ESD-TR-77-186

Technical Note 1977-34

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Theory of the Effects of Pulsed RFI on Coded Communication

22 July 1977

Prepared for the Department of the Navy
under Electronic Systems Division Contract F19628-76-C-0002 by

Lincoln Laboratory

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

LEXINGTON, MASSACHUSETTS

AD A044070

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The work reported in this document was performed at Lincoln Laboratory, a center for research operated by Massachusetts Institute of Technology. The work was sponsored by the Department of the Navy under Air Force Contract F19628-76-C-0002.

This technical report has been reviewed and is approved for publication.

FOR THE COMMANDER

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THEORY OF THE EFFECTS OF PULSED RFI ON CODED COMMUNICATION

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Group 66

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ABSTRACT

This note considers the effects of pulsed RFI (radio frequency interference) on coded communication systems. This problem is of considerable interest in the context of providing reliable satellite communications to military platforms which might suffer degradations due to pulsed RFI sources such as large radars. A general expression relating the computation cut-off rate of a communication channel with RFI to its cut-off rate in the absence of RFI is derived. Several examples are given to show the usefulness of this relationship.

CONTENTS

I. INTRODUCTION

This note considers the effects of pulsed RFI (radio frequency interference) on coded communcation systems. This problem is of considerable interest in the context of providing reliable satellite communications to military platforms which might suffer degradations due to pulsed RFI sources such as large radars. It is not uncommon to find radars with duty cycles of $\approx$ 5% and with pulse durations that may be much longer than the duration of a single information bit.

Uncoded communication will suffer large degradations when the duration of the interfering pulse is nearly as long or longer than an information bit. Indeed, if a fraction f of the bits in such a system are "clobbered" by a long pulse then the receiver will at best have to erase a fraction f of the incoming bits; or, if forced to make decisions, it will do so with probability of error of at least f/2 which will often be unacceptable. On the other hand, short interference pulses (that is, short compared to a bit duration) will, with proper receiver design,* lead to a degradation in SNR of a factor of at most (1-f).

The short-pulse case is not considered further here. The long-pulse case for coded systems is pursued. The following sections set-up the RFI model and then proceed to derive and present results that can be used in a quantitative way to assess the effects of RFI in terms of such system parameters as SNR or channel crossover probabilities in the absence of RFI. A major conclusion is

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*Such as utilizing pulse blanking.

that it is possible to design coding/modulation schemes which not only provide protection from the "usual" channel disturbances such as Gaussian noise, but which also can inherently provide very good protection from pulsed RFI.

II. MODEL AND ASSUMPTIONS

The following assumptions about the RFI source and the receiver prior to the decoder are made:

1. The RFI is strong enough so that any channel symbol hit by a pulse (or part of a pulse) is totally worthless to the decoder. A fraction f of the channel symbols are affected in this way.

2. The receiver is constructed in such a way that channel symbols affected by RFI can be identified as such. (In practice this can be done by: pulse blankers operating in a fairly wide bandwidth, or blanking pulses received from a co-located radar, or observing saturation effects at the outputs of matched filters.) This note will only be concerned with the decoder design and will not dwell on the (crucial) implementation issues implied by this assumption.

3. No advantage will be taken of either the burst nature of the interference (i.e., several consecutive channel symbols being hit) or what may be its periodic nature. Instead, it is assumed that interleaving of the channel symbols has been performed which turns the channel into a memoryless one. (This assumption is obviously going to be a conservative one in many cases; however, it's a good starting point and covers the situation where the RFI environment is not well known. Coding/decoding schemes tailored to specific channel parameters can often be quite fragile with respect to perturbations from the design point.)

4. Lastly, it is assumed that even in the absence of RFI the channel is noisy. The computation cut-off rate for the channel in the absence of RFI is denoted as R_c bits/channel symbol. When RFI is present the cut-off rate degrades to R_c'. Of interest will be finding R_c' as a function of f and R_c. Attention is focused on the cut-off rate for the following reasons:

a. A useful (and usually tight) bound on the probability of error using convolutional coding and Viterbi decoding is a function only of the code structure and cut-off rate [1]. Thus, independent of the channel details, the cut-off rate is usually sufficient to characterize system performance.

b. In a looser sense the cut-off rate indicates the relative behavior of block or convolutionally coded systems through the random coding bound exponents [2, Chapter 6].

c. It gives the practical limit for sequential decoding.

III. DERIVATION OF R_c'

The expression for the computation cut-off rate for the general discrete communication channel with A inputs and Q outputs shown in Fig. 1.

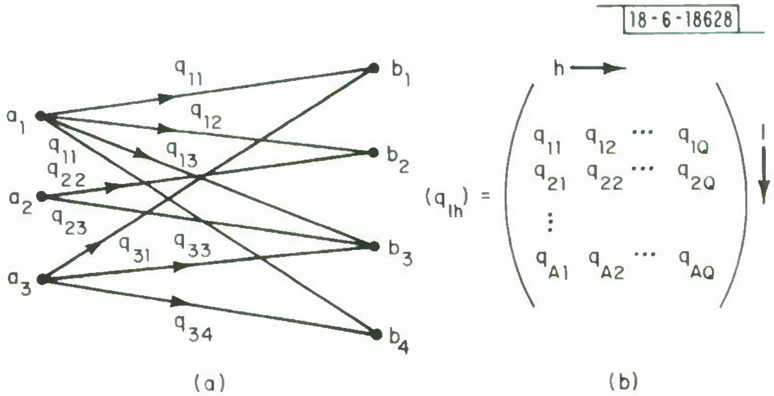

Fig. 1. Transition probability diagram and matrix: (a) A = 3, Q = 4; (b) matrix; the elements in each row sum to one. In the interest of clarity all possible transitions are not shown in the diagram.

This expression is given by [2, Equation 6.62b]:

$$R_c = -\log_2 \sum_{h=1}^{Q} \left[\sum_{\ell=1}^{A} p_\ell \sqrt{q_{\ell h}} \right]^2 \tag{1}$$

where $q_{\ell h}$ are the transition probabilities and p_ℓ are the input letter probabilities. The unprimed parameters appearing in (1) will refer to the channel without RFI. When the channel is disturbed with RFI, an additional output (the 0-th) is defined. The channel (actually the physical channel and the receiver) will show this output only during a disturbed channel symbol. A new discrete channel is thus created with transition probabilities:

$$\begin{aligned} q'_{\ell 0} &= f & \ell &= 1,\ldots,A \\ q'_{\ell h} &= (1-f)q_{\ell h} & \ell &= 1,\ldots,A \\ & & h &= 1,\ldots,Q \end{aligned} \tag{2}$$

The new cut-off rate is then given as:

$$R'_c = -\log_2 \sum_{h=0}^{Q} \left[\sum_{\ell=1}^{A} p_\ell \sqrt{q'_{\ell h}} \right]^2 \tag{3}$$

which, using (2), can be manipulated as follows:

$$\begin{aligned} R'_c &= -\log_2 \left\{ \left[\sum_{\ell=1}^{A} p_\ell \sqrt{f} \right]^2 + (1-f) \sum_{h=1}^{Q} \left[\sum_{\ell=1}^{A} p_\ell \sqrt{q_{\ell h}} \right]^2 \right\} \\ &= -\log_2 \left[f + (1-f) 2^{-R_c} \right] \end{aligned} \tag{4}$$

This is an interesting result, since the new cut-off rate is seen to depend only on the RFI duty cycle and the non-RFI cut-off rate. Thus the degradation due to RFI can be separated explicitly and easily from channel degradations that were previously assumed to exist. Figure 2 is a graphical presentation of these results.

An interesting way to manipulate (4) is to solve for R_c:

$$R_c = R'_c + \log_2 \frac{1-f}{(1-f2^{R'_c})} \tag{5}$$

Equation (5) is useful because the system requirements in terms of error rate will actually lead to a requirement on R'_c (for a given code); when coupled with the design value for f, the resulting value for R_c will determine the required SNR or undisturbed channel transition probabilities. This will be illustrated by the examples of the next section.

IV. EXAMPLES

Two common channels will be used as examples to illustrate the application of the foregoing results.

A. Binary Symmetric Channel (BSC)

Pulsed RFI turns the BSC (with cross-over probability p) into a binary erasures and errors channel. The undisturbed cut-off rate is [2, Equation 6.70]:

$$R_c = 1 - \log_2 [1 + 2\sqrt{p(1-p)}] \tag{6}$$

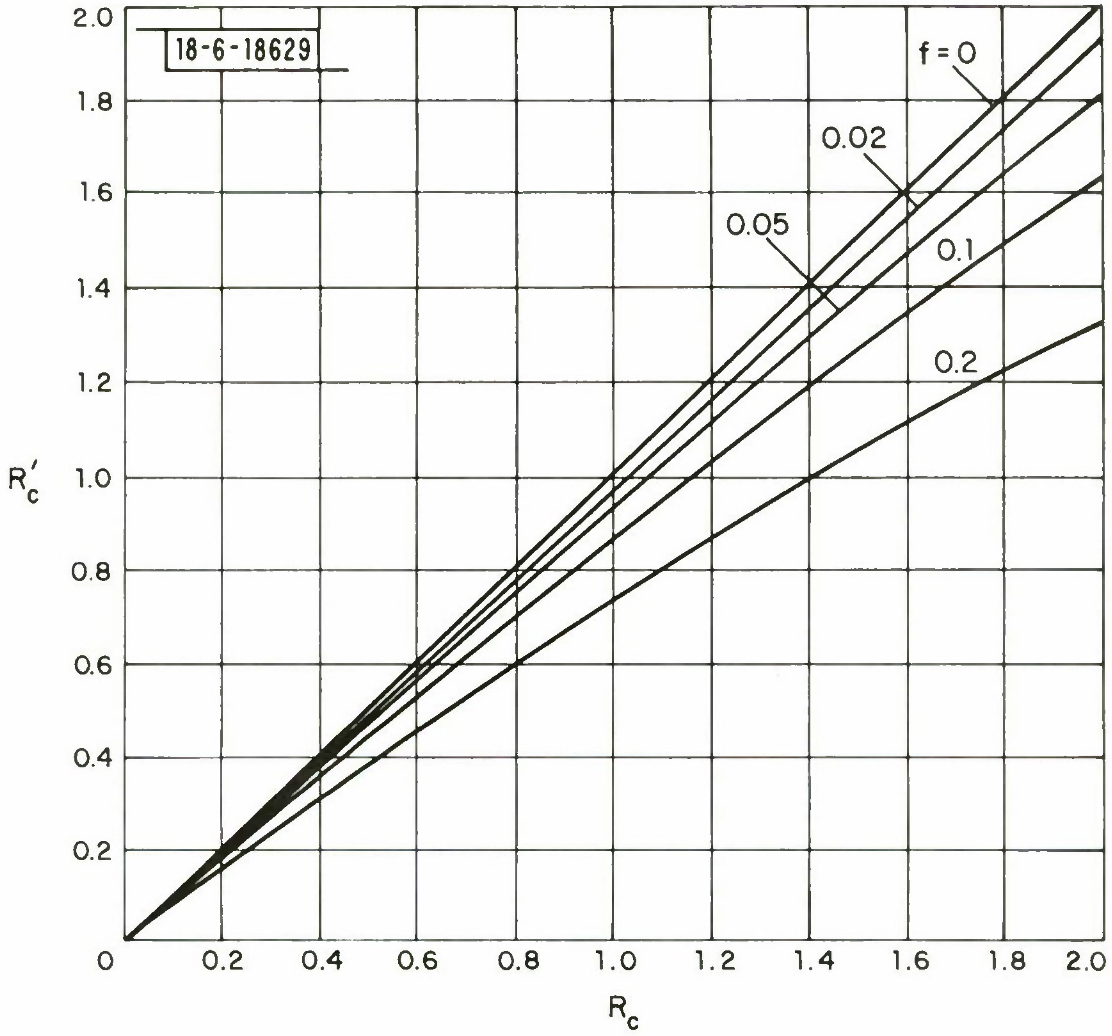

Fig. 2. General relationship between R_c' and R_c (bits/symbol).

With RFI present, using Eq. (6) in Eq. (4) gives:[*]

$$R_c' = 1 - \log_2 [1 + f + 2(1-f)\sqrt{p(1-p)}] \tag{7}$$

Figure 3 shows the required value of p necessary to achieve R_c' for various values of f.

(It is interesting to note in passing that if the BSC were noiseless (p=0) the rate would be given by $R_c' = 1 - \log_2(1+f)$ which is always less than 1-f. Thus even though only a fraction f of the channel symbols are not available for decoding, more than this fraction must be transmitted in redundant check symbols. It is to be expected in all cases that the "loss" either in SNR or channel symbols due to RFI will be greater than a factor of (1-f).)

A common physical source of the BSC is the binary antipodal channel in additive Gaussian noise. In this case $p = Q(\sqrt{2E_s/N_0})$ where E_s/N_0 is the channel symbol energy-to-noise density ratio.[**] Figure 4 shows R_c' as a function of f and E_s/N_0 for this channel. The degradation in SNR due to RFI can be read off directly along lines of constant R_c'. (Recall that decoder performance is a function of R_c', not just R_c.) For example, a code giving acceptable performance in the absence of RFI with $R_c = 1/2$ will degrade 0.4 dB in the presence of 5% duty factor RFI.

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[*] The same expression results from the erasure channel model in [2, pg. 461] with w=f and crossover error probability p(1-f).

[**] $$Q(x) \triangleq \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-x^2/2}\, dx$$

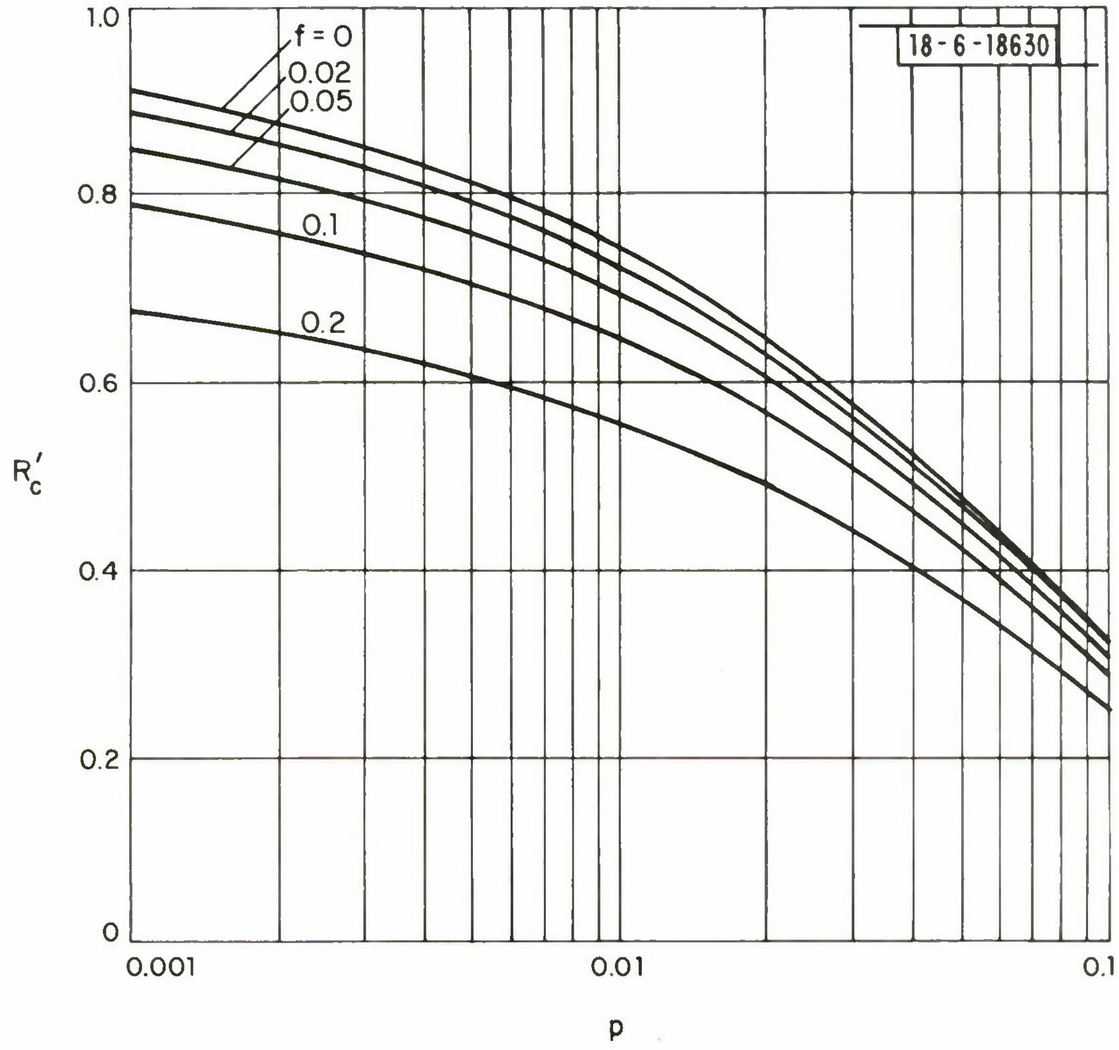

Fig. 3. Channel cut-off rate (bits/symbol) vs. non-RFI crossover probability.

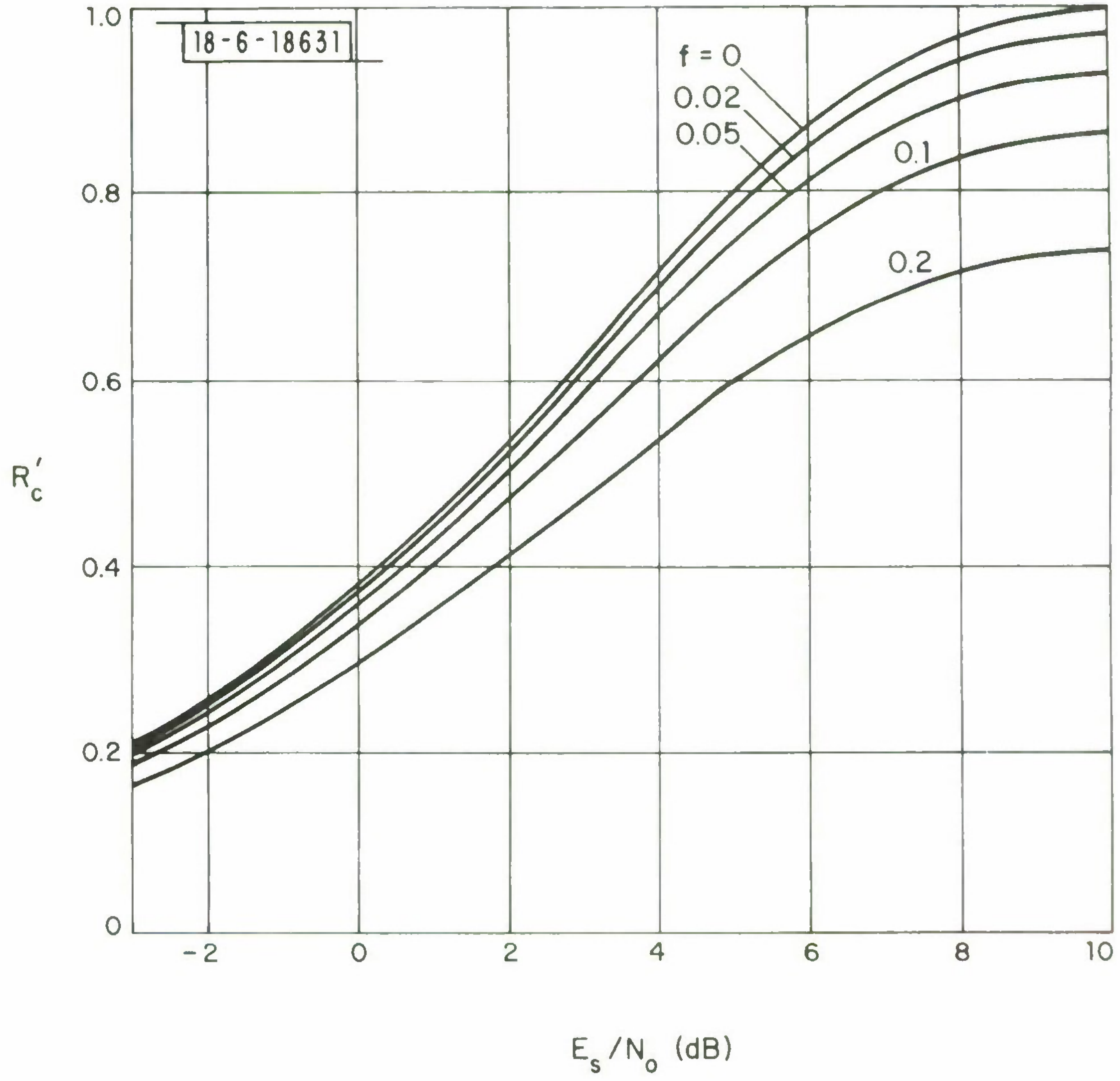

Fig. 4. Channel cut-off rate vs. channel symbol SNR (binary antipodal, hard decision).

B. Binary Soft Decisions

The second example assumes that the receiver utilizes the actual matched filter outputs in making its decision. For binary antipodal signalling [2, 6.67]:

$$R_c = 1 - \log_2 \left[1 + e^{-E_s/N_0} \right] \tag{8}$$

Substituting this expression into Eq. (4) yields:

$$R_c' = 1 - \log_2 \left[1 + f + (1-f) \quad e^{-E_s/N_0} \right] \tag{9}$$

This is shown plotted in Fig. 5. As in the previous example, the loss in SNR due to RFI may be found directly. For example, a code giving acceptable performance in the absence of RFI with $R_c = 1/2$ will degrade 0.4 dB in the presence of 5% duty factor RFI.

For both of these channels it should be noted that the loss in SNR increases significantly when operation at higher values of R_c' is desired, i.e., RFI of a given duty factor will have a greater affect on systems operating at higher code rates.

V. APPLICATION

The work reported here has been used in developing the coding techniques for use with TACS.[3] TACS (Terminal Access Control System) is a Lincoln Laboratory development that will demonstrate demand assigned TDMA with FLTSAT-I. Experimental results with a computer simulation and hardware decoder have been obtained and are being reported separately (e.g., Ref. 4).

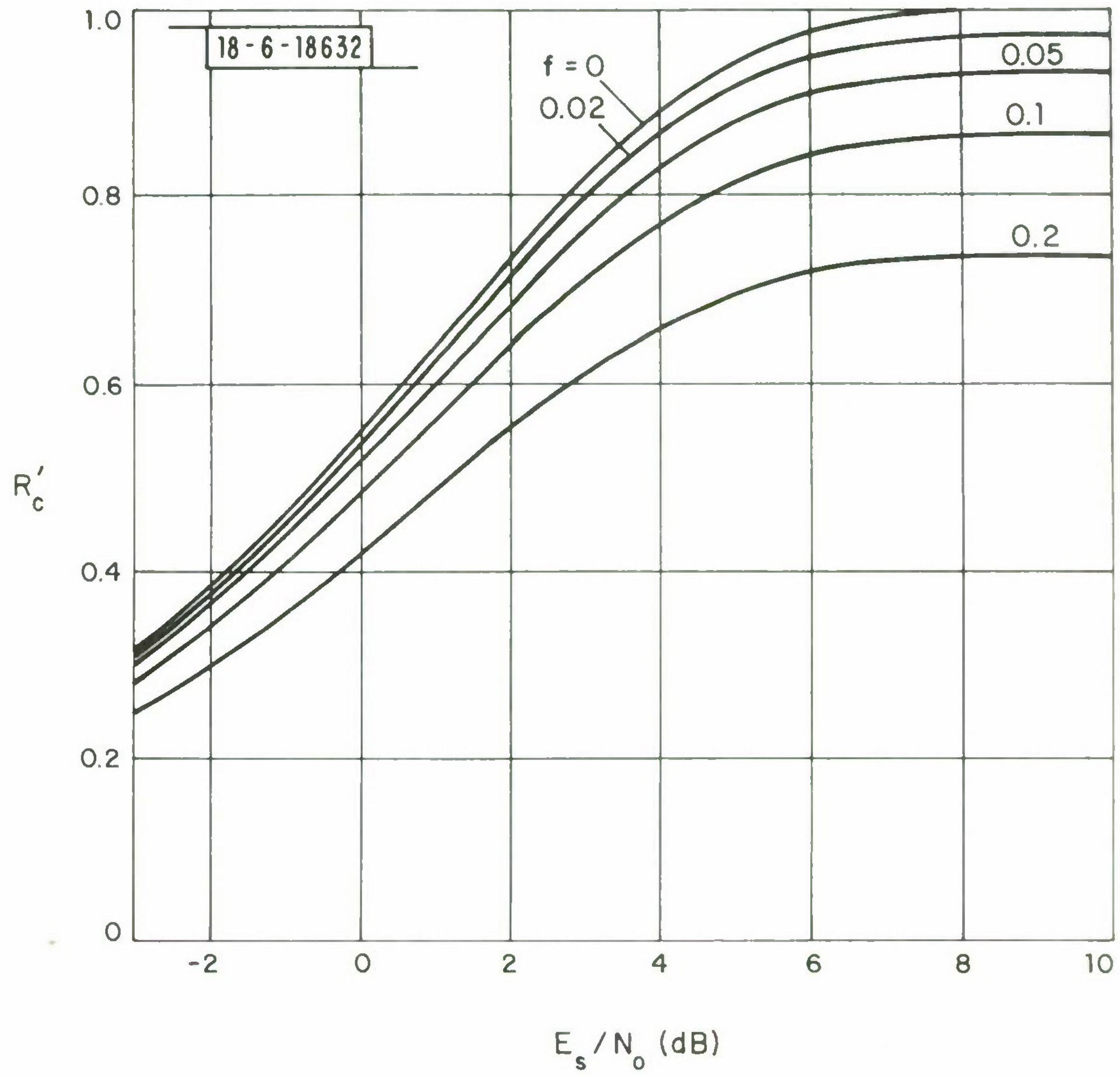

Fig. 5. Channel cut-off rate vs. channel symbol SNR (binary antipodal, soft decision).

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER ESD-TR-77-186	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE *(and Subtitle)* Theory of the Effects of Pulsed RFI on Coded Communication		5. TYPE OF REPORT & PERIOD COVERED Technical Note
		6. PERFORMING ORG. REPORT NUMBER Technical Note 1977-34
7. AUTHOR*(s)* Steven L. Bernstein		8. CONTRACT OR GRANT NUMBER*(s)* F19628-76-C-0002
9. PERFORMING ORGANIZATION NAME AND ADDRESS Lincoln Laboratory, M.I.T. P.O. Box 73 Lexington, MA 02173		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Program Element No. 33109N
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Electronic Systems Command Department of the Navy Washington, DC 20360		12. REPORT DATE 22 July 1977
		13. NUMBER OF PAGES 20
14. MONITORING AGENCY NAME & ADDRESS *(if different from Controlling Office)* Electronic Systems Division Hanscom AFB Bedford, MA 01731		15. SECURITY CLASS. *(of this report)* Unclassified
		15a. DECLASSIFICATION DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT *(of this Report)*

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT *(of the abstract entered in Block 20, if different from Report)*

18. SUPPLEMENTARY NOTES

None

19. KEY WORDS *(Continue on reverse side if necessary and identify by block number)*

pulsed RFI	coded communications	coding/decoding
satellite communications	TDMA	matched filters

20. ABSTRACT *(Continue on reverse side if necessary and identify by block number)*

This note considers the effects of pulsed RFI (radio frequency interference) on coded communication systems. This problem is of considerable interest in the context of providing reliable satellite communications to military platforms which might suffer degradations due to pulsed RFI sources such as large radars. A general expression relating the computation cut-off rate of a communication channel with RFI to its cut-off rate in the absence of RFI is derived. Several examples are given to show the usefulness of this relationship.

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