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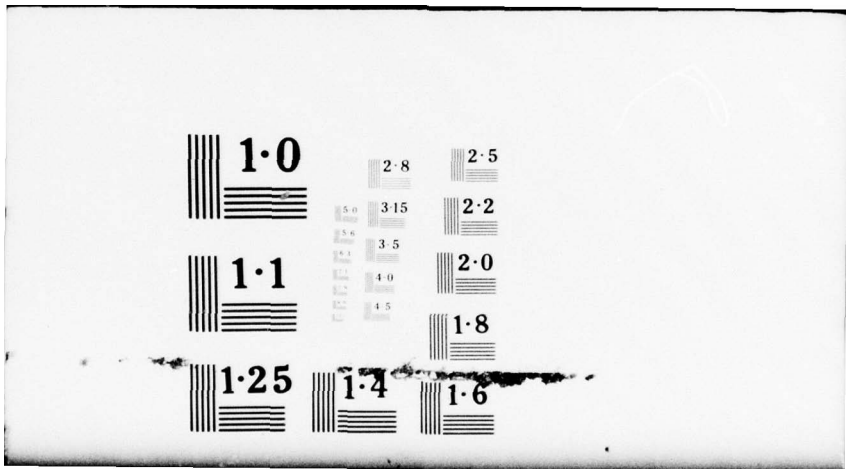
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RF Project 4447
Final Report

the ohio state university
research foundation
1314 kinnear road
columbus, ohio
43212

WORKSHOP ON
VISCOUS INTERACTION AND BOUNDARY-LAYER SEPARATION

Sponsors:

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The Ohio State University

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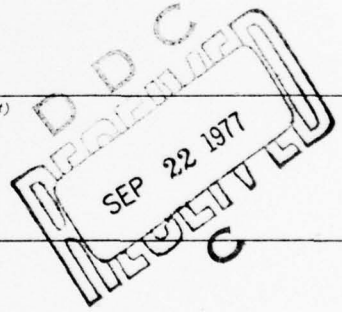

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TRIPLE-DECK

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A technical workshop on viscous interaction and boundary-layer separation was held at The Ohio State University on 16-17 August 1976. The purpose of the workshop was to assess the current state of progress and to stimulate further efforts toward the goal of practical and reliable analysis of viscous flows at high Reynolds numbers. For definiteness, the scope of the workshop was limited to theoretical aspects of the subject, with emphasis placed on fundamental ideas and problem areas rather than on details of computations and results. Only abstracts of the papers were requested in order to promote informality

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and a free interchange of ideas. Each participant was given twenty minutes for a presentation on recent research efforts (in most cases work in progress was reported), followed by approximately ten minutes of lively discussion. The workshop was opened by the keynote lecture presented by Professor Keith Stewartson on "Sychev's Theory of Separation." In addition, an evening session was devoted to unstructured discussion.

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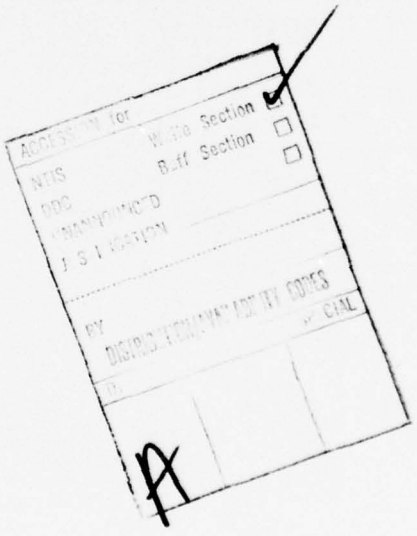

PREFACE

A technical workshop on viscous interaction and boundary-layer separation was held at The Ohio State University on August 16-17, 1976. The purpose of the workshop was to assess the current state of progress and to stimulate further efforts toward the goal of practical and reliable analysis of viscous flows at high Reynolds numbers. For definiteness, the scope of the workshop was limited to theoretical aspects of the subject, with emphasis placed on fundamental ideas and problem areas rather than on details of computations and results.

Only abstracts of the papers were requested in order to promote informality and a free interchange of ideas. Each participant was given twenty minutes for a presentation on recent research efforts (in most cases work in progress was reported), followed by approximately ten minutes of lively discussion. The workshop was opened by the keynote lecture presented by Professor Keith Stewartson on 'Sychev's Theory of Separation'. In addition, an evening session was devoted to unstructured discussion.

Acknowledgement is due Capt. Scott McRae, AFFDL, and Lt. Col. Robert Smith, AFOSR, for their assistance in obtaining the funds that made the workshop possible.

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INTRODUCTION

The collected abstracts of the workshop lectures are presented in these Proceedings. In addition, the Organizing Committee thought it worthwhile to delineate some of the more significant points of the lectures and discussions. These fall naturally into several more or less distinct groupings:

--- Numerical problems;
--- Separation bubbles;
--- Breakaway separation;
--- Trailing edges (steady and unsteady);
--- Tubes and channels;
--- Three-dimensional interactions.

A recapitulation of the main points brought out in the discussion follows under each of the above group headings. It may be remarked that some of this material is controversial even within the Organizing Committee.

Numerical Problems

Approximate analytical theories are not as useful to the engineer as formerly because of the near universal availability of large scale computers and attendant software for solving problems in fluid dynamics. Nevertheless, numerical methods are not without problems, and anlytical solutions are useful not only in understanding the flow processes but also as a guide to resolving the numerical difficulties.

The numerical problems which one encounters in high Reynolds number separated flows fall into several categories. First, the asymptotic analysis has revealed that at high Reynolds number, severe scaling problems exist at and downstream of separation. The scalings indicated by the asymptotic analysis have generally not been honored in Navier-Stokes solution methods to date and therefore there is some doubt about the accuracy of many such solutions and the reliability of the method. None-the-less, some Navier-Stokes solutions have shown good agreement with available experimental data, even though no direct effort was made to honor the asymptotic scale laws. These scalings have been honored in some boundary-layer-like calculations and the results of these calculations tend to confirm the correctness of the triple-deck analysis.

Second, now that the nature of interaction and separation is more clearly understood, it would appear that this knowledge would be useful in formulating numerical techniques. This has been done in boundary-layer solution methods, but little has been done in this direction in terms of solution of the Navier-Stokes equations.

Third, it would appear that significant improvements can still be made in the computational convergence rate of numerical methods for separated flows even when the flow is governed by the simplest equations possible, the lower deck equations. In the subsonic case, for example, the convergence of present methods is extremely slow and requires severe under-relaxation. Some mixed inverse-direct iteration methods show promise in this area. At the same time it would appear that work still needs to be done on developing better difference and switching methods for reverse-flow regions.

Finally, we still need to resolve many remaining questions about whether implicit or explicit, conservative or non-conservative, splitting or non-splitting, methods are most efficient for solving the Navier-Stokes equations at high Reynolds number. For example, some believe that a problem can be solved most efficiently with totally implicit methods while others believe that the viscous portion of the flow should be done with an implicit method and the inviscid portion done with an explicit method.

Closed Separation Bubbles

It appears that a reliable theoretical model is now available for the case of small two-dimensional laminar separation bubbles in steady flow. This model is based on the new asymptotic limit solution obtained using a three-deck model of the interaction/separation process. It remains now to seek ways of solving the resulting governing equations as efficiently as possible.

The triple-deck model provides a sound theoretical basis for the use of the interacting boundary-layer equations which can be viewed as a single set of governing equations uniformly valid across the bottom two decks of the triple-deck model. Evidence to support the use of the interacting boundary-layer approach for steady laminar flows continues to amass for a wide variety of flow configurations and flight regimes. The extension of this approach to the turbulent flow case has shown an equal level of success although, as yet, a helpful asymptotic model has not been developed.

The principle difficulties remaining for closed separation bubbles are those concerned with either developing better methods of solving the currently available model equations, or extending the present models to increase their utility. The following is a list of some of the problems in the first category: (a) solution of the highly nonlinear displacement-interaction equations presently requires use of inefficient local interaction techniques; (b) the global interaction effect (downstream to upstream) of subsonic flows currently involves slow iteration techniques; (c) large separation bubbles develop an inviscid core for which current methods are inefficient; (d) the penetration of shock waves deep into separating supersonic turbulent and hypersonic laminar boundary layers obliterates numerical accuracy in current approaches; and (e) the scale of separation processes are very small and difficult to resolve with current techniques.

For the second category of problems, those involving analytical extension of current models, there are currently three problems that need attention; (a) the inclusion of imbedded shock waves in the supersonic/hypersonic separation model; (b) the extension of asymptotic concepts to the turbulent separation problem; and (c) the development of an extended limit solution for the supersonic separation case to allow application over a more practical range of parameters.

Breakaway Separation

Bluff bodies and airfoils at high angle of attack are characterized by breakaway separation for which the flow does not reattach on the body. The problem is an old one in the literature of fluid mechanics, and the paradoxes associated with the classical Helmholz-Kirchhoff free-streamline theory are well known.* Briefly, the inviscid theory requires the separation point to occur in a region of favorable pressure gradient on a smooth bluff body, contradicting the ideas of classical boundary-layer theory. The paradox appears to be resolved by Sychev's triple-deck theory involving interaction between the viscous and inviscid fluid layers; in this model the triple-deck region with adverse pressure gradient for separation is embedded in the potential flow region of favorable pressure gradient. Up to the time of the workshop, it was uncertain as to whether Sychev's theory is well-founded or not, since no solution of his equations had yet been obtained. However, one speaker did report on progress toward a solution with results suggesting that a solution exists and is unique (F.T. Smith; work in progress, not described in abstract).

It is clear that more theoretical work is needed along these lines; for example, treatment of the case of the turbulent boundary layer, and inclusion of unsteady effects that occur at large Reynolds number even in the laminar case. Nevertheless, certain properties of the present theory may be useful in guiding numerical analyses of bluff-body flows. In particular, the pressure distribution involves a rather rapid rise in the immediate vicinity of the separation point, requiring a rather fine mesh to reproduce by finite differences, together with a slow decay over a large streamwise distance to the "outer" inviscid flow both upstream and downstream, thus requiring the numerical mesh to extend over a large region about the separation point. These opposing trends clarify the well-known difficulty of computing accurate solutions of bluff-body flows at large Reynolds number.

As a final point, there is suggested the possibility of incorporating these triple-deck concepts into ad hoc analyses of, for example, separated wakes of airfoils at high angle of attack.

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* See for example, G. Birkhoff, Hydrodynamics, a Study in Logic, Fact, and Similitude, Princeton University Press (1950).

Trailing Edges

Viscous interaction at a trailing edge results from the abrupt change-over from the no-slip condition on the airfoil surface to the wake-symmetry condition downstream. For the flat plate in incompressible flow at zero incidence, the pressure falls and the skin friction rises (to a value nearly 35% greater than the Blasius value) as the flow is accelerated toward the trailing edge. Downstream the pressure rises very rapidly, overshoots the freestream value, and then decays very slowly over a large distance. These results from the triple-deck theory have strong implications when choosing mesh distributions for solving the Navier-Stokes equations numerically. An even finer substructure than the triple deck was described at the workshop; however, this flow structure is passively driven by that in the triple-deck and consequently does not alter the larger scale properties.

The problem of trailing-edge flow on an airfoil with angle-of-attack has now been treated in the framework of triple-deck theory, both for subsonic and supersonic freestream. The most obvious practical application is the reduction of lift due to viscous interaction through weakening of the Kutta condition. The flow has been computed for angle-of-attack up to the point of trailing-edge separation. For larger angles the flow structure is not known. Obviously more work is needed on this problem.

For turbulent flow, a three-deck asymptotic flow structure exists in the limit of infinite Reynolds number, somewhat like the laminar triple-deck, but different in that the streamwise extent is short compared with the boundary-layer thickness. Experiments were reported in which separation bubbles were observed as short as 20% of the boundary-layer thickness. This short length scale, over which large pressure changes occur, again imposes severe restrictions on a turbulent "Navier-Stokes" computation. As for the laminar case, the flow structure beyond the stall point is not yet known.

The unsteady flow structure was outlined for oscillatory motion. The dominant effect is the unsteady triple-deck at the trailing edge, which produces unsteady contributions to the lift and pitching moment having phase shifts of 45 degrees and 90 degrees. Such phase shifts could have a significant influence on flutter boundaries. Much more work is needed in this area; for example, numerical solutions of the unsteady triple-deck equations have not yet been obtained.

Tubes and Channels

Numerical solutions to parabolized tube and channel flow equations have indicated the possibility of branching behavior in the numerical solutions as the streamwise step size is decreased. This is analagous to the behavior which has been previously found in supersonic interacting boundary layers on flat plates for example.

It has been found that as in the case of an external flow, a triple-deck analysis is applicable to channel and tube flows, but the expansions are different. The asymptotic analysis reveals an upstream influence and therefore provides a mechanism for the branching behavior observed numerically. More work needs to be done to demonstrate the connection between the asymptotic theory and the observed branching behavior of the numerical solutions.

For curved tubes the far downstream solution is not known. A possible formulation of the appropriate asymptotic model and a status report on the numerical solution of the resulting equations was given.

Three-dimensional Interactions

Almost all work in viscous-interaction has been carrried out on two-dimensional flow problems (we include axi-symmetric flows in this category). Indeed, all but two of the abstracts presented for this workshop were concerned with two-dimensional flows. A three-dimensional version of the triple-deck theory was described and, as one application, similarity laws were derived for pressure and shear-stress distributions on simple swept configurations. Preliminary results were presented as well for more general three-dimensional interacting flows. Clearly this field contains a wide variety of problems for future applications.

PROGRAM

WORKSHOP ON VISCOUS INTERACTION AND
BOUNDARY-LAYER SEPARATION

Monday Morning (August 16)

Session 1 (8:30 A.M.)
- K. Stewartson - Sychev Theory of Separation.
- S. Weinbaum - A New Pressure Hypothesis and Approximate Theory for Streaming Motion Past Bluff Bodies at Re from $O(1)$ to $O(10^2)$.
- J. Carter - An Inverse Technique for Viscous-inviscid Interaction.

Coffee Break (10:05 A.M.)

Session 2 (10:20 A.M.)
- R. MacCormack - An Efficient Numerical Method for Solving Time-dependent Compressible Navier-Stokes Equations at High Reynolds Number.
- J. Rom - The Calculation Methods for Flows with Strong Interactions Between Viscous and Inviscid Regions.
- T. Adamson - Shock-wave-turbulent Boundary Layer Interaction at Incipient Separation.
- A. Klüwick - Freely Interacting Transonic Boundary Layers.

Lunch (12:30 P.M.)

Monday Afternoon (August 16)

Session 1 (2:00 P.M.)
- N. Riley - Flow Separation from a Smooth Surface in a Slender Conical Flow.
- F. Blottner - Viscous Slender Channel Flows.
- J.-C. Le Balleur - Viscous Interaction Including Turbulent Boundary Layer Separation in Two-Dimensional Flow.
- P. Roache - Difficulties with Brute-force Numerical Solutions of Navier-Stokes Equations for High - Re Separated Flows.

Coffee Break (3:35 P.M.)

Session 2 (3:50 P.M.)
- R. Melnik - Turbulent Interaction at Trailing Edges.
- S. Brown - Free Interactions in Unsteady Boundary Layers
- M. Werle - Numerical Studies of High Reynolds Number Laminar Separated Flow.
- G. Kuhn - Calculation of Axisymmetric Separated Turbulent Boundary Layers.

Dinner (7:00 P.M.)

Tuesday Morning (August 17)

Session 1 (8:30 A.M.)

S. Dennis - Boundary-layer Calculations for a Circular Cylinder.

F. Smith - Tube-flows: On the Effects of Constrictions, Branchings, Upstream Influence and Separation.

W. Hankey - A Controlling Factor in the Establishment of the Size of the Separation Region.

A. van de Vooren - Viscous Flow Near the Trailing Edge of a Flat Plate.

Coffee Break (10:05 A.M.)

Session 2 (10:20 A.M.)

H. K. Cheng - Study on Unsteady Trailing-edge Stall.

H. McDonald - Viscous Interaction and Separation.

P. Williams - Reversed Flow in Boundary Layers with Prescribed Displacement Thickness.

R. Ackerberg - Separation at a Free Streamline.

Lunch (12:30 P.M.)

Tuesday Afternoon (August 17)

Session 1 (2:00 P.M.)

T. Cebeci - Separated Flows and Their Representation by Boundary-Layer Equations.

M. Holden - Experimental Investigation of Current Problems Involving Boundary Layer Separation.

G. Inger - Upstream Influence in Interacting Non-separated Turbulent Boundary Layers.

S. Dennis and N. Riley - Flow in a Curved Pipe at High Dean Number.

O. Burggraf - The Three-dimensional Triple Deck.

S. Rubin - Viscous Interactions in Three Dimensions. (paper not presented due to illness.)

SEPARATION AT A FREE STREAMLINE

R.C. Ackerberg

Polytechnic Institute of New York
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Brooklyn, New York 11201

ABSTRACT

This talk will deal with the separation that occurs in the neighborhood of a sharp trailing edge to which a free streamline is attached. Previous results [1,2,3] will be summarized, and some recent work dealing with large capillary effects will be discussed.

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1. Ackerberg, R.C., "Boundary-layer separation at a free streamline. Part 1. Two-dimensional flow," J. Fluid Mech., 44, pp. 211-225, 1970.

2. Ackerberg, R.C., "Boundary-layer separation at a free streamline. Part 3. Axisymmetric flow and the flow downstream of separation," *J. Fluid Mech.*, 59, pp. 645-663, 1973.

3. Ackerberg, R.C., "The effects of capillarity on free streamline separation," J. Fluid Mech., 70, pp. 333-352, 1975.

SHOCK WAVE-TURBULENT BOUNDARY LAYER INTERACTION AT INCIPIENT SEPARATION

T. C. Adamson, Jr.

The University of Michigan

ABSTRACT

Asymptotic methods are used to describe the structure of the interaction region formed when a shock wave, in transonic flow, impinges upon a turbulent flat plate boundary layer. The case chosen for study is that of incipient separation. In a previous paper[1] it was shown that just as in the weaker shock cases,[2,3] the interaction region in this strong shock case could be divided into regions such that the flow in the outer region (in the direction normal to the wall) is governed by inviscid flow equations, while in the near wall regions (Reynolds stress sublayer and wall layer) Reynolds and viscous stresses must be taken into account. Unlike the weak shock cases, there are also two regions in the flow direction, one upstream (inner) region with characteristic length ordered by the extent of the upstream influence and thus depending upon the distance from the wall to the undisturbed sonic line, and one downstream (outer) region with characteristic length depending upon the boundary layer thickness. Because the flow is unseparated, it is possible to find solutions in the inviscid flow regions independently of the near wall regions, and these solutions lead to a relation for the wall pressure distribution. The terms in the wall pressure solution are then used in the wall layer solutions to find the relation for the wall shear stress.

The structure of the near wall regions and the derivation of the relation for the wall shear stress, τ_w, are reviewed. The dependence of τ_w upon the closure model employed is discussed as is the variation of τ_w with distance along the wall for both oblique and normal impinging shock waves. The use of the expression for τ_w in forming a separation criterion is illustrated and the problems associated with such a calculation are discussed.

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1. Adamson, Jr., T.C., "Normal Shock Wave-Turbulent Boundary Layer Interactions in Transonic Flow Near Separation," Workshop on Transonic Problems in Turbomachinery, Monterey, California, Feb. 11-13, 1976. Proceedings to be published.

2. Adamson, Jr., T.C. and Feo, A., "Interaction Between a Shock Wave and a Turbulent Boundary Layer in Transonic Flow," SIAM J. Appl. Math., 29, pp. 121-145, 1975.

3. Melnik, R.E. and Grossman, B., "Analysis of the Interaction of a Weak Normal Shock Wave with a Turbulent Boundary Layer," AIAA Paper No. 74-598, 1974.

VISCOUS INTERACTION INCLUDING TURBULENT BOUNDARY LAYER SEPARATION IN TWO DIMENSIONAL FLOWS

by Jean-Claude Le Balleur
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Abstract -

A study is developped to provide a numerical prediction of steady two-dimensional flows with strong viscous interaction, by meansof a matched viscid-inviscid approximation.

In a conventional way, Prandtl equations are used to describe the turbulent boundary layer. Recent developments on laminar separation allow to think that such a simplification is consistent with incipient separation or small separated bubbles. Because of practical considerations, equations are solved by a moment integral method.

As usual for these calculations, the matching consists in searching common boundary conditions at the outer edge of the dissipative layer, thus including both ordinary boundary layer and self-induced separation. Slip condition on displacement surface, or injection condition at the wall are still equivalent, and the more convenient is used.

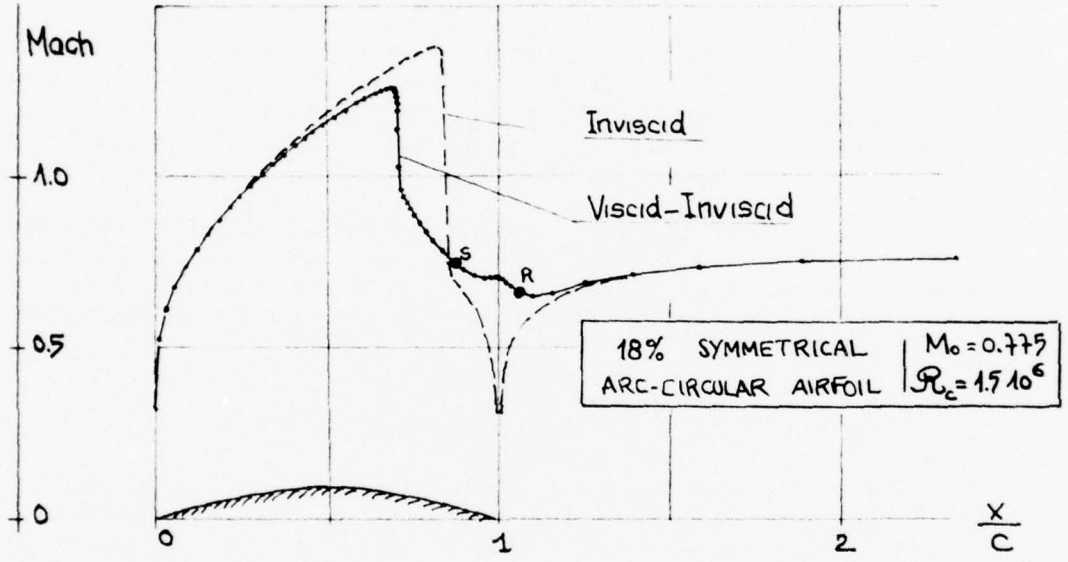
Because of the boundary value nature of the matched problem, emphasis has been given to iterative processes where repeated inviscid and boundary layer calculations are performed.

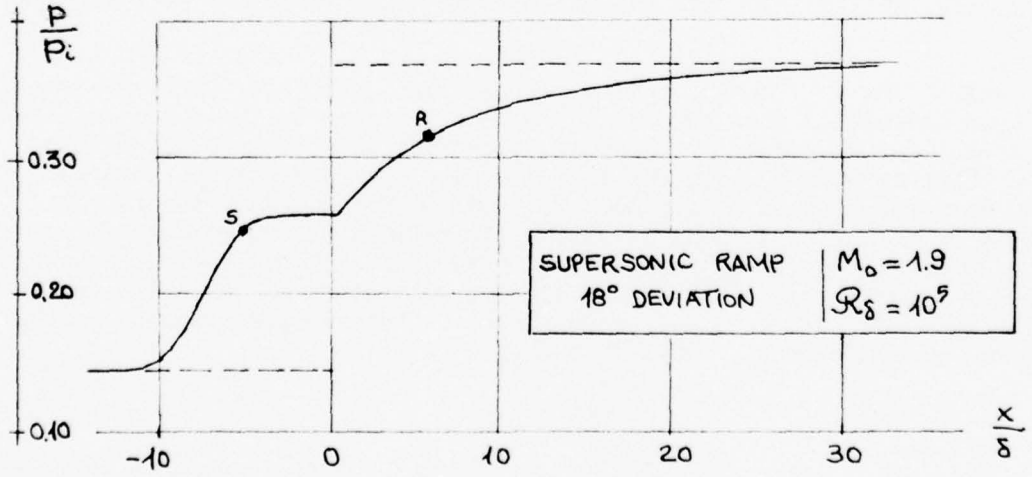
A linear analysis first explains that the classical iteration on the displacement thickness often diverges, but however provides a suitable method if a locally optimal underrelaxation is added. It appears, secondly, for supersonic regime, that the method correctly includes the propagation of upstream influence as long as the boundary layer is subcritical. It is then possible, as in the Werle and Vatsa method, to prescribe a downstream pressure condition, or to respect automatically a downstream critical point condition, without the difficulties associated with an ill-posed initial value problem. Thirdly, convergence of the matching process and of the inviscid algorithm may be simultaneously performed to avoid large computing times.

This method failed at separation because of the boundary layer calculation. However, following Klineberg, inverse calculation is possible, and a linear analysis shows that iteration on the pressure may be adequatly underrelaxed to ensure convergence. Despite this, inverse boundary layer calculation is inconvenient for ordinary attached boundary layer, and is singular at the critical points. Consequently inverse matching must be limited to separated regions, and mixed direct-inverse matching is needed.

Complexity and practical numerical difficulties led us to imagine a new method where only the boundary layer is calculated by a direct or inverse method, whereas the inviscid calculation is always a direct one. When separation occurs, the method provides an automatic change of the displacement surface with regard to viscid and inviscid estimations of the pressure, until convergence. As without separation, supersonic upstream influence is taken into account and simultaneous inviscid resolution - matching iteration is possible.

The method seems very general and applies form subsonic to supersonic regime to any inviscid flow. Qualitative experimental features of viscous effects are found for symmetric arc-circular transonic airfoil with a small transonic perturbation inviscid calculation, and for supersonic ramp with Prandtl-Meyer law.





VISCOUS SLENDER CHANNEL FLOWS

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ABSTRACT

The slender channel approximation has been used successfully by several authors to predict the flow for straight nozzle geometries and to obtain good agreement with experiments. A finite-difference scheme and computer code have been developed[1] for solving the slender channel equations in curved channels of varying height. The governing equations are the incompressible boundary layer equations with longitudinal curvature terms included; the velocity components at the wall are zero from the boundary conditions. New independent and dependent variables are introduced into the governing equations which allow similar solutions to be obtained and the computational domain becomes a rectangle. The resulting transformed continuity and two momentum equations are solved with an implicit second-order difference scheme which requires the solution of block tridiagonal difference equations. The solution procedure determines either similar solutions which can be used as initial conditions or nonsimilar solutions of the slender channel equations.

Numerical solution of the flow in a converging channel have been obtained with various step-sizes to verify the accuracy of the code. The flow has also been calculated in a straight expanding channel that is increasing in height for a distance S_T and then is of constant height $H_f = (n_e/n_o)_f$. The nondimensional channel height variation is

$$n_e/n_o = \frac{1}{2}\,(1 + H_f) + \frac{1}{2}\,(1 - H_f)\,\cos(\pi\xi^2/4S_T)$$

where $\xi^2 = 4s/n_o$, s is distance along the channel centerline and n_o is initial channel height. When $S_T = 10$ and $H_f = 2$, there is a small region of reverse flow. The separation is regular and no numerical problems appear to occur. However for more severe separation and a small step-size in the marching direction, numerical difficulties are encountered which can disappear with a larger step-size.

The entry flow in a curved channel of constant centerline curvature $\kappa n_o = 1/6$ has been calculated when the channel height is given by the above expression with $S_T = 6\pi$. For $H_f = 1$ or 1.5 accurate numerical solutions to the flow are obtained. However with $H_f = 2$, the flow does not separate but the solutions oscillate when the step-sizes are made too small. For boundary layers that interact with the inviscid flow, a similar lower limit on the marching step-size is encountered to avoid branching solutions. These oscillations possibly indicate the slender channel equations are not well posed as an initial value problem as multiple solutions may be possible. A better understanding of the significance of these oscillations is needed.

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1. Blottner, F.G., "Numerical Solutions of Slender Channel Laminar Flows," to be published.

FREE INTERACTIONS IN UNSTEADY BOUNDARY LAYERS

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ABSTRACT

The extension of Lighthill's[1] theory of upstream influence to unsteady supersonic boundary layers was made by Schneider[2] who obtained upstream-propagating waves of small amplitude and high frequency. The Stewartson[3]-Messiter[4] triple-deck theory can be extended in a similar way and applied to unsteady aerodynamics for appropriate orders of magnitude of these parameters. The flow about the trailing edge of a supersonic oscillating aerofoil has been discussed[5], and for the subsonic case a consistent structure obtained[6] which verifies the usual assumption of zero loading at the trailing edge to remove the non-uniqueness in the potential flow. A related situation with application to acoustics is the problem of Orszag and Crow[7] where there are three possible Kutta conditions. Daniels[8] concludes that the choice is not determined but presents a theory for high Strouhal numbers which shows good agreement with the experiments of Bechert and Pfizenmaier[9].

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THE THREE-DIMENSIONAL TRIPLE DECK

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ABSTRACT

When combined with a viscous interaction condition coupling the pressure to the displacement thickness, the boundary-layer equations appear to be an accurate model for certain types of separated flows at moderate Reynolds numbers. Examples of the success of these methods are given by Lees and Reeves[1] and Nielsen.[2] For large Reynolds number, the boundary layer with viscous interaction subdivides itself into the substructure that Stewartson[3] has named the triple deck. Numerous cases of two dimensional triple-deck solutions have been worked out, including those of Jenson, Burggraf, and Rizzetta,[4] and of Rizzetta[5] for the case of a compression ramp with a supersonic mainstream. The theory is here extended to three-dimensional obstacles with supersonic external flow. Applications to swept cylindrical obstacles and to ramps with triangular cross-section are discussed.

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AN INVERSE TECHNIQUE FOR VISCOUS-INVISCID INTERACTION

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ABSTRACT

The problem of 2-D laminar and turbulent boundary-layer separation for flows up through the transonic speed range is being studied. The boundary-layer equations for the viscous flow and the small disturbance potential equation for the inviscid flow are solved by novel techniques which are specifically designed to compute separated flows with strong viscous-inviscid interaction. This procedure is approximate but should provide reasonably accurate results for small separated regions, similar to that found in supersonic viscous-inviscid interactions.[1] Despite the notorious difficulty in numerically matching boundary-layer and inviscid solutions, it is thought that the present approach, if successful, is preferable to a solution of the full Navier-Stokes equations due to lower computer costs. However, it must be remembered that ultimately the success of either of these approaches depends on our capability to model the turbulent flow in regions of large adverse pressure gradient including separation. This topic will not be discussed further here since the present effort, so far, has been directed to the mathematical/numerical difficulties of iteratively solving the boundary-layer and inviscid flow equations.

A regular boundary-layer solution is obtained at the separation point by prescribing the displacement thickness as an input condition to the boundary-layer equations; the pressure is deduced from the resulting solution. Catherall and Mangler[2] were the first to use this approach; later, Carter[3] made a detailed numerical study of this inverse approach using a somewhat different formulation. In this formulation the vorticity, u-component of velocity, and a perturbation stream function are the dependent variables. The perturbation stream function is defined by $\tilde{\psi} = \psi - u\,(y - \delta^*)$ and thus by requiring $\tilde{\psi} \to 0$ as $y \to \infty$ the prescribed displacement thickness controls the mass flow in the boundary layer. Recently a new formulation of the boundary-layer equations for a prescribed displacement thickness has been developed which is simpler since it requires only two dependent variables, u and $\tilde{\psi}$. In making comparisons with turbulent experimental data, it is simpler to work with the u-component of velocity instead of the vorticity, since the latter generally is not measured and must be deduced by differentiating the velocity profile. The streamwise momentum equation is now used in place of the vorticity transport equation which was used in the earlier formulation. The Crank-Nicholson finite-difference scheme, modified by the Reyhner-Flügge-Lotz approximation in regions of reversed flow, is applied to these equations with Newton linearization used to deduce the coupled linear algebraic equations. The unknown pressure gradient is deduced implicitly by a simple modification to the Thomas algorithm for block tridiagonal systems. Alternately, this formulation can be solved in the usual direct approach with the edge velocity, u_e, prescribed instead of the condition $\tilde{\psi} = 0$ at the boundary-layer edge.

An outline of this new inverse boundary-layer procedure will be presented along with comparisons to the turbulent boundary-layer measurements made by Chu and Young near the separation point. This new procedure converges quadratically in 3 or 4 iterations at each streamwise location for laminar flow

and in less than 10 iterations for turbulent flow when the maximum change in either of the dependent variables is required to be less than 10^{-5} between 2 successive iterations. A partial Newton linearization on the inner eddy viscosity law was found to increase the convergence rate in the turbulent calculations.

This inverse boundary-layer technique, suitably modified for compressibility by the Stewartson transformation, is currently being combined with an inverse inviscid program developed by Carlson.[4] This inviscid computation solves the small disturbance potential equation for transonic flow for a prescribed surface pressure distribution which is provided by the inverse boundary-layer solution. The inviscid solution then gives the slope of displacement body which is integrated and combined with the actual surface ordinates to yield the displacement thickness for the input condition to next boundary-layer computation. The form of this inverse-inverse iterative procedure is identical to that presented by Carter and Wornom[5] which was used successfully to solve a low-speed laminar separation problem. In that computation it was found that no numerical smoothing was required as is typically used in matching boundary-layer and inviscid flows. This result occurs because classical matching requires numerical differentiation, whereas the present procedure uses numerical integration of the computed inviscid and boundary-layer results.

At the present time, Carlson's program, now modified to include solid wind-tunnel walls, is being used to compute the inviscid, transonic flow over the bump on the lower tunnel wall which was measured by Alber, et al.[6] The experimental pressure is used as the input condition and it is found that the computed displacement thickness agrees fairly well with that measured by Alber, et al. Further calculations are required, though, as it has been observed that the deduced displacement thickness on the bump on the lower tunnel wall is very sensitive to the slope of the upper tunnel wall. Hence, at the present time it appears that it will be necessary to include, in the usual fashion, the boundary-layer displacement thickness on the upper tunnel wall. For the workshop the current status of the application of this inverse interaction technique will be presented.

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SEPARATED FLOWS AND THEIR REPRESENTATION BY BOUNDARY-LAYER EQUATIONS*

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ABSTRACT

The paper describes two separate approaches for calculating laminar and turbulent boundary layers with positive and/or negative wall shear. The first approach, which we call the "nonlinear eigenvalue method," treats the pressure as an eigenvalue parameter and works with either physical or transformed variables. The second approach treats the pressure as an unknown function and seeks the solution of the governing equations in physical variables. This approach, which we call the "Mechul" function scheme, was used earlier for the Falkner-Skan equation by Cebeci and Keller. Our studies show that, while the first approach only works for flows with positive wall shear, the second approach works extremely well for flows with either positive or negative wall shear. This abstract concentrates on the second approach.

The Mechul method is relevant to the prediction of separation and the calculation of the flow properties within separated flow regions. It has been developed specifically to allow the determination of flow properties in the vicinity of the suction side of airfoils. Boundary-layer equations are solved and are appropriate upstream of separation and more approximate in the separated flow. As is demonstrated, however, when used in the manner described in this paper and outlined below, their solution can lead to results which are sufficiently precise for design purposes and require very short computer times and little storage. The method also allows the testing of various turbulence models for separated flows.

A brief description of the Mechul function method for specified displacement thickness follows. With the concept of eddy viscosity, ε_m, and stream function, ψ, the boundary-layer equations for two-dimensional laminar and turbulent flows can be written in the form,

$$(b\psi'')' - p_x = \psi' \frac{\partial \psi'}{\partial x} - \psi'' \frac{\partial \psi}{\partial x} \tag{1}$$

where primes denote differentiation with respect to y and $b = 1 + \varepsilon_m/\nu$. Eq. (1) is subject to the boundary conditions

$$y = 0 ; \qquad \psi = \psi' = 0 \tag{2a}$$

$$y = y_e ; \qquad \psi' = u_e , \qquad \psi_e = u_e[y_e - \delta^*(x)] \tag{2b}$$

The pressure p is treated as an unknown and equation (1) is written as the first-order system,

$$\psi' = u \tag{3a}$$

$$u' = v \tag{3b}$$

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$$p' = 0 \tag{3c}$$

$$(bv)' - p_x = u \frac{\partial u}{\partial x} - v \frac{\partial \psi}{\partial x} \tag{3d}$$

Similarly, eqs. (2) become

$$y = 0 \; ; \qquad \psi = u = 0 \tag{4a}$$

$$y = y_e ; \qquad u = u_e \; , \qquad \psi_e = u_e[y_e - \delta^*(x)] \tag{4b}$$

The solution of the system (3) and (4) is obtained with Keller's two-point finite-difference method. In the region of negative longitudinal velocity, the FLARE approximation discussed by P. G. Williams was used.

Several flows, which allow the displacement thickness distribution to be specified, have been calculated with this method. These include the laminar separated flows of P. G. Williams, J. E. Carter, and R. Briley; in the last case, it was possible to make comparison with solutions of the Navier-Stokes equations and the agreement is shown to be excellent. The experimentally investigated flow of R. Simpson, et al.was also calculated and a comparison of the laser-annemometer measurements with the present results is presented and shown to be within the limits of experimental uncertainty.

STUDY ON UNSTEADY TRAILING-EDGE STALL

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ABSTRACT

The asymptotic theory of inviscid-viscous interaction near a sharp trailing edge is studied from the viewpoint of aquatic animal propulsion. Development of thrust on the caudal fin (as well as on a bird's wing) depends largely on the (unsteady) Kutta condition, aside from an unseparated smooth flow around the leading edge. The departure from the ideal Kutta condition (of vanishing pressure jump) at the edge at high and low (reduced) oscillating frequencies is of great interest.

The theory for the case corresponding to a flat plate at incidence α^* has been set forth by Brown and Stewartson[1], adopting the basic three-deck structure of Ref. 2 around the trailing edge (T. E.). The theory, as a self-consistent asymptotic analysis, holds as long as

$$\alpha' = O(\epsilon^{1/2}) , \tag{1}$$

where $\epsilon \equiv R_e^{-1/8} \ll 1$. An extension to the case of a rapidly oscillating plate has been made by Brown and Daniels[3]. To match the three-deck structure, a Stokes wall layer and two fore decks are introduced in Ref. 3. Whereas interesting viscous correction to the circulation has been determined (estimated) as in Ref. 1 for the steady case, requirements on α^* and the frequency for the model appear to be rather severe. Namely,

$$\left.\begin{aligned} \alpha^* &= O(\epsilon^{\frac{9}{2}}) , \\ S &\equiv \frac{\omega c}{U} = O(\epsilon^{-2}) \end{aligned}\right\} \tag{2}$$

We note that, in most problems of aero / hydromechanics, α^* must be greater than the main-deck thickness $\delta/L = O(\epsilon^4)$, and that the range $S = O(1)$ is of great practical interest. The work in Ref. 3 nevertheless brings out much insight to the problems and furnishes a valuable guide to the analysis for other domains of α^* and S .

The study on the unsteady trailing flow to be carried out will focus on ranges larger amplitude and lower frequency, especially, domains with

$$\left.\begin{aligned} \epsilon^4 < \alpha^* &\ll 1 , \\ S &= O(1) . \end{aligned}\right\} \tag{3}$$

One of these domains, namely,

$$\left.\begin{aligned} \alpha^* &= O(\epsilon^{\frac{1}{2}}) , \\ S &= O(1) , \end{aligned}\right\} \tag{4}$$

appears tractable, for which a structure similar to, but slightly simplier than, Brown and Daniels'[3] appears to be applicable.

It is interesting to examine the high and low frequency effects as a consequence of short^long wave lengths (compared to wing chord). For high frequency ($S \gg 1$), the influence of the periodic wake vorticity averages^itself out (to a weaker level), and the velocity and pressure on the wing (near the T. E.)^ is dominated by the apparent-mass and the quasi-steady effects. This is found to be indeed the case in Ref. 3, in which the wake-vorticity effect does not enter into the analysis, at least for the fore decks. We expect a more pronounced periodic wake influence for the case under condition (4). The presentation will describe a preliminary examination of the asymptotic structure and scaling problems.

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BOUNDARY-LAYER CALCULATIONS FOR A CIRCULAR CYLINDER

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ABSTRACT

In order to solve the two-dimensional laminar boundary-layer equations it is necessary to specify the external pressure distribution at the outer edge of the boundary layer or some equivalent information. It is clear from the tendency of the surface pressure distributions obtained from numerical solutions of the Navier-Stokes equations as the Reynolds number increases that the external pressure distribution given by potential flow is not an appropriate assumption. In the present note some results are presented which have been calculated by integrating the boundary-layer equations for a circular cylinder using pressure distributions obtained from previous numerical solutions of the Navier-Stokes equations. The object is to check the results for consistency with the trend of the numerical solutions for increasing Reynolds number. The work is a first step in an attempt to devise an interaction scheme between the boundary-layer equations and a model of the external flow obtained from the Navier-Stokes equations.

The integrations of the boundary-layer equations have been carried out in three separate cases utilizing pressure distributions taken from the numerical solutions of Dennis & Chang[1] at R = 40, 70 and 100 respectively. Here $R = 2U_\infty a/\nu$, where a is the radius of the cylinder, U_∞ the stream velocity and ν the coefficient of kinematic viscosity. A calculation of a similar nature has previously been given by Dimopoulos & Hanratty[2] for the single case R = 40 utilizing the pressure distribution calculated from a numerical solution of the Navier-Stokes equations by Kawaguti & Jain[3]. The present work is, however, able to establish some notion of a trend of the results with the changing external pressure distribution as R increases. There is found to be a good measure of consistency between the boundary-layer calculations and the numerical solutions of the Navier-Stokes equations of Dennis & Chang.

Some results are given for the variation of local skin friction over the surface of the cylinder and the variation of velocity throughout the boundary layer. There is a striking illustration of the breakdown of the boundary-layer equations near the point of separation. The consistency of the boundary-layer calculations with the numerical solutions of the full Navier-Stokes equations near the front stagnation point is illustrated by the following results. The pressure coefficient at the front stagnation point is given by boundary-layer theory in the form

$$(p_o - p_\infty)/\tfrac{1}{2}\rho U_\infty^2 \sim 1 + \alpha/R,$$

where ρ is the density and p_o, p_∞ the pressures at the front stagnation point and at large distances, respectively. The estimate $\alpha = 6.35$ is obtained from all three boundary-layer solutions, which is not inconsistent with the trend of the estimates $\alpha = 5.38, 5.76, 5.95, 6.00$ obtained respectively at R = 20, 40, 70 and 100 by Dennis & Chang. Finally, if $c_f = \tau_o / \tfrac{1}{2}\rho U_\infty^2$ is the local coefficient of

skin friction at the surface of the cylinder, where τ_o is the local shear stress, the slope of the curve of $R^{\frac{1}{2}} c_f$ at the front stagnation point with respect to angular distance round the cylinder comes out to be 6.97 from all three boundary-layer solutions. This is quite consistent with the estimates 6.59, 6.70 and 6.89 of this quantity given at R = 40, 70 and 100 by Dennis & Chang.

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FLOW IN A CURVED PIPE AT HIGH DEAN NUMBER

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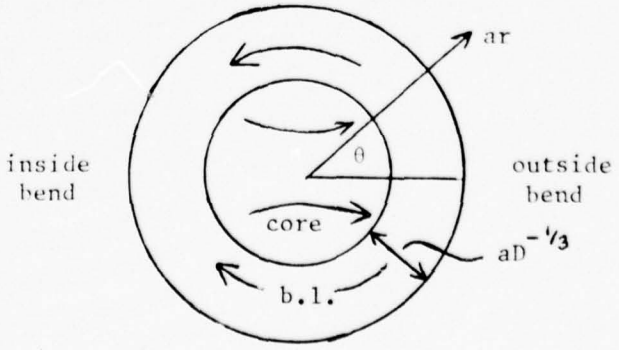
ABSTRACT

The flow down a circular pipe of radius a, coiled in a circle of radius L, under the action of a constant axial pressure gradient G is considered. The first successful analysis of such a flow was presented by Dean[1,2] for the case of small values of the parameter $D = (Ga^2/\mu)(2a^3/\nu^2L)^{\frac{1}{2}}$, where μ, ν represent the viscosity and kinematic viscosity of the fluid respectively, and vanishingly small a/L. Our concern is with the corresponding flow for $D \gg 1$. If, following Collins and Dennis[3], we write the stream function for the (secondary) velocity components in a cross-flow plane as $\Psi = \nu\psi$ and the axial velocity as $W = (\nu^2L/2a^3)^{\frac{1}{2}}w$ then to leading order an inviscid 'core' solution is obtained for which $\psi = D^{1/3} r \sin\theta / w_c'(r\cos\theta)$, $w = D^{2/3} w_c(r\cos\theta)$. The core velocity $w_c(r\cos\theta)$ is not determined at this stage. To determine w_c and satisfy the conditions at r = 1 it is necessary to introduce a boundary layer close to r = 1. Classical arguments indicate that this is of thickness $O(aD^{-1/3})$, which is in accord with numerical results[3] for large but finite D. The core velocity w_c which provides an outer boundary condition for the axial velocity within the boundary layer is itself determined by ensuring continuity of mass flow between the core and boundary layer. In this sense the problem is an interaction problem. By exploiting the mass flow continuity condition, w_c may be formally eliminated from the problem, except that it must be given a prescribed value, w_{c_0} say, at the outside bend $\theta = 0$ (see figure). The asymptotic structure close to $\theta = \pi$ is known, and in principle the solution can be advanced step by step from $\theta = 0$; w_c is determined at each step. The correct value of w_{c_0} is that for which the numerical solution approaches the asymptotic form smoothly as $\theta \to \pi$. Difficulties have been experienced with the above scheme, since although separation of the secondary cross-flow has not been encountered a region of reversed flow close to the outer edge of the boundary layer has. This leads to a breakdown of the forward integration scheme, and an attempt has been made to treat the problem, numerically, as a boundary-value problem. The results which have been obtained are encouraging but, to date, incomplete.



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A CONTROLLING FACTOR IN THE ESTABLISHMENT OF THE SIZE OF THE SEPARATION REGION

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ABSTRACT

Solutions of the complete Navier Stokes equations have been obtained for shock-laminar boundary layer interactions by using MacCormacks finite difference scheme. Turbulent interactions also have been computed with this program by substitution of an eddy viscosity into the equations and treating properties as the time averaged mean values.[1] It was found that a relaxation of the eddy viscosity with a "time constant" of $\lambda \simeq 10\delta$ was necessary to predict the experimentally observed separation length. To further examine the details of this interaction, the total pressure profiles were compared before and after the separation region (Fig. 1). In the outer inviscid region large shock losses are observed, however, an increase in total pressure along the wall is noted. Since static pressure increases through the interaction the total must also rise in that vicinity. To answer the question, "under what conditions can total pressure increase in a viscous region?", the energy equation was examined. Integration of the energy equation along an adiabatic, steady, boundary layer streamline produces the following relationship:

$$\ln \frac{P_{o2}}{P_{o1}} = \int_{x_1}^{x_2} \frac{\tau_y}{\rho} \, dx$$

This integral equation shows that total pressure can be changed according to the sign of τ_y. A local decrease in entropy is possible, however the net total change in entropy must of course increase (in accord with the second law of thermodynamics).

Much can be deduced from the integral relationship concerning the shape of the velocity profile in the vicinity of the recirculation region. For example, the flow just above the dividing streamline experiences an increase in total pressure through the interaction. Therefore τ_y must be positive in some region above the dividing streamline. Also it is obvious that some length $(x_2 - x_1)$ is necessary to provide sufficient time for the viscous forces to act to increase Po.

Examination of similar flows confirms this supposition. Figure 2 snows the locus of the inflection points for similar reversed flows which divide the regions of negative and positive τ_y. Note that a small region does exist above the dividing streamline for which $\tau_y > 0$. Flow in this region will experience an increase in total pressure, while flow above will only undergo a decrease in total pressure.

By assuming locally similar flows it is possible to estimate the extent of laminar separation by employing the previously derived integral relationship for total pressure change. By using a mean value for the integrand an approximation for the length of separation is derived which is similar to the correlation deduced by Ball[2] from six different experimental investigations.

Also the results of the turbulent relaxation model may be analyzed using this integral equation. In reference 1 the separation length was observed to increase dramatically from the equilibrium value when the eddy viscosity was frozen at the upstream value thereby decreasing τ_y near the dividing streamline. One concludes that the length of separation is extremely sensitive to τ_y only in the vicinity of the dividing streamline and therefore the value of the eddy viscosity in other regions is not nearly as important. For example, Shang found the separation length to vary only slightly when the eddy viscosity in the separation bubble was changed between zero and the frozen value. Therefore it can be concluded that what goes on in the recirculation zone is unimportant in establishing the length of the separation zone and analyses of only recirculation then cannot lead to any strong condition (this has been perplexing for years). However an analysis in the region immediately above the recirculation region does produce a useful result.

In summary, a simple integral relation produces a condition which implies that τ_y must be positive above the dividing streamline in order to permit a decrease in entropy (or an increase in total pressure) in the lower portion of the boundary layer. This relationship is felt to be the single most controlling factor in establishing the size of the separation region. Only analyses which properly account for this phenomenon can produce accurate results.

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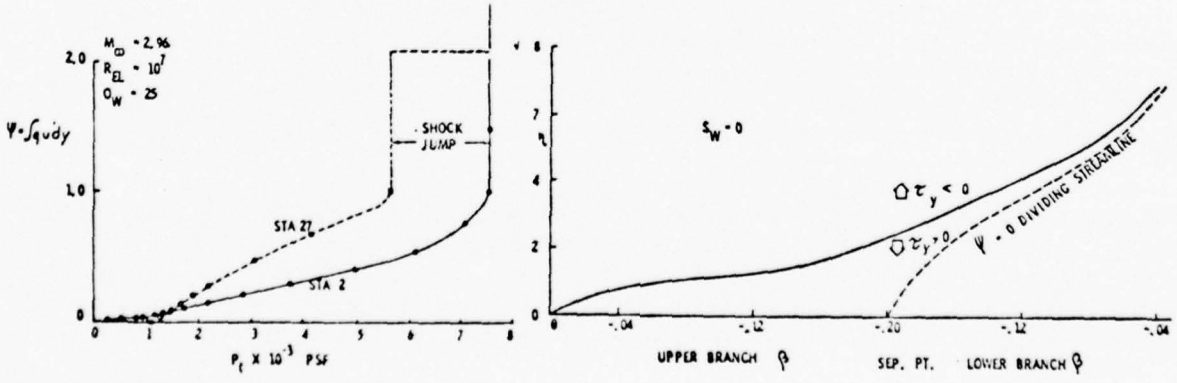

RECENT EXPERIMENTAL STUDIES OF THE MECHANISM OF FLOW SEPARATION IN HIGH MACH NUMBER, HIGH REYNOLDS NUMBER FLOWS

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ABSTRACT

A discussion is given of the results of recent experimental studies which have important implications on the modeling of laminar and turbulent separated flows, and present new problems in this area of research. The results of detailed experimental studies of both wedge- and shock-induced regions of viscous interaction and boundary layer separation are discussed in terms of their implications to the theoretical modeling of these flows. The studies of laminar separation induced "internally" at the junction between a flat plate and a wedge, and "externally" with an oblique shock, demonstrated that at Mach numbers from 12 to 18 the laminar boundary layer can be more easily separated by an externally generated shock. This result is in contrast to earlier findings at lower Mach numbers, and suggests that modeling these flows with a simple "free interaction" theory may be inadequate. Studies of shock- and wedge-induced interaction regions at Mach numbers from 6 to 13 and Reynolds numbers up to 200 x 10^6 suggest that the pressure rise to separation is independent of the mechanism inducing separation. These studies suggest that boundary layer separation is controlled primarily by an interaction which occurs at the base of the boundary layer, thus describing the separation mechanism in terms of the interaction between the growth of a turbulent boundary layer and an external inviscid flow is inaccurate at high Mach numbers. The length of these turbulent interaction regions are comparable with boundary layer thickness, thus it is questionable whether first order boundary layer theory is valid in these flows. Studies of steady and unsteady separated regions over concave nose shapes generated by non-similar ablation are discussed briefly. Finally, we discuss some novel studies where extensive regions of unsteady separated flows were developed when a minute particle launched through the stagnation region of a blunt body pierced the bow shock.

UPSTREAM INFLUENCE IN INTERACTING NONSEPARATING TURBULENT BOUNDARY LAYERS

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ABSTRACT

The important feature of upstream influence in all viscous-inviscid interaction problems is known to be primarily governed by the viscous behavior of the interactive disturbances within a thin "frictional sub-layer" near the body surface.[1] Existing analytical treatments of this region assume that it lies within the linear portion of the incoming flow velocity profile. Although valid for laminar flows, this assumption fails badly for turbulent flows at the higher Reynolds numbers $Re_L \simeq 10^6$-10^{10} typifying full scale flight vehicles. In the present paper we present the formal extension and complete solution for the turbulent non-separating case wherein the velocity profile is not necessarily linear (i. e., the frictional sublayer may penetrate to an arbitrary degree into the logarithmic part of the profile.)

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Freely Interacting Transonic Boundary Layers *

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The inner layer equations of the triple deck structure for transonic free interactions[1], both compressive and expansive, have been studied numerically and analytically. The only restriction placed on the external flow is that it remains supersonic throughout.

In the case of compressive interactions characterized by values of the transonic interaction parameter $K_0 < 2.3$ the assumption of supersonic external flows breaks down before separation occurs. For $K_0 > 2.3$, however, the supersonic flow region extends into the region downstream of the separation point. Calculations based on the Reyhner and Flügge-Lotz[2] approximation in the reversed flow region indicate that for sufficiently large values of K_0 the pressure distributions approach regions of constant pressure which is similar to the supersonic flow case[3].

The results also show that the ultimate upstream decay is identical to that of the supersonic free interaction if $K_0 \neq 0$. For expansive free interactions with $K_0 = 0$, however, the upstream decay is algebraic rather than exponential.

Finally, the asymptotic structure of the flow field near the pressure singularity in transonic expansive flows has been determined, and the structure agrees well with the numerical computations near the singularity.

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CALCULATION OF AXISYMMETRIC SEPARATED TURBULENT BOUNDARY LAYERS

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A method is being developed to analyze compressible viscous attached and separated flow fields for axisymmetric body geometries used for isolated nozzle afterbody models. The work includes coupling of the integral boundary-layer method with a finite-difference, inviscid-flow program using the boundary-layer displacement thickness to modify the shape of the body.

The derivation of the governing equations for the boundary-layer flow has been described in reference 1. That derivation has been modified to use the integral method described in reference 2. The inviscid flow is calculated using the South and Jameson program described in reference 3. That program is also being modified to produce a finite-difference solution of the full potential equation in conservative form.

The boundary-layer method uses the law-of-the-wall and the law-of-the-wake velocity profile formulation with a laminar sub-layer included. This formulation is capable of calculating both laminar and turbulent boundary layers as well as separated boundary layers. For attached flow, the boundary-layer edge velocity, U_e, is prescribed and the boundary-layer thickness, δ, and the friction velocity, u_τ, are calculated. For separated flows, the displacement thickness, δ^*, is prescribed and the thickness, δ, and the edge velocity, U_e, are calculated.

The axisymmetric, conservative, finite-difference inviscid flow program will solve the equation for conservation of mass employing artificial viscosity to assure upwind differencing in supersonic zones. The particular form of the artificial viscosity and the iterative relaxation procedure to be used are currently under development.

Several kinds of iterative procedures are being developed for calculating the interaction of the viscous and inviscid flows. In one approach, the displacement thickness is prescribed on the boundary layer, and the free-stream velocity distribution which results from the boundary-layer calculation is compared with that from the inviscid flow calculation on the augmented body. The displacement thickness is then modified by trial and error until the two pressure distributions agree. Other approaches are being studied which are expected to lead to a calculation method which will iterate without external assistance.

The work described in this abstract is being sponsored by the Arnold Engineering Development Center, Air Force Systems Command, USAF.

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AN EFFICIENT NUMERICAL METHOD FOR SOLVING THE TIME-DEPENDENT COMPRESSIBLE NAVIER-STOKES EQUATIONS AT HIGH REYNOLDS NUMBER

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ABSTRACT

The Navier-Stokes equations adequately describe aerodynamic flows at standard atmospheric temperatures and pressures. If we could efficiently solve these equations, there would be no need for experimental tests to design flight vehicles or other aerodynamic devices. Unfortunately, analytic or closed-form solutions to these equations exist for only a few simple flow problems. During the past decade, the computer has been used to generate many new solutions. However, with existing numerical methods[1] and computer resources, these solutions have been restricted to low Reynolds number or two-dimensional flows. This paper describes a new numerical method[2] that has been used to drastically reduce the computation time required to solve the Navier-Stokes equations at flight Reynolds numbers. Though flows past complete aircraft configurations are still beyond our reach, the new method makes it possible and practical to calculate many important three-dimensional, high Reynolds number flow fields on today's computers.

The unsteady Navier-Stokes equations are difficult to solve because at high Reynolds numbers the magnitude of the inertial forces described by the hyperbolic terms of the equations are much larger than the viscous forces described by the parabolic terms. Conventional numerical methods, whether explicit or implicit, are severely restricted to small time steps by the stability or accuracy requirements imposed by the hyperbolic terms; hence, many time steps may be required before the viscous effects can be determined. The present approach time-splits the equations into a hyperbolic part and a parabolic part, solves the hyperbolic part by using a new explicit numerical method based on characteristics theory, and solves the parabolic part by using a new efficient implicit parabolic method. Both methods are fully conservative and stable with time-steps orders of magnitude larger than those allowed by CFL (Courant, Friedrich, and Lewy) stability criteria.

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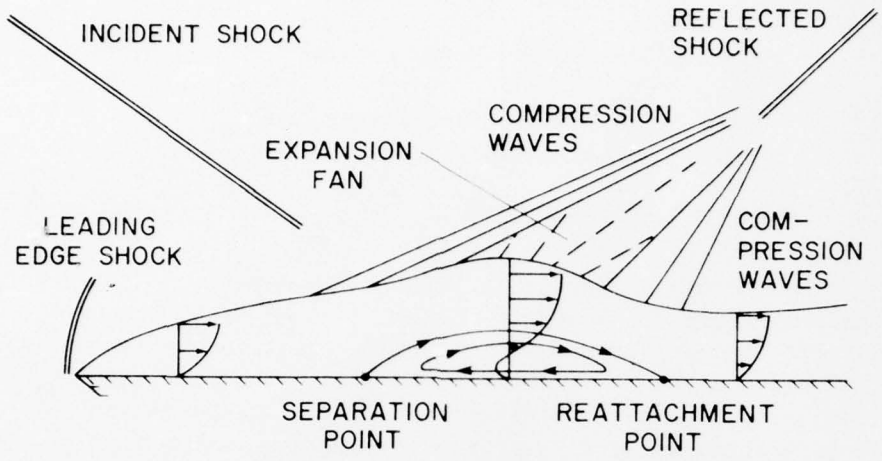

Fig. 1 Shock wave boundary-layer interaction

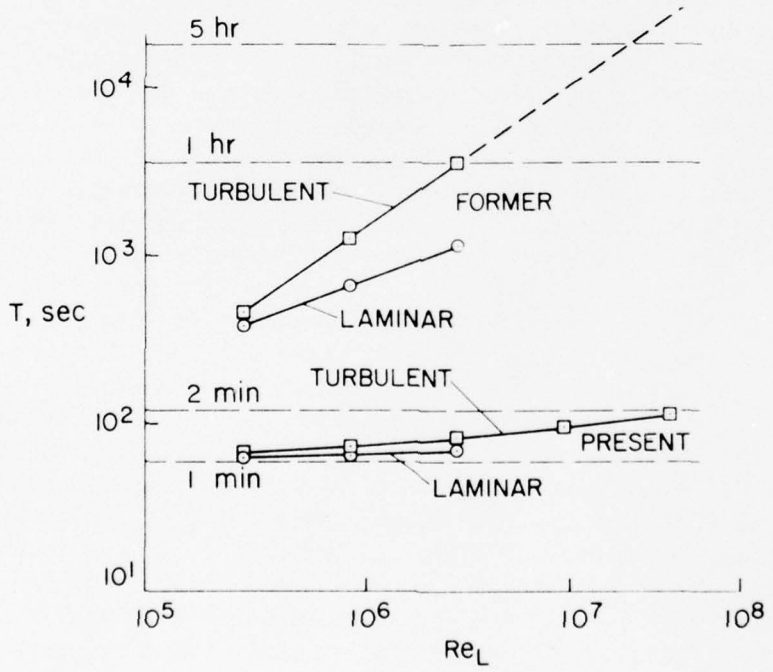

Fig. 2 Comparison of former and present method computing times on the CDC 7600 vs. Reynolds number for several shock boundary-layer interaction calculations

VISCOUS INTERACTION AND BOUNDARY-LAYER SEPARATION

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ABSTRACT

Given the requirements of having to predict the details of the viscous interactions and separated flows over a very wide range of flow conditions, the present authors have concentrated on making predictions by computing efficient and direct numerical solutions to the governing partial differential equations of motion. Aspects of relatively low Reynolds number laminar incompressible separation bubbles have been investigated using this approach by Briley[1]. Of note in this work is the observation of considerable upstream influence of the separation bubble at these relatively low Reynolds numbers and the apparent absence of any separation singularity. Later the incompressible numerical scheme was further developed and improved and higher Reynolds number laminar transitional and turbulent incompressible separation bubbles computed in a time dependent manner by Briley and McDonald[2]. Both the full Navier-Stokes equations and the boundary layer equations were used and compared. Of note in this work was the use of the time dependent approach to allow the interaction of the shear flow region with the outer inviscid flow and the apparent absence of the separation singularity when this interaction is allowed for with the boundary layer equations. Further the highly implicit and non-linear nature of the turbulence model turned out to be amenable to treatment within the computational framework and the predicted results were in good agreement with data. More recent studies have been concerned with the compressible problem and here both the steady boundary layer equations and the time dependent Navier-Stokes equations are being used. Levy, Shamroth and McDonald[3] discuss results obtained in both laminar and turbulent shock wave boundary layer interactions containing small regions of flow separation in supersonic flow using iteratively a spatially marching boundary layer scheme. A companion study on the same problem uses the fully implicit scheme for solving the multi-dimensional Navier-Stokes equations developed by Briley and McDonald[4]. Of particular note in this work are the very modest computer run times attainable with the fully implicit Navier-Stokes scheme, and this should enable future extensive and detailed studies of this problem using the full equations without exorbitant computational expense. Lastly some preliminary work which should eventually lead up to the computation of the flow around an arbitrary airfoil using the Navier-Stokes procedure of Briley and McDonald[4] will be discussed.

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TURBULENT INTERACTIONS AT TRAILING EDGES

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ABSTRACT

We are considering the effect of turbulent boundary layers on the lifting characteristics of airfoils at high Reynolds numbers. The most important effect arises from the local boundary-layer behavior near the trailing edge, which directly effects the Kutta condition and leads to significant reductions in lift. Existing methods for predicting the boundary-layer effects on lift are based on the displacement surface corrections of a second-order boundary-layer theory. The displacement surface is assumed to be a streamline and the corrections to the inviscid flow are determined by standard procedures. Unfortunately, the displacement thickness distribution determined from the solution of the boundary-layer equations develops singularities at trailing edges. The singularities are caused by the discontinuous jump in the surface boundary conditions across the trailing edge and by a singularity in the pressure gradient of the inviscid solution at the trailing edge. Because of these singularities a viscous Kutta condition cannot be imposed and the lift coefficient cannot be determined by standard second-order theory.

To remedy this situation, we have developed an "inner solution" for the strong interaction region at trailing edges.[1] In the present discussion, we will outline the main features of the interaction theory and will describe recent developments leading to a complete solution for the lift decrement due to turbulent boundary layers.

The interaction theory of Ref. 1 is based on a large Reynolds number, asymptotic expansion of the full (and unclosed) Reynolds equations of turbulent flow. The theory leads to a three-layer description of the flow near the trailing edge consisting of: 1) an outer, inviscid, rotational flow region encompassing most of the boundary layer, 2) a thin, conventional, constant stress, wall layer and 3) a blending layer of thickness intermediate between that of the outer and wall layers. In this respect, the theory is superficially similar to the "triple-deck" structure arising in the laminar interaction theory of Brown and Stewartson[2].

Although they both envolve a three-layer structure, the details of the laminar and turbulent solutions are vastly different. In the laminar problem the interaction distance is an order of magnitude larger than a boundary-layer thickness, with boundary-layer approximations and the equivalent displacement-body concept remaining valid in the trailing-edge region. In turbulent flow, the interactions are compact with a streamwise extent of the order of a boundary-layer thickness, the boundary-layer approximation fails and the displacement surface is not a streamline in the trailing-edge region. The reasons for these differences will be discussed during the talk.

We will also discuss how the turbulent-interaction equations for the outer solution are automatically closed to lowest order without the need for empirical closure assumptions. We will outline the procedures for obtaining solution of the inviscid outer equations by analytic function theory. The effect of compressibility on the solution will be discussed and a complete solution for the lift coefficient in a turbulent, compressible flow will be given.

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FLOW SEPARATION FROM A SMOOTH SURFACE IN A SLENDER CONICAL FLOW

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ABSTRACT

When a flat plate delta wing is placed at incidence to an oncoming stream the dominant feature of the resulting flow is the vortex pair which forms over the wing, as in figure 1. The flow separates from each leading edge, S_1, to feed vorticity into these coherent vortices. Efficient calculation methods are available[1,2] for the investigation of this, so-called, primary separation for a slender wing. It is known that the pressure variation over the wing surface due to the presence of these leading-edge vortices induces a secondary separation. It seems natural to explore the possibility of representing this secondary separation, like the primary separated flow, by a conical spiral vortex. The model is shown schematically in a cross-flow plane in figure 2, where S_2 denotes the position of the secondary separation line. Numerical calculations have been carried out[3], in the boundary layer which forms on the surface, to determine the position of S_2 such that boundary-layer separation also occurs at that point. It has been found that separation takes place at or just beyond a pressure minimum in this conical flow. This is reminiscent of recent theories for two-dimensional separation[4]. A local investigation of the inviscid flow structure close to S_2 has been made[5] which shows a close resemblance to the discontinuous potential flow models of separation in two dimensions. The main difference is that in the present case the region beyond S_2 is not a stagnant region. An investigation of the viscous interaction flow structure is currently in progress, exploiting the ideas which have been developed for two-dimensional flow[4].

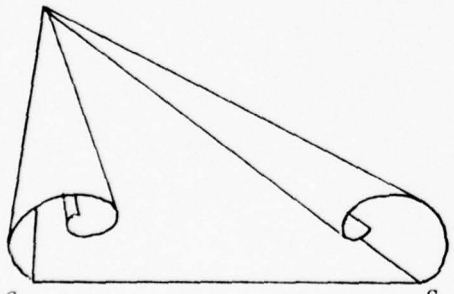

Figure 1

Figure 2

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DIFFICULTIES WITH BRUTE-FORCE NUMERICAL SOLUTIONS OF NAVIER-STOKES EQUATIONS FOR HIGH - RE SEPARATED FLOWS

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ABSTRACT

Discrete numerical solutions in fluid dynamics are well-suited for problems in an intermediate range of Re, which are very difficult to approach by analytical methods. At high Re, difficulties are encountered which were generally unanticipated when computational fluid dynamics was in its infancy.

These difficulties will be viewed from several different perspectives: "balancing" a method; the "swamping" of viscous terms by truncation error from advection terms; spatial oscillations; and most importantly, scaling and the Nyquist frequency limitation. These perspectives give estimates of the maximum Re resolvable in a given mesh. However, computational experience indicates that these estimates are pessimistic if only moderate accuracy is desired. On the other hand, success with a moderately accurate solution can be misleading about the prospects for high accuracy solutions. If a problem can be solved with second-order methods, it can be solved to higher accuracy with high-order methods, but high-order methods do not help the scaling problem at high Re.

Perspectives will also be given on what constitutes acceptable accuracy for some applications, with examples of calculations which are accurate for some parameters (e.g. drag) but not for others (e.g. pressure distribution), or which give an "acceptable" error at one Re, but do not represent variation with Re.

Some possible techniques for obtaining acceptable accuracy will be reviewed, including the following: (1)a judicious choice of problems, which sometimes means that the problem would be more efficiently calculated with some form of parabolized Navier-Stokes equations, (2) for rough accuracy, the use of implicit or explicit "artificial viscosity", which is directional and is better than just calculating at a lower Re, (3) automatic mesh refinement and coordinate alignment, (4) possibly, the inverse approach of Harlow for one-dimensional problems, which is conceptually similar to the isotherm migration technique used in heat conduction problems, (5) possibly, the incorporation of results from perturbation analyses.

THE CALCULATION METHODS FOR FLOWS WITH STRONG INTERACTIONS BETWEEN VISCOUS AND INVISCID REGIONS

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ABSTRACT

In flows with strong interaction, particularly when separated regions are present, small changes in the viscous layer, and conversely small disturbances in the external pressure distribution may precipitate large distortions of the viscous layer and in the flow field. In such cases the most generally valid approach would be a numerical solution of the full Navier-Stokes equations, although such solutions are hampered by the computer size and speed limitations[1]. Therefore, it is recognized that there exists the need for a theory capable of giving good representation of the flow field in the regions of strong interaction, without developing singular behaviour at such points as separation. Such flow fields are very difficult to compute because it is found that equations of hyperbolic, parabolic and elliptic types enter the problem in different regions and have to be joined smoothly on the interface between neighbouring regions. Furthermore, the characteristics of the viscous and inviscid regions and the compatibility relations on the matching line result in an additional scaling parameter for the viscous flow[2]. An experimental result illustrating this effect is shown in Fig. 1. This means that in scaling

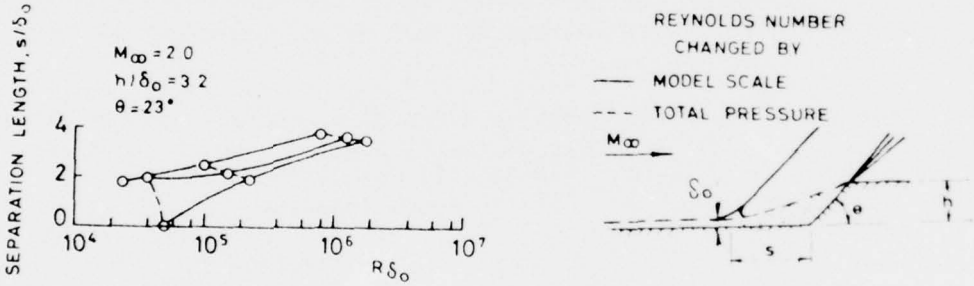


Fig. 1: Variation of separation length with Reynolds number (from Jones - AGARD CP-83, 1971.

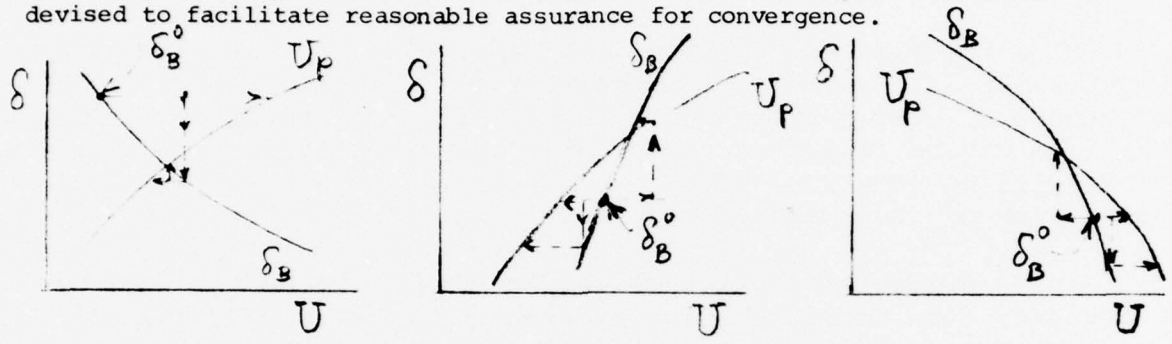
experimental results, the external flow Mach and Reynolds numbers must be matched as well as the relative dimension of the viscous region (including any separated zones). In the separated zone, the proper flow reversal velocities can be obtained by a solution of the Navier-Stokes equations. This will result in a three region model: (1) external flow (inviscid - hyperbolic, elliptic or mixed), (2) shear region (boundary layer-parabolic), (3) separated region (flow reversal-elliptic). In most cases there will be an overlap between regions (2) and (3) and the compatibility relations will be satisfied in this overlap zone.

The solution of the complete flow field will be obtained by an iterative procedure. The key problem for convergence of such iterative procedures are the proper matching methods. The matching can be done either by the schematic procedure suggested in Ref. 3, or by a "Bisection procedure" defined in the following: (1) Assume an initial velocity distribution on the outer edge of the viscous layer $U_B\{X_i\}$. (2) Solve by a finite difference program the boundary layer equations for the imposed $U_B\{X_i\}$ and obtain $\delta_B\{X_i\}$. (3) Assume $\delta_B\{X_i\}=\delta_P\{X_i\}$, calculate the external velocity distribution $Up\{X_i\}$. (4) Define a new velocity for

the outer edge of the boundary layer $U_{BN}\{X_i\} = \frac{1}{2}[U_B\{X_i\} + U_P\{X_i\}]$. (5) Repeat iterative calculation of steps (1) to (4) with $U_B\{X_i\} = U_{BN}\{X_i\}$. This procedure is simpler than that of Ref. 3 in that it does not require the evaluation of the influence matrices $[B_{ij}]$ and $[P_{ij}]$. Furthermore, in cases where the bi-section procedure does not converge, that is when,

$$|U_P^{j+1}\{X_i\} - U_B^{j+1}\{X_i\}| > |U_P^{j}\{X_i\} - U_B^{j}\{X_i\}|,$$

then computation procedures, illustrated in the following sketches, can be devised to facilitate reasonable assurance for convergence.



These procedures are now being applied to the investigations of the following problems: (1) strongly interacting flow in the vicinity of a forward facing step in supersonic flow. (2) calculation of the interaction of a turbulent boundary layer in a transonic flow on a symmetric airfoil.

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VISCOUS INTERACTIONS IN THREE DIMENSIONS

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ABSTRACT

High Reynolds number viscous interactions in three-dimensions can result in a variety of complex flow phenomena. As in two-dimensions, strong pressure interactions and/or mainstream separation may be important; many of the presentations at this workshop will be concerned with free interaction boundary layer modelling of these effects.

Three-dimensional interactions introduce the possibility of secondary flow separation and associated vortex formation, as well as boundary layer interference phenomena associated with geometry, displacement induced secondary motions and interactions near lines of attachment or separation. It is these three-dimensional viscous interactions that have been of interest to me for sometime and about which this presentation is devoted. It should be noted that the procedures described here can also play an important role in the analysis of free interactions. Investigations along these lines are currently in progress.

Specific geometries that have been considered by merged layer theory[1] (single viscous-inviscid system of equations), two-layer theory[2] (coupled viscous-inviscid system) boundary region theory[3,4] (boundary layer within a boundary layer), and viscous slender body theory[5] (second-order evaluation of secondary motions), include axial corners[3,4], cones[1,2,6,7], ogive-cylinders[2], elliptic cylinders[5] and wing-body configurations. Rectangular inlets have also been evaluated but only external interactions are included here. Our main objective in all of these analyses is the evaluation of complex flow phenomena without requiring complete solutions of the Navier-Stokes equations. Some typical secondary motions resulting from these analyses are given below. Figure 1 shows the separated secondary flow profile (q') at the leeplane of a cone at incidence; in reference 7 the inadequacy of conventional boundary layer methods is clearly defined and the odd growth for large η is explained. Figure 2 depicts the low speed secondary flow on a cylinder resembling a wind-body cross-section. This is a recent result obtained with viscous slender body methods.

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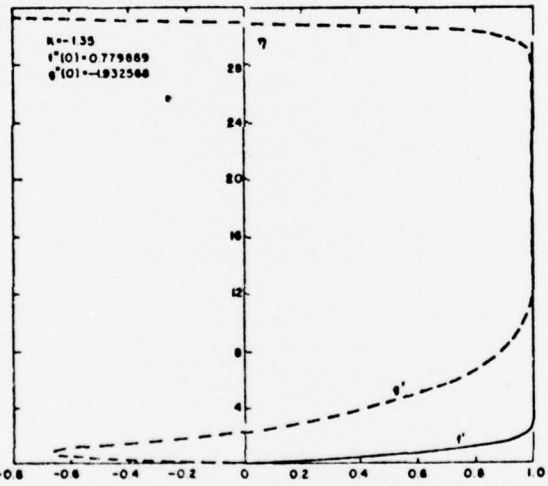

Fig. 1. Similar Boundary Layer: Velocity Profile at Leeplane.

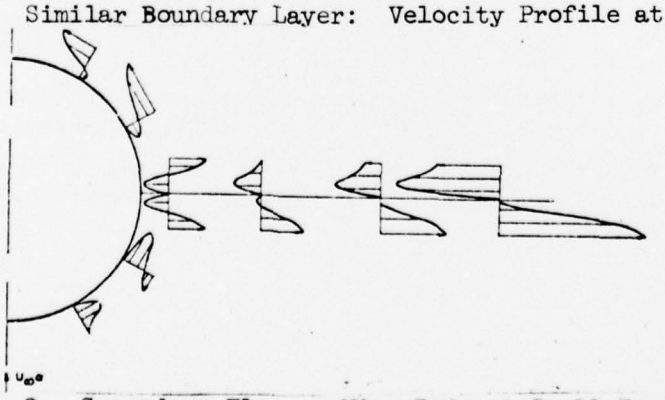

Fig. 2. Secondary Flow on Wing-Body at Small Incidence.

Tube-flows : on the effects of constrictions, branchings, upstream influence and separation

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ABSTRACT

These tubeflows are divided into four main categories: (i) nonsymmetric pipeflows (ii) nonsymmetric channel flows (iii) symmetric pipeflows (iv) symmetric channel flows. The Reynolds number R is large and the incompressible fliud motion is steady and laminar. The aim is to predict flow features in the most realistic case where the obstruction (a constriction or branching of the tube) is ''severe'', i.e. of a size comparable with the undisturbed tubewidth (a).

Flow (ii)[1] provides the most upstream influence then, over a large distance $O(aR^{1/7})$, and upstream separation occurs at $x = x_{sep} = -0.49\, a\, R^{1/7} + O(a)$, independently of the particular form of obstruction. (Here x is downstream distance from the obstruction). The two wall layers interact with the core, one layer expanding, while the other compresses and so sustains the upstream interaction process. Comparisons with experiments and with solutions of the Navier-Stokes equations are favourable overall.

Flows (iii), (iv)[2] are treated together and the upstream influence distance (and hence $-x_{sep}$) is $O(a \ln R)$. For, in "moderate" constrictions of slope $h R^{-1/6}$ ($h \sim 1$), x_{sep} is $O(1)$ but as $h \to \infty$ $x_{sep} \to -\infty$. Hence in severe obstruction x_{sep} is again independent of the obstruction shape, to first order.

Flow (i)[3] is the most physically important, but as yet we can only conclude that, for a ''fine'' contriction (of slope $\sim R^{-1/3}$), upstream influence exists over a distance $O(a)$. The properties for severe obstructions await a numerical investigation of the complete 3D free-interaction that occurs for the fine case. This interaction is due to the jetlike swirl in the wall layer, in the absence of any core displacement.

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SYCHEV'S THEORY OF SEPARATION

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ABSTRACT

The limit of the solution of the Navier-Stokes equations for flow past a bluff body as the Reynolds number $R \to \infty$ is not yet known. Two candidates have been sufficiently well described that one may ask whether they are the leading term, not uniformly valid, in an asymptotic expansion of the solution in descending powers of R. The first, irrotational attached flow, seems to fail this test. The second is Kirchhoff flow of which the most likely has breakaway at a point S of the surface such that the free stream line there has zero curvature and the pressure gradient upstream is favourable. By the use of a triple-deck theory to describe the boundary layer near S, Sychev establishes that the description is viable here provided a certain differential equation, of the boundary layer type, has a solution. Some comments are made about this equation and the relation of the theory to existing studies of flow past bluff bodies at finite values of R.

The properties of flow past finite thin bodies at incidence when $R \gg 1$ is reviewed, it being noted that complete solutions have been found up to but not beyond separation. Possibilities for solutions with separation are touched on.

Sychev went on to claim that the notion of separation being preceded by a finite length of adverse pressure gradient has no place in a theory of flow past a bluff body when $R \gg 1$. The strength of the case for the notion in laminar flow is reviewed and the implications of his claim, if substantiated, is discussed for both laminar and turbulent flows.

VISCOUS FLOW NEAR THE TRAILING EDGE OF A FLAT PLATE

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Netherlands

Re earch, performed during the last year at the Mathematical Institute of the University of Groningen, mainly by dr. Veldman, will be reviewed.

The equations of the triple deck, existing near the trailing edge of a flat plate under zero angle of incidence, have been solved numerically. By aid of suitable transformations the whole range of X-values (from $-\infty$ to $+\infty$ on the triple deck scale $X=Re^{3/8}x$) has been considered without cutting-off. Another difference with other investigator's work is that the solution in the upper deck has been obtained by the aid of a field calculation which is performed simultaneously with the field calculation of the lower deck. This prevents the use of the Cauchy integral whcih either leads to difficulties at X=0 or at $X=\pm\infty$. In the usual boundary layer and in the wake ($|x|>0(Re^{-3/8})$) the displacement thickness determines the pressure distribution, while in the regions inside the triple deck ($|x|<0(Re^{-3/8})$) it is the (self-induced) pressure which determines the displacement thickness. Therefore, for $x=0(Re^{-3/8})$ a transition from the one to the other system occurs and this makes that the direction, in which the Cauchy integral should be applied, also changes.

On an x-scale smaller than that of the usual triple deck an infinite series of triple decks occur. Their sizes are $x=0(Re^{-1/2})$, $x=0(Re^{-5/8})$, etc., finally converging to the Navier-Stokes region, $x=0(Re^{-3/4})$. However, the main term in the shear stress at the wall remains constant from the inner end of the orthodox triple deck until the outer end of the Navier-Stokes region. The discontinuity in the pressure derivative at X=0 disappears in the upper layer of the new triple decks. It remains present in the middle and lower layers of these triple decks until the Navier-Stokes region is reached.

For $X\to\infty$ the expansions of stream function and pressure in the wake contain so-called eigenfunctions. Their coefficients can not be determined from the asymptotic theory but only follow from the numerical determination of the whole flow field.

Finally, a momentum consideration is given which leads to a relation

$$d=-6\,a\,b$$

where d is the coefficient of the $Re^{-7/8}$-term of the drag, $a = f''(0)$ the second derivative of the Blasius profile at the wall and b is the coefficient of the most important eigenfunction.

NUMERICAL STUDIES OF HIGH REYNOLDS NUMBER LAMINAR SEPARATION

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ABSTRACT

The goal of our ongoing effort is to help establish a fundamental set of governing equations and a solution technique for reliable prediction of separation. The approach taken attempts to comprehend as much analytical information as possible in the numerical solution method. Results are discussed for supersonic ramp and injection induced separations and for incompressible flow separation at exterior (convex) and interior (concave) corners.

For supersonic induced separation, Fig. 1 shows that numerical solutions of the triple deck equations (TD) predict the overall scale of separation as obtained from numerical solutions of the interacting boundary layer (IBL) equations as $Re \to \infty$. Most importantly it was found that accurate solutions to the IBL equations could not be achieved unless the finite difference mesh honored the scalings predicted by the TD model. Also it has been found that the parameter groupings that emerge from the supersonic/hypersonic TD model correlate IBL separation solutions thereby reducing the 4 fold dependence on M_∞, T_w, Re_∞ and α to a 2 parameter family - a result that should be helpful to design studies. Fig. 1 also shows results of an integral method (IM) solution of the TD equations obtained using a quadratic velocity profile. The simplicity of the IM approach makes it attractive not only as a simple solution method for studying the fundamental interaction process but also as a vehicle for obtaining (a) initial conditions for relaxation solutions of the IBL model, (b) the overall size of the interaction/separation flow region, and (c) a local slip-boundary condition for more elaborate solution schemes. For the ramp problem there are several areas for future study - these include development of a means for accommodating imbedded shock waves, a better understanding of the mechanism for upstream propagation of information, and a faster relaxation scheme for solving the IBL equations.

Figure 2 shows that application of the IBL model to slot injection into a supersonic stream verifies the TD prediction of separation occurring ahead of the slot and verifies that "blow off" will not occur. IBL plate injection studies indicate the same trend but such solutions were difficult to obtain because of the build up of an inviscid layer near the wall and a slow convergence to an asymptotic downstream flow state. Further study is needed of the numerical method for such viscid/inviscid flows and ultimately correlations based on TD concepts should be developed.

For incompressible flow separations, numerical studies have been conducted of the flow past interior and exterior corners on semi-infinite bodies by mapping the geometries to stagnation point flows. A vorticity stream function formulation of the IBL concept has been followed with typical wall shear distributions for the exterior corner flow shown in Fig. 3. It is found that for low Re solutions are easily obtained whereas for high Re the solutions are

not yet totally reliable due to a lack of TD scaling in the numerical scheme as well as to uncertainty about the mechanism for upstream propagation of disturbances due to interaction. Future work should be aimed at TD solutions for such geometries to provide a standard for assessment of finite difference solutions.

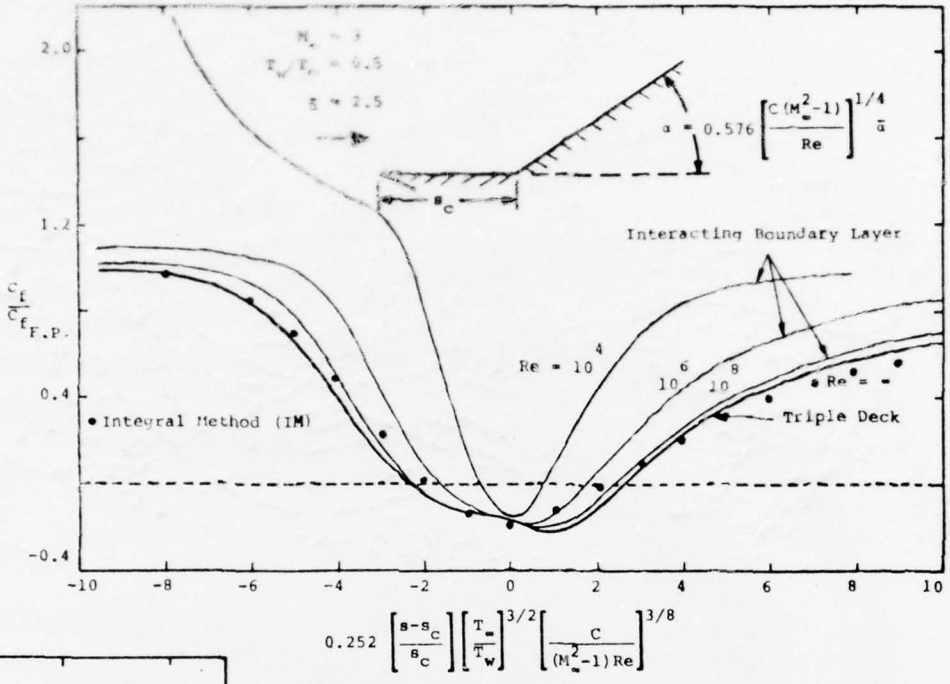

Fig. 1. Triple Deck and Interacting Boundary-Layer Solutions for Ramp Separation.

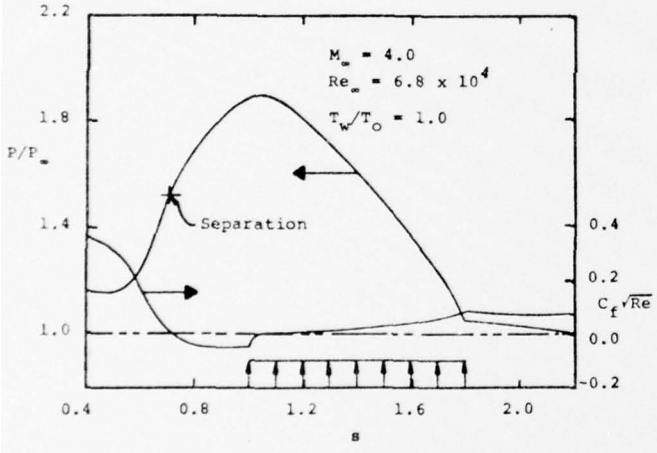

Fig. 3. Exterior Corner Separation.

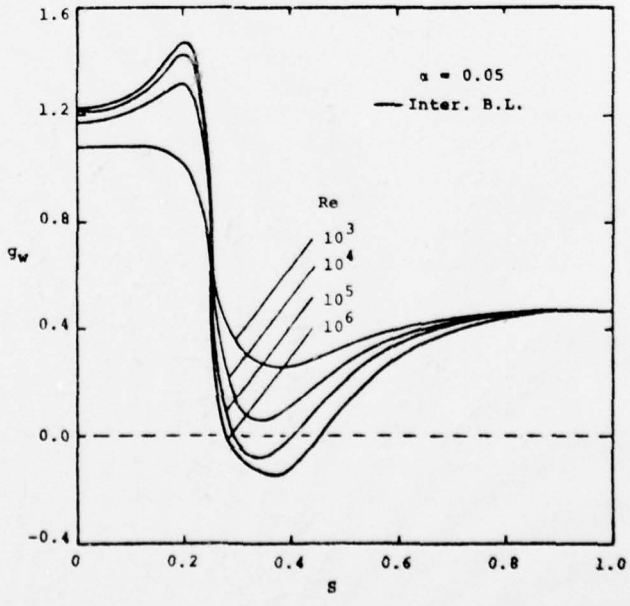

Fig. 2. Surface Injection Induced Separation.

A New Pressure Hypothesis and Approximate Theory for the Streaming Motion Past Bluff Bodies at Re from O(1) to O(10^2)

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ABSTRACT

A new approximate theory will be described for treating the flow past smoothly contoured two-dimensional[1] and axisymmetric[2] bluff bodies in the intermediate Reynolds number range $O(1) < Re < O(10^2)$ where the displacement effect of the thick viscous layer near the surface of the body is large and a steady laminar wake is present. The theory is based on a new pressure hypothesis which enables one to take account of the displacement interaction and centrifugal effects in thick viscous layers using conventional first order boundary layer equations. The basic question asked is how the wall pressure gradient in ordinary boundary layer theory must be modified if the pressure gradient along the displacement surface using the Prandtl pressure hypothesis is to be equal to the pressure gradient along this surface using a higher order approximation to the Navier-Stokes equation in which centrifugal forces are considered. The inclusion of the normal pressure field with displacement interaction is shown to be equivalent to stretching the streamwise body coordinate in first order boundary layer theory. For laminar flow without separation the new theory provides a smooth transition to the solutions obtained from high Reynolds number boundary layer theory.

While the new theory is of a non-rigorous nature, it yields results for the location of separation and detailed surface pressure and vorticity distribution which are in remarkably good agreement with the large body of available numerical Navier-Stokes solutions. These solutions include the flow past parabolic, circular and elliptic cylinders, spheres, speroids and paraboloids of revolution at various Reynolds numbers. A novel feature of the new boundary value problem is the development of a simple but accurate approximate method for determining the inviscid flow past an arbitrary two-dimensional or axisymmetric displacement body with its wake.

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A METHOD FOR REVERSE FLOW BOUNDARY LAYERS WITH PRESCRIBED DISPLACEMENT THICKNESS

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ABSTRACT

A method is described for computing boundary layers with prescribed displacement thickness, together with the modifications needed to implement the iterative procedure introduced in Ref.(1) for dealing with reverse flow regions. The main aim is to investigate more fully the convergence of this iterative procedure on a simpler problem, namely an incompressible boundary layer with displacement thickness prescribed in such a way that a bounded reverse flow region is obtained. This avoids the problem of having to impose a downstream asymptotic profile, which was necessary in Ref.(1), where the convergence of the iterative procedure was not completely convincing. It should be remarked that this may have been due to a low order term in the asymptotic expansion, given in Ref.(2), being incorrect, as has been kindly pointed out to us by Dr. F.T. Smith; the effect should be small, but whether it actually removes the slight difficulty noted in Ref.(1) has not yet been confirmed.

The specific problem considered here has a displacement thickness given by

$$\delta^*(x) = \delta_0(1 + 9e^{-(x-14)^2/16})$$

where δ_0 is the constant displacement thickness of the stagnation point boundary layer. Thus we have basically a stagnation point boundary layer modified in such a way that the displacement thickness has a hump centered at $x=14$ but drops smoothly to the constant δ_0 for large x and also for small x for practical purposes. Actually it was assumed that a stagnation point boundary layer had developed up to $x=1$ and was then allowed to develop in accordance with prescription of $\delta^*(x)$.

The basic step-by-step scheme used is that of Ref.(1) with modifications to account for the different boundary conditions. In Ref.(1) the inner layer of a compressible interaction problem was treated and the outer boundary conditions were $u \to y - A(x)$, where $A = \int_{-\infty} p\,dx$, and $u_y \to 1$. These are here replaced by $u \to U(x)$, where $p + 1/2\,U^2 = \text{constant}$, and $\int_0 (U-u)dy \to \delta^*(x)$. We may remark that the method could also be modified for the corresponding inner layer problem, where $A(x)$ is specified instead of the relation between A and p, which may be used in treating incompressible interaction problems in an iterative manner.

The procedure for the reverse flow region begins by producing a first approximation by the technique first described by Flügge-Lotz

and Reyhner[3] as referred in Ref.(1) as FLARE. This is then corrected successively by downstream-upstream iterations referred to as DUITS. Each upstream iteration starts at the end of the reverse flow bubble and is confined to the reverse flow region produced by the previous downstream iteration and assumes the pressure distribution obtained then as given. Each downstream iteration determines the pressure to satisfy the given $\delta^*(x)$ and in the reverse flow region takes the uu_x term as given by the previous upstream iteration; the first approximation is obtained by setting this equal to zero. A maximum of eight DUITS were performed and convergence appeared to be satisfactory. A reasonable practical accuracy was obtained after three or four DUITS.

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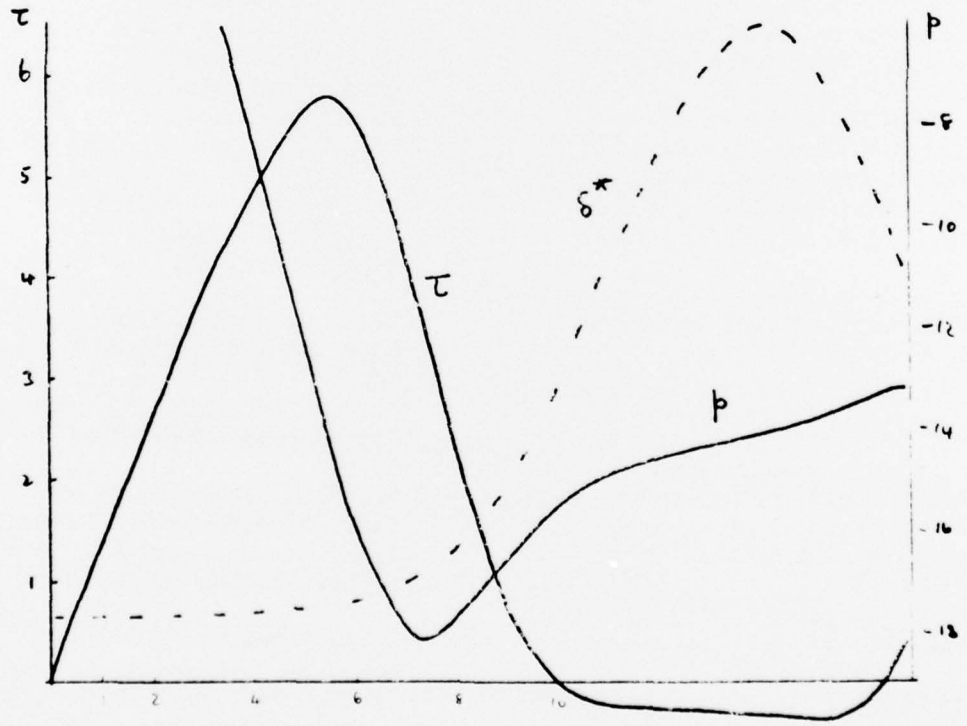

Converged solution for $\Delta x = 0.2$, $\Delta y = 0.2$

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