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LOW RATE ERROR CORRECTION CODING FOR CHANNELS WITH PHASE JITTER--ETC(U)
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ABSTRACT

Many digital communication channels operating at low data rates suffer phase jitter and time varying frequency-offset effects which degrade the signal to noise ratio. Low rate error correction coding is shown to obviate this problem. Experimental results are presented for different low rate coding arrangements and the usefulness of the technique is discussed in relation to both coherent and non-coherent modulation methods.

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FIG

1 Measured 'AWGN' performance of some low rate coding schemes

2 Block schematic of low rate coding scheme

3 Block schematic of experimental system

4 Performance of DCPSK with and without coding (hard decisions)

1. INTRODUCTION

Many digital communication channels are characterised by irreducible, random phase jitter and time varying frequency offsets which become important at low data rates, say less than 40 bits/second. For example on a microwave link operating at 10 bits/second, match filter detection can result in 3 or 4 dB degradation in signal-to-noise ratio.

The method of overcoming phase jitter effects examined in this report consists of transmitting over the channel at a rate much higher than the traffic rate, say 5 to 20 times higher, and detecting each bit individually using a conventional match filter detector. Then, since the bits are much shorter, the effects of phase jitter are much less. However, because the bit rate is now increased to a value substantially higher than normal for the channel, the bit error rate can then only be kept within an acceptable limit by use of a low-rate error correcting code, ie a code with a high level of redundancy. As long as the error correcting code does not exhibit a coding loss, the overall result can be a gain of several dB in SNR.

2. 'GOOD LOW RATE CODES

Figure 1 shows the theoretical performance in terms of error rate versus SNR for an ideal coherent PSK modulation system[1]. It will be seen that for an output error probability of 10^{-4} a signal-to-noise rate of 8.4 dB is required. Using a 1/20 rate error-correcting code, that is transmitting at 20 times the input data rate, the E/N_0 per received bit is 13 dB lower at -4.6 dB and the error probability is .26. To have a net coding gain, the 1/20 rate error-correcting code has to be able to accept an input error probability of .26 and yet deliver output errors at a rate lower than 10^{-4}.

A literature search has revealed very few codes that are capable of correcting errors at very high levels of error rate, eg 25%. For example, a (256,8) maximum length sequence code which is a 1/32 rate code will only correct up to 63 errors in 256 bits and obviously fails at error rates higher than 0.25. The BCH family of (n,k) codes[2] has a minimum Hamming distance of 2t+1 and will correct errors approaching $t/_n$ rate; for example one of the best such codes, (45, 5) only corrects 10 errors in 45. In contrast, the relatively crude method of repeating each bit m-1 times is quite good: for example, a 1/20 rate repeat code with majority vote decoding will correct 9 or fewer errors in 20 bits.

The best coding arrangement found during the course of this work is to use repeat coding concatenated with a Viterbi half rate convolutional code. The repeat coding part improves the error rate from $\simeq .3$ to $\simeq 10^{-1}$ and allows the Viterbi decoder to operate with soft decisions. The Viterbi decoder is then able to deliver an output error rate of less than 10^{-3}. The arrangement is shown in Fig 2. Soft or hard decisions may optionally be made on the bits prior to majority vote decoding, but even with hard decisions into the majority vote circuit, soft decisions are obtained for use in the Viterbi decoder as described below.

When τ ones have been received out of n bits, the probability of this pattern, assuming that that data bit is a 1, is

$$\left[P_\tau\right]_1 = \binom{n}{\tau} e^{\,n-\tau} \, (1-e)^{\tau}$$

where e is the probability of error. The probability of the pattern assuming the data bit is a 0, is

$$\left[P_\tau \right]_0 = \binom{n}{n-\tau} e^\tau (1-e)^{n-\tau}$$

Thus the likelihood function is

$$\frac{\left[P_\tau \right]_1}{\left[P_\tau \right]_0} = \frac{\binom{n}{\tau} e^{n-\tau} (1-e)^\tau}{\binom{n}{n-\tau} e^\tau (1-e)^{n-\tau}} = \left(\frac{e}{1-e}\right)^{n-2\tau}$$

The logarithm of the likelihood is $(n-2\tau) \log \frac{e}{1-e}$ and since scaling is not important, the quantity $n-2\tau$ may be used as the log-likelihood. This has the useful feature that it applies for all error rates. The table below illustrates an example where the soft decisions are quantised to 3 bits and each data bit is sent 7 times.

TABLE 1

Number of ones (τ)	$n-2\tau$	Soft Decision Bits
7	- 7	111
6	- 5	110
5	- 3	101
4	- 1	100
3	1	000
2	3	001
1	5	010
0	7	011

On the other hand when soft decisions are made on the bits prior to majority vote decoding, the log-likelihood is simply proportional[1] to the cross correlation value of the received noisy bits correlated with a pattern of n ones. For example with each bit of unity height sent 7 times, the correlation values are allocated soft decision bits as shown below:-

Cross Correlation Value	Soft Decision Bits
7	111
5	110
3	101
1	100
- 1	000
- 3	001
- 5	010
- 7	011

It should be noted that the above soft decision rules are only applicable to baseband systems or systems using coherent modulation methods since the noise has been assumed to be Gaussian distributed.[3]

For non-coherent modulation methods such as Multi-level Frequency Shift Keying (MFSK) and differentially coherent phase shift keying (DCPSK), the log-likelihood functions have to be calculated for specific cases of SNR and modulation method, taking the number of levels into account.

3. EXPERIMENTAL SYSTEM

An experimental system has been built as shown in Fig 3. This models a communication system at baseband and so the results apply only to coherent demodulation methods. The results were obtained using a DEC PDP 11/40 mini-computer interfaced to a commercially available convolutional encoder/Viterbi decoder. The Linkabit LV 7015 employs a half-rate constraint length 7 convolutional encoder and a Viterbi decoder. Software was written to generate a pseudo random pattern, encode the repeats, corrupt the resulting bit stream according to channel simulator information, decode the repetitive sections of the 'received' bit steam to give soft decision information to the Viterbi decoder and finally to calculate the error rate in the output bit stream. Two types of experiment were performed. In Type I experiments it was assumed that the demodulator could only supply hard decisions to the inner (repeats) decoder, while in Type II experiments it was assumed that the demodulator could reliably supply eight levels of soft decision to the inner decoder.

3.1 Type I Experimental Arrangement

In this arrangement a 511 bit pseudo random data pattern was generated and fed out of the computer to be encoded by the convolutional encoder, the resulting bit stream at twice data rate was then fed into the computer where the repeat encoding was performed so that each bit was sent n times where n was varied from 7 to 19 times. The bit stream, now at 2n times the data rate, was corrupted by channel simulator software to yield a stream with a random errors probability between 0.25 and 0.35. This was then presented to the inner (repeats) decoder subroutine which determined whether a 0 or 1 was more likely to have been transmitted and assigned a confidence or likelihood factor to it by use of Table 1. The soft decision information was sent in parallel to the Viterbi decoder and the decoded output stream was fed into the computer for error probability determination.

3.2 Type II Experimental Arrangement

This arrangement differed in two major aspects from the previous one. Firstly, the channel simulator section was required to assign a likelihood to each demodulated bit and secondly the inner (repeats) decoder was modified to process this information to yield a better soft decision on each output bit than in the case of hard decision demodulation. The channel simulator utilized a look-up table similar to table 2. The repetitions decoder summed the log-likelihoods corresponding to each input bit and normalized the result by dividing by the number of repeats.

4. EXPERIMENTAL RESULTS

The results of the Type I experiment correspond to those labelled 'hard' in Fig 1, because they represent those given by a demodulator yielding only hard decisions on the incoming bit stream. One of the most striking aspects of the results is that the net coding gain is substantially independent of the overall code rate for repetitions in the range 7 to 19. The fact that the points for different numbers of repeats lie on the same straight line means that the extra rate-loss incurred in using 19 repetitions instead of 7, is made up almost exactly by the coding gain of the extra repetitions. It should be pointed out that this is not a fundamental coding theory result but just reflects the shape of the E_b/N_o curve for PSK modulation at the error rates under consideration. In all cases the simulation proceeded until at least 100 errors were observed in the final output stream.

An interesting aspect of these experiments concerned the question of optimising the quantization steps in the output of the inner (repeats) decoder. Heller and Jacobs have indicated that for eight level quantization, evenly spaced threshold steps with spacing = $.5\sigma$ are optimum, where σ = signal noise ratio. During the experiments the quantization thresholds were optimized for a final output error rate = 10^{-2}, the worst at which the system was designed to operate. These trial and error optimizations agreed exactly with the Heller and Jacobs recommendations.

The results of the Type II experiments, labelled 'soft' in Fig 1, indicate that there is approximately 1 dB to be obtained by taking soft decisions (8 level) from the output of a coherent demodulator. This compares with about 2 dB usually gained when using soft decisions with a maximal likelihood decoding scheme. The short fall is assumed to be due to the non-optimality of repeats as a coding technique.

5. APPLICATION OF RESULTS TO NON-COHERENT MODULATION METHODS

Since the experimental results were obtained on a baseband system, they need not always be valid for non-coherent modulation methods. For the situation where hard decisions are made prior to the majority vote circuit, the results are only dependent on the assumption of a specific input error rate and independence between bit errors. This assumption is valid for non-coherent modulation methods making hard decisions and thus the results may be applied to modulation methods such as FSK.

Fig 4 shows the overall signal/noise ratio - error rate curve obtained for a particular non-coherent modulation method, (differentially coherent phase shift keying, DCPSK) by converting the input error rate into a signal/noise ratio per received bit using the relationship $P(e) = \frac{1}{2} e^{-E_b^1/N_o}$, where E_b^1 is the energy per received repeated bit. The energy per data bit, Eb is given by $\frac{E_b^1}{R}$ where R is the rate of the code, and the output error rate, after error correction, has been plotted against E_b/N_o in Fig 4.

Unlike the curve for coherent modulation, the results show that lowering the code rate introduces a greater and greater coding loss and at 10^{-4} error rate, code rates below the 1/8 result in an overall coding loss. The reason for the different results for non-coherent modulation, as opposed to coherent, is due to the much larger E_b/N_o values required for non-coherent modulation when operating at high error rates. For example, at 0.3 error rate, PSK requires only -8.64 dB SNR while DCPSK requires -2.9 dB, a difference of 5.7 dB. At 10^{-4} error rate the difference in SNR between the modulation methods is only 0.9 dB. It follows that an error correcting code that produces a 10^{-4} output error rate at .3 input error rate has, before subtracting the loss in SNR due to the code rate, a coding gain of 17.04 dB for PSK but only 12.2 for DCPSK.

6. EXAMPLE OF LOW RATE CODING

As an example consider a link which has the properties set out in the following table.

Transmitted Speed (bits/sec)	SNR Loss due to Phase Jitter
10	5 dB
20	4 dB
40	2 dB
80	0 dB

The modulation method is assumed to be DCPSK and consequently a 10^{-4} output error rate requires an E_b/N_o of 9.3 dB. For a data rate of 10 bits/sec, the signal power to noise spectral density ratio $\frac{C}{N_o}$ must be 19.3 dB Hz (10 + 9.3).

Taking phase jitter into account as above and using low rate coding with results taken from Fig 4, the overall C/N_o required for different coding rates, is as shown in the table below:-

C/N_o (dB Hz)	Code Rate (R)
25.3	1
21.8	½
20	¼
19	1/8
20.6	1/18

It will be seen that the optimum code rate is 1/8 and that this produces an overall practical gain in SNR of 6.3 dB, and completely overcomes the phase jitter loss.

7. PRACTICAL CONSTRAINTS

From the foregoing it can be seen that for coherent modulation, coding rates of 1/10 to 1/38 may be successfully used to reduce the symbol length by these factors and at the same time achieve coding gains of 3 dB at 10^{-3} error rate and 4.2 dB at 10^{-5} error rate. Although these results look very promising it must be noted that such modulation methods require the receiver to generate a local carrier which is in phase lock with the received signal. As the code rate is reduced the SNR per received bit is correspondingly reduced and extracting a phase reference becomes progressively more difficult. For similar reasons correct bit timing and frequency tuning may also be very difficult to achieve using low rate codes.

For these reasons it is likely that only non-coherent modulation methods would be used with low rate coding as there is then no need to obtain a carrier phase reference. In addition synchronisation would probably be done using decision feedback on the basis of testing possible synchronisation positions and selecting the one that gives the lowest output error rate. For any specific application there should be a best compromise between code rate and SNR loss due to phase jitter, though in practice code rates lower than 1/8 would probably not be useful. It is worth noting that using a low rate code eases the problems associated with frequency variations and frequency tuning, this being because the match filters of the received bits are R times wider and thus that much less sensitive to frequency offset.

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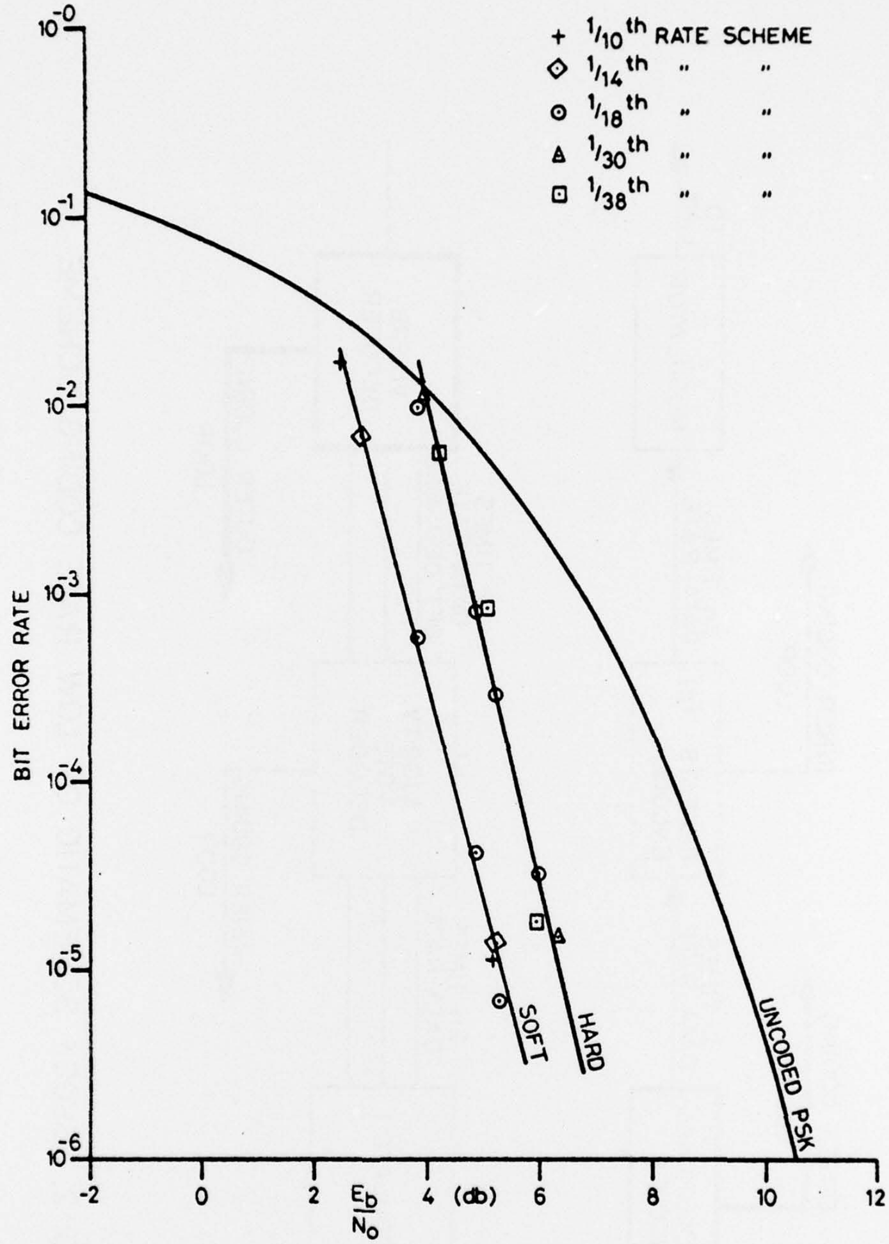

FIG. 1. MEASURED 'AWGN' PERFORMANCE OF SOME LOW RATE CODING SCHEMES.

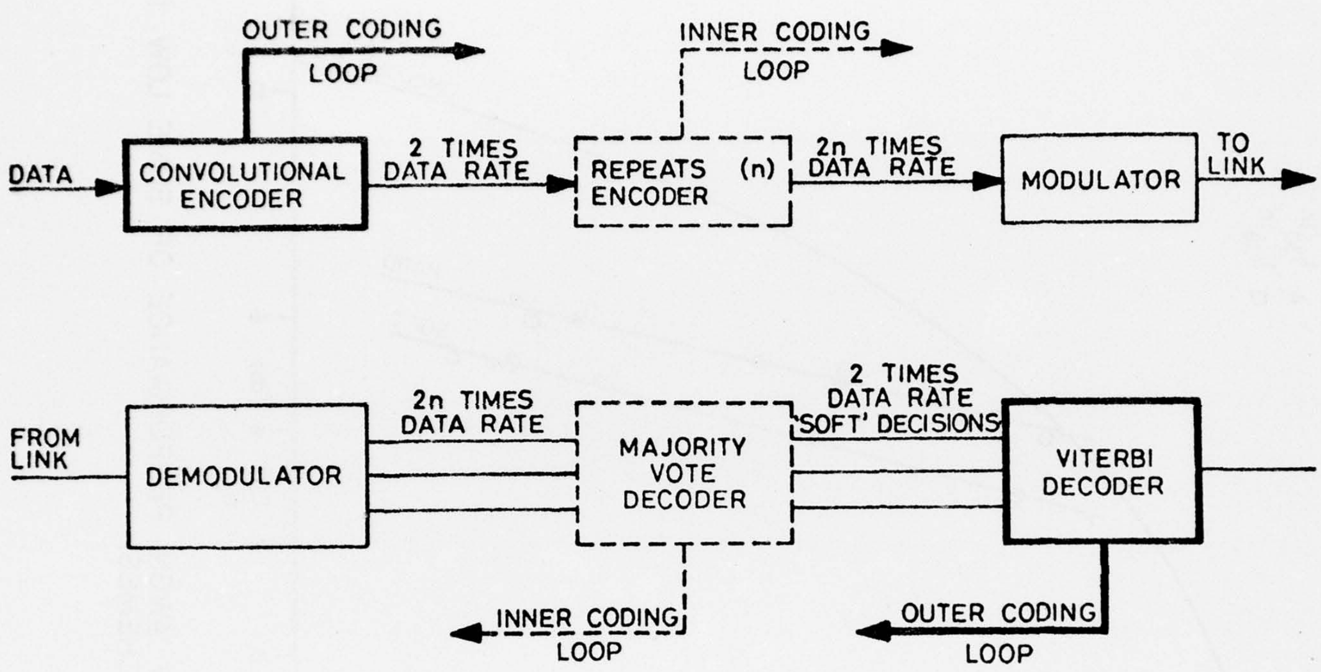

FIG. 2. BLOCK SCHEMATIC OF LOW RATE CODING SCHEME

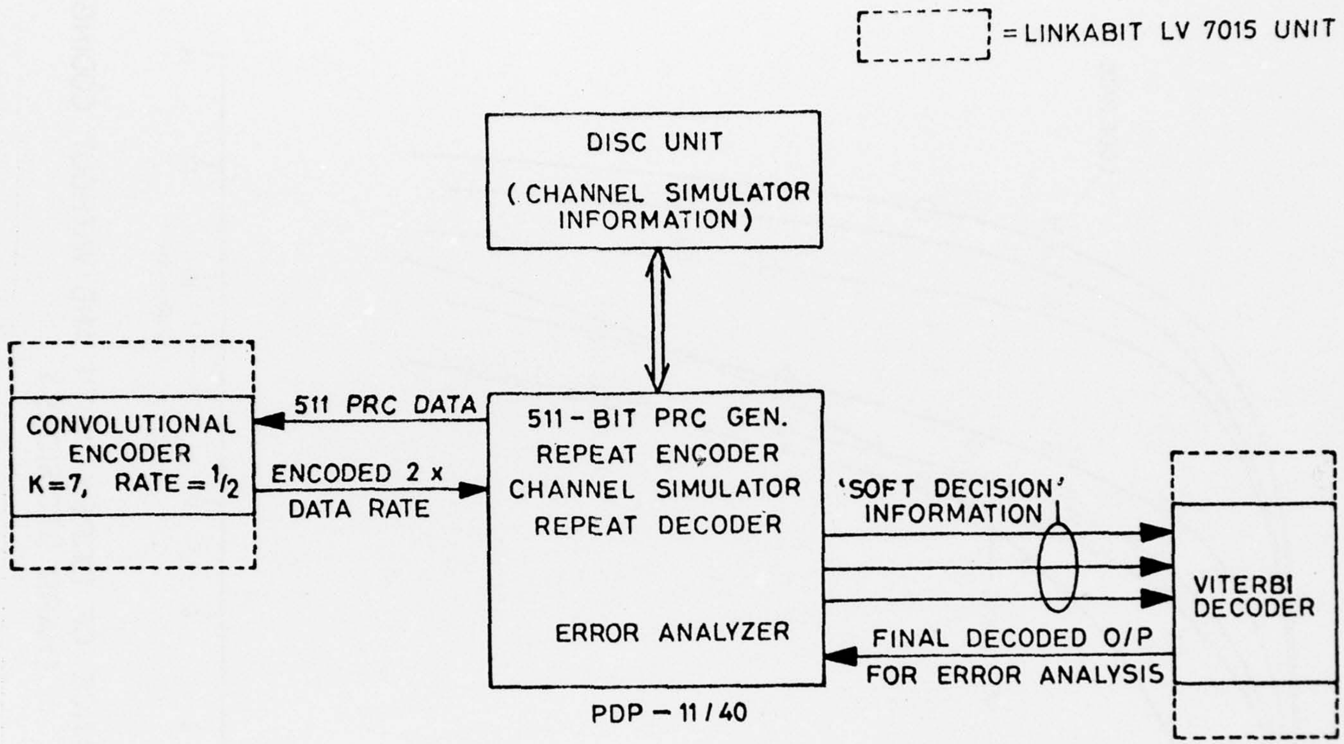

FIG. 3. BLOCK DIAGRAM OF EXPERIMENTAL SYSTEM

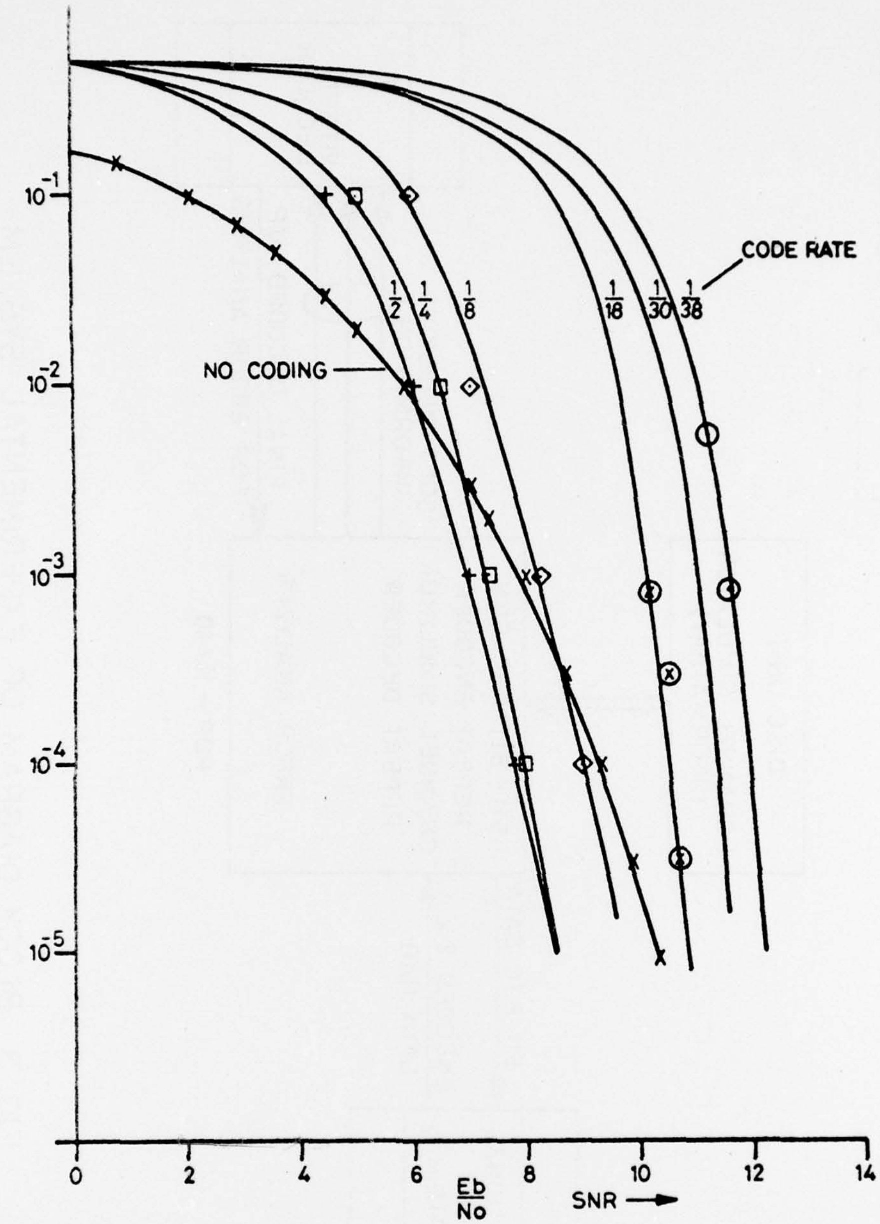

FIG. 4. PERFORMANCE OF D.C.P.S.K. WITH AND WITHOUT CODING (HARD DECISIONS)

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