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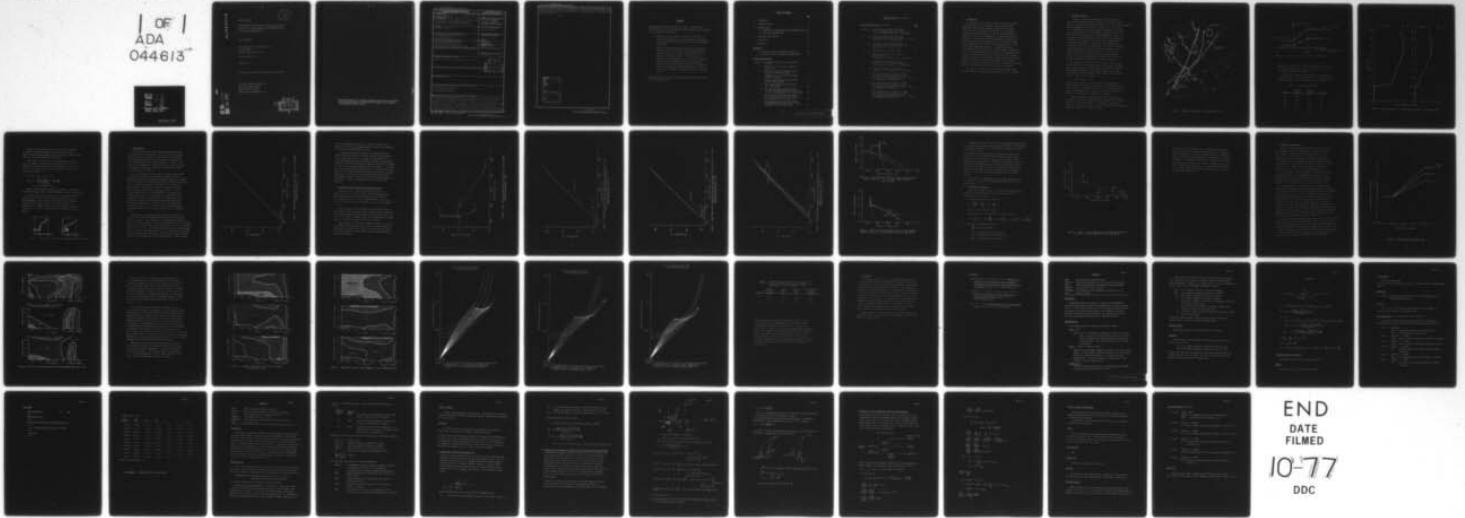
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SOME EXAMPLES OF THE EFFECTS OF THE POLEWARD F-LAYER  
TROUGH WALL ON GROUND RANGE AND AZIMUTH DETERMINATION  
IN AN OTH BACKSCATTER SYSTEM

B.M. Langworthy

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Scientific Report No. 1

February 1977

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errors on the variability of the trough wall, upper and lower quartile values of the model were also used in the computations. Power calculations indicate, that focusing effects > 15 dB can be expected for selected bundles of rays which just enter the trough wall.



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## FOREWORD

This document is the first scientific report on AFGL Contract F19628-76-C-0296. The work described herein represents the completion of the following contract items:

- a. Modify the electron density routine of the existing 3-D ray tracing program to accept a standardized trough wall profile along an arbitrary smooth curve given in geographic coordinates.
- b. Using the trough wall model provided by LIB, compute the effects of 3 to 5 trough wall configurations describing median conditions, and moderate and extreme deviations thereof, on frequencies in the 5 to 10 MHz range. A minimum of four boresights for a narrow beam antenna will be selected for the computations. The range/azimuth errors and the relative energy density of the propagated beam in the landing area will be determined and shown as a function of boresight, frequency, and chosen trough wall model.

Use was made herein of work performed under AFCRL Contracts F19628-73-C-0307 and F19628-76-C-0029.

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## 1. Introduction

The presence of the auroral oval with its sharp horizontal gradients especially in the region of the poleward F-layer trough wall causes large azimuthal deviations from the great circle path for radio waves in the HF band. It furthermore drastically changes the range of a ray to its landing point, if compared with a case of a homogeneous, horizontally stratified ionosphere. For the case of an OTH backscatter situation this results in both range and azimuth errors of the detected target unless corrections for the effect of the gradients are included in the target registration process. Using the AFGL three-dimensional ray tracing computer program (Langworthy and Barrett, 1975), various cross-sections derived from a new trough wall model have been simulated and backscatter results have been tabulated to give an indication of anticipated deviation in ground range and azimuth. The power with which these signals may be returned relative to signals not experiencing effects of gradients associated with the edge of the auroral oval has also been determined.

The data used to simulate the trough wall throughout this report is based on a model derived from Alouette topside ionograms.

## 2. Ionospheric Model

In order to determine the magnitude of range and azimuth errors, which an OTH system will experience if operating in the vicinity of the poleward F-layer trough wall, 3D ray tracing technique was applied to a new model of the trough wall (Pike, 1976). The location of the equatorward edge of the poleward trough wall is specified by a series of points in the geographic coordinate system. These points are then converted to the dipolar geomagnetic system for use with the ray trace program. The location of the trough wall is assumed to be composed of the great circle segments which connect the input points. Variations in the plasma frequency in the horizontal direction then become a function of the distance from the trough wall. Distance is taken to be positive north of the wall base and negative for points to the south. The trough wall used for the current study is shown in Figure 1. Details of the computer program which produces the coordinates for the trough wall, program WALL, are given in Appendix A.

The parameters which vary with the distance from the trough wall are the F-layer and E-layer maximum electron density values. These values are given in terms of critical frequencies ( $f_oE, f_oF2 =$  maximum plasma frequency of the respective layer) or in  $N_e \text{ max}$  (electrons/cm<sup>3</sup> for the respective layer maximum), where the two terms are related by the formula:

$$N_e \text{ max} = 12400 \cdot f_o^2.$$

The  $f_oE$  value varies from 0.1 MHz south of the trough wall to 2.5 MHz north of the trough wall; the height of the maximum electron density ( $h \text{ max}$ ) is assumed to be 120 km. The  $f_oF2$  values north and south of the trough wall are input quantities to the ray trace program as is the width (or steepness) of the wall.

Variation across the trough wall is taken to be linear in the square of the plasma frequency which is equivalent to a linear variation in the electron density. The height of the F2 layer maximum is taken to be constant at 350 km. Electron density variation across the trough wall is given in Figure 2.

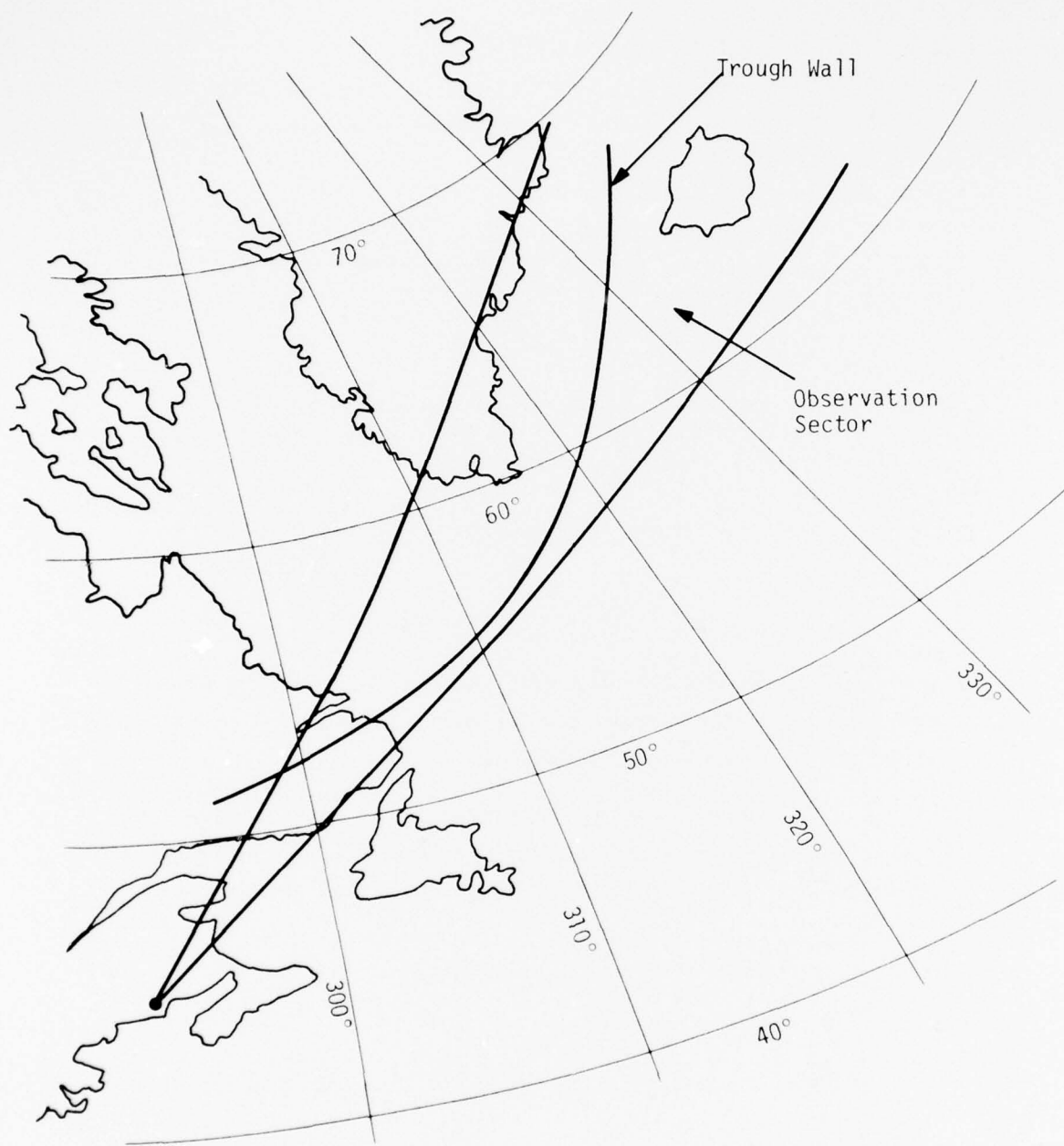


FIGURE 1. GEOGRAPHIC LOCATION OF THE TROUGH WALL AT 06 UT

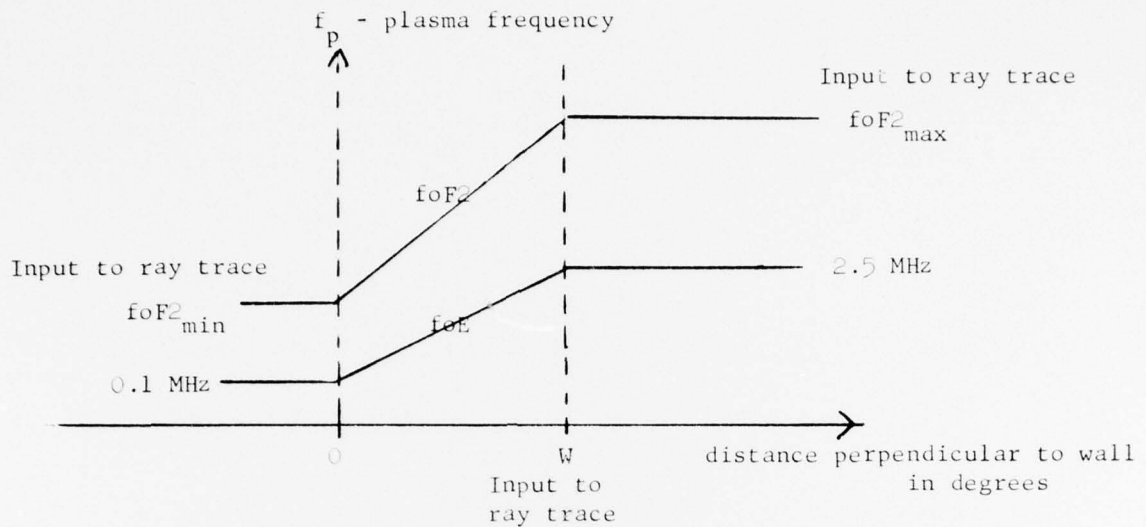


Figure 2. Variation of foE and foF2 across the Trough Wall

Here  $foF2_{min}$  is the foF2 (in MHz) at the bottom of the trough and  $foF2_{max}$  is the foF2 at the top of the trough.  $W$  is the width of the trough wall in degrees.

The various trough wall models examined in this study are listed in Table 1. They were derived from a trough wall model determined by Pike (1976) from Alouette topside ionograms.

Table 1. Trough Wall Models Used

Trough	$foF2_{max}$ in MHz	$foF2_{min}$ in MHz	$W$ in degrees
I	3.81	1.97	3.0
II	3.11	1.97	3.0
III	4.58	1.97	3.0
IV	2.34	1.61	1.5
V	4.92	1.61	3.8

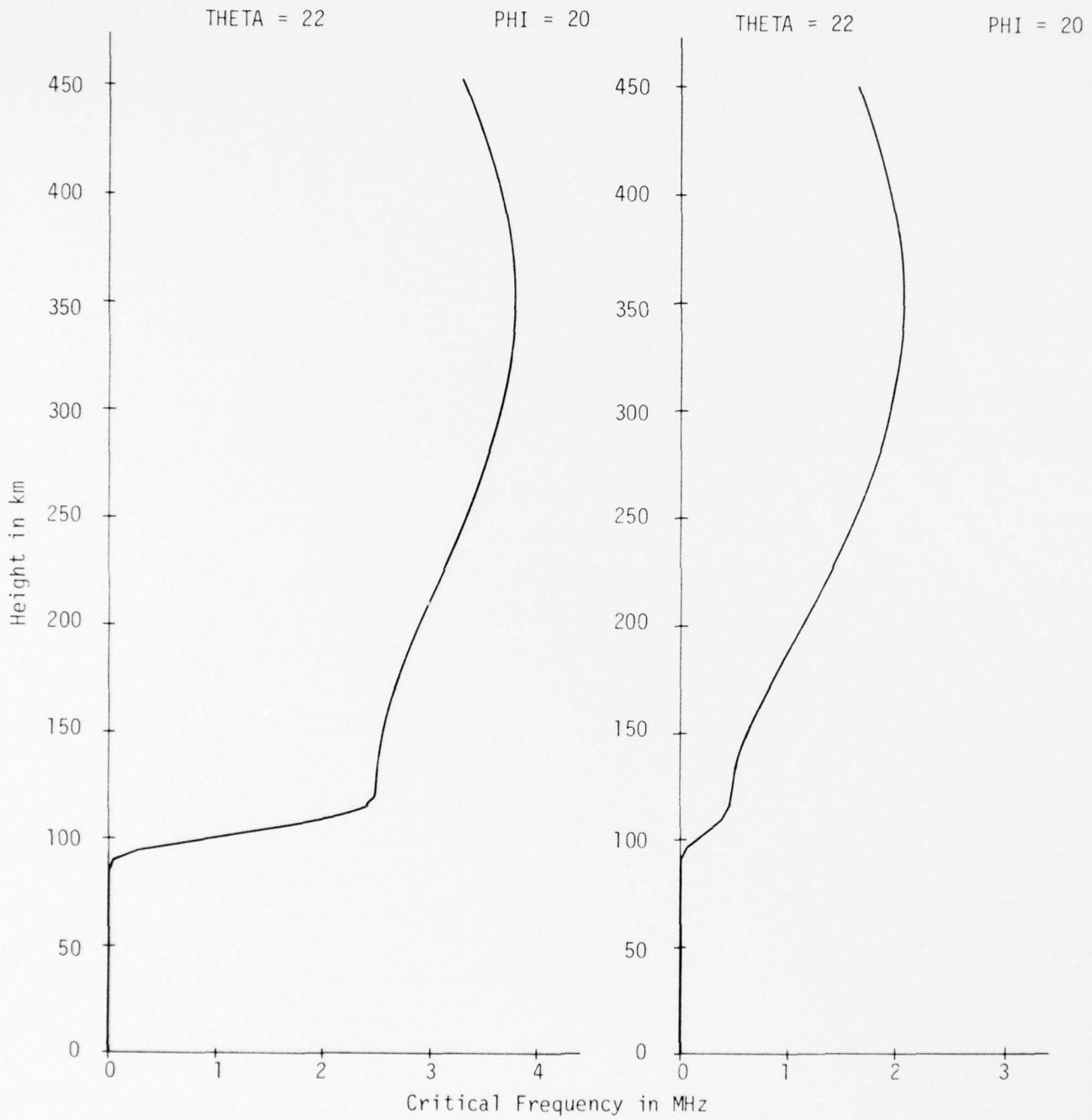


FIGURE 3. VERTICAL PROFILES OF PLASMA FREQUENCY PRODUCED BY SUBROUTINE BCHAP

Model I is the median model, II and III are derived from the upper and lower quartiles of the model gradient, while models IV (most shallow trough), and V (steepest and widest trough wall) are extremes determined from the model.

The vertical variation of the plasma frequency is broken into two segments. Up to the E-layer maximum at 120 km., it is composed of a Chapman layer with a scale height of 10 km. From 120 km. to 350 km. the variation is sine squared in the square of the plasma frequency, i.e.

$$f_p^2 = f_E^2 + (f_{F_{\max}}^2 - f_E^2) \cdot \sin^2 H$$

$$\text{where } H = \frac{\pi}{2} \frac{h - h_{\max E}}{h_{\max F} - h_{\max E}} = \frac{\pi}{2} \frac{h - 120}{230}.$$

$h$  is the height in km.

Sample vertical profiles are given in Figure 3. Details on the electron density model, subroutine BCHAP, are given in Appendix B.

The sine squared segment from 120 to 350 km. was used because it was felt that this is a more realistic presentation of the polar ionosphere. This approach made the model less sensitive to variation in the height of the F2 layer than it would be in a Chapman model. Detailed comparisons of the two models are given in Section 3.1. A sketch of the two types of profiles is given in Figure 4.

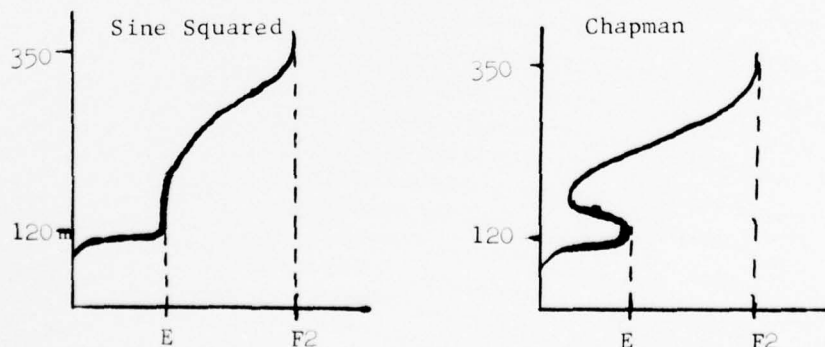


Figure 4. Comparison of Sine Squared and Chapman Vertical Profiles

### 3. Displays Used

Since the quantity which the OTH backscatter system yields as an indication of target range is the group delay, the group delay must then be converted to ground range. As a possible model for determining ground range from group path length, the sine squared model with an F-layer maximum of 3.0 MHz at 350 km. and an E-layer maximum of 0.1 MHz at 120 km. was used. The curve of group path versus ground range for a transmission frequency of 5 MHz is given in Figure 5.

To get a measure of the deviation in azimuth angle and error in ground range caused by the trough wall model, ray tracing results from the disturbed models previously described were compared with results from the horizontally stratified 3 MHz model described above. (3 MHz was considered a reasonable choice, since it is well within the range of foF2 values to the north of the poleward wall model, as shown under foF2<sub>max</sub> in Table 1. It also is a reasonable value, if the standard ITS-78 foF2 map would be considered the basis for the computation of radar range.) The comparison of range and azimuth results for computations using the realistic and the horizontally stratified models was made for several frequencies (4 to 9 MHz), a selected azimuth range of transmissions (26.5° to 42° East of North in the geographic coordinate system), and for elevation angle from 0° to a maximum of 30°.

The procedure for computing range errors was to consider as a reference the range versus group path derived from the 3 MHz model as given in Figure 5. For instance, for a transmission frequency of 5 MHz, an elevation angle of 5°, and an azimuth angle of 36.5°, the model referred to as Trough I gave a group path length of 2644 km. and a ground range of 2295 km. The 3.0 MHz standard model would yield a ground range of 2564 km. for a group path length of 2644 km. meaning that the ground range of the target determined from a horizontally stratified model was actually 269 km. longer than the actual ground range. Thus the range

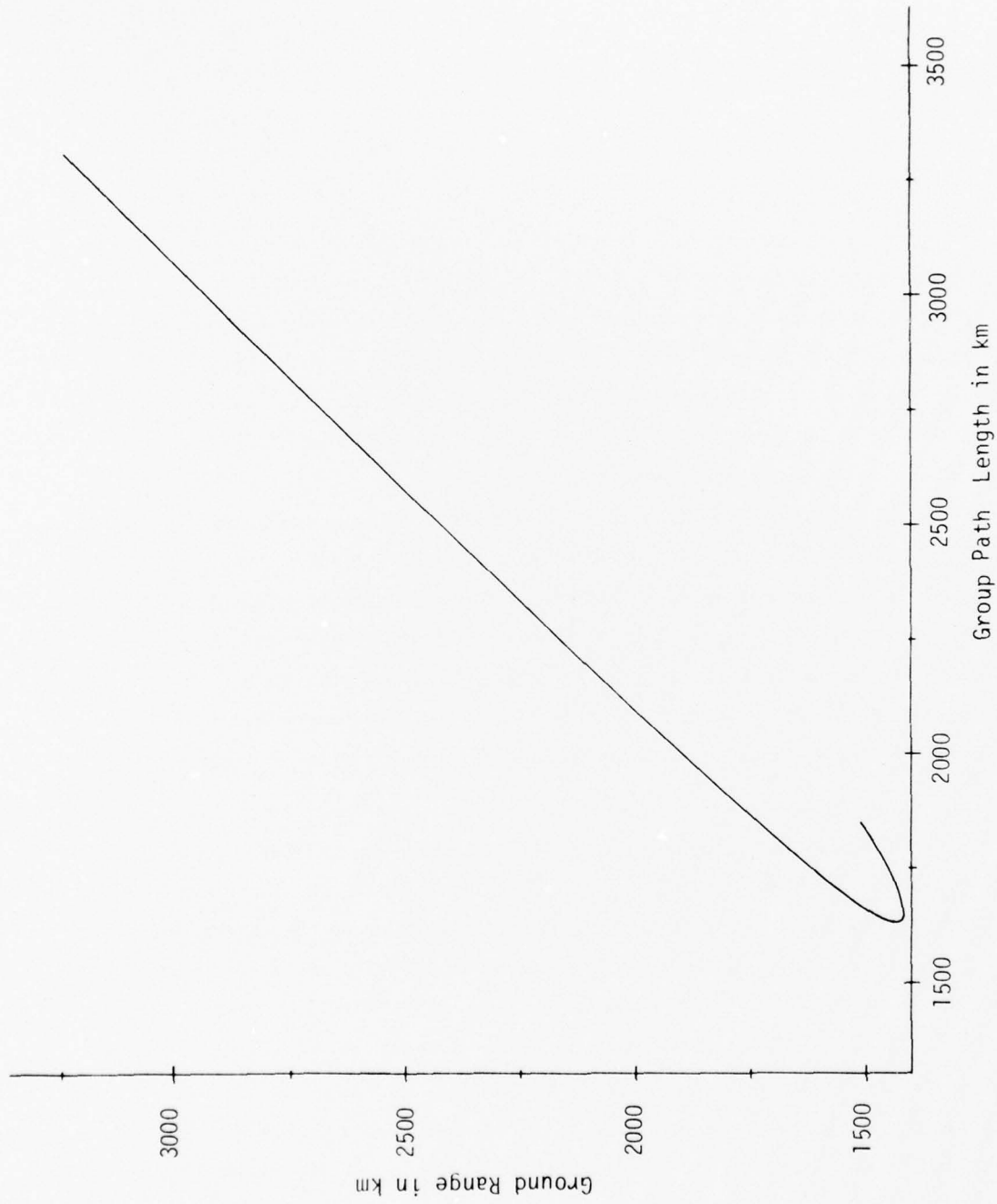


FIGURE 5. GROUND RANGE VERSUS GROUP PATH LENGTH FOR THE 3.0 MHz STANDARD TRANSMISSION FREQUENCY - 5 MHz

error, which would be observed if a realistic model were not available and radar range computations were based on a (reasonable) 3 MHz stratified model, would be +269 km.

A similar procedure was followed in estimating the power difference for the return from a target whose path was influenced by the trough against that of a target that was observed via a 3 MHz horizontally stratified ionosphere (outside the trough wall), using the 3 MHz standard ionosphere. The power level loss was computed as a function of group path length as shown in Figure 6. The power level for any given trough model, frequency, elevation, and azimuth angle was then compared against the standard according to its group path length. In this way it was determined whether the target would appear stronger or weaker as a result of trough bending, and by how much. Details on the power calculations are given in Section 3.2.

### 3.1 Comparison of Sine Squared Versus Chapman Profile

Before using the 3.0 MHz sine squared layer with its maximum at 350 km. as the standard for determining the ground range and power as a function of group path, the curves were also computed for  $f_oF_2 = 2.5$  and 3.5 MHz (Figures 7 and 10) and for the 3.0 MHz case with the maximum at 300 and 400 km. (Figures 8 and 11).

When the value of  $f_oF_2$  was varied, the skip distance varied considerably as could be expected, but the ground range derived from a given group path varied by less than 50 km with the change of  $f_oF_2$  from 2.5 to 3.5 MHz. The power values for a given group path length varied by 5 dB for values away from the skip distance.

When the height of the F layer maximum was varied for the case of 3.0 MHz, the skip distance varied little. The ground range for a given group path varied by less than 30 km in most areas and the power loss varied by less than 2 dB for elevation angles below the skip elevation angles.

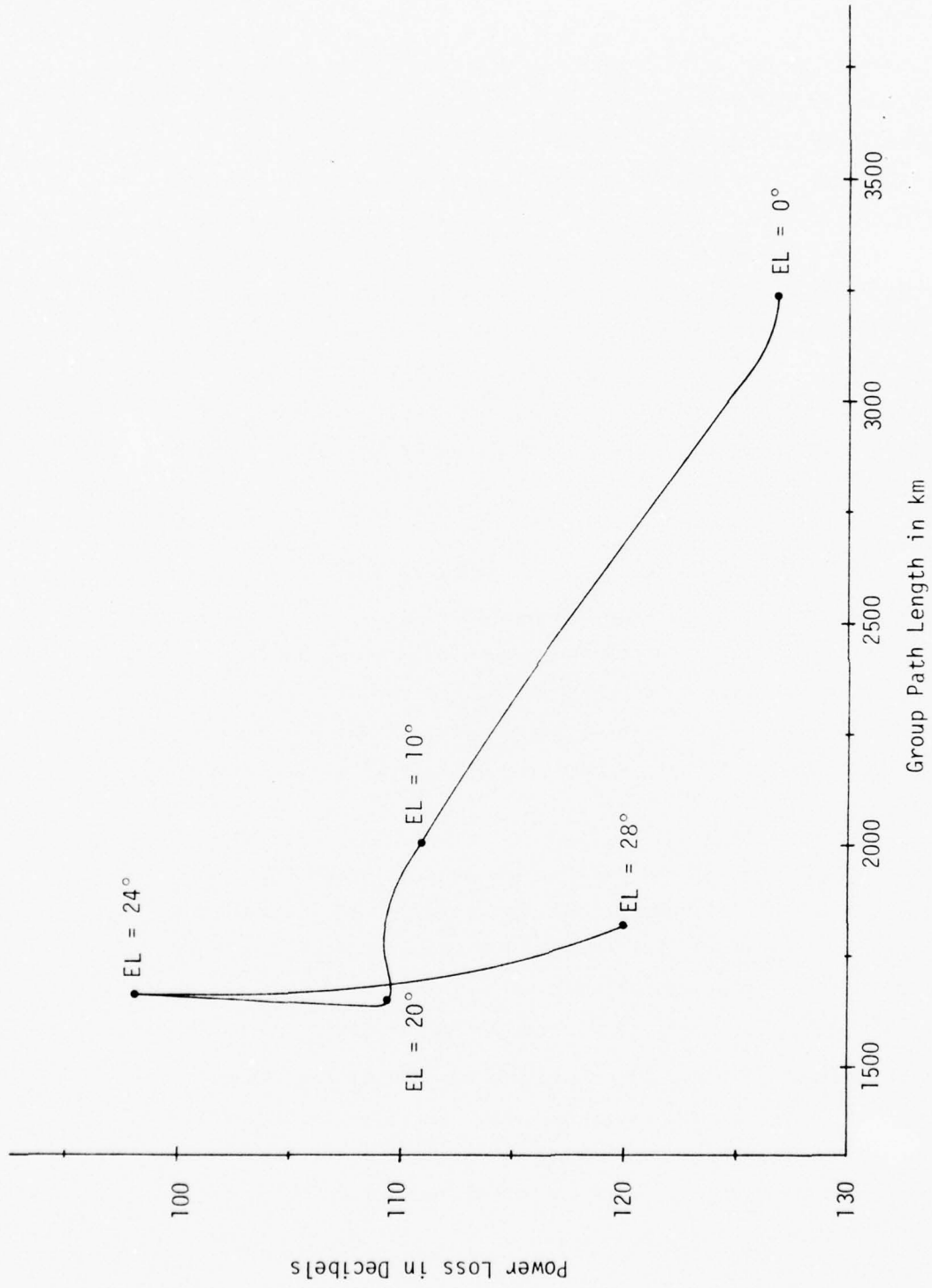


FIGURE 6. POWER LOSS VERSUS GROUP PATH LENGTH FOR THE 3.0 MHz STANDARD TRANSMISSION FREQUENCY - 5 MHz

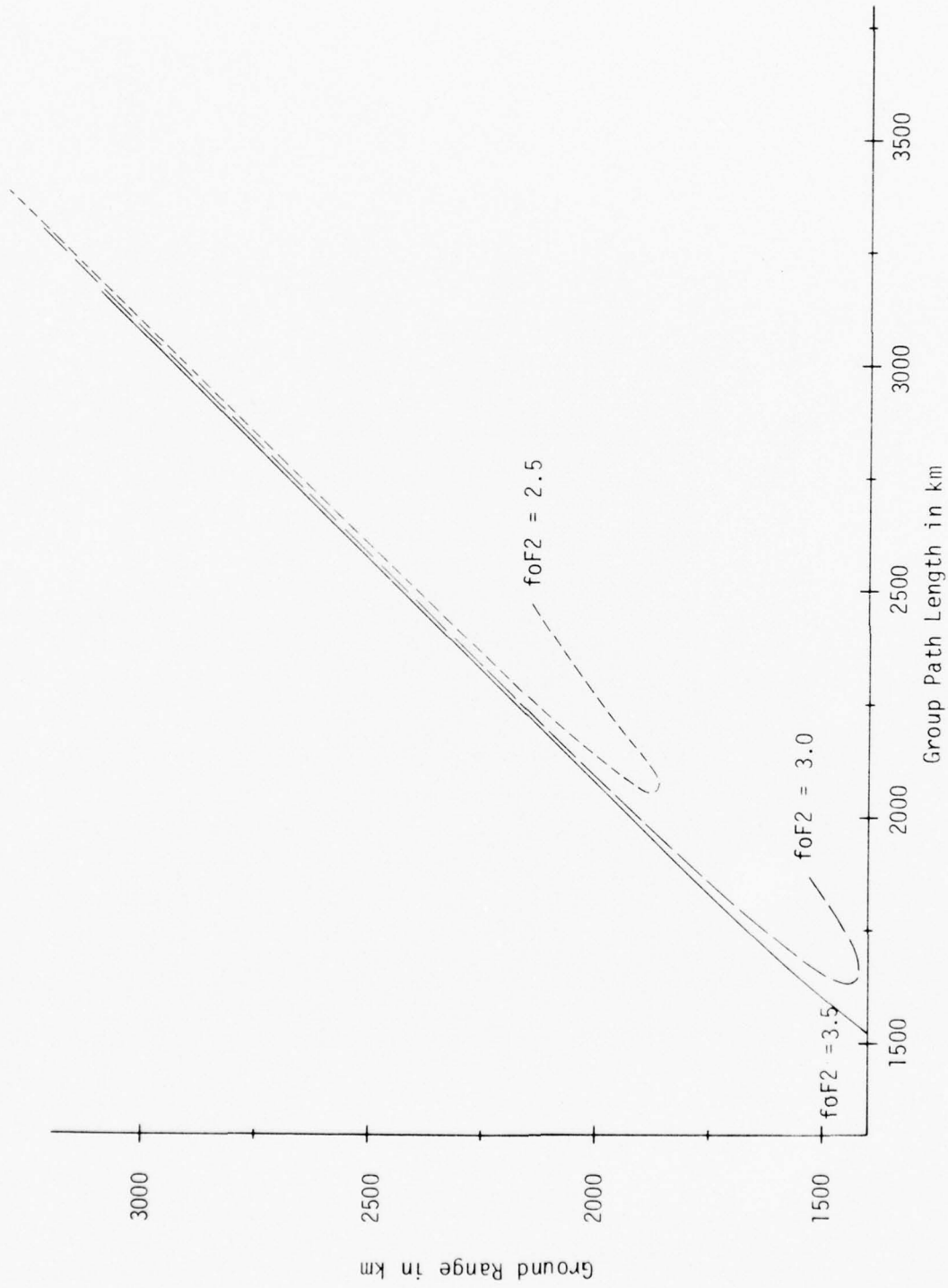


FIGURE 7. GROUND RANGE VERSUS GROUP PATH LENGTH FOR SINE SQUARED PROFILES WITH  
 $f_oF_2 = 2.5, 3.0, \text{ AND } 3.5 \text{ MHz}$  LOCATED AT 350 km.  
 TRANSMISSION FREQUENCY IS 5 MHz.

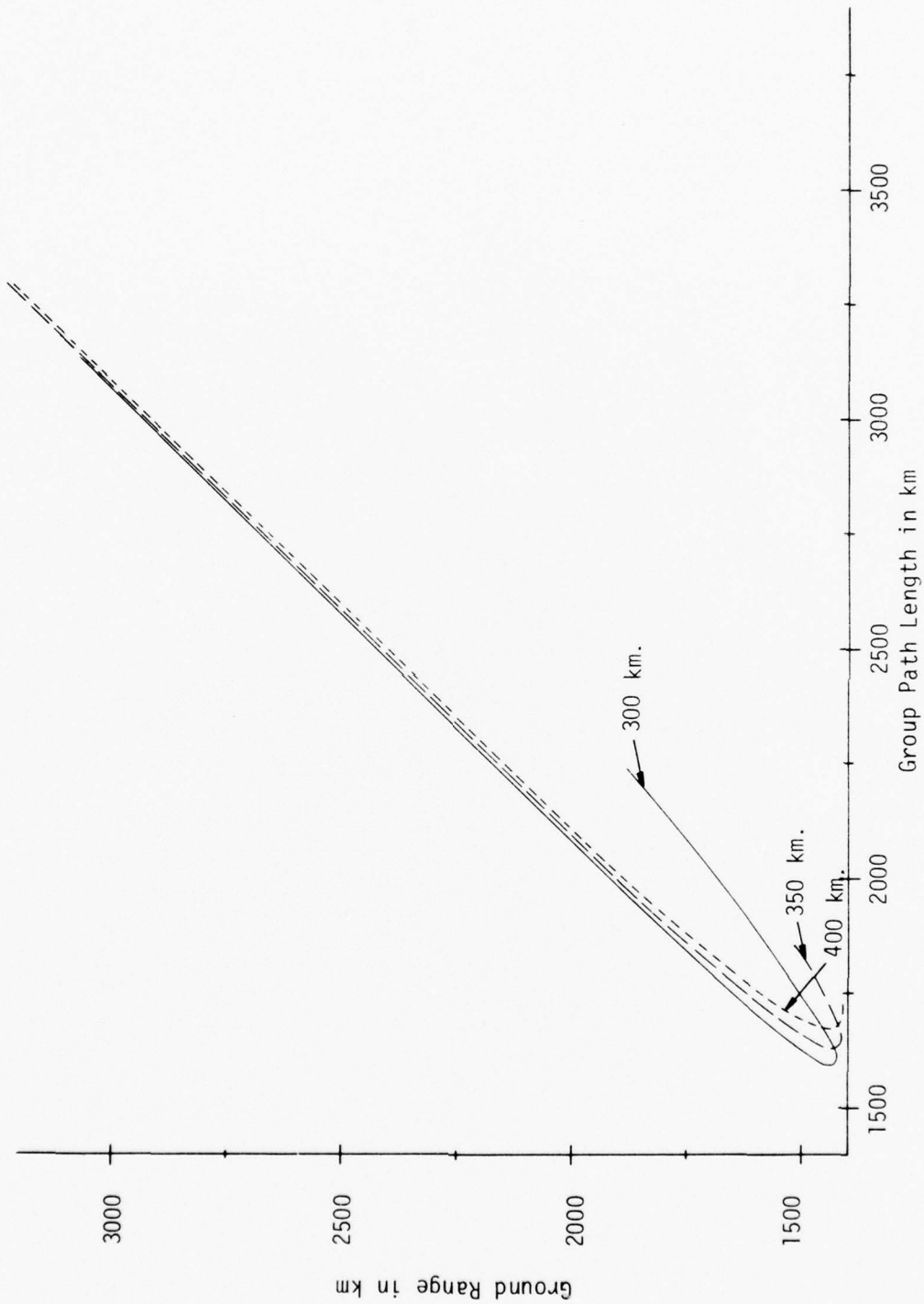


FIGURE 8. GROUND RANGE VERSUS GROUP PATH LENGTH FOR SINE SQUARED PROFILES WITH  $f_0F_2 = 3.0$  MHz LOCATED AT 300, 350, AND 400 km. TRANSMISSION FREQUENCY IS 5 MHz.

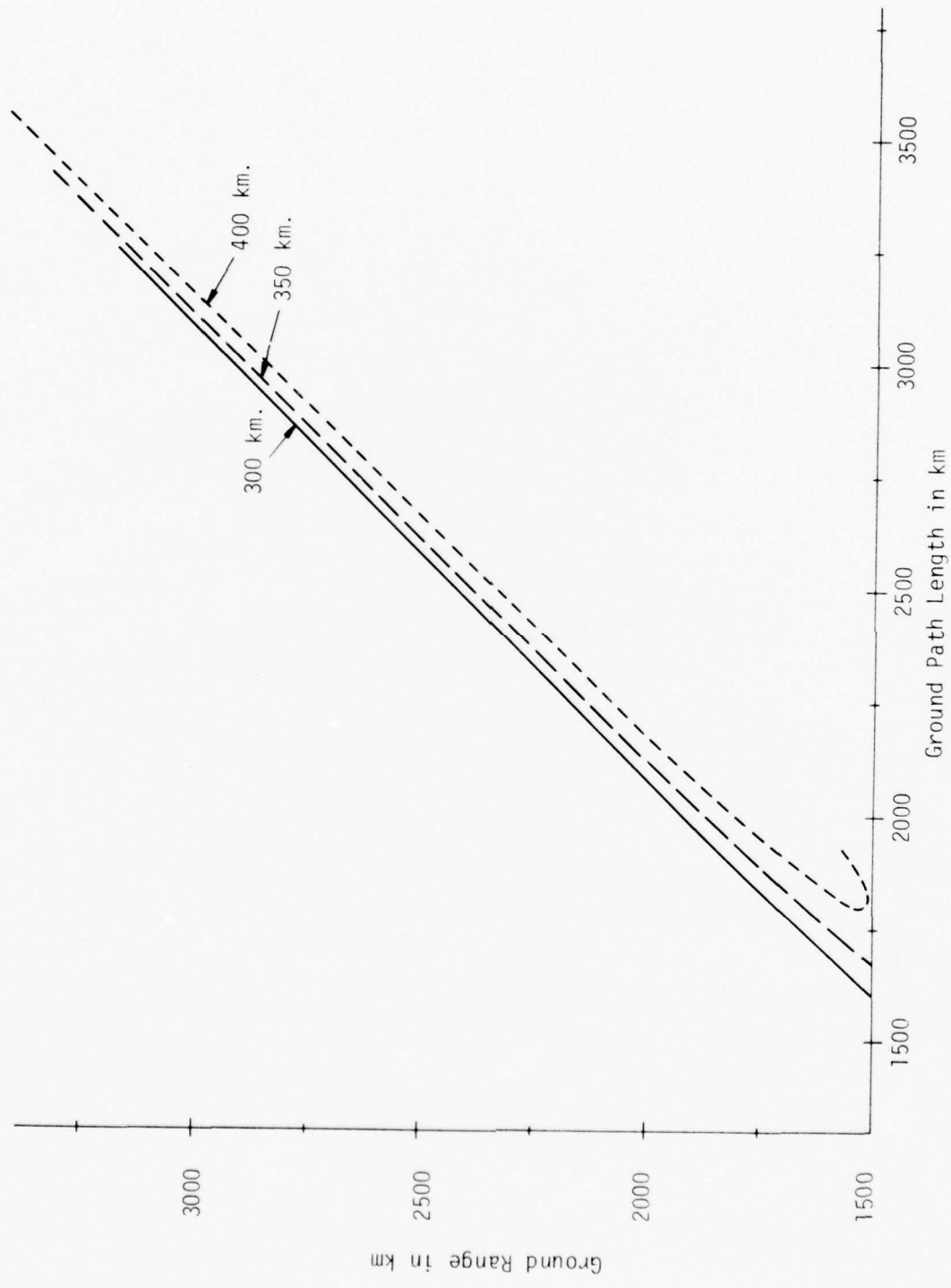


FIGURE 9. GROUND RANGE VERSUS GROUP PATH LENGTH FOR CHAPMAN PROFILES WITH  $f_oF_2 = 3.0$  MHz LOCATED AT 300, 350, AND 400 km. TRANSMISSION FREQUENCY IS 5 MHz.

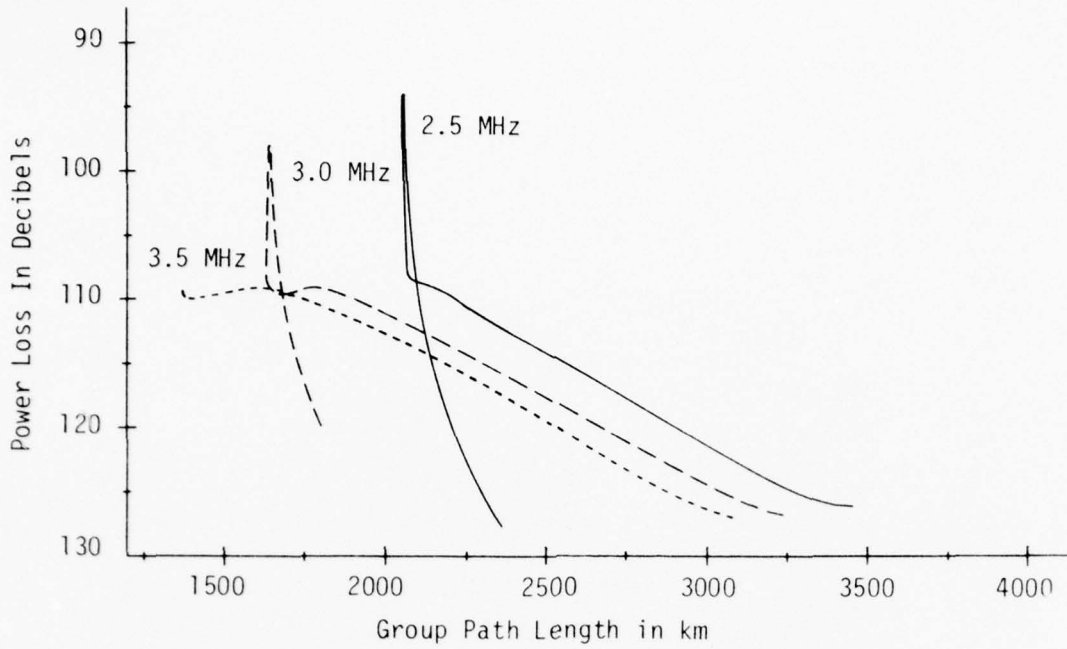


FIGURE 10. POWER LOSS VERSUS GROUP PATH LENGTH FOR SINE SQUARED PROFILES WITH  $f_oF_2 = 2.5, 3.0,$  AND  $3.5$  MHz LOCATED AT 350 km ALTITUDE

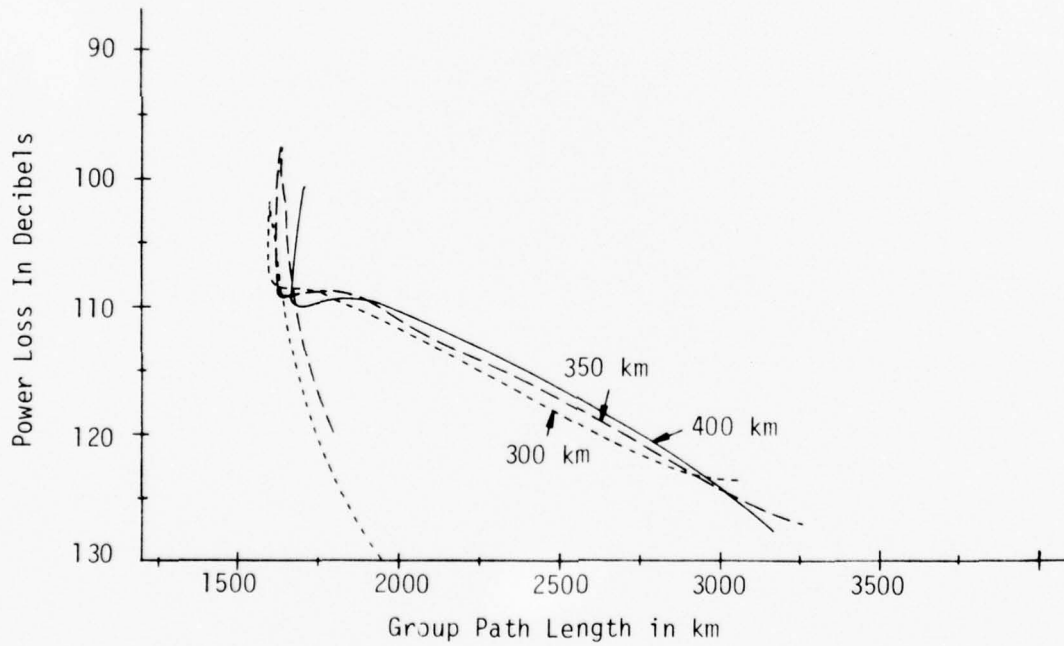


FIGURE 11. POWER LOSS VERSUS GROUP PATH LENGTH FOR SINE SQUARED PROFILES WITH  $f_oF_2 = 3.0$  MHz LOCATED AT 300, 350, AND 400 km

To compare the results derived from the sine squared model with those from a Chapman layer model, rays were traced through a 3.0 MHz Chapman layer with scale height of 67.5 km and with the maximum at heights of 300, 350, and 400 km.

The variation of ground range for a given group path was in the order of 100 km over the three models (Figure 9), three times as sensitive to a 100 km change in  $h_{\max}$  as the sine squared layer. The case for the maximum at 300 km closely followed the 3.0 MHz sine squared standard which was used. The power loss variation for a given group path varied by as much as 8 dB and was considerably different in appearance from the sine squared curves (Figure 12). Again the 300 km case most strongly resembled the sine squared standard.

The chosen sine squared model thus produces results which are less sensitive to  $h_{\max}$  or foF2 changes than those derived from a Chapman model.

### 3.2 Power Loss Calculations

Power loss calculations were for backscattered power returned from ground reflections for the area covered by a beam  $1^\circ$  in elevation and  $1^\circ$  in azimuth.

The power returned,  $P_r$ , is given by:

$$P_r = \frac{P_o \lambda^2}{4\pi} \cdot \left(\frac{\Delta S}{A_p}\right)^2 \cdot \sigma \cdot \frac{P_w}{2\sigma_t}$$

where  $P_o$  is the transmitted power.

The quantity used as the power loss in db is actually

$$10 \log_{10} \left(\frac{P_r}{P_o}\right) = 10 \log_{10} \left(\frac{\lambda^2}{4\pi}\right) + 20 \log_{10} \frac{\Delta S}{A_p} + 10 \log_{10} \sigma + 10 \log_{10} \left(\frac{P_w}{2\sigma_t}\right)$$

where  $\lambda$  is the wave length in km.

$\frac{\Delta S}{A_p}$  is the ray spreading

$\sigma$  is the backscatter cross-section

$P_w$  is the pulse width (50 km. was used)

$\sigma_t$  is the rms group path deviation.

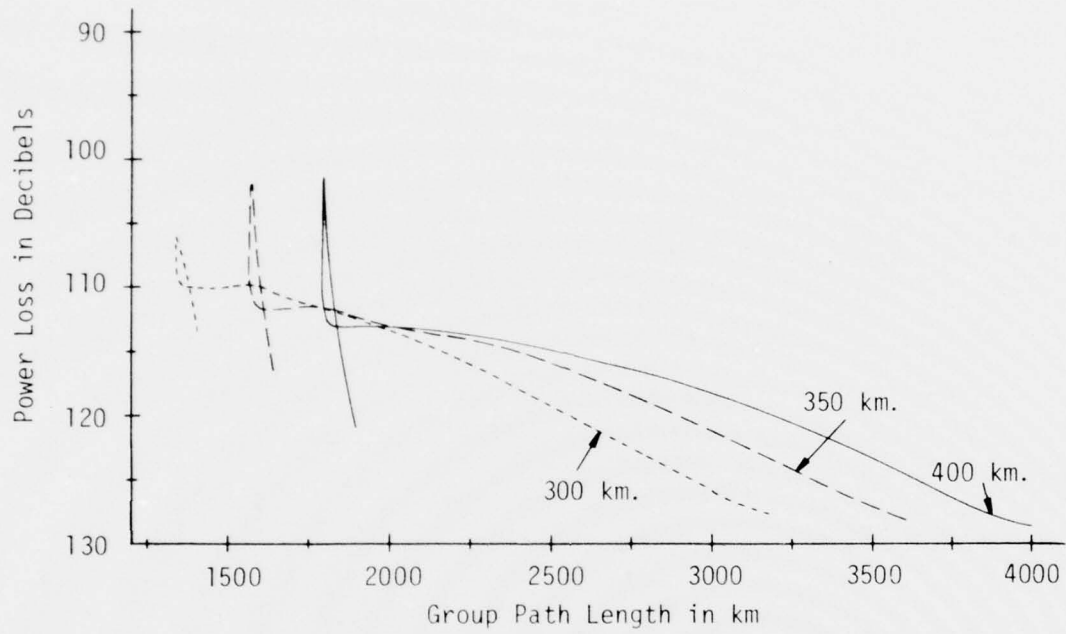


FIGURE 12. POWER LOSS VERSUS GROUP PATH LENGTH FOR CHAPMAN PROFILES WITH  $f_oF_2 = 3.0$  MHz LOCATED AT 300, 350, AND 400 km

Other power loss factors which are not included at the present time are: absorption, transmitter and receiver gain. Absorption was not considered in these model computations, since it would not impact differently on rays propagated via a disturbed or a smooth ionosphere. The transmitter and receiver antenna patterns may be readily imposed as a function of frequency, azimuth, and elevation on the results reported in Section 4. A more detailed discussion of the computation of backscatter and spread losses may be found in Chapter IV of RADC-TR-77-60.

#### 4.0 Significant Observations

Rays were traced through the trough models given in Table 1 and the power differences, range errors, and azimuth deviations were computed. Transmission frequencies used were 4, 5, 6, 7, 8, and 9 MHz. It was found, for the models used, that rays escaped for most elevation angles for frequencies greater than 6 MHz. Trough IV, the model with the weakest gradient and lowest foF2 poleward of the trough wall, also gave poor results for frequencies of 5 and 6 MHz. Rays were traced every degree in elevation from 0 to  $30^{\circ}$  and for azimuth angles from  $26.5^{\circ}$  to  $42^{\circ}$  in the geographic coordinate system. The hypothetical transmitter was located at  $44.8^{\circ}$ N geographic latitude and  $67.8^{\circ}$ W geographic longitude. The dipolar geomagnetic pole was assumed located at  $78.565^{\circ}$ N latitude and  $69.761^{\circ}$ W longitude. Azimuth angles of  $41^{\circ}$  and greater were unaffected by the trough wall.

Using Trough Models II (weakest gradient, lower quartile), I (median) and III (strongest gradient, upper quartile), the effects of an increase in the trough wall steepness on focusing and defocusing can be shown. Figure 14 a, b, and c shows the difference of the backscatter power derived from the realistic trough wall model and that derived from the horizontally stratified 3 MHz model as a function of elevation angle and azimuth angle of the launched ray. In general, the power compared to the 3 MHz standard ionosphere is enhanced over a wide region for Trough II while this enhanced region becomes increasingly narrow in azimuth as the trough wall steepens to Trough III and an ever increasing area of power degradation appears although most relative losses are less than 10 dB.

Relative enhancement of power is greatest for those azimuth angles where the rays just graze the trough wall. This is in part due to the fact that rays heading more directly into the trough wall are bent down fairly sharply giving a group path length which is nearer to the 3.0 MHz skip distance. Even though these backscatter signals do have a power comparable to the more grazing rays, they show less power returned if compared with the horizontally stratified model.

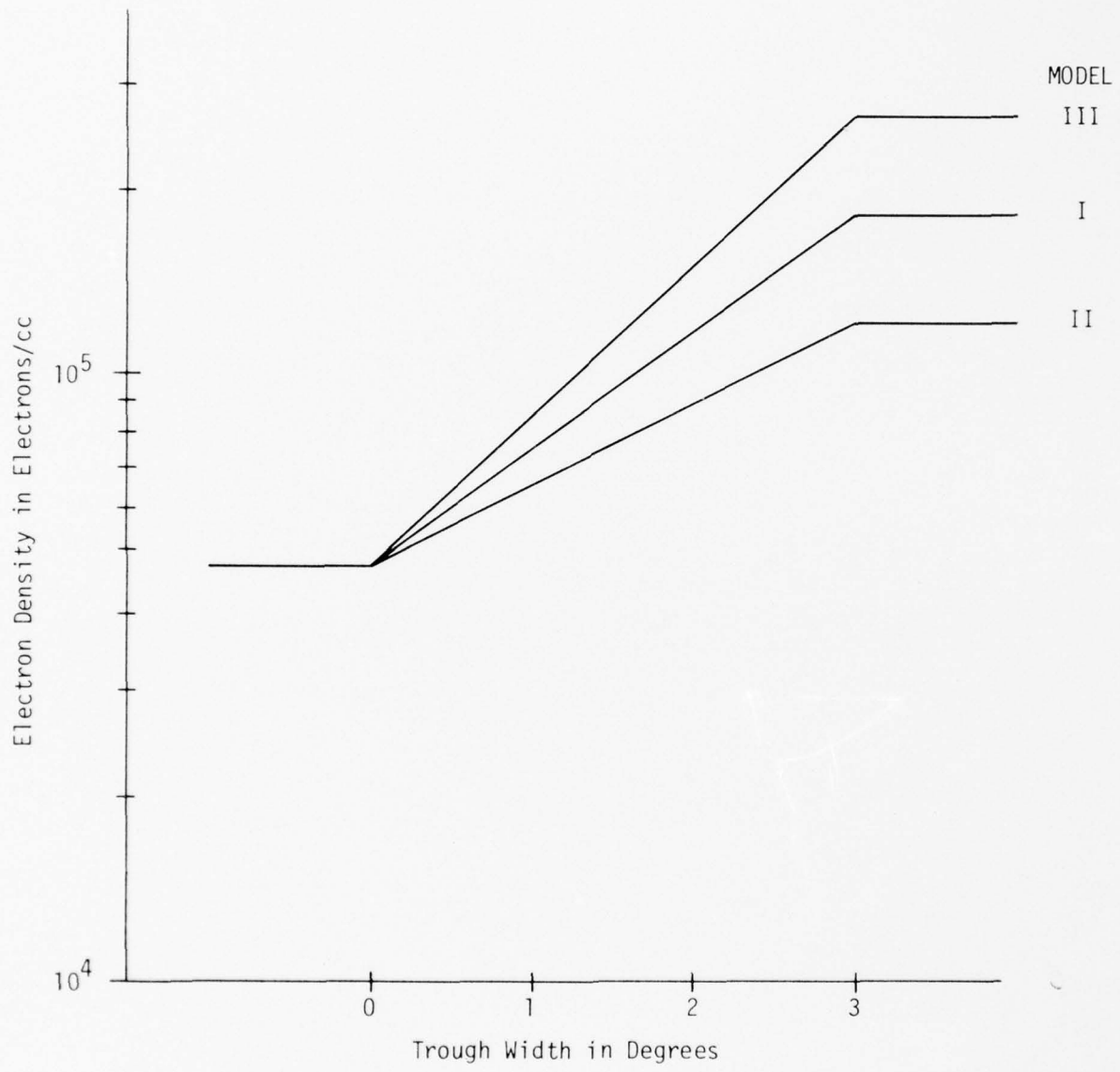


FIGURE 13. MEDIAN TROUGH AND GRADIENT RANGE

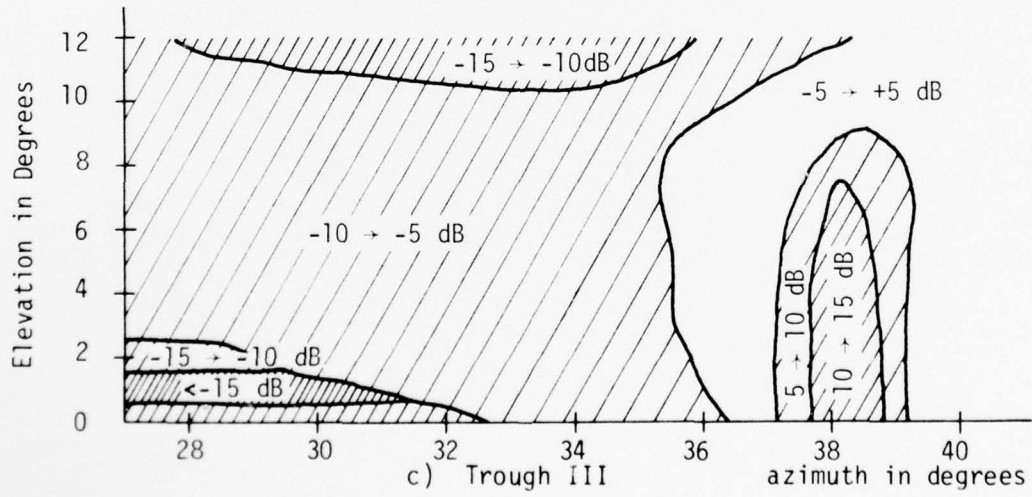
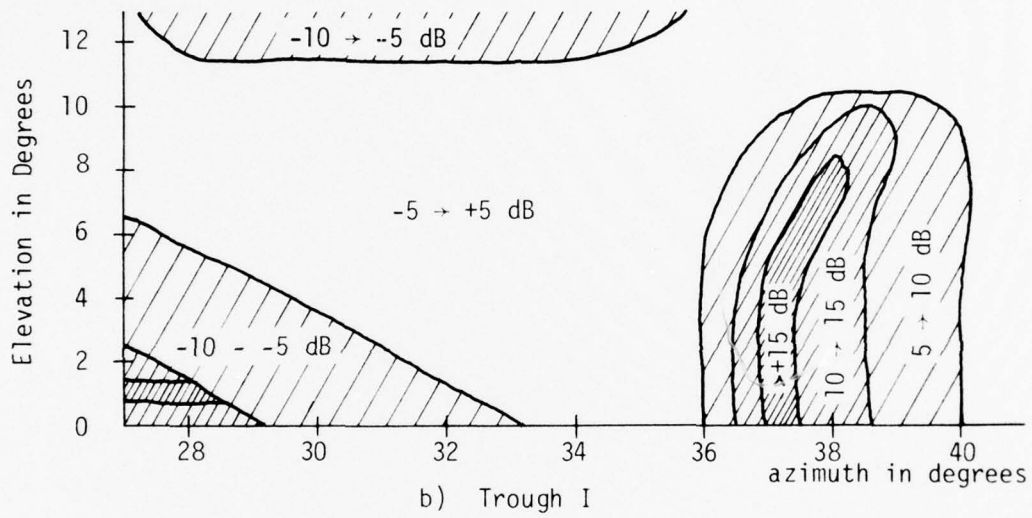
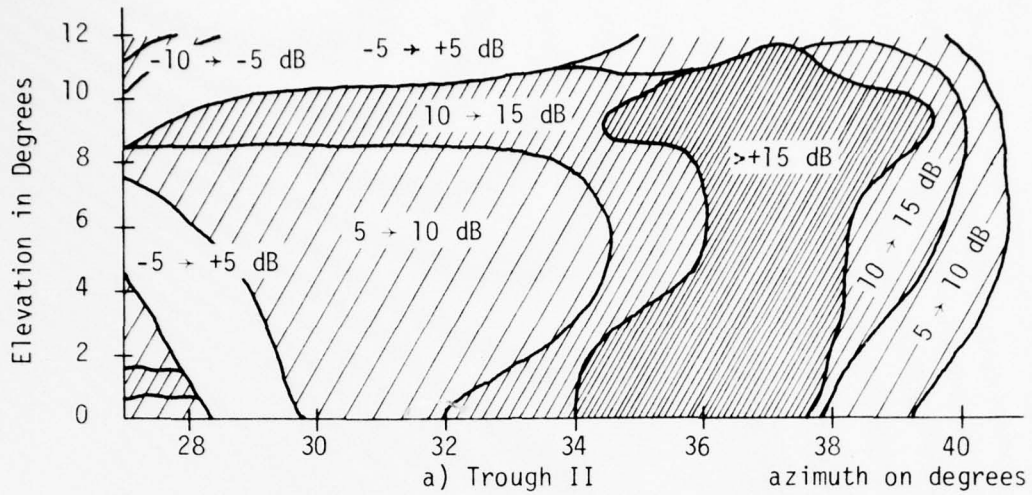


FIGURE 14. POWER LOSS DIFFERENCE WITH 3 MHz FOR SAME GROUP PATH, FREQ. 5 MHz

Rays which escape the effects of the trough wall (those having azimuth angles greater than approximately  $41^{\circ}$ ) experience an apparent enhancement of power if the 3MHz reference model is used. This occurs because, in fact, these rays travel through a 2 MHz horizontally stratified ionosphere. In order to eliminate this false enhancement a 2 MHz reference is used for these rays and for rays which may be slightly influenced by the trough wall but which for the most part travel in the 2MHz ionosphere. Thus for all rays with azimuth angles greater than  $39^{\circ}$ , the 2 MHz reference is used.

The range and azimuth errors shown in Figures 15 and 16 show a behavior similar to one another in that large range errors generally occur for the same azimuth and elevation angles as large azimuth deviations. Range errors decrease with increasing wall steepness mainly due to the fact that rays are bent more sharply downwards and hence travel less far in a direction tangential to the transmitter. The azimuthal deviation also decreases with increasing wall steepness up to a point although rays are bent sideways more sharply as the steepness increases. This is again due to the fact that rays travel farther after they are bent by a weak trough wall. However, as the trough wall continues to increase in steepness, although rays travel less far after bending, they are bent so sharply sideways that the overall azimuth deviation increases.

This behavior can be more clearly understood by looking at the ray paths as a function of range and azimuth from the transmitter which are shown for Troughs II, I, and III in Figures 17, 18, and 19 respectively. The elevation angle in these figures is  $7^{\circ}$ . For the azimuth angle  $35^{\circ}$ , the azimuth deviations, range errors, and total ground range are given in Table 2.

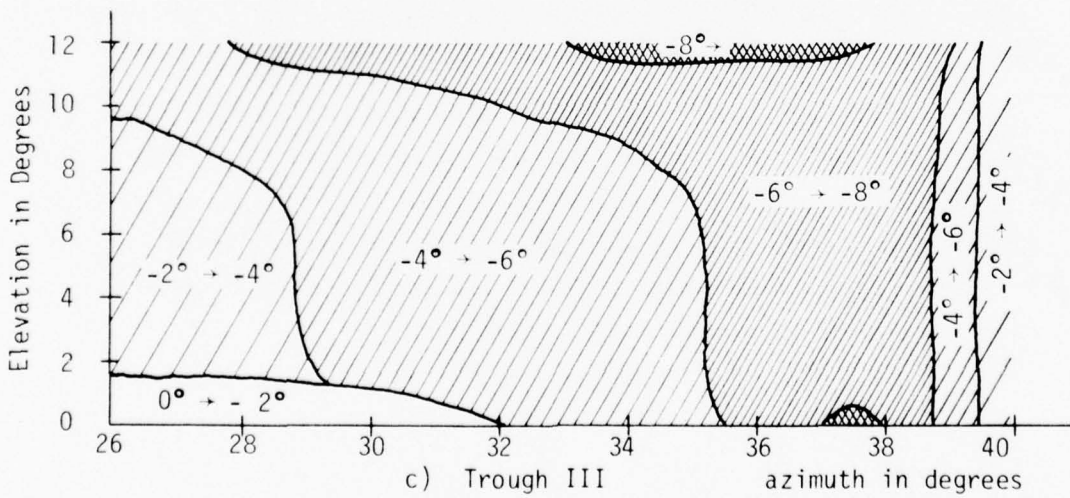
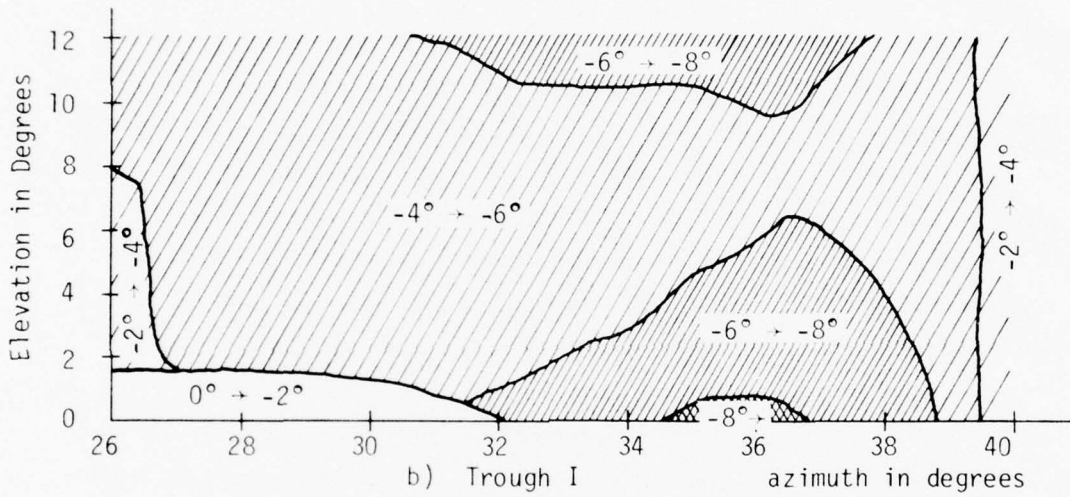
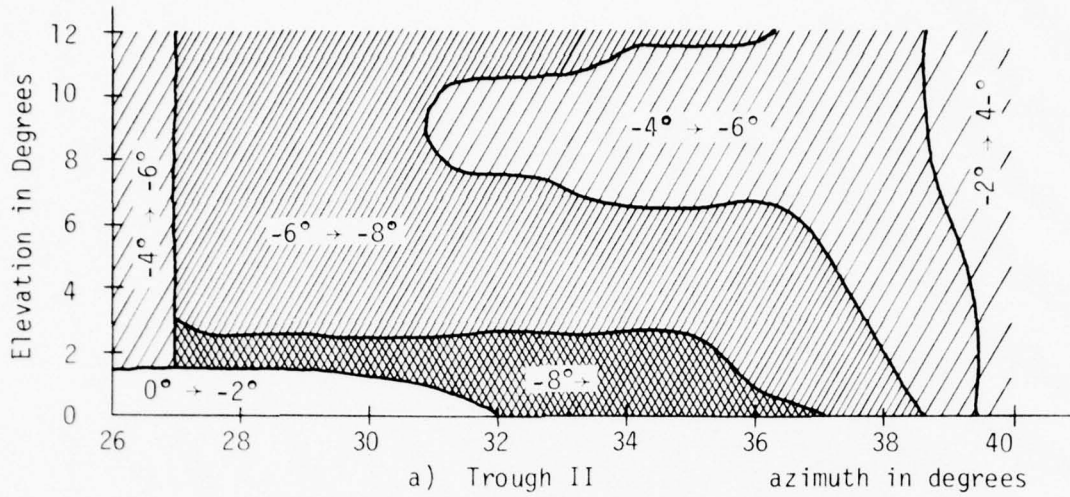


FIGURE 15. AZIMUTHAL DEVIATION AS A FUNCTION OF ELEVATION AND AZIMUTH FOR 5 MHz

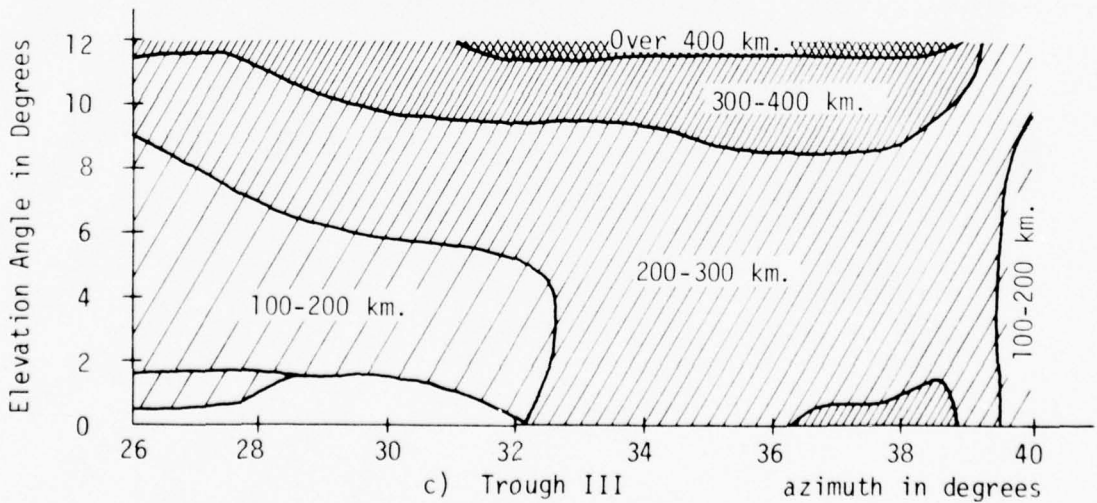
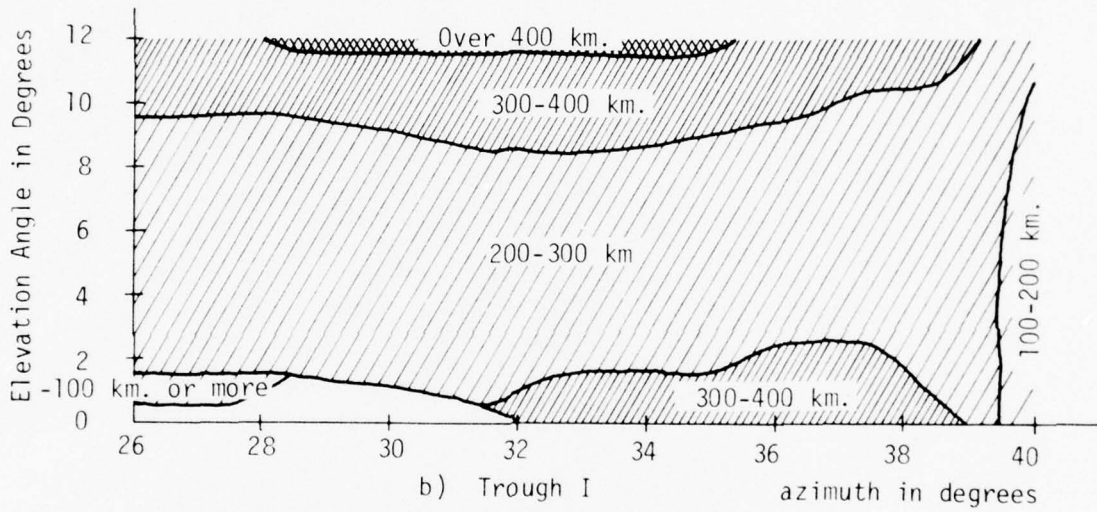
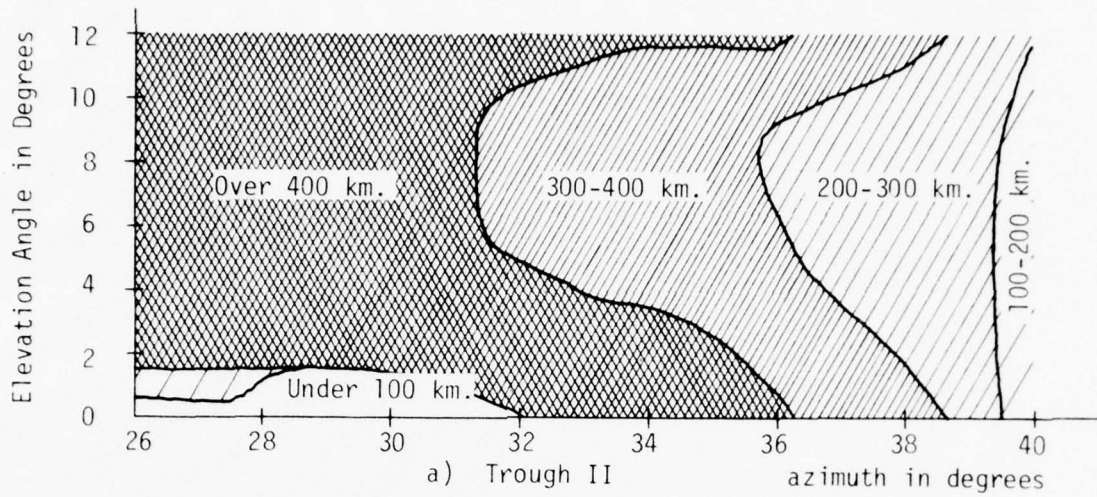


FIGURE 16. RANGE ERRORS USING 3.0 MHz STANDARD FOR 5 MHz TRANSMISSION FREQ.

Plasma Frequency Contours (MHz)  
Are For Height = 350 km

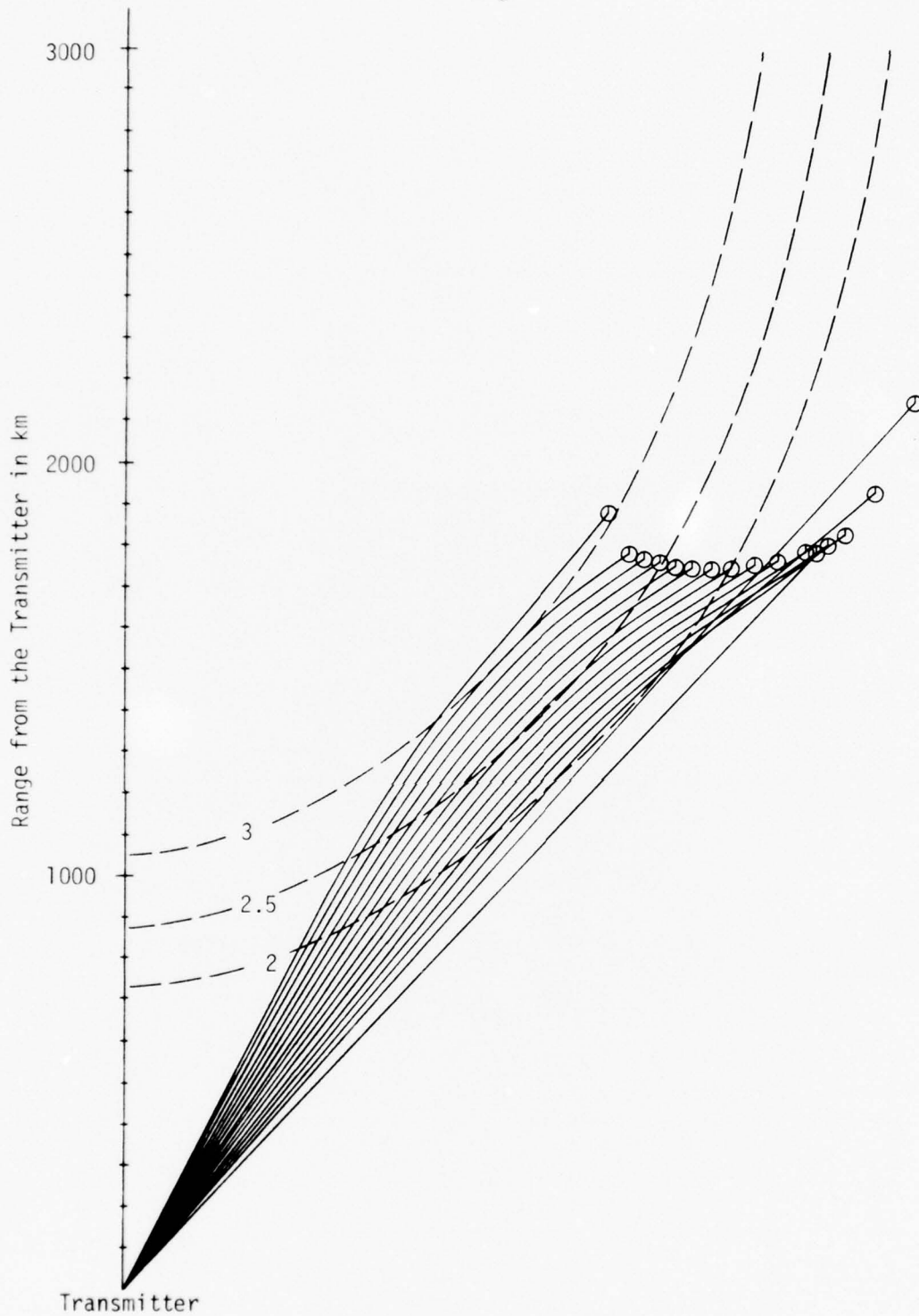


FIGURE 17. RAY PATHS PLOTTED AS A FUNCTION OF RANGE AND AZIMUTH FROM THE TRANSMITTER FOR 7° ELEVATION ANGLE, TRANSMISSION FREQUENCY 5 MHz, AND IONOSPHERIC MODEL TROUGH II

Plasma Frequency Contours (MHz)  
Are For Height = 350 km

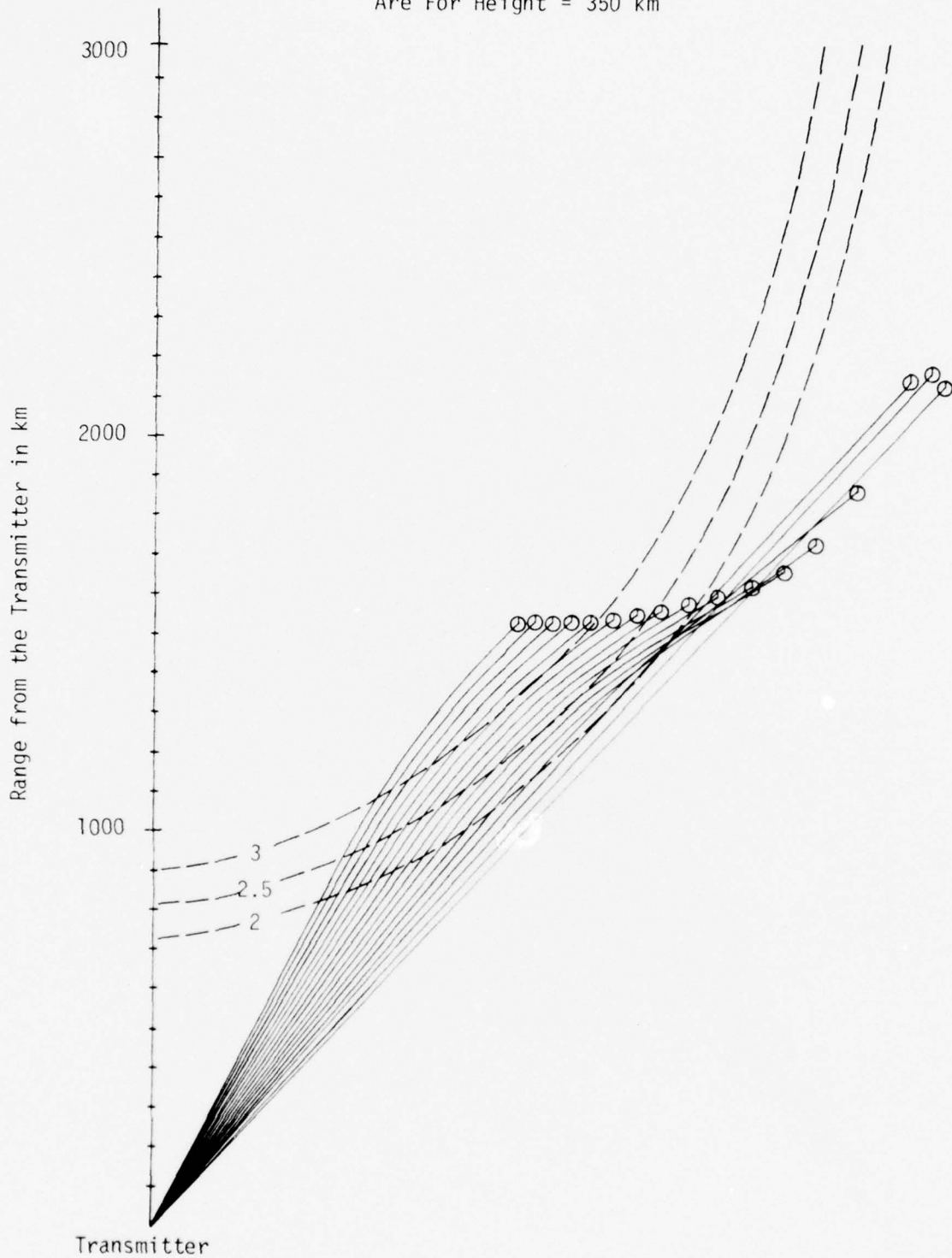


FIGURE 18. RAY PATHS PLOTTED AS A FUNCTION OF RANGE AND AZIMUTH FROM THE TRANSMITTER FOR 7° ELEVATION ANGLE, TRANSMISSION FREQUENCY 5 MHz, AND IONOSPHERIC MODEL TROUGH I

Plasma Frequency Contours (MHz)  
Are For Height = 350 km

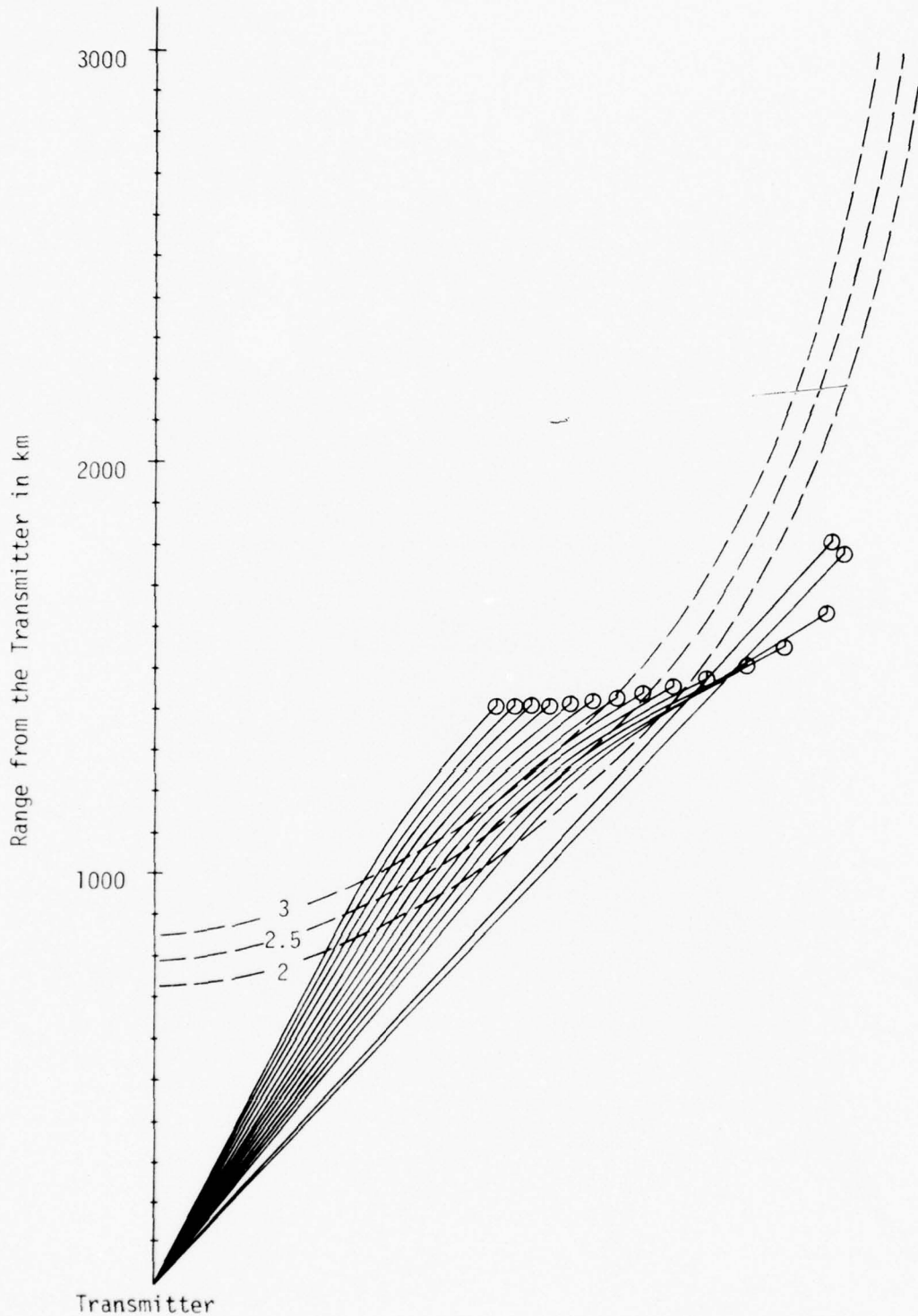


FIGURE 19. RAY PATHS PLOTTED AS A FUNCTION OF RANGE AND AZIMUTH FROM THE TRANSMITTER FOR 7° ELEVATION ANGLE, TRANSMISSION FREQUENCY 5 MHz, AND IONOSPHERIC MODEL TROUGH III

Table 2. Comparison of Azimuth Deviations and Range Errors for Three Trough Models at Elevation Angle -  $7^{\circ}$ , Azimuth Angle -  $35^{\circ}$ , and Frequency - 5 MHz.

Model	Azimuth Deviation in degrees	Range Error in km.	Ground Range in km.	Ground Reflection Azimuthal Angle
II	-5.8	300	2348	50.88
I	-5.6	270	2115	51.47
III	-5.9	270	1962	53.87

The ground reflection azimuthal angle given in Table 2 is the azimuth angle East of North geomagnetic at which the ray was traveling when a ground reflection occurred. In this case, while the ray for Model II was bent to an azimuth angle less than Model I, its azimuth deviation was greater due to the increased ground range gained (particularly that gained after off great circle bending). However, for Model III the azimuth angle to which the ray was bent was great enough that there was an increase in azimuth deviation over Models II and I even though the ray went less far.

## 5.0 Conclusions

For the great majority of the rays examined the power return from targets located at their landing points should be great enough to allow detection, if detection at the same frequency under smooth ionospheric conditions would have been possible. The assumptions made about the target's position, if a trough model is not considered in such a determination, could be off in range by up to 500 km and have an azimuth deviation of up to  $9^{\circ}$  (the equivalent of 270 km at the ranges we are considering). If a trough model is considered, target positions are still strongly dependent on the chosen trough wall gradient. Ranges vary from +100 to -50 km relative to ranges derived from the median model, if the gradient is changed to the lower quartile or the upper quartile value respectively.

There is also a region close to the trough wall boundary where multi-path occurs thus giving rise to the appearance of two targets where one actually exists.

6. References

- Langworthy, B.M., and T.B. Barrett (1975) Implements and Techniques for the Analysis of Radio Propagation Through the Arctic Ionosphere Using Three-Dimensional Ray Tracing, AFCRL-TR-75-0319, Hanscom AFB, Mass.
- Langworthy B., T. Barrett, D. Bandes, and L. Calabi (1977) Analysis and Synthesis of Model Ionograms Using 3-D Ray Tracing Techniques, RADC-TR-77-60, Griffiss AFB, New York.
- Pike, C.P., (1976) An Analytical Model of the Main F-Layer Trough, AFGL-TR-76-0098, Hanscom AFB, Mass.

APPENDIX A

NAME: WALL, revision 0, program, PML 152  
CATEGORY: Preprocessing program for use with ray-tracing programs  
TITLE: Trough wall preprocessing program for subroutine BCHAP  
LANGUAGE: CDC extended Fortran - version 4  
PROGRAMMER: B.M. Langworthy, Parke Mathematical Laboratories, Inc.  
DATE: October 21, 1976

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DESCRIPTION

Program WALL accepts coordinates of a path in the geographical coordinate system which specifies the poleward side of the arctic trough bottom. It then converts these points to the dipolar geomagnetic system and computes certain quantities needed in the computation of plasma frequency by ray-trace subroutine BCHAP. These quantities are put out on a tape or permanent file for use by the ray-trace program.

INSTRUCTION SET

To use program WALL, the following data must be entered:

Card 1 (A10)

NAME - This is a 10-character alphanumeric name for the trough wall. It will be used by subroutine BCHAP as an electron density subroutine name and will appear on each page of ray-trace output. This name must be punched in columns 1 through 10.

Card 2 (2F10.2) GLAT(1), GLON(1)

GLAT(1) is the geographic latitude in degrees of the first point.  
GLON(1) is the geographic longitude in degrees of the first point.  
GLAT must be punched in columns 1 to 10 and must contain a decimal point. GLON must be punched in columns 11 to 20 and must also contain a decimal point.

Cards 3 and on

contain subsequent points along the trough wall and data should be entered in the same manner as on Card 2.

Output from program WALL includes printed output and an unformatted tape or permanent file referred to as TAPE6. TAPE6 is described under FILE DESCRIPTIONS. A sample of the printed output is shown in Attachment 1. For each input point the following information is given:

- GLAT - the geographic latitude of the point in degrees
- GLON - the geographic longitude of the point in degrees
- $\Theta$  - the dipolar geomagnetic colatitude in degrees
- $\Psi$  - the dipolar geomagnetic longitude in degrees
- a - the length in degrees of the great circle segment connecting  $(\Theta, \Psi)$  with the next point
- $\beta$  - the azimuth angle in degrees of the great circle segment connecting  $(\Theta, \Psi)$  with the next point
- $\delta$  - one half the angle in degrees formed by the two great circles which meet at  $(\Theta, \Psi)$

The message "TAPE6 HAS BEEN WRITTEN" will appear after the table to indicate that the program has completed writing TAPE6.

#### STORAGE REQUIRED

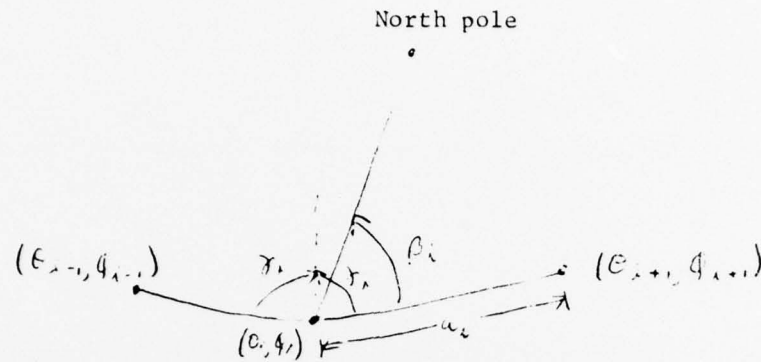
Program WALL requires 40000 octal words of core storage.

#### ALGORITHM

Points inputted in the geographic coordinate system are converted to points  $(\Theta_i, \Psi_i)$  where:

- $\Theta_i$  is the dipolar geomagnetic colatitude of the  $i^{\text{th}}$  entry
- $\Psi_i$  is the dipolar geomagnetic longitude of the  $i^{\text{th}}$  entry

Several other quantities are computed at this point which are useful for electron density subroutine BCHAP. They are  $a_i$ ,  $\beta_i$ , and  $\delta_i$  which are defined and illustrated on the following page.



$$a_i = \cos^{-1}(\cos \theta_i \cos \theta_{i+1} + \sin \theta_i \sin \theta_{i+1} \cos(\phi_{i+1} - \phi_i))$$

$$\beta_i = \cos^{-1} \left( \frac{\cos \theta_{i+1} - \cos \theta_i \cos a_i}{\sin \theta_i \sin a_i} \right)$$

If  $i > 1$

$$a_{13i} = \cos^{-1}(\cos \theta_{i-1} \cos \theta_{i+1} + \sin \theta_{i-1} \sin \theta_{i+1} \cos(\phi_{i+1} - \phi_{i-1}))$$

$$\gamma_i = 1/2 \cos^{-1} \left( \frac{\cos a_{13i} - \cos a_{i-1} \cos a_i}{\sin a_{i-1} \sin a_i} \right)$$

$$\beta_{13i} = \cos^{-1} \left( \frac{\cos \theta_{i+1} - \cos \theta_{i-1} \cos a_{13i}}{\sin \theta_{i-1} \sin a_{13i}} \right)$$

If  $\beta_{13i} > \beta_{i-1}$ ,  $\gamma_i = \pi - \gamma_i$

If  $i = 1$ ,  $\gamma_i = \frac{\pi}{2}$

\*If  $i = \text{NENT}$  or the last entry,  $a_i = 1$  radian,  $\beta_i = \frac{\pi}{2}$ , and  $\gamma_i = \frac{\pi}{2}$ .

#### SPECIAL CAUTIONS AND FEATURES

The number of point entries should not exceed 60.

#### TIMING

Execution time is .05 seconds for 10 points

ERROR MESSAGES

THERE IS NO INPUT DATA.

This message will be printed in data cards if point entries are inadvertently omitted.

SUBROUTINES

DICOORD - The subroutine converts points from the geographic to the dipolar geomagnetic system.

ACCURACY

The accuracy of the trough wall outputted by WALL is dependent on the accuracy and closeness of the input points.

FILE DESCRIPTIONS WALL(INPUT,OUTPUT,TAPE4=INPUT,TAPE6)

Program WALL writes data onto TAPE6 to describe the trough wall to subroutine BCHAP. TAPE6 contains the following data in unformatted form.

Record 1    NAME,NENT  
           NAME    - an alphanumeric name for the wall data set  
           NENT    - the number of entry points ( $\leq 30$ )

Record 2    (DCLAT(i), i = 1,NENT)  
           DCLAT   - array of dipolar geomagnetic colatitudes in radians

Record 3    (DLON(i), i = 1,NENT)  
           DLON    - array of dipolar geomagnetic longitudes in radians

Record 4    (SDA(i), i = 1,NENT)  
           SDA    - length of great circle segments connecting points  
                   in radians

Record 5    (BETA(i), i = 1,NENT)  
           BETA    - the azimuth angle of the line segment in radians

Record 6    (GAM(i), i = 1,NENT)  
           GAM    - one half the angle between consecutive segments  
                   in radians

DECK SETUP

	NO.	NAME
LANWG,T10,CM50000.		
FTN.		
REQUEST(TAPE6,*PF)		
LGO.		
CATALOG(TAPE6,UTO6WALLX3693818, ID=LANGW, RP=999)		
7/8/9		
Deck of program WALL and subroutine DICOORD		
7/8/9		
Data cards		
6/7/8/9		

## TROUGH WALL 060T

GEOG LATITUDE	GEOG LONG.	DIP COLAT.	DIP LONG.	SLIDE A	DELTA	GAMMA
51.500	295.000	27.119	6.500	3.230	72.110	91.000
52.500	300.000	26.292	13.471	3.230	70.977	89.120
53.700	305.000	25.388	20.593	3.380	64.761	87.687
55.400	310.000	24.112	28.027	3.500	64.403	86.643
57.200	315.000	22.667	35.718	3.591	57.735	83.139
59.600	320.000	21.174	44.050	3.350	60.375	81.450
61.900	325.000	19.723	52.722	3.290	60.664	80.987
64.300	330.000	18.326	61.890	2.891	67.781	84.221
66.300	335.000	17.432	70.661	2.700	70.966	87.330
68.200	340.000	16.741	79.787	27.290	91.000	91.000

TAPE6 HAS BEEN WRITTEN.

Attachment 1. Sample Output from Program WALL

NAME: BCHAP, revision 0, subroutine, PML 151  
CATEGORY: Electron density subroutine for Ray-Tracing Program  
TITLE: Double Chapman Layer for Trough Simulation  
LANGUAGE: CDC Extended Fortran - version 4  
PROGRAMMER: B.M. Langworthy, Parke Mathematical Laboratories, Inc.  
DATE: October 21, 1976

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DESCRIPTION

The electron density model, BCHAP, is a replacement for electron density subroutine CHPTRH, PML 128. Whereas in subroutine CHPTRH, the trough wall was located along a line of constant dipolar geomagnetic colatitude, subroutine BCHAP can use an arbitrary trough wall orientation. The electron density model is a double Chapman layer with variations as a function of perpendicular distance from the trough wall. At present the only Chapman layer parameters which vary are the critical frequencies of the two layers. The layer height and thickness of both layers remain fixed. The location of the trough wall and other parameters needed by subroutine BCHAP are supplied on input TAPE6. This file is created by program WALL, PML 152.

INSTRUCTION SET

The use of CHPTRH is similar to that of other electron density subroutines used by the ray-trace program. In the event that the selective load method described in PML 121 and AFCRL-TR-75-0319 is being used, the SLOAD card will be:

SLOAD(BSUBS, BCHAP, and any desired magnetic field, collision  
frequency, and perturbation subroutines)

No auxiliary subroutines are required by subroutine BCHAP.

Input required by the subroutine is in two forms. First, the location of base of the trough wall is defined by coordinates given on TAPE6. This information is explained in detail under FILE DESCRIPTIONS. For further information on it, refer to PML 152 which describes the process of generating TAPE6. The other information required by subroutine BCHAP is input to the

W-array of the ray-trace program. Three locations must be defined as follows:

<u>W-array location</u>	<u>Quantity</u>	
101	F2MX	The maximum plasma frequency of the F2 layer in MHz at the top of the trough wall.
102	F2MN	The maximum plasma frequency of the F2 layer in MHz at the bottom of the trough.
103	PMAX	The width in radians of the trough wall. If PMAX is entered in degrees, a "1" must be placed in column 18 of the input card.

Subroutine BCHAP uses the following information from the ray-trace program:

R(1) - $r$ -	radius in km.
R(2) - $\theta$ -	dipolar geomagnetic colatitude in radians.
R(3) - $\phi$ -	dipolar geomagnetic longitude in radians.
EARTH R -	(also W(2)) the radius of the earth in km.
F -	(also W(3)) the transmission frequency in MHz.
PI - $\pi$ -	} constants.
PIT2 - $2\pi$ -	
PID2 - $\frac{\pi}{2}$ -	

The subroutine then returns the following quantities:

MODX(1)	- an alphanumeric identifier which is supplied by TAPE6, identifying the location of the trough wall.
X	- refractivity due to plasma frequency (dimensionless).
PXPR	- partial derivative of refractivity with respect to the radius (/km).
PXPTH	- partial derivative of refractivity with respect to colatitude (/radian).
PXPPH	- partial derivative of refractivity with respect to longitude (/radian).
HMAX	- the maximum height of the electron density in km. (This is used to determine when penetration has occurred.)

STORAGE REQUIRED

Subroutine BCHAP requires 700 octal words. When used in the ray-tracing program with index of refraction AHWFC and magnetic field DIPOLY, 50000 octal words are required.

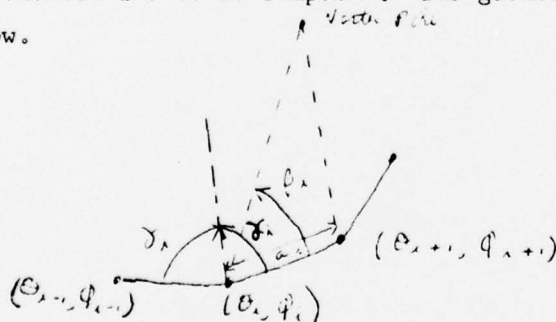
ALGORITHM

The plasma frequency (or electron density) model given by BCHAP can be broken into three separate computational areas:

1) computation of wall location parameters; 2) computation of perpendicular distance from the wall and angular components relative to the wall; and 3) computation of the trough wall and the vertical profile. The first area of computation is performed by program WALL but will be briefly presented here since the definition of terms will be needed subsequently.

1) Computation of wall location parameters

The great circle segments which give the location of the trough wall are specified by their endpoints in the dipolar geomagnetic system. The  $i$ th endpoint will be referred to by  $(\theta_i, \phi_i)$  where  $\theta$  is the dipolar magnetic colatitude and  $\phi$  is the dipolar magnetic longitude. The point  $(\theta, \phi)$  without subscripts is the point for which the plasma frequency and its derivatives are to be computed. The geometry of the situation is given below.



The quantities which will be needed for later computations are:

$a_i$  - the great circle path length in radians between points  $i$  and  $i + 1$ .

$\beta_i$  - the clockwise positive angle in radians with vertex at point  $i$  between due north and the path connecting points  $i$  and  $i+1$ .

$\delta_i$  - one half the angle in radians formed by points  $i-1$ ,  $i$ , and  $i+1$ .

Excluding exceptions for end points,

$$a_i = \cos^{-1}(\cos \theta_i \cos \theta_{i+1} + \sin \theta_i \sin \theta_{i+1} \cos(\psi_{i+1} - \psi_i))$$

$$\beta_i = \cos^{-1} \left( \frac{\cos \theta_{i+1} - \cos \theta_i \cos a_i}{\sin \theta_i \sin a_i} \right)$$

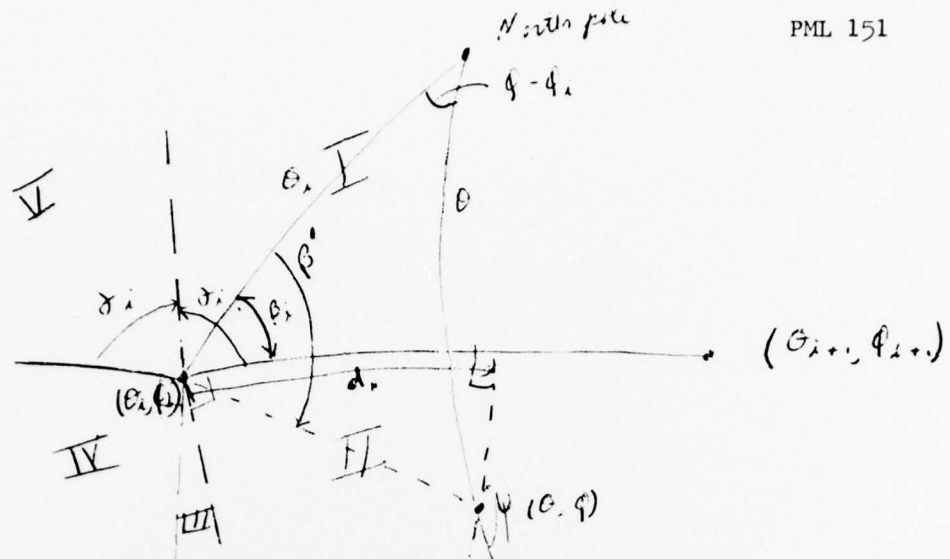
$$\delta_i = 1/2 \cos^{-1} \left( \frac{\cos a' - \cos a_{i-1} \cos a_i}{\sin a_{i-1} \sin a_i} \right)$$

$$\text{where } \cos a' = \cos \theta_{i-1} \cos \theta_{i+1} + \sin \theta_{i-1} \sin \theta_{i+1} \cos(\psi_{i+1} - \psi_{i-1})$$

2) Computation of the perpendicular distance from the wall to point  $(\theta, \psi)$

To eliminate any ambiguity, perpendicular distance from the wall,  $P$ , will be considered positive if the point  $(\theta, \psi)$  is on the north poleward side of the wall and negative if it is on the other side. Since all points except those lying on a given great circle will have at least two perpendiculars to the great circle, it must be stated that the perpendicular must occur with the segment of any great circle which forms a part of the trough wall. There may also be cases where there is no such perpendicular due to "kinks" in the wall where two segments join. The first problem, then, is that of finding the proper great circle segment.

To find  $P$ , the distance from the curve, and  $\psi$ , the angle with respect to positive  $(\theta, \psi)$ , each segment is tried beginning with  $i = 1$  for the first time and beginning with the preceding  $i$  value thereafter.



$C$  = distance from  $(\theta, \phi)$  to  $(\theta_i, \phi_i)$

$$C = \cos^{-1}(\cos \theta_i \cos \theta + \sin \theta_i \sin \theta \cos(\phi - \phi_i))$$

$\beta'$  = angle from due north with vertex at  $(\theta_i, \phi_i)$  to  $(\theta, \phi)$

$$\beta' = \cos^{-1}\left(\frac{\cos \theta - \cos \theta_i \cos C}{\sin \theta_i \sin C}\right)$$

If  $\beta' \leq \beta_i$  and  $\beta_i - \beta' < \delta$ , then  $B = \beta_i - \beta'$  }  
 P is positive } Region I  
 proceed to \* }

If  $\beta' < \beta_i$  and  $\beta_i - \beta' \geq \delta$ , then try next lowest segment. } Region V

If  $\beta' < \beta_i$  and  $\beta' - \beta_i \leq \frac{\pi}{2}$ , then  $B = \beta_i - \beta'$  }  
 P is negative } Region II  
 proceed to \* }

If  $\beta' < \beta_i$  and  $\frac{\pi}{2} \leq \beta' - \beta_i \leq \frac{3\pi}{2} - 2\delta_i$ , then  $B = \beta_i - \beta'$ , }  
 P = C } Region III  
 proceed to \* }

If  $\beta' > \beta_i$  and  $\beta' - \beta_i > \frac{3\pi}{2} - 2\delta_i$ , then next lowest segment } Region IV

\*  $P = \sin^{-1}(\sin B \sin C)$

$d_1$  = distance along the great circle connecting  $(\theta_i, \phi_i)$  and  $(\theta_{i+1}, \phi_{i+1})$  to the perpendicular from  $(\theta, \phi)$

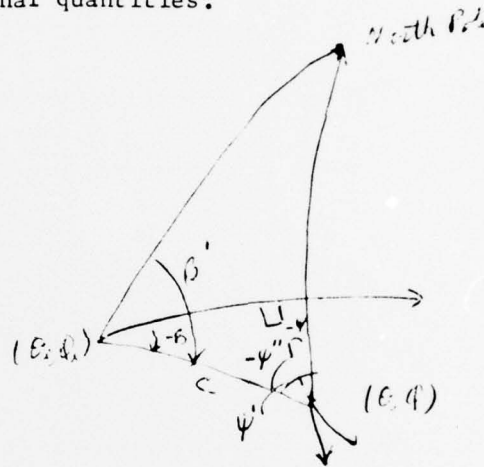
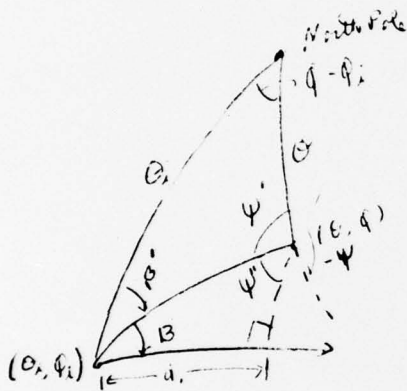
$$d_1 = \cos^{-1} \left( \frac{\cos C}{\cos P} \right)$$

If  $d_1 > a$ , the perpendicular does not fall within the desired segment and the next highest (in  $i$ ) segment is tried.

If the perpendicular occurs below the point  $(\theta_1, \phi_1)$ , the segment is taken along a constant latitude of  $\theta_1$ . Here  $P = \theta_1 - \theta$  and

$$d_i = - \cos^{-1} \left( \frac{\cos C}{\cos P} \right)$$

To find  $\psi$ , the angle between the perpendicular,  $P$ , and the positive  $\theta$  direction, we compute two additional quantities.



$$\psi^I = \cos^{-1} (- \cos(\phi - \phi_i) \cos \beta^I + \sin(\phi - \phi_i) \sin \beta^I \cos \theta_i)$$

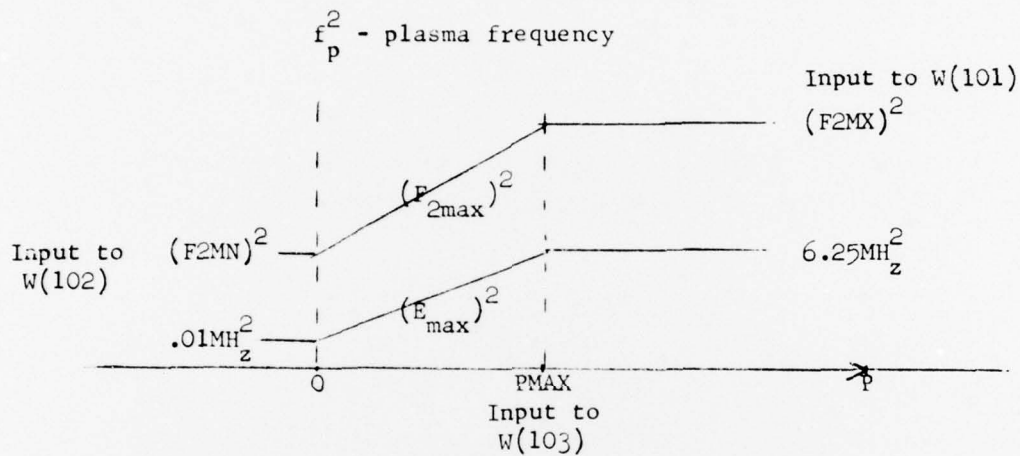
$$\psi^{II} = \cos^{-1} (\sin B \cos d_1)$$

$$\psi = - \psi^{II} - \psi^I$$

For the region below  $(\theta_1, \phi_1)$ ,  $\psi = \pi$

3) Computation of the trough wall and the vertical profile

The trough wall currently in BCHAP varies as a function of P, the perpendicular distance on inputted curve, in the maximum values of the E- and F- layer. At this time the values of height and layer width remain fixed. The height of the E- layer is 120 km and its scale height is 10 km. The height of the F- layer is 350 km. The E- and F- layer variations are given below.



Given P and  $\psi$  from the previous computation we can now compute the plasma frequency and its derivatives at the point  $(r, \theta, \phi)$ .

$$h = r - r_e \quad \text{where } r_e \text{ is the radius of the earth.}$$

$$\text{If } 70 \leq r \leq 120,$$

$$f_p^2 = f_{c_1}^2 \exp(1/2 (1 - z_1 - e^{-z_1})) \quad \text{where } z_1 = \frac{h - 120}{10}.$$

$$\frac{\partial (f_p^2)}{\partial r} = f_p^2 \cdot \frac{1}{20} (e^{-z_1} - 1)$$

$$\frac{\partial (f_p^2)}{\partial P} = \frac{\partial (f_{c_1}^2)}{\partial P} \cdot \exp(1/2(1 - z_1 - e^{-z_1}))$$

$$\frac{\partial (f_p^2)}{\partial \psi} = \frac{\partial (f_p^2)}{\partial P} \cos \psi$$

$$\frac{\partial (f_p^2)}{\partial \phi} = \frac{\partial (f_p^2)}{\partial P} \sin \theta \sin \psi$$

If  $120 < h \leq 350$ ,

$$f_p^2 = f_{c1}^2 + (f_{c2}^2 - f_{c1}^2) \sin^2 H$$

$$\text{where } H = \frac{\pi}{2} \frac{h - 120}{230}$$

$$\frac{\partial (f_p^2)}{\partial r} = \frac{\pi}{230} (f_{c2}^2 - f_{c1}^2) \sin H \cos H$$

$$\frac{\partial (f_p^2)}{\partial P} = \frac{\partial (f_{c1}^2)}{\partial P} + \left( \frac{\partial (f_{c2}^2)}{\partial P} - \frac{\partial (f_{c1}^2)}{\partial P} \right) \sin^2 H$$

$$\frac{\partial (f_p^2)}{\partial \theta} = \frac{\partial (f_p^2)}{\partial P} \cdot \cos \psi$$

$$\frac{\partial (f_p^2)}{\partial \phi} = \frac{\partial (f_p^2)}{\partial P} \sin \theta \sin \psi$$

$$\begin{aligned} f_{c1}^2 &= .01 \quad \text{if } P \leq 0 \\ &= .01 + 6.24 \frac{P}{P_{\max}} \quad \text{if } 0 < P < P_{\max} \\ &= 6.25 \quad \text{if } P \geq P_{\max} \end{aligned}$$

$$\frac{\partial f_{c1}}{\partial P} = \frac{6.24}{P_{\max}}$$

$$\begin{aligned} f_{c2}^2 &= (F2MN)^2 \quad \text{if } P \leq 0 \\ &= (F2MN)^2 + (F2MX^2 - F2MN^2) \frac{P}{P_{\max}} \quad \text{if } 0 < P \leq P_{\max} \\ &= (F2MX)^2 \quad \text{if } P \geq P_{\max} \end{aligned}$$

$$\frac{\partial f_{c2}}{\partial P} = \frac{(F2MX^2 - F2MN^2)}{P_{\max}}$$

SPECIAL CAUTIONS AND FEATURES

This subroutine as all other electron density subroutines for RAYTRACEBL assumes that values for  $\theta$  and  $\phi$  are in dipolar geomagnetic or computational coordinates.

If the number of entries from TAPE6 exceeds 30, a new dimension statement must be inserted.

TIMING

For a large number of rays computed using BCHAP execution time was about 1 second per ray. This will approach 1.7 seconds per ray if only low elevations angles are considered. Timing was done on 1-hop rays at frequencies from 5 to 9 MHz.

ERROR MESSAGES

None.

SUBROUTINES

No nonsystem subroutines are required.

ACCURACY

The accuracy of the trough in BCHAP is dependent on the closeness and accuracy of the points describing the trough wall on input TAPE6.

FILE DESCRIPTIONS

BCHAP reads data off of TAPE6 during program initialization. TAPE6 contains the following data in unformatted form generated by WALL(PML 152). (Symbols in parentheses refer to symbols used in the algorithm on pages 3-4 .

FILE DESCRIPTIONS (continued)

- Record 1    MODX(1), NENT  
             MODX(1) - an alphanumeric name for the wall data set  
             NENT     - the number of point entries ( $\leq 30$ )
- Record 2    (TH(i), i = 1, NENT)  
             TH - array of dipolar geomagnetic colatitudes in radians ( $\theta_i$ )
- Record 3    (PH(i), i = 1, NENT)  
             PH - array of dipolar geomagnetic longitudes in radians ( $\phi_i$ )
- Record 4    (SDA(i), i = 1, NENT)  
             SDA - length of great circle segments connecting points  
                   in radians ( $a_i$ )
- Record 5    (BETA(i), i = 1, NENT)  
             BETA - the azimuth angle of the line segment in radians ( $\beta_i$ )
- Record 6    (GAM(i), i = 1, NENT)  
             GAM - one half the angle between consecutive segments in  
                   radians ( $\gamma_i$ )

DECK SETUP

Runs using BCHA: would be similar to other ray-trace runs except that a TAPE6 must be attached and the SLOAD card given on page 1 should be used