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SPECTRUM OF A SIGNAL REFLECTED FROM A TIME-VARYING STOCHASTIC SURFACE[1]

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by

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INTRODUCTION

The purpose of this study is to determine the frequency spread caused by reflection of an acoustic signal off a time-varying stochastic surface. Scattering from a rough surface has been investigated by many scientists, of which we will mention only a few. Eckart, using the method of Physical Optics and applying the Helmholtz integral, obtained the intensity of the scattered sound; from this quantity, the scattering coefficient of the surface was obtained. Although Eckart had the time variation of the surface in his preliminary integral, he did not compute any quantities involving the time variation. Beckmann, in his book, also applied the Helmholtz integral to a rough stochastic surface, but did not consider a time-varying surface. Marsh, on the other hand, utilized the Rayleigh method and considered the scattered wave to consist of an infinite set of outgoing plane waves. By applying boundary conditions at the surface, the scattering coefficient of the surface was obtained.

The number of investigations of a time-varying surface is considerably less than that of a fixed surface. Of these, we will mention only the works of Roderick and Parkins. Roderick considered a sinusoidal traveling surface wave. For a single-frequency incident acoustic wave, the frequency of the reflection in the specular direction is the same

[1]Paper presented at the 78th Meeting of the Acoustical Society of America, San Diego, Calif, November 3-7, 1969.

as the incident frequency, and the frequencies in the non-specular directions are equal to the incident frequency plus a multiple of the surface frequency. Parkins considered a random time-varying surface, a joint Gaussian distribution of heights, and a Neumann-Pierson surface height spectrum. For both small and large surface roughness, he obtained the scattered acoustic spectrum.

In our work, we start with the assumptions used by others: the method of Physical Optics is used; the insonified area is in the far field of the source; the observation point is in the far field of the insonified area; the surface is pressure release; no shadowing or multiple reflections occur; a directional source is used; and variation of the surface is slow compared to the speed of sound.

(Figure 1) Starting with the Helmholtz integral, and a single-frequency acoustic source, the complex pressure at the observation point is obtained. It is assumed that the surface is random, homogeneous, and stationary. That is, for a given instant of time, first- and second-order properties of the surface are independent of absolute position, and depend only on the differences of the x and y coordinates, these differences being specified by u and v. In time, the properties depend only on the difference in time, specified by τ. The autocorrelation of the received complex pressure is obtained and is denoted by $R_p(\tau)$. C depends on the geometry of the situation. a, b and c are the sums of the direction cosines in the x, y and z directions, respectively, of the incident and scattered waves.

k_a is the acoustic wave number.

σ is the rms wave height.

$\rho(u, v, \tau)$ is the normalized surface correlation function, which will be discussed on the next slide.

$\overline{p(t)}$ is the expected value of the received complex pressure, and is referred to as the coherent component of the received acoustic pressure waveform. We have assumed a joint Gaussian distribution of heights in this equation.

The Fourier transform of $R_p(\tau)$ gives the received acoustic power spectrum. The Fourier transform of the first term of $R_p(\tau)$ results in a line component located at the transmitted frequency. This line component has appreciable value mainly in the specular direction and only for a slightly rough surface. For a given geometry and given surface correlation function, the double integral in the slide could be evaluated by numerical methods.

(Figure 2) $\zeta(x,y,t)$ represents the surface wave height at the point x,y at time t, and $\rho(u,v,\tau)$ is the normalized spatial-temporal correlation of the surface wave heights. As stated earlier, Parkins has obtained the received acoustic power spectrum for the case of the Neumann-Pierson surface height spectrum and a fully-aroused sea. This corresponds to a wide-band spectrum, with a Q approximately equal to 1.4. We, instead, consider a narrow-band surface spectrum, which corresponds to the case of swell waves. For the narrow-band assumption, the correlation of surface heights is in the form given by the second equation. The magnitude and phase of the complex envelope $\underline{\rho}(u,v,\tau)$ vary slowly in τ with respect to cos ($2\pi f_s \tau$). If, in addition, the iso-values of the correlation function is an ellipse, the phase of $\underline{\rho}$ (u,v,0) is identically zero. In the computations to be performed, an elliptical iso-correlation surface is assumed.

(Figure 3) In the expression for the received power spectrum, there is a term that contains an exponential of the correlation function shown in the previous slide. For the narrow-band case, this correlation is approximately sinusoidal, and the expression can be expanded in a series of Bessel functions of imaginary argument. An examination of the resulting expression shows that the received acoustic spectrum consists of a series of spectral lobes or sidebands each separated from the incident acoustic frequency by multiples of the center frequency f_s of the surface variation plus a spike at the acoustic frequency f_a. The coherent component occurs mainly in the specular direction and only has an appreciable value for small surface roughness. We will now find the power in each sideband component.

(Figure 4) The relative scattered power in each sideband can be obtained in terms of only two parameters of the surface, namely, α and β . β is the usual measure of the roughness of the surface.

L_x and L_y are the (correlation) distances along the x and y axes, at which the correlation of surface heights is $1/e$ of its peak value.

$\underline{\rho}$ in this equation represents the spatial dependence of the complex envelope of the surface correlation function.

δ is the Kronecker delta.

$t_m(0)$ gives the relative scattered power in the m^{th} sideband. $m = 0$ corresponds to the lobe centered at acoustic frequency f_a. The scattering coefficient σ_m is defined by Eckart, and the correction factor c_0, as given by Horton and Muir, are shown in the second equation. This equation contains the factors from which the scattering strength could be evaluated for a specific geometry. B is a factor which depends only on geometry.

$t_m(0)$ is not integrable in closed form. A computer program has been written to numerically evaluate $t_m(0)$ for given values of α and β. An investigation into the accuracy of the numerical integration by Simpson's rule shows that from 6 to 8 places of accuracy are obtained. The number of summations for each value of α and β ranged from 400 to 4,000. With the advent of modern computers, this numerical integration is feasible and economical.

(Figure 5) In our computations, we consider two types of auto-correlations of surface height, namely, exponential- and Gaussian-modulation of an oscillatory function. In this figure, we have examined Gaussian-modulation of a sinusoid for α equal to zero; this corresponds to a and b, the direction cosines with respect to the x and y axes, equal to zero, and thus to the specular direction. The power in the first sideband is plotted as a function of β. $\beta = 0$ corresponds to a flat surface; for this value of β, there is no scattered power in the sidebands, as all the power appears in the coherent component. As β increases, there is less power in the coherent component and more scattered power in the first sideband. The power in the first sideband reaches a maximum value when the surface roughness parameter β is approximately unity, and then decreases. For these larger values of β, the surface is rougher, and more of the scattered power will appear in higher order sidebands.

(Figure 6) In this final figure, a three-dimensional plot of the scattered power in the first sideband is drawn vs α and β^2. For any value of α, as β^2 increases, the power in a particular sideband will first increase and then decrease. For a given value of β^2, i.e. roughness, as α increases, again the power first increases and then decreases. Three-dimensional plots of this type were obtained for several sidebands and for both Gaussian- and exponentially-modulated oscillatory functions.

Before the final conclusion and summary, we would like to discuss the difference between the narrow-band Gaussian surface and the fixed-amplitude sinusoidal surface considered by Roderick. We have considered a narrow-band spectrum for the height variation at a point of the surface, and assumed the joint probability density of the surface heights to be Gaussian. As the bandwidth of the surface variation decreases and approaches zero, the properties of this surface process do <u>not</u> approach the fixed-amplitude sinusoidal surface case. For example, the distribution of heights remains Gaussian and is, therefore, different from the probability density associated with a sinusoid. The scattered powers in the sidebands as given by the narrow-band Gaussian theory are different from those of the single-frequency sinusoidal theory.

In mathematical terms, the first-order characteristic function of a fixed-amplitude sinusoid is a Bessel function. If we average this Bessel function with the Rayleigh distribution of sine wave amplitudes, we obtain the Gaussian first-order characteristic function. Thus, our results can be viewed as an average over the sinusoidal surface case.

On the other hand, a narrow-band time function with center frequency f_s and bandwidth W resembles a sine wave of frequency f_s over a short period of time, but will differ from a sine wave over a longer period of time, the difference showing up on the order of W^{-1} sec. Thus, if we obtain a <u>short</u> sample of the reflected signal from a narrow-band surface, and determine the received acoustic spectrum from it, the single-frequency theory (with the <u>current</u> surface height amplitude) should accurately predict that spectrum. However, for a long time average over the statistical properties of the surface, the narrow-band Gaussian theory, not the fixed-amplitude single-frequency theory, must be used to predict the expected behavior of the spectrum.

In summary, we have considered a time-varying stochastic surface, stationary in both space and time. The reflected acoustic spectrum was obtained in terms of the first- and second-order characteristic functions of the surface height variations. For the special case of surface wave height variation possessing a narrow-band spectrum at a point, joint Gaussian distributions of heights, an elliptical spatial correlation, the integral expression obtained for the reflected acoustic sidelobe powers at the observation point was derived and evaluated. The integral was computed in terms of two fundamental parameters, and curves of the results were shown.

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ACOUSTIC AUTOCORRELATION

$$R_p(\tau) = \overline{p(t)}\ \overline{p^*(t-\tau)} + C\iint du\, dv\, \exp\left[ik_a(au+bv)\right]$$

$$\left\{\exp\left[-\beta^2\left(1-\rho(u,v,\tau)\right)\right] - \exp(-\beta^2)\right\}$$

$$\beta = k_a c\sigma$$

Figure 1

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SURFACE CORRELATION

$$\rho(u,v,\tau) = \frac{\overline{\zeta(x,y,t)\,\zeta(x-u,\,y-v,\,t-\tau)}}{\sigma^2}$$

NARROW-BAND ASSUMPTION

$$\rho(u,v,\tau) = |\underline{\rho}(u,v,\tau)|\cos\left(2\pi f_s \tau + \arg\left[\underline{\rho}(u,v,\tau)\right]\right)$$

FOR ELLIPTICAL CASE, $\arg\left[\underline{\rho}(u,v,0)\right] \equiv 0$

Figure 2

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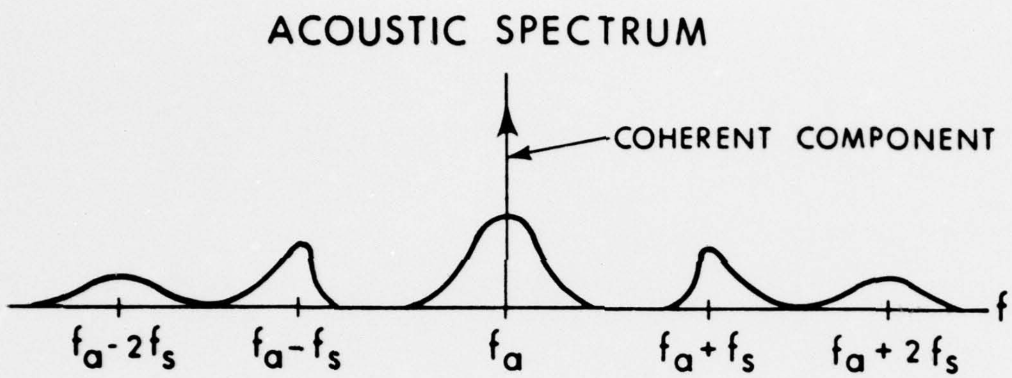

Figure 3

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SCATTERED POWER IN THE m^{th} SIDEBAND

$$t_m(o) = 2\pi \exp(-\beta^2) \int_0^\infty du\, u J_o(u) \left\{ I_m \left[\beta^2 \rho(u/\alpha) \right] - \delta_{om} \right\}$$

SCATTERING STRENGTH OF m^{th} SIDEBAND

$$\frac{\sigma_m}{c_Q} = \frac{|B|^2}{c_Q} \frac{t_m(o)}{\lambda_a^2}$$

$$\alpha = k_a \sqrt{a^2 L_x^2 + b^2 L_y^2}$$

$$\beta = k_a c \sigma$$

Figure 4

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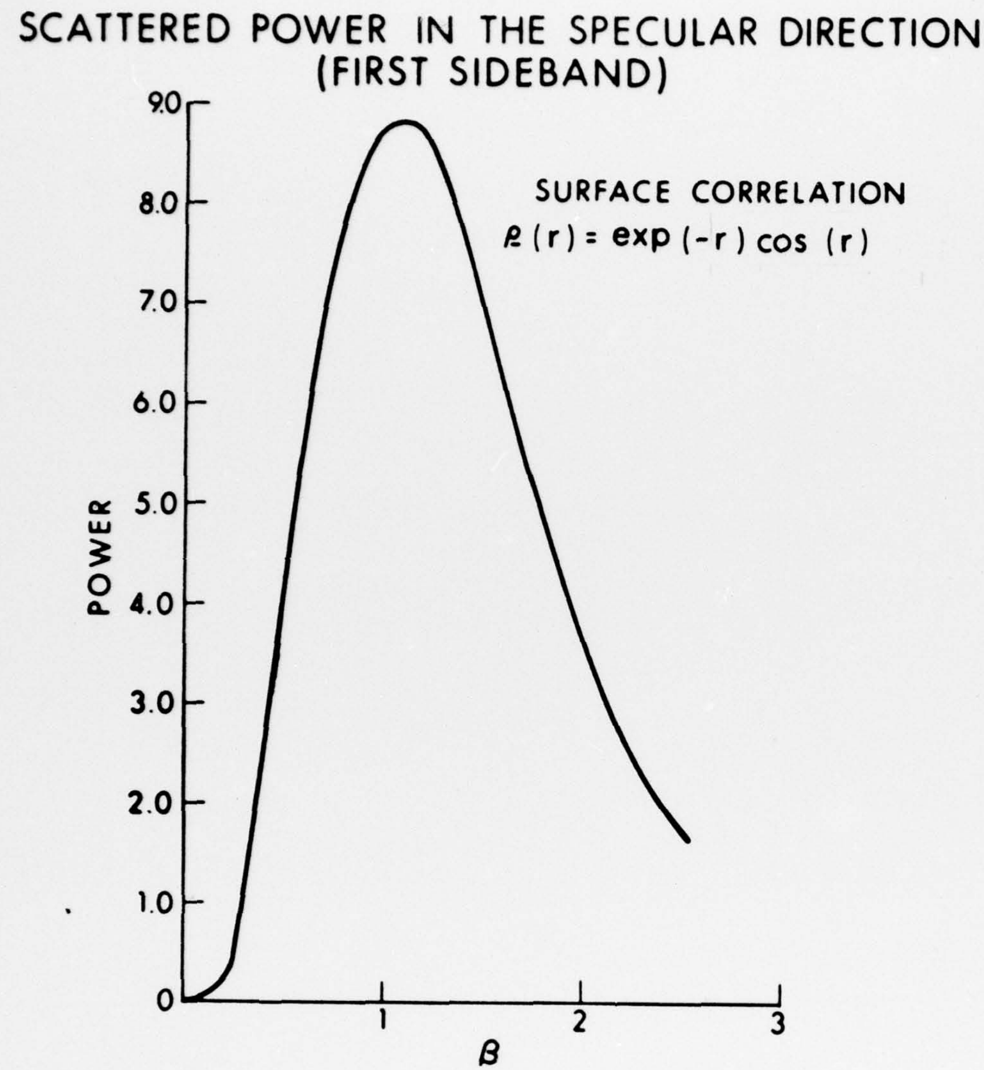

Figure 5

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SCATTERED POWER IN THE FIRST SIDEBAND

GAUSSIAN
MODULATED
SINUSOID

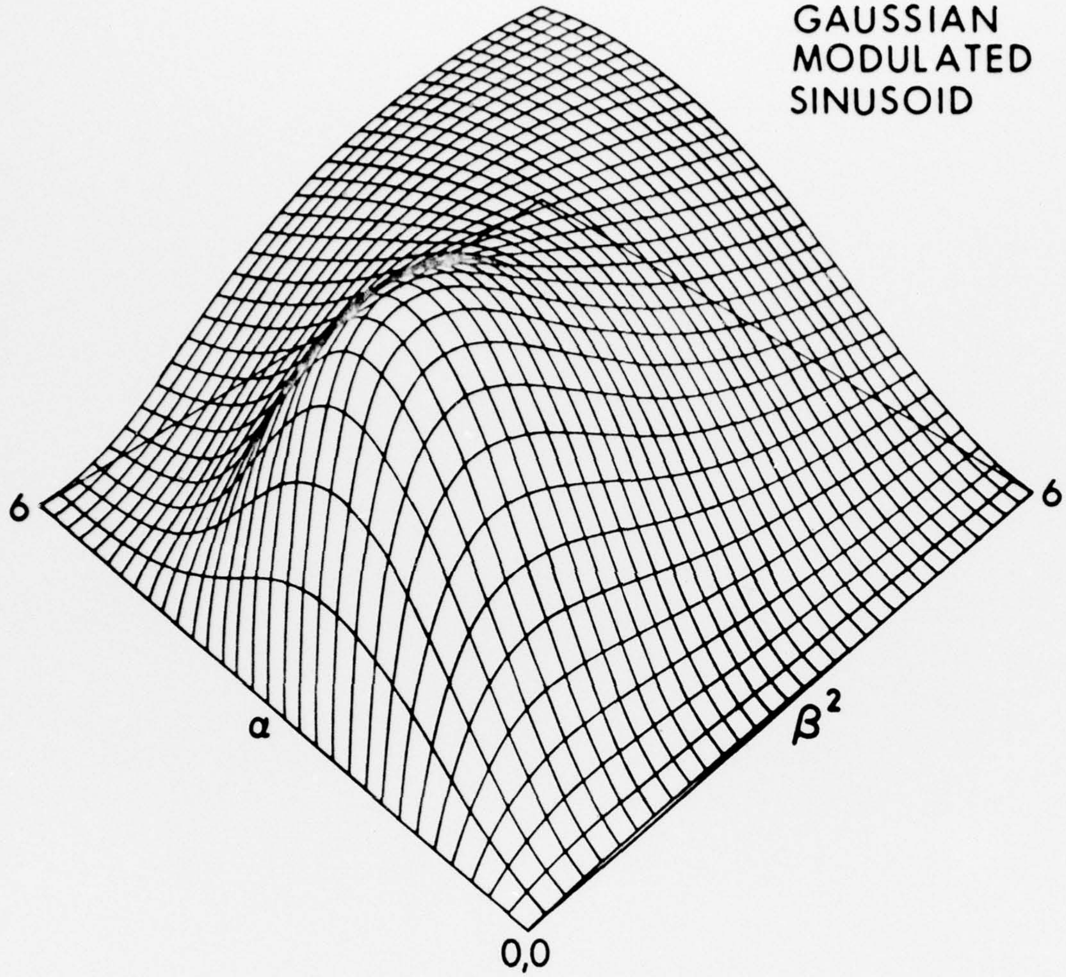

Figure 6

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