 + DY**2)
      CSN = DX/DL
      SNE = DY/DL
      ANG = 3.1415
      IF(ABS(CSN-0.) .LT. 0.0000001) GO TO 190
      TANG = SNE/CSN
      ANG = 2.*ATAN(TANG)
  150 SS = GAM(NB1TYP)*(HTB - YAVG)
      SR = AO(NB1TYP)*SS
      SIG(MM,1) = 0.5*(SS + SR) + 0.5*(SS -SR)*CHS(ANG)
      STFN(MM) = 100000000.
  170 STFS(MM) = 10.
```

```
      GO TO 300
  180 MG = NCTYP
      IF(L .GT. NCE) MG = NB1TYP
      IF(L .GT. N1) MG = NB2TYP
      IL(MM,5) = MG
      IF(IL(MM,3) .EQ. NB2TYP .OR. IL(MM,3) .EQ. NB1TYP) GO TO 200
      GU(MM) = GUI(NCTYP)
      BMOD(MM) = EI(NCTYP)/(2.*(1. + GUI(NCTYP))*1. - 2.*GUI(NCTYP)))
      EE(MM)=E(NCTYP)
      GO TO 220
  200 Q(2) = GAM(MG)*(HTB-YAVG)
      Q(1) = AO(MG)*Q(2)
      Q(3) = 0.0
      Q(4) = 0.0
      CALL MODCAL(MM)
      IF(IL(MM,3) .EQ. NB2TYP) GO TO 201
       EE(MM)=EE(MM) *1.001
  201 SIG(MM,2) = Q(2)
      SIG(MM,1) = Q(1)
      SIG(MM,4) = 0.0
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C  * GENERATE GRAVITY LOADING FOR BUILD-UP ELEMENTS *
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
  220 AREA = 0.0
      I = IL(MM,1)
      J = IL(MM,2)
      K = IL(MM,4)
      GO TO 260
  240 I = IL(MM,3)
      J = IL(MM,4)
      K = IL(MM,2)
      ITR = I + J
  260 AREA = AREA + 0.5*(X(J)*(Y(K)-Y(I)) + X(I)*(Y(J)-Y(K)) +
     1 X(K)*(Y(I) - Y(J)))
      IF(I .EQ. IL(MM,1)) GO TO 240
      IF (ITR) 264, 262, 264
  262 IQT=3
      GO TO 266
  264 IQT = 4
  266 FI = IQT
      CK =-AREA*GAM(MG)/FI
      DO 280 I = 1,IQT
      J = IL(MM,I)
  280 FY(J) = FY(J) + CK
      WRITE(3,1000)MM,EE(MM)
  300 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE SEEP
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      PARAMETER MEL=690
      PARAMETER MND=761
      PARAMETER MBW=120
      PARAMETER MW=80
      PARAMETER NEE=2,NBE=100,N4=20,N5=2
      COMMON       HED(20),LL,NUMNP,NUMEL,NUMMAT,NUMSOL,ITRD,LORUL,NUMJT
     1,NUMAPP,INIT,MTYPE,NDY,NOX,NOXY,NCE,NLEL,NB1E,NONP,HTB,NCTYP,NB1TY
     2P,NB2TYP,NATYP,NEL,NTRIP,NLL,NF,NWPF,NUL,MS,NC,MQ,NQ,NWP,NT,MBAND,
     3TEMP,NUMBLK,II,JJ,KK,VOL,UW,SL,XC,YC,RDF,RD,PATM,GAMW,RKS(20),COJ(
     420),PHJ(20),COEFJ(20),EXPJ(20),STFN(320),STFS(320),GAM(20),E(20),G
     5UI(20),GUF(20),AO(20),ALPHA(20),PHI(20),CONE(20),HCOEF(20),AXP(20)
     6,ULCOER(20),FR(20),Q(4),PSTR(5),SIG(MEL,4),EE(MEL),BROD(MEL),GU(ME
     7L),X(MND),Y(MND),DISP(MND,2),FX(MND),FY(MND),PP(MND),DP(MND),TC(30
     80),IL(MEL,5),LUL(NEE,8),LEL(NBE),NP(N4),IBC(N5),JBC(N5),PR(N5),F(2
     9,4),XY(2,4),B(MBW),A(MBW,MW),S(10,10),R(80),C(4,4),LM(8),BA(3,2),A
     1BC(3,4,2),DD(3,10),SV(3,10)
      REAL HED
C . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .
C     . SET UP NEW PORE PRESSURE .
C . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .
      DO 100 N=1,NUMNP
      PTEM = PR(N) + DR(N)
      IF(PTEM . GE . 0.) GO TO 50
      DP(N) = -PP(N)
      PP(N) = 0.
      GO TO 100
   50 PP(N) = PTEM
  100 CONTINUE
      DO 200 NI = 1,NUMEL
      MTYPE = IL(NI,5)
      IF(MTYPE .GT. NUMSOL .OR. MTYPE .EQ. NCTYP .OR. MTYPE .EQ. NATYP)
     1GO TO 200
      DO 120 I = 1,4
      NN = IL(NI,I)
      C(3,I) = 0.0
      DO 120 J = 1,2
      C(J,I) = DP(NN)*GAMW
  120 CONTINUE
      CALL FONDFO(NI)
******  1457 DO LOOP INDEX NI MAY NOT BE REDEFINED IN CALL OR ABNORMAL FUNCTION
C . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .
C     . DISTRIBUTE DIFFERENCE IN NODAL LOADS .
C . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .
  150 ITR = IL(NI,3) - IL(NI,4)
      IF (ITR) 154, 152, 154
  152 IOT = 3
      GO TO 155
  154 IOT = 4
  155 FI = IOT
      HH = 0.
```

```
52              VV = 0.
53              DO 157 I = 17 4
54              HH = F(1,I) + HH
55              VV = F(2,I) + VV
56          157 CONTINUE
57              HH = HH/FI
58              VV = VV/FI
59              DO 160 I = 1,IDT
60              XY(1,I) = HH
61          160 XY(2,I) = VV
62          180 DO 187 J = 1,IDT
63              NN = IL(NI,J)
64              FX(NN) = FX(NN) + XY(1,J)
65              FY(NN) = FY(NN) + XY(2,J)
66          187 CONTINUE
67          200 CONTINUE
68              RETURN
69              END
```

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```
C • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • •
      SUBROUTINE STRSTF
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      PARAMETER MEL=690
      PARAMETER MND=761
      PARAMETER MBW=120
      PARAMETER MW=60
      PARAMETER NEE=2,NBE=100,N4=20,N5=2
      COMMON        HED(20),LL,NUMNP,NUMEL,NUMMAT,NUMSOL,ITRD,LORUL,NUMJT
     1,NUMAPP,INIT,MTYPE,NOY,NOX,NOXY,NCE,NLEL,NB1G,NONP,HTB,NCTYP,NB1TY
     2P,NB2TYP,NATYP,NEL,NTRIP,NLL,NF,NWPF,NUL,MG,NC,MQ,NQ,NWP,NT,MBAND,
     3TEMP,NUMBLK,II,JJ,KK,VOL,UW,SL,XC,YC,HDF,ROD,RATH,BADW,RNS(20),COJ(
     420),PHJ(20),COEFJ(20),EXPJ(20),STFN(320),STFD(320),GAM(20),E(40),G
     5OI(20),GOF(20),AO(20),ALPHA(20),PHI(20),OME(20),HCOEF(20),AXP(20)
     6,ULCOEF(20),PR(20),Q(4),PSTR(5),SIG(MEL,4),TEE(MEL),BMOD(MEL),BU(ME
     7L),X(MND),Y(MND),DISP(MND,2),FX(MND),FY(MND),PP(MND),DP(MND),IC(30
     80),IL(MEL,9),LUL(NEE,8),LEL(NBE),NP(N4),IBC(N5),JBC(N5),PR(N5),F(2
     9,4),XY(2,4),B(MBW),A(MBW,MW),S(10,10),P(10),Q(4,4),LM(8),BA(3,8),A
     1BC(3,4,2),DD(3,10),SS(3,10)
      COMMON/NXY/NNPX,NNPY
      REAL HED
C • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • •
C     • INITIALIZE CONSTANTS TO CONTROL EQUATION BLOCKS •
C • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • •
      REWIND 5
      NB = MW/2
      ND = 2*NB
      ND2 = 2*ND
      NUMBLK = 0
      STOP = 0.
C • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • •
C     • INITIALIZE STIFFNESS MATRIX AND LOAD VECTOR •
C • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • •
      DO 100 N = 1,ND2
      B(N) = 0.0
      DO 100 M = 1,ND
  100 A(N,M) = 0.0
C • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • •
C     • FORM STRUCTURAL STIFFNESS MATRIX A(MBAND/2*DOF) IN BLOCKS •
C • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • • •
  105 NUMBLK = NUMBLK + 1
      NH = NB*(NUMBLK + 1)
      NM = NH - NB
      NL = NM - NB + 1
      KSHIFT = 2*NL - 2
      DO 260 N = 1,NUMEL
      IF(IL(N,5) .LT. 0) GO TO 260
      DO 120 I = 1,4
      IF(IL(N,I) - NL)120,110,110
  110 IF(IL(N,I) - NM)140,140,120
  120 CONTINUE
      GO TO 260
```

```
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   53      C . . . . . .
   54      C     . FORM ELEMENT STIFFNESS MATRIX S(10,10) .
   55      C . . . . . .
   56       140  MTYPE = IL(N,5)
   57            IF (MTYPE .LE. NUMSOL) GO TO 150
   58            CALL JTSTF(MT)
*****W  1457 DO LOOP INDEX N MAY NOT BE REDEFINED IN CALL OR ABNORMAL FUNCTION
   59            GO TO 200
   60       150  CALL QUAD(N,0)
*****W  1457 DO LOOP INDEX N MAY NOT BE REDEFINED IN CALL OR ABNORMAL FUNCTION
   61            IF(VOL .GT. .0001) GO TO 200
   62            WRITE(3,2000) N
   63            STOP = 1.0
   64      C . . . . . .
   65      C     . ADD S(8,8) TO STRUCTURAL STIFFNESS AS DIRECT STIFFNESS
   66      C     METHOD = STORE ONLY ABOVE DIAGONAL TERMS IN A( ) .
   67      C . . . . . .
   68       200  IL(N,5) = MTYPE
   69            DO 210 I = 1,4
   70       210  LM(I) = 2*IL(N,I) - 2
   71            DO 290 I = 1,4
   72            DO 250 K = 1,2
   73            II = LM(I) + K - KSHIFT
   74            KK = 2*I - 2 + K
   75            B(II) = B(II) + Q(KK)
   76            DO 250 J = 1,4
   77            DO 250 L = 1,2
   78            JJ = LM(J) + L - II + 1 - KSHIFT
   79            LL = 2*J - 2 + L
   80            IF(JJ) 250,250,220
   81       220  IF(ND - JJ) 230,240,240
   82       230  WRITE(3,2000) N
   83            STOP = 1.
   84            GO TO 260
   85       240  A(II,JJ) = A(II,JJ) + S(KK,LL)
   86       250  CONTINUE
   87       260  CONTINUE
   88      C . . . . . .
   89      C     . ADD INCREMENTAL CONCENTRATED LOADS TO LOAD VECTOR B( ) .
   90      C . . . . . .
   91            NX = NM
   92            IF(NM .GT. NUMNP-NX) NUMNP
   93            DO 270 N = NL,NX
   94            K = 2*N - KSHIFT
   95            B(K) = B(K) + FY(N)
   96       270  B(K - 1) = B(K - 1) + FX(N)
   97            IF(NWP .EQ. 0) GO TO 275
   98      C . . . . . .
   99      C     . ADD BOUNDARY PRESSURE LOADS .
  100      C . . . . . .
  101            DO 500 L = 1,NWP
  102            I = IBC(L)
```

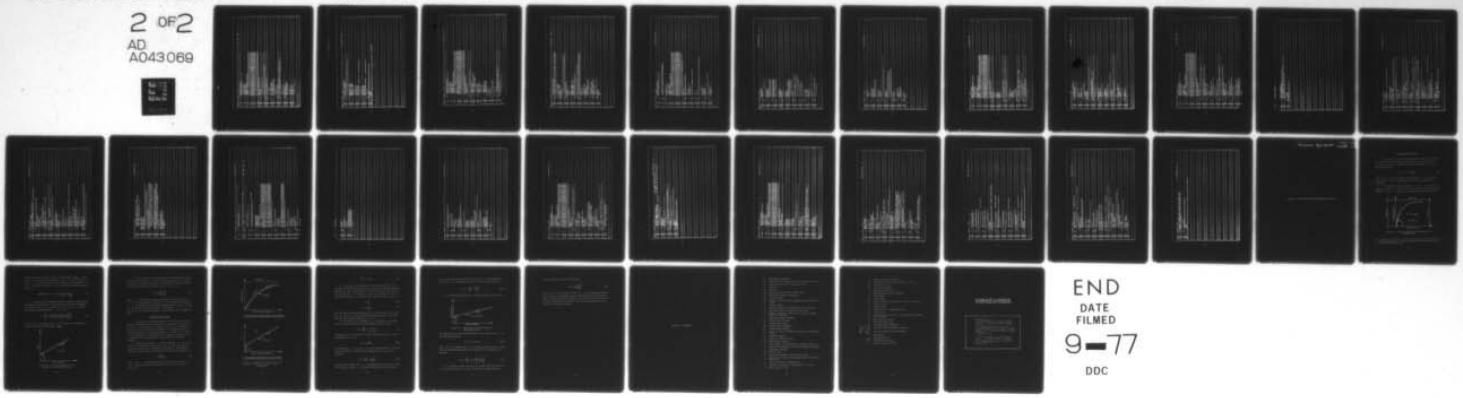
```
          J = JBC(L)
          WP = PR(L)/2.
          DY = (Y(I) - Y(J))*WP
          DX = (X(J) - X(I))*WP
          II = 2*I-KSHIFT
          JJ = 2*J-KSHIFT
          IF(II .LE. 0) GO TO 450
          IF(II-ND .GT. 0) GO TO 450
          B(II-1) = B(II-1) + DY
          B(II) = B(II) + DX
    450   IF(JJ .LE. 0) GO TO 500
          IF(JJ-ND .GT. 0) GO TO 500
          B(JJ-1) = B(JJ-1) - DY
          B(JJ) = B(JJ) - DX
    500   CONTINUE
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C         * MODIFY EQUATIONS FOR BOUNDARY CONDITIONS
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
    295   IN = NOY + NDX + 1
          IM=NOY+NOX+NGXT+NNPX
          IM1=IM+NNPX
          IM2=IM+NNPY
          DO 310 MM = 1,IM2
          M = IC(MM)
          IF(MM .GT. NOY .AND. MM .LT. IN) GO TO 280
          IF(MM.GT.IM1.AND.MM.LE.IM) GO TO 280
          KK = 0
          U = FY(M)
          IF(MM.GT.IM) U=0.0
          N = 2*M - KSHIFT
          GO TO 290
    280   U = FX(M)
          KK = 1
          N = 2*M - 1 - KSHIFT
    290   IF(M .LT. NL) GO TO 310
          IF(M .GT. NH .OR. M .GT. NUMNP) GO TO 310
          DO300 NN = 2,NBAND
          K = N - NN + 1
          IF(K .LE. 0) GO TO 299
          B(K) = B(K) - A(K,NN)*U
          A(K,NN) = 0.0
    299   K = N + NN - 1
          IF(ND2 .LT. K) GO TO 300
          B(K) = B(K) - A(N,NN)*U
          A(N,NN) = 0.0
    300   CONTINUE
          A(N,1) = 1.0
          B(N) = U
           IF(MM.GE.IN.AND.MM.LE.IM1.AND.KK.EQ.0) GO TO 280
    310   CONTINUE
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C         *WRITE 60X60 BLOCK OF EQUATIONS ON TAPE AND SHIFT UP LOWER BLOCK*
```

```
R0588 01  11-18-76  17:346  * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * LABEL STRSTF  PAGE     4
  155       C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
  156             WRITE(5) (B(N),(A(N,M),M = 1,MBAND),N = 1,ND)
  157       C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
  158       C     * INITIALIZE FOR NEXT BLOCK OF EQUATIONS *
  159       C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
  160             DO 320 N = 1,ND
  161             K = N + ND
  162             B(N) = B(K)
  163             B(K) = 0.0
  164             DO 320 M =1,ND
  165               A(N,M) = A(K,M)
  166       320   A(K,M) = 0.0
  167       C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
  168       C     * CHECK FOR LAST BLOCK *
  169       C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
  170             IF(NM .LT. NUMEQ) GO TO 105
  171        340  CONTINUE
  172             IF (ISTOP .EQ. 1) CALL EXIT
  173             RETURN
  174        2000 FORMAT(1H1,'NEGATIVE AREA ELEMENT = ',I5)
  175        3000 FORMAT(3I5)
  176        3010 FORMAT(2I5,F15.5)
  177        3020 FORMAT(10E11.4)
  178             END
000000W  1470 EQUALITY OR NON-EQUALITY COMPARISON MAY NOT BE MEANINGFUL IN LOGICAL IF EXPRESSIONS
```

AD-A043 069 ARMY ENGINEER WATERWAYS EXPERIMENT STATION VICKSBURG MISS F/G 8/13
FINITE ELEMENT ANALYSIS OF A REINFORCED EARTH WALL.(U)
JUL 77 M M AL-HUSSAINI, L D JOHNSON

UNCLASSIFIED WES-TR-S-77-6 NL

2 OF 2
AD
A043069



END
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R0588 01 11-18-76 17:349 LABEL JTSTF PAGE 1

```
      C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
            SUBROUTINE JTSTF(N)
            IMPLICIT DOUBLE PRECISION (A-H,O-Z)
            PARAMETER MEL=690
            PARAMETER MND=761
            PARAMETER MBW=120
            PARAMETER MW=660
            PARAMETER NEE=2,NBE=600,N4=20,N5=2
            COMMON      HED(20),LL,NUMAP,NUMEL,NUMMAT,NUMSOL,ITRD,LORUL,NUMJT
           1,NUMAPP,INIT,MTYPE,NDY,NDX,NDXY,NCE,NLEL,NBIE,NQNP,NTB,NCTYP,NB1TY
           2P,NB2TYP,NATYP,NEL,NTRIP,NLL,NF,NWPF,NUL,NMS,NC,MQ,NQ,NWP,NT,MBAND,
           3TEMP,NUMBLK,II,JJ,KK,VOL,UW,SL,XC,YC,RDF,RD,RATH,GAUN,RMS(20),COJ(
           420),PHJ(20),DOEFJ(20),EXPJ(20),STFN(3,3),SVFB(3,3),GAM(20),E(20),G
           5DI(20),GDF(20),AG(20),ALPHA(20),PHI(20),GONE(20),HCOEF(20),AXE(20)
           6,ULCOEF(20),FR(20),QI(4),PSTR(9),SIGMEL,4),TEE(MEL),BMOD(MEL),GU(ME
           7L),X(MND),Y(MND),DISR(MND,2),FX(MND),FY(MND),PP(MND),DP(MND),IC(30
           80),IL(MEL,5),LUL(NEE,8),LEL(NBE),NP(N4),IBC(N5),JBC(N5),PR(N5),F(2
           9,4),XY(2,4),B(MBW),A(MBW,MW),S(10,10),R(10),G(4,4),LM(8),BA(8,2),A
           1BC(3,4,2),DDI3,10),ST(3,10)
            REAL MED
      C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
            DIMENSION BC(4,4),COSIN(2,2)
            DATA BC/2.0D0,-1.0D0,-2.0D0,1.0D0,2.0D0,-2.0D0,-1.0D0,-1.0D0,
           1 -2.0D0,2.0D0,1.0D0,-2.0D0,-1.0D0,1.0D0,2.0D0/
      C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      C     * ESTABLISH ELEMENT GEOMETRY *
      C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
            II = IL(N,1)
            JJ = IL(N,2)
            DX = X(JJ) - X(II)
            DY = Y(JJ) - Y(II)
            DL = DSQRT(DX**2 + DY**2)
            IF(DL .GT. 0.) GO TO 100
            WRITE(3,2000) N
            CALL EXIT
        100 JTYPE = IL(N,5)
      C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      C     * SET UP ELEMENT STIFFNESS IN LOCAL COORDINATE SYSTEM *
      C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
            DO 150 II = 1,8
            P(II) = 0.
            DO 150 JJ = 1,8
        150 S(II,JJ) = 0.
            CKS = STRS(N)*DL/8.
            CKN = STFN(N)*DL/8.
            DO 160 II = 1,4
            IS = 2*II - 1
            IN = 2*IS
            DO 160 JJ = 1,4
            JS = 2*JJ - 1
            JN = 2*JJ
            S(IS,JS) = CKS*BC(II,JJ)
```

```
  160 S(IN,JN) = CKN*BC(II,JJ)
      IF(DY .EQ. 0.) RETURN
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C     * ROTATE STIFFNESS TO GLOBAL COORDINATE * (AT)*(K)*(A) *
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      COSIN(1,1) = DX/DL
      COSIN(1,2) = DY/DL
      COSIN(2,1) = -DY/DL
      COSIN(2,2) = DX/DL
      DO 190 NN = 1,4
      DO 170 II = 1,8
      JJ = 2*NN - 1
      TEMP = S(II,JJ)
      DO 170 KK = 1,2
      S(II,JJ) = TEMP    *COSIN(1,KK) + S(II,2*NN)*COSIN(2,KK)
  170 JJ = JJ + 1
      DO 180 II = 1,8
      JJ = 2*NN - 1
      TEMP = S(JJ,II)
      DO 180 KK = 1,2
      S(JJ,II) = COSIN(1,KK)*TEMP     + COSIN(2,KK)*S(2*NN,II)
  180 JJ = JJ + 1
  190 CONTINUE
 2000 FORMAT(1H1/'NEGATIVE AREA JOINT ELEMENT = ',I5)
      RETURN
      END
*****W  1470 EQUALITY OR NON-EQUALITY COMPARISON MAY NOT BE MEANINGFUL IN LOGICAL IF EXPRESSIONS
```

```
R0588 01  11-18-76   17,348    . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .  LABEL QUAD    PAGE    1
C . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .
      SUBROUTINE QUAD(N,IZ)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      PARAMETER MEL=690
      PARAMETER MND=761
      PARAMETER MBW=120
      PARAMETER MW=60
      PARAMETER NEE=2,NBE=100,N4=20,N5=2
      COMMON        HED(20),LL,NUMNP,NUMEL,NUMMAT,NUMSOL,ITRD,LORUL,NUMJT
     1,NUMAPP,INIT,MTYPE,NOY,NOX,NOXY,NCE,NLEL,NB1E,NONP,UTB,NCTYP,NB1TY
     2P,NB2TYP,NATYP,NBL,NTRIP,NLL,NF,NWPF,NUL,MS,NC,MQ,NQ,NMP,NT,MBAND,
     3TEMP,NUMBLK,II,JJ,KK,VOL,UW,SL,XC,YC,RBF,RO,RATM,GAMW,RRS(20),COJ(
     420),PHJ(20),COEFJ(20),EXPJ(20),STFN(320),STFS(320),GAM(20),E(20),G
     5UI(20),GUF(20),AO(20),ALPHA(20),PHI(20),GOME(20),MCOEF(20),AMP(20)
     6,ULCOEF(20),FR(20),D(4),PSTR(5),SIG(MEL,4),TEE(MEL),BMOD(MEL),GU(ME
     7L),X(MND),Y(MND),DISP(MND,2),FX(MND),FY(MND),PP(MND),DP(MND),IC(30
     80),IL(MEL,5),LUL(NEE,8),LEL(NBE),NP(N4),IBB(N5),JBC(N5),PR(N5),F(2
     9,4),XY(2,4),B(MBW),A(MBW,MW),S(10,10),R(80),E(4,4),CM(8),BA(3,2),A
     1BC(3,4,2),DD(3,10),SS(3,10)
      DIMENSION TT(8)
      REAL HED
C . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .
C     . SET UP STRESS-STRAIN RELATIONSHIP .
C . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .
      MTYPE = IL(N,5)
      DEVMOD = EE(N)/(2.*(1. + GU(N)))
      C(1,1) = BMOD(N) + DEVMOD
      C(1,2) = BMOD(N) - DEVMOD
      C(3,3) = DEVMOD
   88 C(1,3) = 0.0
      C(2,1) = C(1,2)
      C(2,2) = C(1,1)
      C(3,2) = 0.0
      C(3,1) = 0.0
      C(2,3) = 0.0
      C(1,3) = 0.0
      ITR=IL(N,3)-IL(N,4)
      IF(ITR) 84, 82, 84
   82 IOT=3
      GO TO 86
   84 IOT=4
   86 PI=IOT
C
      IF (IZ) 100, 100, 90
C
   90 TEMP = 0.
      GO TO 160
C . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .
C     . FORM QUADRILATERAL STIFFNESS MATRIX .
C . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .
  100 CALL ISOPAR (N)
      IF(VOL .LE. .0001) GO TO 300
```

```
      IF(INIT .EQ. 0) GO TO 130
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C     * PERFORM GRAVITY LOADING TO CALCULATE INITIAL VERTICAL STRESSES*
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      CK = VOL*GAM(MTYPE)
      DO 120 I = 1,IOT
  120 P(2*I) = -CK/FI
  130 CONTINUE
      TEMP = 0.0
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C     * REDUCE 10X10 QUAD STIFFNESS TO 8X8 BY STATIC CONDENSATION *
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      IF(C(1,1))140,160,140
  140 CONTINUE
      DO 150 K = 1,2
      IH=10-K
      ID=IH+1
      DO 150 I = 1,IH
      S(ID,I)=S(ID,I)/S(ID,ID)
      DO 150 J = 1,IH
  150 S(J,I) = S(J,I) - S(J,ID)*S(ID,I)
  160 IF(INIT .EQ. 1) GO TO 300
      IF(NT .EQ. 0 .OR. MTYPE .NE. NCTYP) GO TO 300
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C     * APPLY TEMPERATURE LOADING TO STRUCTURE MATERIAL IN NT .EQ. 1 *
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      DTAVG=0.
      DO 162 I=1,IOT
      J=IL(N,I)
      DTAVG=DP(J)+DTAVG
  162 CONTINUE
      DTAVG=DTAVG/FI
      TEMP = DTAVG*ALPHA(MTYPE)
      IF (IZ) 170, 170, 300
  170 DO 200 I = 1,4
      II = IN(N,I)
      TT(2*I) = Y(II)*TEMP
  200 TT(2*I-1) = X(II) * TEMP
      DO 250 I = 1,8
      DO 250 K = 1,8
  250 P(I) = P(I) + S(I,K)*TT(K)
  300 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE ISOPAR(N)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C
C
C     THIS SUBROUTINE COMPUTES THE STIFFNESS MATRIX
C     ACCORDING TO THE FIVE-POINT ISOPARAMETRIC ELEMENT.
C
C
      PARAMETER MEL=690
      PARAMETER MND=761
      PARAMETER MBW=120
      PARAMETER MW=60
      PARAMETER NEE=2,NBE=100,N4=20,N5=2
      COMMON      HED(20),LL,NUMNP,NUMEL,NUMMAT,NUMSOL,ITRD,LORUL,NUMJT
     1,NUMAPP,INIT,MTYPE,NOY,NOX,NOXY,NCE,NLEL,NB1E,NONP,UTB,NCTYP,NB1TY
     2P,NB2TYP,NATYP,NEL,NTRIP,NLL,NF,NWPF,NUL,MB,NC,MQ,NQ,NWP,NT,MBAND,
     3TEMP,NUMBLK,II,JJ,KK,VOL,UW,SL,XC,YC,RDF,RD,RATM,GAMW,RKS(20),COJ(
     420),PHJ(20),COEFJ(20),EXPJ(20),STFN(320),STFB(320),GAM(20),E(20),G
     5UI(20),GOF(20),AO(20),ALPHA(20),PHI(20),GOME(20),HCOEF(20),AXP(20)
     6,ULCOEF(20),PR(20),Q(4),PSTR(5),SIG(MEL,4),EB(MEL),BMOD(MEL),GU(ME
     7L),X(MND),Y(MND),DISP(MND,2),FX(MND),FY(MND),PQ(MND),DP(MND),IC(30
     80),IL(MEL,5),LUL(NEE,8),LEL(NBE),NP(N4),IBC(N5),JBC(N5),PR(N5),F(2
     9,4),XY(2,4),B(MB4),A(MBW,MW),S(10,10),R(10),C(4,4),LM(8),BA(3,2),
     1WF(2),PT(2),FJ(2,2),AA(2,3),HP(2,5),DD(3,10),ST(3,10)
      REAL HED
C
C     DEFINE CONSTANTS OF INTEGRATION.
C
      WF(1)=1.
      WF(2)=1.
      PT(2)=SQRT(3.)/3.
      PT(1)=-PT(2)
C
C     ESTABLISH THE THICKNESS OF THE MATERIAL.
C
      TH=1.
C
      VOL = 0.
C
C     INITIALIZE THE STIFFNESS MATRIX(S) AND THE FORCE VECTOR(P).
C
      DO 100 I=1,10
      P(I) = 0.
      DO 100 J=1,I
      S(I,J)=0.
  100 CONTINUE
C
C     PERFORM THE NUMERICAL INTEGRATION.
C
      DO 1000 NT=1,2
      TTT=PT(NT)
C
```

```
      DO 1000 NS=1,2
      SSS=PT(NS)
C
C     COMPUTE THE P PRIME MATRIX.
C
      DO 200 I=1,4
      SI=-1.+I+I+SSS
      TI=I/3.-2.1
      PP(1,I)=(TTT+TI+I.)+SI+0.25
      PP(2,I)=(SSS+SI+I.)+TI+0.25
200   CONTINUE
      PP(1,5)=(TTT+TTT+1.)+SSS+2.
      PP(2,5)=(SSS+SSS+1.)+TTT+2.
C
C     COMPUTE THE JACOBIAN MATRIX.
C
      DO 300 I=1,2
      DO 300 J=1,2
      FJ(I,J)=0.
300   CONTINUE
C
      DO 400 I=1,4
      ND=IL(N,I)
      XX=X(ND)
      YY=Y(ND)
      FJ(1,1)=PP(2,I)+YY+FJ(1,1)
      FJ(1,2)=-PP(1,I)+YY+FJ(1,2)
      FJ(2,1)=-PP(2,I)+XX+FJ(2,1)
      FJ(2,2)=PP(1,I)+XX+FJ(2,2)
400   CONTINUE
C
C     COMPUTE THE VOLUME OF THE ELEMENT.
C
      DET=FJ(1,1)+FJ(2,2)-FJ(1,2)+FJ(2,1)
      VOL=WF(NS)+WF(NT)+DET+TH+VOL
C
      IF(DET) 500, 5000, 500
C
C     COMPUTE THE P MATRIX.
C
500   DO 600 J=1,5
      T1 = PP(1,J)
      T2 = PP(2,J)
      PP(1,J)=0.
      PP(2,J)=0.
      DO 600 I=1,2
      PP(I,J)=FJ(I,1)+T1+FJ(I,2)+T2+PP(I,J)
600   CONTINUE
C
C     COMPUTE THE STIFFNESS MATRIX.
C
      T3=WF(NS)+WF(NT)/DET
```

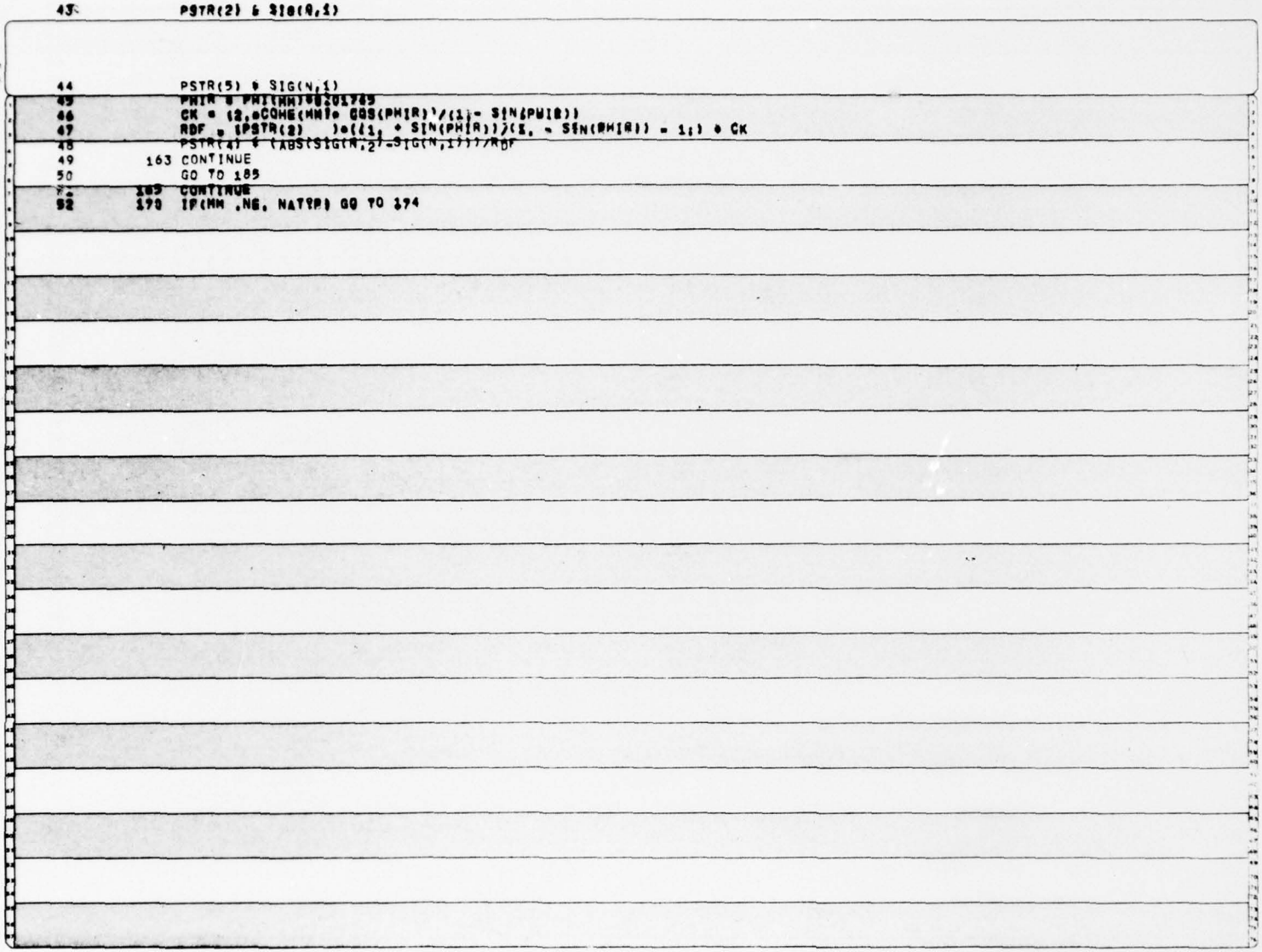
```
      C
            DO 900 I=1,5
            II=I*2-1
            III=II+1
      C
            T1=PP(1,I)*T3
            T2=PP(2,I)*T3
            DO 800 L=1,3
            AA(1,L)=C(1,L)*T1+C(3,L)*T2
            AA(2,L)=C(2,L)*T2+C(3,L)*T1
      800   CONTINUE
      C
            DO 900 J=1,I
            JJ=J*2-1
            JJ1=JJ+1
      C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      C     *  CALCULATE NEW INTERFACE STIFFNESS VALUES  *
      C
            T1=PP(1,J)
            T2=PP(2,J)
            S(II,JJ)=AA(1,1)*T1+AA(1,3)*T2+S(II,JJ)
            S(III,JJ)=AA(2,1)*T1+AA(2,3)*T2+S(III,JJ)
            S(II,JJ1)=AA(1,2)*T2+AA(1,3)*T1+S(II,JJ1)
            S(III,JJ1)=AA(2,2)*T2+AA(2,3)*T1+S(III,JJ1)
      C
      900   CONTINUE
      C
      1000  CONTINUE
      C
            DO 2000 I=1,9
            IP1=I+1
            DO 2000 J=IP1,10
            S(I,J)=S(J,I)
      2000  CONTINUE
      C
      9000  RETURN
            END
```

```
      SUBROUTINE BANSOL
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      PARAMETER MEL=690
      PARAMETER MND=761
      PARAMETER MBW=120
      PARAMETER MW=60
      PARAMETER NEE=2,NBE=500,N4=20,N5=2
      COMMON       MED(20),LL,NUMNP,NUMEL,NUMMAT,NUMSOL,ITRD,LORUL,NUMJT
     1,NUMAPP,INIT,MTYPE,NOY,NOX,NOXY,NCE,NLEL,NBIE,NONP,BTB,NCTYP,NBITY
     2P,NB2TYP,NATYP,NEL,NTRIP,NLL,NF,NWPF,NULEM,MC,MQ,ND,NWR,NT,MBAND,
     3TEMP,NUMBLK,II,JJ,KK,VOL,UW,SL,XC,YC,RDF,RD,PATM,GAMW,RKS(20),COJ(
     420),PHJ(20),DOEPJ(20),EXPJ(20),STFN(320),STFS(320),GAM(20),E(20),S
     5UI(20),QUF(20),A0(20),ALPHA(20),PHI(20),COME(20),HCOEF(20),AXP(20)
     6,ULCOEF(20),P(120),Q(4),PSTR(3),IG(MEL,4),TEE(MEL),BMOD(MEL),SU(ME
     7L),X(MND),Y(MND),DISP(MND,2),FX(MND),FS(MND),PP(MND),DP(MND),IC(30
     80),IL(MEL,5),LDL(NEE,8),LEL(NBE),NP(N4),IBB(N5),JBC(N5),PR(N5),F(2
     9,4),XY(2,4),B(MBW),A(MBW,MW),S(10,10),R(10),G(4,4),LM(8),BA(3,2),A
     1BG(3,4,2),DD(3,10),SV(3,10)
      REAL MED
C . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .
      MM = MBAND
      NN=MW
      NL=NN+1
      NH=NN+NN
      REWIND 4
      REWIND 5
      NB=0
      GO TO 150
C . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .
C     REDUCE EQUATIONS BY BLOCKS
C . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .
C . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .
C     1. SHIFT BLOCK OF EQUATIONS
C . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .
  100 NB=NB+1
      DO 125 N=1,NN
      NM=NN+N
      B(N)=B(NM)
      B(NM)=0.0
      DO 125 M=1,MM
      A(N,M)=A(NM,M)
  125 A(NM,M)=0.0
C . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .
C     2. READ NEXT BLOCK OF EQUATIONS INTO CORE
C . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .
      IF (NUMBLK-NB) 150,200,150
  150 READ (5) (B(N),(A(N,M),M=1,MM),N=NL,NH)
  151 IF (NB) 200,100,200
C . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .
C     3. REDUCE BLOCK OF EQUATIONS
C . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .
  200 DO 300 N=1,NN
```

```
      IF (A(N,I)) 225,300,225
  225 B(N)=B(N)/A(N,I)
      DO 275 L=2,NM
      IF (A(N,L)) 230,275,230
  230 D=A(N,L)/A(N,1)
      I=N+L-1
      J=0
      DO 250 K=L,NM
      J=J+1
  250 A(I,J)=A(I,J)-D*A(N,K)
      B(I)=B(I)-A(N,L)*B(N)
      A(N,L)=D
  275 CONTINUE
  300 CONTINUE
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C     4; WRITE BLOCK OF REDUCED EQUATIONS ON TAPE 2
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      IF (NUMBLK-NB) 325,400,375
  375 WRITE (4) (B(N),(A(N,M),M=2,NM),N=1,NN)
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      GO TO 100
C     BACK-SUBSTITUTION
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
  400 DO 450 N=1,NN
      NONN=1-N
      DO 425 K=2,NM
      L=N+K-1
  425 B(N)=B(N)-A(N,K)*B(L)
      NM=N-NN
      D=B(N)/B(N)
  450 A(NM,NB)=B(N)
      NB=NB-1
      IF (NB) 475,500,475
  475 BACKSPACE 4
      READ (4) (B(N),(A(N,M),M=2,NM),N=1,NN)
      BACKSPACE 4
      GO TO 400
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C     ORDER UNKNOWNS IN B ARRAY
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
  500 K=0
      DO 600 NB=1,NUMBLK
      DO 600 N=1,NN
      NB=N+NN
      K=K+1
  600 B(K)=A(NM,NB)
C
      RETURN
 3000 FORMAT(1915)
C
      END
```

```
R0188 01  11-18-76   17:355                                   LABEL STRESS  PAGE     1

      1            SUBROUTINE STRESS
      2            IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      3            PARAMETER MEL=690
      4            PARAMETER MND=761
      5            PARAMETER MBW=120
      6            PARAMETER MW=60
      7            PARAMETER NEE=2,NBE=100,N4=20,N5=2
      8            COMMON       MED(20),LL,NUMNP,NUMEL,NUMMAT,NUMSOL,ITRD,LORUL,NUMJT
      9           1,NUMAPP,INIT,MTYPE,NDY,NDX,NDXY,NCE,NLEL,NB1P,NONP,MTB,NCTYP,NB1TY
     10           2P,NB2TYP,NATYP,NEL,NTRIP,NLL,NF,NWPF,NUL,MB,NC,MU,NB,NWP,NT,MBAND,
     11           3TEMP,NUMBLK,II,JJ,KK,VOL,UW,SL,XC,YC,RDF,RD,PATM,GAMW,RKS(20),COJ(
     12           420),PHJ(20),COEFJ(20),EXPJ(20),STFN(320),STFS(320),GAM(20),E(20),G
     13           5UI(20),GUF(20),A0(20),ALPHA(20),PHI(20),COHE(20),HCOEF(20),AXP(20)
     14           6,ULCOEF(20),PR(20),Q(4),PSTR(5),SIG(MEL,4),EB(MEL),BMOD(MEL),BU(ME
     15           7L),X(MND),Y(MND),DISP(MND,2),FX(MND),FS(MND),PP(MND),DP(MND),IC(30
     16           80),IL(MEL,5),LUL(NEE,8),LEL(NBE),NP(N4),IBC(N5),JBC(N5),PR(N5),F(2
     17           9,4),XY(2,4),B(MBW),A(MBW,MW),S(10,10),R(10),C(4,4),LM(8),BA(3,2),A
     18           1BC(3,4,2),DD(3,3),BT(3,10)
     19            COMMON/PUNCH/IPUNCH
     20            REAL MED
     21       C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
     22       C     * CALCULATE ELEMENT STRESSES AT CENTER NODE *
     23       C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
     24            IF(INIT .EQ. 1) GO TO 162
     25            IF(ITRD .EQ. 0 .AND. NO .NE. NUMAPP) GO TO 162
     26            WRITE(3,2020) MO,NO
     27            IPRINT=0
     28       162  DO 300 N = 1,NUMEL
     29            ICHECK=0
     30            ISOIL=0
     31            IL(N,5) = IABS(IL(N,5))
     32            MM = IL(N,5)
     33            IF(MM .GT. NUMSOL) GO TO 300
     34            DO 173 I = 1,4
     35       173  Q(I) = SIG(N,I)
     36            LL = 0
     37            IF(NLL .EQ. 0) GO TO 170
     21       C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
     22       C     * CALCULATE ELEMENT STRESSES AT CENTER NODE *
     23       C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
     24            IF(INIT .EQ. 1) GO TO 162
     25            IF(ITRD .EQ. 0 .AND. NO .NE. NUMAPP) GO TO 162
     26            WRITE(3,2020) MO,NO
     27            IPRINT=0
     28       162  DO 300 N = 1,NUMEL
     29            ICHECK=0
     30            ISOIL=0
     31            IL(N,5) = IABS(IL(N,5))
     32            MM = IL(N,5)
     33            IF(MM .GT. NUMSOL) GO TO 300
     34            DO 173 I = 1,4
     35       173  Q(I) = SIG(N,I)
     36            LL = 0
     37            IF(NLL .EQ. 0) GO TO 170
     38            DO 169 I = 1,NLEL
     39            IF(LEL(I) .NE. N) GO TO 165
     40            IF(MM .EQ. NCTYP) ICHECK=1
     41            IF(MM .EQ. NCTYP) GO TO 163
     42            ISOIL=1
```



```
      PSTR(2) = SIG(N,1)

      PSTR(5) = SIG(N,1)
      PHIR = PHI(MM)*.01745
      CK = (2.*COHE(MM)*COS(PHIR))/(1.- SIN(PHIR))
      RDF = (PSTR(2)    )*((1. + SIN(PHIR))/(1. - SIN(PHIR)) - 1.) + CK
      PSTR(4) = (ABS(SIG(N,2)-SIG(N,1)))/RDF
  163 CONTINUE
      GO TO 185
  183 CONTINUE
  170 IF(MM .NE. NAT?P) GO TO 174
```

```
R0588 01  11-18-76   17:355                                   LABEL STRESS  PAGE    2

    53              GO TO 240
    54         174  CONTINUE
    55              IF(INIT .EQ. 1 .OR. NQ .EQ. NUMAPP) GO TO 185
    56              LL = 1
    57              GO TO 240
    58         175  PSTR(4) = 2.*RD/WDF
    59              PSTR(5) = PSTR(2)
    60         185  CALL QUAD (N71)
*****W  1457 DO LOOP INDEX N MAY NOT BE REDEFINED IN CALL OR ABNORMAL FUNCTION
    61        C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
    62        C     * SET UP ELEMENT DISPLACEMENT VECTOR *
    63        C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
    64              DO 195 I = 1,4
    65              II = 2*I
    66              JJ = 2*IL(N,I)
    67              P(II - 1) = B(JJ - 1)
    68         195  P(II) = B(JJ)
    69        C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
    70        C     * SOLVE FOR STRESSES *
    71        C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
    72              IF(ISOIL.EQ.0)CALL ISOSTR(N)
*****W  1457 DO LOOP INDEX N MAY NOT BE REDEFINED IN CALL OR ABNORMAL FUNCTION
    73              IF(ICHECK.EQ.1)Q(1)=GU(N)*Q(2)/(1.0-GU(N))
    74              LL = 0
    75         240  CONTINUE
    76              IF(INIT .EQ. 1) GO TO 270
    77        C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
    78        C     * CALCULATE NEW MODULUS AND POISSON RATIO VALUES *
    79        C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
    80        C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
    81              CALL MODCAL(N)
*****W  1457 DO LOOP INDEX N MAY NOT BE REDEFINED IN CALL OR ABNORMAL FUNCTION
    82              IF(LL .EQ. 1) GO TO 175
    83              IF(INIT .EQ. 1) GO TO 300
    84              IF(ITRD .EQ. 0 .AND. NQ .NE. NUMAPP) GO TO 300
    85              IPRINT=IPRINT+1
    86              IF(IPRINT.LT.51) GO TO 275
    87              IPRINT=1
    88              WRITE(3,2020)NQ,NQ
    89         275  CONTINUE
    90              WRITE(3,2030) N,XC,YC,( Q(I),I=1,3),   (PSTR(I),I=1,3),EE(N),GU(N)
    91             1,SL,UW
    92              IF(NQ .NE. NUMAPP) GO TO 300
    93              IF(IPUNCH.LT.2) GO TO 270
    94              PUNCH 3020,XC,YC,PSTR(1),PSTR(2),PSTR(3)
    95         270  DO 280 I = 1,4
    96         280  SIG(N,I) = Q(I)
    97         300  CONTINUE
    98              IF(NUMJT .EQ. 0) GO TO 340
    99              IF(INIT .EQ. 1)GO TO 310
   100              IF(ITRD .EQ. 0 .AND. NQ .NE. NUMAPP) GO TO 310
   101              WRITE(3,2040)
```

```
      310 DO 320 N = 1,NOMEL
          IF(IL(N,5) .GT. NUMSOL) CALL JSTRES(N)
****** 1457 DO LOOP INDEX N MAY NOT BE REDEFINED IN CALL OR ABNORMAL FUNCTION
      320 CONTINUE
      340 CONTINUE
          IF(NLL .EQ. 0 .OR. NO .EQ. NUMAPP) GO TO 355
          DO 350 N = 1,NCEL
          MM = LEL(N)
          MG = IL(MM,5)
          SIG(MM,4) = 0.0
          IF(MG .GT. NUMSOL) GO TO 345
          IF(MG.NE.NCTYP)EE(MM)=EE(MM)*0.001
          GO TO 350
      345 IF(ABS(STFS(MM)-100000000.) .LT. 1.)GO TO 350
          STFS(MM) = 10.
      350 CONTINUE
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C       * CALCULATE THE DISPLACEMENTS *
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      355 CONTINUE
          IF(INIT .EQ. 1 .OR. NO .LT. NUMAPP) GO TO 160
          WRITE(3,2000) NO
          J = 1
          IF(IPUNCH.LT.2) GO TO 700
          PUNCH 3050,HED,NUMNP
      700 CONTINUE
          IPRINT=0
          DO 150 N = 1,NUMNP
          FX(N) = 0.
          FY(N) = 0.
          IF(NP(J) .NE. N) GO TO 130
          J = J + 1
          DISP(N,1) = 0.
          DISP(N,2) = 0.
          GO TO 150
      130 IF(ABS(B(2*N)) .LT. 100. .AND. ABS(B(2*N-1)) .LT. 100.) GO TO 140
          DISP(N,1) = 0.0
          DISP(N,2) = 0.0
          GO TO 150
      140 DISP(N,1) = DISP(N,1) + B(2*N-1)
          DISP(N,2)= DISP(N,2) + B(2*N)
          IF(IPUNCH.LT.2) GO TO 701
          PUNCH 3060,N,X(N),Y(N),DISP(N,1),DISP(N,2)
      701 CONTINUE
          IPRINT=IPRINT+1
          IF(IPRINT.LT.51) GO TO 702
          IPRINT=1
          WRITE(3,2000)NO
      702 CONTINUE
      150 WRITE(3,2010)N,X(N),Y(N),(DISP(N,I),I=1,2),B(2*N-1),B(2*N),PP(N),N
      160 CONTINUE
          IF(NO.NE.NUMAPP.OR.INIT.EQ.1.OR.MG.NE.NC) GO TO 360
```

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R0588 01  11-18-76   17.355                                   LABEL STRESS  PAGE    4

  153           WRITE(2,3010) ((SIG(N,M),M=1,4),N=1,NUMEL)
  154           WRITE(2,3020) ((DISP(N,M),M=1,2),PP(N),N=1,NUMNP)
  155           WRITE(2,3030) (IL(N,5),N=1,NUMEL)
  156           WRITE(2,3040) (STFS(N),STFN(N),N=1,NUMJT)
  157       3010 FORMAT(8F10.2)
  158       3020 FORMAT(9F8.3)
  159       3030 FORMAT(1515)
  160       3040  FORMAT(6E12.5)
  161       2000 FORMAT(1H1,30X,'DISPLACEMENT RESULTS FOR LOADING CASE NO. ',I5///
  162          1  2X,'N. P.',4X,'X',6X,'Y',6X,'TOTAL UX',3X,'TOTAL UY',
  163          13X,'INCREM. UX',2X,'INCREM. UY',1X,'PORE PRES.',2X,'N,P')
  164       2010 FORMAT(I5,2F8.2,1P3E12.4,I5)
  165       2020 FORMAT(1H1,20X,'EFFECTIVE STRESSES,MODULUS VALUES AND PORE PRESSUR
  166          1ES'/35X,'FOR CASE',I2,2X,'-ITERATION-',I2//  ,'EL NO'  ,6X,'X',6X,'
  167          2Y',4X,'X-STRESS',3X,'Y-STRESS',3X,'XY-STRESS',1X,'MAX-STRESS',1X,'
  168          3MIN-STRESS',2X,'ANGLE',4X,'MODULUS',1X,'POIS RAT' ,1X,'S LEVEL',
  169          4 1X,'POR PRES')
  170       2030 FORMAT(I8,2F8.2,1P3E11.3,0PF7.2,1PE11.3,0PF8.3,1PE10.2)
  171       2040 FORMAT(1H1,40X,'INTERFACE ELEMENT RESULTS'///9X,'EL NO',3X,'X',6X,
  172          1 'Y',9X,'NORM STRESS',3X,'TAN STRESS',3X,'NORM DISP',3X,'TAN DISP'
  173          2,3X,'TAN STIFF',3X,'NORM STIFF',3X,'PORE PRESS'////)
  174       3000 FORMAT(I5)
  175       3050 FORMAT(17A4,A1,241,2I4)
  176       3060 FORMAT(1I10,4E12.5)
  177       3070 FORMAT(4E10.3,3E8.1,E10.3)
  178       360  CONTINUE
  179            RETURN
  180            END
```

A41

```
R0588 01   11-18-76   17.356   . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .  LABEL ISOSTR  PAGE   1
      C . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .
            SUBROUTINE ISOSTR (N)
            IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      C
      C
      C     THIS SUBROUTINE COMPUTES THE STRESS AT THE FIFTH NODE
R0588 01   11-18-76   17.356   . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .  LABEL ISOSTR  PAGE   1
      C . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .
            SUBROUTINE ISOSTR (N)
            IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      C
      C
      C     THIS SUBROUTINE COMPUTES THE STRESS AT THE FIFTH NODE
      C     OF THE FIVE-NODE ISOPARAMETRIC ELEMENT;
      C
      C
            PARAMETER MEL=690
            PARAMETER MND=761
            PARAMETER MBW=120
            PARAMETER MW=60
            PARAMETER NEE=2,NBE=100,N4=20,N5=2
            COMMON       HED(20),LL,NUMNP,NUMEL,NUMMAT,NUMSOL,ITRD,LORUL,NUMJT
           1,NUMAPP,INITSMTYPE,NBY,NOX,NOXY,NCE,NCEL,NB1S,NONP,NTB,NCTYP,NB1TY
           2P,NB2TYP,NATYP,NEL,NTRIP,NLL,NF,NWPF,NUL,MS,NC,MQ,NQ,NWP,NT,MBAND,
           3TEMP,NUMBLK,II,JJ,KK,VOL,UW,SL,XC,YC,RDF,RD,PATM,GAMW,RKS(20),COJ(
           420),PHJ(20),COEFJ(20),EXPJ(20),STPN(320),STPS(320),GAM(20),E(20),G
           5UI(20),GUF(20),AO(20),ALPHA(20),PHI(20),COME(20),HCOEF(20),AXP(20)
           6,ULCOEF(20),FR(20),SIG(4),PSTR(5),Z(MEL,4)TEE(MEL),BMOD(MEL),GU(ME
           7L),X(MND),Y(MND),DISR(MND,2),FX(MND),FY(MND),PQ(MND),DP(MND),IC(30
           80),IL(MEL,5),LOL(NEE,8),LEL(NBE),NP(N4),IBC(N5),JBC(N5),PR(N5),F(2
           9,4),XY(2,4),B(MBW),A(MBW,MW),S(10,10),U(80),Q(4,4),LM(8),BA(3,2),A
           1BC(3,4,2),ST(30),PP(2,4),PJ(2,2),AI(18)
            REAL HED
      C
      C     COMPUTE THE P PRIME MATRIX,
      C
            DO 200 I=1,4
            SI=-I*I+I*5-5
            TI=I/3*2-1
            PP(1,I)=SI*0.25
           7L),X(MND),Y(MND),DISR(MND,2),FX(MND),FY(MND),PQ(MND),DP(MND),IC(30
           80),IL(MEL,5),LOL(NEE,8),LEL(NBE),NP(N4),IBC(N5),JBC(N5),PR(N5),F(2
           9,4),XY(2,4),B(MBW),A(MBW,MW),S(10,10),U(80),Q(4,4),LM(8),BA(3,2),A
           1BC(3,4,2),ST(30),PP(2,4),PJ(2,2),AI(18)
            REAL HED
      C
      C     COMPUTE THE P PRIME MATRIX,
      C
            DO 200 I=1,4
            SI=-I*I+I*5-5
            TI=I/3*2-1
            PP(1,I)=SI*0.25
            PP(2,I)=TI*0.25
  200       CONTINUE
      C
      C     COMPUTE THE JACOBIAN MATRIX;
      C
            DO 300 I=1,2
```

```
      DO 300 J=1,2

      FJ(I,J)=0.
300   CONTINUE
C
      DO 400 I=1,4
      ND = IL(N,I)
      XX=X(ND)
      YY=Y(ND)
      FJ(1,1)=PP(2,I)+YY*FJ(1,1)
      FJ(1,2)=PP(I,I)+YY*FJ(1,2)
      FJ(2,1)=PP(2,I)+XX*FJ(2,1)
      FJ(2,2)=PP(1,I)+XX*FJ(2,2)
400   CONTINUE
```

```
C
      DET=FJ(1,1)*FJ(2,2)-FJ(1,2)*FJ(2,1)
C
      IF(DET) 500, 5000, 500
C
C     COMPUTE THE P MATRIX.
C
500   DO 600 J=1,4
      T1 = PP(1,J)
      T2 = PP(2,J)
      PP(1,J) = 0.
      PP(2,J) = 0.
      DO 600 I=1,2
      PP(I,J)=FJ(I,1)*T1+FJ(I,2)*T2+PP(I,J)
600   CONTINUE
C
C     COMPUTE THE STRAIN VECTOR.
C
      ST(1)=0.0
      ST(2)=0.0
      ST(3)=0.0
      ST(4)=TEMP
      ST(5)=TEMP
      ST(6)=0.0
      DO 800 I=1,4
      II = I*2
      ST(1)=U(II-1)*PP(1,I)+ST(1)
      ST(2)=U(II)*PP(2,I)+ST(2)
      ST(3) = U(II-1) *PP(2,I) + U(II) *PP(1,I) + ST(3)
800   CONTINUE
C
C     COMPUTE THE STRESS VECTOR.
C
      DO 1000 J=1,3
      STT =-ST(J)/DET
      STT=STT+ST(J+3)
      DO 1000 I=1,3
      SIG(I)=C(I,J)*STT  +SIG(I)
1000  CONTINUE
C
5000  RETURN
      END
```

```
R0588 01  11-18-76   17,357                                                    LABEL  JSTRES  PAGE     1

      1            SUBROUTINE JSTRES(N)
      2            IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      3            PARAMETER MEL=690
      4            PARAMETER MND=761
      5            PARAMETER MBW=120
      6            PARAMETER MW660
      7            PARAMETER NEE=2,NBE=100,N4=20,N5=2
      8            COMMON       HED(20),LL,NUMNP,NUMEL,NUMMAT,NUMSOL,IPRD,LORUL,NUMJT
      9           1,NUMAPP,INIT,MTYPE,NDY,NDX,NDXY,NCE,NLEL,NBIE,NONP,MTB,NCTYP,NBITY
     10           2P,NB2TYP,NATYP,NGL,NTRIP,NLL,NF,NWPF,NUL,MS,NC,MQ,NQ,NWP,NT,MBAND,
     11           3TEMP,NUMBLK,II,JJ,KK,VOL,UW,SL,XC,YC,RDF,RD,PATM,GAMW,RKS(20),COJ(
     12           420),PHJ(20),COEFJ(20),EXPJ(20),STFN(320),STFS(320),GAM(20),E(20),G
     13           5UI(20),GUF(20),AO(20),ALPHA(20),PHI(20),COHE(20),HCOEF(20),AXP(20)
     14           6,ULCOEF(20),FR(20),Q(4),PGTR(5),SIG(MEL,4),YEB(MEL),BMOD(MEL),GU(ME
     15           7L),X(MND),Y(MND),DISP(MND,2),FX(MND),FY(MND),PP(MND),DP(MND),IC(30
     16           80),IL(MEL,5),ILOL(NEE,8),LEL(NBE),NP(N4),IBC(N5),JBC(N5),PR(N5),F(2
     17           9,4),XY(2,4),B(MBW),A(MBW,MW),S(10,10),P(10),C(4,4),LM(8),BA(3,2),A
     18           1DD(3,4,2),DD(3,10),SY(3,10)
     19           REAL HED
     20           JTYPE = IL(N,5)
     21     C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
     22     C     * ROTATE GLOBAL DISPLACEMENTS TO LOCAL COORDINATE SYSTEM *
     23     C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
     24           I = IL(N,1)
     25           J = IL(N,2)
     26           XC = (X(I) + X(J))/2.
     27           YC = (Y(I) + Y(J))/2.
     28           DX = X(J) -X(I)
     29           DY = Y(J) - Y(I)
     30           DL = SQRT(DX**2 + DY**2)
     31           CSN = DX/DL
     32           SNE = DY/DL
     33           DO 100 K = 1,4
     34           L = IL(N,K)
     35           P(2*K)   = -B(2*L-1)*SNE + B(2*L)*CSN
     36       100 P(2*K-1) = B(2*L-1)*CSN + B(2*L)*SNE
     37           RDN = 0.5*(P(8)-P(2) + P(6)*P(4))
     38           RDS = 0.5*(P(7)-P(1) + P(5)*P(3))
     39           Q(1) = SIG(N,1)
     40           Q(2) = SIG(N,2)
     41           IF(NLL .EQ. 0) GO TO 110
     42           DO 105 K=1,NLEL
     43           IF(LEL(K) .NE. N) GO TO 105
     44           GO TO 120
     45       105 CONTINUE
     46     C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
     47     C     * COMPUTE STRESSES IN INTERFACE ELEMENT *
     48     C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
     49       110 AA = 0.5
     50           IF(INIT .EQ. 1 .OR. NO .EQ. NUMAPP) AA = 1
     51           Q(1) = SIG(N,1) + AA*STFN(N)*RDN
     52           Q(2) = SIG(N,2) + AA*STFS(N)*RDS
```

```
      IF(NO .NE. NUMAPP) GO TO 120
      SIG(N,1) = SIG(N,1) - STFN(N)*RDN
      SIG(N,2) = SIG(N,2) - STFS(N)*RDS
  117 IF (ABS(STFS(N)-100000000.) .LT. 1.) GO TO 120
      IF(SIG(N,1) .GT. 0.) GO TO 120
      STFS(N) = 10.
      STFN(N)=10.
      GO TO 130
  120 IF(ABS(STFS(N)-100000000.).LT. 1.) GO TO 130
      CALL MODCAL(N)
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C *   PRINT INTERFACE STRESSES AND RELATIVE DISPLACEMENTS   *
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
  130 CONTINUE
      IF(ITRD .EQ. 0 .AND. NO .NE. NUMAPP) GO TO 140
      IF(INIT .EQ. 1) GO TO 140
      WRITE(3,2010) N,XC,YC,Q(1),Q(2),RDN,RDS,STFS(N),STFN(N),UW
  140 CONTINUE
      RETURN
 2010 FORMAT(I10,2F8.2,5X,4E11.4,4X,1P3E12.3)
      END
```

IF(N.LE. ROS) GO TO 117
IF(STFN(N). LT. 1.) GO TO 130

```
      SUBROUTINE MODCAL(N)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      PARAMETER MEL=690
      PARAMETER MND=761
      PARAMETER MBW=120
      PARAMETER MW=460
      PARAMETER NEE=2,NBE=100,N4=20,N5=2
      COMMON       HED(20),LL,NUMNP,NUMEL,NUMMAT,NUMSOL,IORD,LORUL,NUMJT
     1,NUMAPP,INIT,NTYPE,NBY,NOX,NOXY,NCE,NLEL,NB1B,NONP,HTB,NCTYP,NB1TY
     2P,NB2TYP,NATYP,NGL,NTRIP,NLL,NF,NWPF,NOL,MG,NC,MQ,NQ,NWP,NT,MBAND,
     3TEMP,NUMBLK,II,JJ,KK,VOL,UW,SL,XC,YC,RDF,RD,PATM,GAMW,RKS(20),COJ(
     420),PHJ(20),BOEFJ(20),EXPJ(20),STFN(320),STFS(320),GAM(20),E(20),G
     5UI(20),GUF(20),AG(20),ALPHA(20),PHI(20),COME(20),MCOEF(20),AXP(20)
     6,ULCOEF(20),PR(20),Q(4),PSTR(3),SIG(MEL,4),EB(MEL),BMOD(MEL),GU(ME
     7L),X(MND),Y(MND),DISP(MND,2),FX(MND),FY(MND),PP(MND),DP(MND),IC(30
     80),IL(MEL,5),LUL(NEE,8),LEL(NBE),NP(N4),IBB(N5),JBC(N5),PR(N5),F(2
     9,4),XY(2,4),B(MBW),A(MBW,MW),S(10,10),R(10),G(4,4),LM(8),BA(3,2),A
     1BC(3,4,2),DD(3,10),SS(3,10)
      REAL HED
      TAN(X)=SIN(X)/COS(X)
C . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .
      XC=0.
      YC=0.
      DO 10 I=1,4
      L=IL(N,I)
      XC=X(L)+XC
      YC=Y(L)+YC
   10 CONTINUE
      XC=XC*0.25
      YC=YC*0.25
      MM = IL(N,5)
C
C . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .
C     * SET UP PORE PRESSURE FOR ELEMENT *
C . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .
      UU = 0.0
      DO 100 I = 1,4
      L = IL(N,I)
  100 UU = UU + PP(L)
      UW = UU/4.*GAMW
      IF(MM .GT. NUMSOL) GO TO 300
C . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .
C     * CALCULATE PRINCIPAL STRESSES AND MODULI FOR 2-D ELEMENT *
C . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .
      IF(MM .NE. NATYP) GO TO 140
      Q(1) = 0.0
      Q(2) =0.0
      Q(3) = 0.0
      PSTR(1) = 0.0
      PSTR(2) = 0.0
      PSTR(3) = 0.0
      GU(N) = GUI(NATYP)
```

```
      EE(N) = E(NATYP)
      BMOD(N) = E(NATYP)
      SL = 0.0
      DO 120 I = 1,4
  120 SIG(N,I) = 0.0
      GO TO 400
  140 CD = (Q(1) + Q(2))/2.
      DXY = (Q(1) - Q(2))/2.
      RD = SQRT(DXY**2 + Q(3)**2)
      PSTR(1) = CD + RD
      PSTR(2) = CD - RD
      SL = 0.0
      IF(ABS(DXY ) .GT. 0.000001) GO TO 145
      RAD = 0.0
      GO TO 150
  145 RAD = ATAN2(SIG(N,3),DXY)/2.
  150 PSTR(3) = 57.296*RAD
C
C     IF E(MM).GT. 0.0 THEN LINEAR ELASTIC MATERIAL
C
      IF(E(MM).GT. 0.0) GO TO 270
      PHIR = PHI(MM)*0.01745
      CK = (2.*COHE(MM)* COS(PHIR))/(1. - SIN(PHIR))
      RDF = (PSTR(2) - PSTR(1) * (1. + SIN(PHIR))/(1. - SIN(PHIR)) - 1.) * CK
      IF(LL .EQ. 1) RETURN
      IE(NO .NE. NOMAPR)GO TO 160
      PSTR(1) = PSTR(2)
      PSTR(4) = 2.*RD/RDF
  160 SL = (PSTR(4) + 2.*RD/RDF)/2.
C     IF PSTR(1) AND PSTR(2) ARE ZERO, TH PROGRAM WOULD CALCULATE
C     A MODULUS OF BEING OF ZERO FOR NONLINEAR SOIL ELEMENTS
C       THIS CHECK ALLOWS THE PROGRAM TO FIND THIS AND SUBSTITUTE
C       A VALUE OF THE INITIAL SLOPE OF THE MODULUS CURVE,
C     THIS WILL OCCUR IF THERE IS NO GRAVITY STRESSES IN A
C     NONLINEAR SOIL MASS OR IF THE STRESSES IN A NONLINEAR SOIL
C       ELEMENT WERE EXACTLY RELIEVED,
        IF(PSTR(1),EQ.0.0.AND.PSTR(2).EQ.0.0.AND.PSTR(3),EQ.0.0)
     1  GO TO 470
        GO TO 180
C
C     TENSION FAILURE
C
  270   EE(N)=HCOEF(MM)*PAYM/100.0
        GU(N) = GUF(MM)
        BMOD(N)=EE(N)
        Q(4) = RD
        GO TO 400
  180  CONTINUE
C
C     IF INIT = 1, CHECK LORUL FOR LOADING-UNLOADING
C     IF INIT = 0, CHECK RD AGAINST Q(4) FOR LOAD-UNLOAD
C         NOTE= INIT = 0 FOR ANY CASE AFTER INITIALIZATION
```

```
      C
            IF(INIT.EQ.1) GO TO 182
            IF(RD+0.5*U(4))185,190,190
  182       IF(LORUL-1)185,190,190
      C
      C     UNLOADING MODULUS CALCULATED
      C
  185 CH = (PSTR(2) + RSTR(5))/2.
      C
      C     CHECK FOR TENSION FAILURE   CH LESS THAN ZERO
      C
            IF(CH .LT. 0.0) GO TO 170
      C
      C     CHECK FOR EXPONENT OF ZERO
      C
            IF(AXP(MM).GT.0.01)GO TO 188
  186 EE(N) = ULCOEF(MM)*PATM
            GO TO 189
  188 CONTINUE
            EE(N) = (ULCOEF(MM)*PATM)*((PSTR(2) + RSTR(5))/(2.*PATM))**AXP(MM)
  189 CONTINUE
            BB = EE(N)
      C
      C     CHECK FOR YIELD CONDITION     SL GREATER THAN 1.0
      C
            IF(SL .GT. 1.) GO TO 210
            SMOD(N) = BB/(1.+(1. - GUI(MM))*(1. -2.*GUI(MM)))
            GU(N) = GUI(MM)
            GO TO 400
      C
      C     LOADING MODULUS CALCULATED
      C
  190 Q(4) = RD
            CH = (PSTR(2) + PSTR(5))/2.
      C
      C     CHECK FOR TENSION FAILURE   CH LESS THAN ZERO
      C
            IF(CH .LT. 0.0) GO TO 170
      C
      C     CHECK FOR EXPONENT OF ZERO
      C
            IF(AXP(MM).GT.0.001)GO TO 192
  191 BB = WCOEF(MM)*PATM
      C
      C     CHECK FOR YIELD CONDITION     SL GREATER THAN 1.0
      C
            IF(SL.GT.1.0) GO TO 210
            EE(N) = WCOEF(MM)*PATM*(1.0-FR(MM)*SL)**B
            GO TO 199
  198 CONTINUE
            BB = (WCOEF(MM)*PATM)*((PSTR(2)+PSTR(5))/(2.*PATM))**AXP(MM)
      C
```

```
      C     CHECK FOR YIELD CONDITION    SL GREATER THAN 1.0
      C
            IF(SL .GT. 1.) GO TO 210
            EE(N) = (HCOEF(MM)*PATM)*((PSTR(2)*PSTR(5))/(2.*PATM))**AXP(MM)*
           1 (1.-FR(MM)*SL)**2
  199       CONTINUE
            GU(N) = GUI(MM)
            IF(ABS(GU(N)-0.99 .LT. 0.0001) GU(N) = 0.4999
            BMOD(N) = BB/(2.*(1.+GU(N))*(1.-2.*GU(N)))
            GO TO 400
      C
      C     SHEAR FAILURE
      C
  210   EE(N) = 10.
            GU(N) = GUF(MM)
            BMOD(N) = BB/(2.*(1.+GU(N))*(1.-2.*GU(N)))
            GO TO 400
      C
      C     LINEAR GLASTIC MATERIAL
      C
  270   EE(N) = E(MM)
            IF(MM .EQ. NCTYP) GO TO 271
            IF(MM .EQ. NB2TYP) GO TO 272
  271       IF(Q(1).GE. ABS(1.04E+06)) EE(N)=10.
            GO TO 273
  272       IF(Q(2).GE. ABS(5.04E+05)) EE(N)=10.
  273       GU(N)=GUI(MM)
            SL = 0.0
            BB = EE(N)
  280       BMOD(N) = BB/(2.*(1. + GU(N))*(1. - 2.*GU(N)))
            GO TO 400
      C*********************************************************
      C     * CALCULATE STIFFNESSES FOR 1-D ELEMENT *
      C*********************************************************
  300   IF(ABS(STFS(N)-100000000.) .LT. 1.) GO TO 400
            PHIR = PHI(MM)*0.01745
            SHRST = COH(MM) + ABS(Q(1))*TAN(PHIR)
            STFS(N) = RKS(MM)
            STFN(N)=RKS(MM)
            IF(ABS(Q(2)) .ST. SHRST .OR. Q(1) .LE. 0.) GO TO 400
            SD = (1.-( ABS(Q(2))*FR(MM)/(COH(MM) + Q(1)* TAN(PHIR))))**2
            IF(ABS(EXPJ(MM)).GT.0.001) GO TO 380
            SU = COEFJ(MM)*GAMW
            GO TO 395
  380   CONTINUE
            SU = (COEFJ(MM)*GAMW)*((Q(1)/PATM)**EXPJ(MM))
  390   CONTINUE
            STFS(N) = SU*SD
            STFN(N) = 100000000.
  400   CONTINUE
            RETURN
      C
```

```
      C  SETS EE(N) AND BMOD(N) WHEN A NONLINEAR SOIL ELEMENT HAS NO
      C   STRESSES .   SETS THEM EQUAL TO THE INITIAL SLOPE OF THE
      C   INITIAL MODULUS CURVE.
      C
       470   EE(N)=200000.0
             BMOD(N)=EE(N)/(2.0*(1.0+GU(N)))*(1.0-2.0*GU(N))
             RETURN
            END
*****W  1470 EQUALITY OR NON-EQUALITY COMPARISON MAY NOT BE MEANINGFUL IN LOGICAL IF EXPRESSIONS
```

APPENDIX B: NONLINEAR STRESS-STRAIN BEHAVIOR FOR SOIL

Homogeneous Soil Element

1. A simple and practical expression for describing the nonlinear soil behavior has been suggested by Kondner[8*] who assumed that the stress-strain curve as obtained from triaxial compression tests is the following hyperbola:

$$(\sigma_1 - \sigma_3) = \frac{\varepsilon_1}{a + b\varepsilon_1} \tag{B1}$$

where $(\sigma_1 - \sigma_3)$ is the principal stress difference, ε_1 is the axial strain, and a and b are parameters that can be determined by experiment.

2. The physical meaning attached to a and b can be seen in Figure B1 in which a is equal to the reciprocal of the initial tangent modulus E_i and b is equal to the asymptotic value of the stress

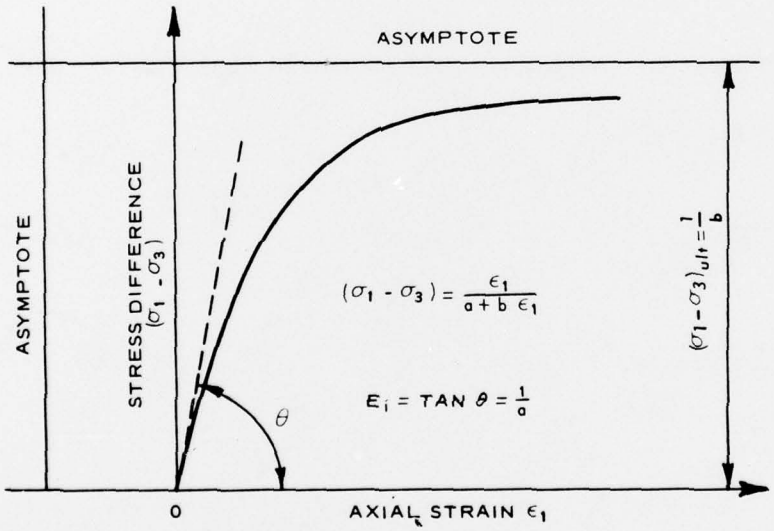

Figure B1. Kondner hyperbola for representation of stress-strain

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* Raised numerals refer to similarly numbered items in the References at the end of the main text.

difference called the ultimate stress difference $\left(\sigma_1 - \sigma_3\right)_{ult}$. Equation B1 can be simplified by expressing $\varepsilon_1/(\sigma_1 - \sigma_3)$ as a linear function of ε_1 , which would enable direct evaluation of the parameters a and b as depicted in Figure B2. The linear form of Equation B1 may be written as:

$$\frac{\varepsilon_1}{(\sigma_1 - \sigma_3)} = a + b\varepsilon_1 = \frac{1}{E_i} + \frac{\varepsilon_1}{\left(\sigma_1 - \sigma_3\right)_{ult}} \tag{B2}$$

3. Duncan and Chang[6] expanded Kondner's hyperbolic stress-strain relationship and utilized it for incremental FE analysis by expressing the tangent modulus E_t as a function of $(\sigma_1 - \sigma_3)$, E_i , and the Mohr-Coulomb strength parameters c and ϕ as:

$$E_t = \left[1 - \frac{R_f(1 - \sin\phi)(\sigma_1 - \sigma_3)}{2c\cos\phi + 2\sigma_3 \sin\phi}\right]^2 E_i \tag{B3}$$

in which R_f is the ratio of $(\sigma_1 - \sigma_3)$ at failure to the asymptotic value of the stress difference $\left(\sigma_1 - \sigma_3\right)_{ult}$.

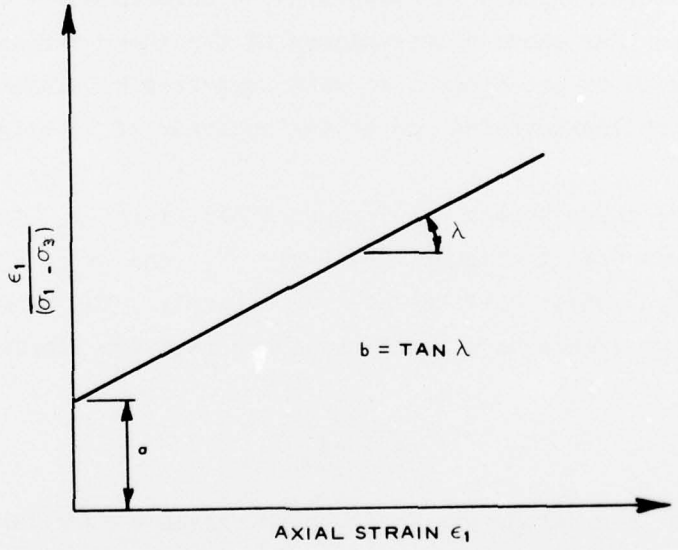


Figure B3. Linear representation of the hyperbolic stress-strain function

4. Another characteristic of many soils is the dependency of the initial tangent modulus E_i upon the confining pressure σ_3. Janbu[9] has suggested and Duncan and Chang[6] have employed the following exponential relation between the confining pressure σ_3 and E_i:

$$E_i = KP_a\left(\frac{\sigma_3}{P_a}\right)^n \tag{B4}$$

where P_a is the atmospheric pressure expressed in the same units as σ_3, K is a dimensionless number, and n is a dimensionless exponent.

5. The hyperbolic stress-strain relationship as expressed by Equations B3 and B4 is useful and simple. The parameters K, n, c, ϕ, and R_f are all readily determined by conventional triaxial compression tests.

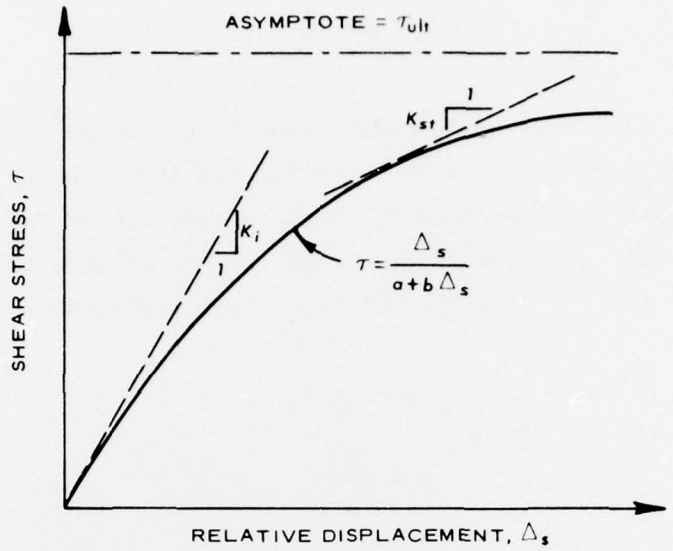
Interface Soil Elements

6. Although the stress-displacement curve from direct shear tests cannot be used directly for determining the stress-strain relationship for soil, it seems likely that a curve for such a test is indicative of the form of the stress-strain relationship. A comprehensive study by Clough and Duncan[4] has shown a resemblance of the shear stress relative to the displacement to the hyperbolic form suggested by Kondner,[8] which is appropriate for characterization of the behavior of interface elements.

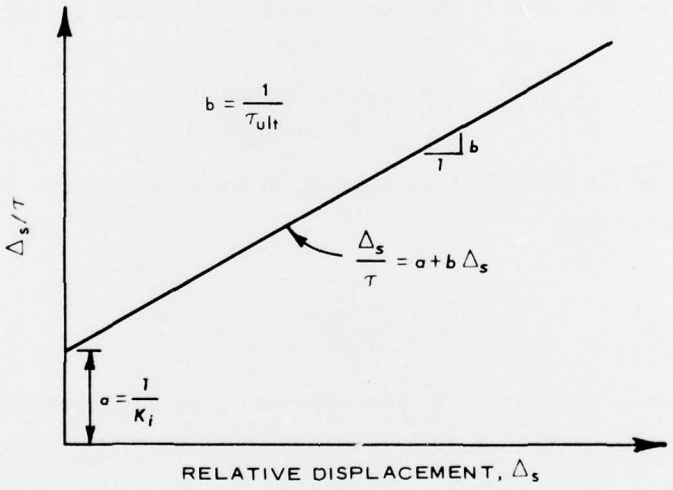
7. Typical shear stress τ versus displacement Δ_s for a direct shear test is presented in Figure B3a, where K_i and K_{st} are the initial and tangent shear stiffnesses, respectively. The hyperbolic form for the shear stress as a function of the relative displacement is:

$$\tau = \frac{\Delta_s}{a + b\Delta_s} \tag{B5}$$

where a and b are parameters that can be obtained from laboratory tests. Equation B5 may be transformed to a linear form as shown in Figure B3b:



a. CONVENTIONAL SHEAR STRESS - DISPLACEMENT



b. TRANSFORMED SHEAR STRESS - DISPLACEMENT

Figure B3. Hyperbolic representation of the variation of shear stress with relative displacement

$$\frac{\Delta_s}{\tau} = a + b\Delta_s \tag{B6}$$

8. It has also been suggested by Clough and Duncan[4] that it is desirable to determine the straight line for the test data by connecting points on the curve corresponding to 70 and 95 percent of the shear resistance mobilized. The constants a and b are related to the initial shear stiffness K_i and the ultimate shear resistance τ_{ult} as:

$$a = \frac{1}{K_i} \tag{B7}$$

$$b = \frac{1}{\tau_{ult}} \tag{B8}$$

The relationships expressed by Equations B7 and B8 can be used to determine K_i and τ_{ult} from the transformed linear hyperbolic plot shown in Figure B3b by simply evaluating a and b.

9. The tangent shear stiffness K_{st} at any point on the hyperbolic curve may be obtained by differentiating Equation B5 as:

$$K_{st} = \frac{d\tau}{d\Delta_s} = \frac{a}{(a + b\Delta_s)^2} \tag{B9}$$

From Equation B5 it is possible to obtain an expression for Δ_s by rearranging terms to read:

$$\Delta_s = \frac{a\tau}{1 - b\tau} \tag{B10}$$

By substituting a, b, and Δ_s as expressed in Equations B7, B8, and B10, respectively, in Equation B9, an expression for K_{st} can be obtained:

$$K_{st} = K_i\left(1 - \frac{\tau}{\tau_{ult}}\right)^2 \tag{B11}$$

Experiments have shown that τ_{ult} overestimates the actual value of the shear stress at failure τ_f, and the ratio between two shear stresses

R_f does not vary significantly for a given soil. By introducing the failure ratio R_f into Equation B11, the following may be obtained:

$$K_{st} = K_i\left(1 - \frac{R_f\tau}{\tau_f}\right)^2 \tag{B12}$$

10. From Mohr-Coulomb failure criteria, presented in Figure B4,

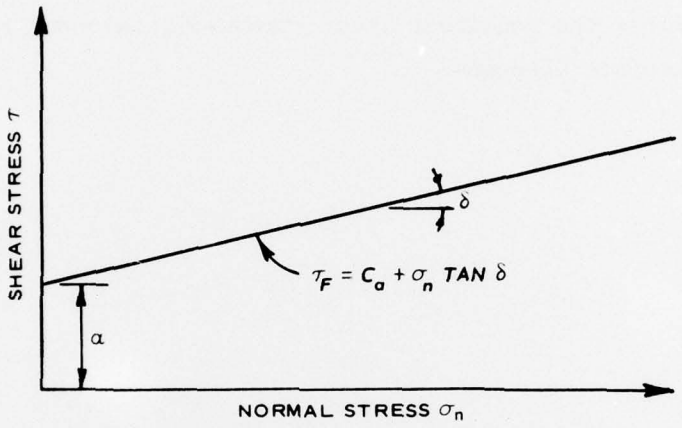


Figure B4. Variation of friction resistance with normal stress

the shear stress at failure may be related to the normal stress σ_n by the following expression:

$$\tau_f = C_a + \sigma_n \tan \delta \tag{B13}$$

where C_a is the adhesion and δ is the angle of skin friction. Therefore, by substituting Equation B13 in Equation B12, the following is obtained:

$$K_{st} = K_i\left(1 - \frac{R_f\tau}{C_a + \sigma_n \tan \delta}\right)^2 \tag{B14}$$

11. Laboratory studies by Clough and Duncan[4] have shown that the initial shear stiffness K_i may be related to the normal stress σ_n

by the following exponential relationship:

$$K_i = K_j \gamma_w \left(\frac{\sigma_n}{P_a}\right)^m \tag{B15}$$

where γ_w is the unit weight of water, P_a is the atmospheric pressure, and K_j and m are parameters that can be determined from laboratory tests. Equations B14 and B15 define a simplified and practical relationship which describes the nonlinear shear stress-displacement relationship for the interface elements.

APPENDIX C: NOTATION

a,b	Experimental parameters
A_e	Equivalent cross-sectional area of reinforcing strips
A_s	Cross-sectional area of reinforcing strip
c	Cohesion
C_a	Adhesion
E_a	Modulus of elasticity of aluminum panel
E_e	Equivalent modulus of elasticity
E_i	Initial modulus
E_s	Modulus of elasticity of galvanized steel reinforcing strips
E_t	Tangent modulus
I_a	Moment of inertia of aluminum panel per unit width
I_e	Equivalent moment of inertia per unit width
I_x	Moment of inertia of metal sheet at back of aluminum panel per unit width
K	Hyperbolic loading parameter
K_i	Initial shear stiffness
K_j	Dimensionless number
K_r	Residual shear stiffness
K_{st}	Tangent shear stiffness
L_a	Length of beam (S_z) between any two rows of reinforcing strips
L_e	Equivalent length
L_s	Length of reinforcing strip
m_f	Magnification factor
n	Pure number; also total number of strips in each row
N	Number of reinforcing ties in each reinforced elevation
P_a	Atmospheric pressure
R_f	Failure ratio
S	Total axial stiffness of reinforcing strip
S_e	Equivalent stiffness of plate substituting for reinforcing strips
t	Actual thickness of reinforcing tie
t_e	Equivalent thickness of reinforcing tie in FE mesh
w	Water content of soil

W	Actual width of reinforcing tie
W_e	Equivalent width of reinforcing tie in FE mesh
γ_d	Dry density of soil
γ_w	Unit weight of water
δ	Angle of skin friction
Δ	Shear deformation or displacement
Δ_s	Displacement
ε_1	Axial strain
λ_1, λ_2	Stress levels; see Equations 1c and 2c, respectively
ν	Poisson's ratio
σ_m	Normal stress
σ_t	Tensile stress in reinforcing strip
σ_y	Yield stress
σ_{te}	Equivalent tensile stress in reinforcing strip computed by FE analysis
σ_1	Major principal stress
σ_3	Minor principal stress
$\sigma_1 - \sigma_3$	Principal stress difference
$(\sigma_1 - \sigma_3)_f$	Principal stress difference at failure
$(\sigma_1 - \sigma_3)_{ult}$	Ultimate principal stress difference
τ	Shear stress
τ_{max}	Peak shear stress
τ_{ult}	Ultimate shear stress
ϕ	Angle of internal friction

In accordance with ER 70-2-3, paragraph 6c(1)(b), dated 15 February 1973, a facsimile catalog card in Library of Congress format is reproduced below.

Al-Hussaini, Mosaid M

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1. Finite element method. 2. Reinforced earth. 3. Walls. I. Johnson, Lawrence D., joint author. II. U. S. Army. Corps of Engineers. (Series: U. S. Waterways Experiment Station, Vicksburg, Miss. Technical report S-77-6)

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