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SIMULATION MODEL FOR AIR TRAFFIC CONTROL COMMUNICATIONS.(U)
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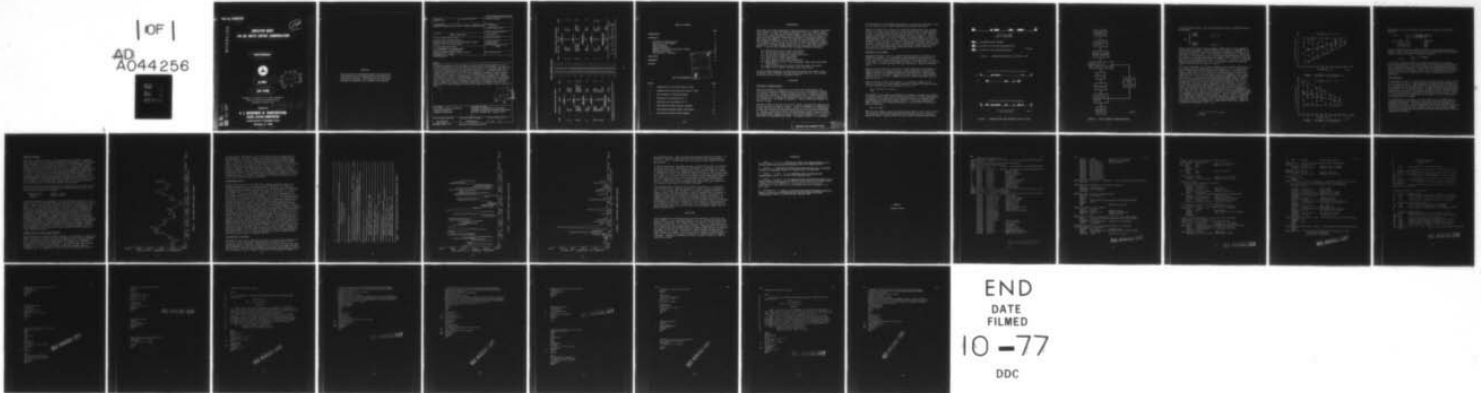
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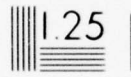
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**SIMULATION MODEL
FOR AIR TRAFFIC CONTROL COMMUNICATIONS**

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Robert Mulholland



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JUL 1977

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FINAL REPORT

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<p>16. Abstract</p> <p>A computer simulation model designed to mimic second-by-second behavior of air/ground communications in an air traffic control sector is described. The model can be used to simulate any one of nine sector functions (e.g., high-altitude enroute, low-altitude transitional, radar-arrival control, etc.). The model exists as a computer program written in the GPSS V and FORTRAN IV languages. Input variables include aircraft arrival rate into sector, distribution of transmission length, distribution of number of transmissions in an air/ground exchange, etc. Response variables include sector aircraft loading, channel utilization, and communications delay. Model output can be obtained in the form of time series (e.g., minute-by-minute averages of channel utilization) exhibiting the dynamics of sector communications or simple averages of such series taken over several hours of simulated time.</p>					
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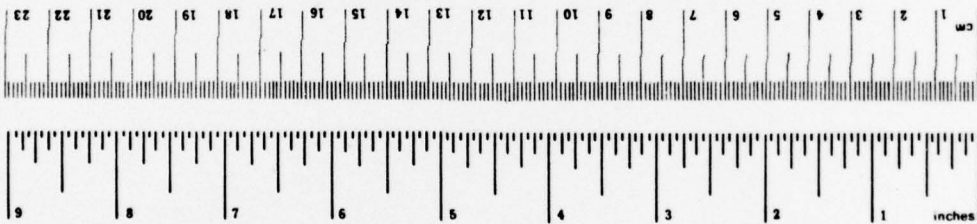
METRIC CONVERSION FACTORS

Approximate Conversions to Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
LENGTH				
in	inches	2.5	centimeters	cm
ft	feet	30	centimeters	cm
yd	yards	0.9	meters	m
mi	miles	1.6	kilometers	km
AREA				
in ²	square inches	6.5	square centimeters	cm ²
ft ²	square feet	0.09	square meters	m ²
yd ²	square yards	0.8	square meters	m ²
mi ²	square miles	2.6	square kilometers	km ²
	acres	0.4	hectares	ha
MASS (weight)				
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2000 lb)	0.9	tonnes	t
VOLUME				
teaspoon	teaspoons	5	milliliters	ml
fl oz	fluid ounces	30	milliliters	ml
c	cup	0.24	liters	l
qt	quarts	0.97	liters	l
gal	gallons	3.8	liters	l
ft ³	cubic feet	0.03	cubic meters	m ³
yd ³	cubic yards	0.76	cubic meters	m ³
TEMPERATURE (exact)				
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C

Approximate Conversions from Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
LENGTH				
mm	millimeters	0.04	inches	in
cm	centimeters	0.4	inches	in
m	meters	3.3	feet	ft
km	kilometers	0.6	miles	mi
AREA				
cm ²	square centimeters	0.16	square inches	in ²
m ²	square meters	1.2	square yards	yd ²
km ²	square kilometers	0.4	square miles	mi ²
ha	hectares (10,000 m ²)	2.5	acres	
MASS (weight)				
g	grams	0.035	ounces	oz
kg	kilograms	2.2	pounds	lb
t	tonnes (1000 kg)	1.1	short tons	
VOLUME				
ml	milliliters	0.03	fluid ounces	fl oz
l	liters	2.1	pints	pt
l	liters	1.06	quarts	qt
l	liters	0.26	gallons	gal
m ³	cubic meters	35	cubic feet	ft ³
m ³	cubic meters	1.3	cubic yards	yd ³
TEMPERATURE (exact)				
°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature	°F



*1 in = 2.54 exactly. For other exact conversions and more detailed tables, see NBS Misc. Publ. 286, Units of Weights and Measures, Price \$2.25, SD Catalog No. C13.10-286.

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INTRODUCTION

This report is a brief description of the main aspects of a computer simulation model designed to mimic air/ground communications in an air traffic control (ATC) sector. The model was developed through a joint effort of the National Aviation Facilities Experimental Center (NAFEC) and Princeton University to apply fast time simulation techniques and methods of time series analysis to ATC problems (references 1 to 4). The model has been validated with field data obtained in 1969 from the New York Control Area and additional information collected from the Houston Control Area in 1971. The model is presently available in a software package adapted for use in computer facilities at NAFEC. It can be used to simulate the following nine sector functions:

- HI = High-altitude enroute (radar controllers)
- LE = Low-altitude enroute (radar controllers)
- LT = Low-altitude transitional (radar controllers)
- GN = Ground control (tower controllers)
- LC = Local control (tower controllers)
- LG = Local ground control (tower controllers)
- DP = Radar departure control (IFR (instrument flight rules) Room radar controllers)
- AD = Radar arrival/departure control (IFR Room radar controllers)
- AR = Radar arrival control (IFR Room radar controllers)

In the following paragraphs, we describe the structure of the model, input variables, output variables, and some possible applications. More detailed descriptions can be found in references 2 and 3.

DISCUSSION

STRUCTURE OF COMMUNICATIONS.

Before proceeding to a discussion of the model, some formulation of the structure of air/ground communications is in order. An air/ground conversation normally consists of several transmissions (TR's) alternately initiated by pilot and controller. In keeping with earlier work, a whole conversation is referred to as a communication transaction (CT), so that a CT consists of one or more TR's. The reader is advised to keep the distinction between TR's and CT's in mind.

Obviously, while an aircraft is in sector, there are several CT's between controller and pilot as shown in figure 1. From the standpoint of communications, we adopt the attitude that an aircraft arrives at a sector with the beginning of the first CT with the ground and leaves the sector at the end of the final CT. Between these points in time, the communication between a single aircraft and ground usually consists of several CT's and gaps between CT's. We refer to these gaps as intercommunication gaps. Referring to figure 1, it is apparent

that the number of intercommunication gaps is one less than the number of CT's, so that specification of either automatically determines the other.

Figure 1 is essentially a picture of air/ground communications from the point of view of a pilot. Since there are usually many aircraft in sector simultaneously, the communications picture from the point of view of the controller is just a superposition of several diagrams like that of figure 1. This is demonstrated in figure 2 for the case where the maximum number of aircraft in sector between time 1 and 2 is two. Obviously, the gaps between CT's from the viewpoint of the controller are normally shorter than those experienced by the pilot of a single aircraft. In order to distinguish the former from the latter, we refer to the gaps experienced by the controller as intertransaction gaps. Thus, the pilot experiences intercommunication gaps, and the controller experiences intertransaction gaps.

STRUCTURE OF THE PROGRAM.

The simulation model exists as a computer program written in the GPSS V and FORTRAN IV languages. It is designed to mimic second-by-second behavior of sector air/ground communications over periods of time in the order of hours. A flow chart of the model is displayed in figure 3. As indicated by the blocks of the chart, the model performs nine basic operations. These are described below.

Aircraft arrivals (block 1) at the sector under investigation correspond to a sample function from a Poisson process. Stated another way, interarrival times are modeled as independent exponential variates with a common average (AMEAN) expressed in seconds. AMEAN is an input variable that specifies the average rate, in this case, 1/AMEAN aircraft per second, with which aircraft enter the sector.

When an aircraft arrives at the sector, it is assigned a number of CT's (block 2) by means of a random sample drawn from a negative binomial distribution with shifted origin and parameters P and K, i.e.,

$$\frac{K+r-2}{K-1} p^K (1-p)^{r-1}; r=1,2,3,\dots$$

For example, P and K might be estimated by the method of moments from historical data collected from a sector or group of sectors of the type being studied. In addition to the number of CT's, an incoming aircraft is assigned a mean intercommunication gap length (MGAP) (block 3). The natural logarithm of MGAP is generated from a normal distribution. The mean (XM) of this distribution is given by,

$$XM = A1 + A2XN$$

where N is the number of gaps, one less than the number of CT's, and A1 and A2 are regression coefficients determined from historical records of mean gap length and number of gaps obtained for many aircraft passing through sectors

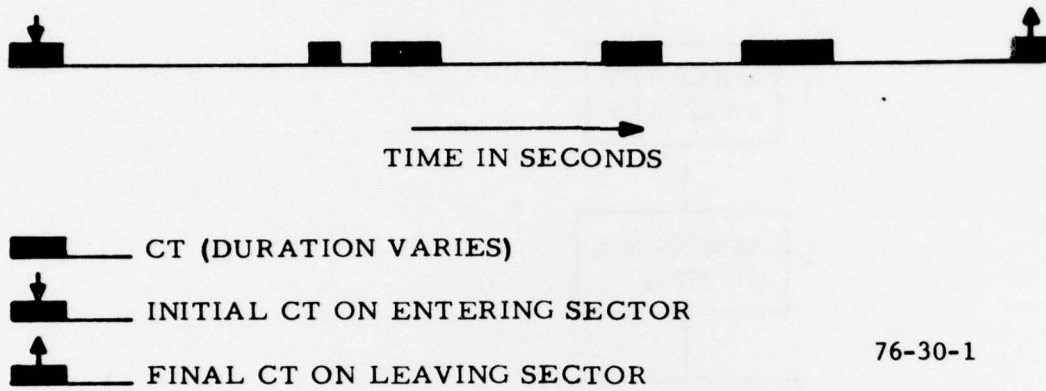


FIGURE 1. COMMUNICATIONS FROM PILOT POINT OF VIEW

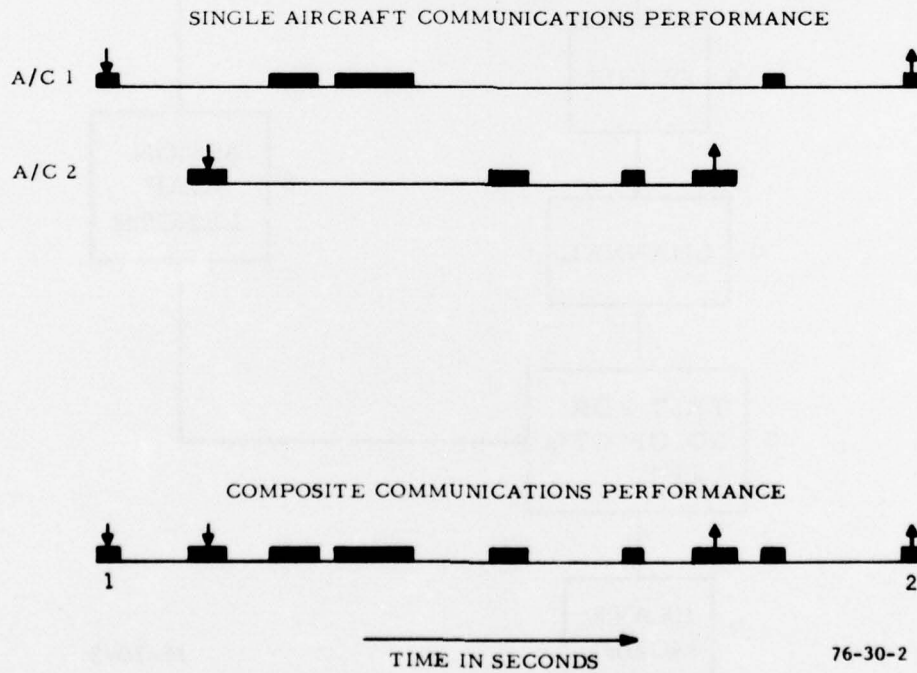
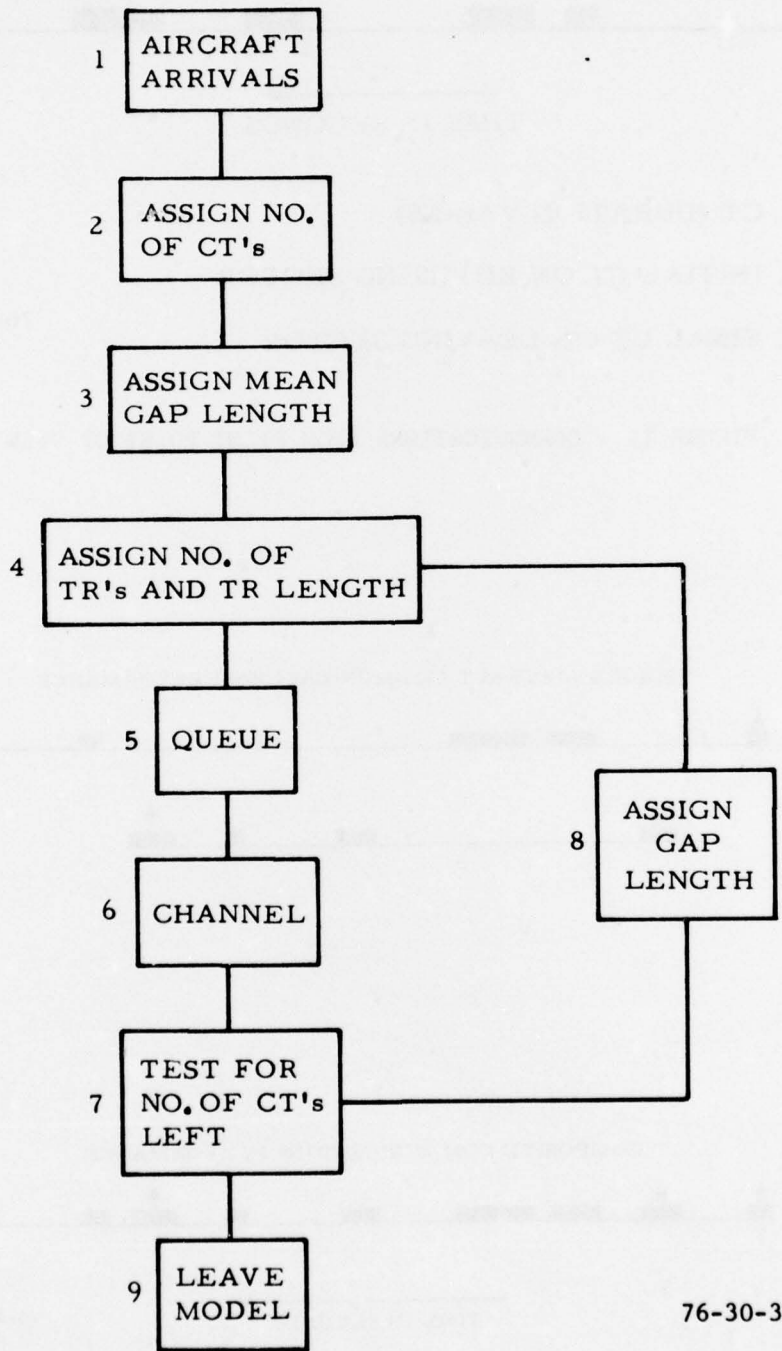


FIGURE 2. COMMUNICATIONS FROM CONTROLLER POINT OF VIEW



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FIGURE 3. BLOCK DIAGRAM OF SIMULATION MODEL

of the type being studied. The standard deviation (SD) is determined from the relationship,

$$SD = \begin{cases} \frac{CU - XM}{2.5758} & \text{if } A2 > 0 \\ \frac{XM - CL}{2.5758} & \text{if } A2 \leq 0 \end{cases}$$

where CU and CL are upper and lower bounds, respectively, of the logarithm of recorded mean gap lengths. These formulas are the result of examinations of historical records that indicate the existence of patterns in the data of the type shown in figures 4 and 5. The constant 2.5758 was chosen to insure that 99 percent of the probability mass of the normal distribution used to generate the logarithm of the mean gap length lies between $2 XM - CU$ and CU in the case where $A2 > 0$, and between CL and $2 XM - CL$ when $A2 < 0$. In either case, whenever sampling from this distribution results in a negative number, the mean gap length is arbitrarily set equal to 1 second.

The first CT assigned to an aircraft commences as the aircraft enters the sector provided that the channel (block 6) is available, i.e., a conversation between another aircraft and ground is not taking place. Otherwise, the CT enters a queue (block 5) on a first-come, first-serve basis. As each CT is completed, the model ascertains whether or not all assigned CT's have taken place (block 7). If not, then an intercommunication gap length is randomly selected from an exponential distribution with mean MGAP seconds (block 8). If it is determined that the CT just completed is indeed the last of the assigned CT's, then the aircraft leaves the sector (block 9). At any rate, the end time of the most recently completed CT and the length of the subsequent intercommunication gap determines the start time of the next CT. Of course, in the event that the channel is busy when the CT is scheduled to take place, then the CT enters a queue and waits in turn for transmission service. When the start time of a CT is established, the corresponding CT length is determined in a two-step process (block 4). The first step of the process is to establish the number of TR's involved in the CT. This is accomplished by a random sample drawn from an empirical distribution based upon data collected from one or more sectors of the type being investigated. Thereafter, the length of each TR is obtained as a random sample drawn from a gamma distribution with parameters α and λ , i.e.,

$$\frac{1}{\lambda \alpha \Gamma(\alpha)} t^{\alpha-1} e^{-t/\lambda} \quad 0 < t < \infty$$

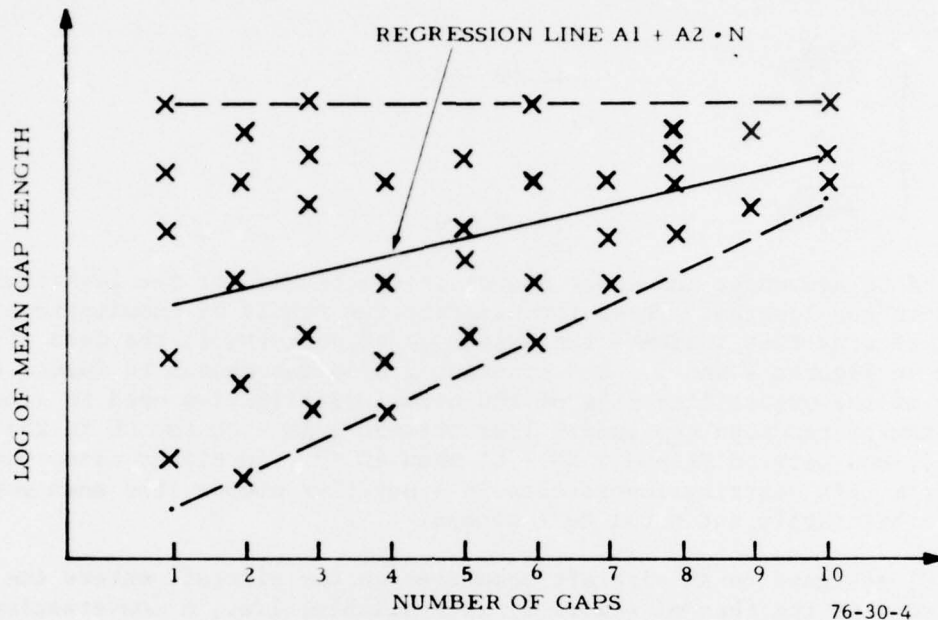


FIGURE 4. XM VERSUS N IN CASE WHERE $A_2 > 0$

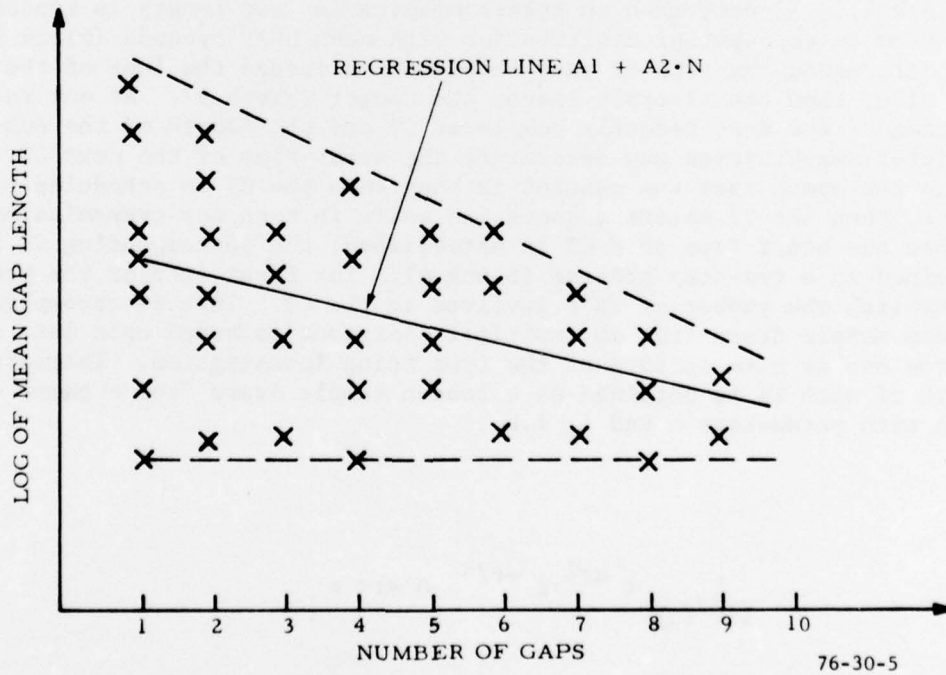


FIGURE 5. XM VERSUS N IN CASE WHERE $A_2 > 0$

The parameter values are obtained from two master equations (i) and (ii). These are,

$$(i) \alpha \cdot \lambda \quad = 3.70 - 56.88/AMEAN \text{ (General)} \\ = 2.97 - 15.12/AMEAN \text{ (LC)}$$

$$(ii) (\alpha - 1)\lambda \quad = 2.0 \quad \text{(General)} \\ (\alpha - 1)\lambda \quad = 1.7 \quad \text{(GN)} \\ (\alpha - 1.13)\lambda = 1.72 \quad \text{(DP)} \\ (\alpha - 1.5)\lambda = 1.0 \quad \text{(AD)}$$

where, as already indicated, AMEAN is the mean aircraft interarrival time in seconds. Thus, if the LC sector function is to be simulated, then appropriate values for α and λ are obtained from the two equations,

$$\alpha \cdot \lambda \quad = 2.97 - 15.12/AMEAN \\ (\alpha - 1) \lambda \quad = 2.0$$

A listing of the program available at NAFEC is provided in the appendix. This listing together with the description of the program structure presented here is merely intended to provide the reader with some general idea of the existing simulation capability. Further details including possible modifications, variations in the manner in which inputs can be supplied to the model, justification of model formulation, etc., can be obtained from the cited references.

INPUT VARIABLES.

As described, the model is characterized by 10 input variables, namely, AMEAN, P, K, A1, A2, CU, CL, α , λ , and an empirical distribution for the number of TR's contained in one CT. AMEAN prescribes the mean interarrival time between successive aircraft entering the sector. P and K determine the distribution of the number of CT's, or equivalently, the number of intercommunication gaps, experienced by an aircraft as it passes through the sector. The four parameters, A1, A2, CU, and CL, determine the distribution of gap length. Finally, α , λ , and the empirical distribution of the number of TR's in a CT specify the distribution of CT length.

In an application of the model, all input parameters except AMEAN might be assigned fixed values to represent a particular sector in some center such as Houston or New York. Then several simulations could be performed corresponding to decreasing values of AMEAN to ascertain the effect of increasing aircraft arrival rate on sector communications. By the same token, AMEAN could be held constant and the effect of some other input parameter on sector communications observed.

RESPONSE VARIABLES.

Three basic response variables are observed during each second of simulated time. They are the sector aircraft loading (i.e., the number of aircraft that have been handed off to the sector, but have not yet been handed off by the sector), the state of the air/ground channel (i.e., busy or idle), and the queuing state of each aircraft in the sector (i.e., in queue or otherwise). An aircraft is defined to be in queue if either the controller or pilot desires to converse with the other, but is prevented from doing so by virtue of the fact that a conversation is presently taking place between another aircraft and ground. Thus, insofar as the computer simulation is concerned, the third response variable measures the lag between the instant that the first TR of a CT is scheduled to occupy the channel and the time that it is actually carried by the channel. In this sense, the queuing state of the channel represents delay in the transfer of information between air and ground.

The next few paragraphs discuss each basic response variable in greater detail in the context of a specific simulation run. Values selected for input parameters in this example are as follows:

AMEAN=86 seconds	P=0.495	K=3.88
A1=4.336	A2=0.032	CU=6.0
CL=3.1		

The example experiment was designed to simulate New York LC sector 510. As a result, α and λ were determined from the second of master equations (i) and the first of master equations (ii). Moreover, the empirical distribution of the number of TR's in a CT was determined from historical records obtained from sector 510. The simulation was allowed to run for 1 hour of simulated time prior to the accumulation of any data in order to dissipate the influence of transient phenomena generated by boundary conditions that exist at the beginning of the experiment. Thereafter, data were gathered for 2 hours of simulated time. Consequently, results obtained for the 2-hour observation period can be viewed as representative of sample functions drawn from stationary random phenomena. Of course, in the event that a transient effect persists or that values assigned to input parameters result in an explosive situation, then the underlying stochastic processes are far from stationary. However, such circumstances are usually reflected by very definite trends in the time series generated by one or more of the basic response variables during the 2-hour observation period.

CHANNEL UTILIZATION AND AIRCRAFT LOADING.

From observation of the first response variable, it is possible to compute the number of aircraft in sector per second averaged over each minute of simulated time. The average over the t th minute of simulated time is represented by n_t , and there are 120 such averages; namely, n_1 through n_{120} . These are illustrated for LC sector 510 in figure 6. The number of aircraft in sector averaged over 2 hours of simulated time is just the arithmetic mean of n_1 through n_{120} . For example, the 2-hour average corresponding to figure 6

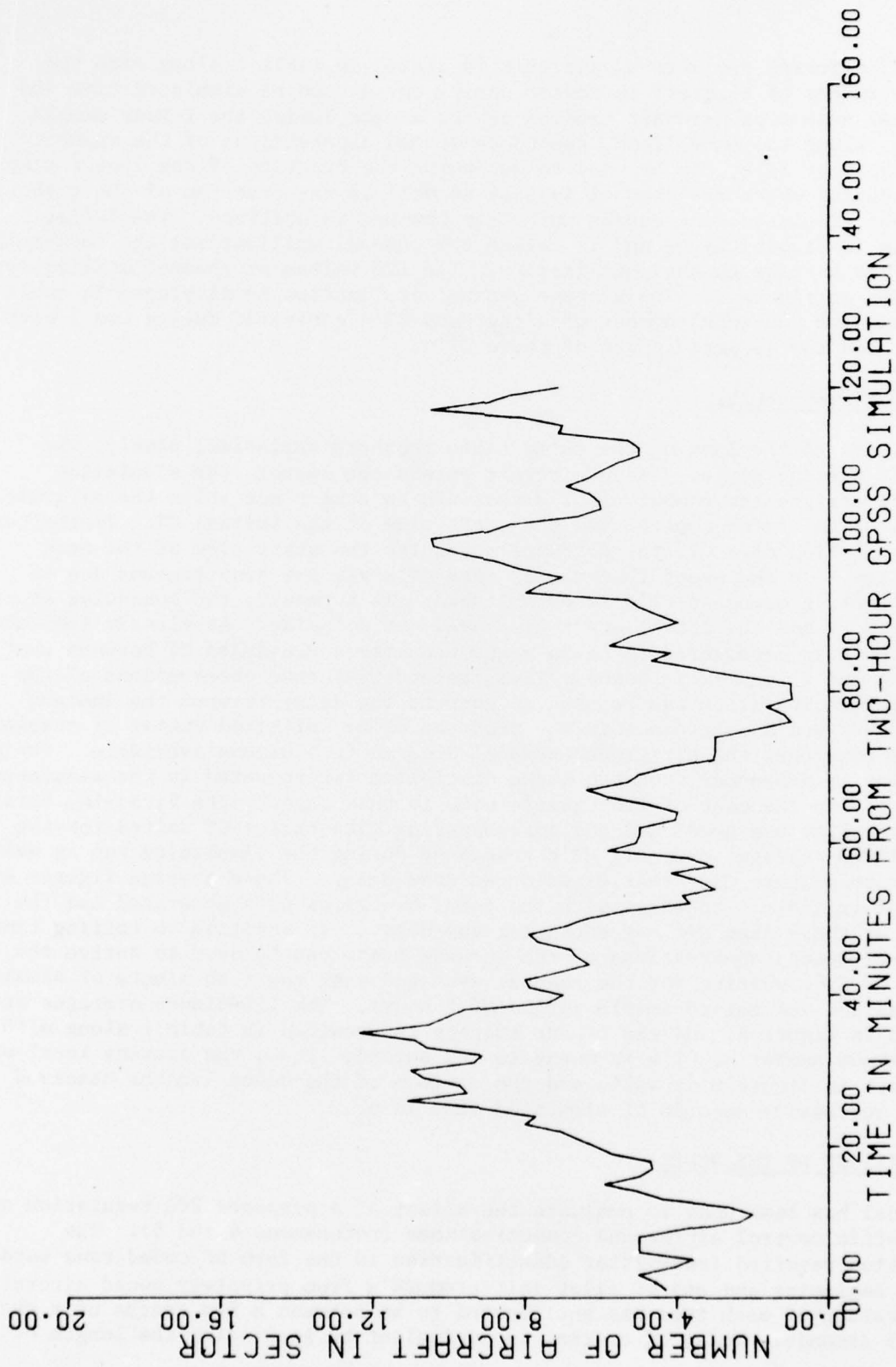


FIGURE 6. SIXTY-SECOND AVERAGE AIRCRAFT LOADINGS

is 5.983 aircraft per second, and this is listed in table 1 along with the maximum number of aircraft in sector during any second of simulated time and the total number of aircraft handled by the sector during the 2-hour sample period. Along the same lines, second-by-second observations of the channel state, busy or idle, can be used to determine the fraction of the 2-hour sample period during which the channel is busy as well as the fraction of the t th minute of simulated time during which the channel is utilized. The latter fraction is denoted by c_t and is called the channel utilization; the former is called the average channel utilization. The 120 values of channel utilization are shown in figure 7. The average channel utilization is displayed in table 1 together with the total number of air/ground CT's generated during the 2-hour period, and the average length of these CT's.

COMMUNICATIONS DELAY.

We now turn to the last of the three basic response variables; namely, the aircraft queuing state. As an aircraft enters the sector, the simulation software assigns the number of CT's that are to take place while the aircraft is subject to sector control and the start time of the initial CT. Thereafter, upon completion of a CT, the software schedules the start time of the next CT, if any. In the event that two or more CT's vie for simultaneous use of the channel, a queue of CT's is established. As a result, the scheduled start time of a CT and the actual start time need not coincide. As already indicated, an aircraft is considered to be in queue whenever a scheduled CT between that aircraft and ground is in queue. Thus, second-by-second observations of the aircraft queuing state can be used to compute the delay between the instant that the CT would take place in the presence of an unlimited number of channels and the time that the air/ground channel does in fact become available. Obviously, the delay is dependent upon the queue discipline incorporated in the simulation software. In the case of the example used in this report, the first-in, first-out discipline was used, and the corresponding time that a CT waited for the channel was averaged over all CT's generated during the simulation run as well as over only those CT's that experienced some delay. These average figures are recorded in table 1 together with the total number of CT's generated and the number of those that did not encounter any delay. In addition to waiting times, second-by-second observations of the queuing state can be used to derive the number of CT's waiting for the channel averaged over the t th minute of simulated time and the entire sample period of 2 hours. The 120-minute averages are plotted in figure 8, and the 2-hour average is provided in table 1 along with the maximum number of CT's in queue in any second. Thus, the average level of the graph in figure 8 is .426, and the maximum of the queue lengths observed during successive seconds of simulated time is 6.

APPLICATIONS OF THE MODEL.

The model has been used to evaluate the effect of a proposed FCC regulation on air traffic control air/ground communications (references 4 and 5). The regulation required transmitter identification in the form of coded tone bursts at the beginning and end of pilot initiated TR's from privately owned aircraft. The duration of each tone was anticipated to be between a few tenths of a second and 1.5 seconds. This, of course, is equivalent to increasing the length of

GPSS SIMULATION MODEL FOR ATC VERBAL COMMUNICATIONS SYSTEM

TRANSPORTATION PROGRAM
DEPT. OF CIVIL ENGINEERING
PRINCETON UNIVERSITY
MARCH, 1974

INPUT PARAMETERS - SETUP 510

- (1) AIRCRAFT INTERARRIVAL TIMES: EXPONENTIAL WITH MEAN = 90.00 SECONDS
- (2) TRANSACTIONS PER AIRCRAFT: SHIFTED NEGATIVE BINOMIAL WITH $K = 3.879$ AND $p = 0.494$
- (3) TRANSACTIONS PER TRANSACTION: EMPIRICAL DISTRIBUTION
- (4) TRANSMISSIONS LENGTHS: γ WITH $\rho = 1.2591$ AND $\alpha = 3.5183$
- (5) INTERCOMMUNICATION GAP LENGTHS ARE A FUNCTION OF TRANSACTIONS PER AIRCRAFT

SYSTEM-TYPE RESPONSE - 2 HOUR ANALYSIS

(1) SPEC OF AIRCRAFT TOGETHER

- NUMBER OF AIRCRAFT IDENTIFIED IN SETUP = 84
- AVERAGE NUMBER OF AIRCRAFT PER SECOND = 5.583
- MAXIMUM NUMBER OF AIRCRAFT PER SECOND = 13

(2) COMMUNICATIONS CHANNEL LOADING

- AVERAGE CHANNEL UTILIZATION = .469
- TOTAL NUMBER OF TRANSACTIONS = 426
- AVERAGE LENGTH OF TRANSACTIONS = 7.932 SECONDS

(3) CHANNEL QUEUING EFFECTS

- AVERAGE TIME IN QUEUE = 7.215 SECONDS
- AVERAGE TIME INCLUDING ZERO ENTRIES = 12.566 SECONDS
- TOTAL ENTRIES INTO QUEUE = 425
- NUMBER OF ZERO ENTRIES (NOW-WAITING) = 191
- PERCENT OF ZERO ENTRIES = 42.4
- AVERAGE NUMBER OF AIRCRAFT IN QUEUE = .426
- MAXIMUM NUMBER OF AIRCRAFT IN QUEUE = 6

TABLE 1. GPSS SIMULATION MODEL FOR ATC VERBAL COMMUNICATIONS SYSTEM

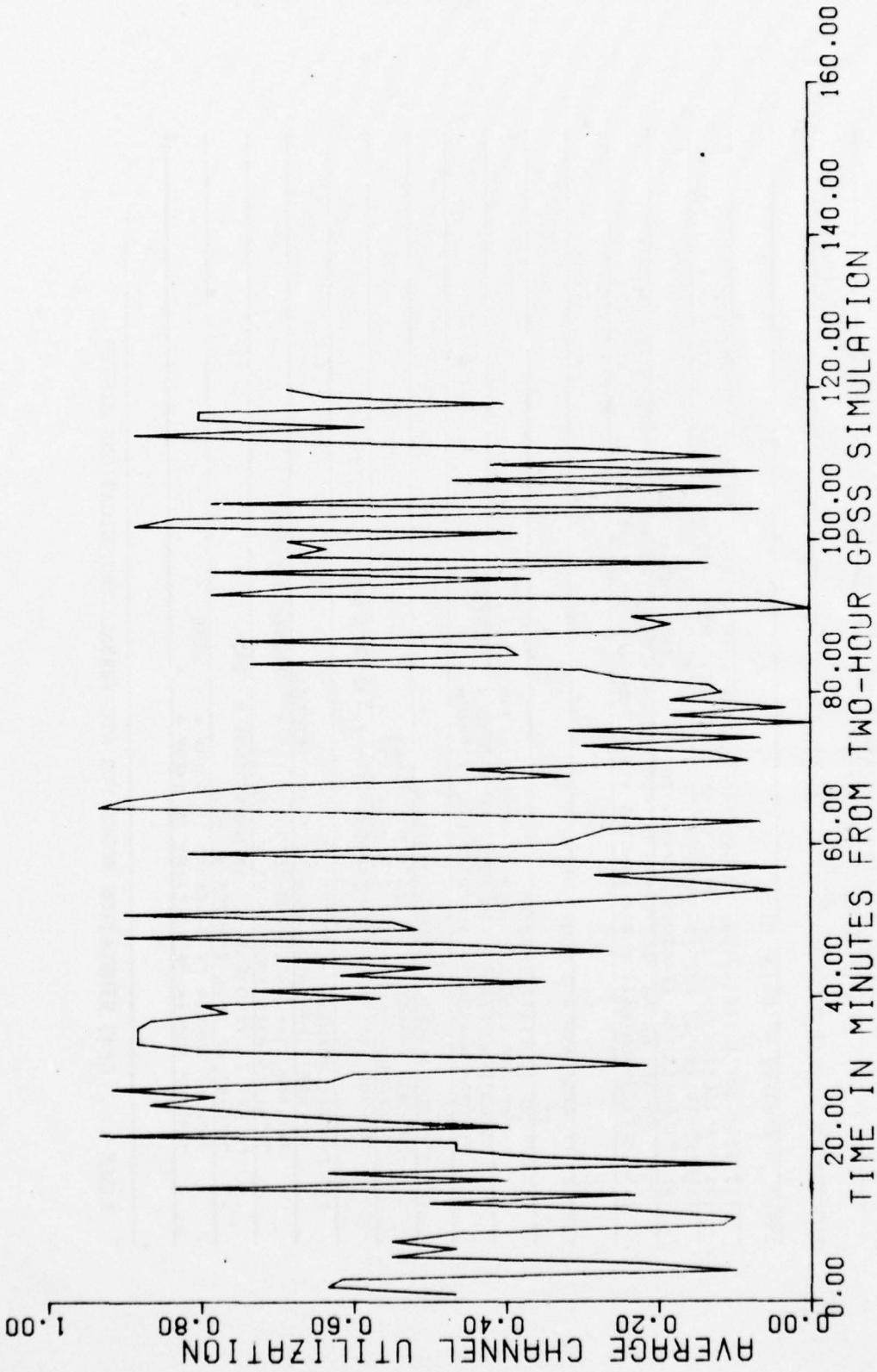


FIGURE 7. SIXTY-SECOND AVERAGE CHANNEL UTILIZATION

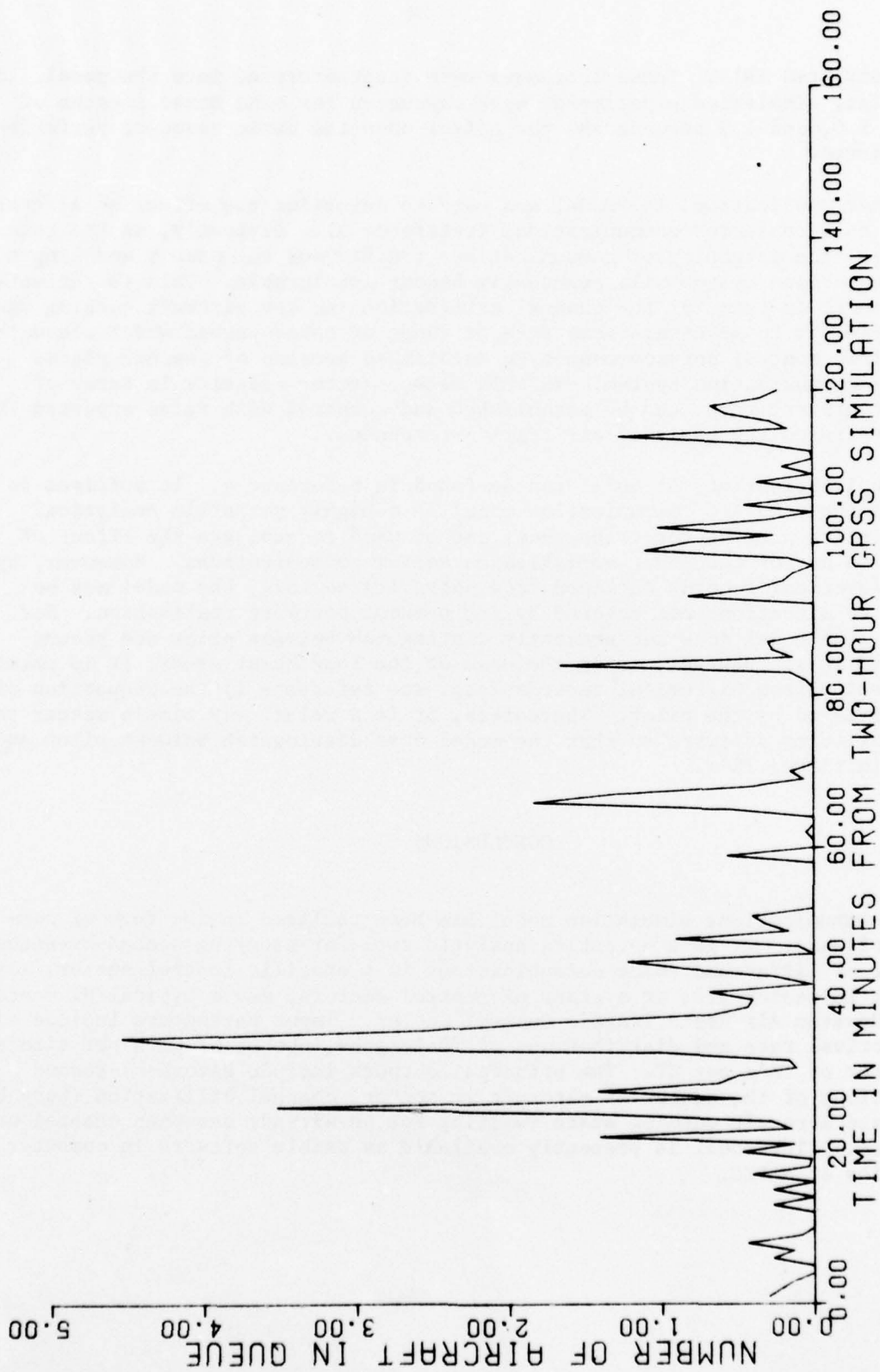


FIGURE 8. SIXTY-SECOND AVERAGE QUEUE LOADINGS

pilot-initiated TR's. These increases were incorporated into the model. In particular, simulation experiments were conducted for tone burst lengths of 0, 0.5, 1.0, and 1.5 seconds and the effect upon the basic response variables was observed.

In another application, the model was used to determine the effect of aircraft arrival rate on sector communications (reference 4). Obviously, as the rate increases, the intensity of communications traffic and the number and length of communication delays will eventually become intolerable. This is reflected by the model in terms of the channel utilization and the aircraft queuing state. The problem is to ascertain that rate or range of rates beyond which adequate air traffic control service cannot be maintained because of demands placed upon the communication system. In this sense, sector capacity in terms of aircraft arrival rates can be established and compared with rates expected in future years on the basis of air traffic forecasts.

Other applications of the model can be found in reference 4. It suffices to say here that the ATC communication model is a highly versatile analytical tool. In its present form, the model can be used to evaluate the effect of changes in any of the input variables on sector communications. Moreover, by using historical records obtained from operating sectors, the model may be adapted to situations not covered by its present software realization. For example, the model does not presently distinguish between pilot and ground initiated TR's. However, as in the case of the tone burst study, it is possible to determine from historical records (e.g. see reference 1) the proportion of TR's initiated by the pilot. Thereafter, it is a relatively simple matter to modify existing software so that the model does distinguish between pilot and ground-initiated TR's.

CONCLUSIONS

An ATC communications simulation model has been realized in the form of computer software. It is a versatile analytic tool for studying second-by-second behavior of air/ground voice communications in a specific control sector, say New York LC sector 510, or a class of control sectors, say a typical HI sector in the Houston Air Route Traffic Control Center. Input parameters include aircraft arrival rate and distributions of TR lengths, number of CT's per aircraft, and number of TR's per CT. The principal outputs include second-by-second observations of the number of aircraft in sector, channel utilization (busy or idle), and aircraft queuing state (waiting for an already occupied channel or otherwise.) The model is presently available as usable software in computer facilities at NAFEC.

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APPENDIX
PROGRAM LISTING

```

REAL LOCATE COM,30000,HMS,12
***** FUNCTIONS *****
TRCT FUNCTION RN1,013
.195,1/.485,2/.741,3/.853,4/.925,5/.958,6/.978,7/.985,8
.991,9/.993,10/.996,11/.998,12/1,13
***** STORAGES *****
SCTR STORAGE 150
***** MATRICES *****
1 MATRIX MH,600,1 NAC IN SECTOR
2 MATRIX MH,120,1 ARRIVAL TIMES
3 MATRIX MH,120,1 CT'S PER AIRCRAFT
4 MATRIX MH,400,1 TR'S PER CT
5 MATRIX MH,1200,1 TR LENGTHS
6 MATRIX MH,400,1 CT LENGTHS
7 MATRIX MH,400,1 INTERCOM. GAP LENGTHS
8 MATRIX MH,120,1 AC TIME IN SECTOR
9 MATRIX MH,120,1 DEPARTURE TIMES
10 MATRIX MH,600,1 NAC IN QUEUE
11 MATRIX MH,400,1 QUEUEING TIMES
12 MATRIX MH,600,1 CHANNEL STATUS
***** VARIABLES *****
LAMBDA FVARIABLE 1.0/(0.97-15.12/XL$AMEAN)
ALPHA FVARIABLE (1.0/XL$PGAM2+2.0)*XL$PGAM2
***** SAVEVALUES *****
INITIAL XL$PNR1,3.88 K FOR CT/AC
INITIAL XL$PNR2,0.495 P FOR CT/AC
INITIAL XL$PGAP1,4.336 A1 FOR GAP LENGTHS
INITIAL XL$PGAP2,0.032 A2 FOR GAP LENGTHS
INITIAL XL$PNBT1,0.0 COMPILING ONLY
INITIAL XL$PNBT2,0.0 COMPILING ONLY
INITIAL XL$PGAM1,0.0 COMPILING ONLY
INITIAL XL$PGAM2,0.0 COMPILING ONLY
INITIAL XL$AMEAN,86.00 MEAN INTERARRIVAL TIME
INITIAL XH10,510 SECTOR NUMBER
INITIAL XF$ONE,48949 RANDOM NUMBER GENERATORS
INITIAL XF$TWO,25701
INITIAL XF$THREE,85261
INITIAL XF$FOUR,52693
INITIAL XF$FIVE,2529
INITIAL XF$SIX,18623
INITIAL XF$SEVEN,44035
INITIAL XL$MGAP,0.0 MEAN GAP LENGTH
INITIAL XF$YAC,0 # OF CTS PER AC
INITIAL XF$TIME,0 CT LENGTHS
INITIAL XF$TRL,0 TR LENGTHS
INITIAL XF$TGAP,0 INTERCOM. GAPS
INITIAL XF$NTR,0 # OF TR'S PER CT
INITIAL XF$IAT,0 INTERARRIVAL TIMES
INITIAL XF$COUNT,0 SECOND COUNTER FOR MATRIX
INITIAL XH1,10 MATRIX UNIT COUNTER

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```

INITIAL XH2,22 STAT OUTPUT UNIT COUNTER
INITIAL XH3-XH9,0 MATRIX OUTPUT COUNTERS
INITIAL MH1(1-600,1),0 INITIAL MATRICES
INITIAL MH2(1-120,1),0
INITIAL MH3(1-120,1),0
INITIAL MH4(1-400,1),0
INITIAL MH5(1-1200,1),0
INITIAL MH6(1-400,1),0
INITIAL MH7(1-400,1),0
INITIAL MH8(1-120,1),0
INITIAL MH9(1-120,1),0
INITIAL MH10(1-600,1),0
INITIAL MH11(1-400,1),0
INITIAL MH12(1-600,1),0

```

***** SIMULATION *****

SIMULATE

* DETERMINE GAMMA PARAMETERS AT BEGINNING OF SIMULATION

```

GENERATE ,,,1,25,0
SAVEVALUE PGAM2,V$LAMDA,XL
SAVEVALUE PGAM1,V$ALPHA,XL
TERMINATE

```

* GENERATE POISSON ARRIVALS AT SPECIFIED RATE

```

GENERATE ,,,,15,4PF,1PL
GATE LR 1
LOGIC S 1
HEL PR EXPON,AMFAN$XL,IAT$XF,SEVEN$XF,ONE$XF,TWO$XF,FOUR$XF
ADVANCE XF$IAT
LOGIC R 1
MARK MARK TIME OF ARRIVAL

```

* SAVE AND TABULATE TIME OF ARRIVAL

```

ASSIGN 3,AC1,PF
SAVEVALUE 3+,1,XH ARRIVAL COUNTER
MSAVEVALUE 2,XH3,1,C1,MH TABULATE ARRIVAL TIME
ENTER SCTR ENTER THE SECTOR

```

* DETERMINE THE NUMBER OF CT FROM A NEG. BIN. DSN. WITH SHIFTED ORIGIN

```

HEL PR SIBNRJ,PNR1$XL,PNR2$XL,XAC$XF,ONE$XF,TWO$XF,THREE$XF
SAVEVALUE XAC+,1,XF
MSAVEVALUE 3,XH3,1,XF$XAC,MH TABULATE CTS PER AIRCRAFT
ASSIGN 1,XF$XAC,PF

```

* GENERATE MEAN GAP LENGTH FOR ENTERING AIRCRAFT, AS A FUNCTION

```

* OF THE NUMBER OF CTS ASSIGNED.
SAVEVALUE XAC-,1,XF NUMBER OF GAPS
HEL PR GAPM,PGAP1$XL,PGAP2$XL,XAC$XF,MGAP$XL,TWO$XF,THREE$XF
ASSIGN 1,XL$MGAP,PL
TRANSFER ,SCTS SKIP GAP FOR FIRST CT

```

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```

*
* ADVANCE INTERCOMMUNICATION GAP TIME, THEN ENTER QUEUE.
* SEIZE CHANNEL WHEN AVAILABLE.
SGAP ADVANCE XFS*GAP WAIT TIL GAP COMPLETED
SCTS MARK 4PF MARK TIME IN QUEUE
      QUEUE SCTR
      GATE LR 2
      LOGIC S 2
SCTR SEIZE CNTRL SEIZE THE CHANNEL
      DEPART SCTR
*
* TABULATE TIME IN QUEUE
  SAVEVALUE 9+,1,XH NUMBER OF QUEUEING TIMES
  MSAVEVALUE 11,XH9,1,MP4PF,MH TABULATE TIME IN QUEUE
*
* GENERATE CT LENGTH
* TR'S PER CT FROM EMPIRICAL DISTRIBUTION
* TR LENGTHS FROM GAMMA DSN.
  SAVEVALUE TIME,0,XF
  SAVEVALUE NTR,FN*TRCT,XF TR'S PER CT
  SAVEVALUE 4+,1,XH COUNT OF CT'S
  MSAVEVALUE 4,XH4,1,XF*NTR,MH TABULATE TR'S PER CT
  ASSIGN 2,XF*NTR,PF
STRL HELPB MSSC,PGAM1$XL,PGAM2$XL,NTR$XF,TRL$XF,FIVE$XF,SIX$XF
  SAVEVALUE 5+,1,XH COUNT OF TR'S
  MSAVEVALUE 5,XH5,1,XF*STRL,MH TABULATE TR LENGTHS
  SAVEVALUE TIME+,XF*STRL,XF ADD TR LENGTH TO CT LENGTH
  LOOP 2PF,STRL LOOP FOR MORE TR'S IN CT
  MSAVEVALUE 6,XH4,1,XF*TIME,MH TABULATE CT LENGTHS
*
* HOLD CHANNEL FOR LENGTH OF CT, THEN FREE CHANNEL.
  ADVANCE XFS*TIME WAIT TIL CT COMPLETED
* IMPOSE MANDATORY GAP OF 1 SECOND BEFORE CHANNEL IS AVAILABLE.
  SPLIT 1,TEST1
  RELEASE CNTRL FREE CHANNEL
  ADVANCE 1 KEEP CHANNEL CLEAR FOR 1 SECOND
  LOGIC R 2
  TERMINATE
*
* GENERATE INTERCOMMUNICATIONS GAP FROM EXPON. DSN. GIVEN MEAN
* GAP LENGTH FOR THE AIRCRAFT.
* GAPS MUST BE AT LEAST ONE SECOND AND NO MORE THAN 700.
TEST1 TEST NE PF1,K1,SKIPR NO GAP IF NO CT'S REMAIN
  SAVEVALUE MGAP,PL1,XL RETRIEVE MEAN GAP LENGTH
TEST2 HELPB EXPON,MGAP$XL,TGAP$XF,THREE$XF,TWO$XF,FOUR$XF,FIVE$XF
  TEST LE XFS*GAP,700,TEST2 IF OVER 700, TRY AGAIN
  TEST LE XFS*GAP,1,SAVE6 IF LESS THAN 1, SET TO 1
  SAVEVALUE TGAP,1,XF
SAVE6 SAVEVALUE 6+,1,XH COUNT GAPS
  MSAVEVALUE 7,XH6,1,XF*TGAP,MH TABULATE GAP LENGTHS

```

```

      LOOP      1PF,SGAP      LOOP TO MAKE MORE CT'S
*
* TABULATE TIME IN SECTOR IF AIRCRAFT ENTERED DURING SAMPLE PERIOD
SKIPR TEST G      PF3,3600,SKIP]
  SAVEVALUE 7+,1,XH      COUNT OF TIMES IN SECTOR
  MSAVEVALUE 8,XH7,1,M1,MH      TABULATE TIMES IN SECTOR
*
* TABULATE DEPARTURE TIMES
SKIP] SAVEVALUE 8+,1,XH      COUNT OF DEPARTURES
  MSAVEVALUE 9,XH8,1,C1,MH      TABULATE DEPARTURE TIME
*
* LEAVE THE SECTOR
LEAVE SCTR
TERMINATE
***** MAC STATS *****
* THIS SECTION OF THE PROGRAM KEEPS TRACK OF THE NUMBER OF
* AIRCRAFT IN THE SECTOR IN EACH SECOND AND HAS THE DATA PUNCHED
* FOR LATER ANALYSIS.
*
GENERATE 1,,3601,,2,0      SECOND TIMER
SAVEVALUE COUNT+,K1,XF      COUNT SECONDS
MSAVEVALUE 1,XF$COUNT,1,S1,MH      TABULATE NAC IN SECTOR
MSAVEVALUE 10,XF$COUNT,1,O1,MH      TABULATE NAC IN QUEUE
MSAVEVALUE 12,XF$COUNT,1,F1,MH      TABULATE CHANNEL STATUS
TERMINATE
GENERATE 600,,4200,,1,0      TEN-MINUTE TIMER
SAVEVALUE 1+,K1,XH      MATRIX OUTPUT UNIT
HELPC TIME5,MH1(1,1),MH10(1,1),MH12(1,1),XH1,XF$ONE,XF$TWO
SAVEVALUE COUNT,K0,XF      SET SECOND COUNT TO ZERO
TERMINATE
GENERATE 3600,,7200,,3,0      HOUR TIMER
SAVEVALUE 2+,K1,XH      STAT OUTPUT UNIT
HELPC PASS2,XH2,XF$ONE,XF$TWO,XF$THREE,XF$FOUR,XF$FIVE
SAVEVALUE 3-9,0,XH      SET COUNTS BACK TO ZERO
TERMINATE
GENERATE 3600,,3600,1,4,0      INITIAL MATRIX CLEARER
SAVEVALUE 3-9,0,XH      SET COUNTS TO ZERO
TERMINATE
***** TIMER *****
GENERATE 3600
TERMINATE 1
START 1,NP
RESET
START 2
***** OUTPUT *****
REPORT
EJECT
***** GPSS SIMULATION MODEL FOR ATC VERTICAL COMMUNICATIONS SYSTEM *****
TRANSPORTATION PROGRAM
DEPT. OF CIVIL ENGINEERING

```

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PRINCETON UNIVERSITY
MARCH, 1974

*
*
*
*
2 TEXT INPUT PARAMETERS - SECTOR #XH10,2/XXX#
*
5 TEXT (1) AIRCRAFT INTERARRIVAL TIMES: EXPONENTIAL WITH MEAN#
M = #XL1,2/XXXX.XXX# SECONDS
5 TEXT (2) TRANSACTIONS PER AIRCRAFT: SHIFTED NEGATIVE BINOM#
IAL WITH K = #XL3,2/XXX.XXX# AND P = #XL4,2/XXX.XXX#
5 TEXT (3) TRANSMISSIONS PER TRANSACTION: EMPIRICAL DISTRIBU#
TION
5 TEXT (4) TRANSMISSIONS LENGTHS: GAMMA WITH P = #XL2,2/XXX.#
XXXX# AND ALPHA = #XL9,2/XXX.XXXX#
5 TEXT (NOTE: GAMMA PARAMETERS DETERMINED FROM EXPECT#
ED ARRIVAL RATE)
5 TEXT (5) INTERCOMMUNICATION GAP LENGTHS ARE A FUNCTION OF #
TRANSACTIONS PER AIRCRAFT

*
*
* SIMULATION RESPONSE - 2 HOUR ANALYSIS
*

* (1) SECTOR AIRCRAFT LOADING
10 TEXT NUMBER OF AIRCRAFT IDENTIFIED IN SECTOR = #S1,5/XXXX#
10 TEXT AVERAGE NUMBER OF AIRCRAFT PER SECOND = #S1,3/XXX.XXX#
*
10 TEXT MAXIMUM NUMBER OF AIRCRAFT PER SECOND = #S1,8/XXX#
*
* (2) COMMUNICATIONS CHANNEL LOADING
10 TEXT AVERAGE CHANNEL UTILIZATION = #F1,2/X.XXX#
10 TEXT TOTAL NUMBER OF TRANSACTIONS = #F1,3/XXXX#
10 TEXT AVERAGE LENGTH OF TRANSACTIONS = #F1,4/XXX.XXX# SECON#
DS
*
* (3) CHANNEL QUEUING EFFECTS
10 TEXT AVERAGE TIME IN QUEUE = #Q1,7/XXXX.XXX# SECONDS
10 TEXT AVERAGE TIME EXCLUDING ZERO ENTRIES = #Q1,8/XXXX.XXX#
SECONDS
10 TEXT TOTAL ENTRIES INTO QUEUE = #Q1,4/XXXX#
10 TEXT NUMBER OF ZERO ENTRIES (NON-WAITING) = #Q1,5/XXXX#
10 TEXT PERCENT OF ZERO ENTRIES = #Q1,6/XXX.X#
10 TEXT AVERAGE NUMBER OF AIRCRAFT IN QUEUE = #Q1,3/XXX.XXX#
10 TEXT MAXIMUM NUMBER OF AIRCRAFT IN QUEUE = #Q1,2/XXX#
EJECT

***** FND *****
FND

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```

SUBROUTINE EXPON(FX,IX,I1,I2,I3,I4)
R=RANDB(I1)
X=(-FX)*ALOG(R)
IX=X+0.5
RETURN
END
```

```

FUNCTION RANDU(IX)
IY=IX*65539
IF(IY)5,6,6
5 IY=IY+2147483647+1
6 RANDU=IY
RANDU=RANDB(.4656613E-9)
IX=IY
RETURN

END
```

```

SUBROUTINE SUBNR1(K,P,NX,I1,I2,I3)
IMPLICIT REAL*8(D)
REAL K
DP=P
DK=K
R=RANDB(I1)
DR=R
I=0
X=0.0
DCUM=DP**DK
DY=DCUM
IF (DCUM .LT. DR) GO TO 10
NX=(X+0.5)
RETURN
10 I=I+1
X=1.0*I
```

```

DX=X
DY=DY*((DK+DX-1.000)/DX)
DCUM=DCUM+DY*(1.000-DP)**DX
IF (DCUM .LT. DR) GO TO 10
NX=(X+0.5)
RETURN
END
```

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```

SUBROUTINE CAPM(A1,A2,N,Y,I1,I2)
CII=6.0
CL=3.1

XM=A1+A2*N
IF(A2 .LE. 0.) GO TO 1
SD=(CII-XM)/2.5758
GO TO 2
1 SD=(XM-CL)/2.5758

2 X=RNORM(I1)
XY=(X*SD)+XM
IF(XY .LE. 0.) XX=0.
Y=EXP(XX)
RETURN
END

```

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```

FUNCTION RNORM(IX)
SIIM=0.0
DO 5 I=1,12
SIIM=SIIM+RANDU(IX)
RNORM=SIIM-6.0
RETURN
END

```

```

SUBROUTINE MSSG(P1,P2,N,NTIME,I1,I2)
CALL SURGAM(P1,P2,X,I1,I2)
NTIME=X+0.5
IF (NTIME .LT. 1) NTIME=1

RETURN
END

```



```

SUBROUTINE TIME5(IVALUE, ISAVFF, ISAVEH, IFAC, ISTD, FSTD, IOUF,
* FOUF, ILOG, ITAR, FTAR, IUSE, IUSEF, FUSE, IMAX, IMAXR, IMAXH, IMAXRH,
* FSAVEL, IMAXL, FMAXRL)
INTEGER*2 ISAVEH, ILOG, IUSE, IMAXBH
REAL*8 FOUF, FUSE
REAL*4 FSTD, FSAVEL, FMAXRL
DIMENSION IVALUE(6), ISAVFF(2), ISAVEH(2), IFAC(2), ISTD(2), FSTD(2),
* IOUF(2), FOUF(2), ILOG(2), ITAR(2), FTAR(2), IUSE(2), IUSEF(2), FUSE(2),
* IMAX(2), IMAXR(2), IMAXH(2), IMAXRH(2), FSAVEL(2), IMAXL(2), FMAXRL(2)
INTEGER Z(600), KEY(3)/1,10,12/
NUNIT=ISAVEH(1)
IC=COLUMN NUMBER
IR=ROW NUMBER
IC=1
ICN=1
K=6
L=1
DO 200 IJK=1,3
N=KEY(IJK)
JK=K*(N-1)+L
DO 100 IR=1,600
J=(IMAXH(JK)+2*(ICN*(IR-1)+(IC-1)))/2
100 Z(IR)=IMAXRH(J)
WRITE(NUNIT,101) Z
101 FORMAT(40I2)
200 CONTINUE
RETURN
END

```

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```

SUBROUTINE PASS2(IVALUE,ISAVFF,ISAVEH,IFAC,ISTO,FSTO,IOUF,
* FOUE,ILOG,ITAR,FTAR,IUSE,IUSFF,FUSE,IMAX,IMAXR,IMAXH,IMAXRH,
* FSAVEL,IMAXL,FMAXRL)
INTEGER*2 ISAVEH,ILOG,IUSE,IMAXBH
REAL*8 FOUE,FUSE
REAL*4 FSTO,FSAVEL,FMAXRL
DIMENSION IVALUE(6),ISAVFF(2),ISAVEH(2),IFAC(2),ISTO(2),FSTO(2),
* IOUF(2),FOUE(2),ILOG(2),ITAR(2),FTAR(2),IUSE(2),IUSFF(2),FUSE(2),
* IMAX(2),IMAXR(2),IMAXH(2),IMAXRH(2),FSAVEL(2),IMAXL(2),FMAXRL(2)
INTEGER Z(1200),N(9),KEYN(9)/3,3,4,5,4,6,7,8,9/
INTEGER KEYM(9)/2,3,4,5,6,7,8,9,11/
NUNIT=ISAVEH(2)
IC=1
ICN=1
K=6
L=1
DO 100 I=1,9
KN=KEYN(I)
100 N(I)=ISAVEH(KN)
WRITE(NUNIT,101) N
101 FORMAT(9I5)
DO 200 J=1,9
KM=KEYM(I)
NR=N(I)
JK=K*(KM-1)+L
DO 300 IR=1,NR
J=(IMAXH(JK)+2*(ICN*(IR-1)+(IC-1)))/2
300 Z(IR)=IMAXRH(J)
WRITE(NUNIT,102) (Z(M),M=1,NR)
102 FORMAT(16I5)
200 CONTINUE
RETURN
END

```

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```

SUBROUTINE EXPON(FX,IX,I1,I2,I3,I4)
R=RANDU(I1)
X=(-FX)*ALOG(R)
IX=X+0.5
RETURN
END

```

```

FUNCTION RANDU(IX)
IY=IX*65539
IF(IY)5,6,6
5 IY=IY+2147483647+1
6 RANDU=IY
RANDU=RANDU*.4656613E-9
IX=IY
RETURN

```

END

```

SUBROUTINE SUBNR1(K,P,NX,I1,I2,I3)
IMPLICIT REAL*8(D)
REAL K
DP=P
DK=K
R=RANDU(I1)
DR=R
I=0
X=0.0
DCUM=DP**DK
DY=DCUM
IF (DCUM .LT. DR) GO TO 10
NX=(X+0.5)
RETURN
10 I=I+1
X=1.0*I

DX=X
DY=DY*((DK+DX-1.000)/DX)
DCUM=DCUM+DY*(1.000-DP)**DX
IF (DCUM .LT. DR) GO TO 10
NX=(X+0.5)
RETURN
END

```

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```

SUBROUTINE GAPM(A1,A2,N,Y,I1,I2)
  CII=6.0
  CL=3.1

  XM=A1+A2*N
  IF(A2 .LE. 0.) GO TO 1
  SD=(CII-XM)/2.5758
  GO TO 2
1 SD=(YM-CL)/2.5758

2 X=RNORM(I1)
  XX=(X*SD)+XM
  IF(XX .LE. 0.) XX=0.
  Y=EXP(XX)
  RETURN
END

```

```

5 FUNCTION RNORM(IX)
  SIIM=0.0
  DO 5 I=1,12
  SIIM=SIIM+RANDU(IX)
  RNORM=SIIM-6.0
  RETURN
END

```

```

SUBROUTINE MSSG(P1,P2,N,NTIME,I1,I2)
  CALL SURGAM(P1,P2,X,I1,I2)
  NTIME=X+0.5
  IF (NTIME .LT. 1) NTIME=1

  RETURN
END

```

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SUBROUTINE SURGAM(K,A,X,I3,I4)

REAL K

THIS SUBROUTINE GENERATES RANDOM VARIATES FROM A GAMMA DSN. WITH
GENERAL FORM

$$f(x) = \frac{A^k x^{k-1} \exp(-Ax)}{\Gamma(k)}$$

WHEN K IS AN INTEGER, THE GAMMA DSN. IS EQUIVALENT TO THE ERLANG
DSN., WHICH ARISES AS A SUM OF K EXPONENTIAL VARIATES WITH EXPECTED VALUE
1/A. THEREFORE, THE ERLANG VARIATE X IS EQUAL TO 1/A TIMES THE LOG OF THE
PRODUCT OF K RANDOM VARIATES FROM A UNIFORM (0,1) DSN..

WHEN K IS NOT AN INTEGER, AN APPROXIMATE TECHNIQUE MUST BE USED.
LET $K=M+Q$ WHERE M IS THE SMALLEST INTEGER CONTAINED IN K AND Q IS THE
REMAINDER. SINCE THE EXPECTED VALUE, VARIANCE, AND THIRD CENTRAL MOMENT
OF GAMMA VARIATES ARE LINEAR FUNCTIONS OF K, AN APPROXIMATE TECHNIQUE
FOR GENERATING GAMMA VARIATES WITH PARAMETER K IS TO GENERATE A MIXTURE
OF GAMMA VARIATES, CHOOSING M WITH PROR. (1-Q) AND M+1 WITH PROR. Q.
THE APPROXIMATION IMPROVES WITH INCREASING K.

REF. - 'COMPUTER SIMULATION TECHNIQUES'
NAYLOR, BALINTFY, BURDICK, & CHU
JOHN WILEY & SONS, 1966 PP. 87-90

M1=K
M2=M1+1
Q=K-FLOAT(M1)
KK=M1
IF (Q.EQ. 0.0) GO TO 10
T=RANU(I3)
IF (T.LE. Q) KK=M2
10 TR=1.0
DO 20 I=1, KK
R=RANU(I4)
20 TR=TR*R
X=(-ALOG(TR))/A
RETURN
END

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```

SUBROUTINE TIME5(IVALUE, ISAVFF, ISAVEH, IFAC, ISTO, FSTO, IOUF,
* FOUF, ILOG, ITAR, FTAR, IUUF, IUUFF, FIUSE, IMAX, IMAXR, IMAXH, IMAXRH,
* FSAVEL, IMAXL, FMAXRL)
INTEGER*2 ISAVEH, ILOG, IUUF, IMAXBH
REAL*8 FOUF, FIUSE
REAL*4 FSTO, FSAVEL, FMAXRL
DIMENSION IVALUE(6), ISAVFF(2), ISAVEH(2), IFAC(2), ISTO(2), FSTO(2),
* IOUF(2), FOUF(2), ILOG(2), ITAR(2), FTAR(2), IUUF(2), IUUFF(2), FIUSE(2),
* IMAX(2), IMAXR(2), IMAXH(2), IMAXRH(2), FSAVEL(2), IMAXL(2), FMAXRL(2)
INTEGER Z(600), KEY(3)/1,10,12/
NUMIT=ISAVEH(1)
IC=COLUMN NUMBER
IR=ROW NUMBER
IC=1
ICN=1
K=6
L=1
DO 200 IJK=1,3
N=KEY(IJK)
JK=K*(N-1)+L
DO 100 IR=1,600
J=(IMAXH(JK)+2*(ICN*(IR-1)+(IC-1)))/2
100 Z(IR)=IMAXRH(J)
WRITE(NUMIT,101) Z
101 FORMAT(40I2)
200 CONTINUE
RETURN
END

```

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